

Aeroelastic Analysis of Highly Flexible Wings with Linearized Frequency-Domain Aerodynamics

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Flutter analysis of configurations with geometric structural nonlinearities typically is done with time-domain analysis. The results from this process are computationally expensive and can yield cumbersome results that may be difficult to manage/interpret. Compared to time-domain methods, frequency-domain flutter analysis can provide additional insight into the characteristics of a flutter stability problem. By linearizing the aeroelastic problem about the nonlinear equilibrium state, this work applies frequency-domain aeroelastic analysis to the Pazy wing, the subject of the Large Deformation Working Group in the Aeroelastic Prediction Workshop. Generalized aerodynamic forces (GAFs) are computed with both a doublet-lattice method and a computational fluid dynamics solver at a range of reduced frequencies as well as a range of dynamic pressures to account for the dependence of the mode shapes on the nonlinear equilibrium state. These GAFs are used in a $p - k$ flutter solver, which is modified to handle the dependence of the stiffness matrix and GAFs on the dynamic pressure.

I. Introduction

Subsonic aeroelastic analysis of long slender wings undergoing large deflections has a rich history in the recent literature (see for example Refs. 1-4, among many others). The computational emphasis in these papers is typically on the structural analysis, which requires geometrical nonlinearities to properly account for the large deformations and rotations. Aeroelastic results are typically obtained in the time-domain, with low-to-medium fidelity aerodynamic tools such as strip theory,⁵ or the unsteady vortex lattice method (UVLM).⁶ Traditional linear aeroelastic analyses however (using software such as NASTRAN,⁷ ASTROS,⁸ ZAERO,⁹ etc.), utilize frequency-domain aeroelasticity, namely the doublet-lattice method (DLM). The generalized aerodynamic forces (GAFs) computed from these tools, and their application in the $p - k$ eigen-equations, can provide useful insight into the aeroelastic flutter mechanism.

This work seeks to apply frequency-domain aerodynamics to the large-deformation aeroelastic response of a cantilevered wing with large deformations. This is done by implementing a static aeroelastic coupling loop between a nonlinear structural representation and a steady airloads model, a process that is iterated to convergence via nonlinear block Gauss-Seidel (NLBGS). The vibration eigen-data are then computed for the deformed structure, the mode shapes are mapped onto the aerodynamic mesh, and finally the GAFs can be computed. These GAFs will be a strong function of dynamic pressure q_{dyn} (i.e., of the steady loading), a feature that is not seen in linear aeroelastic modeling. Finally, a modified version of the $p - k$ flutter equations, which account for the GAF- q_{dyn} dependence, can be implemented to compute the flutter point.

Aspects of the process outlined above have been considered in the literature,^{10,11} using DLM aerodynamics. In addition to DLM, the current work also considers a Reynolds-averaged Navier-Stokes (RANS) solver, FUN3D/SFE^{12,13}, where GAFs can be computed with a recently-developed linearized frequency-domain (LFD) method.¹⁴

As an application case, this work considers the nonlinear aeroelastic analysis of a subsonic cantilevered wing, named the Pazy wing. This wing experiences relatively large deflections, and preliminary wind tunnel testing efforts are presented in Ref. 15, as well as a description of the wing geometrical, inertial, and stiffness

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properties. This test case is under study as part of the third Aeroelastic Prediction Workshop (AePW)^a, within the Large Deformation Working Group (LDWG). Existing computational predictions as part of the LDWG can be found in Refs. 16-18.

II. Aeroelastic Modeling

The extended design structure matrix (XDSM) diagram¹⁹ for the aeroelastic analysis procedure used here, is shown in Fig. 1. For a given dynamic pressure q_{dyn} and angle of attack AoA, an iterative loop (nonlinear block Gauss-Seidel) is implemented to compute the static aeroelastic convergence. Aerodynamic pressures (C_p) are converted into nodal beam loads and moments (f_{struct}), which are then applied to a geometrically-nonlinear beam model in NASTRAN (SOL106). The beam deformations (u_{struct}) are mapped onto the aerodynamic mesh (u_{aero}), which is then fed to the steady aerodynamic solver to compute C_p . This process continues until the difference in u_{struct} between consecutive iterations is below some threshold.

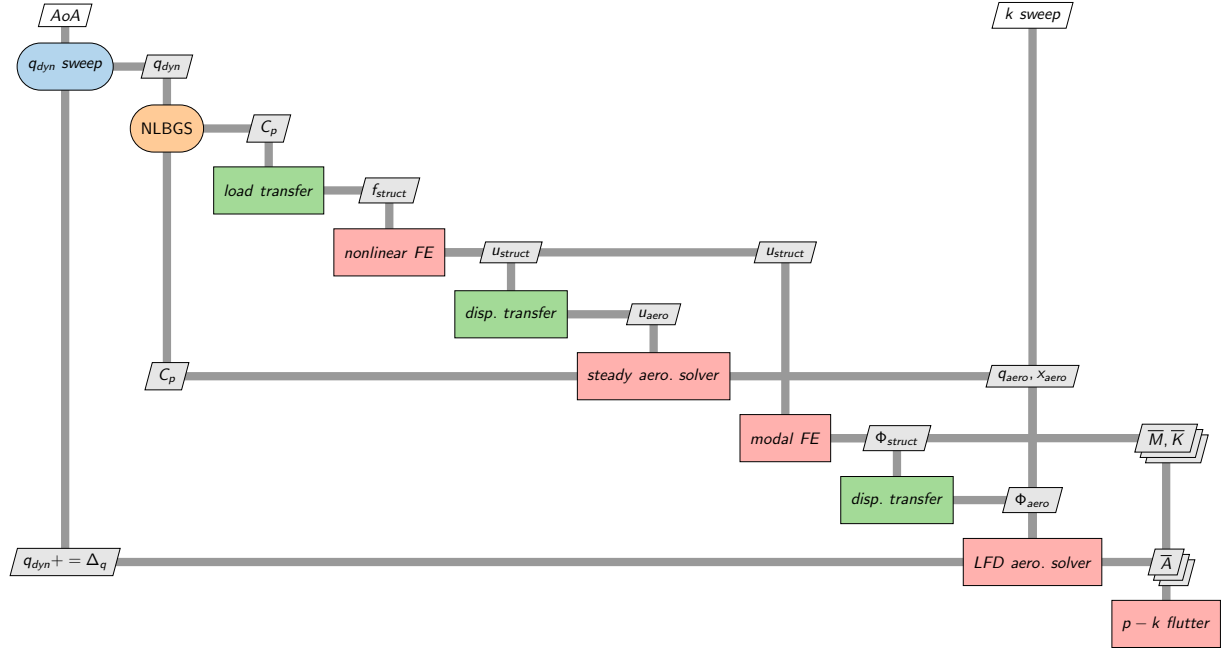


Figure 1: XDSM diagram of the aeroelastic flutter process.

Once converged, a modal vibration analysis is conducted by linearizing about the static deformed shape, again using NASTRAN (SOL106). The structural eigenmodes, Φ_{struct} , are each mapped onto the aerodynamic surface to produce Φ_{aero} , and the linearized frequency-domain aerodynamic solver computes the GAFs, $\bar{A}$, at a prescribed list of reduced frequencies, k . Then, the dynamic pressure is augmented by some amount, and the entire process repeats again. Once the entire array of dynamic pressures is completed, the final GAF matrix is of size $N_{modes} \times N_{modes} \times N_k \times N_{q_{dyn}}$. Similarly, the reduced mass ($\bar{M}$) and stiffness ($\bar{K}$) matrices are matrices of size $N_{modes} \times N_{modes} \times N_{q_{dyn}}$. The reduced stiffness matrix contains the square of the vibration frequencies along the diagonal, and these frequencies are dependent upon u_{struct} , for the geometrically nonlinear structure. The reduced mass matrix is assumed independent of u_{struct} , simply set as the identity matrix.

