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Calculating Factors of Safety and Margins of Safety From Interaction Equations for Preloaded Bolts

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June 2022

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Summary

This report presents the derivation of the factored interaction equations for preloaded bolts in two- or three-dimensional load-ratio space. This is an extension of the previous work of the first author, which covered factored interaction equations consisting of load or stress ratios that were either solely constant or solely varying quantities. In this report, the factored interaction equations for preloaded bolts consist of an axial load ratio that is a combination of a constant quantity, comprising the preload and thermal load at preload, and a varying quantity, comprising the tensile and nonplastic bending loads; this was not covered in the previous work. Two forms of the factored interaction equations for preloaded bolts, arising from two perspectives, are presented and compared. Closed-form solutions for the tensile, shear, and bending loads as functions of the other concurrent loads are presented that produce a margin of safety of 0.

Introduction

Reference 1 presented the derivation of the relationship between the factor of safety (*FS*), margin of safety (*MS*), and a given interaction equation in two- or three-dimensional stress-ratio (or load- or moment-ratio) space. Interaction equations characterize combinations of stress, loads, or moments that cause structural failure. These equations have been expressed in terms of load ratios, moment ratios, or stress ratios and produce a *FS* that is, in turn, used to calculate the *MS*. Reference 1 covered interaction equations that consisted of stress ratios that were either exclusively constant or exclusively varying quantities. Preloaded bolts with nonplastic bending are characterized by interaction equations that have an axial load ratio that is a combination of a constant term comprising the preload and thermal load at preload and a varying term comprising the tensile and bending loads. This work extends that of Reference 1 to cover the derivation of the factored interaction equations for preloaded bolts with nonplastic bending in two- or three-dimensional load-ratio space.

The factored interaction equations, and thus the *MS* for preloaded bolts, may be obtained from two different perspectives. From the first perspective, the axial load ratio starts from 0 at the stress-free or load-free state of the bolt, denoted herein as the standard perspective. From the second perspective, the axial load ratio starts from 0 at the preload and thermal load condition, with the allowable axial load reduced accordingly; this is denoted herein as the alternate perspective. In both perspectives, all load ratios are 0 at the origin of load-ratio space.

The factored interaction equations from the two perspectives are derived along with closed-form solutions for the tensile, shear, and bending loads as functions of the other concurrent loads that produce a *MS* of 0. A comparison of these load equations from the standard and alternate perspectives is presented.

The symbols used within this report are listed in Appendix A.

Interaction Equations

The mathematical developments in this work leading up to the factored interaction equations largely follow that of Reference 1. As will be shown herein, changes result in the factored interaction equations of Reference 1 due to the inclusion of preload and thermal load in the tensile load ratio.

The interaction of one load ratio R with others at failure can be characterized using an interaction equation expressed as

$$\left(\Sigma R_i'\right)^a + \left(\Sigma R_j'\right)^b + \left(\Sigma R_k'\right)^c = 1 \qquad \text{for } i \neq j \neq k \tag{1}$$

where the exponents a, b, and c are determined from experimental test results and/or theory. The load ratios R_i', R_j', and R_k' characterize the failure load state. The summations and corresponding indices permit the combination of load ratios as dictated by experiment or theory.

Referring to Figure 1, the load state at point P has coordinates (ΣR_i, ΣR_j, and ΣR_k), and it lies within the bounds of the failure (or interaction) surface (or equation), which indicates that the structure has reserve strength. The load state at point P' has coordinates ($\Sigma R_i'$, $\Sigma R_j'$, and $\Sigma R_k'$) and lies on the failure (or interaction) surface, indicating that the structure does not have reserve strength at point P'.

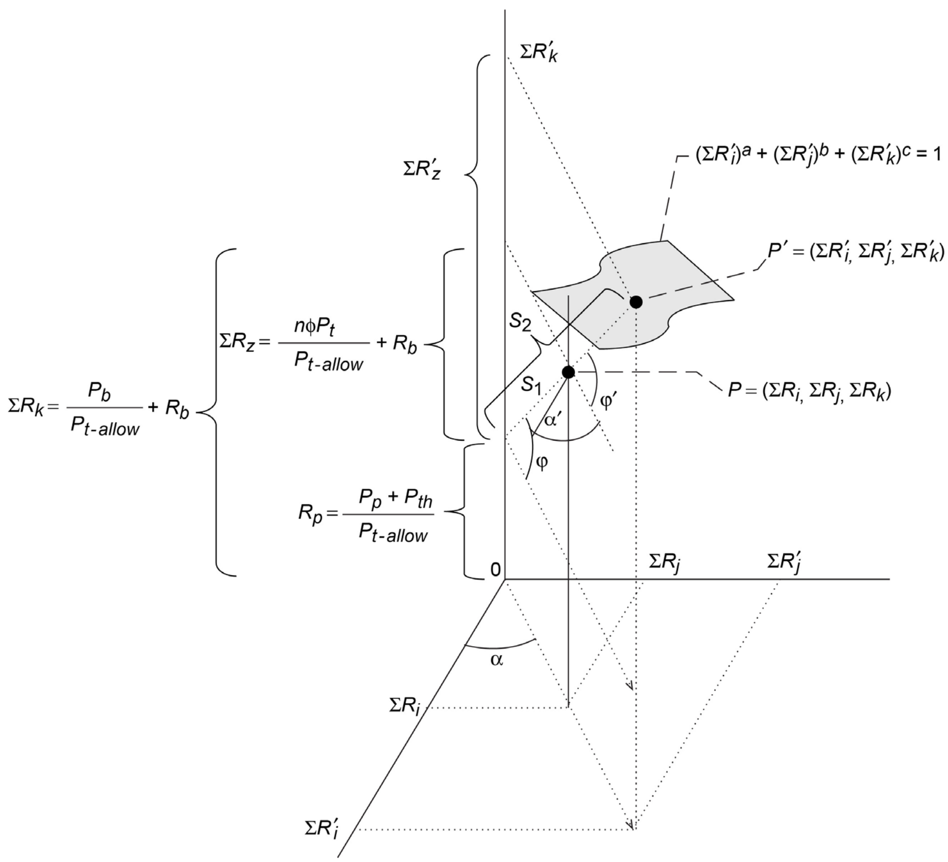

Figure 1.—Generic failure surface (in three-dimensional load-ratio space), illustrating magnitude S_1 of current load-ratio state P and magnitude S_2 of load-ratio state at failure P' for case where all load-ratio sums ΣR are proportional up to failure; here, n refers to a load introduction factor; ϕ, to a joint stiffness factor; P_t, to external, axial load; $P_{t\text{-}allow}$, to bolt tensile load allowable; P_0, to preload; P_{th}, to thermal load; R_b, to bending load ratio; R_p, to combined preload and thermal load ratio.

Margin of Safety

One of the most concise definitions of the *MS* is given in Reference 2 as "the ratio of excess strength to the required strength," where, in terms of load, the excess strength is the difference between the allowable load and the required load. The required load is the product of the factor of safety $\mathcal{FS}$ (factor of safety definition 2, see Ref. 1), any additional factors, and the calculated load. This is expressed mathematically as

$$MS = \frac{(\text{allowable load}) - (\text{required load})}{(\text{required load})} = \frac{(\text{allowable load})}{(\mathcal{FS})(\text{additional factors})(\text{calculated load})} - 1 \qquad (2)$$

Referring to Figure 1, the allowable load ratio combination corresponds to point P' on the failure surface or curve, and the required load ratio combination corresponds to point P. The preloaded bolt load trajectory starts at zero load and traverses the vertical axis to the point of combined preload and thermal load equilibrium, designated as load ratio R_p, prior to external loads being applied. As external loads are applied, the load ratio trajectory extends from R_p to a load-ratio combination of interest at point P. As the loads are further increased, the load-ratio trajectory extends to the allowable (failure) load ratio combination at point P'. It is assumed that the load ratios continue beyond point P up to point P' with the same proportionality to each other as they had from R_p to point P. This assumption determines the point of intersection of the load-ratio trajectory with the failure envelope or curve. The allowable and required load ratios are load-trajectory dependent.

