4D Point-to-Point Ray Tracing

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Outline

- Objective and motivation
- A means to this objective -> 4D Rays
- Description of 4D Rays and what that buys us
- 4D system of equations
- Point-to-Point ray trace equations
- Current status and conclusions

Objective and Motivation
A Balanced Analytical System with Propagation

As understanding of source acoustics advances (rotorcraft, UAVs, aircraft noise, etc.), acoustic propagation must advance as well in order to effectively propagate these sources through realistic atmospheres.

…4D Ray Tracing is the next giant leap in propagation.

Advancing Propagation

During the last two decades, NASA has made advances to account for effects of wind and temperature gradients on propagation. We have recently integrated these effects into NASA system prediction codes and found that we need a more advanced propagation scheme.

In order to integrate seamlessly into the existing codes, propagation calculations need to be point-to-point, or from source to observer, not just a ground contour.

- Homogeneous atmosphere -> It is a straight line.
- Inhomogeneous atmosphere -> It is a curved ray (no longer point-to-point).

NASA's Ray Tracing Program (RTP), an inhomogeneous ray code, modified to find a ray that connects a source to an observer, but …

- It is an iterative process.
- It requires bookkeeping and caution when a caustic is approached.

The acoustic community could benefit from a propagation code that connects two locations in an inhomogeneous medium much like the straight ray propagation codes do.

4D Point-to-Point Ray Code

Point-to-point curved ray propagation can be achieved.

- The method involves solving the full 4D (space-time/relativistic) geodesic ordinary differential equation.
- Our RTP propagation code is limited to stratified linear temperature and linear wind variations. A 4D ray propagation code would be more versatile (i.e., allow predictions for various sound speeds, wind conditions, shear layers and other viscous effects).
- Spreading loss is easily computed with a 4D ray method. Spreading loss is a direct result of the deviation vectors connected to the main ray.

What 4D General Relativity Buys You …

Given a coordinate system [t, x, y, z] and a covariant metric

$$g_{ij} = \begin{pmatrix} -c^2 + v^2 & -v_j \\ -v_i & \delta_{ij} \end{pmatrix}$$

You automatically get all the physics:

- contra_metric - contravariant metric
- det_met - determinant of the covariant metric component matrix
- C1 - Christoffel symbols of the first kind
- C2 - Christoffel symbols of the second kind

  Describes how the vectors change as we move from location to location, a “connection coefficient”
- Rm - covariant Riemann tensor - Curvature of the space-time manifold
- Rc - covariant Ricci tensor
- R - Ricci scalar

  Describes shape transformation as an object moves along a geodesic in space, i.e., change in volume
- G - covariant Einstein tensor - Space-time curvature of stress-energy tensor effects, i.e., gravitational potential
- C - covariant Weyl tensor - Space-time curvature when Ricci tensor is zero, i.e., tidal force felt

4D System of Equations for Propagation

The system equations are:

The geodesic equations

- Rays= $\frac{\partial^2 [t,x,y,z]}{\partial l^2}$ + DTangent=0

Parallel propagation of internal frames

- FrameEqns= $\frac{\partial E}{\partial l}$ + DFrame=0
- Frame2Eqns= $\frac{\partial F}{\partial l}$ + DFrame2=0

The deviation equations

- Deviation=

  $\frac{\partial^2 W1(l)}{\partial l^2}$ + SecK11*W1(l) + SecK21*W2(l) = 0, and

  $\frac{\partial^2 W2(l)}{\partial l^2}$ + SecK21*W1(l) + SecK22*W2(l) = 0

Reference: David Bergman, Application of Differential Geometry to Acoustics: Development of a Generalized Paraxial Ray-Trace Procedure from Geodesic Deviation, Naval Research Laboratory, Washington, DC, NRL/MR/7140-05-8835, January 18, 2005.

