

Generalized Predictive Control for Active Stability Augmentation and Vibration Reduction on an Aeroelastic Tiltrotor Model

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Tiltrotor aircraft are defining the state-of-the-art in vertical lift technology as they have the potential to greatly expand rotary-wing operational boundaries. However, they are often limited in forward flight speed due to complex coupled rotor and wing dynamic instabilities. The U.S. Army and NASA have been developing a new wind tunnel model, the TiltRotor Aeroelastic Stability Testbed (TRAST), to test proprotors in the NASA Langley Research Center Transonic Dynamics Tunnel (TDT) to investigate aeroelastic stability in cruise. The test is intended to provide high-quality research data for analytical tool development and validation. In addition, the TRAST model will support, develop, and mature new technologies for the design of advanced proprotor aircraft. Stability augmentation and vibration reduction during testing is planned with the use of an active control methodology known as Generalized Predictive Control (GPC). GPC is an autoregressive adaptive control law that experimentally determines the input-output relation of controls and sensors to derive the system identification parameters. This type of control law is especially useful for complex dynamic interactions that are difficult to explicitly model such as proprotor pylon instability, often referred to as whirl flutter. GPC has been successfully employed on other tiltrotor vehicles to suppress whirl flutter instabilities and vibrations. To aid in the characterization of the wind-tunnel model and in tool development, an analytical representation of the wind-tunnel model was developed using the rotorcraft comprehensive analysis system (RCAS) that simulates structural dynamics and aerodynamics. RCAS was used to derive state-space estimates of the physical plant at various flight conditions to test control law effectiveness. This paper will present an overview of the test article development, a description of RCAS, an explanation of the GPC methodology, and results of GPC being applied to the RCAS state-space plant estimates of the TRAST model. In these simulations, GPC was effective at stabilizing the aircraft beyond the

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whirl-flutter boundary while simultaneously reducing vibrations across the flight regime. Additionally, a modern advancement to GPC, termed advanced GPC (AGPC), is introduced that enables a self-adapting system identification. Preliminary results show that AGPC is successful at self-correction as the plant changes from what was used for system identification.

I. Nomenclature

AGPC	=	advanced generalized predictive control
ARX	=	autoregressive moving average model with exogenous input
GPC	=	generalized predictive control
h_c	=	control horizon
h_p	=	prediction horizon
k	=	time step index parameter
m	=	number of response measurements (outputs)
p	=	order of the ARX equation, number of past inputs and responses in model
r	=	number of control inputs
s	=	number of time steps used in system identification
TDT	=	Transonic Dynamics Tunnel, NASA Langley Research Center
TRAST	=	TiltRotor Aeroelastic Stability Testbed

II. Introduction

The U.S. Army and NASA have been developing a new wind tunnel model, the TiltRotor Aeroelastic Stability Testbed (TRAST), to test proprotors in the NASA Langley Research Center Transonic Dynamics Tunnel (TDT) for aeroelastic stability investigations. The semispan model has an 8.0 ft diameter rotor and is dynamically scaled to be representative of vehicles such as the XV-15. The test is intended to provide high-quality research data for analytical tool development and validation and to support, develop, and mature new technologies for design of advanced proprotor aircraft. This model is designed for the TDT since that wind tunnel offers unique capabilities for rotorcraft performance and dynamic stability testing [1].

Tiltrotor aircraft are defining the state-of-the-art in revolutionary vertical lift technologies as they have not yet reached a full potential. One factor limiting the speed potential of modern tiltrotor aircraft is an aeroelastic instability known as proprotor pylon instability, often referred to as whirl flutter. Due to the complex dynamics and degrees of freedom of the rotor, flutter typically limits high-speed forward flight in the airplane mode. The driving mechanisms of this instability are introduced by Kvaternik [2], [3] and are identified as the aerodynamic forces acting in-plane to the rotor disk in contrast to the airload moments as is the case with classical propeller whirl flutter [3]. These in-plane forces result from asymmetric aerodynamic loads acting on the rotor blades, are aggravated by gyroscopic effects, and are interacted with by inertial coupling of the nacelle pylon and wing structure. As Kvaternik [2] mentions, this instability can be contained in one plane and is not required to whirl as is the case with a stiff propeller. The frequency feedback between the structure and the rotor is dependent upon structural frequencies, aerodynamic damping applied to the wing and rotor system, and rotor rpm. As rpm changes, the system dynamics change considerably, and stability boundaries are often very dependent upon rpm in addition to flight vehicle velocity. Proprotor pylon instability will have a cross-coupling between pitch and yaw modes if a hub-spring or hinge-offset exists. Many tiltrotor designs incorporate at least one of these mechanisms to stiffen the rotor hub. Therefore, proprotor pylon instability will usually manifest with a whirl motion of the rotor disk, and it is often convenient to simply refer to it as whirl flutter. Unlike classical whirl flutter with a stiff propeller, proprotor pylon instability can cause whirl in either direction [2], [3].

The complex nature of tiltrotor dynamic control does not lend itself well to conventional control theory as the equations of motion become prohibitively complex and nonlinear by comparison to fixed wing aircraft or many other dynamic systems. The advent of predictive/adaptive control theory using AutoRegressive moving average models with eXogenous input (ARX) has been successful at controlling dynamic systems without the requirement to explicitly solve the equations of motion [4]. The system plant is modeled by a series of coefficients relating the system states at the next time step to the current system state, current controls, and a history of past states and controls. The input/output relationship is determined experimentally through a system identification (system ID) process to derive the coefficients that solve the ARX equation [4]. Relatively new algorithms have been developed using a feed-forward prediction of future controls and outputs that drive the response to zero over a prediction horizon [5]. One of these algorithms was further manipulated to enable tracking in addition to regulation and to efficiently determine the system ID. Using these latter changes, generalized predictive control (GPC) is an advanced form of the ARX model that

shares similarities to conventional optimal control theory by designation of weighting matrices and an objective function. The objective function is minimized to achieve optimized controller performance with minimal actuator workload [5], [6]. GPC was successfully employed in the wing and rotor aeroelastic test system (WRATS) tiltrotor model tested in the late 1990s and early 2000s [5], [7]. Some challenges have occurred in the past employing GPC because as flight conditions changed, the system ID was no longer valid in some cases. Introduced in this paper is the concept of an advanced GPC algorithm to enable a self-adapting system ID as flight conditions change.

This paper explores the application of a GPC control algorithm to the TRAST using state-space models derived by the rotorcraft comprehensive analysis system (RCAS) [8]. RCAS provides a fully nonlinear dynamic model of the structure in addition to the aerodynamics and enables simulation of aeroelastic instabilities [8]. A modal reduction is used to create the state-space models for control law development and feasibility studies. As previously mentioned, accurate modeling of tiltrotor dynamic aeroelastic behavior is difficult, and the RCAS model may behave differently than the physical experiment. However, these inaccuracies are less important since the system ID parameters are acquired experimentally and the analytical plant will not be used during the physical experiment. Therefore, the simulated plant should provide a good testbed to experiment with GPC control weighting, step size optimizations, and development of advanced GPC algorithms. Traditional control methods that rely on equations of motion or analytical models to determine control law application are far more sensitive to inaccurate plant models than GPC.

This paper will provide an overview of the physical TRAST model and the analytical TRAST model used for whirl flutter analyses. A description of the GPC algorithm will follow including the derivation of the algorithm as applied in the current investigation. Results of the GPC control law applied to the analytical model of TRAST will be presented. Results show that GPC is capable of significantly reducing vibration and simultaneously stabilizing the system as it exceeds the flutter boundary. Finally, an advanced GPC concept is introduced that enables a self-adaptive system ID as flight conditions change.

III. Description of TRAST

For over 30 years, WRATS has been used in the TDT to acquire data needed to support research, design, and flight tests of advanced tiltrotors, notably the V-22 [7]. Although significantly modified over time, WRATS is limited in the types of rotors and wings that it can support and data that it can acquire. Its utility for research is further limited by restrictions on the use and release of design and test data pertaining to the V-22. The aeroelastic phenomena WRATS was intended to explore remain highly relevant, even critical, to the success of future tiltrotor rotorcraft.

A new wind-tunnel test rig, TRAST, was developed by the U.S. Army and NASA to further explore the physics of dynamic and aeroelastic coupling between the tiltrotor, wing, and control system and to support analysis and design of advanced proprotor aircraft without the limitations and restrictions of WRATS. TRAST can accommodate a gimballed rotor operating at hover and cruise tip speeds typical of tiltrotors or a hingeless rotor. Unpowered whirl flutter experimentation can also be conducted with the TRAST. The baseline rotor system of TRAST is based upon the XV-15 design and has three blades of 4.0-ft radius that are Aeroelastically-scaled for heavy gas. The wing has 4.4-ft length and 1.5-ft chord. The main spar is centered on the quarter chord line of the wing, swept forward by 6.5 deg and tilted upward by 2.0 deg. At the tip of the wing there is a tilting nacelle (pylon), the conversion angle of which can be changed manually to test helicopter mode, airplane mode, and various transitions in-between. The conversion axis is coincident with the driveshaft along the wing, parallel to the wing spar at 45% chord. The driveshaft has flex couplings to alleviate any moment that is induced by wing deflections. The pylon mounts to the spar through two key springs. The first is a diaphragm spring, which is a segmented disk that can have its stiffness tuned to control pylon yaw and roll motions for simulation of a range of tiltrotor configurations. The second spring tunes the pitching motion of the pylon and can simulate locked or unlocked pylon configurations that occur at the end of a transition between airplane and helicopter modes. TRAST includes a full rotor control system with cyclic and collective controls capable of high-frequency inputs for active control studies. Provisions exist to include an active flaperon for future studies. Finally, blade flap-pitch coupling ($\delta\alpha$) angles are adjustable in order to explore the effects of control system geometry on whirl flutter.

To accommodate different rotors and operating conditions, the wing and nacelle have provisions to adjust frequencies and mode shapes through adjustable tuning masses and spring stiffnesses. Replaceable aerodynamic panels on the wings are adopted to enable high efficiency rotor and wing studies. The wing and nacelle design allows the addition of a winglet or wing extension to investigate performance and stability improvements for wing extensions. Figure 1 shows the baseline TRAST installed in the TDT, and Fig. 2 is an illustration of the TRAST model showing the structural elements, actuators, and wing flaperon.

