

# Transition Analysis for the CRM-NLF Wind Tunnel Configuration using Transport Equation Models and Linear Stability Correlations

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Transition models based on auxiliary transport equations augmenting the Reynolds-averaged Navier-Stokes (RANS) framework rely upon transition correlations that were derived from a limited number of low-speed experiments. Furthermore, these models often account for only a subset of the relevant transition mechanisms and/or cannot accurately predict the sensitivity of those mechanisms to the changes in significant flow parameters. A preceding investigation had targeted the assessment of the transport-equation-based transition models in NASA's OVERFLOW 2.3b solver, namely, the amplification factor transport (AFT-2017b) equation model coupled with the Spalart-Allmaras RANS model and the Langtry-Menter transition models (LM2009 without crossflow effects and LM2015 including the modeling of crossflow transition) implemented with Menter's shear-stress transport equation (SST2003) RANS model. Comparisons with recent measurements at transonic freestream conditions on the Common Research Model with Natural Laminar Flow (CRM-NLF) reinforced our earlier finding that all three of the above models significantly underpredict the reported extent of the laminar flow region over the entire span of the wing, regardless of the dominant instability mechanism(s) underlying the onset of the transition process. The underprediction of the laminar flow extent was attributed to the failure of the above models in accounting for the stabilizing effect of compressibility on the amplification of Tollmien-Schlichting instabilities. Based on previous linear stability studies related to compressibility effects, the present work proposes modifications to the two classes of transition models that reduce to the original form of each model at low subsonic speeds and do not require any nonlocal flow information or additional transport equation(s). The modifications are shown to significantly improve the predicted laminar extent of the flow and compare well against the data from the CRM-NLF experiment. Additionally, a previous assessment of transition prediction based on the the dual, nonparallel  $N$ -factor method in conjunction with linear parabolized stability equations (PSE) is extended to additional angles of attack to provide the first comprehensive assessment of transition models based on nonparallel disturbance amplification over the CRM-NLF. In general, the transition criterion based on the dual, nonparallel  $N$ -factor method with  $N_{TS} = N_{CF} = 6$  is reasonably successful at correlating with the measured transition fronts at  $Re_{MAC} = 15$  million for all angles of attack investigated herein and provides additional validation of the improved predictions from the compressibility-corrected transition models.

## Nomenclature

|       |   |                                            |
|-------|---|--------------------------------------------|
| $C_D$ | = | drag coefficient [nondimensional]          |
| $C_f$ | = | skin-friction coefficient [nondimensional] |
| $C_L$ | = | lift coefficient [nondimensional]          |

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|                    |   |                                                                                                      |
|--------------------|---|------------------------------------------------------------------------------------------------------|
| $C_m$              | = | pitching moment coefficient [nondimensional]                                                         |
| $C_p$              | = | surface pressure coefficient [nondimensional]                                                        |
| $f$                | = | disturbance frequency [Hz]                                                                           |
| $M$                | = | Mach number [nondimensional]                                                                         |
| $N$                | = | logarithmic amplification factor                                                                     |
| $\bar{q}$          | = | vector of base flow variables                                                                        |
| $\tilde{q}$        | = | vector of perturbation variables                                                                     |
| $\hat{q}$          | = | vector of amplitude variables                                                                        |
| $Re$               | = | unit Reynolds number [ $m^{-1}$ ]                                                                    |
| $Re_{MAC}$         | = | Reynolds number based on freestream variables and mean aerodynamic chord [nondimensional]            |
| $T$                | = | temperature [K]                                                                                      |
| $(u, v, w)$        | = | streamwise, wall-normal, and spanwise velocity components [ $m\ s^{-1}$ ]                            |
| $(x, y, z)$        | = | Cartesian coordinates [m]                                                                            |
| $x/c$              | = | chordwise coordinate scaled by reference chord length [nondimensional]                               |
| $y^+$              | = | near wall grid spacing in wall units [nondimensional]                                                |
| $\alpha$           | = | angle of attack [deg], streamwise wavenumber [ $m^{-1}$ ]                                            |
| $\beta$            | = | spanwise wavenumber [ $m^{-1}$ ]                                                                     |
| $\lambda$          | = | spanwise wavelength [m]                                                                              |
| $\mu$              | = | dynamic viscosity [ $kg/(m.s)$ ]                                                                     |
| $\mu_t$            | = | turbulent eddy viscosity [ $kg/(m.s)$ ]                                                              |
| $\omega$           | = | disturbance angular frequency [ $rad\ s^{-1}$ ]                                                      |
| $\rho$             | = | density [ $kg\ m^{-3}$ ]                                                                             |
| $\sigma$           | = | spatial growth rate [ $m^{-1}$ ]                                                                     |
| $(\xi, \zeta)$     | = | streamwise, and spanwise coordinates [m]                                                             |
| $\eta$             | = | wall-normal coordinate [m], semispan location nondimensionalized by semispan length [nondimensional] |
| <b>Subscripts</b>  |   |                                                                                                      |
| $\infty$           | = | farfield reference state                                                                             |
| $0$                | = | neutral/critical point                                                                               |
| $c$                | = | critical value                                                                                       |
| $e$                | = | boundary layer edge value                                                                            |
| <b>Superscript</b> |   |                                                                                                      |
| $T$                | = | transpose                                                                                            |

## I. Introduction

Given the push for greener and/or safer technologies in the arena of aerospace research, accurate prediction of viscous flows with laminar-to-turbulent boundary-layer transition has assumed increased significance for applications such as the design of natural laminar flow wings, unmanned aerial vehicles, and crewed reentry vehicles. Consequently, the prediction of laminar-to-turbulent boundary-layer transition has been identified as a critical and pacing item for computational fluid dynamics (CFD) in the NASA CFD Vision 2030 [1] study.

Direct numerical simulations (DNS) and wall-resolved large-eddy simulations (WRLES) with suitable subgrid scale models are the only approaches capable of simulating the complete set of physical processes ranging from the onset of transition to the breakdown of laminar boundary layers into turbulent flow. However, these approaches remain impractical because of their excessive computational cost and the need for an accurate specification of the initial and boundary conditions required to capture the excitation of the flow instabilities responsible for transition. Adequate information about the latter conditions is often unavailable from most wind-tunnel and flight tests.

Semiempirical correlations based on the linear stability theory (LST), such as the  $e^N$  method introduced by Smith and Gamberoni [2] and van Ingen [3], or the parabolized stability equations (PSE) [4] represent the most widely used approaches for the prediction of transition onset during aircraft-design computations. Both

LST and PSE can account for the amplification of the dominant primary instabilities encountered on aircraft wings (all speed regimes), namely, the Tollmien-Schlichting (TS) waves and the stationary/traveling crossflow vortices at subsonic and transonic Mach numbers. However, the application of these approaches toward a coupled computation of laminar, transitional, and fully turbulent parts of a flowfield entails an increased computational cost as well as complexity; it also suffers from a lack of robustness and often requires adequate understanding of transition physics and hydrodynamic stability theory that typical users of CFD solvers may not possess. To alleviate these shortcomings, surrogate models are often used in lieu of the direct computation of stability characteristics based on the laminar flow information provided by the flow solver. There are several successful implementations of CFD solvers that allow for automated transition predictions based on LST, for instance, Refs. 5–12. However, a majority of these models suffer from the need to have sufficiently well resolved boundary-layer profiles in the laminar baseflow computations, including the wall-normal derivatives of velocities and temperature, or at least, the need to compute integral boundary-layer parameters that may be used as a proxy to the laminar profiles themselves. These restrictions are difficult to overcome in modern CFD solvers that rely upon massive parallelization and the use of unstructured grids [13].

Efforts to develop models that incorporate the underlying physics of the transition process and, furthermore, can be embedded into a RANS framework have gained substantial ground [14–25]. These models typically require only local information to model transition, in contrast to LST-based models that require detailed boundary-layer profiles or the metamodels for LST analysis that require integral boundary-layer parameters. The RANS-based transition models often rely on solving additional transport equations that are similar to the RANS equations but include correlations and other modifications to source terms in the turbulence equations to mimic the behavior of transition, allowing the codes to traverse seamlessly from the laminar to fully turbulent flows. This approach is amenable to easy implementation in modern, unstructured-grid based CFD solvers. This approach clearly overcomes some of the limitations of the stability-based transition prediction and is well-suited for generalizing the established process for turbulent flow computations in a cost-effective manner. However, the inherent averaging process in the RANS procedure makes it difficult to capture the development of the linear disturbances and its subsequent nonlinear growth and eventual breakdown. Thus, the transport-equation-based models are incapable of accurately modeling the process of laminar-to-turbulent boundary-layer transition and, hence, are reasonably accurate only under limited regimes. Despite this limitation, the transport-equation-based transition models remain an attractive option because the vehicle performance is often characterized in terms of global flow metrics such as the force and moment coefficients and/or the pressure and skin-friction distributions, which are relatively insensitive to the detailed physics of the transition to turbulence. Earlier RANS-based transition models were designed to account for the transition processes initiated by TS instability waves, or some form of bypass transition, or due to inviscid instabilities in the regions of local flow separation; however, they did not account for crossflow (CF) effects. In the past decade, there has been an increased interest to incorporate the effects of CF in the transition models [26–31]. Perhaps the most significant limitation of the transport-equation-based transition models is that the increased empiricism renders them less amenable to an extrapolation to new configurations, making it more important to validate their predictions on a case-by-case basis. More details on the various approaches currently being used to predict/model transition can be found in Refs. [24, 25].

The overarching goal of the present research is to blend the so-called physics-based methodology for transition prediction with the RANS-based approaches, providing a reliable and cost efficient, yet robust and user-friendly approach for the integrated modeling of laminar-turbulent transition. Toward achieving that goal, the data obtained from NASA's recent experiments on the Common Research Model with Natural Laminar Flow (CRM-NLF) [32–35] in the National Transonic Facility (NTF) provide a suitable platform for evaluating the performance of the tools being developed under the present research supporting NASA's Transformational Tools and Technologies (TTT) project. In a typical transport aircraft wing with high sweep, transition occurs because of CF instability and/or TS instability, provided that the attachment line remains laminar. The CRM-NLF wing has been designed to modify the surface pressure distribution in such a way that the overall amplification of both instabilities is reduced, thereby allowing laminar flow to be maintained over a substantial region of the wings and thus helping reduce its drag.

As a first step, we had carried out studies on the CRM-NLF by using both RANS-based transition models [36] available within NASA's OVERFLOW 2.3b solver [38] and linear stability analysis at various levels of fidelity [37]. The results of Paredes et al. [37] showed that linear stability analysis could reasonably explain the transition fronts inferred from the CRM-NLF experiment (TS effects and shock-limiting being the

dominant mechanisms). The results by Venkatachari et al. [36] showed that the transport-equation-based transition models significantly underpredict the laminar extent of the flow, with the most likely reason being the inability of these models to account for the stabilizing influence of compressibility on the boundary-layer flow for Mach numbers up to approximately  $M_\infty = 2.0$  [39]. This observation is in line with the fact that the bulk of the existing studies based on the transport-equation-based transition models have been carried out for low Mach number configurations ( $M_\infty < 0.3$ ) and/or for modest Reynolds numbers based on the wing chord length and with pressure distributions that may not be optimal for wing designs for subsonic transport aircraft flying at Mach numbers between 0.75 to 0.90. Additionally, many of these transition models rely upon empirical correlations that were based on low-speed flows.

The main objectives of this current work are the following: (i) to modify some of the correlations within these transport-equation-based transition models to account for compressibility effects, by drawing from the previous works on linear stability analysis for canonical flows, and then evaluate the resulting improvements in the context of transition modeling for the CRM-NLF configuration; (ii) to see if the dual  $N$ -factor criterion utilized in the previous work by Paredes et al. [37] is able to perform comparably well at the higher angles of attack that were not included in that earlier work; and (iii) use the stability analysis results in addition to the NTF measurements to evaluate the improvements due to the modified transport-equation-based transition models.

Details about the model configuration, the flow solver, transition models, and the stability analysis tools are given in sections II and III. In section IV, details about the compressibility-correction related modifications to the transition models are described. Details about the flow conditions and the mesh used for the computations are discussed in section V. The results obtained with the modified transition models and the stability-analysis/dual  $N$ -factor criterion are given in sections VI and VII. The conclusions are provided in section VIII.

## II. CFM-NLF Model

The CRM is an open geometry representation of a generic transport vehicle and has been used in a multitude of studies [40]. The CRM-NLF builds upon this geometry by replacing the CRM wing with a new outer mold line (OML) that supports a significant region of NLF on the upper surface of the wing. This modified wing configuration was designed by using the Constrained Direct Iterative Surface Curvature (CDISC) design process [40, 41]. The wind-tunnel model is a 5.2% scaled semispan model of the CRM-NLF. The model has a semispan length of 1527.83 mm (60.151 inches), mean aerodynamic chord (MAC) of 364.29 mm (14.342 inches) and a leading-edge sweep of 37.3 degrees over the majority of the wingspan. The leading-edge sweep is reduced to 12.9 degrees over the inboard 10% of the wing to avoid attachment-line transition via turbulent contamination from the fuselage boundary layer. Data acquired during the test include total forces and moments, surface static pressures, model deformation, and transition visualization data. More details on the model geometry and the wind-tunnel measurements can be found on the CRM website.<sup>††</sup>

## III. Flow Solver, Transition Models, and Stability Analysis Tools

The NASA OVERFLOW 2.3b [38] is an implicit structured overset grid Navier-Stokes solver that is capable of computing time-accurate and steady-state solutions via a variety of options for spatial and temporal discretization. The RANS-based transition models utilized in this study (and available in OVERFLOW 2.3b) are the following: (i) the two-equation Langtry-Menter transition model (LM2009) [18] based on the year 2003 version of Menter's shear-stress transport (SST) RANS model [43,44], along with the modifications proposed by Langtry et al. [27] to account for crossflow induced transition (LM2015); and (ii) Coder's [22, 23] 2017b version of the amplification factor transport equation model (AFT 2017b) that uses the Spalart-Allmaras (SA) model [45]. The AFT model, as implemented in OVERFLOW 2.3b, can only account for transition due to TS waves in 2D boundary layers and cannot account for crossflow effects (this has been addressed in a recent work [46]). Terms incorporating rotation correction (RC) [47] were included in the underlying turbulence models. According to the nomenclature of the NASA turbulence modeling resource,<sup>‡‡</sup>

<sup>††</sup> <https://Commonresearchmodel.larc.nasa.gov/crm-nlf/>

<sup>‡‡</sup> <https://turbmodels.larc.nasa.gov>

the transition models considered here should ideally be referred to as SST-2003RC-LM2015 and SA-RC-AFT2017b, respectively. However, for sake of brevity, they will be referred to as LM2015 and AFT2017b throughout this work.

The basic state for the linear stability analysis is computed via the OVERFLOW 2.3b flow solver, by using the SA turbulence model with an imposed transition front. The LASTRAC.3d (nonorthogonal coordinate system) module of the NASA LASTRAC transition analysis software [48, 49] is used to calculate the instability characteristics by using the PSE formulation.