The $p - k$ flutter equations are then given as:

$$\left(\frac{U^2}{b^2} \cdot p^2 \cdot \bar{M} + \bar{K}(q_{dyn}) - q_{dyn} \cdot \bar{A}(i \cdot k, q_{dyn}) \right) \cdot v = 0 \quad (1)$$

where U is the flow speed, b is a reference length, v is the aeroelastic eigenvector, and p is the corresponding eigenvalue, with $imag(p) = k$. These equations are solved across a range of dynamic pressures with the

^a<https://nescacademy.nasa.gov/workshops/AePW3/public/>

noniterative scheme from Ref. 20, adapted to account for the fact that both stiffness and the GAFs are functions of dynamic pressure. In the traditional $p - k$ equations, the GAFs only exist at the sampled k -sweep values, and so linear interpolation is used to compute matrices at other k values. The same idea is used here to interpolate GAFs (and the stiffness matrices) across dynamic pressures.

As noted above, the aerodynamic solvers in Fig. 1 may utilize linear DLM, or nonlinear FUN3D/LFD. For DLM, NASTRAN’s SOL144 is used to compute steady pressures, and SOL145 is used to compute unsteady oscillatory GAFs. NASTRAN is also used as a structural solver, but the static aeroelastic coupling in Fig. 1 is conducted outside of NASTRAN via file I/O, in order to properly account for the nonlinear physics. NASTRAN’s own aeroelastic solvers⁷ can conduct static aeroelastic coupling internal to the software, but only linear mechanisms (namely, linear structures) can be captured with this option. It is also important to note that Fig. 1 shows the flow solution vector $\mathbf{q}_{aero}$, and the deformed volume mesh $\mathbf{x}_{aero}$, being fed from the static solver to the LFD solver. This is only needed for FUN3D/LFD, as this process represents an oscillatory linearization about a steady flow. For DLM, this link can be removed, as the DLM solver is a purely linear computation, and the steady flow state has no impact on the unsteady flow state.

III. Results

A picture of the Pazy wing surface is shown on the left of Fig. 2. This is an unswept untapered wing with a length of 0.55 m, chord of 0.1 m, a constant NACA0018 airfoil, and a slender cylindrical body attached to the tip. Additional geometrical details can be found in Ref. 15. The detailed built-up wing structure has been reduced to a beam model, shown on the right of Fig. 2: additional details regarding this equivalent beam process can be seen in Ref. 16. The first natural frequencies of this beam model are 4.19 Hz (first bending, 1B), 28.33 Hz (second bending, 2B), 41.88 Hz (first torsion, 1T), 81.92 Hz (third bending, 3B), 110.69 (first edgewise / inplane bending, 1E). The DLM mesh is also shown with 20 chordwise panels and 40 spanwise panels. Aerodynamic modeling of the tip body is done with slender body theory. The FUN3D mesh used here has 2.5 million nodes, with the surface mesh detail shown in Fig. 3. FUN3D results are performed with the negative version of the Spalart-Allmaras turbulence model.²¹

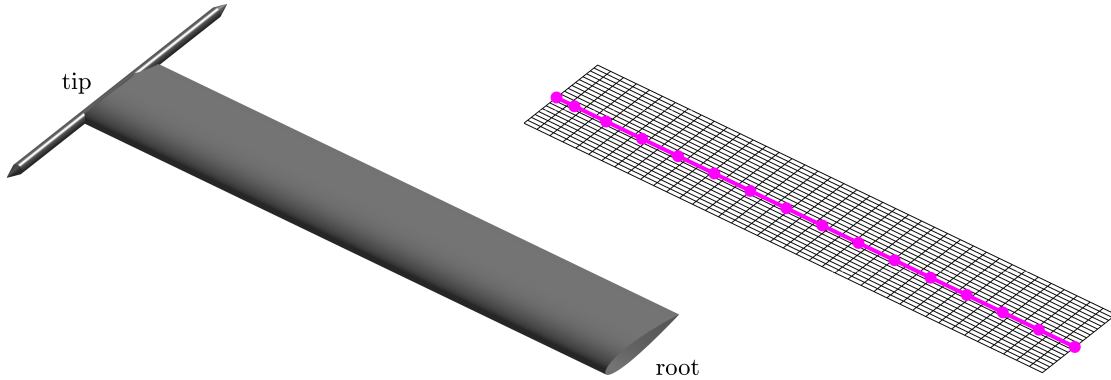


Figure 2: Pazy wing surface (left); DLM mesh and beam mesh (right).

A. Static Aeroelasticity

Using the process of Fig. 1, tip deflection and tip twist results are shown in Fig. 4 for the DLM-based solver and the FUN3D-based solver at 0° , 1° , and 5° AoA. At low flow speeds, the wing sags under its own weight, with tip displacements equal to 3.4% of the wing length. For non-zero angles of attack, tip deformations return to 0 near 38 m/s (1° AoA) and 18 m/s (5° AoA). At the highest AoA, the DLM results are taken through a flow speed of 75 m/s, at which point the vertical tip deformations are over 60% of the wing length, and the tip twist is over 3° . The FUN3D results are only taken through 60 m/s (after which negative volumes appear in the mesh due to the limitations of the linear elasticity model used for mesh deformation), and display moderately larger deformations relative to DLM at a given flow speed. Deformed wing snapshots and surface pressures are shown for the two solvers in Fig. 5. The DLM results in this figure are only differential pressures between the upper and lower surfaces, as opposed to FUN3D, which provides a true

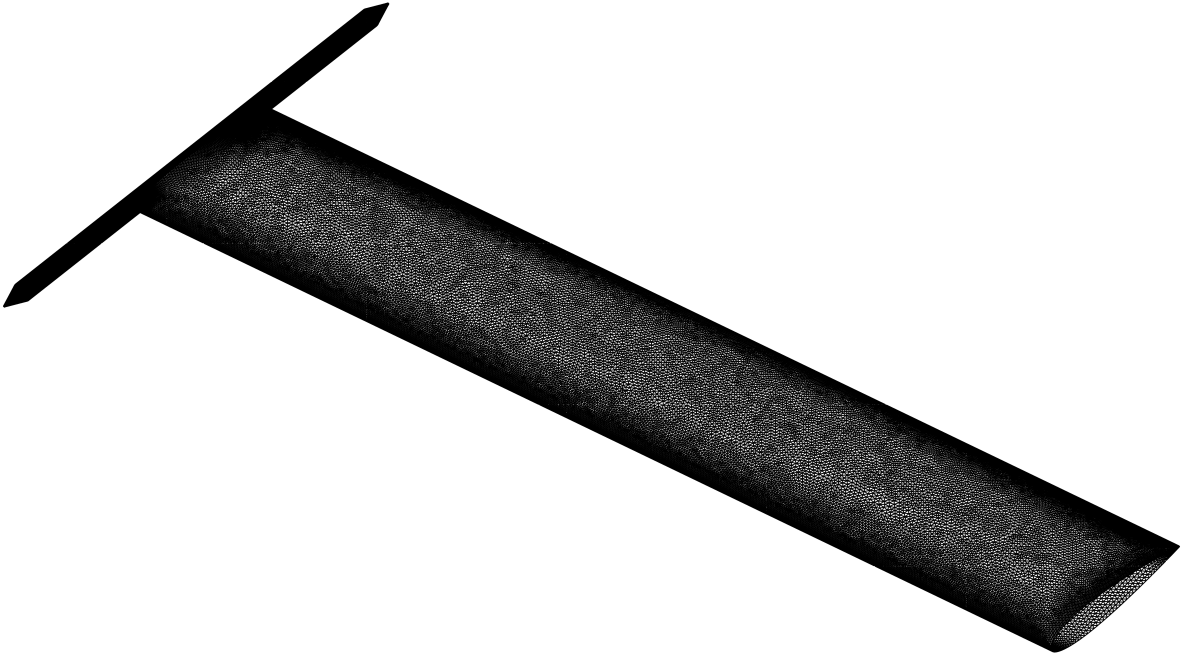


Figure 3: FUN3D surface mesh.