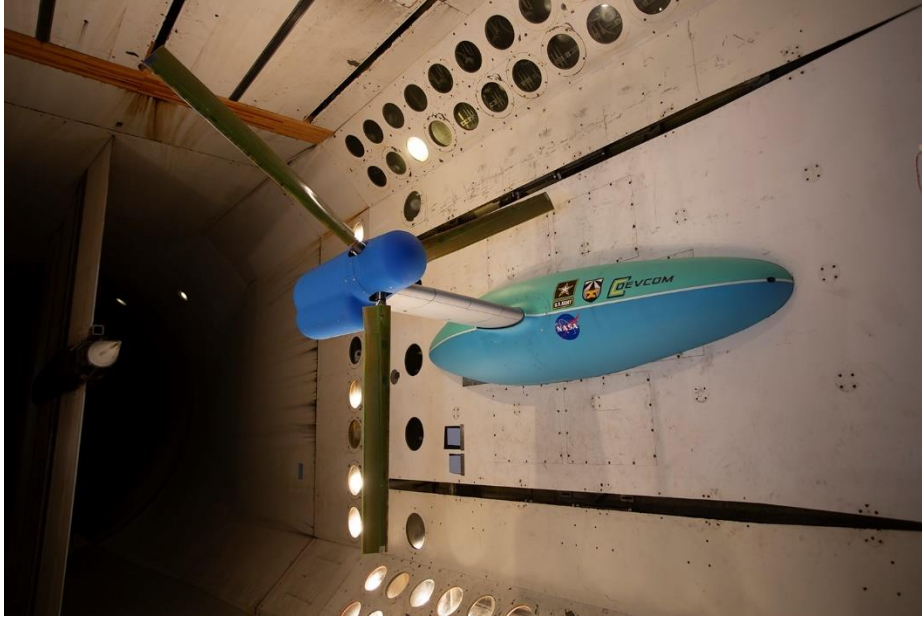


Fig. 1 TRAST installed in the NASA Langley Transonic Dynamics Tunnel.

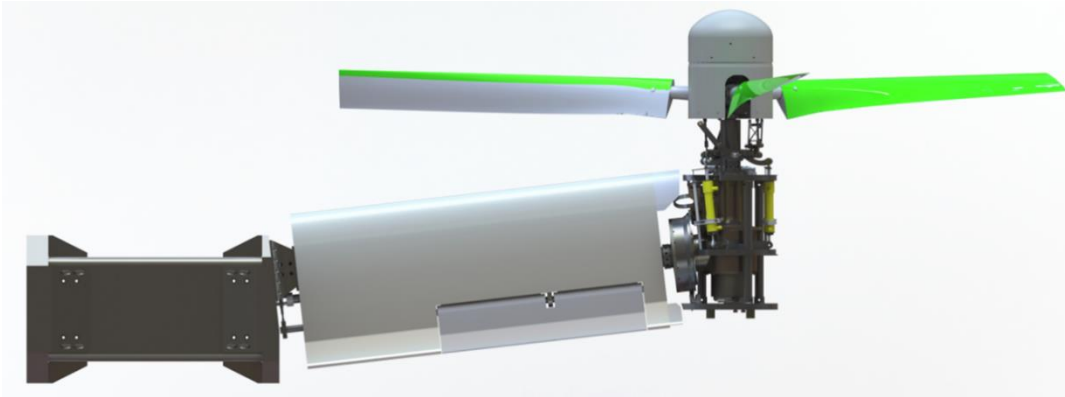


Fig. 2 Illustration of the TRAST model.

IV. Analytical TRAST Model for Aeroelastic Stability Analysis

The analytical model for TRAST whirl flutter stability predictions was developed using RCAS. RCAS is a flexible multibody dynamics-based program for rotorcraft research, engineering, and design [8]. The RCAS structural model employs a hierarchical, finite element, multibody dynamics formulation for coupled rotor-body systems. It includes a library of primitive elements including nonlinear beams, rigid body masses, rigid bars, springs, dampers, hinges, and slides to build arbitrarily complex models [8]. The aerodynamics of RCAS are simulated with lifting line theory based on airfoil look-up tables combined with an inflow model. Unsteady airloads are based on several linear and nonlinear modeling options. Linear unsteady airloads include classical Theodorsen theory and a finite state airfoil theory of Peters, Hsieh, and Torrero. Nonlinear unsteady modeling includes the ONERA and Leishman-Beddoes models with options for dynamic stall, unsteady trailing edge separation effects, and vortex shedding. Inflow modeling include uniform, finite state dynamic wake, prescribed vortex wake, and free vortex wake models. In RCAS, the nonlinear equations are solved by direct time domain integration for trim and maneuver problems. A harmonic balance method is also available for trim analysis [8].

The analysis presented in this paper uses the baseline TRAST model which is a semi-span tiltrotor with a three-bladed stiff-in-plane gimballled rotor with hub springs. The RCAS nonlinear beam element with blade pitch bearings

is used to model the elastic blade and the rotor mast. Inputs to RCAS are the geometry of structures and their inertia and stiffness properties. The rotor hub, pitch horns, and swashplate are modeled as rigid bodies with their appropriate inertial properties. The pitch links are flexible and modeled using a linear spring. The hub gimbal is modeled using the constant-velocity (CV) joint. The connections between the pitch link to pitch horn and to swashplate are modeled as spherical joints. Figure 3 is an illustration of the rotor structural model.

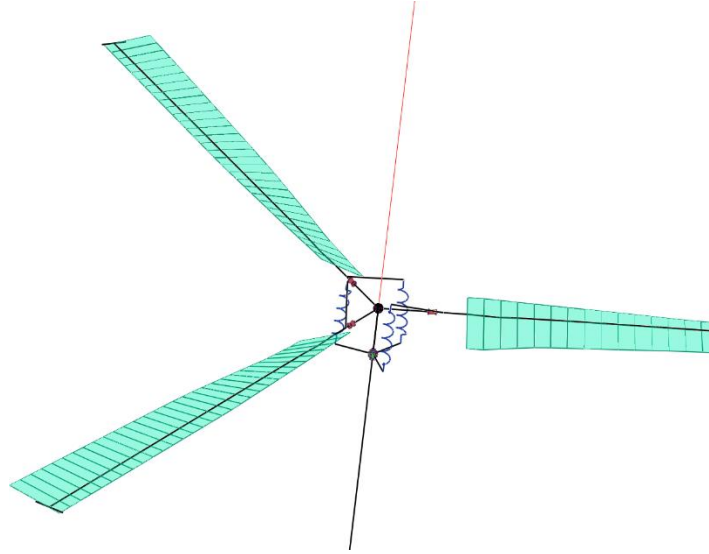


Fig. 3 Gimbaled rotor modeling of TRAST in analytical model.

The TRAST pylon is modeled using rigid bars and point masses representing the pylon frame, the gearbox, and the swashplate non-rotating actuators. The diaphragm, which connects the wing tip and the pylon, is modeled using 3-DOF joints with rotational springs and dampers. The pylon pitch spring, which also connects the wing and the pylon, is modeled using a linear spring. The wing consists of the wing spar, the driveshaft, the wing flap and flap control shaft, and the support ribs. These components are modeled using nonlinear beam elements connected through joints or bearings as shown in Fig.4.

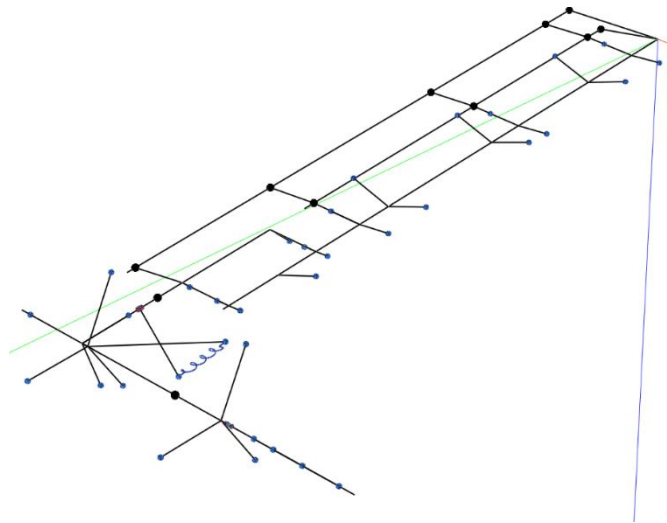


Fig. 4 Wing and pylon structural modelling of TRAST in analytical model.

The airloads on the rotor blades and wing are calculated using a quasi-steady (airfoil table look-up) airloads model with linear unsteady flow effects based upon classical Theodorsen theory. Rotor inflow is calculated by a dynamic wake model, and the wing induced velocity is calculated assuming uniform inflow. Aerodynamic lift (F_z), drag (F_x), and pitching moment (M_y) on the pylon are calculated using a closed-form expressions as:

$$\begin{aligned} F_x &= -D_0 - D_1 |\sin(\alpha)| - D_2 \sin(\alpha)^2 \\ F_z &= -L_0 - L_1 \sin(\alpha) \\ M_y &= M_0 + M_1 \sin(\alpha) \end{aligned} \quad (1)$$

where, D_0 , D_1 , and D_2 are drag coefficients, L_0 and L_1 are lift coefficients, and M_0 and M_1 are pitch moment coefficients.

The rotor induced velocity is very small compared to the inflow from forward speed. Therefore, no aerodynamic interference is modeled between the rotor, the wing, and the pylon. An illustration of the complete TRAST model used for analysis is shown in Fig. 5.

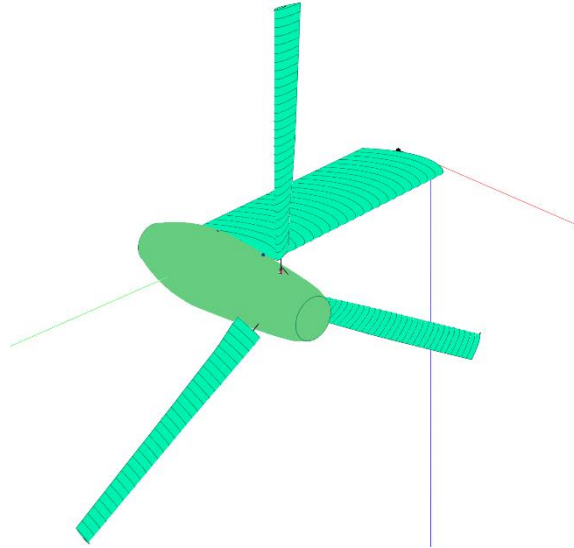


Fig. 5 Complete RCAS model of TRAST.

