Minimum-Variance Control Allocation Considering Parametric Model Uncertainty

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Presentation Outline

- Background
- Problem statement and solution
- Examples
  - NASA T-2 open-loop pitching dynamics
  - F-16 AFTI closed-loop lat/dir dynamics
- Summary and conclusions

Control Allocation Overview

Multiple / redundant effectors require control allocation

Examples:

- Conventional ailerons
- X-56A trailing-edge flaps

Opportunity to do something useful with the extra design DOFs

(Credit: NASA / Carla Thomas)

Weighted Pseudo Inverse (WPI) Allocator

For the system $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + \boldsymbol{\tau}(\mathbf{u})$,

find $\mathbf{u}$ to minimize $J(\mathbf{u}) = \frac{1}{2}\mathbf{u}^T\mathbf{W}\mathbf{u}$ subject to $\boldsymbol{\tau} = \mathbf{B}\mathbf{u}$

The solution is $\mathbf{u} = \mathbf{P}\boldsymbol{\tau}_c$,

where $\mathbf{P} = \mathbf{W}^{-1}\mathbf{B}^T\left(\mathbf{B}\mathbf{W}^{-1}\mathbf{B}^T\right)^{-1}$

and where $\mathbf{W}$ is a weighting matrix to design/tune

A Practical Question

But what happens if we (only) have the estimate $\mathbf{B}(\hat{\boldsymbol{\theta}})$ and $\mathbb{E}[\hat{\boldsymbol{\theta}}\hat{\boldsymbol{\theta}}^T]$?

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Many sources of error

- Linearization
- Unmodeled dynamics
- Uncertainty in parameters

Sources of information

- System identification and Cramér-Rao bounds
- Dispersion models
- Historical data

Several robust/adaptive approaches

- Cui and Yang, 2011
- Choi, Kim, and Kim; 2014
- Duan and Okwudire, 2017
- Schwager, Annaswamy, and Lavretsky; 2005
- Tjønnås and Johansen, 2008
- Falconi and Holzapfel, 2016

Minimum-Variance (MV) Allocator

Find $\mathbf{u}$ to minimize $J(\mathbf{u}) = \frac{1}{2}\mathbb{E}\left[\tilde{\boldsymbol{\tau}}^T \mathbf{S} \tilde{\boldsymbol{\tau}}\right]$ subject to $\boldsymbol{\tau}_c = \mathbb{E}\left[\mathbf{B}(\hat{\boldsymbol{\theta}})\mathbf{u}\right]$

The solution is also $\mathbf{u} = \mathbf{P}\boldsymbol{\tau}_c$,

where $\mathbf{P} = \mathbf{W}^{-1}\mathbf{B}^T(\hat{\boldsymbol{\theta}})\left[\mathbf{B}(\hat{\boldsymbol{\theta}})\mathbf{W}^{-1}\mathbf{B}^T(\hat{\boldsymbol{\theta}})\right]^{-1}$

but where here $\mathbf{W} = \sum_{i=1}^{n_x}\sum_{j=1}^{n_x} s_{ij}\,\mathbb{E}\left[\mathbf{b}_i(\hat{\boldsymbol{\theta}})\mathbf{b}_j^T(\hat{\boldsymbol{\theta}})\right]$

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WPI allocator with a specific weighting matrix

Provides an optimal trade-off between effectiveness and uncertainty

Analogous to Kalman filter and minimum-variance regulator

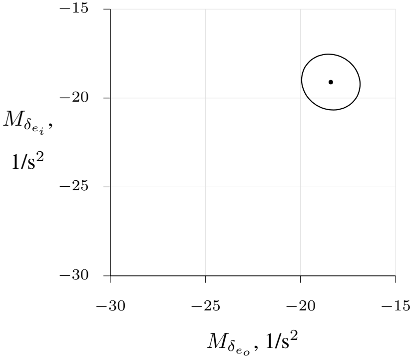
NASA T-2/GTM Subscale Airplane Model

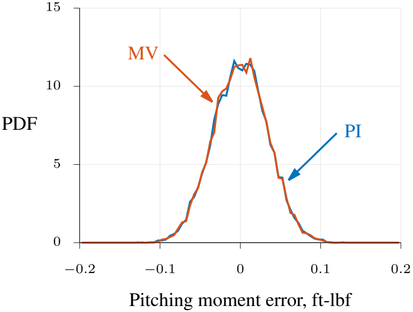
(Credit: NASA Langley Research Center)