surface pressure: a more direct comparison is shown in Fig. 6 in terms of steady sectional lift distributions at 60 m/s. The largest differences between the solvers is seen at the highest AoA, as expected, where the larger tip twist of the FUN3D results (Fig. 4) provides a higher outboard lift. The foreshortening of the wing due to geometric nonlinearities is also clearly seen at the highest AoA.

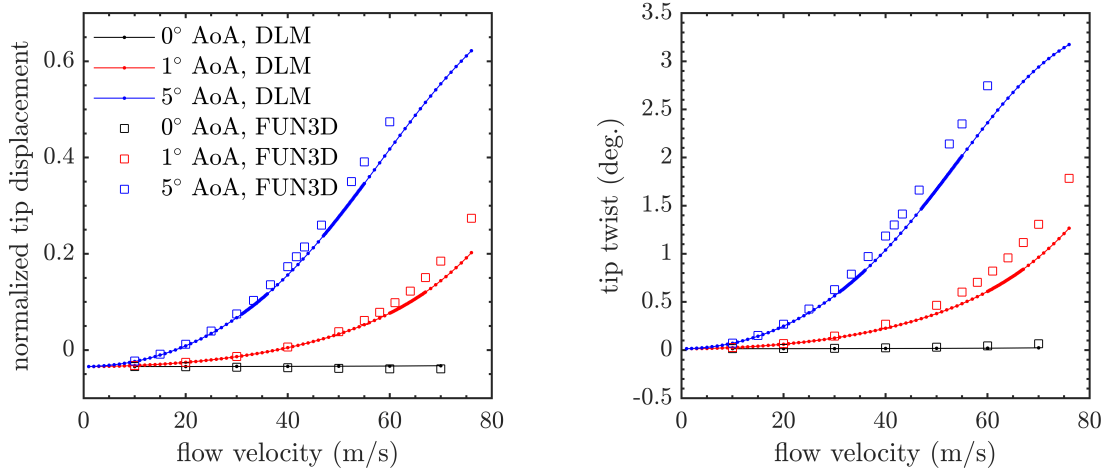


Figure 4: Static aeroelastic deformation at the wing tip.

B. Vibration Frequencies

Results in Figs. 4-6 represent the NLBGS portion of the XDSM in Fig. 1. Next, a modal analysis is conducted; the migration of the first five structural vibration frequencies is shown in Fig. 7. These results are conducted by linearizing about the static aeroelastic results at 0°, 1°, and 5° AoA. As expected, the geometrical nonlinearities in the structure have a substantial impact on the natural frequencies (except the first bending mode), with a greater migration of frequencies seen for larger AoA and flow velocity (i.e.,

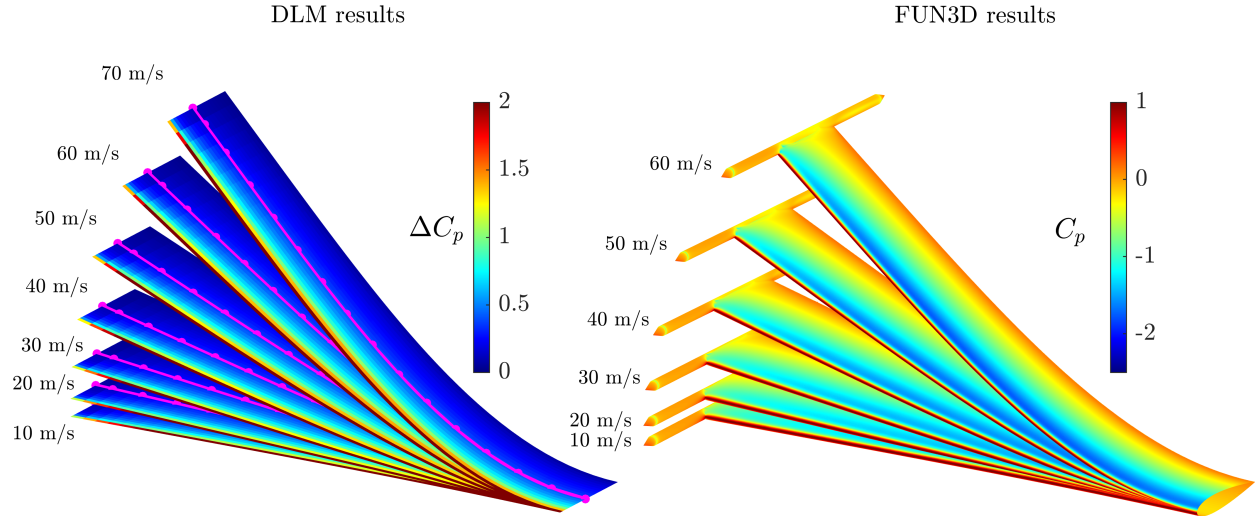


Figure 5: Converged static aeroelastic deformation history across a range of flow velocities: deformed wing snapshots and pressures at 5° AoA, as computed by DLM (left) and FUN3D (right). DLM results are shown as the pressure differential between the upper and lower surfaces.

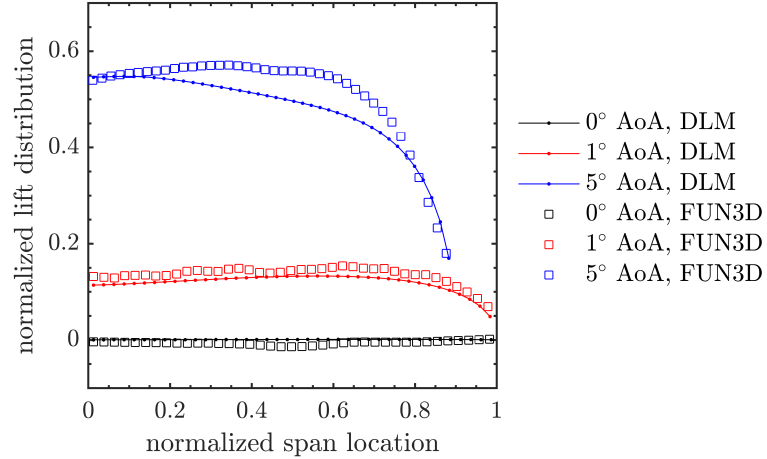


Figure 6: Steady spanwise lift distributions at 60 m/s.

larger static deformations). FUN3D- and DLM-based results generally agree well in Fig. 7, with moderate differences at higher speeds and AoAs, owing to the differences noted in Fig. 4 also.