The rotor system is normally operated in a windmill state (zero torque) in the airplane-mode for wind-tunnel stability testing, because this state is typically more conservative than powered operation. For the analytical results presented in the current paper, the rotor is first trimmed to zero torque at a given wind velocity and tip speed. Once the rotor is trimmed to a specified operating condition, RCAS linearizes the equations about the trim solution and calculates eigenvalues at multiple azimuth angles using a constant coefficient approximation. A constant-coefficient approximation is suitable for axial flow conditions. System frequencies and damping are determined by averaging the eigenvalues over one rotor revolution.

Figure 6 shows aeroelastic stability analysis results (frequency and damping) for the baseline TRAST with the airspeed ranging from 30 to 210 knots. The analyses were conducted for a baseline rotor speed of 909 rpm and under sea level standard atmospheric conditions. Positive damping indicates a stable system, and negative damping indicates an instability. Mode shapes for the first three modes are shown in Fig. 7.

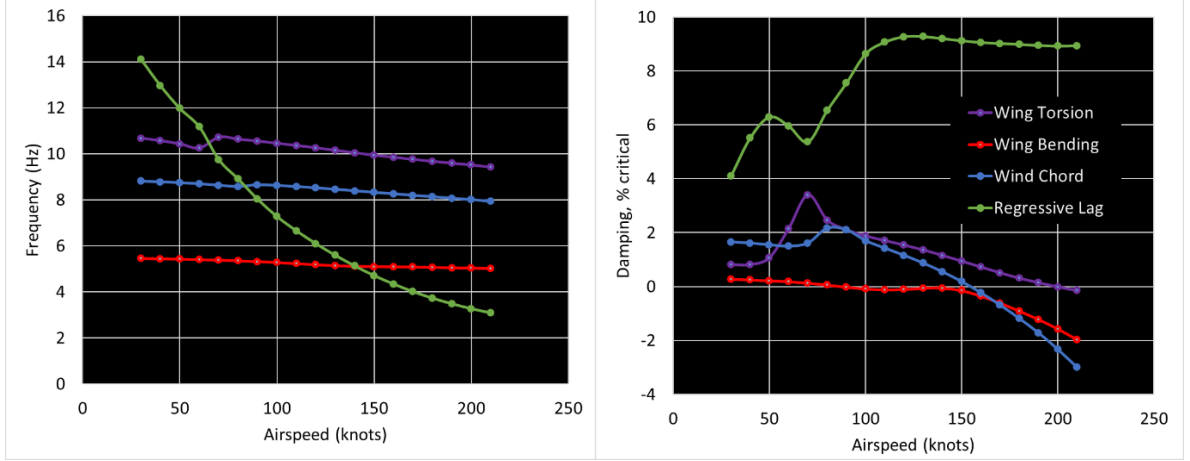


Fig. 6 TRAST aeroelastic stability analysis using RCAS analytical model.

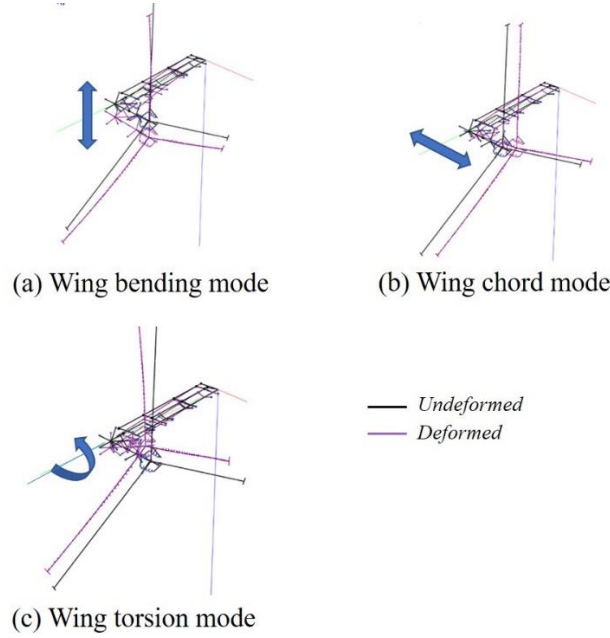


Fig. 7 TRAST mode shapes from RCAS analytical model.

To examine the effectiveness of the GPC for TRAST stability augmentation, 19 linear equations were generated using the numerical model at the conditions described above. These linear equations represent the system at airspeeds of 30, 40, 50, 60, 70, 80, 90, 100, 110, 120, 130, 140, 150, 160, 170, 180, 190, 200, and 210 knots, respectively.

In RCAS, a numerical linearization method is used to linearize the nonlinear equations governing the dynamic response of a rotorcraft as:

$$\Gamma(u, X, \dot{X}, \ddot{X}, t) = 0 \quad (2)$$

where X are the system states or the system degrees of freedom, $\dot{X}$ are the first time derivatives of the states, $\ddot{X}$ are the second time derivatives of the states, u is the control input, t is the time, and Γ is the general nonlinear function governing the equations of motion.

The perturbations about the trim condition are defined as shown in Eqn. 3.

$$\begin{aligned}
X_l &= X - X_0 \\
\dot{X}_l &= \dot{X} - \dot{X}_0 \\
\ddot{X}_l &= \ddot{X} - \ddot{X}_0 \\
u_l &= u - u_0
\end{aligned} \tag{3}$$

where “₀” denotes the trim condition.

Substituting the variables in Eqn. 2 with Eqn. 3, the dynamic response of a rotorcraft can be written in the linearized form and then the linearized equations can be written in the conventional first order form as follows:

$$\begin{aligned}
\dot{X} &= AX + Bu \\
Y &= CX + Du
\end{aligned} \tag{4}$$

There are more than 300 states for the TRAST model. To reduce the computational cost for GPC application, a reduced order model was developed. Assuming that the eigensolver outputs the eigenvalues and eigenvector matrix in ascending order and the highest desired frequency is mode number $\bar{m}$, then the modally reduced linear model may be extracted using the following equations:

$$\begin{aligned}
A_{\bar{m}} &= A_m(1:\bar{m}, 1:\bar{m}) \\
B_{\bar{m}} &= B_m(1:\bar{m}, 1:N_u) \\
C_{\bar{m}} &= C_m(1:N_y, 1:\bar{m}) \\
D_{\bar{m}} &= D_m
\end{aligned} \tag{5}$$

where N_u is the number of inputs and N_y is the number of outputs.

An assumption is made that the model can be transformed into modal space and portioned into low and high frequency mode sets as shown in Eqn. 6.

$$\begin{aligned}
\begin{Bmatrix} \dot{\eta}_1 \\ \dot{\eta}_2 \end{Bmatrix} &= \begin{bmatrix} \Lambda_1 & 0 \\ 0 & \Lambda_2 \end{bmatrix} \begin{bmatrix} \eta_1 \\ \eta_2 \end{bmatrix} + \begin{bmatrix} B_{m1} \\ B_{m2} \end{bmatrix} u \\
Y &= \begin{bmatrix} C_{m1} & C_{m2} \end{bmatrix} \begin{bmatrix} \eta_1 \\ \eta_2 \end{bmatrix} + D_{mat} u
\end{aligned} \tag{6}$$

In the above equations η_1 , Λ_1 , B_{m1} , C_{m1} and D_{mat} are associated with the low frequency modes and η_2 , Λ_2 , B_{m2} , and C_{m2} are associated with the high frequency modes. These equations may be separated as follows. The low frequency mode equations are:

$$\begin{aligned}
\dot{\eta}_1 &= \Lambda_1 \eta_1 + B_{m1} u \\
Y_1 &= C_{m1} \eta_1 + D_{mat} u + Y_2
\end{aligned} \tag{7}$$

The high frequency mode equations are:

$$\begin{aligned}
\dot{\eta}_2 &= \Lambda_2 \eta_2 + B_{m2} u \\
Y_2 &= C_{m2} \eta_2
\end{aligned} \tag{8}$$

Y_1 and Y_2 are the output from each respective set. If one assumes the dynamics of the high frequency modes are very fast and respond instantaneously to inputs, setting $\dot{\eta}_2 = 0$, then Eqn. 8 reduces to the following:

$$Y_2 = -C_{m2} \Lambda_2^{-1} B_{m2} u \tag{9}$$

The outputs from Eqns. 7 and 9 can be superimposed. The resulting modally reduced linear model equation set is:

$$\begin{aligned}
\dot{\eta}_1 &= \Lambda_1 \eta_1 + B_{m1} u \\
Y_1 &= C_{m1} \eta_1 + D_1 u
\end{aligned} \tag{10}$$

where,

$$D_1 = (D_{mat} - C_{m2}\Lambda_2^{-1}B_{m2}) \quad (11)$$

For the problem addressed in this paper, collective and cyclic controls of the tiltrotor were selected as the inputs and the moments in the in-plane and out-of-plane bending and the torsion direction at the wing root were selected as the outputs for the model linearization. Eight modes, including the wing in-plane and out-of-plane bending modes and the torsional mode and the rotor blade flap and lag modes, were selected in the modal reduction. The matrices Λ_1 , B_{m1} , C_{m1} , and D_1 of the modally reduced linear models were then used as the plant for the stability analysis of the TRAST with GPC. This enables an evaluation of the GPC concept for use on TRAST across a range of test conditions to evaluate its feasibility.

V. GPC and Application to the TRAST Model

A. Background of GPC and Governing Equations

The concept of GPC was originally introduced by Clarke, Mohtadi, and Tuffs [6] as a potential adaptive control algorithm. Reference [6] proposed the concepts of a prediction horizon (h_p), a control horizon (h_c) after which controls become zero, and a least-squares observer parameter estimator (p). However, no details are presented in Ref. [6] regarding the parameter computation or system identification. Additionally, Ref. [6] describes using continuous parameter estimation at every time step to enable adaptive control as the plant changes, or the option of a single system identification and then only conducting the computation of the optimal control input for each time step. The former method offers the opportunity for self-adjustment and shows great promise in the introductory paper. However, the authors of Ref. [6] conducted their analyses in the absence of external disturbances and noise, the model tracked large displacement step functions of a single input single output (SISO) controller simplifying continuous system identification, and the practical limitations of continuous system identification in a real-time controller were not discussed. Juang and Phan [4] expanded this effort and explored various system identification processes of ARX models with multiple input multiple output (MIMO) systems and enabled unknown disturbance rejection. Building upon the ARX foundations of Ref. [4] and the GPC introduction of Ref. [6], Kvaternik, Juang and Bennet [5] developed an objective function similar to optimal control theory and turned GPC into a practical and powerful MIMO controller. Reference [5] is a foundational, albeit informal, paper describing the state-of-the-art in GPC. Reference [9] explored the use of GPC, as described by Ref. [5], as a tracker for rigid-body control of a flight vehicle.