Referring to Figure 1, the allowable load beyond preload is represented by the line segment $\overrightarrow{R_p P'}$, and the required load ratio beyond preload is represented by the line segment $\overrightarrow{R_p P}$. Equation (2), in terms of these line segments, becomes

$$MS = \frac{\overrightarrow{R_p P'} - \overrightarrow{R_p P}}{\overrightarrow{R_p P}} \qquad (3)$$

Referring to Figure 2, it is not correct to assume the required and allowable bolt loads are associated with a load trajectory originating at the origin of the load-ratio space and extending through point P to point P'', because this load-ratio trajectory does not appropriately incorporate the preload procedure. This erroneous load trajectory results in an erroneous proportionality between axial load ratio, herein denoted as $\sum R_k$, and the shear load ratio, herein denoted as $\sum R_i$. This in turn results in the erroneous allowable load ratio at point P''. However, it is possible to define a *MS* in terms of the line segments $\overrightarrow{0P}$ and $\overrightarrow{0P'}$, as shown below in Equation (4) if the analyst is only interested in determining the load ratios at a $MS = 0$:

$$MS = \frac{\overrightarrow{0P'} - \overrightarrow{0P}}{\overrightarrow{0P}} \qquad (4)$$

As shown in Figure 2, as point P reaches point P' along the true load trajectory segment, $\overrightarrow{0P}$ becomes equivalent to segment $\overrightarrow{0P'}$, the *MS* from Equation (4) equaling 0 concurrently with that of Equation (3). Aside from the point where $MS = 0$, the nonzero margins per Equations (3) and (4) are generally different; Equation (3) assumes load ratios remain proportional up to failure, whereas Equation (4) does not.

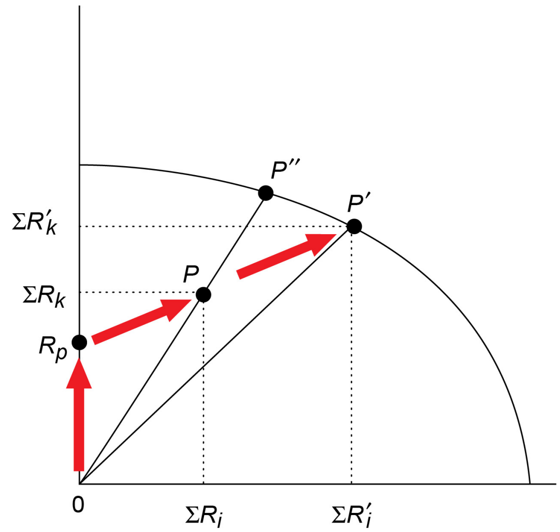

Figure 2.—Generic failure curve (in two-dimensional load-ratio space) illustrating several load paths. Point *P* designates load-ratio state of interest. Point *P′* designates load-ratio state at failure for case where all load-ratio sums Σ*R* maintain same proportionality that existed from point R_p (see Figure 1) to point *P*. Point *P″* is erroneous allowable load-ratio state.

Referring to Figure 1, the following relationships may be observed:

$$S_1 \cos\varphi \cos\alpha = \Sigma R_i \rightarrow S_1 C_1 = \Sigma R_i \tag{5}$$

$$S_1 \cos\varphi \sin\alpha = \Sigma R_j \rightarrow S_1 C_2 = \Sigma R_j \tag{6}$$

$$R_p + S_1 \sin\varphi = \Sigma R_k \rightarrow R_p + S_1 C_3 = \Sigma R_k \tag{7}$$

$$S_2 \cos\varphi \cos\alpha = \Sigma R'_i \rightarrow S_2 C_1 = \Sigma R'_i \tag{8}$$

$$S_2 \cos\varphi \sin\alpha = \Sigma R'_j \rightarrow S_2 C_2 = \Sigma R'_j \tag{9}$$

$$R_p + S_2 \sin\varphi = \Sigma R'_k \rightarrow R_p + S_2 C_3 = \Sigma R'_k \tag{10}$$

where the trigonometric terms are constants for a given load state, and for conciseness these are denoted as C_1, C_2, and C_3 as shown above.

It should also be mentioned that the load ratios that are located within the bounds of the failure envelope or failure curve are those load ratios without the prime, and these may include factors of safety and other proprietary factors as products of the load-ratio numerator, although this is not explicitly shown herein.

The load ratios with the prime are the load ratios at the point of failure. Because there is no uncertainty about the point of failure, there are no factors of safety or other proprietary factors incorporated into these load ratios. These load ratios are located on the failure surface or curve.

Utilizing the equations for the distance between two points in three-dimensional space, the ratio in Equation (3) becomes

$$\frac{\overrightarrow{R_pP'}-\overrightarrow{R_pP}}{\overrightarrow{R_pP}}=\frac{\sqrt{\left(\Sigma R_i'-\Sigma R_i\right)^2+\left(\Sigma R_j'-\Sigma R_j\right)^2+\left(\Sigma R_k'-\Sigma R_k\right)^2}}{\sqrt{\left(\Sigma R_i-0\right)^2+\left(\Sigma R_j-0\right)^2+\left(\Sigma R_k-R_p\right)^2}} \tag{11}$$

Substituting Equations (5) to (10) into Equation (11) gives

$$\frac{\overrightarrow{R_pP'}-\overrightarrow{R_pP}}{\overrightarrow{R_pP}}=\frac{\sqrt{\left(S_2C_1-S_1C_1\right)^2+\left(S_2C_2-S_1C_2\right)^2+\left[R_p+S_2C_3-R_p-S_1C_3\right]^2}}{\sqrt{\left(S_1C_1\right)^2+\left(S_1C_2\right)^2+\left(R_p+S_1C_3-R_p\right)^2}} \tag{12}$$

Factoring out terms in Equation (12) yields

$$\frac{\overrightarrow{R_pP'}-\overrightarrow{R_pP}}{\overrightarrow{R_pP}}=\frac{\sqrt{\left(S_2-S_1\right)^2\left(C_1^2+C_2^2+C_3^2\right)}}{\sqrt{S_1^2\left(C_1^2+C_2^2+C_3^2\right)}} \tag{13}$$

Rearranging Equation (13) simplifies to

$$\frac{\overrightarrow{R_pP'}-\overrightarrow{R_pP}}{\overrightarrow{R_pP}}=\frac{\left(S_2-S_1\right)\sqrt{\left(C_1^2+C_2^2+C_3^2\right)}}{S_1\sqrt{\left(C_1^2+C_2^2+C_3^2\right)}}=\frac{S_2-S_1}{S_1} \tag{14}$$

Substituting Equation (14) into Equation (3), the *MS* becomes

$$MS=\frac{S_2-S_1}{S_1}=\frac{S_2}{S_1}-1 \tag{15}$$

As can be seen from Equation (15), the *MS* is expressed in terms of the distances S_1 and S_2 as defined in Figure 1.

Factor of Safety

Reference 3 defines the factor of safety to be the calculated ratio of S_2 to S_1. The factor of safety in this regard will be denoted as *FS* (definition 1, see Ref. 1), thus differentiating it from the specified factors of safety definition 2, denoted generically in this work as $\mathcal{FS}$. Therefore,

$$FS=\frac{S_2}{S_1} \tag{16}$$

Substituting Equation (16) into Equation (15) gives the *MS* in terms of the factor of safety (definition 1):

$$MS=FS-1 \tag{17}$$

The factor of safety *FS* must be determined in order to solve for the *MS* in Equation (17). A factored interaction equation (which will be described in the next section, "Factored Interaction Equations") along with Equation (17) form a system of two equations in two unknowns, *FS* and *MS*. Once the factored interaction equation is solved for *FS*, it is substituted into Equation (17) to determine the *MS*. The *FS* as defined in Equation (16) could be determined graphically from the load-ratio space as shown in Figure 1, Figure 2, and Figure 3, but it is more convenient to determine the *FS* from factored interaction equations. The following sections derive the factored interaction equations, which consist of the factor of safety *FS*, and various load ratios. The *FS* can be determined using a root-finding algorithm, where the *FS* is the lowest positive root of the factored interaction equation.