It is important to note that, even though the data in Fig. 7 are plotted versus flow velocity, these are not aeroelastic eigenvalues. These are the wind-off eigenvalues (i.e., vibration frequencies) computed about a wing deformed into a static aeroelastic equilibrium, a situation that can only be realized numerically, not experimentally.

It can also be seen in Fig. 7 that the vibration frequencies are driven toward each other at certain dynamic pressures: at 1° AoA, modes 4 and 5 meet near a speed of 64 m/s. At 5° AoA, the 4/5 coalescence happens earlier near 35 m/s, and modes 2 and 3 meet also near 50 m/s. An algorithmic choice can be made to allow these modes to cross and reorder (i.e., second bending mode 2 is reidentified as mode 3, etc.), or not. In this work, modes are tracked using the scheme of Ref. 22, and if the velocity step is small enough, the modes will not reorder, as seen in Fig. 7. For larger steps, however, the modes can reorder, and this has important ramifications for the GAFs, as will be discussed below. The DLM-based mode shapes are shown in Fig. 8: cleanly-classified modes (i.e., a second-bending mode) are only available for small elastic deformations (top row of Fig. 8).

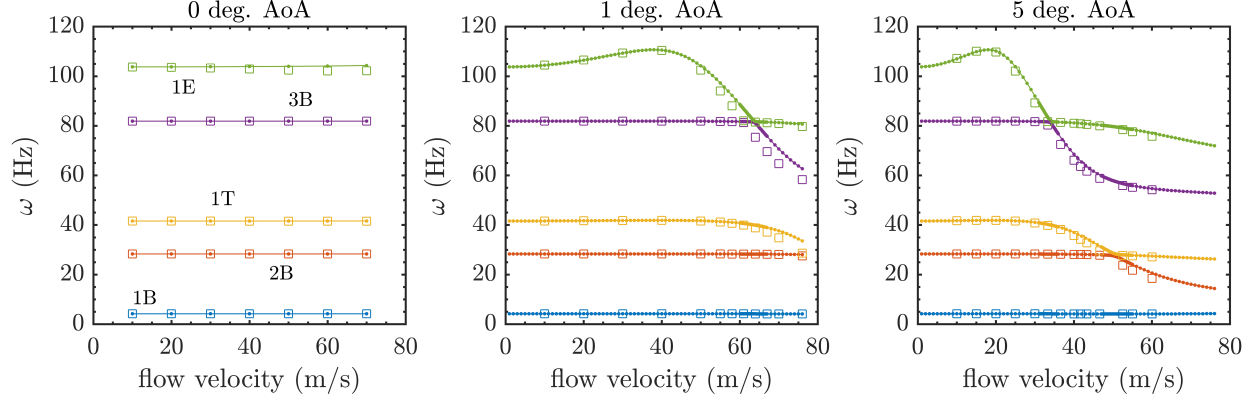


Figure 7: Migration of wind-off vibration frequencies: DLM data (dots) and FUN3D data (squares).

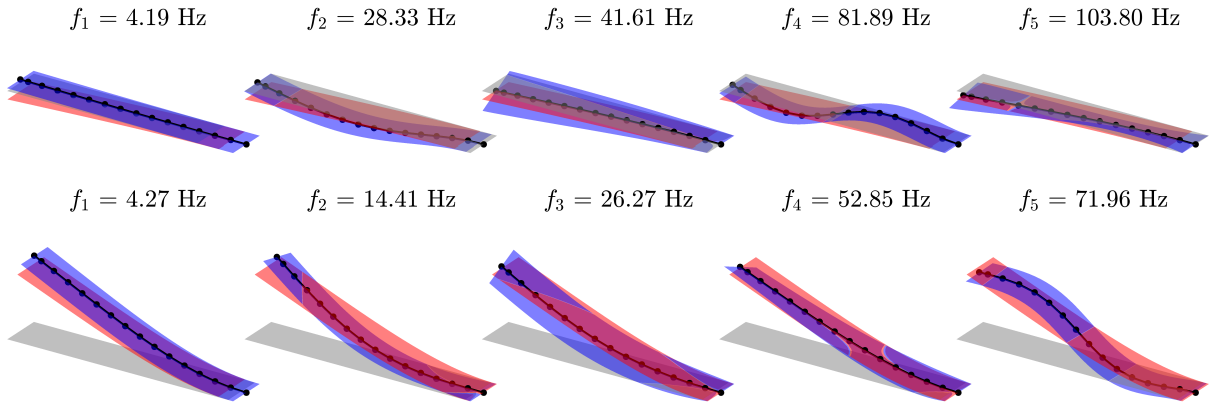


Figure 8: DLM-based wind-off mode shapes at 5° AoA; 1 m/s (top row), 50 m/s (bottom row); grey surface is the undeformed shape, red surface is the static aeroelastic shape, blue surface includes the modal deformation.

C. Generalized Aerodynamic Forces

The next step in the XDSM is to interpolate the beam mode shapes onto the aerodynamic mesh and compute GAFs. The first 1×3 submatrix of the complex-valued GAFs is shown in Fig. 9 at 1° AoA, and Fig. 10 at 5° AoA, as computed by the DLM solver. Data in these plots is shown as the changes in the imaginary part of the GAF versus changes in the real part, in response to a sweep across reduced frequencies between 0.0 and 1.0. Different colors in these plots correspond to different flow speeds: this, as noted, is the hallmark of a nonlinear aeroelastic problem, as linear aeroelastic GAFs are independent of dynamic pressure. Indeed, at 0° AoA, the GAFs (not shown here) are entirely independent of flow speed, as also inferred from the nonexistent frequency migration on the left of Fig. 7. At 1° AoA (Fig. 9), moderate shifts in the GAFs are noted, particularly for the $\bar{A}_{13}$ term, which is the aerodynamic force produced by mode 3 (torsion), as seen by mode 1 (bending). At 5° AoA (Fig. 10), the dependence of the GAFs on dynamic pressure is very strong, with large changes in sign, amplitude, and phase.

The exception to these trends is $\bar{A}_{11}$, the generalized force on bending mode 1 due to itself; this GAF is largely independent of dynamic pressure at both angles of attack. A GAF matrix is computed by pre- and post-multiplying the aerodynamic influence coefficient (AIC) by the matrix of mode shapes, where the AIC is a function of the deformed wing shape, and entirely independent of the modal quantities. A change in a GAF term could be due to changes in either the AICs or changes in the mode shapes, but the insensitivity of $\bar{A}_{11}$ would indicate that GAF shifts shown in these figures are more strongly driven by changes in the mode shapes. This is because mode 1 has been shown to be insensitive to nonlinear deformations (Fig. 7), but the deformed aerodynamic surface (used to compute the AICs) is very large, as shown in Fig. 5.