ARX models are frequently used for adaptive control and are often represented with the equation shown in Eqn. 12 (Ref. [4]). In this ARX equation, y is the output, u is the input, and the α and β terms are time-varying coefficients that define the system behavior and are referred to as the observer Markov parameters. The parameter p is a fixed integer that defines the number of past parameters included in the model and is referred to as the order of the ARX equation. The parameter k refers to the time step in this discrete-time scheme such that k is the current time step and $k-1$ refers to the previous time step. Notice, therefore, that β_0 is a direct transmission term that relates the current control input to the current response output.

$$y(k) = \alpha_1 y(k-1) + \alpha_2 y(k-2) + \dots + \alpha_p y(k-p) \dots \\ + \beta_0 u(k) + \beta_1 u(k-1) + \beta_2 u(k-2) + \dots + \beta_p u(k-p) \quad (12)$$

The primary assumption is that the dynamic system can be modeled by a series of past inputs and past outputs in addition to the current input. This assumption is reasonable for a dynamic system that exhibits quasi-periodic response if the order p is high enough to capture a history of the dynamics and that the time step size is small enough to capture the dynamic behavior. Quasi-periodic is used to describe the dynamics of a system rather than periodic in that the addition of disturbances or signal noise will alter the behavior represented by the output(s) y . With the conditions of p and step size being adequate, quasi-periodic behavior should be adequately predicted by analyzing past behavior as Eqn. 12 serves as a higher order curve fit to this behavior. Adequate modelling is assured if the dynamic system can be represented by a linear state-space representation and if the selection of p and step size are properly chosen.

Equation 12 can be expanded for future time steps beyond step k , as described by Refs. [4] and [5], to enable the matrix relation shown below in Eqn. 13. In this equation, the prediction horizon (h_p) is the limit of future state estimates and the control horizon (h_c) is the limit of the finite control horizon after which the controls are assumed to be zero [5]. Determining the required future controls u_{h_c} in order to drive the states y to the target value y_T is

described below. Guidance regarding the selection of h_p , h_c , and p is provided in Ref. [5] with the constraint that $h_p \geq h_c$.

$$y_{h_p}(k) = Tu_{h_c}(k) + Ay_p(k-p) + Bu_p(k-p) \quad (13)$$

where, the subscripts h_p , h_c , and p denote the size of the respective vectors, and the parenthetical values represent the starting index.

The values of past and future outputs and controls and the Tau (T), Alpha (A), and Beta (B) matrices (not to be confused with the state-space matrices A and B) are shown in Eqns. 14 and 15.

$$y_{h_p}(k) = \begin{bmatrix} y(k) \\ y(k+1) \\ \vdots \\ y(k+h_p-1) \end{bmatrix} \quad u_{h_c}(k) = \begin{bmatrix} u(k) \\ u(k+1) \\ \vdots \\ u(k+h_c-1) \end{bmatrix} \quad (14)$$

$$y_p(k-p) = \begin{bmatrix} y(k-1) \\ \vdots \\ y(k-p+1) \\ y(k-p) \end{bmatrix} \quad u_p(k) = \begin{bmatrix} u(k-1) \\ \vdots \\ u(k-p+1) \\ u(k-p) \end{bmatrix}$$

$$T = \begin{bmatrix} \beta_o & 0 & \dots & 0 \\ \beta_o^{(1)} & \beta_o & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \beta_o^{(s-1)} & \beta_o^{(s-2)} & \dots & \beta_o \end{bmatrix}$$

$$A = \begin{bmatrix} \alpha_1 & \dots & \alpha_{p-1} & \alpha_p \\ \alpha_1^{(1)} & \dots & \alpha_{p-1}^{(1)} & \alpha_p^{(1)} \\ \vdots & \vdots & \ddots & \vdots \\ \alpha_1^{(s-1)} & \dots & \alpha_{p-1}^{(s-1)} & \alpha_p^{(s-1)} \end{bmatrix} \quad (15)$$

$$B = \begin{bmatrix} \beta_1 & \dots & \beta_{p-1} & \beta_p \\ \beta_1^{(1)} & \dots & \beta_{p-1}^{(1)} & \beta_p^{(1)} \\ \vdots & \vdots & \ddots & \vdots \\ \beta_1^{(s-1)} & \dots & \beta_{p-1}^{(s-1)} & \beta_p^{(s-1)} \end{bmatrix}$$

The components of the T , A , and B matrices are recursive coefficients as defined in Eqn. 16.

$$\begin{aligned}
\alpha_1^{(1)} &= \alpha_1 \alpha_1 + \alpha_2 & \beta_1^{(1)} &= \alpha_1 \beta_1 + \beta_2 \\
\alpha_2^{(1)} &= \alpha_1 \alpha_2 + \alpha_3 & \beta_2^{(1)} &= \alpha_1 \beta_2 + \beta_3 \\
&\vdots & &\vdots \\
\alpha_{p-1}^{(1)} &= \alpha_1 \alpha_{p-1} + \alpha_p & \beta_{p-1}^{(1)} &= \alpha_1 \beta_{p-1} + \beta_p \\
\alpha_p^{(1)} &= \alpha_1 \alpha_p & \beta_p^{(1)} &= \alpha_1 \beta_p
\end{aligned} \tag{16}$$

and

$$\beta_o^{(1)} = \alpha_1 \beta_o + \beta_1$$

Multiple methods are available to acquire the T , A , and B matrices through an experimental system identification process. References [4] and [5] propose various methods and one of those is described here. If one were to acquire s time steps of input/output data, then input and output matrices can be developed as shown by Eqns. 17 and 18, with $s \gg h_p$. Using these matrices and expanding upon Eqn. 13, one arrives at Eqn. 19. Then, the T , A , and B matrices can be determined as shown in Eqn. 20.

$$\begin{aligned}
Y_{h_p}(k) &= [y_{h_p}(k), y_{h_p}(k+1) \dots y_{h_p}(k+s-1)], \text{ and} \\
U_{h_c}(k) &= [u_{h_c}(k), u_{h_c}(k+1) \dots u_{h_c}(k+s-1)]
\end{aligned} \tag{17}$$

$$\begin{aligned}
Y_p(k-p) &= [y_p(k-p), y_p(k-p+1) \dots y_p(k-p+s-1)], \text{ and} \\
U_p(k-p) &= [u_p(k-p), u_p(k-p+1) \dots u_p(k-p+s-1)]
\end{aligned} \tag{18}$$

With these matrix definitions, the combined relation by applying Eqn. 13 yields the result shown in Eqn. 19.

$$Y_{h_p}(k) = T U_{h_c}(k) + A Y_p(k-p) + B U_p(k-p) \tag{19}$$

It is obvious that to acquire and compute the matrices in Eqn. 20, the time steps k through $k+h_p-1$ are no longer in the future. Correspondingly, the time steps $k-p$ are now in the distant past.

$$[T \quad A \quad B] = Y_{h_p}(k) * \text{pinv} \begin{bmatrix} U_{h_c}(k) \\ Y_p(k-p) \\ U_p(k-p) \end{bmatrix} \tag{20}$$

With the establishment of the coefficient matrices, there is now a discrete model of the dynamic system. With this model, the effect of control inputs can be predicted to track a target value of outputs or to regulate (minimize) the output response. There are several methods for computing acceptable control commands. An improved method employed by Ref. [5] uses an objective function similar to optimal control theory using a classical linear quadratic regulator. The objective function presented by Kvaternik et al. [5] is shown below as Eqn. 21.

$$J = \varepsilon^T R \varepsilon + u_{h_c}^T Q u_{h_c} \tag{21}$$

where, ε is the error between the target response and the predicted response $\varepsilon = y_T - y_{h_p}$, R is the weighting matrix for error, and Q is the weighting matrix for control input.

In the literature, the weighting matrices R and Q are diagonal of constant value such as $R = rI$ where I is the identity matrix and r is a positive scalar value [4]-[6], and [9]. Similarly, Q can be expressed as $Q = qI$. This representation of

an objective function uses similar weighting nomenclature to linear quadratic regulators. It should also be noted that the specification of both R and Q for the objective function defined in Eqn. 21 can be redundant (when the diagonal elements are of constant value) since the ratio of the two is the defining feature. This is expressed in the previous work by Clarke et al. [6] since they only specify the equivalent of Q in their work. However, the designation of R and Q independently enables the user to specify different weighting values to different input/output parameters. An example application is presented in latter sections of this report.

Kvaternik et al. [5] show the expression for the required control input to minimize the objective function, but they do not show the computation to arrive at said expression. Therefore, the computation is offered here. To find the control input, u_{h_c} , that minimizes the objective function, one can simply evaluate the function where the derivative with respect to the control, denoted as J_u , is zero. Since the objective function is a quadratic equation with positive coefficients, the condition where the derivative is zero is indeed the global minimum. Using the chain rule, J_u can be calculated as shown in Eqn. 22. Recall that the error is $\varepsilon = y_T - y_{h_p}$ where y_{h_p} is also a function of u_{h_c} as shown in Eqn. 13. Therefore, one must first expand J , as done in the first step of Eqn. 22, before taking the derivative.

$$J = [y_T - Tu_{h_c} - Ay_p - Bu_p]^T R [y_T - Tu_{h_c} - Ay_p - Bu_p] + u_{h_c}^T Q u_{h_c} \quad (22)$$

$$J_u = [-T]^T R [y_T - Tu_{h_c} - Ay_p - Bu_p] + [y_T - Tu_{h_c} - Ay_p - Bu_p]^T R [-T] + Qu_{h_c} + u_{h_c}^T Q = 0$$

Grouping like terms with regards to u and y yields Eqn. 23.

$$T^T R T u_{h_c} + [T u_{h_c}]^T R T + Q u_{h_c} + u_{h_c}^T Q + T^T R A y_p + [A y_p]^T R T + T^T R B u_p + [B u_p]^T R T - T^T R y_T - y_T^T R T = 0 \quad (23)$$

With diagonal Q and R , this can be simplified as shown in Eqn. 24.

$$2T^T R T u_{h_c} + 2Q u_{h_c} + 2T^T R A y_p + 2T^T R B u_p - 2T^T R y_T = 0 \quad (24)$$

Solving for u_{h_c} results in Eqn. 25, which is the control input solution to minimize the objective function defined in Eqn. 21. The result of Eqn. 25 will yield a column vector with a length equal to the control horizon times the number of control inputs r . To determine the current time step's control, only the first r rows of u_{h_c} are used since this computation is repeated every time step. To expedite the computation, only the first r rows of the pseudo-inverse need to be retained.

$$u_{h_c} = -pinv[T^T R T + Q] T^T R [-y_T + A y_p + B u_p] \quad (25)$$

Equation 25 is identical to that shown by Ref. [5] for the computation of the required control inputs to minimize the objective function.