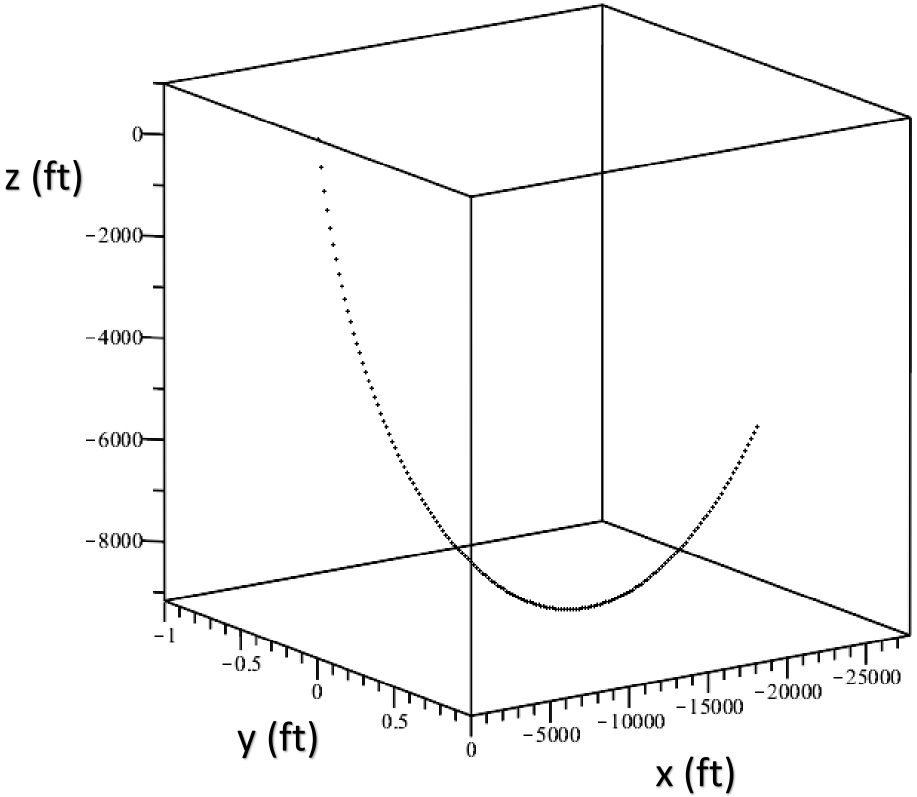
MAPLE 4D Ray Resulting Plot

System Equations: Rays, FrameEqns, Frame2Eqns, Deviation

Numerically solve the system of equations with the initial conditions for
X, Y, Z, T and initial launch angles and unit time derivative along the ray

Second and Third frames along the ray (the first is the Tangent vector)
(E1, E2, E3, E4) and
(F1, F2, F3, F4)

And the two deviation vectors
W1 and W2, and their derivatives, define the spreading loss.



(x, y, z) plot of ray trace

Reference: David Bergman, Application of Differential Geometry to Acoustics: Development of a Generalized Paraxial Ray-Trace Procedure from Geodesic Deviation, Naval Research Laboratory, Washington, DC, NRL/MR/7140-05-8835, January 18, 2005.

Point-to-Point Ray

In order to solve a point (source) to point (observer) propagation problem, the following changes in the initial conditions must be made to the 4D Ray. The key is that slowness vectors, sx and sy, remain constant along a given ray in stratified media.

Launch Angle Initial Conditions

(x,y,z)_source
Dx_(Phi_0,Theta_0)
Dy_(Phi_0,Theta_0)
Dz_(Phi_0,Theta_0)
T0_source = 0
Dt(0) = 1

Solve for t(l), x(l), y(l), z(l)

Two Point Ray Initial Conditions (in stratified media, along a null geodesic line)

(x,y,z)_source
(x,y,z)_observer
(dx/dl)_source=sx*ceff_source
(dy/dl)_source=sy*ceff_source
(dx/dl)_observer=sx*ceff_observer
(dy/dl)_observer=sy*ceff_observer
T0_source = 0
Dt(0) = 1

Solve for t(l), x(l), y(l), z(l), sx, sy

Conclusion

- The acoustic community would benefit from advanced propagation codes for all aircraft sectors and all weather conditions

- Acoustic propagation needs codes that are “More Versatile” and “Easier to Use”

- The 4D (relativistic) propagation takes effort
  - the mathematical physics is intense (time to learn and ramp up)
  - find the right analysis tool(s)
  - collaboration with others in this field to develop the methods and codes

- Current status requires further work to obtain the complete solution to these equations with given point-to-point initial conditions

- 4D relativistic rays could be a game changer for propagation