### A. Stability Analysis

Stability analysis of the boundary-layer flow over the wing is performed in a nonorthogonal, curvilinear coordinate system as implemented in the LASTRAC.3D code [48]. The above coordinate system is chosen to align one of the surface coordinates with an approximate streamwise direction and the other surface coordinates along the (approximately) spanwise grid lines that are aligned with the leading and trailing edges in the spanwise direction. The remaining 3<sup>rd</sup> coordinate is defined to be along the wall normal direction. Results for boundary layers over infinite-swept-wing configurations have been shown to be identical between the orthogonal and nonorthogonal coordinate systems [48]. In conventional quasiparallel LST, the boundary layer is assumed to be locally parallel by dropping the streamwise derivative terms and setting the wall-normal velocity derivative equal to zero. However, we use the PSE approach, wherein the effects of the streamwise and spanwise curvatures and the nonparallel effects are also considered.

The PSE approximation is based on isolating the rapid phase variations in the streamwise direction from the slow evolution of the disturbance mode shape due to weak effects of mean-flow nonparallelism and the surface curvature. The effects of instability wave propagation within a fully three-dimensional boundary layer are evaluated via a parabolic integration along suitable trajectories such as inviscid streamlines based on the mean flow, or group velocity lines associated with the disturbance of interest. The variables  $(\xi, \eta, \zeta)$  denote the streamwise, wall-normal, and spanwise directions in the nonorthogonal coordinates and  $(u, v, w)$  represent the corresponding velocity components. Density and temperature are denoted by  $\rho$  and  $T$ . The Cartesian coordinates are denoted as  $(x, y, z)$ , respectively. The vector of the basic state fluid variables corresponds to  $\bar{\mathbf{q}}(\xi, \eta, \zeta) = (\bar{\rho}, \bar{u}, \bar{v}, \bar{w}, \bar{T})^T$ . The perturbations to the basic state have the form

$$\tilde{\mathbf{q}}(\xi, \eta, \zeta, t) = \hat{\mathbf{q}}(\xi, \eta) \exp \left[ i \left( \int_{\xi_0}^{\xi} \alpha(\xi') d\xi' + \beta \zeta - \omega t \right) \right], \quad (1)$$

where the perturbation vector is  $\tilde{\mathbf{q}}(\xi, \eta, \zeta, t) = (\tilde{\rho}, \tilde{u}, \tilde{v}, \tilde{w}, \tilde{T})^T$  and  $\hat{\mathbf{q}}(\xi, \eta, \zeta) = (\hat{\rho}, \hat{u}, \hat{v}, \hat{w}, \hat{T})^T$  denotes the vector of amplitude functions. The streamwise and spanwise wave numbers are  $\alpha$  and  $\beta$ , respectively, and  $\omega$  is the angular frequency of the perturbation. The spanwise wavelength is defined as  $\lambda = 2\pi/\beta$ . The unknown, streamwise-varying wavenumber  $\alpha(\xi)$  is determined in the course of the solution by imposing a normalization condition. This condition makes the decomposition in Eq. (1) unique, by imposing a slow variation of the amplitude functions  $\hat{\mathbf{q}}(\xi, \eta)$  in the streamwise direction and absorbing all the rapid variations in the phase term  $\exp \left[ i \int_{\xi_0}^{\xi} \alpha(\xi') d\xi' \right]$ . Substituting Eq. (1) into the linearized Navier-Stokes equations and invoking the scale separation to neglect the higher-order terms involving a suitable subset of the viscous and/or streamwise derivative terms, one obtains the PSE in the form

$$\left( \mathbf{L} + \mathbf{M} \frac{\partial}{\partial \xi} \right) \hat{\mathbf{q}}(\xi, \eta) = 0. \quad (2)$$

The entries of the coefficient matrices for  $\mathbf{L}$  and  $\mathbf{M}$  with a more detailed description of the method can be found in the Refs. [4, 48, 49]. In this work, the PSE marching is performed along preidentified streamline paths along the surface. More details on this approach can be found in Ref. [48]. The nonparallel growth rate of the disturbances along a streamline for a selected pair of  $\omega$  and  $\beta$  is defined as

$$\sigma_{\text{PSE}}(\xi, \omega, \beta) = -\Im(\alpha) + \frac{1}{2} \frac{d\hat{\mathbf{K}}}{d\xi}, \quad (3)$$

where  $\hat{\mathbf{K}}$  denotes the norm of the disturbance kinetic energy.

### 1. The Dual $N$ -factor Method

The onset of laminar-turbulent transition is estimated using the logarithmic amplification ratio, the so-called  $N$ -factor, relative to the lower neutral branch location  $\xi_{lb}$  where the disturbance first becomes unstable,

$$N(\omega, \beta) = - \int_{\xi_{lb}}^{\xi} \sigma(\xi', \omega, \beta) d\xi'. \quad (4)$$

Here, we assume that transition onset is likely to occur when a function of the envelope  $N$ -factors based on stationary crossflow instabilities ( $N_{CF}$ ) and the Tollmien-Schlichting instability waves ( $N_{TS}$ ) reaches a certain threshold [37,50–51]. This dual  $N_{TS} - N_{CF}$  criterion can be written as

$$\left( \frac{N_{TS}}{N_{TS,c}} \right)^{a_{TS}} + \left( \frac{N_{CF}}{N_{CF,c}} \right)^{a_{CF}} = 1, \quad (5)$$

where the subscript  $c$  refers to the critical value, and the exponents  $a_{TS}$  and  $a_{CF}$  control the nature of interaction between the two types of instability waves.

## IV. Compressibility-related Modifications to the LM2009/2015 and AFT2017b Models

In this section, we describe how the two selected transition models LM2015 and AFT2017b have been modified to account for compressibility effects.

### A. Langtry-Menter Model

In the case of the Langtry-Menter model, Kaynak and Gurdamar [52] had first identified the need for incorporating compressibility effects into this model and did so by using the approach of Narasimha [53] to multiply the incompressible transition-onset momentum thickness Reynolds number with a correction factor involving the boundary-layer edge Mach number. Pecnick et al. [54] investigated the uncertainty due to compressibility effects on a transonic turbine guide vane under high turbulence intensity and found huge influence on uncertainty due to compressibility effects. Zhang and Gao [55] introduced a compressibility correction into the model, based on some wind tunnel data at hypersonic Mach numbers. However, the focus on hypersonic flow conditions requires separate considerations regarding the transition physics. Several researchers [56,57] have adopted the path of replacing the Abu-Ghannam and Shaw [58] type correlation for transition-onset momentum-thickness Reynolds number,  $Re_{\theta t}$ , with the Arnal, Habiballah, and Delcourt (AHD) criterion [59] that has been extended up to a Mach number of 4. Such an approach will render the model incapable of handling bypass scenarios, routinely seen in turbomachinery applications where this model finds wide use, as the AHD criterion is based on data from stability analysis. Therefore, some of the other correlations used within the model may also need to be recalibrated. Additionally, this approach requires information about the spatial history of the pressure gradient parameter. Such nonlocal terms are difficult to implement in unstructured flow solvers used in massively parallel settings. To overcome this issue of nonlocality, Str  er et al. [56] and Pascal et al. [57] have introduced additional transport equations to compute the spatial average required by the AHD criterion via a purely local formulation. This approach incurs additional cost and increased complexity due to additional boundary conditions and numerical stiffness.

In this work, we adopt a simpler modification that enables the Langtry-Menter model to return to its original behavior at low subsonic speeds and also retain its ability to handle bypass transition scenarios. The original form of the Langtry-Menter model [18] includes two transport equations, one for intermittency variable,  $\gamma$ , and another for a local transition onset momentum-thickness Reynolds number,  $\overline{Re_{\theta t}}$ . Given that this model relies on several correlations, the key elements that were modified to account for compressibility effects are the following: (i) the relation between the vorticity Reynolds number,  $Re_v$ , and the momentum thickness Reynolds number,  $Re_{\theta}$ ; (ii) the empirical correlation used to determine the transition onset momentum-thickness Reynolds number,  $Re_{\theta t}$ ; and (iii) the empirical correlation  $F_{length}$  that controls the length of the transition region and appears in the production term of the  $\gamma$  – equation. While an increase in the Mach number is expected to increase the transition length [60], we currently choose to not modify the original correlation for this term because of a lack of reliable data concerning the transition zone, and instead focus on the first two relations. Given previous findings [61,62] that indicate the compressibility correction

to crossflow instability to be small, at least up to the transonic regime, we do not modify the stationary crossflow terms included in the LM2015 model.

### 1. Relationship between $Re_v$ and $Re_\theta$

Borrowing from the work of van Driest and Blumer [63], the Langtry-Menter model uses the vorticity Reynolds number,

$$Re_v = \frac{\rho d^2}{\mu} \left| \frac{\partial u}{\partial y} \right| = \frac{\rho y^2}{\mu} S, \quad (6)$$

as a surrogate for estimating the momentum thickness Reynolds number,  $Re_\theta$ , through the relation based on the Blasius solution for the zero pressure gradient boundary layer:

$$Re_\theta = \frac{\max(Re_v)}{2.193}. \quad (7)$$

In general, the ratio between the maximum of vorticity Reynolds number and the momentum-thickness Reynolds number is a function of the pressure gradient as well as the compressibility effects. With an increase in the Mach number, the peak location of the vorticity Reynolds number moves toward the edge of the boundary layer [63]. Given that the original model is calibrated using data that includes pressure gradient effects, it seems unnecessary to include any additional corrections to account for the impact of pressure gradients in compressible flows [64]; only the effect of the Mach number can be included as a first-order correction for  $O(1)$  Mach numbers, which can be accomplished by using compressible boundary-layer profiles obtained with a compressible boundary-layer code. Such corrections were computed in the works of Fehrs [65] and Qiao et al. [66] and we use the correlation derived by Qiao et al. [66] in this work, which is given by:

$$Re_\theta = \frac{\max(Re_v)}{2.193 * C(M_e)} \quad (8)$$

$$C(M_e) = 1.0 - 0.06124M_e + 0.2402(M_e)^2 - 0.00346(M_e)^3 \quad (9)$$

with  $M_e$  being the local edge Mach number. The procedure to obtain  $M_e$  will be given in a subsequent section.

### 2. Correlation for $Re_{\theta t}$

The original correlation for  $Re_{\theta t}$  in the Langtry-Menter model was expressed as a function of the pressure-gradient parameter and turbulence intensity alone and was calibrated to account for both natural and bypass transition scenarios. Its predictions were similar to those of the Abu-Ghannam and Shaw correlation [58] for flows with nonzero pressure gradients. However, we know from the studies of Mack [39] and Narasimha [53] that compressibility enhances the stability of the boundary layer up to approximately  $M_\infty = 2.0$ , thereby delaying transition and making  $Re_{\theta t}$  a function of the edge Mach number. Our modification was based upon the idea that the Mach number correction must retain the original structure to remain valid for the test cases used during the calibration as well as the validation of the model developed for low-speed flows. This was achieved in Ref. [55] by borrowing an idea from Narasimha [53] that the Mach number effects could be incorporated by applying a multiplicative correction factor,  $f(M_e)$ , to the incompressible estimate for  $Re_{\theta t}$ . However, as the correction functions suggested by Narasimha were based on a limited set of experiments, we have chosen to derive our own correction factor on the basis of the stability computations by Masad and Malik [67] and Masad and Abid [68], who investigated the combined effects of pressure gradient, suction/blowing at the surface, and wall cooling on boundary-layer transition over a flat plate at subsonic through supersonic Mach numbers. Given that the original formulation for  $Re_{\theta t}$  in the Langtry-Menter model,  $(Re_{\theta t})_{\text{Original}}$ , was defined to be a product of separate functions of  $Tu$  and  $\lambda_\theta$  (see Eqs. 35-38 in Ref. [18]), we decided to keep the modification purely a function of the edge Mach number as a first step in our quest to incorporate compressibility effects into the model. This was achieved by focusing only on zero pressure gradient boundary layers involving adiabatic wall boundary conditions without any suction or blowing, and then determining a polynomial curve-fit for the desired Mach correction factor, using the data available from

their work on the influence of Mach number on the transition Reynolds number. The polynomial curve fit corresponds to

$$f(M_e) = 1.0105 - 0.3046M_e + 1.1646(M_e)^2 - 0.3605(M_e)^3 \quad (10)$$

As previously mentioned, the coefficient  $f(M_e)$  is used to correct the value of  $Re_{\theta t}$  from the original model according to

$$(Re_{\theta t})_{New} = (Re_{\theta t})_{Original} \cdot f(M_e) \quad (11)$$

In the future, we hope to achieve a more general correction so that the pressure gradient influence on the compressibility-correction can be accurately accounted for. However, the present restricted modification can still be applied to the CRM-NLF configurations, as it has nearly flat roof-top pressure distribution across the majority of the span. Figure 1 shows how the current modification for  $Re_{\theta t}$  compares against the original correlation and other similar modifications available in the literature for a zero-pressure gradient flow with varying Mach number and a  $Tu = 0.07\%$ , clearly showing the stabilizing effect of compressibility. As expected, the current correlation closely matches the AHD criterion for this low- $Tu$  case, as both correlations are based on stability data. We have chosen to use the data from Masad and Malik [67], because it enables the effects of wall temperature, i.e., cooling/heating to be included into the correlation if and when necessary. Figure 1 also indicates the substantial underprediction of the transition Reynolds number by the original LM2009 model when the Mach number reaches the high subsonic through transonic range. The same is also true of the Narasimha-Dey correlation, albeit to a lesser extent. On the other hand, the Zhang correlation seems to overpredict the compressibility effect until the Mach number becomes supersonic.