B. Baseline GPC Applied to TRAST

The dSPACE system was chosen as the real-time controller for use on the TRAST wind-tunnel model in TDT. In order to exercise the dSPACE real-time system prior to application to the wind-tunnel model, the physical plant was replaced with the RCAS linear simulations of the plant as described in section IV. The controller outputs command voltages through digital-to-analog (D-to-A) hardware that is then wired into secondary parallel A-to-D inputs into the same controller. These secondary inputs are applied to the RCAS-derived state-space model of TRAST that produces a predicted response Y . These response voltages are then wired from the secondary D-to-A outputs into the primary A-to-D inputs of the controller. This simulation setup is of value because it exercises the real-time controller hardware and no software changes are required to substitute the physical plant for the analytical plant. Recall, the results presented in this report all use the simulated analytical plant. Testing with the physical model and a GPC control law

is planned for future wind-tunnel entries. A screen shot of the controller with the parallel linearized analytical plant from RCAS is shown in Fig. 8a. Details about the linearized plant are shown in Fig. 8b. When used with the experimental TRAST model, the analytical plant in Fig. 8b (bottom part of Fig. 8a) will not be used. GPC command signals and the random excitation used for system ID are generated in the GPC Controller block in the top of Fig. 8a.

For realism, the analytical plant contains external disturbances, representing things such as wind gusts and/or actuator chatter, and signal noise representing instrumentation noise. Disturbances and noise are simulated by the addition of random disturbance/signal blocks as shown. For simplicity, the external disturbances are simulated as an “unknown” addition to the command signal. Finally, rate limiters are applied to the command inputs to simulate actuator performance.

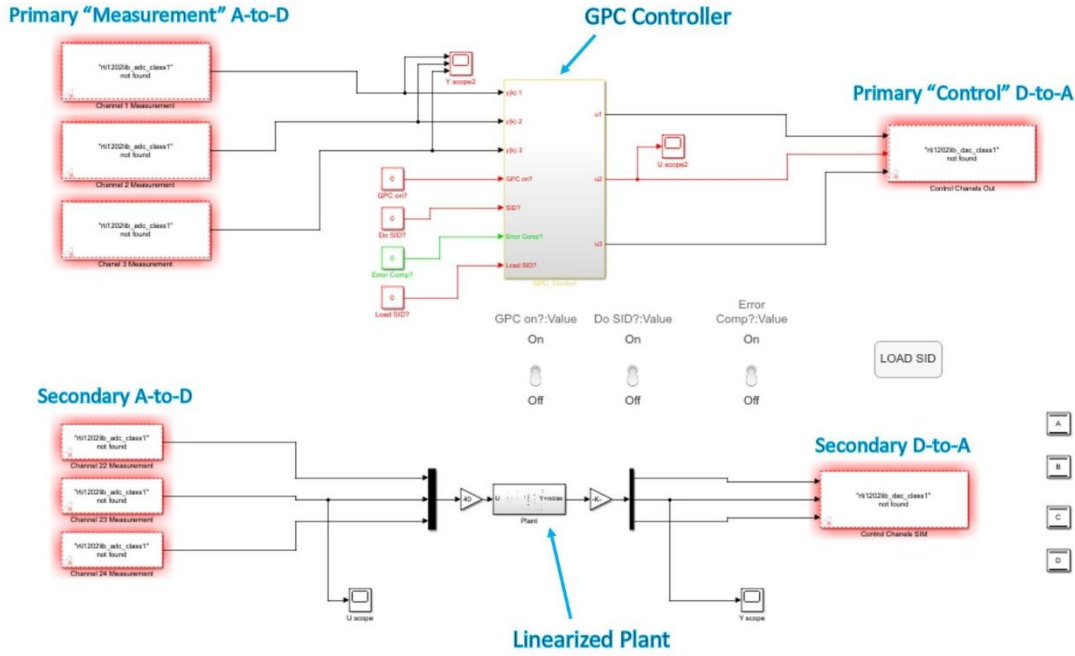


Fig. 8a Real-time controller, control law and analytical model with linearized plant.

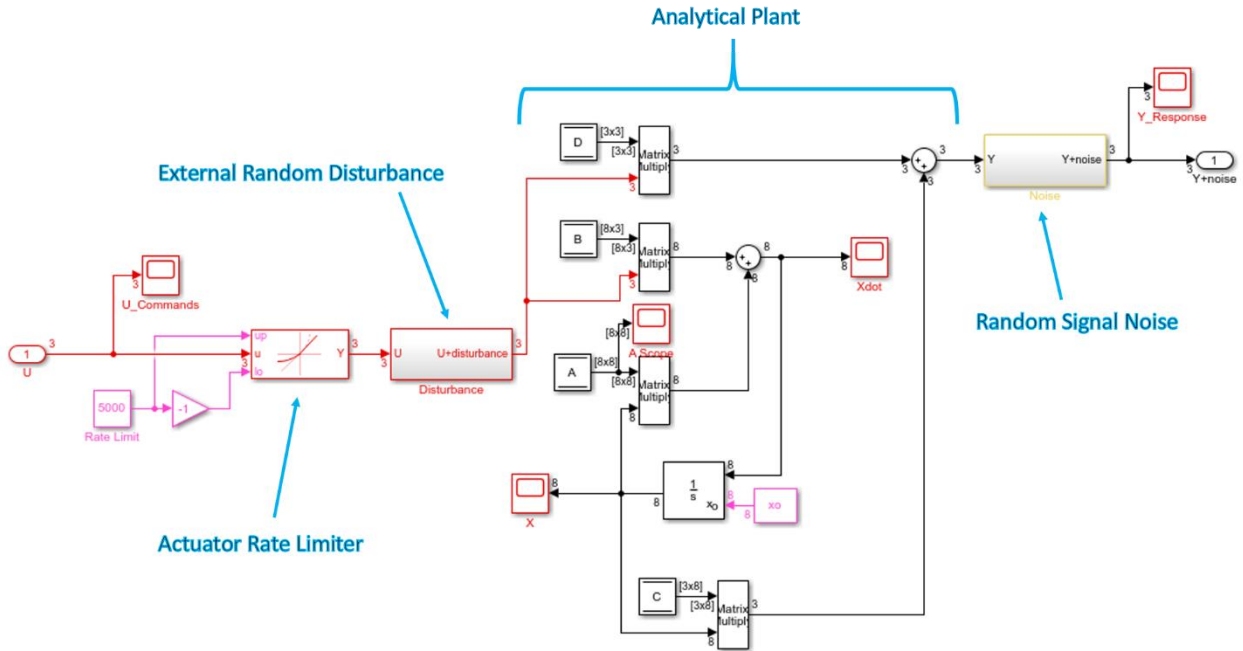


Fig. 8b Real-time controller, details of analytical linearized plant model.

As described in section IV, RCAS was used to derive linearized state-space systems for a fixed RPM of 909 and a velocity sweep from 30 to 210 knots. As seen in Fig. 6, the wing bending mode experiences a slightly unstable hump-mode flutter instability at about 90 knots and continues until approximately 160 knots when a hard crossing of the wing chord mode occurs as a result of a propotor-pylon instability or whirl-flutter.

Figure 9 is an open loop response of the system at an airspeed of 160 knots where the TRAST is unstable. The response measurements are wing bending moments (ft-lbs) in the out-of-plane, in-plane chord, and torsion directions respectively. With no control inputs, one can see that the system is indeed unstable, and the oscillations are growing rapidly. The control commands are degrees of blade pitch in lateral cyclic, longitudinal cyclic, and collective respectively.

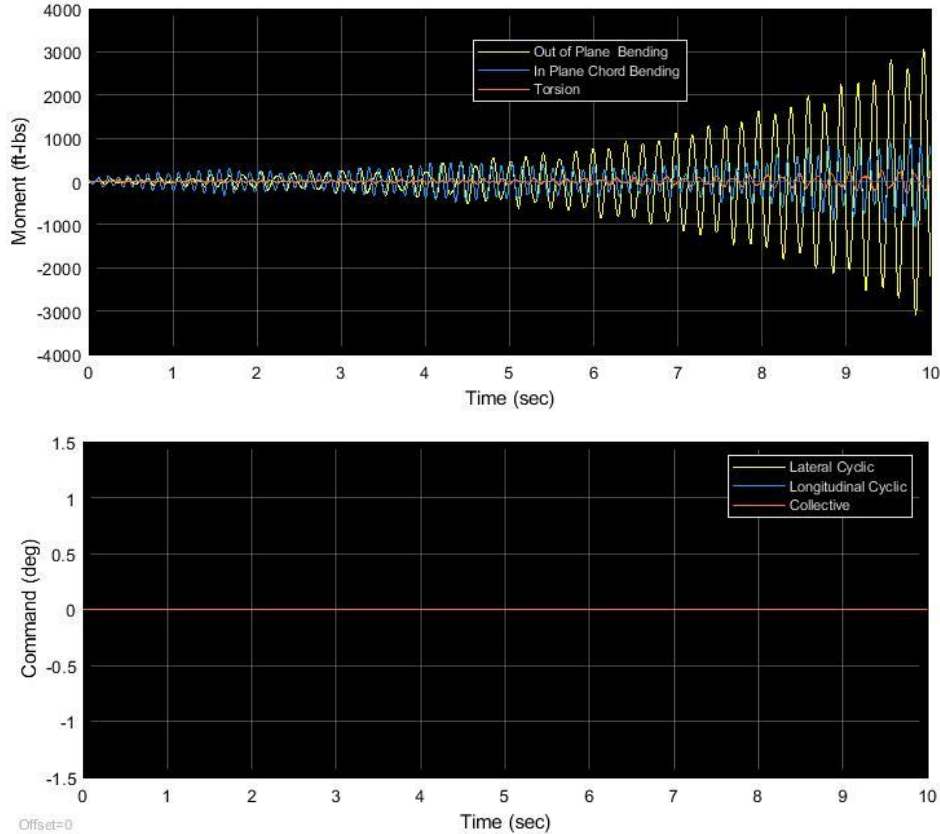


Fig. 9 TRAST open loop response at 160 knots.