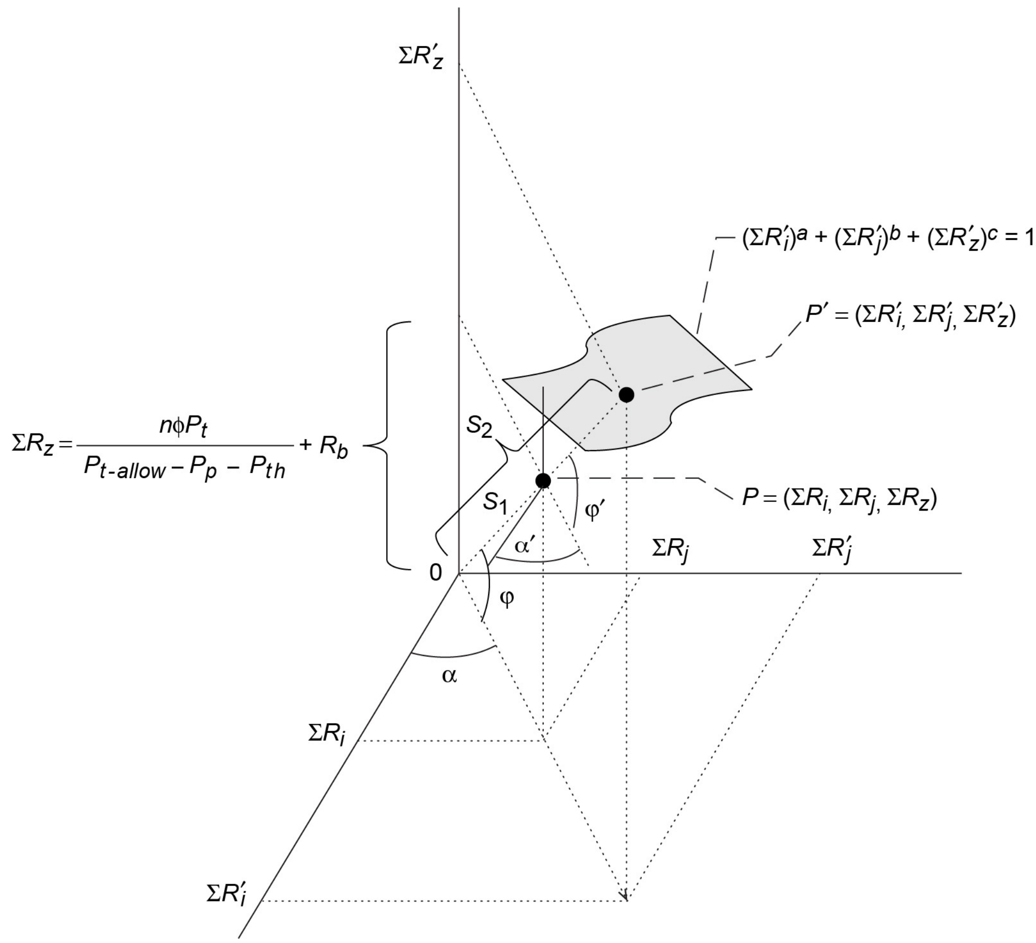

Figure 3.—Generic interaction curve (in three-dimensional load-ratio space) involving three load ratios ΣR, illustrating current load-ratio state *P* and load-ratio state at failure *P′*, from the alternate perspective.

Factored Interaction Equations

In the following mathematical development through Equation (24), the load ratios are assumed to be proportional to each other. This means that $\varphi' = \varphi$ and $\alpha' = \alpha$ in Figure 1. Multiplying both sides of Equation (16) by $\cos\varphi\cos\alpha$ and rearranging gives

$$(FS)S_1 \cos\varphi\cos\alpha = S_2 \cos\varphi\cos\alpha \tag{18}$$

and upon substituting Equations (5) and (8) into Equation (18),

$$(FS)\Sigma R_i = \Sigma R_i' \tag{19}$$

Multiplying both sides of Equation (16) by $\cos\varphi\sin\alpha$ and rearranging leads to

$$(FS)S_1 \cos\varphi\sin\alpha = S_2 \cos\varphi\sin\alpha \tag{20}$$

and upon substituting Equations (6) and (9) into Equation (20),

$$(FS)\Sigma R_j = \Sigma R_j' \tag{21}$$

Multiplying both sides of Equation (16) by $\sin\varphi$ and rearranging gives

$$(FS)S_1 \sin\varphi = S_2 \sin\varphi \tag{22}$$

and upon substituting Equations (7) and (10) into Equation (22),

$$(FS)\left(\Sigma R_k - R_p\right) = \Sigma R_k' - R_p \tag{23}$$

Substituting Equations (19), (21), and (23) into Equation (1) generates Equation (24), which describes the relationships between load ratios in terms of the *FS* for the scenario that all load ratios beyond preload maintain proportionality prior to and at failure:

$$\left[(FS)\Sigma R_i\right]^a + \left[(FS)\Sigma R_j\right]^b + \left[(FS)\left(\Sigma R_k - R_p\right) + R_p\right]^c = 1 \tag{24}$$

Equation (24) explicitly accounts for the preload; is valid for all failure surfaces in three-dimensional load-ratio space, including exponents with noninteger values; and corresponds to the failure path from points P to P' shown in Figure 1. This type of equation describing the relationships between the load ratios prior to, and at, failure in terms of the factor of safety *FS* was denoted as the “factored interaction equation” in Reference 1 and will be denoted as such herein.

Standard Perspective

As shown in Appendix A.8 of Reference 4, it is assumed the tensile load ratio R_t and the bending stress ratio R_b can be summed in both the linear elastic range and for the ultimate condition. Although it is not theoretically correct to do this for the ultimate condition, it is conservative.

Therefore,

$$\sum R_k = R_t + R_b = \frac{P_{tb}}{P_{t\text{-}allow}} + \frac{f_b}{F_t} \tag{25}$$

where $P_{t\text{-}allow}$ is the tensile load allowable, f_b is the stress due to bending, and F_t is the maximum allowable tensile stress in the bolt. Recall the equation of tensile load P_{tb} for a bolt with a preload P_p and applied external tensile load P_t, as shown in Equation (8) of Reference 4, with thermal load P_{th} added:

$$P_{tb} = P_p + P_{th} + n\phi P_t \tag{26}$$

where n is a load introduction factor, and ϕ is the stiffness factor.
Upon substituting Equation (26) into Equation (25),

$$\sum R_k = \frac{P_p + P_{th} + n\phi P_t}{P_{t\text{-}allow}} + \frac{f_b}{F_t} = R_p + \frac{n\phi P_t}{P_{t\text{-}allow}} + \frac{f_b}{F_t} \tag{27}$$

where the load ratio for the combined preload and thermal load state is given as

$$R_p = \frac{P_p + P_{th}}{P_{t\text{-}allow}} \tag{28}$$

When including various factors into the numerator of the load ratios, it is acknowledged that they do not usually apply to the preload term because the uncertainty in the preload has typically already been accounted for in the specified preload value. It is straightforward to include various factors in the load ratios as desired, and the equations presented herein may be modified accordingly.

The shear load ratio R_s has been assigned to the "i" axis of the load-ratio space and is given as

$$\sum R_i = R_s = \frac{P_s}{P_{s\text{-}allow}} \tag{29}$$

where P_s and $P_{s\text{-}allow}$ are the bolt shear load and allowable bolt shear load, respectively.