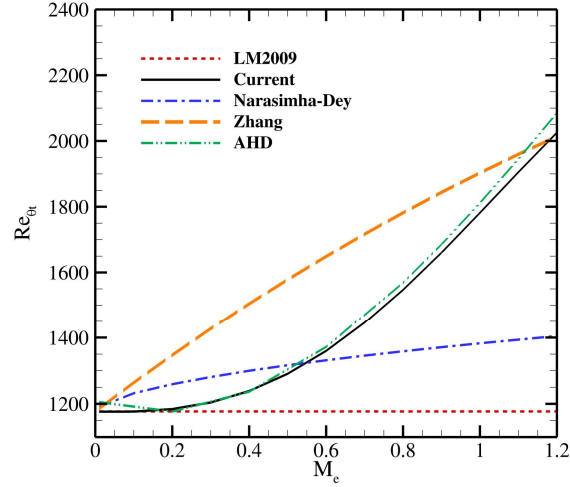


**Fig. 1. Variation of transition-onset momentum-thickness Reynolds number with Mach number based on various correlations.**

## B. The Amplification Factor Transport Equation Model

Sturdza [69] had proposed alternate corrections to account for compressibility effects, including the switchover in the dominant instability modes from two-dimensional TS waves at low Mach numbers to oblique instability waves at supersonic speeds. He developed corrections to the  $N$ -factor correlations of Drela and Giles [5] by using the results of linear stability analysis of compressible Falkner-Skan boundary-layer profiles and several biconvex airfoils at freestream Mach numbers between  $1.5 < M < 2.4$ . Given that the AFT model is based on the Drela and Giles correlations [5], we implemented Sturdza's correlations into the SA-AFT2017b model in this work. Sturdza's modifications targeted two correlations, namely, the correlation for the momentum-thickness Reynolds number at the onset of instability and that for the slope of the  $N$ -factor envelope. The momentum-thickness Reynolds number at the critical point (i.e., at the onset of TS instability) in Sturdza's work was expressed as:

$$\log_{10} \left( \frac{Re_{\theta 0}}{K_0} \right) = \left( \frac{1.415}{H_k - 1} - 0.489 \right) \tanh \left( \frac{20}{H_k - 1} - 12.9 \right) + \frac{3.295}{H_k - 1} + 0.44 , \quad (12)$$

where the value of  $K_0$  was changed from unity in the original AFT model to

$$K_0 = \frac{2}{\pi} \operatorname{atan} \left( 10 \frac{T_w}{T_e} - 10 \right) + 1 . \quad (13)$$

The next modification involved the correlation for the slope of the  $N$ -factor envelope. Whereas the original AFT model was based on the exact correlation proposed by Drela and Giles [5], the newer versions such as AFT2017b [23] use a modified version of the Drela and Giles correlation. As it is not evident from the literature [23] what additional data were used to arrive at the modified version, we will adopt the correlation by Sturdza [69], as this approach guarantees that the model will revert to the original Drela and Giles correlation at low subsonic speeds. The form for the slope of the  $N$ -factor envelope as given by Sturdza is:

$$\frac{dN}{dRe_{\theta}} = \frac{0.01}{K_b} \left( 2.4 H_k K_a - 3.7 + 2.5 \left( \frac{T_w}{T_e} \right) \tanh[1.5(H_k - 3.1)] + 0.125 + K_c - K_d \right) \quad (14)$$

$$K_a = 1 + 0.2(H_k - 2.5918) \left( 1 - \frac{T_e}{T_w} \right); K_b = 4.7 \left( \frac{T_w}{T_e} - 1 \right); K_c = 1.2 \left( \frac{T_w}{T_e} - 1 \right)^{1.5}; K_d = 0. \quad (15)$$

The terms  $K_a$ ,  $K_b$ ,  $K_c$ ,  $K_d$ , and the wall temperature ratio  $T_w/T_e$ , seen in Eq. (14) did not appear in the original correlation by Drela and Giles.  $T_w/T_e$  for adiabatic thermal boundary conditions can be estimated by using the local edge Mach number  $M_e$ , which, in turn, can be determined from the local pressure via the compressible form of the Bernoulli equation (see subsection IV.C below). However, this approach for estimating the wall temperature ratio is likely to fail in regions involving a shock, where an alternate approach would become necessary. The term  $K_d$  was introduced by Sturdza [69] to incorporate history effects that are necessary in regions of linear pressure gradients. However, in this work, we set it to 0 in an ad-hoc manner, as for accurately incorporating history effects in the manner originally proposed by Sturdza requires one to bookkeep a running average of the kinematic shape-factor,  $H_k$ , from the critical point. Such nonlocal terms are difficult to implement in unstructured flow solvers used in massively parallel settings, and therefore, it is desirable for the transition models to use purely local formulations. However, the limitation of such an ad-hoc approach needs to be evaluated via application to a range of configurations. Here, we take an initial step in that direction by assessing the overall model against measurements for the CRM-NLF configuration, in the expectation that, the region starting downstream of the suction peak up to the shock exhibits a weak pressure gradient, across the span of the wing, possibly making the results relatively insensitive to pressure history effects.

### C. Estimation of edge Mach number $M_e$

One of the key parameters necessary for the modification of both the transition models to account for compressibility effects is the local edge Mach number  $M_e = \frac{u_e}{a_e}$ , which can be easily extracted in a boundary-layer code. However, in an unstructured RANS-based CFD solver, the determination of the edge Mach number becomes rather nontrivial if the formulation is required to be strictly local. One possible resolution is to use Bernoulli's equation in conjunction with the isentropic relations:

$$|u_e| = \sqrt{u_{\infty}^2 + \frac{2\tilde{\gamma}}{\tilde{\gamma}-1} \left[ 1 - \left( \frac{p}{p_{\infty}} \right)^{1-1/\tilde{\gamma}} \right] \frac{p}{\rho_{\infty}}} \quad (16)$$

$$a_e = \sqrt{\tilde{\gamma} R T_{\infty} \left( \frac{p}{p_{\infty}} \right)^{\frac{\tilde{\gamma}-1}{\tilde{\gamma}}}} . \quad (17)$$

However, this approach is likely to fail in regions involving a shock, as the isentropic relations are no longer valid in that case. In transonic applications, even when the flow develops shocks following a region of local

supersonic flow, the deviation from the isentropic relations is not expected to be significant if the local Mach number does not exceed approximately 1.2 [56,70]. That restriction should be met in the majority of transonic cruise conditions we are interested in. Therefore, we will use Eqs. (16) and (17) to evaluate the compressibility corrections.

## V. Flow Conditions and Computational Grids

The flow conditions investigated as part of this work, listed under Table 1, are taken from the problem statement specified by the AIAA Transition Modeling and Prediction workshop organizers.<sup>§§</sup> This problem statement prescribes the angle of attack instead of fixing the lift coefficient based on the experimentally measured value. The freestream turbulence intensity (FSTI) in the experiment was expected to be 0.24%, based on previous characterization of the NTF tunnel [71]. The surface roughness of the model was specified to be  $2.28 \times 10^{-5}$  mm (0.9  $\mu$ inch) [34].

**Table 1 Flow Conditions.**

| Condition | AOA (deg.)        | Mach No.          | $T_\infty$ (K) | Re (per m) | Re <sub>MAC</sub> |
|-----------|-------------------|-------------------|----------------|------------|-------------------|
| 1         | $1.448 \pm 0.001$ | $0.856 \pm 0.001$ | 241.7          | 40.73M     | 14.84M            |
| 2         | $1.980 \pm 0.001$ | $0.856 \pm 0.001$ | 242.0          | 40.66M     | 14.81M            |
| 3         | $2.461 \pm 0.001$ | $0.856 \pm 0.001$ | 242.0          | 40.56M     | 14.78M            |
| 4         | $2.938 \pm 0.001$ | $0.856 \pm 0.001$ | 243.0          | 40.41M     | 14.72M            |

The mesh for the semispan geometry used in these computations had seven overset near-body blocks (three on the fuselage including its nose and tail, one wing-body collar grid, and three on the wing, including its tip). The generation of off-body grids and hole cutting were carried out by using OVERFLOW's domain connectivity function (DCF) approach. The near-body grids over the wing and the fuselage had 201 points in the wall-normal direction with a near-wall spacing of  $2.54 \times 10^{-3}$  mm ( $1 \times 10^{-5}$  inches) ( $y^+ \sim 0.5$  based on quarter MAC), a growth rate of 1.02, and 10 of the initial grid points near the wall were uniformly spaced. The mesh contained 651 points around the wing in the chordwise direction and 401 points in the spanwise direction. The near-body grid had an overall grid count of approximately 120 million points and extended outward from the body by a distance of approximately 1 mean aerodynamic chord, with the bulk of the grid points focused within a distance of two boundary-layer thicknesses (estimated at the midchord location at selected spanwise locations). The far-field boundaries of the outer-body grids were placed at approximately 100 semispans, i.e., 419 times the mean aerodynamic chord. This mesh corresponded to the mesh with the finest grid resolution in our previous study [36]. As part of the previous study [36], grids with three different resolutions (same topology; but the grid refinement was not done in a uniform manner) were generated in-house and used along with the fine grid provided by the AIAA Transition Modeling and Prediction Workshop committee. Comparison of sectional pressure coefficients and the transition front on the upper surface of the wing against measured data was used to show that the solutions from the different transition models were reasonably grid converged and that the fine grid should be sufficient for carrying out the assessment of the transition models. An overview of the mesh block topology, and the general grid evolution in the wall-normal direction in the wing section are provided in Fig. 2. Adiabatic wall, symmetry, and freestream boundary conditions were used for all computations.

## VI. Results from the Models with Compressibility-Correction

The solutions presented here were obtained by running the flow solver in a steady-state manner by using the 3<sup>rd</sup>-order Roe upwind scheme [72] and the unfactored, successive symmetric overrelaxation (SSOR) implicit solution algorithm [73]. In the experiment, boundary layer on the fuselage was tripped at a location downstream of the nose (approximately 1.5% of the fuselage length) and the lower surface of the wing was tripped at approximately 5% local chord (see Ref. for details). However, because of the way the transition models are implemented in OVERFLOW, activating the transition model may override the trips. Therefore, in this work, the computations are performed assuming free-transition without any trips everywhere.

<sup>§§</sup>[https://transitionmodeling.larc.nasa.gov/wp-content/uploads/sites/109/2020/02/TransitionMPW\\_CaseDescriptions.pdf](https://transitionmodeling.larc.nasa.gov/wp-content/uploads/sites/109/2020/02/TransitionMPW_CaseDescriptions.pdf)

The computational results are evaluated against the measured data by using two separate metrics, namely, surface pressure variation at different spanwise locations as indicated in Fig. 3 and the predicted transition front on the upper wing surface against the transition front inferred from the experimental measurements.

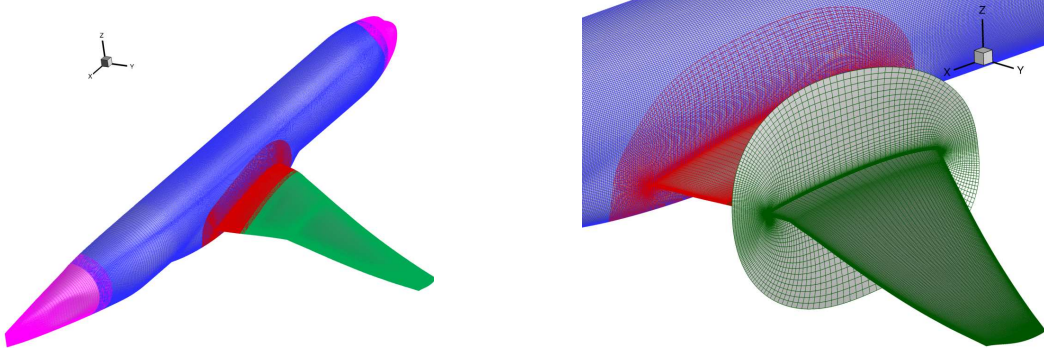


Fig. 2. Overview of the near-body mesh.

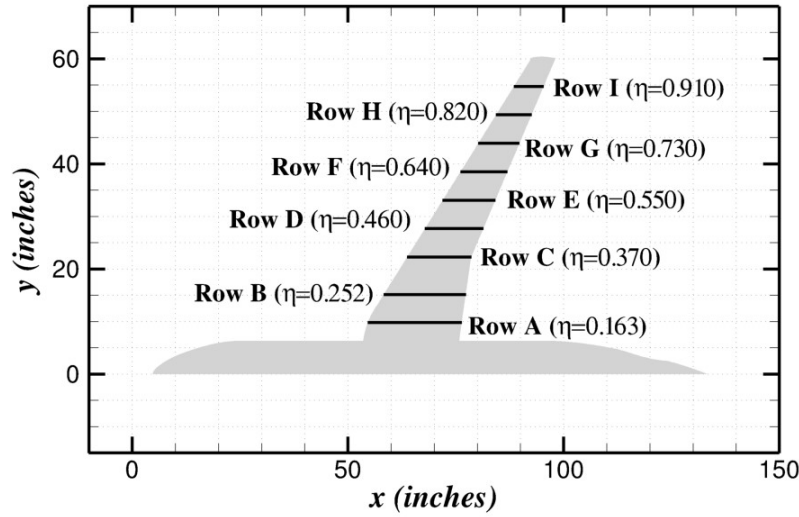


Fig. 3. Spanwise stations where pressure data were available from experiments.

With FSTI being a critical parameter influencing the boundary-layer transition characteristics, specifying it accurately in the computation such that the FSTI value near the leading edge of the wing corresponds to that measured in the experiment is an important prerequisite to meaningful transition predictions [74]. This is even more important for the SST-2003RC-LM2009/2015 models that exhibit a large sensitivity to the FSTI parameter. One approach for the SST-based Langtry-Menter series of transition models is to specify a sufficiently high inlet value for the FSTI and allow it to decay (according to the specified value of  $(\mu_t/\mu)_{\text{inlet}}$ ), such that the FSTI evolves to the correct value in the near-field region. However, given the size of the far-field domain in this problem, extremely large values of the eddy viscosity ratio were required at the inlet ( $(\mu_t/\mu)_{\text{inlet}} > 100$ ) to achieve the expected FSTI of 0.24 % in the near field. Langtry [75] had advised against using such large values of  $(\mu_t/\mu)_{\text{inlet}}$  because of the resulting errors in predicted skin-friction coefficients. Thus, we adopt the alternate approach of sustaining the desired FSTI level in the near field by adding source terms to the SST model, as proposed by Spalart and Rumsey [76]. To that end, we had initially set  $\text{FSTI} = 0.24\%$  at the inlet, along with a typical value for the inlet viscosity ratio,  $(\mu_t/\mu)_{\text{inlet}} = 1$  [36].

However, Venkatachari et al. [36] had found that this choice led to the SST-based LM model predicting incorrect surface pressure distributions (due to how the sustaining terms influenced the behavior of the model within the boundary layer) and, hence, to an incorrect prediction of the laminar extent. This discrepancy in the surface pressure distributions vanished when the FSTI level was reduced to 50% or lower of the original desired value. The SA-based AFT model is insensitive to the inlet FSTI parameter, as the latter is used only to set the critical  $N$ -factor value needed for the determination of transition onset. Thus, to have the correct surface pressure distribution for both the SST-based Langtry Menter models and the AFT models and, hence, to enable a direct comparison between the two groups of models, computations with both models are carried out using  $FSTI=0.12\%$ .

#### A. Computations for $\alpha = 1.44^\circ$

Surface contours of pressure gradient along the  $x$ -direction obtained using the SA-based, baseline AFT model (i.e., without any compressibility corrections) is shown in Fig. 4 along with the inferred transition front from the experiment. The computations indicate a dual shock system, with the first shock extending from the root of the wing up to 70% of the span, and a second shock in the outboard region of the wing, extending up to the wing-tip. The inferred transition front appears to be shock-limited in the outboard region of the wing, jumping from the first shock to the second one following the termination of the first shock. In the region from the wing root to slightly outboard of the Yehudi break, the transition occurs significantly upstream of the shock, and within a region that experiences a weak adverse pressure gradient according to the measured surface pressure data. The sectional pressure distributions obtained by using the baseline and compressibility-corrected models are shown in Figs. 5 and 6, respectively. Similar to previous computations [36], the results in Fig. 5 clearly show that a reduction in the FSTI level enables the pressure distribution as predicted by the SST-2003RC-based LM model to closely match the measured pressured data across the entire wingspan. From the standpoint of transition prediction, we believe that a correct baseflow computation is more important than matching the experimentally measured FSTI. Based on Figs. 5(d) and (e), the baseline model predicts a slightly upstream location of the first shock measured along rows E and F, respectively, but this aspect appears to be improved with the incorporation of the compressibility correction. Apart from this change, the modification to account for the compressibility effects does not have any additional influence on the predicted pressure distribution. Similarly, from Fig. 6, one can infer that the SA-AFT2017b model with or without the compressibility correction predicts a good match in pressure distribution with the measured data across all measurement rows. Overall, the models are accurately capturing the suction peak levels, the rapid acceleration near the leading edge, the mild adverse pressure gradient in the midchord section of the upper surface, and the shock location(s).