Frequency-banded white noise was applied to each of the commands to excite the system in order to acquire a system ID. Different seed values were used for the white noise generator for each of the three different controls to ensure that the inputs differed and were distinguishable. Plotted in Fig. 10 are the response and commands for the system ID for the same flight condition shown in Fig. 9. Using the guidance from Ref. [5], the order (p) was set to 20, the control horizon (h_c) and prediction horizon (h_p) were both set to 25, and the sample length (s) was set to 1000. For this simulation, the time step is 1/200 to provide ample resolution of the system dynamics that range from 3 to 10 Hz. With these settings, a system ID of 1000 time steps will take five seconds to acquire. These settings are used for all of the examples presented in this report unless otherwise stated.

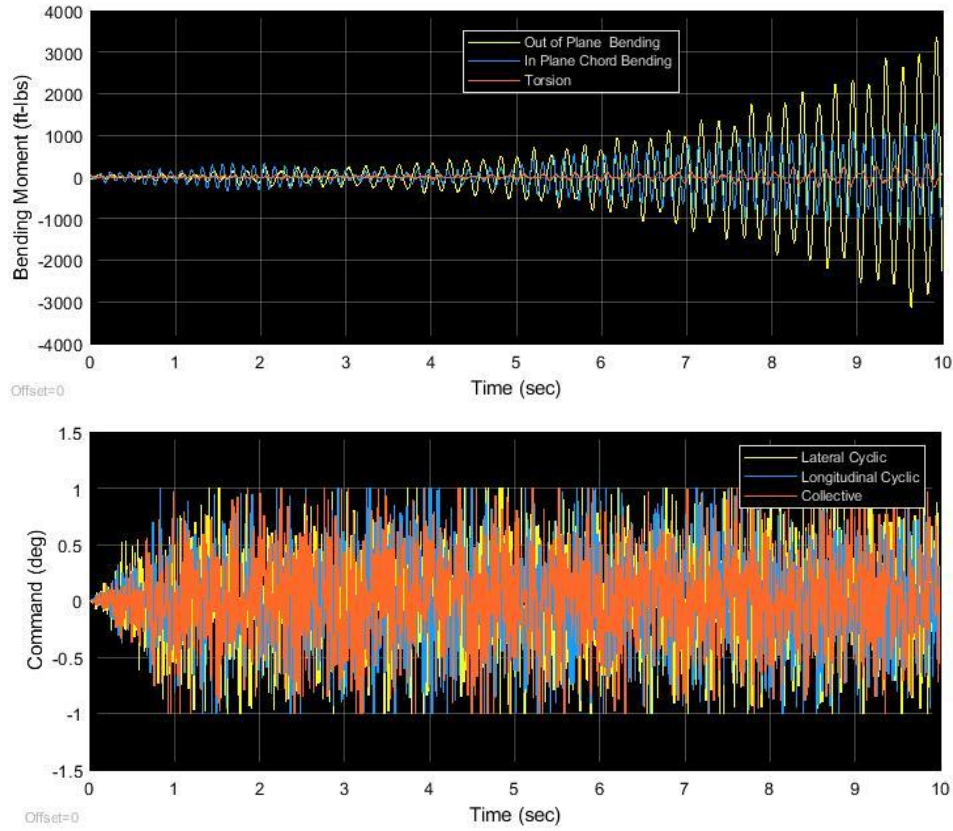


Fig. 10 TRAST system identification at 160 knots.

Following the system ID and derivation of the ARX observer parameters, the GPC algorithm was applied closed loop to the system for the same unstable flight condition shown in Figs. 9 and 10. The closed loop response is shown in Fig. 11. The beginning of the time series is open loop to allow the response to build, and at a time of approximately five seconds, the loop is closed. One can see that the system is stabilized, the vibrations are minimized even in the presence of external disturbances, and the required control commands are small. To help ensure model safety as this control law is implemented on TRAST, a command limiter of one deg was instituted. This can be observed by the clipping of the command signal at a magnitude of one degree. Additionally, the command amplitude is ramped in gradually over a period of one second to avoid sudden inputs when the loop is closed. This can be observed by the gradually increasing amplitude of the command in the first second after the loop is closed or in the beginning of the system ID in Fig. 10.

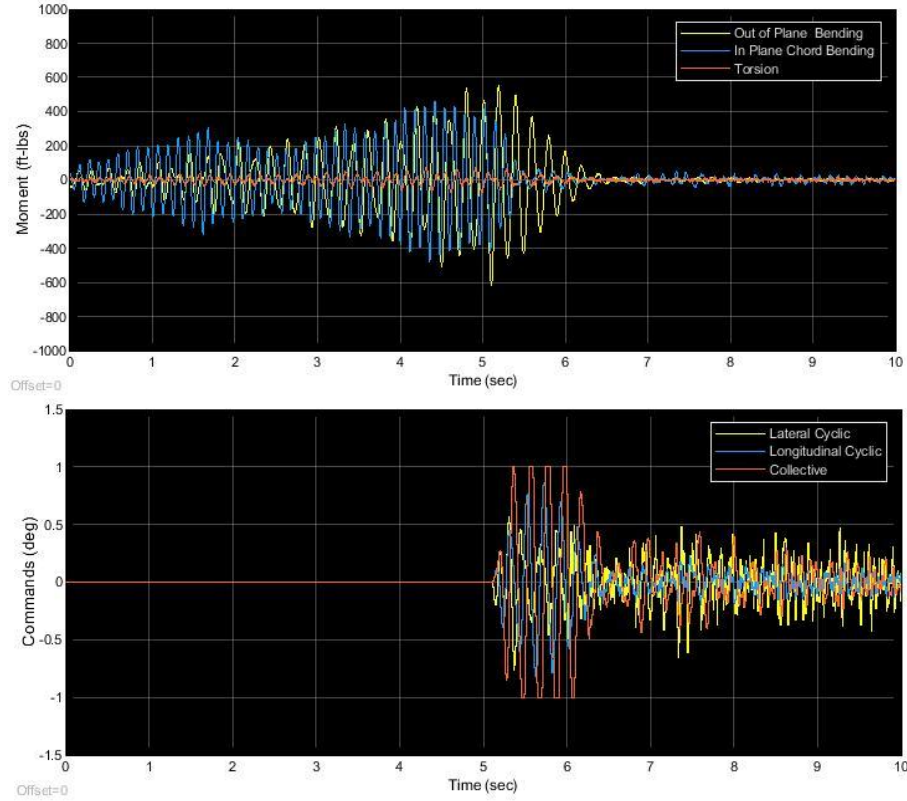


Fig. 11 TRAST closed loop response at 160 knots.

The weighting matrices Q and R , as shown in Eqn. 21, were selected so that comparable “importance” was given to control effort and minimizing response error. Unity values of Q and R do not necessarily imply equal importance since one degree of control input results in hundreds or thousands of ft-lbs of response moment. Therefore, one must be cautious in the selection of Q and R and do so within an appropriate order of magnitude to achieve the desired controller performance. For the simulations presented in this report, the controller inputs and outputs used to define the system ID were measured as volts in lieu of dimensional units. The voltage inputs/outputs were gained such that all signals were in the same order of magnitude to avoid signal clipping while also ensuring adequate signal resolution. As a result, Q and R matrices close to unity work well with the current implementation. In the examples provided above, Q was the identity matrix and R was a diagonal matrix of value 0.5.

To explore the significance of the weighting matrices, Q was left as the identity matrix and R was reduced to a diagonal matrix of value 0.1. Reducing the magnitude of R reduces the magnitude of the commands at the expense of the response performance. The closed loop response with these weighting matrices is shown in Fig. 12. Comparing Fig. 12 with Fig. 11, one can see that the commands are no longer clipped as the control loop is closed when the weighting on response error is lowered. However, the out-of-plane bending moment response is now poorly controlled. Also noteworthy is that the collective command is larger than the others.

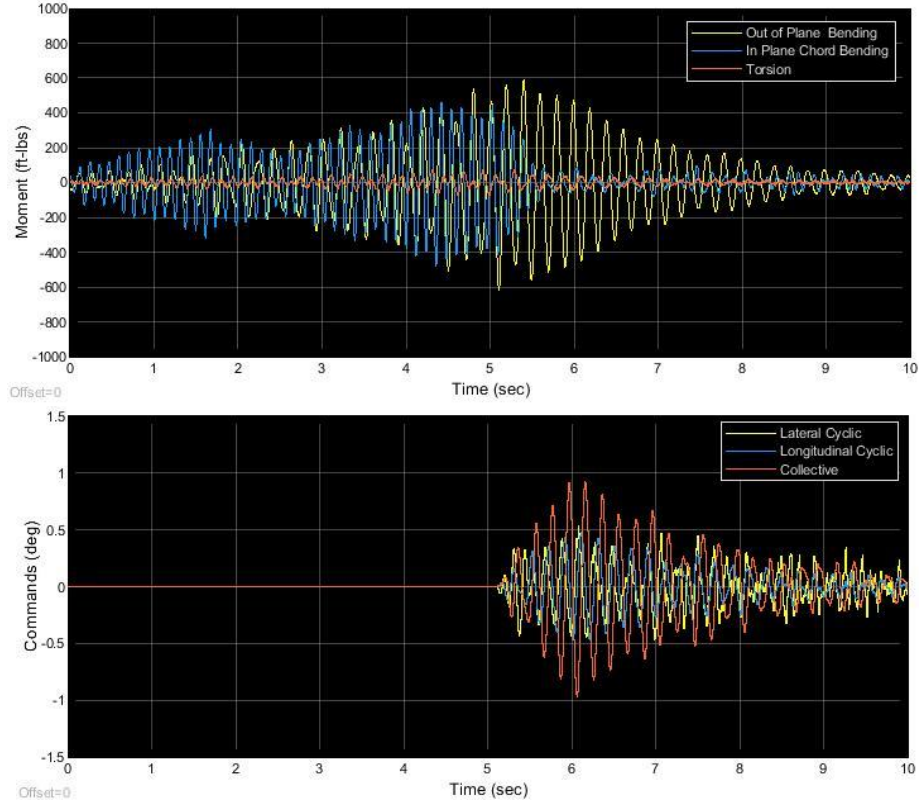


Fig. 12 TRAST closed loop response at 160 knots, altered weight matrices with $R=[0.1;0.1;0.1]$.

An additional option is to provide different weighting values to the various command and response parameters in order to fine-tune the controller performance. To explore this option, a slight increase to the weighting values were applied to the out-of-plane response and collective command beyond that used for the simulation presented in Fig. 12. Figure 13 contains the time history response and commands with a weighting of 0.3 on out-of-plane bending and 0.1 on the other responses, and a weight of 2.0 on collective command and unity on the other commands. In this figure, the response is considerably improved, and the control effort is still below the one-degree limit even when initially engaged. The ability to individually weight response and control parameters was not explored in Refs. [5], [6] or [9]; however, the benefit of doing so is clear. This benefit is increasingly magnified if response or command parameters operate at different magnitudes and/or have different sensitivities.