Substituting Equations (27) to (29) into Equation (24) gives the final form of the factored interaction equation for tension, shear, and bending in the elastic range:

$$\left[(FS)\frac{P_s}{P_{s\text{-}allow}}\right]^a + \left[(FS)\left(\frac{n\phi P_t}{P_{t\text{-}allow}} + \frac{f_b}{F_t}\right) + \frac{P_p + P_{th}}{P_{t\text{-}allow}}\right]^c = 1 \tag{30a}$$

where the terms associated with the "j" axis in Equation (24) were omitted because there were no load ratios to be associated with it.

By multiplying the numerator and denominator of the bending stress ratio by the tensile stress area A, the bending stress ratio is converted into a bending load ratio:

$$\left[(FS)\frac{P_s}{P_{s\text{-}allow}}\right]^a + \left[(FS)\left(\frac{n\phi P_t + Af_b}{P_{t\text{-}allow}}\right) + \frac{P_p + P_{th}}{P_{t\text{-}allow}}\right]^c = 1 \tag{30b}$$

where $P_{t\text{-}allow} = AF_t$. In Equations (30a) and (30b) the bending stress ratios (implicitly in Eq. (30b)) may be converted to their equivalent bending moment ratios using the flexure formula.

Alternate Perspective

The factored interaction Equations (30a) and (30b) assumed the initial axial load or stress state to be the unstressed bolt. The alternate perspective assumes the axial load ratio starts from 0 at the preload and thermal load condition at the origin of the load-ratio space. That is, the preload and thermal load are subtracted from the axial load, offsetting the origin of the load-ratio space along the axial coordinate axis as shown in Figure 3, with a corresponding reduced allowable axial load as shown below in the denominator of Equation (37). In both perspectives all load ratios are 0 at the origin of load-ratio space.

The axial stress in the bolt, assuming a linear elastic system, is the sum of the stresses due to preload f_p, thermal load f_{th}, tensile load above preload f_t, and bending f_b. Whereas the sum is theoretically correct in the linear elastic range, it is not theoretically correct for the ultimate condition; rather, it is conservative:

$$\sigma = f_p + f_{th} + f_t + f_b \tag{31}$$

Assuming the bolt stress is at the maximum allowable tensile stress F_t, whether that be a yield or ultimate value as the case may be, Equation (31) becomes

$$F_t = f_p + f_{th} + (f_t + f_b)' = f_p + f_{th} + f_{allow} \tag{32}$$

where f_{allow} is the allowable combined stress due to both the external tension load above preload and the bending $(f_t + f_b)'$. Solving Equation (32) for the allowable stress,

$$F_t - f_p - f_{th} = f_{allow} \tag{33}$$

Multiplying and dividing Equation (33) by the tensile stress area A, the allowable stress is obtained in terms of loads:

$$\frac{A\left(F_t - f_p - f_{th}\right)}{A} = \frac{P_{t\text{-}allow} - P_p - P_{th}}{A} = f_{allow} \tag{34}$$

The axial stress in the bolt due to tensile load and bending moment may be obtained by dividing Equation (26) by the tensile stress area A and adding the bending stress:

$$\sigma = \frac{P_p + P_{th} + n\varphi P_t}{A} + f_b \tag{35}$$

Since in this alternate view only the loads above preload are of interest, Equation (35) becomes

$$\sigma = \frac{n\varphi P_t}{A} + f_b \tag{36}$$

One can determine the axial load ratio, which is equivalent to the axial stress ratio, by dividing Equation (36) by Equation (34):

$$\frac{\sigma}{f_{allow}} = \Sigma R_z = R_t + R_b = \frac{n\phi P_t}{\left(P_{t\text{-}allow} - P_p - P_{th}\right)} + \frac{A\,f_b}{\left(P_{t\text{-}allow} - P_p - P_{th}\right)} \tag{37}$$

R_b is called a bending load ratio as opposed to a bending moment ratio, because the numerator and denominator are loads and not moments. The numerator is the product of the bolt cross-sectional area and the bolt bending stress, giving units of load. Since the units are in load and since the stress comes from bending, the terminology "bending load" is used.

In this alternate view, Equations (19), (21), and (23) can be substituted into Equation (1) with the subscript "*k*" changed to subscript "*z*" corresponding to the alternate view of the origin of the load-ratio space as shown in Figure 3, and with R_p set to 0 in Equation (23), resulting in the following factored interaction equation:

$$\left[(FS)\Sigma R_i\right]^a + \left[(FS)\Sigma R_j\right]^b + \left[(FS)\Sigma R_z\right]^c = 1 \tag{38}$$

Substituting Equations (29) and (37) into Equation (38) yields the alternate form of the factored interaction equation:

$$\left[(FS)\frac{P_s}{P_{s\text{-}allow}}\right]^a + \left[(FS)\left(\frac{n\phi P_t + Af_b}{P_{t\text{-}allow} - P_p - P_{th}}\right)\right]^c = 1 \tag{39}$$

Equations (30a) and (30b) are the factored interaction equations for a preloaded bolt where the load ratios are defined with respect to the origin of the load-ratio space, which is at a point of zero load or stress in the bolt. Equation (39) is the factored interaction equation for a preloaded bolt where the load ratios are defined with respect to an origin of the load-ratio space corresponding to a point of combined preload and thermal load and where the allowable axial load has been reduced accordingly. The bending stress ratio may be converted to an equivalent bending moment ratio using the flexure formula, in this case with the allowable bending moment being based on f_{allow} (Eq. (34)).

One of these interaction equations (Eqs. (30a), (30b), or (39)) and the *MS* Equation (Eq. (17)) form a system of two equations in two unknowns; upon solving one of the factored interaction equations for the *FS* and substituting it into Equation (17), the *MS* is determined. A root-finding algorithm using a combination of Newton-Raphson and bisection (Ref. 5), has been used successfully at the NASA Glenn Research Center for solving for the *FS* from the factored interaction equations.

The question naturally arises as to which factored interaction equation should be used to determine the *FS* and thus the *MS*. Usually, an analyst will want to use, or at least start out using, the factored interaction equation leading to the most conservative *MS*. This would correspond to the factored interaction equation producing the lowest *FS*.

The following section compares the factored interaction equations—Equations (30b) and (39), corresponding to the standard and alternate perspectives, respectively—to determine which factored interaction equation, and thus which perspective, leads to the most conservative *MS*.

It should be mentioned that the factored interaction equations are equivalent for the case where the preload and thermal load are not active at bolt failure. This is the case where the joint separates before bolt failure. In this situation, the preload and thermal load terms are 0 in the interaction equations.

Comparison of Standard and Alternate Forms of Interaction Equations

Tension and Bending Without Shear

It should be mentioned that the factor of safety, *FS*, based on either Equations (30a) and (30b) or Equation (39) for the case of tension and bending without shear, results in identical margins of safety, *MS*. To show the equivalence of the standard and alternate forms of the factored interaction equations for the case of tension and bending without shear, first consider the standard form, Equation (30b), without the shear load term:

$$\left[(FS)\left(\frac{n\phi P_t + Af_b}{P_{t\text{-}allow}}\right) + \frac{P_p + P_{th}}{P_{t\text{-}allow}}\right]^c = 1 \tag{40}$$

Solving for the factor of safety *FS*,

$$FS = \frac{P_{t\text{-}allow} - \left(P_p + P_{th}\right)}{n\phi P_t + Af_b} \tag{41}$$

Next consider the alternate form of the factored interaction equation, for the case of tension and bending without shear:

$$\left[(FS)\left(\frac{n\phi P_t + Af_b}{P_{t\text{-}allow} - P_p - P_{th}}\right)\right]^c = 1 \tag{42}$$

Solving for the factor of safety *FS*,

$$FS = \frac{P_{t\text{-}allow} - P_p - P_{th}}{n\phi P_t + Af_b} \tag{43}$$

As can be seen by comparing Equations (41) and (43), the *FS* is the same for either form of interaction equation for tension and bending without shear, and in either case, the *MS* is the same and is

$$MS = FS - 1 = \frac{P_{t\text{-}allow} - P_p - P_{th}}{n\phi P_t + Af_b} - 1 \tag{44}$$

Now Equation (30b) is compared with Equation (39) for the case of interaction of tension, bending, and shear to see which factored interaction equation is more conservative; that is, which equation predicts a lower external tension failure load, shear failure load, and bending failure load as a function of the other concurrent load ratios at the point of *MS* = 0. This exercise also has the benefit of providing closed-form solutions for the tensile, shear, and bending failure loads, which produces a *MS* of 0 as a function of the other concurrent load ratios.