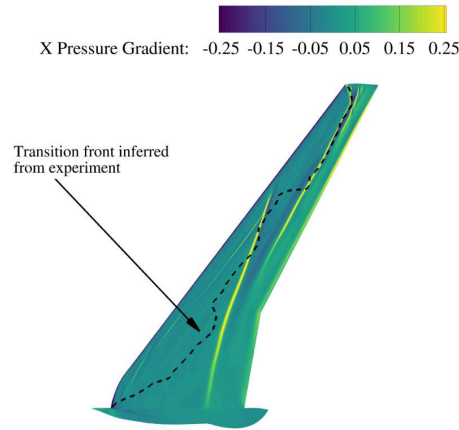


**Fig. 4. Surface contours of  $x$ -pressure gradient superimposed with inferred transition front from the experiment for  $\alpha = 1.44^\circ$ ,  $M = 0.856$ , and  $Re_{MAC} = 15 \times 10^6$ .**

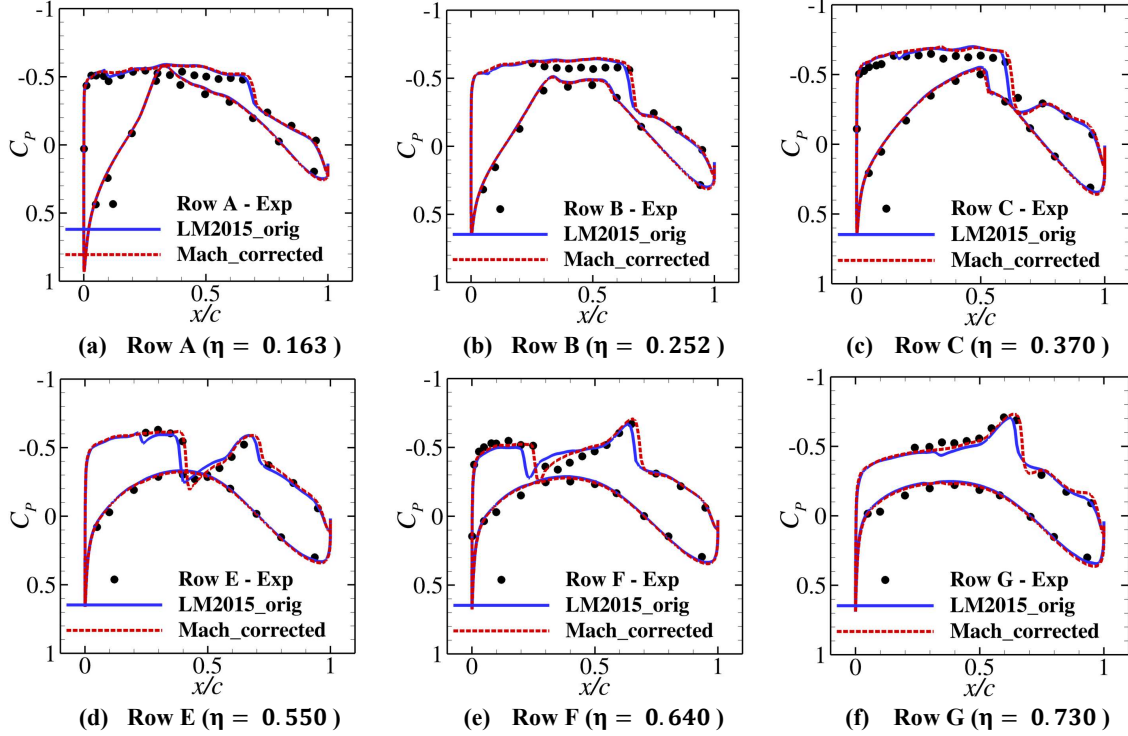


**Fig. 5. Comparison of measured and computed pressure distributions at various stations on the CRM-NLF wing. Results are shown for both the original and the compressibility-corrected versions of the SST-2003RC-LM2015 transition model for  $\alpha = 1.44^\circ$ ,  $M = 0.856$ ,  $FSTI = 0.12\%$ , and  $Re_{MAC} = 15 \times 10^6$ .**

It was observed in the previous study [36] that a good match of the pressure distribution alone does not automatically translate into good prediction of the transition front by these models. Therefore, to evaluate the performance of these models, Figs. 7(a) and (b) overlay the predicted transition fronts based on both LM2015 and AFT models onto the temperature sensitive paint (TSP) images of the upper surface of the wing as obtained from the experiment [34, 35]. The predicted transition fronts have been extracted by using turbulence index contours. The turbulence index is an indicator for transition in the CFD computations, the definition for which was originally proposed by Spalart [45]. The definition of the index differs between the SA [45] and the SST [77] models. In SA-based transition models, that value always lies between 0 and 1, while for SST-based transition models, its value may exceed unity in the fully turbulent regions (values around 2.5 were seen for the computations shown herein). A turbulence index value of 0.95 (value of 0 indicating laminar regions) and above appears to reasonably indicate the location of transition for this configuration (as it coincides with the location around which the skin friction distribution jumps from its laminar value to the turbulent level).

Figures 7(a) and (b) demonstrate that the baseline models underpredict the laminar extent of the flow across the entire span of the wing, always predicting a significantly earlier onset of transition in comparison with the transition front inferred from the experiment. According to the boundary-layer stability analysis study on the CRM-NLF by Lynde et al. [35] and Paredes et al. [37], large TS  $N$ -factors were seen in the inboard portion of the wing (up to the break) and in the farthest outboard region of the wing, while relatively large crossflow  $N$ -factors were also observed on either side of the break and in most of the outboard sections of the wing. In the farthest outboard region of the wing alone, where the transition front appears to be shock limited (see Fig. 4), the unmodified AFT2017b model performs slightly better than the LM2015 model. Accounting for crossflow effects by the LM2015 model could be causing it to trigger an earlier transition in this region, as stability analysis indicated to large CF  $N$ -factors in this region. But the fact the predicted laminar extent is small even in the regions dominated by TS effects is a clear indication of the inability of the models to account for the stabilizing effect of compressibility.

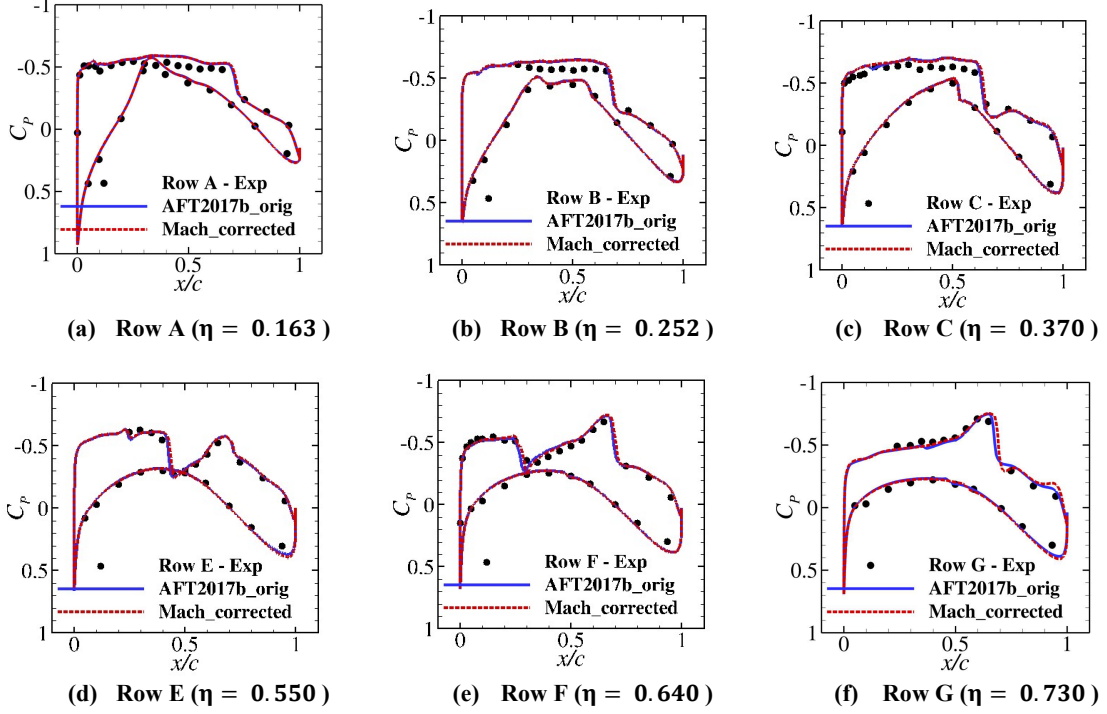


**Fig. 6.** Comparison of measured and computed pressure distributions at various stations on the CRM-NLF wing. Results are shown for both the original and the compressibility-corrected versions of the SA-RC-AFT2017b transition model for  $\alpha = 1.44^\circ$ ,  $M = 0.856$ ,  $FSTI = 0.12\%$ , and  $Re_{MAC} = 15 \times 10^6$ .

In the case of the LM2015 model, the result obtained by incorporating the compressibility correction clearly results in a larger laminar flow extent and leads to a better match with the experiment, as seen from Fig. 7(a). Apart from the region between the wing root and the break, the results from the modified LM2015 model compares very well with the inferred front from the experiment. Even in the inboard section there is substantial improvement when compared to the baseline model and as shown in the subsequent section, the result in this region compares well with the predictions based on the stability analysis. In the case of the modified AFT model, there is an excellent agreement in the shock-dominated region ( $y \geq 1.1 m$ ), but the improvements are not significant enough elsewhere, as seen from Fig. 7(b). One possible reason could be that this model does not account for crossflow and crossflow effects do play a substantial role in the region on either side of the break. It is also not clear as to what was causing the zig-zag front in the inboard portion, as the surface pressure distribution does not display such variations.

The predicted force and moment coefficients and their comparison with the measured values are given in Table 2. In general, one sees smaller drag coefficients and large lift and moment coefficients being predicted by the different models (apart from baseline LM2015) when compared to those from the experiment. One possible reason could be that in the experiment, the fuselage and the lower portion of the wing surface were tripped but not in the computations, resulting in a different laminar extent for the entire geometry. However, one can clearly see an increase in lift and pitching moments, along with a reduction in the drag coefficient with the addition of compressibility correction into the models. Also, in general, the force and moment coefficients from the AFT models are higher than that from the LM2015 models.

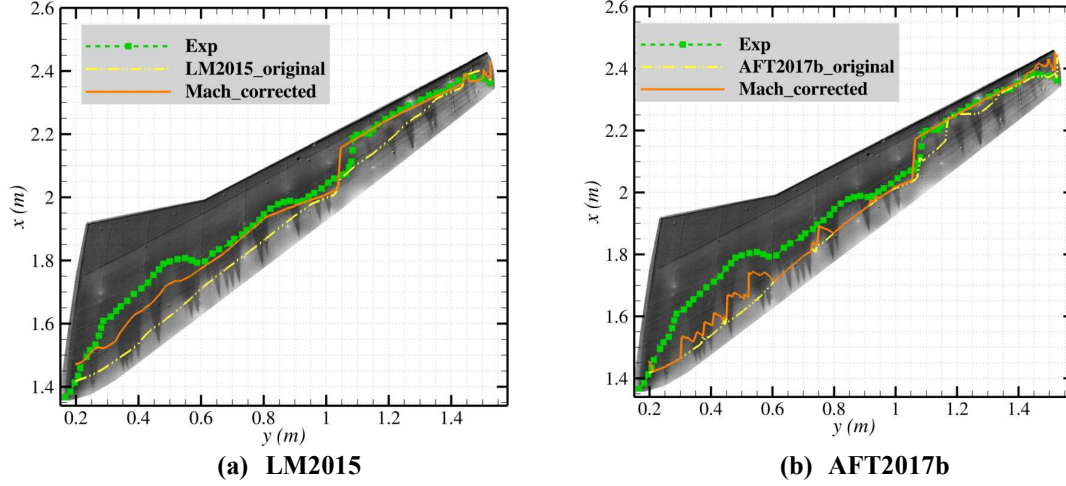


**Fig. 7.** Transition fronts as predicted by the original and the compressibility-corrected versions of the different transition models overlaid on a TSP image obtained from the experiment for  $\alpha = 1.44^\circ$ ,  $M = 0.856$ ,  $FSTI=0.12\%$ , and  $Re_{MAC} = 15 \times 10^6$ .

**Table 2 Force and Moment Coefficients.**

|                                    | $C_L$ | $C_D$  | $C_m$  |
|------------------------------------|-------|--------|--------|
| <b>Exp</b>                         | 0.312 | 0.0174 | -0.083 |
| <b>SST2003 – LM2015 (Original)</b> | 0.307 | 0.0160 | -0.071 |
| <b>LM2015 (Mach corrected)</b>     | 0.325 | 0.0158 | -0.082 |
| <b>SA-AFT2017b (Original)</b>      | 0.337 | 0.0165 | -0.097 |
| <b>AFT (Mach corrected)</b>        | 0.345 | 0.0161 | -0.103 |

## B. Computations for $\alpha = 1.98^\circ$

Figure 8 shows the surface contours of pressure gradient along the  $x$ -direction obtained using the SA-based baseline AFT along with the inferred transition front from the experiment. Similar to the observations for  $\alpha = 1.44^\circ$ , the computations here also indicate the presence of a dual shock system and that the transition front inferred from the experiment is shock-limited in the outboard region of the wing. However, unlike at  $\alpha = 1.44^\circ$ , the transition front appears to be shock-limited over a larger region across the wing span, starting at a small distance from the break and ending near the wing-tip. In the region from the root of the wing to slightly outboard of the break, transition onset occurs much upstream of the shock and the laminar extent appears to be smaller than that in the  $\alpha = 1.44^\circ$  case.

The sectional pressure distributions obtained with both the baseline and the compressibility-corrected models are shown in Figs. 9 and 10, respectively. Once again, a good match between the pressure coefficients predicted by both the baseline and the modified models, and the measured values is seen along the inboard rows A, B, and C. As one moves outboard, small differences between the baseline and the modified versions of the model, as well as between the LM2015 and AFT2017b models start to appear, especially for the shock location on the upper surface of the wing. In the case of the LM2015 model, the compressibility corrections appear to indicate a better match with the experiment as seen from Figs. 9(d)–(f), while the baseline model had predicted the shock to be slightly upstream in the outboard sections of the wing. In the case of the AFT model, the baseline model resulted in an exact match of the shock location, while the compressibility correction pushed the shock further downstream, as indicated by Figs. 10(d)–(f).

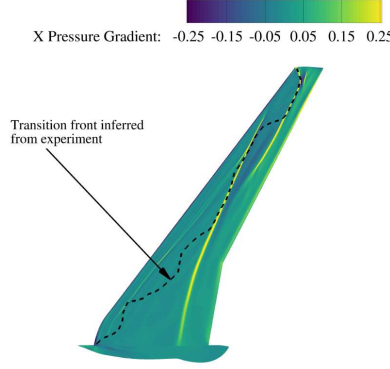


Fig. 8. Surface contours of x-pressure gradient superimposed with the transition front inferred from the experiment for  $\alpha = 1.98^\circ$ ,  $M = 0.856$ , and  $Re_{MAC} = 15 \times 10^6$ .