Model of pitching moment from elevator pairs,

$$\tau = M_{\delta_{e_o}} \delta_{e_o} + M_{\delta_{e_i}} \delta_{e_i}$$

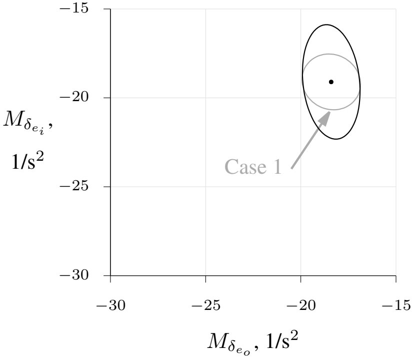
Case 1: Baseline Scenario

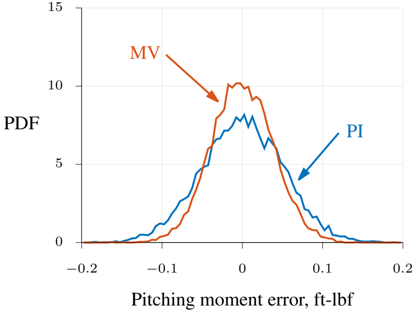




MV allocator reduced to PI allocator

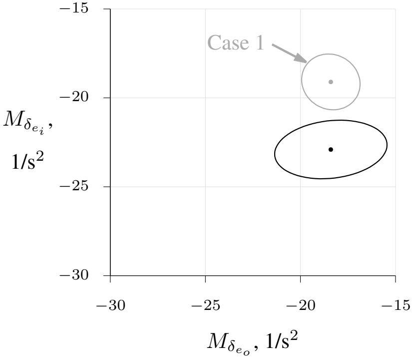
Case 2: Increased Uncertainty in $M_{\delta_{e_i}}$

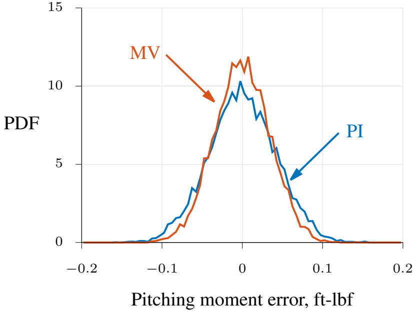




MV had 24% lower error standard deviation than PI

Case 4: Increased $M_{\delta_{e_i}}$ and Uncertainty in $M_{\delta_{eo}}$



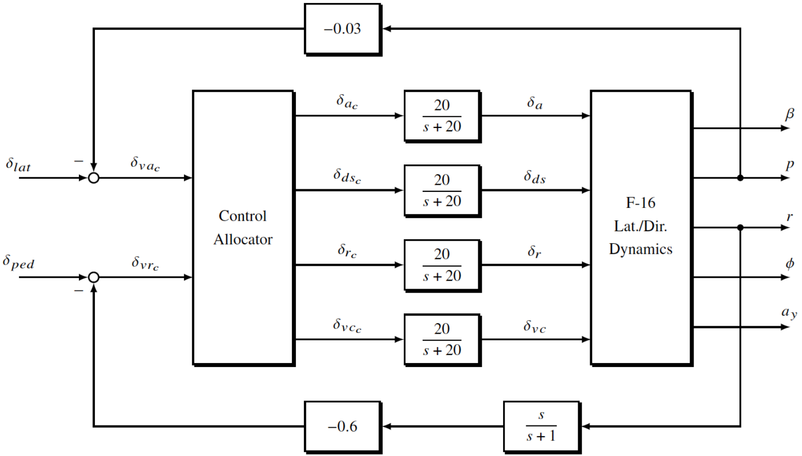


MV had 16% lower error standard deviation than PI

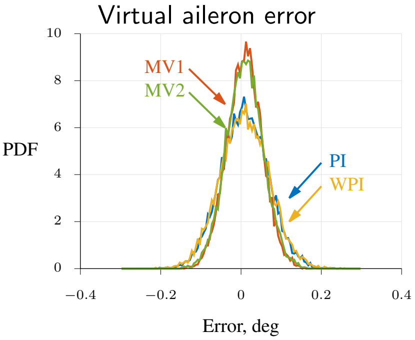
F-16 AFTI Lat/Dir Model

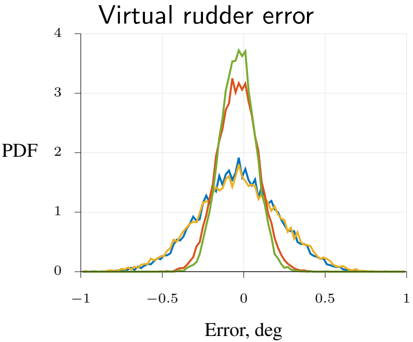
(Credit: NASA)

Stability Augmentation System Design



Monte Carlo Simulation Results





MV had 25% and 54% lower error standard deviations than PI

Summary:

- Approach for designing control allocators given parametric uncertainty in the plant model
- Minimizes the error covariance in applied forces and moments

Main Findings:

- MV is more likely to produce the intended forces and moments
- Could improve handling qualities, closed-loop performance, safety
- Only applicable for known, significant, and unevenly distributed uncertainty among the control inputs
- Cost is decreased allocation efficiency

Applications:

- New vehicle configurations
- Initial flight tests
- Adaptive control based on real-time system identification