Tension, Bending, and Shear

Appendix B presents the derivation of the tension failure load that produces a *MS* of 0 as a function of the concurrent shear and bending loads. Appendix C presents the derivation of the shear failure load that produces a *MS* of 0 as a function of the concurrent tension and bending loads, and Appendix D presents the derivation of the bending failure load that produces a *MS* of 0 as a function of the concurrent tension and shear loads. All of the failure load equation derivations are provided from the standard and alternate perspectives, and these are presented in Table I. Appendix B, Appendix C, and Appendix D also provide a comparison of the failure load equations resulting from the standard and alternate perspectives. The tension, shear, and bending loads resulting from these load equations correspond to the coordinates of S_2 on the failure envelope as shown in Figure 1 and Figure 3. These comparisons indicate that the failure envelope loads of tension, shear, and bending resulting from factored interaction Equations (30a) and (30b) are lower than those resulting from factored interaction Equation (39). This means that S_2 and thus the *FS* (Eq. (16)) resulting from Equations (30a) and (30b) is less than that resulting from Equation (39). Therefore, the *MS* (Eq. (17)) as determined from the standard perspective, factored interaction Equations (30a) and (30b), is more conservative than the *MS* as predicted from the alternate perspective (i.e., factored interaction Equation (39)—when the preload and thermal load are active at bolt failure). This is the case when the bolt ruptures before the joint separates.

TABLE I.—EQUATIONS FOR TENSION, SHEAR, OR BENDING FAILURE LOAD AS FUNCTION OF OTHER CONCURRENT LOAD RATIOS AT MARGIN OF SAFETY $MS = 0$

[Failure loads are shown as derived from factored interaction equations resulting from both standard and alternate perspective.]

Failure load corresponding to $MS = 0$	From standard factored interaction Equation (30b)	From alternate factored interaction Equation (39)
Tension	$P_t = \frac{P_{t\text{-}allow}}{n\phi}\left[1-\left(\frac{P_s}{P_{s\text{-}allow}}\right)^a\right]^{(1/c)} - \frac{Af_b + P_p + P_{th}}{n\phi}$	$(P_t)_{alternate} = \frac{P_{t\text{-}allow} - P_p - P_{th}}{n\phi}\left[1-\left(\frac{P_s}{P_{s\text{-}allow}}\right)^a\right]^{(1/c)} - \frac{Af_b}{n\phi}$
Shear	$P_s = P_{s\text{-}allow}\left[1-\left(\frac{n\phi P_t + Af_b + P_p + P_{th}}{P_{t\text{-}allow}}\right)^c\right]^{(1/a)}$	$(P_s)_{alternate} = P_{s\text{-}allow}\left[1-\left(\frac{n\phi P_t + Af_b}{P_{t\text{-}allow} - P_p - P_{th}}\right)^c\right]^{(1/a)}$
Bending	$Af_b = P_{t\text{-}allow}\left[1-\left(\frac{P_s}{P_{s\text{-}allow}}\right)^a\right]^{(1/c)} - n\phi P_t - P_p - P_{th}$	$(Af_b)_{alternate} = \left(P_{t\text{-}allow} - P_p - P_{th}\right)\left[1-\left(\frac{P_s}{P_{s\text{-}allow}}\right)^a\right]^{(1/c)} - n\phi P_t$

Conclusion

The factored interaction equations for preloaded bolts have been derived from two perspectives: (1) the standard perspective, where the axial load ratio starts from 0 at the stress-free or load-free state of the bolt, and (2) the alternate perspective, where the axial load ratio starts from 0 at the preload and thermal load condition and where the allowable axial load has been reduced accordingly. All of the load ratios from both perspectives are 0 at the origin of load-ratio space.

Based upon the factored interaction equations resulting from the standard and alternate perspectives, closed-form equations are derived for the tensile, shear, and nonplastic bending failure loads as functions of the other concurrent load ratios that produce margins of safety equal to 0.

A comparison of these closed-form equations reveals that for cases where the preload and thermal load are active at bolt failure, the standard perspective (where the axial load ratio starts from 0 at the stress-free or load-free state of the bolt) is more conservative for bolt loading than the alternate perspective (where the axial load ratio starts from 0 at the preload and thermal load condition and where the allowable axial load has been reduced accordingly). Note that preload and thermal load are active at bolt failure when the joint does not separate before the bolt ruptures.

Where the preload and thermal load are not active at bolt failure, the standard and alternate perspectives are equivalent.

Appendix A.—Nomenclature

A	tensile area of bolt
a,b,c	exponents in interaction Equation (1)
$C_{1,2,3}$	constants in Equations (5) to (10) and (12) to (14), representing trigonometric functions for a given load state (cf. Figure 1)
FS	factor of safety, definition 1: a calculated ratio, typically of two quantities of the same character. Calculated ratio S_2/S_1.
$\mathcal{FS}$	factor of safety, definition 2: a specified multiplying factor applied to calculated loads for the purpose of creating a margin of safety
F_t	maximum allowable axial stress in the bolt, either a yield or ultimate value
f_{allow}	allowable stress above the stress due to the combination of preload and thermal load
f_b	bending stress
f_p	stress due to preload
f_t	stress due to tensile load above preload
f_{th}	stress due to thermal load
MS	margin of safety. A measure of a structure's predicted reserve strength in excess of the product of the load under consideration, the applicable factor of safety (definition 2 of factor of safety), and additional factors
n	load introduction factor
P	point in load-ratio space that is interior to the failure surface or failure curve
P_p	bolt preload
P_s	bolt shear load
$P_{s\text{-}allow}$	bolt shear load allowable, either a yield or ultimate condition
P_t	external tensile load applied to preloaded bolts
$P_{t\text{-}allow}$	bolt tensile load allowable, either a yield or ultimate condition; $P_{t\text{-}allow} = A(F_t)$
P_{tb}	bolt tensile load
P_{th}	bolt thermal load
P'	point in load-ratio space on the failure surface or failure curve
P''	point of erroneous allowable load ratio state on the failure surface or failure curve in load-ratio space
$\tilde{P}_{tb}$	sum of bolt tensile and bending loads (see Eq. (C.7))
R	load ratio, ($\mathcal{FS}$)(additional factors)(calculated load)/(allowable load)
R_b	bending load ratio (see Eq. (37))
R_p	combined preload and thermal load ratio
R_s	shear load ratio
R_t	tension load ratio
S_1	magnitude of load state corresponding to point P, interior to the failure surface in load-ratio space (see Figure 1 and Figure 3)
S_2	magnitude of load state corresponding to point P' on the failure surface in load-ratio space (see Figure 1 and Figure 3)
α	angle defined in Figure 1 and Figure 3
φ	angle defined in Figure 1 and Figure 3
ϕ	joint stiffness factor defined as bolt stiffness divided by the sum of the bolt stiffness and the joint stiffness

σ	axial stress
$(\ )_{alternate}$	quantity associated with the alternate perspective (Eq. (39))
$(\ )'$	quantity on the failure surface or failure curve indicating structural failure, either a yield or ultimate condition
$(\ )_{i,j,k,z}$	indices representing different load types, load states, or failure modes and associated axes in load-ratio space