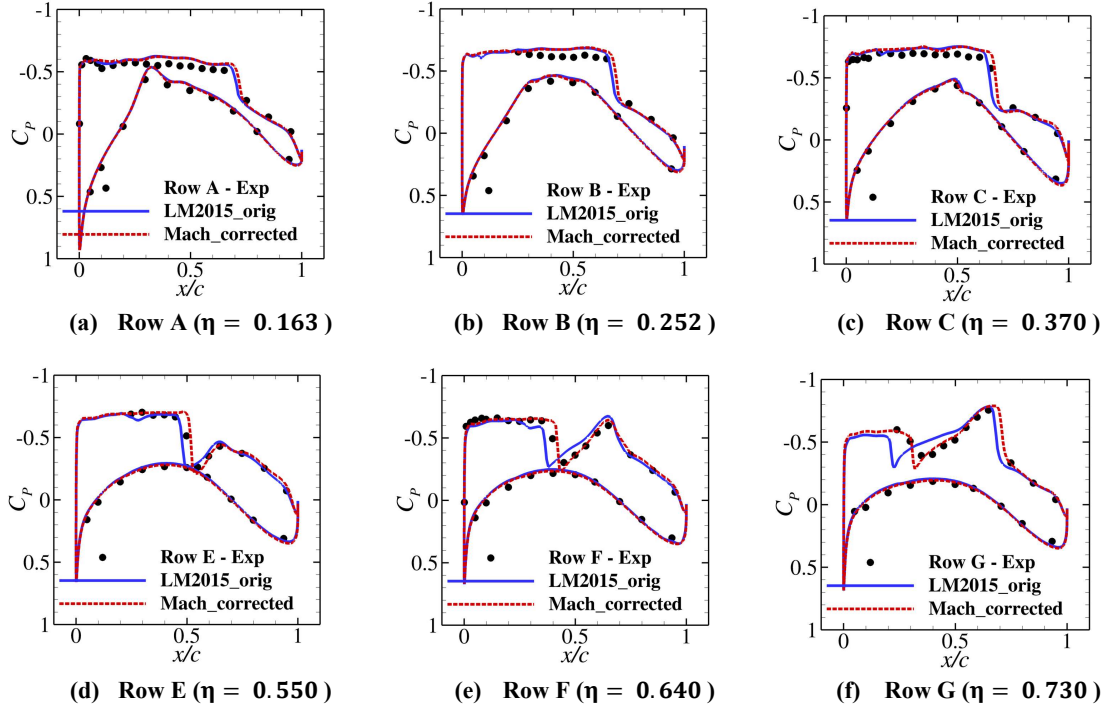


Fig. 9. Comparison of measured and computed pressure distributions at various stations on the CRM-NLF wing. Results are shown for both the original and the compressibility-corrected versions of the SST-2003RC-LM2015 transition model for  $\alpha = 1.98^\circ$ ,  $M = 0.856$ ,  $FSTI = 0.12\%$ , and  $Re_{MAC} = 15 \times 10^6$ .

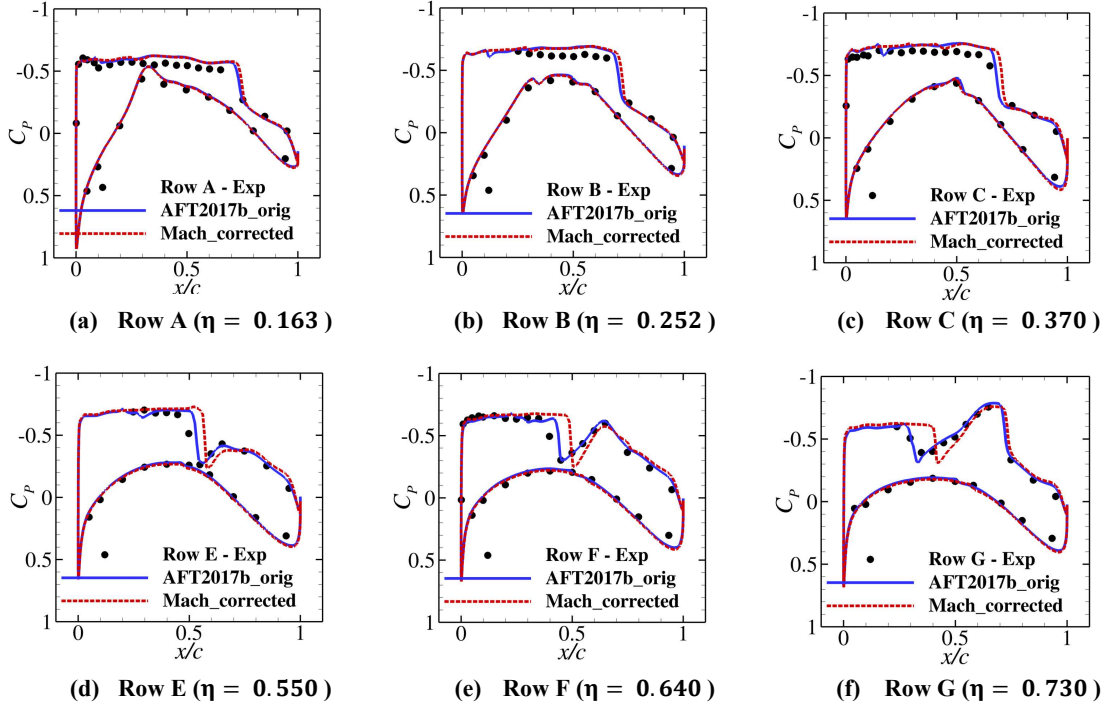


**Fig. 10.** Comparison of measured and computed pressure distributions at various stations on the CRM-NLF wing. Results are shown for both the original and the compressibility-corrected versions of the SA-RC-AFT2017b transition model for  $\alpha = 1.98^\circ$ ,  $M = 0.856$ ,  $FSTI = 0.12\%$ , and  $Re_{MAC} = 15 \times 10^6$ .

The comparison between the transition fronts predicted by the different models and that inferred from the experiment is shown in Fig. 11. For this flow condition, the study by Lynde et al. [35] and Paredes et al. [37] had indicated that the crossflow effects were significant in a small region in the vicinity of the break and in the outboard most regions of the wing, while TS growth was substantial all the way from the root up to  $y = 0.9$  m. Therefore, the inability of the AFT model to account for stationary crossflow effects was not expected to play a significant role in the overall prediction of transition onset at  $\alpha = 1.98^\circ$ . As seen earlier for  $\alpha = 1.44^\circ$ , the baseline predictions from the LM2015 and AFT2017b models are comparable to each other, indicating a significantly smaller laminar extent than the experiment across the entire span of the wing (see Figs. 11(a) and (b)). However, with the modification to account for the compressibility effects, the results predicted by both models improve significantly and match very well with the transition front inferred from the experiment, except for a few spanwise locations. The improved prediction of transition front within the outboard region of shock-dominated transition (with the exception of a modest spanwise shift in shock location relative to the measured data as seen from the pressure distribution profiles) points to the significant viscous-inviscid interaction in this case. In the case of the modified LM2015 model, the predicted front appears to be slightly upstream of the inferred front from the experiment in the region  $0.4 \text{ m} \leq y \leq 0.6 \text{ m}$  and the switchover in the transition front from one shock to the other happens slightly inboard (near  $y = 1.2$  m) with respect to the measured data, as seen from Fig. 11(a). Figure 11(b) indicates that for the case of the modified AFT model, the predicted front is slightly downstream of the experiment in the farthest inboard region of  $0.2 \text{ m} \leq y \leq 0.4 \text{ m}$ , while being upstream of that from the experiment in the region  $0.4 \text{ m} \leq y \leq 0.6 \text{ m}$ . Also, the location where the front switches from one shock to the other (near  $y = 1.3$  m) is farther outboard in comparison with the experiment.

The forces and moment coefficients obtained with the various models and their comparison against measured values are given in Table 3. Just as in the case with  $\alpha = 1.44^\circ$ , all models predict smaller drag coefficients and larger lift and moment coefficients when compared to those from the experiment. However, the compressibility correction leads to a significant increase in the lift and in the pitching moments, along

with a reduction in the drag coefficient. Also, the force and moment coefficients at  $\alpha = 1.98^\circ$  are higher than those at  $\alpha = 1.44^\circ$ .

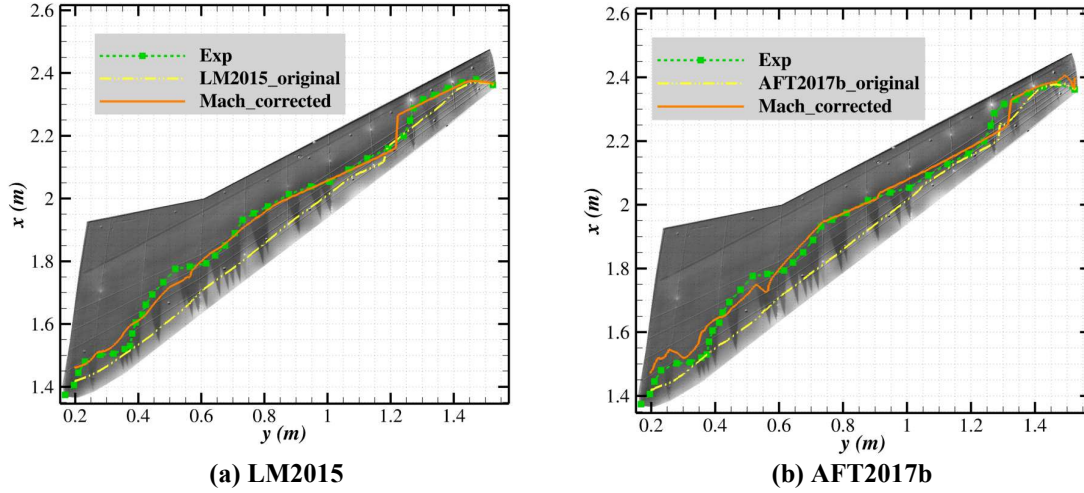


**Fig. 11. Transition fronts as predicted by the original and the compressibility-corrected versions of the various transition models overlaid on a TSP image obtained from the experiment for  $\alpha = 1.98^\circ$ ,  $M = 0.856$ ,  $FSTI=0.12\%$ , and  $Re_{MAC} = 15 \times 10^6$ .**

**Table 3. Force and Moment Coefficients.**

|                                    | $C_L$ | $C_D$  | $C_m$  |
|------------------------------------|-------|--------|--------|
| <b>Exp</b>                         | 0.378 | 0.0192 | -0.091 |
| <b>SST2003 – LM2015 (Original)</b> | 0.380 | 0.0185 | -0.082 |
| <b>LM2015 (Mach corrected)</b>     | 0.402 | 0.0184 | -0.095 |
| <b>SA-AFT2017b (Original)</b>      | 0.416 | 0.0192 | -0.110 |
| <b>AFT (Mach corrected)</b>        | 0.437 | 0.0189 | -0.123 |

### C. Computations for $\alpha = 2.46^\circ$

Figure 12 shows the computed surface contours of pressure gradient along the  $x$ -direction along with the transition front inferred from the experiment. Unlike in the two earlier cases with lower angles of attack, the  $\alpha = 2.46^\circ$  case shows a stronger single shock extending across the entire wing span with a weaker second shock appearing at a small distance from the primary shock in the region close to the wing tip. Furthermore, the transition front also appears to be shock-limited by the primary shock across the full span, starting from a small distance from the break. In the region from the root of the wing to slightly outboard of the break, the onset of transition occurs much upstream of the shock and the laminar extent appears to be significantly smaller than that in the previous cases and the front is nearly parallel to the leading edge in the inboard section.

The sectional pressure distributions obtained using the baseline and compressibility-corrected models are shown in Figs. 13 and 14, respectively. A good match between the predicted pressure coefficients and the measured values is seen across the entire wing span for both of the baseline models (with the exception of Row G for LM2015). Figure 13 indicates that the compressibility correction to the LM2015 model results in a slightly downstream shift in the shock with respect to the measured shock location on the upper surface. For the modified AFT model, the only differences from the measured pressure data are seen for Row A, where the shock system appears to be replaced by a system of compression waves, and for Row G, where the shock location has shifted downstream by approximately 7% of the local chord length (see Figs. 14(a) and (f)).

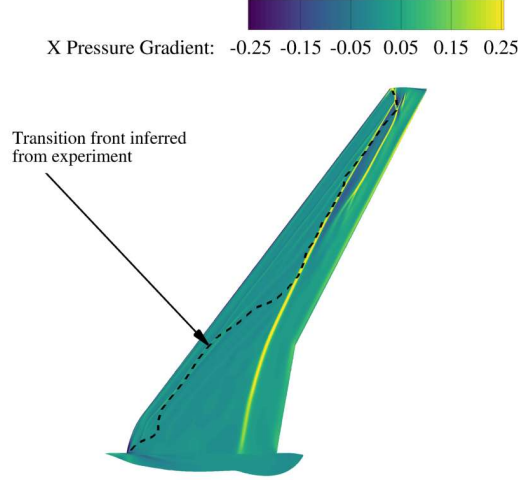


**Fig. 12.** Surface contours of x-pressure gradient superimposed with the transition front inferred from the experiment for  $\alpha = 2.46^\circ$ ,  $M = 0.856$ , and  $Re_{MAC} = 15 \times 10^6$ .

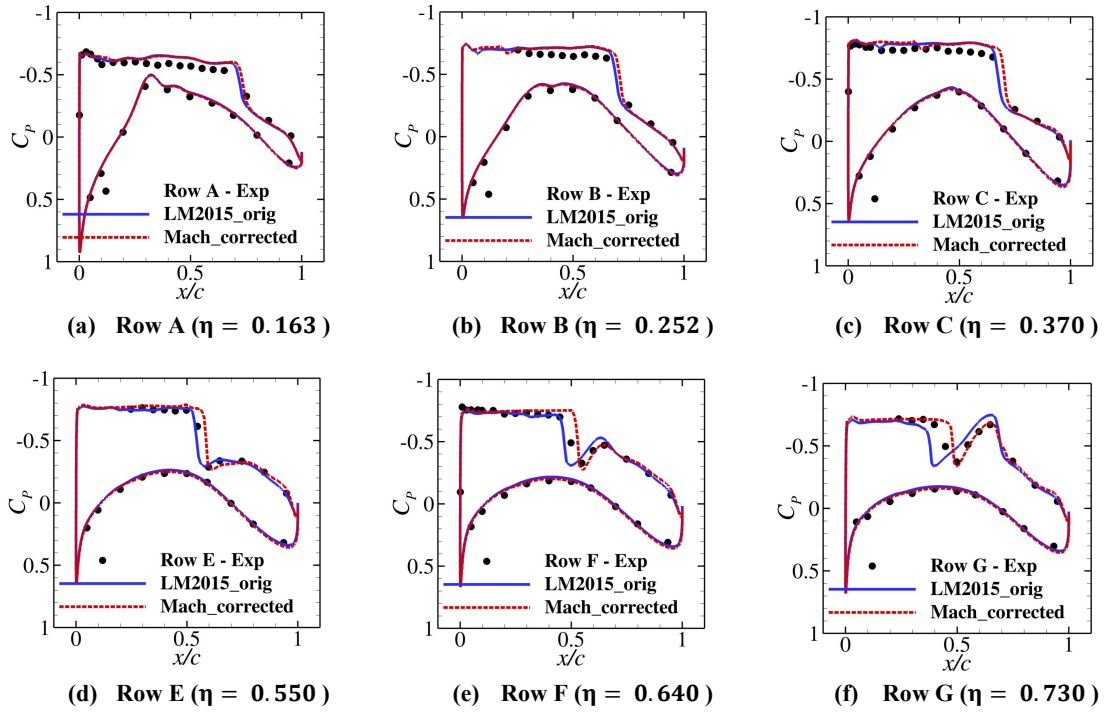


**Fig. 13.** Comparison of measured and computed pressure distributions at various stations on the CRM-NLF wing. Results are shown for both the original and the compressibility-corrected versions of the SST-2003RC-LM2015 transition model for  $\alpha = 2.46^\circ$ ,  $M = 0.856$ ,  $FSTI = 0.12\%$ , and  $Re_{MAC} = 15 \times 10^6$ .