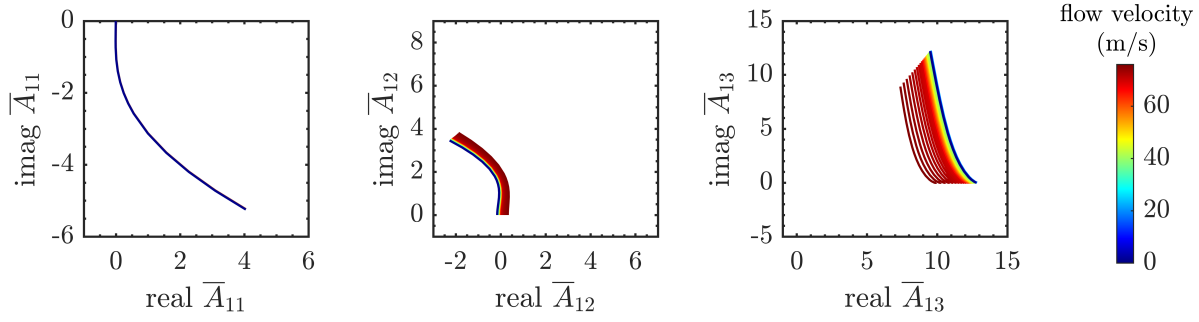


Figure 9: Migration of complex-valued GAFs, as computed by the DLM solver at 1° AoA. Each line in the plots shows a migration of real versus imaginary GAF due to an increase in the reduced frequency, k , between 0 and 1. The color of each line indicates the flow velocity at which it was computed.

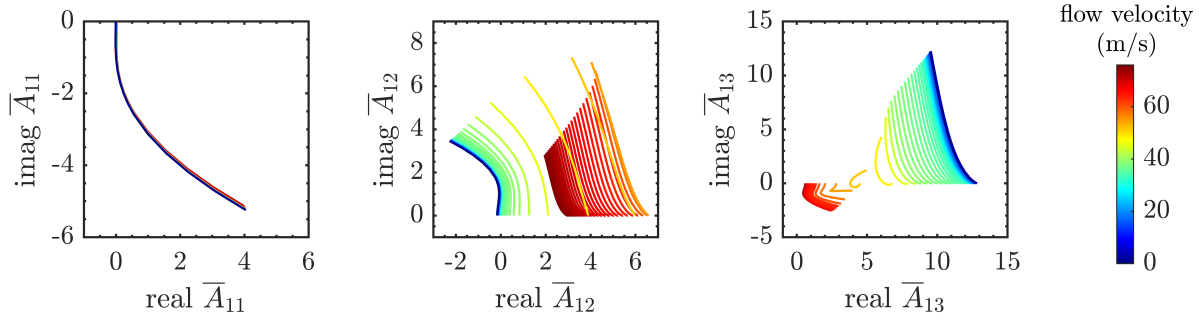


Figure 10: Migration of complex-valued GAFs, as computed by the DLM solver at 5° AoA. Each line in the plots shows a migration of real versus imaginary GAF due to an increase in the reduced frequency, k , between 0 and 1. The color of each line indicates the flow velocity at which it was computed.

As noted above, with increases in velocity (and therefore increases in wing deformation), the vibration frequencies can be driven towards each other, as seen in Fig. 7. An important algorithmic choice must be made as to whether or not the modes should be allowed to cross and thus be re-labeled. This choice has important ramifications for the GAFs, as shown in Figs. 11 and 12. These figures plot the DLM-based frequency and GAF migration for the 4th and 5th modes, at 1° AoA. Only the real part of the GAFs at $k=0$ are shown here. For Fig. 11, a relatively large step size of 1.0 m/s is taken across the plot, and the method of Ref. 22 allows the modes to cross (i.e., reorder). This is accompanied by a discontinuity in the GAFs, particularly the $\bar{A}_{54}$ and $\bar{A}_{55}$ terms. If a finer step size of 0.1 m/s is taken, the frequencies do not cross, but instead smoothly separate (these data are reproduced from Fig. 7), as shown in the inset plot of 12. For this latter case, the GAFs smoothly vary across the meeting speed of 64 m/s, albeit with dramatic changes in sign.

It should be noted that the GAFs in Figs. 11 and 12 are the same, with the exception of a relabeling of the two modes, and also a change in sign. A smooth variation in GAFs across flow speed is important for the aeroelastic framework used here, because GAFs are interpolated across flow speeds to compute flutter, as discussed in Eq. 1. Discontinuities in GAFs may lead to nonphysical flutter behavior, or convergence difficulties in the solution of the $p - k$ equations.

A comparison of the DLM-computed GAFs with the FUN3D-computed GAFs is shown in Fig. 13, at 5° AoA and a reduced frequency of 0.1, and in Fig. 14 at a higher reduced frequency of 0.5. For brevity, only the diagonal part of the GAFs are shown. The comparison between the two flow solvers is mixed, with good correlation across the entire velocity range for the $\bar{A}_{22}$ and $\bar{A}_{55}$ terms. The good correlation in the 5th mode is surprising, given that this is an in-plane bending mode, whose aerodynamics will not be well-predicted by the DLM (which is driven entirely by downwash perturbations). For higher velocities, however, there is enough mixing of the mode shapes that mode 5 no longer contains a strong in-plane component, as seen in Fig. 8.

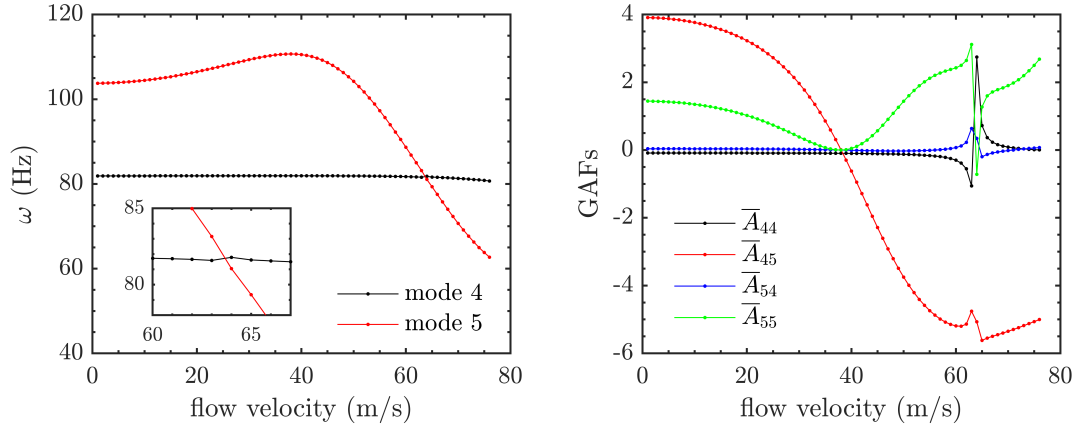


Figure 11: Migration of the 4th and 5th natural frequencies (left) and the 4/5 GAF submatrix at $k=0$ (right), at 1° AoA, with a 1.0 m/s velocity step size.

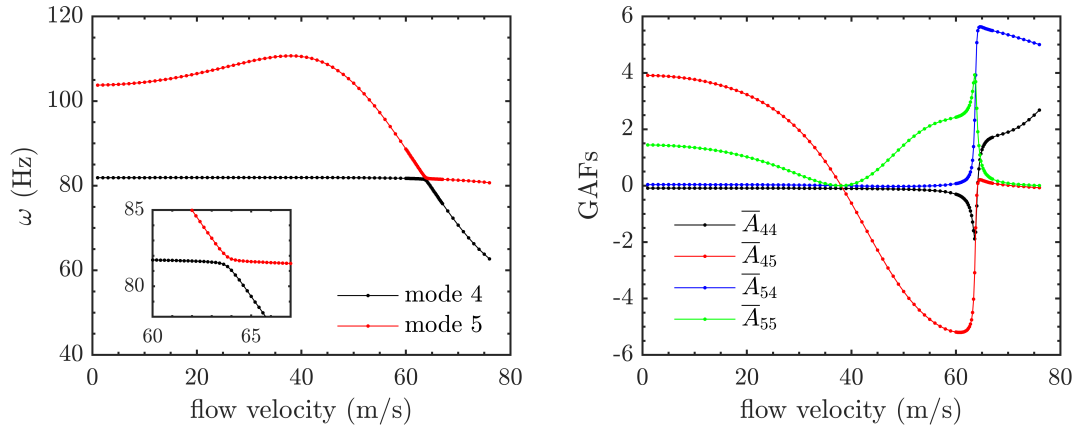


Figure 12: Migration of the 4th and 5th natural frequencies (left) and the 4/5 GAF submatrix at $k=0$ (right), at 1° AoA, with a 0.1 m/s velocity step size.