Appendix B.—Tension Failure Load as a Function of Shear Load and Bending Stress at Margin of Safety (*MS*) of 0

This appendix derives the equation for the bolt tensile failure load, as a function of bolt shear and bending load ratios, corresponding to a *MS* of 0. This is done using the standard factored interaction Equation (30b) and the alternate factored interaction Equation (39). A comparison is made to determine which interaction equation—Equation (30b)[1] or (39)—is more conservative in determining the external tension load at *MS* = 0 for any combination of shear and bending load ratios.

$$\left[(FS)\frac{P_s}{P_{s\text{-}allow}}\right]^a + \left[(FS)\left(\frac{n\phi P_t + Af_b}{P_{t\text{-}allow}}\right) + \frac{P_p + P_{th}}{P_{t\text{-}allow}}\right]^c = 1 \tag{30b}$$

$$\left[(FS)\frac{P_s}{P_{s\text{-}allow}}\right]^a + \left[(FS)\left(\frac{n\phi P_t + Af_b}{P_{t\text{-}allow} - P_p - P_{th}}\right)\right]^c = 1 \tag{39}$$

where

A	tensile area of bolt
a, b, c	exponents in interaction equations
f_b	bending stress
FS	factor of safety, definition 1: a calculated ratio, typically of two quantities of the same character
n	load introduction factor
P_t	external tensile load applied to preloaded bolts
P_p	bolt preload
P_s	bolt shear load
$P_{s\text{-}allow}$	bolt shear load allowable
$P_{t\text{-}allow}$	bolt tensile load allowable
P_{th}	bolt thermal load
ϕ	joint stiffness factor defined as bolt stiffness divided by the sum of the bolt stiffness and the joint stiffness

From Equation (30b), *MS* = 0 when *FS* = 1. Setting the *FS* = 1 in Equation (30b) results in

$$\left(\frac{P_s}{P_{s\text{-}allow}}\right)^a + \left[\left(\frac{n\phi P_t + Af_b}{P_{t\text{-}allow}}\right) + \frac{P_p + P_{th}}{P_{t\text{-}allow}}\right]^c = 1 \tag{B.1}$$

[1]Equivalent to Equation (30a): $\left[(FS)\frac{P_s}{P_{s\text{-}allow}}\right]^a + \left[(FS)\left(\frac{n\phi P_t}{P_{t\text{-}allow}} + \frac{f_b}{F_t}\right) + \frac{P_p + P_{th}}{P_{t\text{-}allow}}\right]^c = 1.$

Rearranging,

$$\left[\left(\frac{n\phi P_t + Af_b}{P_{t\text{-}allow}}\right) + \frac{P_p + P_{th}}{P_{t\text{-}allow}}\right]^c = 1 - \left(\frac{P_s}{P_{s\text{-}allow}}\right)^a \tag{B.2}$$

Raising both sides of Equation (B.2) to the ($1/c$) power,

$$\frac{n\phi P_t}{P_{t\text{-}allow}} + \frac{Af_b + P_p + P_{th}}{P_{t\text{-}allow}} = \left[1 - \left(\frac{P_s}{P_{s\text{-}allow}}\right)^a\right]^{1/c} \tag{B.3}$$

Rearranging further,

$$\frac{n\phi P_t}{P_{t\text{-}allow}} = \left[1 - \left(\frac{P_s}{P_{s\text{-}allow}}\right)^a\right]^{1/c} - \frac{Af_b + P_p + P_{th}}{P_{t\text{-}allow}} \tag{B.4}$$

Solving for the external axial force that produces an *MS* of 0,

$$P_t = \frac{P_{t\text{-}allow}}{n\phi}\left[1 - \left(\frac{P_s}{P_{s\text{-}allow}}\right)^a\right]^{1/c} - \frac{Af_b + P_p + P_{th}}{n\phi} \tag{B.5}$$

From the alternate form of the factored interaction Equation (39), $MS = 0$ when $FS = 1$. Setting $FS = 1$ in Equation (39),

$$\left(\frac{P_s}{P_{s\text{-}allow}}\right)^a + \left(\frac{n\phi P_t + Af_b}{P_{t\text{-}allow} - P_p - P_{th}}\right)^c = 1 \tag{B.6}$$

Rearranging,

$$\left(\frac{n\phi P_t + Af_b}{P_{t\text{-}allow} - P_p - P_{th}}\right)^c = 1 - \left(\frac{P_s}{P_{s\text{-}allow}}\right)^a \tag{B.7}$$

Raising both sides of Equation (B.7) to the ($1/c$) power and rearranging again,

$$\frac{n\phi P_t}{P_{t\text{-}allow} - P_p - P_{th}} = \left[1 - \left(\frac{P_s}{P_{s\text{-}allow}}\right)^a\right]^{1/c} - \frac{Af_b}{P_{t\text{-}allow} - P_p - P_{th}} \tag{B.8}$$

Solving for the external force that produces a *MS* of 0,

$$\left(P_t\right)_{alternate} = \frac{P_{t\text{-}allow} - P_p - P_{th}}{n\phi}\left[1 - \left(\frac{P_s}{P_{s\text{-}allow}}\right)^a\right]^{1/c} - \frac{Af_b}{n\phi} \tag{B.9}$$

where the subscript "alternate" designates that this external tension force is from the alternate form of the factored interaction equation.

By subtracting Equation (B.9) from Equation (B.5), it can be determined which value of the external tensile load corresponding to $MS = 0$ is smaller and then which corresponding factored interaction Equation (30b)[1] or (39) is more conservative in predicting the tensile load corresponding to $MS = 0$ for the case of combined tension, bending, and shear.

Subtracting Equation (B.9) from Equation (B.5) and canceling some terms,

$$\begin{aligned} P_t - \left(P_t\right)_{alternate} = &\left\{\cancel{\frac{P_{t\text{-}allow}}{n\phi}\left[1 - \left(\frac{P_s}{P_{s\text{-}allow}}\right)^a\right]^{1/c}} - \frac{Af_b + P_p + P_{th}}{n\phi}\right\} \\ &-\left\{\frac{\cancel{P_{t\text{-}allow}} - P_p - P_{th}}{n\phi}\left[1 - \left(\frac{P_s}{P_{s\text{-}allow}}\right)^a\right]^{1/c} - \frac{Af_b}{n\phi}\right\} \end{aligned} \tag{B.10}$$

Canceling more terms gives

$$\begin{aligned} P_t - \left(P_t\right)_{alternate} = &\left[-\frac{\cancel{Af_b} + P_p + P_{th}}{n\phi}\right] \\ &-\left\{\frac{-\left(P_p + P_{th}\right)}{n\phi}\left[1 - \left(\frac{P_s}{P_{s\text{-}allow}}\right)^a\right]^{1/c} - \cancel{\frac{Af_b}{n\phi}}\right\} \end{aligned} \tag{B.11}$$

The final difference becomes

$$P_t - \left(P_t\right)_{alternate} = -\frac{\left(P_p + P_{th}\right)}{n\phi} + \frac{\left(P_p + P_{th}\right)}{n\phi}\left[1 - \left(\frac{P_s}{P_{s\text{-}allow}}\right)^a\right]^{1/c} \tag{B.12}$$

---

[1]Equivalent to Equation (30a): $\left[\left(FS\right)\frac{P_s}{P_{s\text{-}allow}}\right]^a + \left[\left(FS\right)\left(\frac{n\phi P_t}{P_{t\text{-}allow}} + \frac{f_b}{F_t}\right) + \frac{P_p + P_{th}}{P_{t\text{-}allow}}\right]^c = 1$.