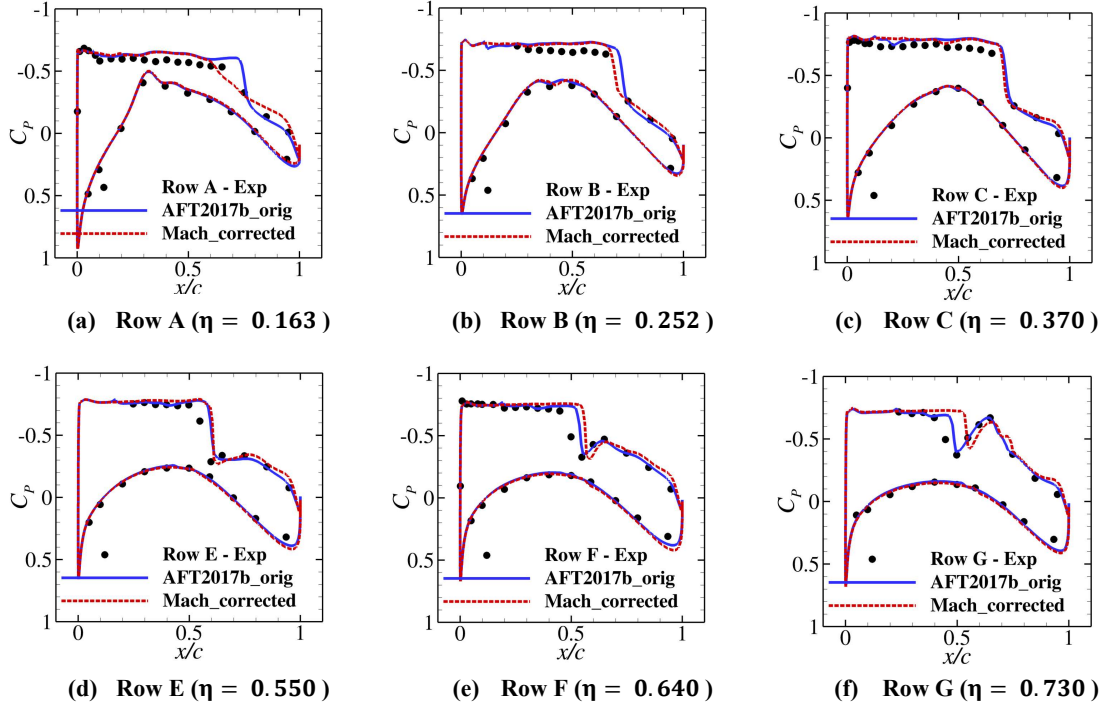


**Fig. 14.** Comparison of measured and computed pressure distributions at various stations on the CRM-NLF wing. Results are shown for both the original and the compressibility-corrected versions of the various transition models for  $\alpha = 2.46^\circ$ ,  $M = 0.856$ ,  $FSTI = 0.12\%$ , and  $Re_{MAC} = 15 \times 10^6$ .

The comparison between the predicted transition front and the front inferred from the experiment for the various models are shown in Fig. 15. For this flow condition, crossflow effects are expected to be insignificant and, therefore, the inability of the AFT model to account for stationary crossflow effects is not expected to play a significant role. Once again, as seen in the earlier case, the baseline LM2015 and AFT2017b models predict significantly smaller laminar extents when compared to the experiment in the region between the break and the tip of the wing. However, the baseline predictions within the inboard region match closely with the transition front inferred from the experiment, possibly due to the front being very close to the leading edge and immediately downstream of the suction peak, where there is a small region of adverse pressure gradient as seen from Figs. 13 and 14. With the modification to account for compressibility effects, the results predicted by both the models improve significantly and match well with that from the experiment in the region  $y \geq 0.6 m$ . In the inboard region of  $0.3 \leq y \leq 0.6 m$ , both the models predict a transition front that is further aft of that from the experiment, with a larger laminar extent in the case of the compressibility-corrected AFT model when compared to the compressibility corrected LM2015. Also, in the case of the compressibility corrected AFT model, the predicted transition front is downstream of the measured front starting from the wing root itself and the associated viscous-inviscid interaction may have caused the discrepancy in pressure distribution along Row A in Fig. 14(a). The force and moment coefficients obtained with the various models and their comparison against measured values are given in Table 3. The trend is similar to what was discussed for cases corresponding to  $\alpha = 1.45^\circ$  and  $\alpha = 1.98^\circ$ .

Overall, the results shown for the three different angles of attack conditions here, clearly confirm that the modifications to the two transition models are working as desired, with resulting downstream shift in the predicted transition front narrowing the gap with respect to the experimental measurements.

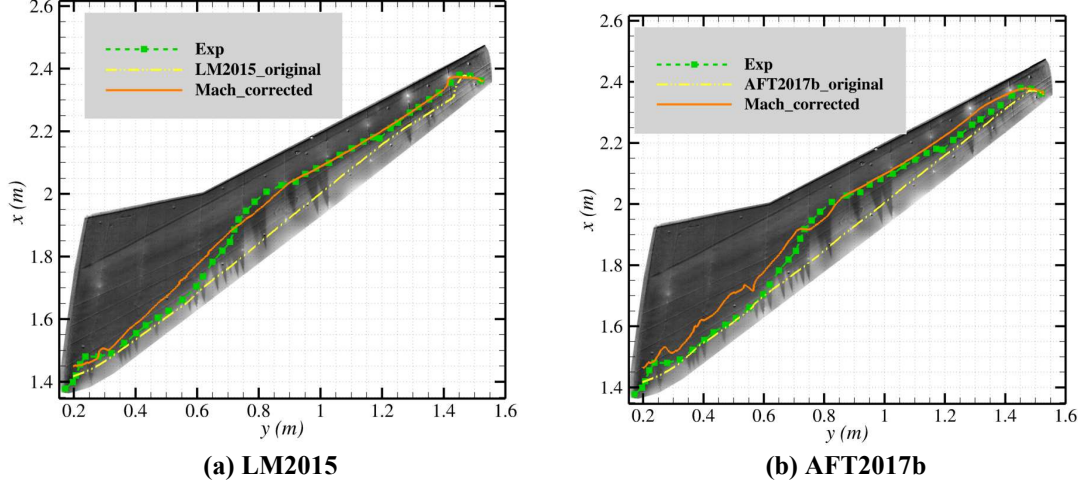


**Fig. 15.** Transition fronts as predicted by the original and the compressibility-corrected versions of the SA-RC-AFT2017b 5 transition model overlaid on a TSP image obtained from the experiment for  $\alpha = 2.46^\circ$ ,  $M = 0.856$ ,  $FSTI=0.06\%$ , and  $Re_{MAC} = 15 \times 10^6$ .

**Table 4 Force and Moment Coefficients.**

|                                    | $C_L$ | $C_D$  | $C_m$  |
|------------------------------------|-------|--------|--------|
| <b>Exp</b>                         | 0.439 | 0.0212 | -0.099 |
| <b>SST2003 – LM2015 (Original)</b> | 0.447 | 0.0213 | -0.092 |
| <b>LM2015 (Mach corrected)</b>     | 0.473 | 0.0211 | -0.108 |
| <b>SA-AFT2017b (Original)</b>      | 0.485 | 0.0222 | -0.121 |
| <b>AFT (Mach corrected)</b>        | 0.491 | 0.0214 | -0.133 |

## VII. Results from Stability Analysis - Dual $N$ -factor Criterion

In this section, we discuss the stability results obtained via line marching PSE in a nonorthogonal coordinate system and the transition front predicted using the dual  $N$ -factor criterion. In the previous study by Parades et al.[37], the PSE marching was carried out along gridlines, while in this study it is carried out by marching along the streamlines, as the latter represent a physical property of the flow and, hence, are not dependent on the computational grid. Furthermore, the streamline marching is expected to better correlate with the propagation of instability waves and, therefore, the resulting predictions are expected to be more accurate than those based on the gridline marching. As a consequence, the results for flow conditions 1 and 2 (see Table 1) were computed again along with other flow conditions. The basic state for the linear stability analysis is computed by using the SA turbulence model, along with an imposed transition front and the same baseline grid that was described earlier. The transition front was imposed slightly downstream of the front inferred from the experiment or at shock locations along the spanwise stations where the inferred front was shock-limited. The small downstream shift was designed to enable a fully laminar flow all the way up to the measured transition front, without any upstream influence of the imposed front. Overall, the above strategy for imposing a transition front helped ensure that the pressure distributions closely matched the experimentally measured surface pressures. Details about the mean flow and the grid convergence of the results were presented in the previous study by Paredes et al. [37] and will not be repeated here for brevity.

The streamlines used for the PSE marching and the stability computations were computed in a nearly automated manner using the Python-based interface tool suite called PyLASTRAC [78]. A total of 32 streamlines were used for the computations as shown in Fig. 16. Based on the earlier work by Crouch et al. [71], the critical TS  $N$ -factor for the NTF facility was determined to be 6 and based on the roughness of the CRM-NLF, the critical CF  $N$ -factor was also set to equal 6.

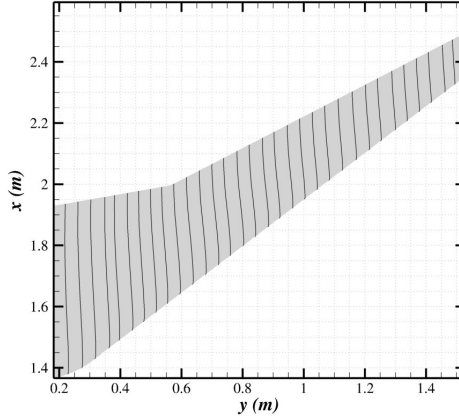


**Fig. 16. Streamlines used for PSE marching.**

Although in this work the PSE analysis was carried out by marching along the streamlines as opposed to the grid lines as was done in Ref. [37], for flow conditions 1 and 2, the instability characteristics of the flow differed only slightly from the results presented in Ref. [37] and hence will not be described in detail in the present paper. However, for the benefit of the reader, for flow condition 1, large TS  $N$ -factors were seen within the inboard portion of the wing (up to the break) and also in the far outboard region of the wing. On the other hand, the significant crossflow  $N$ -factors were also seen in the region starting from around the break, all the way to the tip of the wing. For flow condition 2, large crossflow  $N$ -factors were observed only in a small region in the vicinity of the break and in the most outboard region of the wing, whereas large TS  $N$ -factors were seen all the way from the root up to  $y = 0.9$  m. These predictions are qualitatively consistent with the those reported by Lynde et al. [35] and Fischer et al. [79], who both used the classical LST to characterize the instability amplification characteristics. The agreement between the present PSE results (which include the leading nonparallel effects and the abovementioned LST predictions indicates that the neglect of nonparallel effects does not alter the transition front characteristics at the conditions of interest.

For flow condition 3 ( $\alpha = 2.46^\circ$ ), the  $N$ -factor envelope contours for TS and crossflow obtained with PSE marching along the streamlines are shown in Fig. 17 within the laminar region of the solution. High  $N_{CF}$  values ( $>6$ ) are only seen near the wing tip, while large  $N_{TS}$  values are seen across most of the wing span, indicating that this configuration is dominated by TS amplification. In general,  $N_{TS} \approx 6$  contour from Fig. 17(b) shows a good comparison with the transition front inferred from the experiment, except in a small region  $0.7 \leq y \leq 0.9$  m near the break where  $N_{TS}$  values along the measured transition front are larger than six.

For flow condition 4 ( $\alpha = 2.94^\circ$ ), the  $N$ -factor envelope contours obtained with PSE marching along the streamlines are shown in Fig. 18 within the laminar region of the computed basic state. Once again, moderate  $N_{CF}$  values ( $\approx 5$ ) are only seen near the wing tip (see Fig. 18(a)), while large  $N_{TS}$  values are seen across the wing span, indicating once again that this configuration is also dominated by TS effects.  $N_{TS}$  contours shown in Fig. 18(b) indicates a good comparison with the transition front inferred from the experiment for values of  $N_{TS} \approx 7$ , except in a small region  $0.8 \leq y \leq 1.0$  m, where  $N_{TS}$  reaches larger values. The PSE based instability characteristics predicted for flow conditions 3 and 4 agree with the findings of LST calculations by Lynde et al. [35] and Fischer et al. [79].

The dual  $N_{TS} - N_{CF}$  criterion of Eq. (5) (detailed in section III A) is evaluated by using the PSE results obtained for all four angles of attack and the results are summarized in Fig. 19, wherein the transition fronts predicted by the dual  $N$ -factor criterion as well as those from the experiment and the compressibility-corrected transition models are overlaid on top of the TSP images from the experiment. Similar to Ref. [37], the dual  $N$ -factor criterion was evaluated using  $N_{TS,c} = N_{CF,c} = 6$  and  $a_{TS} = a_{CF} = 3$ . In these plots, the predicted transition locations from the dual  $N$ -factor criterion are available only for selected streamline locations along the span where the data reach the critical value within the laminar region of the solution.

For flow condition 1 ( $\alpha = 1.45^\circ$ ) in Fig. 19(a), the transition onset predictions based on the dual  $N$ -factor criterion closely follows the front inferred from the experiment in the region outboard of the break. In the inboard region from the root to the break, the dual  $N$ -factor criterion results in a slightly earlier transition than the front inferred from the experiment. The results from the compressibility-corrected LM2015 model also follow the results from the dual  $N$ -factor criterion, while the results from the compressibility-corrected

AFT2017b model only match the results from the dual  $N$ -factor criterion in the outboard region  $y \geq 1.0$  m, possibly due to lack of accounting for the CF effects.

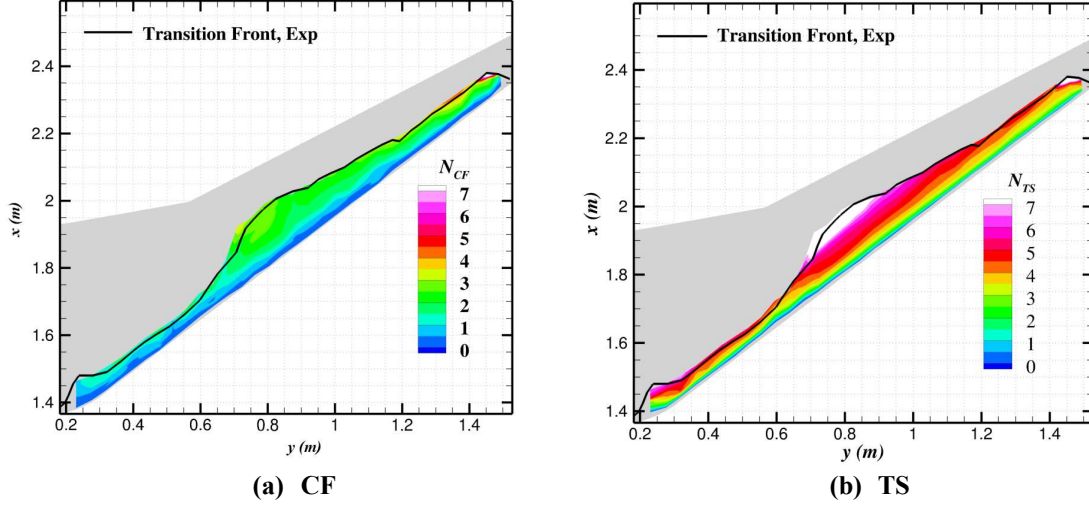


**Fig. 17.**  $N$ -factor contours of (a) stationary CF waves ( $f = 0$ ) and (b) TS waves ( $\beta = 0$ ) calculated with nonorthogonal PSE for configuration 3 ( $Re_{MAC} = 15 \times 10^6$ ,  $\alpha = 2.46^\circ$ ).