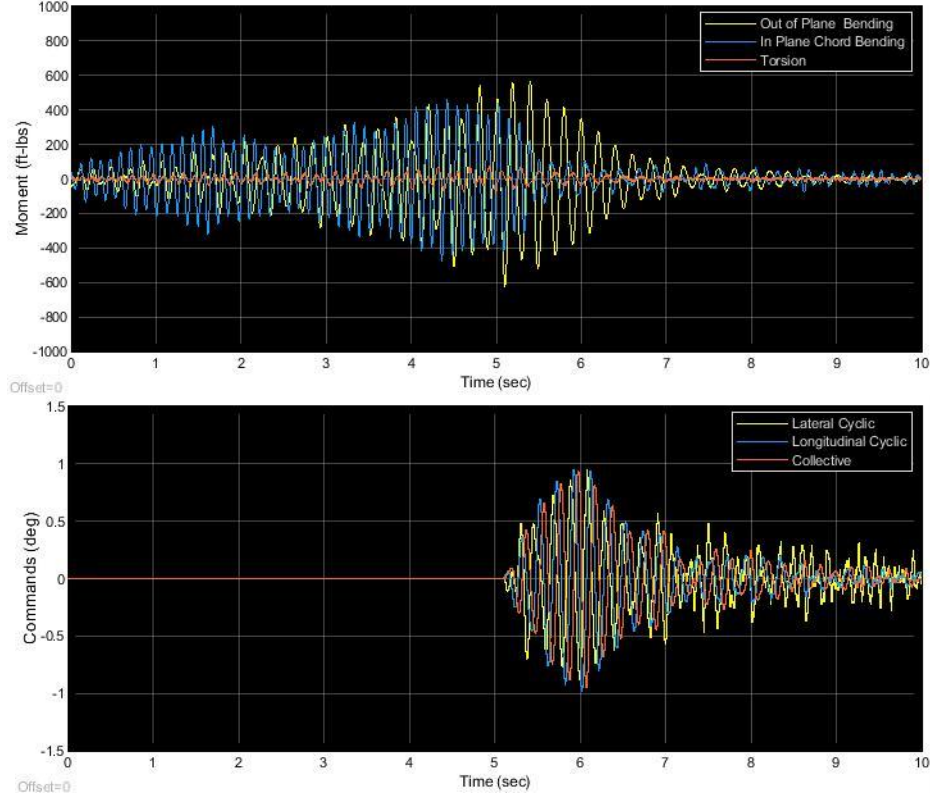


Fig. 13 TRAST closed loop response at 160 knots, altered weight matrices with $R=[0.3;0.1;0.1]$ and $Q=[1;1;2]$.

VI. Self-Adapting Advanced GPC

The GPC algorithm is very effective at stabilizing the unstable TRAST system when the system ID parameters are acquired at the same flight condition. However, it is desirable to have a controller that is stable across a range of flight conditions. To explore this concept, a system ID was done at a velocity of 140 knots. The performance of the controller at 140 knots is shown in Fig. 14. At 140 knots the system is only slight unstable, and GPC substantially reduces the vibration and improves system damping with minimal control effort. Similar to previous examples, the beginning of the time history is open loop, and the loop is closed at approximately 5 seconds.

This same system ID parameters acquired at 140 knots were used for the TRAST model at 150 knots and is presented in Fig. 15. The degradation in controller performance is apparent as the airspeed is increased and the controller experiences a large increase in controller workload albeit the system remains stable. Recall that the open loop system at this velocity has a slightly unstable out-of-plane mode. It should be noted that plant behavior changes significantly as a stability boundary is approached and maintaining control with a fixed set of observer parameters is especially challenging. At subcritical conditions, the observer parameters are expected to perform better over a wider range of velocity.

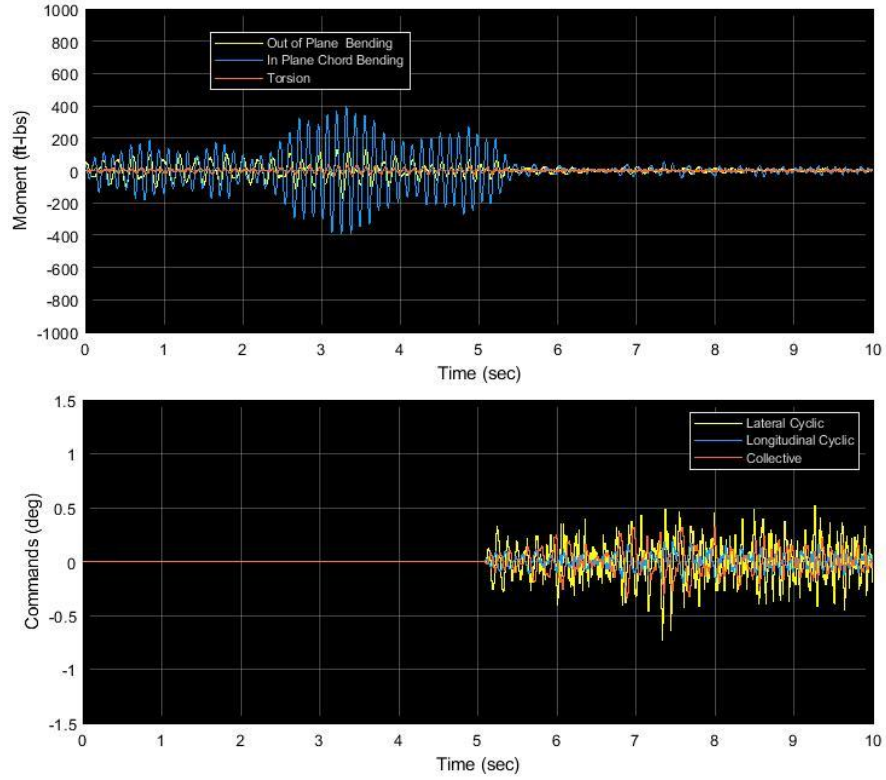


Fig. 14 TRAST closed loop response at 140 knots.

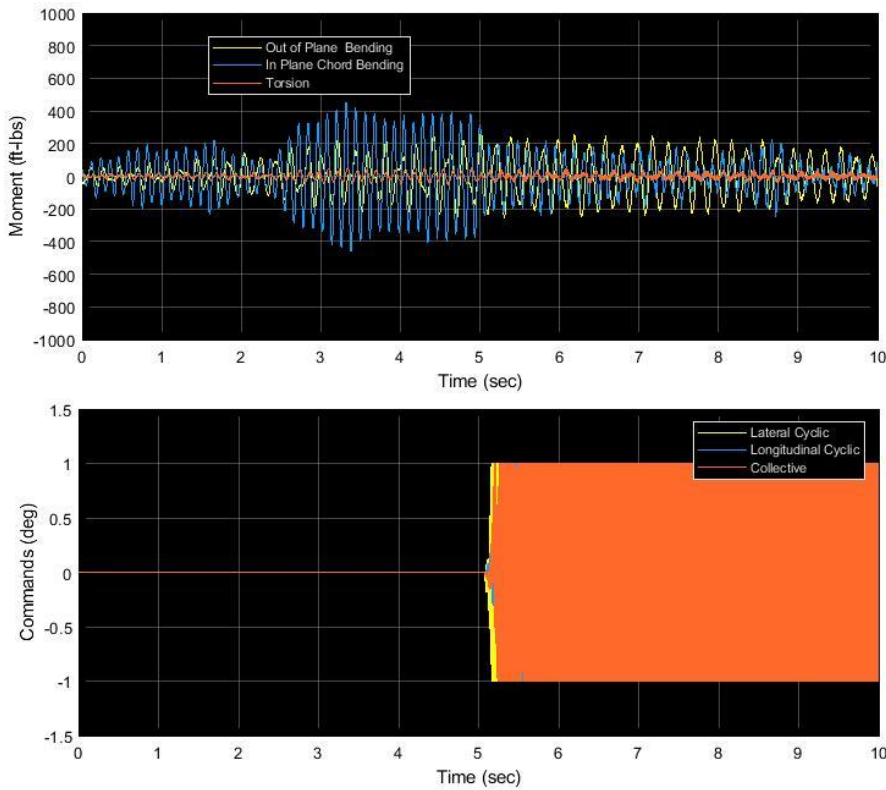


Fig. 15 TRAST closed loop response at 150 knots, using system ID conducted at 140 knots.

Progressing farther in velocity, the response at 160 knots with a system ID conducted at 140 knots is shown in Fig. 16. For this condition, the controller is no longer able to stabilize the system, but the response is slightly less violent than the open loop system presented in Fig. 9 indicating that the controller is still providing some useful input.

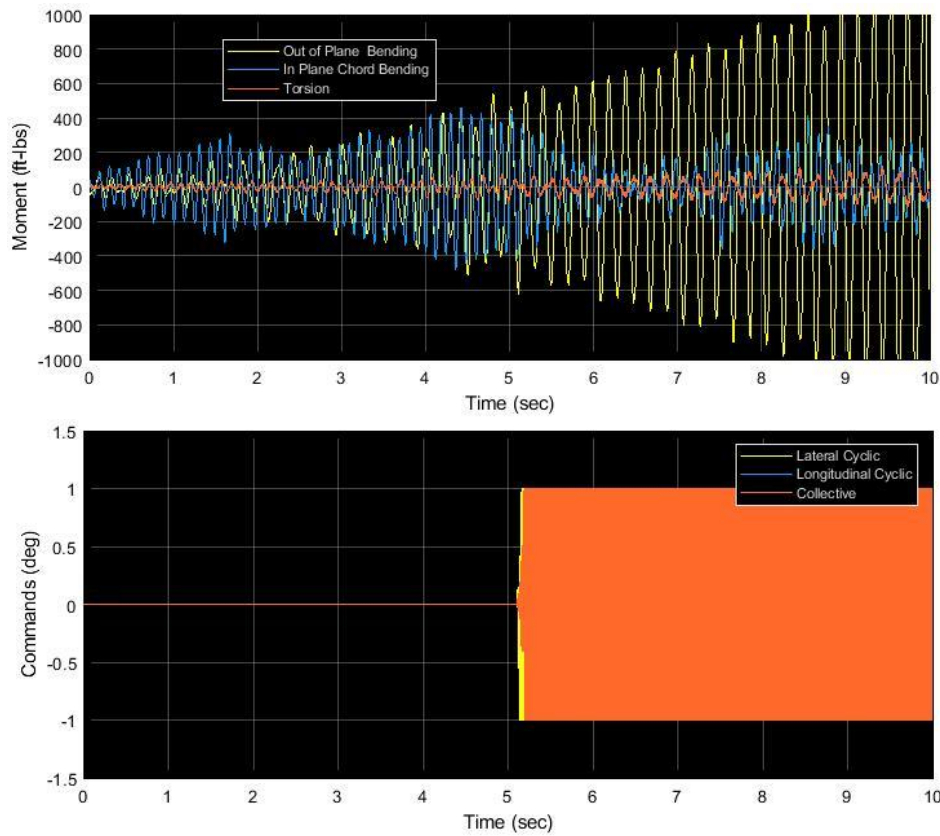


Fig. 16 TRAST closed loop response at 160 knots, using system ID conducted at 140 knots.