Since $P_p + P_{th} > 0$, for practical situations, and since $0 < \left(P_s / P_{s\text{-}allow}\right) \leq 1$, the right-hand side of Equation (B.12) is always negative, and as a result, the external load corresponding to the standard factored interaction equation is always less than the external load corresponding to the alternate form of the interaction equation at *MS* = 0,

$$\left(P_t\right)_{alternate} > P_t \tag{B.13}$$

Therefore, the standard factored interaction equations, Equations (30a) and (30b), are more conservative than the alternate factored interaction Equation (39), for determining the external tensile failure load that produces a *MS* of 0 for interaction of tension, shear, and bending.

Appendix C.—Shear Failure Load as a Function of Tension and Bending Load Ratios at Margin of Safety (*MS*) of 0

This appendix derives the equation for the bolt shear failure load, as a function of bolt tension and bending load ratios, corresponding to a $MS = 0$. This is done using the standard factored interaction Equation (30b) and the alternate factored interaction Equation (39). A comparison is made to determine which interaction equation (Eq. (30b)[1] or (39)) is more conservative in determining the external shear failure load at $MS = 0$ for any combination of tension and bending load ratios.

$$\left[(FS)\frac{P_s}{P_{s\text{-}allow}}\right]^a + \left[(FS)\left(\frac{n\phi P_t + Af_b}{P_{t\text{-}allow}}\right) + \frac{P_p + P_{th}}{P_{t\text{-}allow}}\right]^c = 1 \tag{30b}$$

$$\left[(FS)\frac{P_s}{P_{s\text{-}allow}}\right]^a + \left[(FS)\left(\frac{n\phi P_t + Af_b}{P_{t\text{-}allow} - P_p - P_{th}}\right)\right]^c = 1 \tag{39}$$

where

A	tensile area of bolt
a, b, c	exponents in interaction equations
FS	factor of safety, definition 1: a calculated ratio, typically of two quantities of the same character
f_b	bending stress
n	load introduction factor
P_p	bolt preload
P_s	bolt shear load
$P_{s\text{-}allow}$	bolt shear load allowable
P_t	external tensile load applied to preloaded bolts
$P_{t\text{-}allow}$	bolt failure load
P_{th}	bolt thermal load
ϕ	joint stiffness factor defined as bolt stiffness divided by the sum of the bolt stiffness and the joint stiffness

From Equation (30b) the $MS = 0$ when the $FS = 1$. Setting the $FS = 1$ in Equation (30b),

$$\left(\frac{P_s}{P_{s\text{-}allow}}\right)^a + \left[\left(\frac{n\phi P_t + Af_b}{P_{t\text{-}allow}}\right) + \frac{P_p + P_{th}}{P_{t\text{-}allow}}\right]^c = 1 \tag{C.1}$$

---

[1]Equivalent to Equation (30a): $\left[(FS)\frac{P_s}{P_{s\text{-}allow}}\right]^a + \left[(FS)\left(\frac{n\phi P_t}{P_{t\text{-}allow}} + \frac{f_b}{F_t}\right) + \frac{P_p + P_{th}}{P_{t\text{-}allow}}\right]^c = 1$.

Rearranging,

$$\left(\frac{P_s}{P_{s\text{-}allow}}\right)^a = 1 - \left[\left(\frac{n\phi P_t + Af_b}{P_{t\text{-}allow}}\right) + \frac{P_p + P_{th}}{P_{t\text{-}allow}}\right]^c \tag{C.2}$$

Raising both sides of Equation (C.2) to the power ($1/a$) and solving for the shear load that corresponds to $MS = 0$,

$$P_s = P_{s\text{-}allow}\left[1 - \left(\frac{n\phi P_t + Af_b + P_p + P_{th}}{P_{t\text{-}allow}}\right)^c\right]^{(1/a)} \tag{C.3}$$

From the alternate form of the factored interaction Equation (39) the $MS = 0$ when the $FS = 1$. Setting the $FS = 1$ in Equation (39),

$$\left(\frac{P_s}{P_{s\text{-}allow}}\right)^a + \left(\frac{n\phi P_t + Af_b}{P_{t\text{-}allow} - P_p - P_{th}}\right)^c = 1 \tag{C.4}$$

Rearranging,

$$\left(\frac{P_s}{P_{s\text{-}allow}}\right)^a = 1 - \left(\frac{n\phi P_t + Af_b}{P_{t\text{-}allow} - P_p - P_{th}}\right)^c \tag{C.5}$$

Raising both sides of Equation (C.5) to the power ($1/a$) and solving for the shear load that corresponds to a $MS = 0$,

$$\left(P_s\right)_{alternate} = P_{s\text{-}allow}\left[1 - \left(\frac{n\phi P_t + Af_b}{P_{t\text{-}allow} - P_p - P_{th}}\right)^c\right]^{(1/a)} \tag{C.6}$$

where the subscript "alternate" designates that this shear load is from the alternate factored interaction equation.

In order to simplify the comparison of the standard and alternate factored interaction equations, Equations (C.3) and (C.6), the equations are made more concise by defining the variable $\tilde{P}_{tb}$ to be the sum of the bolt axial and bending loads as shown:

$$\tilde{P}_{tb} = P_p + P_{th} + n\phi P_t + Af_b \tag{C.7}$$

Equation (C.7) is equivalent to multiplying Equation (35) by the tensile stress area A.

Substituting Equation (C.7) into Equation (C.3),

$$P_s = P_{s\text{-}allow}\left[1-\left(\frac{\tilde{P}_{tb}}{P_{t\text{-}allow}}\right)^c\right]^{(1/a)} \tag{C.8}$$

Substituting Equation (C.7) into Equation (C.6),

$$\left(P_s\right)_{alternate} = P_{s\text{-}allow}\left\{1-\left[\frac{\tilde{P}_{tb}-\left(P_p+P_{th}\right)}{P_{t\text{-}allow}-\left(P_p+P_{th}\right)}\right]^c\right\}^{(1/a)} \tag{C.9}$$

Let us assume $\left(P_s\right)_{alternate} > P_s$. For this to be true, the ratio on the right-hand side of Equation (C.8) must be larger than the ratio on the right-hand side of Equation (C.9):

$$\frac{\tilde{P}_{tb}}{P_{t\text{-}allow}} > \frac{\tilde{P}_{tb}-\left(P_p+P_{th}\right)}{P_{t\text{-}allow}-\left(P_p+P_{th}\right)} \tag{C.10}$$

Equation (C.10) is multiplied through by the denominator of the expression on the right-hand side, followed by a multiplication by the denominator on the left-hand side:

$$\tilde{P}_{tb}\left[P_{t\text{-}allow}-\left(P_p+P_{th}\right)\right] > P_{t\text{-}allow}\left[\tilde{P}_{tb}-\left(P_p+P_{th}\right)\right] \tag{C.11}$$

Canceling terms in Equation (C.11) gives

$$-\tilde{P}_{tb}\left(P_p+P_{th}\right) > -P_{t\text{-}allow}\left(P_p+P_{th}\right) \tag{C.12}$$

Upon reducing,

$$\tilde{P}_{tb} < P_{t\text{-}allow} \tag{C.13}$$

Since $P_{t\text{-}allow} > \tilde{P}_{tb}$, and these quantities cannot be equal because of the assumed interaction with shear,[2] the inequality in Equation (C.13) holds, and therefore the inequality from which it originated,

[2]Rewriting Equations (30b) and (39) in terms of $\tilde{P}_{tb}$ for $FS = 1$ we have $\left(\frac{P_s}{P_{s\text{-}allow}}\right)^a + \left(\frac{\tilde{P}_{tb}}{P_{t\text{-}allow}}\right)^c = 1$ and $\left(\frac{P_s}{P_{s\text{-}allow}}\right)^a + \left(\frac{\tilde{P}_{tb}-P_p-P_{th}}{P_{t\text{-}allow}-P_p-P_{th}}\right)^c = 1$. We see that $P_{t\text{-}allow} > \tilde{P}_{tb}$ to satisfy the interaction equations for nonzero shear load ratios.