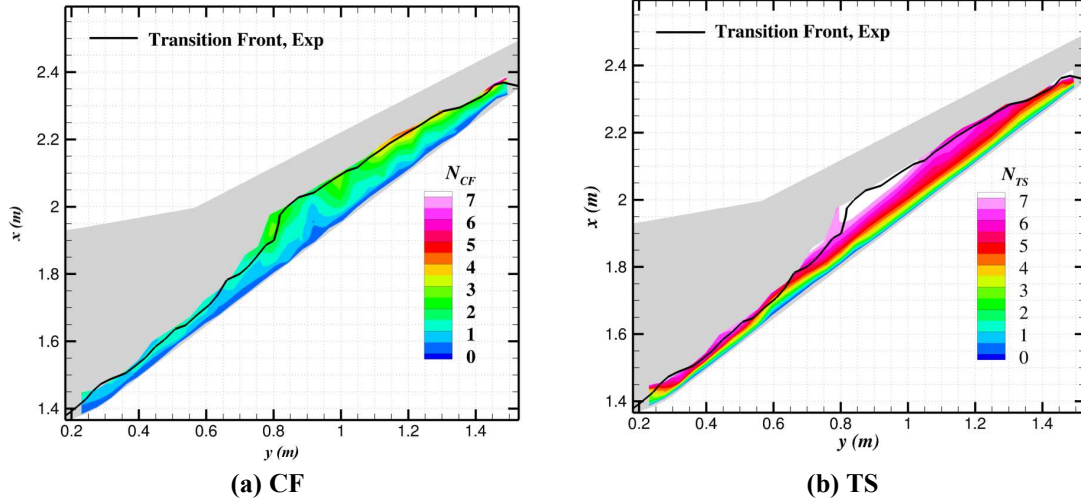


**Fig. 18.**  $N$ -factor contours of (a) stationary CF waves ( $f = 0$ ) and (b) TS waves ( $\beta = 0$ ) calculated with nonorthogonal PSE for configuration 4 ( $Re_{MAC} = 15 \times 10^6$ ,  $\alpha = 2.94^\circ$ ).

Figure 19(b) shows a similar comparison for flow condition 2 ( $\alpha = 1.98^\circ$ ). The match between the transition front predicted by the dual  $N$ -factor criterion and that from the experiment is good across most of the wing span, with some small differences near the wing tip ( $1.2 \leq y \leq 1.35$  m), and a larger predicted laminar extent near the wing root. Given that the CF effects are strong only within a narrow region around the break and the wing tip, the predictions of both compressibility-corrected models match very well with the transition front predicted from the dual  $N$ -factor criterion across the entire span of the wing; near the wing-tip ( $1.2 \leq y \leq 1.35$  m), the result from the compressibility-corrected AFT2017b model shows a better match with the dual  $N$ -factor criterion than the compressibility-corrected LM2015 model.

The transition front predicted by the dual  $N$ -factor criterion for flow condition 3 ( $\alpha = 2.46^\circ$ ) is shown in Fig. 19(c). Here again, a good match is observed between the predictions from the compressibility-corrected transport-equation-based transition models and the stability-based prediction across the entire span of the wing. In the region outboard of the break, the match between the experimental front and that from the dual  $N$ -factor criterion is good. In the region inboard of the break, the dual  $N$ -factor criterion indicates a slightly

larger laminar extent as indicated by the lack of data points (the compressibility-corrected models also indicated a later transition in this region).

For flow condition 4 ( $\alpha = 2.94^\circ$ ), the results from the dual  $N$ -factor criterion show a good match with the inferred front from the experiment across the span of the wing, apart from the region around  $0.8 \leq y \leq 1.0$  m, where the dual  $N$ -factor criterion results indicate an earlier transition.

Overall, the results predicted by the dual  $N$ -factor criterion indicate a good agreement with the transition fronts inferred from the experiment. Additionally, they also help confirm that the compressibility-corrected transition models are able to predict transition happening in the regions where the TS  $N$ -factors is predicted to be large by the PSE analysis thereby accounting for the increased stability of the boundary layer due to compressibility effects.

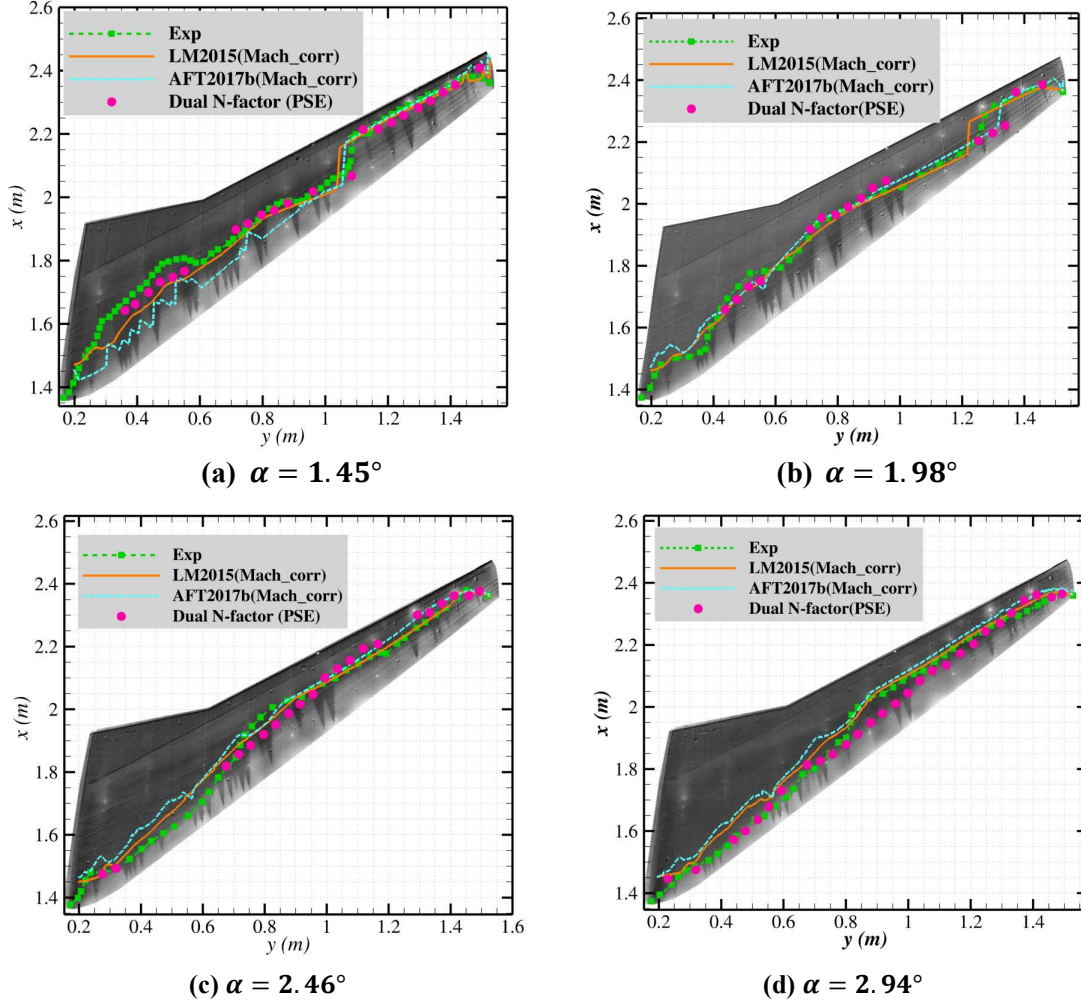


**Fig. 19.** Predicted transition locations using the dual  $N_{TS}$ - $N_{CF}$  criteria at available span locations over the TSP images shown along with experimental transition front and transition front predicted by compressibility-corrected transition models for flow conditions (a) 1, (b) 2, (c) 3, and (d) 4.

## VIII. Summary and Concluding Remarks

The findings presented in this paper underline the shortcomings of both the Langtry-Menter and the Amplification Factor Transport-equation-based transition models in accounting for the stabilizing effect of compressibility on the boundary-layer flow over a transport aircraft configuration with a natural laminar flow wing design, namely, the CRF-NLF configuration. The formulation of a potential path to alleviate the shortcomings is also presented. Based on a database of LST computations from the literature, we have proposed modifications to some of the correlations used within these models to capture the stabilizing

influence of compressibility. The modified models reduce to the original formulations of the respective model at zero Mach number, so as to recover the ability of the respective, original models to predict natural transition (both LM and AFT models) and bypass transition scenarios (LM models). For the Langtry-Menter model, one part of the proposed modification targets the relationship between the vorticity Reynolds number and the momentum-thickness Reynolds number that plays a key role in triggering the onset of transition within the model. An additional aspect of the modification alters the correlation that determines the transition onset momentum-thickness Reynolds number based on freestream turbulence level, pressure gradient, and compressibility effects. Modifications to the AFT model also include two main corrections derived from the work by Sturdza [69], one involving the correlation for the momentum-thickness Reynolds number at the onset of instability and the other for the slope of the  $N$ -factor envelope. Even though crossflow transition is also included in the LM2015 model, the effects of compressibility are deemed to be most significant for the amplification of the TS instabilities. Accordingly, the compressibility corrections for both models are restricted to the TS mechanism alone. For both LM and AFT models, the modifications retain the critical requirement of a dependence on purely local data, which makes it easier to implement them in any other CFD code, either for the abovementioned transition models or any of their variants without requiring any significant extra effort.

The resulting improvement in the accuracy of transition prediction at transonic speeds was demonstrated in the context of the generic CRM-NLF cruise wing configuration by comparing the results of both original and modified models with the data obtained from the NASA's experiments in the National Transonic Facility as well as with the transition predictions based on linear stability analysis. The compressibility-corrected, transport-equation-based transition models lead to a substantially improved prediction of the laminar extent in the flow (vis-a-vis a similar comparison for the baseline models) for the three angles of attack studied herein. The results confirm that accounting for the stabilizing effect of compressibility is an important prerequisite to the routine application of these models for the design of transonic vehicles.

Additionally, nonparallel stability analysis of the three dimensional boundary-layer flow over the CRM-NLF aircraft configuration was carried out for the mean flow solutions obtained with an imposed transition front at each of the four angles of attack that were the primary focus during the CRM-NLF experiment. These computations represent a major extension of the previous work by Paredes et al. [37], by carrying out the PSE marching along streamlines (i.e., flow based marching trajectories as against grid dependent trajectories based on computational coordinates) and by applying the dual  $N_{TS} - N_{CF}$  criterion based on the computed  $N$ -factor envelopes for TS and CF amplification to determine the transition onset front for the four different angles of attack studied in the CRM-NLF experiment. The transition fronts based on the dual  $N$ -factor criterion matched reasonably well with those inferred from the experiment at all angles of attack for the Reynolds number of interest. In that regard, these PSE computations provide the first comprehensive assessment of transition prediction based on nonparallel disturbance amplification for the CRM-NLF configuration. Additionally, the nonparallel, PSE analysis serves as a higher fidelity verification of the transition predictions based on other quasiparallel, LST studies, as well as providing further insights into what instability mechanisms may have been at play during the CRM-NLF experiment. The stability analysis also confirmed that the compressibility-corrections incorporated into the LM2015 and AFT2017b models were working as desired, predicting the delay in transition due to increased stability due to compressibility effects. In certain regions of the flow where the results based on the compressibility-corrected models differed from the experimental measurements, the corresponding results based on the dual  $N$ -factor criterion also predicted a similar trend.

In general, the modifications to the transport-equation-based transition model proposed here are expected to hold up to a Mach number of 1.2 (the deviation from the isentropic relations is not expected to be significant) but can be retained up to a Mach number of 2.0 (as the LST data utilized in the development involved flow speeds in this range), provided a reliable way to estimate the edge Mach number at conditions involving shocks can be identified as part of future work. Beyond a Mach number of 2.0 additional models of instability needs to be considered, and the stabilizing effects of compressibility starts to vanish. However, a careful evaluation of the limitations of some of the assumptions made during the development of the modifications is warranted. The proposed modifications also provide a template for the inclusion of wall temperature effects on transition that become important at supersonic flight conditions and also in turbomachinery applications. Both the compressibility corrections to the transport-equation-based transition models and the dual  $N_{TS} - N_{CF}$  criterion developed here need to be validated against reliable measurements for a more extensive set of test problems, but the present work has paved the way for this criterion to be utilized in our ongoing work on developing a suite of automated, CFD-integrated transition prediction tools.