One consistent and notable discrepancy between DLM and FUN3D is seen in the $\bar{A}_{11}$, $\bar{A}_{33}$, and $\bar{A}_{44}$ terms, where the FUN3D solver predicts a spike in the GAFs through 40 m/s, but the DLM-based solver does not. For the $\bar{A}_{11}$ term in particular, the DLM-based result is largely independent of static wing bending (seen in Fig. 10 also). The cause of this trend can be seen in Fig. 15, which shows the oscillatory pressure coefficients over the wing, namely the imaginary part of those pressures due to the first bending mode. The spatial integration of the product of these pressures and the mode shapes, produces the GAFs. Two pressure fields are shown: one for the lightly-loaded case of 10 m/s, and one for the case of 40 m/s, which nearly coincides with the GAF spikes seen in Fig. 13 and Fig. 14. Over the wing, the oscillatory pressures are very

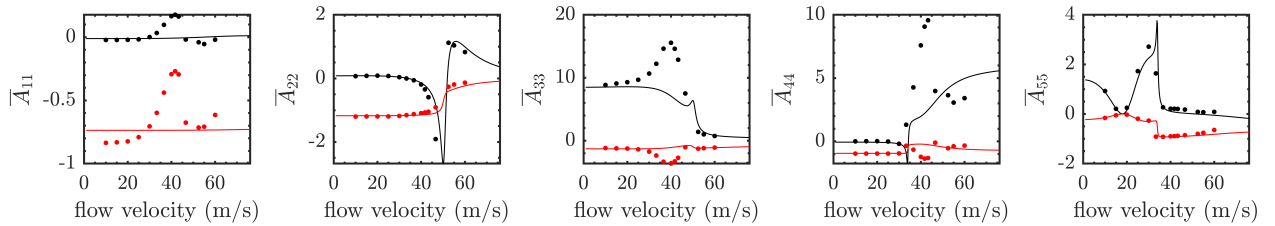


Figure 13: Comparison of DLM-computed (lines) and FUN3D-computed (dots) GAFs, at 5° AoA, $k=0.1$: real part is in black, imaginary part is in red.

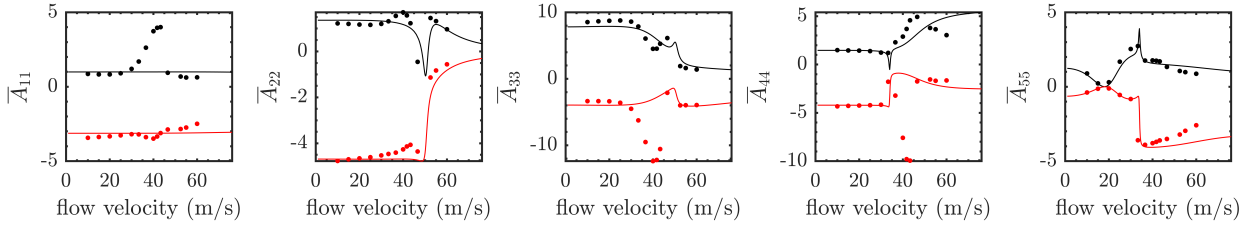


Figure 14: Comparison of DLM-computed (lines) and FUN3D-computed (dots) GAFs, at 5° AoA, $k=0.5$: real part is in black, imaginary part is in red.

similar for the two cases, with positive imaginary pressures on the upper surface, and negative on the lower; these pressures follow the bending mode distribution, with higher pressures outboard.

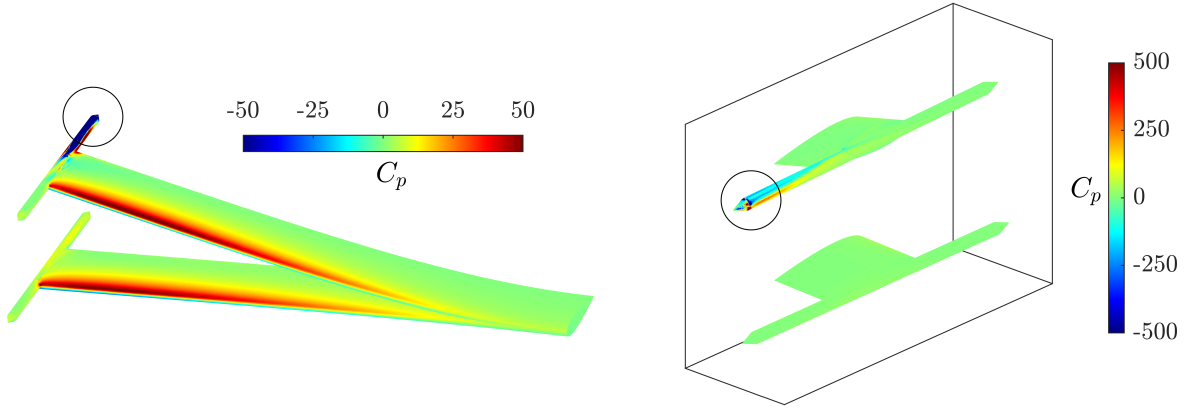


Figure 15: FUN3D-based oscillatory pressure coefficients (imaginary part due to the first bending mode) over the wing at 10 m/s (bottom) and 40 m/s (top), 5° AoA, $k=0.1$: global view at restricted contour limits (left), and tip view at the full contour limits (right).

Large discrepancies for the two cases in Fig. 15 are seen along the trailing edge of the slender cylindrical body, with oscillatory pressures an order-of-magnitude higher than those seen on the wing, particularly at the geometrical intersection between the cylinder and the trailing edge cone. Additional research is needed to better understand whether the results shown here are physical (unsteady interactions between the wing and the slender body, manifesting itself via the GAFs), or a numerical artifact (i.e., an under-resolved mesh). With this knowledge, the FUN3D GAFs can be re-computed, but ignoring the oscillatory pressures on the slender body. These new GAFs are shown in Fig. 16, and are directly comparable to the data in Fig. 14. As expected, the GAF spike near 40 m/s is now gone, and the comparison with DLM is much better. DLM also models the cylindrical body (via slender body theory), but adding or removing this aspect of the model has no discernible impact on the GAFs.