Equation (C.10), is verified. Therefore, the quantity on the right-hand side of Equation (C.8) is less than the quantity on the right-hand side of Equation (C.9), resulting in

$$\left(P_s\right)_{alternate} > P_s \tag{C.14}$$

Therefore, the standard factored interaction Equations (30a) and (30b) are more conservative than the alternate factored interaction Equation (39) for determining the shear failure load that produces a $MS = 0$ for the interaction of tension, shear, and bending.

Appendix D.—Bending Load as a Function of Tension and Shear Loads at Margin of Safety (*MS*) of 0

This appendix derives the equation for the bolt bending failure load, as a function of bolt tension and shear load ratios, corresponding to a *MS* = 0. This is done using the standard factored interaction Equation (30b) and the alternate factored interaction Equation (39). A comparison is made to determine which interaction equation—Equation (30b)[1] or (39)—is more conservative in determining the bending failure load at *MS* = 0 for any combination of tension and shear load ratios.

$$\left[(FS)\frac{P_s}{P_{s\text{-}allow}}\right]^a + \left[(FS)\left(\frac{n\phi P_t + Af_b}{P_{t\text{-}allow}}\right) + \frac{P_p + P_{th}}{P_{t\text{-}allow}}\right]^c = 1 \tag{30b}$$

$$\left[(FS)\frac{P_s}{P_{s\text{-}allow}}\right]^a + \left[(FS)\left(\frac{n\phi P_t + Af_b}{P_{t\text{-}allow} - P_p - P_{th}}\right)\right]^c = 1 \tag{39}$$

where

A	tensile area of bolt
a, b, c	exponents in interaction equations
FS	factor of safety, definition 1: a calculated ratio, typically of two quantities of the same character
f_b	bending stress
n	load introduction factor
P_p	bolt preload
P_s	bolt shear load
$P_{s\text{-}allow}$	bolt shear load allowable
P_t	external tensile load applied to preloaded bolts
$P_{t\text{-}allow}$	bolt failure load
P_{th}	bolt thermal load
ϕ	joint stiffness factor defined as bolt stiffness divided by the sum of the bolt stiffness and the joint stiffness

From Equation (30b) the *MS* = 0 when the *FS* = 1. Setting the *FS* = 1 in Equation (30b),

$$\left(\frac{P_s}{P_{s\text{-}allow}}\right)^a + \left[\left(\frac{n\phi P_t + Af_b}{P_{t\text{-}allow}}\right) + \frac{P_p + P_{th}}{P_{t\text{-}allow}}\right]^c = 1 \tag{D.1}$$

[1]Equivalent to Equation (30a): $\left[(FS)\frac{P_s}{P_{s\text{-}allow}}\right]^a + \left[(FS)\left(\frac{n\phi P_t}{P_{t\text{-}allow}} + \frac{f_b}{F_t}\right) + \frac{P_p + P_{th}}{P_{t\text{-}allow}}\right]^c = 1$.

Rearranging,

$$\left[\left(\frac{n\phi P_t}{P_{t\text{-}allow}}+\frac{Af_b}{P_{t\text{-}allow}}\right)+\frac{P_p+P_{th}}{P_{t\text{-}allow}}\right]^c=1-\left(\frac{P_s}{P_{s\text{-}allow}}\right)^a \tag{D.2}$$

Raising both sides of Equation (D.2) to the power (1/*c*) and rearranging,

$$\left(\frac{Af_b}{P_{t\text{-}allow}}\right)=\left[1-\left(\frac{P_s}{P_{s\text{-}allow}}\right)^a\right]^{(1/c)}-\frac{n\phi P_t}{P_{t\text{-}allow}}-\frac{P_p+P_{th}}{P_{t\text{-}allow}} \tag{D.3}$$

Solving for the bending load that corresponds to a *MS* = 0,

$$Af_b=P_{t\text{-}allow}\left[1-\left(\frac{P_s}{P_{s\text{-}allow}}\right)^a\right]^{(1/c)}-n\phi P_t-\left(P_p+P_{th}\right) \tag{D.4}$$

From the alternate form of the factored interaction Equation (39), the *MS* = 0 when the *FS* = 1. Setting the *FS* = 1 in Equation (39),

$$\left(\frac{P_s}{P_{s\text{-}allow}}\right)^a+\left[\left(\frac{n\phi P_t+Af_b}{P_{t\text{-}allow}-P_p-P_{th}}\right)\right]^c=1 \tag{D.5}$$

Rearranging,

$$\left[\left(\frac{n\phi P_t+Af_b}{P_{t\text{-}allow}-P_p-P_{th}}\right)\right]^c=1-\left(\frac{P_s}{P_{s\text{-}allow}}\right)^a \tag{D.6}$$

Raising both sides of Equation (D.6) to the power (1/*c*) and rearranging again,

$$\frac{Af_b}{P_{t\text{-}allow}-P_p-P_{th}}=\left[1-\left(\frac{P_s}{P_{s\text{-}allow}}\right)^a\right]^{(1/c)}-\frac{n\phi P_t}{P_{t\text{-}allow}-P_p-P_{th}} \tag{D.7}$$

Solving for the bending load required to create a *MS* of 0,

$$\left(Af_b\right)_{alternate}=\left(P_{t\text{-}allow}-P_p-P_{th}\right)\left[1-\left(\frac{P_s}{P_{s\text{-}allow}}\right)^a\right]^{(1/c)}-n\phi P_t \tag{D.8}$$

where the subscript “alternate” designates that this is from the alternate factored interaction equation.

Subtracting Equation (D.8) from Equation (D.4) and canceling some terms,

$$Af_b - \left(Af_b\right)_{alternate} = \cancel{P_{t\text{-}allow}\left[1-\left(\frac{P_s}{P_{s\text{-}allow}}\right)^a\right]^{(1/c)}} - \cancel{n\phi P_t} - \left(P_p + P_{th}\right)$$
$$-\left\{\left(\cancel{P_{t\text{-}allow}} - P_p - P_{th}\right)\left[1-\left(\frac{P_s}{P_{s\text{-}allow}}\right)^a\right]^{(1/c)} - \cancel{n\phi P_t}\right\} \tag{D.9}$$

Rewriting,

$$Af_b - \left(Af_b\right)_{alternate} = -\left(P_p + P_{th}\right) + \left(P_p + P_{th}\right)\left[1-\left(\frac{P_s}{P_{s\text{-}allow}}\right)^a\right]^{(1/c)} \tag{D.10}$$

Since interaction is assumed, the shear load ratio on the right-hand side of Equation (D.10) exists and therefore is also less than 1 and greater than 0, the quantity on the right-hand side in square brackets never exceeding 1. Therefore, the right-hand side of Equation (D.10) is less than 0 (i.e., is negative), which means that

$$\left(Af_b\right)_{alternate} > Af_b \tag{D.11}$$

and therefore the standard factored interaction Equations (30a) and (30b) are more conservative than the alternate factored interaction Equation (39) for determining the bending failure load that produces a *MS* of 0 for interaction of tension, shear, and bending.

References

1. Ramsey, John K.: Calculating Factors of Safety and Margins of Safety From Interaction Equations. NASA/TM—2019-220153, 2019. https://ntrs.nasa.gov
2. Peery, David J.: Aircraft Structures. McGraw-Hill Book Co., New York, NY, 1950.
3. National Aeronautics and Space Administration: Astronautic Structures Manual. Vol. 1, Sect. A3, NASA TM X–73305, 1975. https://ntrs.nasa.gov
4. National Aeronautics and Space Administration: Requirements for Threaded Fastening Systems in Spaceflight Hardware. NASA–STD–5020B, 2021.
5. Press, William H., et al.: Numerical Recipes in Fortran 77: The Art of Scientific Computing. Cambridge University Press, New York, NY, 1989, p. 258. https://websites.pmc.ucsc.edu/~fnimmo/eart290c_17/NumericalRecipesinF77.pdf Accessed Feb. 11, 2022.