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## References

- [1] Slotnick, J., Khodahoust, A., Alonso, J., Darmofal, D., Gropp, W., and Mavriplis, D., "CFD Vision 2030 Study: A Path to Revolutionary Computational Aerosciences," NASA/CR-2014-218178, 2014.
- [2] Smith, A. M. O., and Gamberoni, N., "Transition, Pressure Gradient and Stability Theory," Douglas Aircraft Company, Long Beach, Calif. Rep. ES 26338, 1956.
- [3] van Ingen, J. L., "A Suggested Semi-Empirical Method for the Calculation of the Boundary Layer Transition Region," University of Delft, Dept. of Aerospace Engineering, Delft, The Netherlands, Rep. VTH-74, 1956.
- [4] Herbert, T., "Parabolized Stability Equations," *Annu. Rev. Fluid Mech.*, 29:245-283, 1997.
- [5] Drela, M. and Giles, M. B., "Viscous-Inviscid Analysis of Transonic and Low Reynolds Number Airfoils," *AIAA Journal*, Vol. 25, No. 10, 1987, pp. 1347-1355.
- [6] Drela, M., "Implicit Implementation of the Full  $e^N$  Transition Criterion," AIAA Paper 2003-4066, 2003.
- [7] Cliquet, J., Houdeville, R., and Arnal, D., "Application of Laminar-Turbulent Transition Criteria in Navier-Stokes Computations," *AIAA Journal*, Vol. 46, No. 5, 2008, pp. 1182-1190.
- [8] Perraud, J., Arnal, D., Casalis, G., Archambaud, J.-P., and Donelli, R., "Automatic Transition Predictions Using Simplified Methods," *AIAA Journal*, Vol. 47, No. 11, 2009, pp. 2676-2684.
- [9] Krumbein, A., Krimmelbein, N., and Schrauf, G., "Automatic Transition Prediction in a Hybrid Flow Solver – Part 1: Methodology and Sensitivities," *Journal of Aircraft*, Vol. 46, No. 4, 2009, pp. 1176 – 1190.
- [10] Krumbein, A., Krimmelbein, N., and Grabe, C., "Streamline-Based Transition Prediction Techniques in an Unstructured Computational Fluid Dynamics Code," *AIAA Journal*, Vol. 55, No. 5, 2017, pp. 1548-1564.
- [11] Campbell, R., Campbell, M., and Streit, T., "Progress Toward Efficient Laminar Flow Analysis and Design," AIAA 2011-3527.
- [12] Davis, M. B., Reed, H. L., Youngren, H., Smith, B., and Bender, E., "Transition Prediction Method Review Summary for the Rapid Assessment Tool for Transition Prediction (RATTraP)," Technical Report AFRL-VA-WP-TR-2005-3130, Air Force Research Lab, Wright-Patterson AFB, OH, 2005.
- [13] Stock, H. W., and Haase, W., "Navier-Stokes Airfoil Computations with  $e^N$  Transition Prediction Including Transitional Flow Regions," *AIAA Journal*, Vol. 38, No. 11, 2000, pp. 2059-2066.
- [14] Warren, E. S., and Hassan, H. A., "Transition Closure Model for Predicting Transition Onset," *Journal of Aircraft*, Vol. 35, No. 5, September- October 1998, pp. 769-775.
- [15] Edwards, J. R., Roy, C. J., Blottner, F. G., and Hassan, H. A., "Development of a One-Equation Transition/Turbulence Model," *AIAA Journal*, Vol. 39, No. 9, 2001, pp. 1691-1698.
- [16] Walters, D. K., and Leylek, J. H., "A New Model for Boundary-Layer Transition Using a Single-Point RANS Approach," *ASME J. of Turbomach.*, Vol. 126(1), 2004, pp. 193-202.
- [17] Menter, F. R., Langtry, R. B., and Völker, S., "Transition Modelling for General Purpose CFD Codes," *J. Flow, Turbulence and Combustion*, Vol. 77, 2006, pp. 277-303.
- [18] Langtry, R. B., and Menter, F. R., "Correlation-Based Transition Modeling for Unstructured Parallelized Computational Fluid Dynamics Codes," *AIAA Journal*, Vol. 47, No. 12, 2009, pp. 2894-2906.
- [19] Medida, S., and Baeder, J., "Application of the Correlation-based  $\gamma$ - $Re_{\theta t}$  Transition Model to the Spalart-Allmaras Turbulence Model," AIAA Paper 2011-3979, 2011.
- [20] Ge, X., Arolla, S., and Durbin, P., "A Bypass Transition Model Based on the Intermittency Function," *J. Flow Turbulence and Combustion*, Vol. 93, 2014, pp. 37-61.
- [21] Song Fu, A., Wang, L., "RANS modeling of High-Speed Aerodynamic Flow Transition with Consideration of Stability Theory," *Progress in Aerospace Sciences*, Vol. 58, 2013, pp. 36-59.
- [22] Coder, J.G., and Maughmer, M.D., "Computational Fluid Dynamics Compatible Transition Modeling Using an Amplification Factor Transport Equation," *AIAA Journal*, Vol. 52, No. 11, 2014, pp. 2506-2512.
- [23] Coder, J.G., "Further Development of the Amplification Factor Transport Transition Model for Aerodynamic Flows," AIAA Paper 2019-0039, 2019.

- [24] Medida, S., "Correlation-based Transition Modeling for External Aerodynamic Flows," University of Maryland PhD Dissertation, 2014.
- [25] Krumbein, A., Krimmelbein, N., Grabe, C. and Shengyang, N., "Development and Application of Transition Prediction Techniques in an Unstructured CFD Code," AIAA Paper 2015-2476, 2015.
- [26] Choi, J. H., and Kwon, O. J., "Recent Improvement of a Correlation-based Transition Model for Simulating Three-Dimensional Boundary Layers", *AIAA Journal*, Vol. 55, No. 6, 2017, pp. 2103–2108.
- [27] Langtry, R. B., Sengupta, K., Yeh, D. T., and Dorgan, A. J., "Extending the  $\gamma$ - $Re_{\theta t}$  Correlation based Transition Model for Crossflow Effects," AIAA Paper 2015-2474, 2015.
- [28] Medida, S., and Baeder, J., "A New Crossflow Transition Onset Criterion for RANS Turbulence Models," AIAA Paper 2013-3081, 2013.
- [29] Nie, S., Krimmelbein, N., Krumbein, A., and Grabe, C., "Extension of a Reynolds-Stress-Based Transition Transport Model for Crossflow Transition," *Journal of Aircraft*, Vol. 55, 2018, pp. 1641–1654.
- [30] Grabe, C., Shengyang, N., and Krumbein, A., "Transport Modeling for the Prediction of Crossflow Transition," *AIAA Journal*, Vol. 56, No. 8, 2018, pp. 3167–3178.
- [31] Choi, J., and Kwon, O. J., "Enhancement of a Correlation-Based Transition Turbulence Model for Simulating Crossflow Instability", AIAA Paper 2014-1133, 2014.
- [32] Lynde, M. N., and Campbell, R. L., "Computational Design and Analysis of a Transonic Natural Laminar Flow Wing for a Wind Tunnel Model," AIAA Paper 2017-3058, 2017.
- [33] Rivers, M., Lynde, M., Campbell, R., Viken, S., Chan, D., Watkins, A., and Goodliff, S., "Experimental Investigation of the NASA Common Research Model with a Natural Laminar Flow Wing in the NASA Langley National Transonic Facility", AIAA Paper 2019-2189, 2019.
- [34] Lynde, M. N., Campbell, R. L., Rivers, M. B., Viken, S. A., Chan, D. T., Watkins, A. N., and Goodliff, S. L., "Preliminary Results from an Experimental Assessment of a Natural Laminar Flow Design Method," AIAA Paper 2019-2298, 2019.
- [35] Lynde, M. N., Campbell, R. L., and Viken, S. A., "Additional Findings from the Common Research Model Natural Laminar Flow Wind Tunnel Test", AIAA Paper 2019-2298, 2019.
- [36] Venkatachari, B.S., Paredes, P., Derlaga, J. M., Buning, P. G., Choudhari, M., Li, F. and Chang, C.L., "Assessment of RANS-based Transition Models based on Experimental Data of the Common Research Model with Natural Laminar Flow," AIAA Paper 2021-1430, 2021.
- [37] Paredes, P., Venkatachari, B., Choudhari, M., Li, F., Hildebrand, N., and Chang, C. L., "Transition Analysis for the CRM-NLF Wind Tunnel Configuration," AIAA Paper 2021-1431, 2021.
- [38] Nichols, R. H. and Buning, P. G., "User's Manual for OVERFLOW 2.3, Version 2.3," NASA Langley Research Center, Hampton, VA, Oct 2019.
- [39] Mack, L., "Linear Stability Theory and the Problem of Supersonic Boundary- Layer Transition," *AIAA Journal*, vol. 13, Mar. 1975, pp. 278–289.
- [40] Vassberg, J.C., DeHaan, M.A., Rivers, M.S. and Wahls, R.A., "Retrospective on the Common Research Model for Computational Fluid Dynamics Validation Studies," *Journal of Aircraft*, Vol. 55, 2018, pp.1325–1337.
- [41] Campbell, R. L., "Efficient Viscous Design of Realistic Aircraft Configurations," AIAA Paper 1998-2539, 1998.
- [42] Campbell, R. L., and Lynde, M. N., "Natural Laminar Flow Design for Wings with Moderate Sweep," AIAA Paper 2016-4326, 2016.
- [43] Menter, F. R., "Two-Equation Eddy-Viscosity Turbulence Models for Engineering Applications," *AIAA Journal*, Vol. 32, No. 8, 1994, pp. 1598-1605.
- [44] Menter, F. R., Kuntz, M., and Langtry, R., "Ten Years of Industrial Experience with the SST Turbulence Model," *Turbulence, Heat and Mass Transfer*, Vol. 4 (1), 2003, pp. 625 - 632.
- [45] Spalart, P. R. and Allmaras, S. R., "A One-Equation Turbulence Model for Aerodynamic Flows," *Recherche Aerospatiale*, No. 1, 1994, pp. 5-21.
- [46] Carnes, J. A., and Coder, J. G., "Effect of Crossflow on the S-76 and PSP Rotors in Hover," AIAA Paper 2020-0773, 2020.
- [47] Shur, M. L., Strelets, M. K., Travin, A. K., and Spalart, P. R., "Turbulence Modeling in Rotating and Curved Channels: Assessing the Spalart-Shur Correction," *AIAA Journal*, Vol. 38, No. 5, 2000, pp. 784–792.
- [48] Chang, C.-L., "LASTRAC.3d: Transition Prediction in 3D Boundary Layers," AIAA Paper 2004-2542, 2004.
- [49] Chang, C.-L., "Langley Stability and Transition Analysis Code (LASTRAC) Version 1.2 User Manual," NASA TM-2004-213233, 2004.
- [50] Horstmann, K., Redeker, G., Quast, A., Dressler, U., and Bieler, H., "Flight Test with a Natural Laminar Flow Glove on a Transport Aircraft," AIAA Paper 90-3044, 1990.
- [51] Arnal, D., "Boundary Layer Transition: Prediction Based on Linear Theory," Special course on Progress in Transition Modeling. AGARD. R-793, 1994, pp. 1–62.
- [52] Kaynak, Ü., and Gürdamar, E., "Boundary-Layer Transition Under the Effect of Compressibility for the Correlation Based Transition Model," AIAA Paper 2008-, 2008.
- [53] Narasimha, R., "The Laminar-Turbulent Transition Zone in the Boundary Layer," *Prog. Aerospace Sci.*, Vol. 22, No. 9, 1985, pp.29–80.

- [54] Pecnik, R., Witteveen, J. A. S., and Iaccarino, G., "Assessment of Uncertainties in Modeling of Laminar to Turbulent Transition for Transonic Flows," *Flow, Turbulence and Combustion*, vol. 91, Jul. 2013, pp. 41–61.
- [55] Zhang, X., and Gao, Z., "A Numerical Research on a Compressibility-correlated Langtry's Transition Model for Double Wedge Boundary Layer Flows," *Chinese Journal of Aeronautics*, vol. 24, Jun. 2011, pp. 249–257.
- [56] Ströer, P., Krimmelbein, N., Krumbein, A., and Grabe, C., "Stability-Based Transition Transport Modeling for Unstructured Computational Fluid Dynamics at Transonic Conditions," *AIAA Journal*, Vol. 59, No. 9, 2021, pp. 3585–3597.
- [57] Pascal, L., Delattre, G., Deniau, H., and Cliquet, J., "Stability-Based Transition Model Using Transport Equations," *AIAA Journal*, Vol. 58, No. 7, 2020, pp. 2933–2942.
- [58] Abu-Ghannam, B. J. and Shaw, R., "Natural Transition of Boundary Layers: The Effects of Turbulence, Pressure Gradient, and Flow History," *Journal of Mechanical Engineering Science*, Vol. 22, No. 5, 1980, pp. 213–228.
- [59] Arnal, D., Casalis, G., and Houdeville, R., "Practical Transition Prediction Methods: Subsonic and Transonic Flows," AVT-151RTOAVT/ VKI Lecture Series, No. 8, Von Karman Inst., Rhode St. Genèse, Belgium, 2008.
- [60] Boyle, R. J., and Simon, F. F., "Mach Number Effects on Turbine Blade Transition Length Prediction," NASA/TM-1998-208494, 1998.
- [61] Mack, L., "On the Stability of the Boundary Layer on a Transonic Swept Wing," American Institute of Aeronautics and Astronautics, 1979.
- [62] Dagenhart, J. R., and Saric, W. S., "Crossflow Stability and Transition in Swept Wing Flow," NASA TP-1999-209344, 1999.
- [63] van Driest, E. R., and Blumer, C. B., "Boundary Layer Transition: Freestream Turbulence and Pressure Gradient Effects," *AIAA Journal*, Vol. 1, No. 6, 1963, pp. 1303–1306.
- [64] Menter, F. R., Smirnov, P. E., Liu, T., and Avancha, R., "A One-Equation Local Correlation-Based Transition Model," *Flow, Turbulence and Combustion*, Vol. 95, Issue 4, Dec. 2015, pp. 583–619.
- [65] Fehrs, M., "Boundary Layer Transition in External Aerodynamics and Dynamic Aeroelastic Stability," Ph.D. Dissertation, Technical University Braunschweig, ISSN 1434-8454, ISRN DLR-FB-2018-11, also NFL-FB 2017-27, Braunschweig, Germany, 2018.
- [66] Qiao, L., Bai, J.-Q., Xu, J.-K., Xu, J.-L., and Zhang, Y., "Modeling of Supersonic/Hypersonic Boundary Layer Transition Using a Single-Point Approach," *International Journal of Nonlinear Sciences and Numerical Simulation*, vol. 19, Jun. 2018, pp. 263–274.
- [67] Masad, J. A., and Malik, M. R., "Transition Correlation in Subsonic Flow over a Flat Plate," *AIAA Journal*, Vol. 31, No. 10, 1993, pp. 1953–1955.
- [68] Masad, J. A., and Abid, R., "On Transition in Supersonic and Hypersonic Boundary Layers," *Int. J. Engng Sci*, Vol. 33, No. 13, 1995, pp. 1893–1919.
- [69] Sturdza, P., "An Aerodynamic Design Method for Supersonic Natural Laminar Flow Aircraft," Ph.D. Thesis, Stanford University, California, USA, 2003.
- [70] Lee, J., *The Gas Dynamics of Explosions*, Cambridge Univ. Press, Cambridge, England, U.K., 2016, Chap. 2. <https://doi.org/10.1017/CBO9781316226926>.
- [71] Crouch, J., Sutanto, M., Witkowski, D., Watkins, A., Rivers, M., and Campbell, R., "Assessment of the National Transonic Facility for Natural Laminar Flow Testing," AIAA Paper 2010-1302, 2010.
- [72] Roe, P. L., "Approximate Riemann Solvers, Parameter Vectors, and Difference Schemes," *J. Comput Phys.*, Vol. 43, No. 2, 1981, pp. 357–372.
- [73] Nichols, R. H., Tramel, R. W., and Buning, P. G., "Solver and Turbulence Model Upgrades to OVERFLOW 2 for Unsteady and High-Speed Applications," AIAA Paper 2006-2824, 2006.
- [74] Langel, C.M., Chow, R., van Dam, C.P., Maniaci, D., Ehrmann, R.S., and White, E.B., "A Computational Approach to Simulating the Effects of Realistic Surface Roughness on Boundary Layer Transition," AIAA Paper No. 2014-0234, 2014.
- [75] Langtry, R. B., "A Correlation-Based Transition Model Using Local Variables for Unstructured Parallelized CFD Codes," Ph.D. Thesis, Univ. of Stuttgart, Stuttgart, Germany, 2006.
- [76] Spalart, P. R. and Rumsey, C. L., "Effective Inflow Conditions for Turbulence Models in Aerodynamic Calculations," *AIAA Journal*, Vol. 45, No. 10, 2007, pp. 2544–2553.
- [77] Sclafani, A., Slotnick, J., Vassberg, J. and Pulliam, T., "Extended OVERFLOW Analysis of the NASA Trap Wing Wind Tunnel Model," AIAA Paper 2012-2919, 2012.
- [78] Chang, C.-L., "Development of Physics-Based Transition Models for Unstructured-Mesh CFD Codes using Deep Learning Models," AIAA Paper 2021-2828, 2021.
- [79] Fischer, J.S., Soemarwoto, B.I. and van der Weide, E.T., "Automatic Transition Prediction in a Navier–Stokes Solver Using Linear Stability Theory," *AIAA Journal*, Vol. 59, No. 7, 2021, pp. 2409–2426.