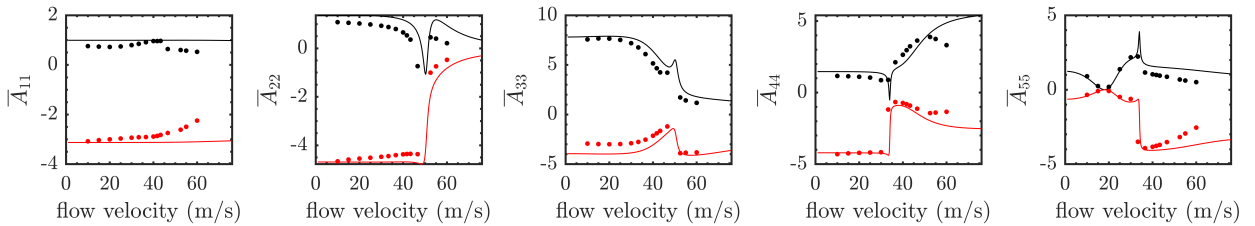


Figure 16: Comparison of DLM-computed (lines) and FUN3D-computed (dots) GAFs, at 5° AoA, $k=0.5$: real part is in black, imaginary part is in red. Pressures on the slender body are ignored.

D. Flutter

Finally, results of the $p - k$ eigen-equations are shown in Fig. 17 for 0° , 1° , and 5° AoA, as computed by the DLM solver. At 0° AoA, which presents a largely linear aeroelastic problem, modes 2 and 3 lightly coalesce, leading to an instability mechanism in mode 3 (torsion) near 68 m/s. This flutter mechanism is driven to lower speeds as the AoA increases, and further manifests as a restabilizing hump mode. At the highest AoA, the hump mode destabilizes at 44 m/s and restabilizes at 51 m/s. Another subcritical flutter mechanism also appears at this highest AoA (coupling of the first 2 modes), destabilizing at 73 m/s. For the imaginary parts in Fig. 17, the mode tracking realized by the scheme in Ref. 22 (which was also used for the wind-off frequencies in Fig 7) is inconsistent. At the highest AoA, the imaginary parts do not cross and re-label, as seen in Fig. 12 also. But at 1° AoA, modes 4 and 5 do intersect (similar to Fig. 11), and this occurs despite extremely small velocity steps.

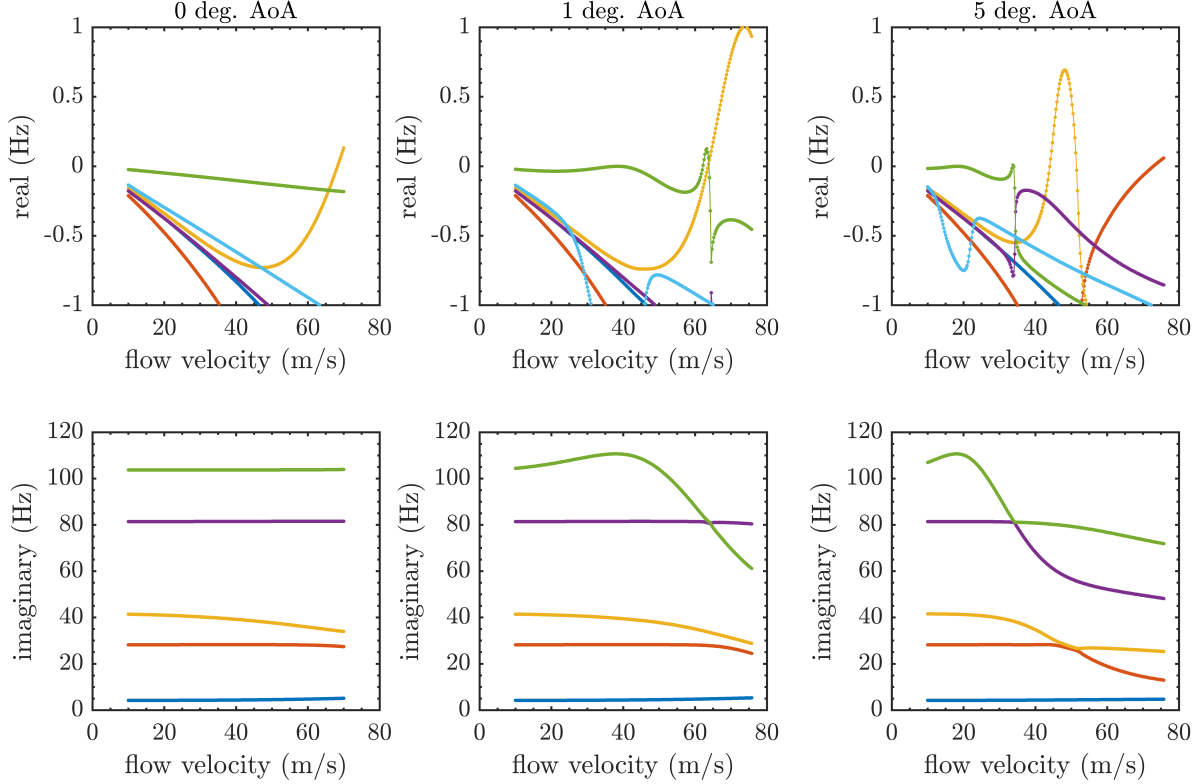


Figure 17: Real and imaginary aeroelastic eigenvalue migration, as computed by DLM.

Flutter results from DLM are compared with FUN3D in Fig. 18, at 5° AoA. The DLM results are repeated from Fig. 17; the FUN3D results utilize the GAFs shown in Figs. 13 and 14; results are also shown via GAFs where the pressures on the slender tip body ignored, via the GAFs in Fig. 16. DLM results extend from speeds of 10 m/s to 75 m/s, but FUN3D results were only obtained between 25 m/s and 60 m/s. The upper limit is because, as noted above, negative volumes appear in the mesh above 60 m/s, due to excessive static deformations. Obtaining results at lower velocities requires GAFs to be computed at relatively high reduced frequency ($k = \omega \cdot b/2U$) values. For DLM, a high k value of 10 could easily be included, but for FUN3D, in order to reduce the cost of the LFD tool, the highest k value was kept to 2.

The basic features of the three flutter diagrams in Fig. 18 are the same, with each showing a hump mode between 45 and 50 m/s. The FUN3D results show a much higher instability peak than DLM, by over a factor of 2. Furthermore, the center plot in Fig. 18 (FUN3D with the tip pressures included) shows that mode 2 destabilizes, rather than mode 3. Fig. 17 indicates that these two modes have basically coalesced across this velocity range, however, and so the wind-off identity of the fluttering mode is immaterial. It can also be seen in the center plot that the damping of mode 1 shows a moderate peak through 40 m/s, though

never destabilizes; once the slender tip body pressures are removed from the GAFs (right plot of Fig. 18), this peak is gone, and is presumably driven by the same tip mechanisms seen in Figs. 13 and 14.

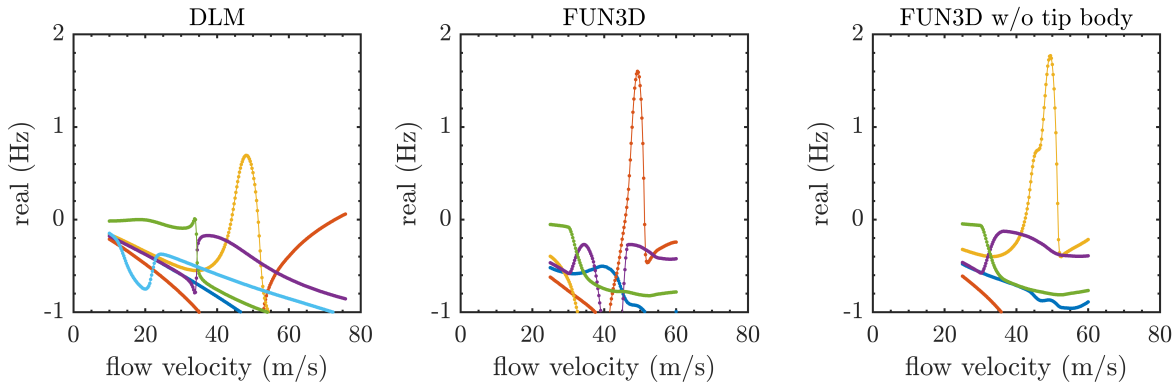


Figure 18: Aeroelastic eigenvalue migration (real part) at 5° AoA, as computed by DLM, FUN3D, and FUN3D with pressures on the slender tip body ignored.