If a good system identification has occurred, the GPC controller should drive the system to minimal response with minimal control input cost. As a result, continually adaptive GPC is difficult to achieve since the signal to noise ratio, once optimized and employed, is usually very small. Additionally, with a MIMO controller the system ID process can struggle to identify the input/output relationship from nominal closed-loop operation since the control input may not be distinct with minimal correlation across channels when used in practice. Adequate system identification can be conducted with a working GPC controller employed if additional orthogonalized control input, or dither, is added to the control to create additional excitation of the system enabling input/output characterization above background noise levels. The problem with this approach is that it shakes the system that it is otherwise trying to suppress through active controls and defeats a key purpose of employment. Therefore, development of a capability to provide self-correction without the need for dither is desired.

The concept of a self-adaptive system identification while the GPC controller is active is introduced here as “advanced GPC” (AGPC). Two approaches were explored in the development of AGPC. In the first, the system identification process described by Eqn. 20 was used after an additional s time steps have progressed. Existing control inputs and responses were used to develop new observer matrices T , A , and B that replaced the previous values. This method is effective if large control inputs are being utilized as it completely replaces all observer matrices. In the second approach, the concept of a system error (ξ) is introduced as defined by Eqn. 26 after at least h_p timesteps into the future have been completed.

$$\begin{aligned}\xi &= y_{h_p actual} - y_{h_p predicted} + \{\varphi\} \\ &= y_{h_p actual} - \left[T u_{h_c actual}(k) + A y_p(k-p) + B u_p(k-p) \right] + \{\varphi\}\end{aligned}\quad (26)$$

where φ represents noise and external disturbances.

The subscript *actual* in Eqn. 26 indicates the actual measurements of response and control inputs after the h_p timesteps into the future. If one were to assume that the noise and disturbances follow a random pattern, then repeated computations should cancel these disturbances. If we continue to an additional s timesteps, then one would have $s-h_p$ computations of ξ to minimize the effect of φ . If φ is minimized and all the values of y and u shown in Eqn. 26 are now measured, then the only uncertainty that should remain is the error (or “delta” values) in the observer matrices T , A , and B . After these s timesteps, Eqn. 26 can be expanded into the matrix form shown in Eqn. 27 where the delta operator indicates the error in the observer T , A , and B matrices where the control and state matrices are of width $s-h_p$. The delta matrix values can then be determined by Eqn. 28 and the updated values of T , A , and B by Eqn. 29.

$$[\xi] = [\Delta T \mid \Delta A \mid \Delta B] \begin{Bmatrix} [u_{h_c actual}] \\ [y_p(k-p)] \\ [u_p(k-p)] \end{Bmatrix} \quad (27)$$

$$[\Delta T \mid \Delta A \mid \Delta B] = [\xi] pinv \left(\begin{Bmatrix} [u_{h_c actual}] \\ [y_p(k-p)] \\ [u_p(k-p)] \end{Bmatrix} \right) \quad (28)$$

$$\begin{aligned}T_{new} &= T_{old} + \Delta T \\ A_{new} &= A_{old} + \Delta A \\ B_{new} &= B_{old} + \Delta B\end{aligned} \quad (29)$$

The potential advantage of the second approach to AGPC is that it employs incremental changes to an existing system ID that was adequately performing at nearby flight conditions. It is theorized that this may be more robust than the first approach of replacing the T , A , and B matrices. However, initial exploration of this concept has revealed that it too can be destabilizing if the control inputs are undistinguishable or if the system response and control magnitudes decrease to levels that are masked by background noise. Ongoing research is aimed at refining metrics that measure control orthogonality and system performance. The intent is that AGPC will not alter a good performing controller, but rather update the system ID only when it is shown to deteriorate below a threshold and only when control orthogonality metrics are satisfied. Preliminary results show great promise to this concept and an example is presented in Fig. 17. In this example, the system is operating at 160 knots with a system ID conducted at 140 knots (same condition as that presented in Fig. 16). The simulation begins with the controller closed loop and the self-correcting algorithm active with the second approach described above that computes system error. After five seconds, the self-correcting system ID is applied. One can observe that the controller performance is substantially improved, and the self-correction methodology is effective at stabilizing the system and reducing vibration with little control effort.

Future work will continue to refine AGPC and quantify its robustness to changing systems and will investigate the use of orthogonal multisine inputs to improve system ID efforts. Additionally, the addition of orthogonal multisine signals are being investigated to improve the self-adapting algorithm.

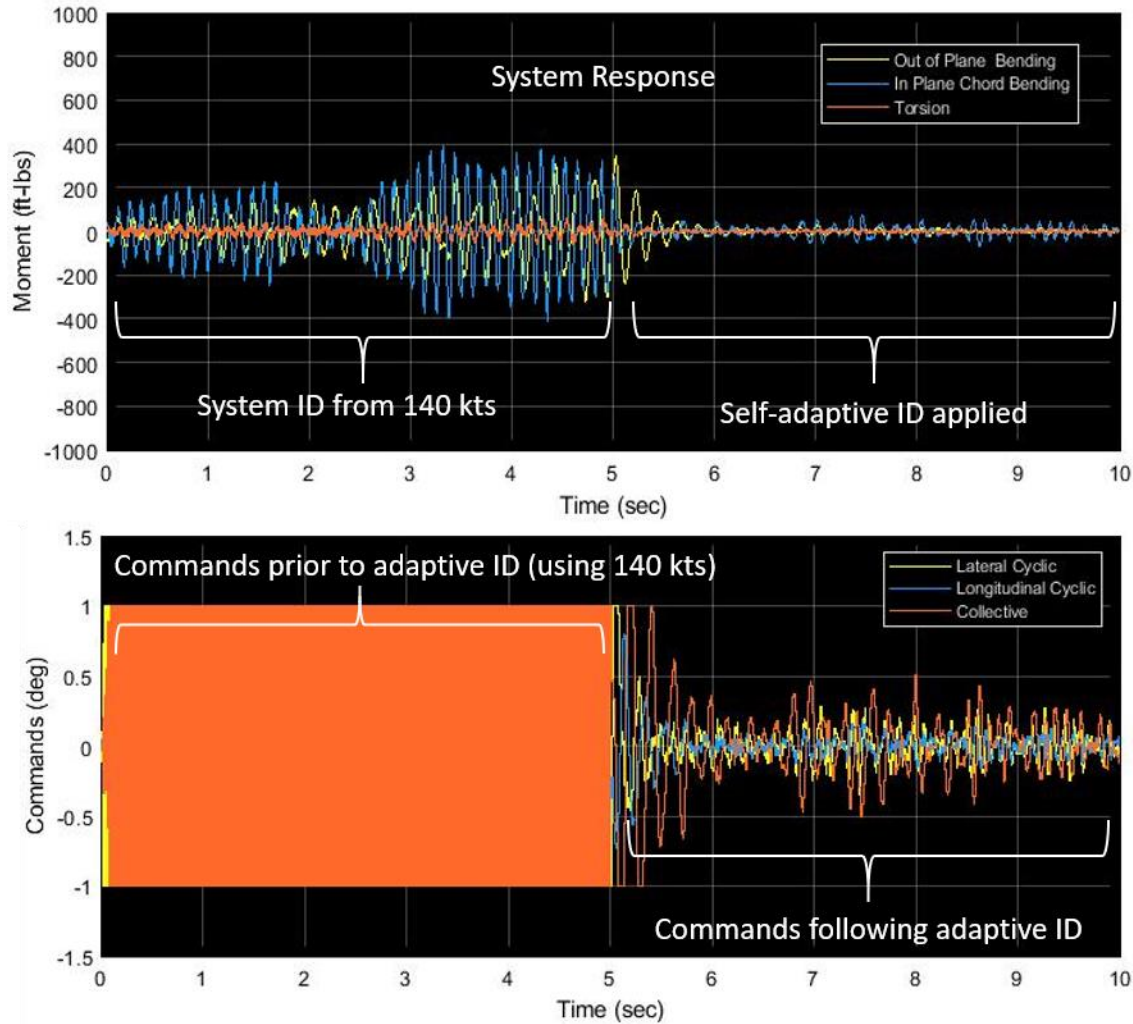


Fig. 17 TRAST Closed loop response at 160 knots, using system ID conducted at 140 knots, with Advanced GPC self-correcting algorithm.

VII. Concluding Remarks

A new wind-tunnel model test program is underway for aeroelastic and dynamic response testing of tiltrotor aircraft known as the TiltRotor Aeroelastic Stability Testbed or TRAST. This model will serve to investigate tiltrotor dynamic phenomena and stability to further the state-of-the-art in tiltrotor aircraft. The present investigation characterized the use of GPC upon TRAST assessing its ability to suppress vibrations and whirl flutter using a linear state-space representation of TRAST. The linear models were produced for various flight conditions using the RCAS comprehensive analysis tool. Simulations have shown that GPC can indeed suppress whirl flutter and significantly reduce vibration across a range of flight conditions. Therefore, it is feasible to use GPC to substantially expand the flight envelope of tiltrotor aircraft. However, since the system ID parameters are experimentally acquired, as the flight conditions migrate away from those used to for the system ID, the controller performance degrades. A newly introduced concept termed advanced GPC or AGPC shows great promise in the ability to enable a self-adapting system ID such that the controller self-corrects as flight conditions change. Ongoing research aims to refine and quantify metrics that measure and monitor control orthogonality and controller performance to ensure a robust self-adapting algorithm. Finally, the use of orthogonal multisine inputs are being investigated to improve system ID methodologies and/or reduce the correlation between input controls in the self-adapting algorithm.

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