IV. Conclusions

This work has considered the static and dynamic aeroelastic analysis of a highly-flexible, subsonic, unswept wing with a cylindrical tip mass. The aeroelastic response is driven by nonlinear mechanisms, namely the large geometrically-nonlinear deformations. Two numerical frameworks are described here: the first couples linear doublet lattice aerodynamics to a nonlinear beam solver for a static aeroelastic equilibrium. Oscillatory modal perturbations are then considered about the static point, producing generalized aerodynamic forces. Because the modal quantities (both frequencies and mode shapes) change so drastically with large static deformations of the wing, the generalized forces must be computed and stored at a large range of flow dynamic pressures. An altered form of the flutter eigen-equations can finally be solved to compute flutter mechanisms, where generalized forces are interpolated across not only reduced frequencies (as is typically done) but also dynamic pressures. The second numerical framework swaps out the linear doublet lattice aerodynamics for nonlinear RANS computations, and a linearized frequency-domain method used for generalized aerodynamic forces.

Results show a largely good agreement between the two solvers, with the RANS tool showing overall larger static deformations, and also more aggressive flutter mechanisms. The RANS tool also showed unsteady interactions between the wing and the slender body mounted to the tip, at certain oscillation frequencies, angles of attack, and flow speeds. These interactions led to substantial pressure fluctuations on the slender body, which had a noticeable impact on the generalized aerodynamic forces, though less of an impact on the final flutter mechanism. More work is needed to understand the cause of these interactions. The work shown here has also demonstrated the need for careful handling of wind-off structural modes as they migrate due to large static deformations, and how these migrations can impact generalized aerodynamic forces and downstream flutter computations.

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References

¹Patil, M., Hodges, D., “On the Importance of Aerodynamic and Structural Geometrical Nonlinearities in Aeroelastic Behavior of High-Aspect-Ratio Wings,” *Journal of Fluids and Structures*, Vol. 19, No. 7, pp. 905-915, 2004.

- ²Su, W., Cesnik, C., “Dynamic response of Highly Flexible Flying Wings,” *AIAA Journal*, Vol. 49, No. 2, pp. 324-339, 2012.
- ³Tang, D., Dowell, E., “Experimental Aeroelastic Models Design and Wind Tunnel Testing for Correlation with New Theory,” *Aerospace*, Vol. 3, No. 2, pp. 1-25, 2016.
- ⁴Xie, C., Liu, Y., Yang, C., Cooper, J., “Geometrically Nonlinear Aeroelastic Stability Analysis and Wind Tunnel Test Validation of a Very Flexible Wing,” *Shock and Vibration*, Vol. 2016, ID 5090719, 2016.
- ⁵Peters, D., Karunamoorthy, S., Cao, W., “Finite State Induced Flow Models Part I: Two-Dimensional Thin Airfoil,” *Journal of Aircraft*, Vol. 32, No. 2, pp. 313-322, 1995.
- ⁶Del Carre A., Munoz A., Goizueta N., Palacios R., “SHARPy: A Dynamic Aeroelastic Simulation Toolbox for Very Flexible Aircraft and Wind Turbines” *Journal of Open Source Software*, Vol. 4, No. 44, pp. 1885-1895, 2019.
- ⁷Rodden, W., Johnson, E., *MSC/NASTRAN Aeroelastic Analysis User’s Guide*, MacNeal-Schwendler, Los Angeles, 1994.
- ⁸Neill, D., Herendeen, D., Venkayya, V., “ASTROS Enhancements Volume III - ASTROS Theoretical Manual”, WL-TR-95-3006, 1995.
- ⁹Zona Technology, “ZAERO Version 9.2 User’s Manual,” , 2017.
- ¹⁰Strong, D., Kolonay, R., Huttzell, L., Flick, P., “Flutter Analysis of Wing Configurations using Pre-Stressed Frequencies and Model Shapes,” *AIAA Structures, Structural Dynamics, and Materials Conference*, Austin, TX, April 2005.
- ¹¹Anand, H., “Nonlinear Aeroelastic Analysis of a High Aspect Ratio Wing in NASTRAN,” Masters of Science Thesis Report, Delft University of Technology, April 2019.
- ¹²Biedron, R., Carlson, J., Derlaga, J., Gnoffo, P., Hammond, D., Jacobson, K., Jones, W., Kleb, B., Lee-Rausch, E., Nielsen, E., Park, M., Rumsey, C., Thomas, J., Thompson, K., Walden, A., Wang, L., Wood, W., “FUN3D Manual: 13.7,” NASA TM-2020-5010139.
- ¹³Anderson, W., Newman, J., Karman, S., “Stabilized Finite Elements in FUN3D,” *Journal of Aircraft*, Vol. 55, No. 2, pp. 696-714, 2017.
- ¹⁴Jacobson, K., Stanford, B., Wood, S., Anderson, W., “Frequency-Domain Flutter Analysis and Sensitivities with Stabilized Finite Elements,” *AIAA SciTech Forum*, Orlando, FL, January 6-10, 2020.
- ¹⁵Avin, O., Raveh, D., Drachinsky, A., Ben-Shmuel, Y., Tur, M., “An Experimental Benchmark of a Very Flexible Wing,” *AIAA SciTech Forum*, virtual event, January 2021.
- ¹⁶Riso, C., Cesnik, C., “Correlations Between UM/NAST Nonlinear Aeroelastic Simulations and the Pre-Pazy Wing Experiment,” *AIAA SciTech Forum*, virtual event, January 2021.
- ¹⁷Ritter, M., Hilger, J., Zimmer, M., “Static and Dynamic Simulations of the Pazy Wing Aeroelastic Benchmark by Nonlinear Potential Aerodynamics and Detailed FE Model,” *AIAA SciTech Forum*, virtual event, January 2021.
- ¹⁸Goizueta, N., Drachinsky, A., Wynn, A., Raveh, D., Palacios, R., “Flutter Predictions for Very Flexible Wing Wind Tunnel Test,” *AIAA SciTech Forum*, virtual event, January 2021.
- ¹⁹Lambe, A., Martins, J., “Extensions to the Design Structure Matrix for the Description of Multidisciplinary Design, Analysis, and Optimization Processes” *Structural and Multidisciplinary Optimization*, Vol. 46, pp. 273-284, 2012.
- ²⁰van Zyl, L., Maserumule, M., “Divergence and the p-k Flutter Equation,” *Journal of Aircraft*, Vol. 38, No. 3, pp. 584-586, 2001.
- ²¹Allmaras, S., Johnson, F., Spalart, P., “Modifications and Clarifications for the Implementation of the Spalart-Allmaras Turbulence Model” *Seventh International Conference on Computational Fluid Dynamics*, Big island, HI, July 9-13, 2012.
- ²²Eldred, M., Venkayya, V., Anderson, W., “Mode Tracking Issues in Structural Optimization,” *AIAA Journal*, Vol. 33, No. 10, pp. 1926-1933, 1995.