

# Secondary Sonic Boom Ray Path Prediction using sBOOM

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Sonic boom ray path equations are solved numerically to show the extent of secondary booms compared to primary sonic boom carpet. Secondary booms are shown to reach altitudes of  $\approx 125$  km before turning toward the ground and impinging at lateral distances of  $\approx 50$  km and beyond. Exact closed-form solutions of the ray paths and their partial derivatives with respect to certain relevant parameters are derived for the simplified case of windless atmospheres and compared against numerical solutions. Ray tube areas are computed using the Jacobian of a single ray as opposed to the traditional technique of using 4 rays that are temporally and azimuthally perturbed. The ray tube areas corresponding to secondary booms can get 1000 times larger than those from primary booms. The numerically computed ray tube areas are verified using exact expressions for the special case of windless atmospheres. Finally, pressure scaling due to ray tube area changes and atmospheric stratification is presented and the shortcomings associated with the current atmospheric interpolation techniques are discussed. It is determined that piece-wise linear interpolation of the atmospheric temperature may be inadequate and could lead to numerical noise in the prediction of secondary booms.

## Nomenclature

|             |   |                                                                                               |
|-------------|---|-----------------------------------------------------------------------------------------------|
| $c$         | = | Speed of sound (m/s)                                                                          |
| $c_a$       | = | Speed of sound at the last atmospheric layer interface along the ray path (m/s)               |
| $D$         | = | Doppler shift parameter                                                                       |
| $J$         | = | Jacobian of the ray paths                                                                     |
| $L$         | = | Atmospheric lapse rate (K/km)                                                                 |
| $\hat{n}$   | = | Unit vector normal to the wavefront                                                           |
| $n_h$       | = | Horizontal component of the unit vector normal to the wavefront                               |
| $\vec{q}$   | = | Slowness vector                                                                               |
| $\vec{q}_a$ | = | Slowness vector at the last atmospheric layer interface along the ray path                    |
| $q_{z,a}$   | = | Z-component of the slowness vector at the last atmospheric layer interface along the ray path |
| $R$         | = | Gas constant (287.026)                                                                        |
| $s$         | = | Ray path length (m)                                                                           |
| $s_a$       | = | Ray path length at the last atmospheric layer interface along the ray path (m)                |
| $t$         | = | Time step during propagation (s)                                                              |
| $T$         | = | Atmospheric temperature (K)                                                                   |
| $T_a$       | = | Atmospheric temperature at the last atmospheric layer interface along the ray path (K)        |
| $V$         | = | Aircraft velocity (m/s)                                                                       |
| $\vec{w}$   | = | Wind velocity vector                                                                          |
| $\vec{x}$   | = | Ray path vector                                                                               |
| $x, y, z$   | = | Cartesian components of the ray paths                                                         |
| $\rho$      | = | Density (kg/m <sup>3</sup> )                                                                  |
| $\phi$      | = | Propagation azimuthal angle (degrees)                                                         |
| $\gamma$    | = | Ratio of specific heats (1.4)                                                                 |

## I. Introduction

SONIC boom prediction involves three key pieces: a) determining the complex flow-field around the aircraft (termed as near-field) using Computational Fluid Dynamics (CFD), b) propagating those pressure disturbances over long

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distances to reach the ground (termed as far-field) and c) computing the loudness metrics to quantify the impact of sonic booms. Three sonic boom prediction workshops have been organized in the past to investigate best practices in sonic boom prediction, both in CFD and atmospheric propagation. Readers interested in knowing more about the workshops should refer to past publications [1–17].

Although the complexity of the cases has increased from the first to the third workshop for both the near-field and ground signature prediction, computations were limited to primary sonic booms. Pressure disturbances generated by aircraft flying at supersonic speeds propagate from the aircraft in all directions. Disturbances that are initially directed downward from the aircraft toward the ground generate what is called a primary boom carpet. Most sonic boom analysis and mitigation exercises attempt to minimize only the primary sonic boom strength, as it generally would cause the most startle and human annoyance. Pressure waves that initially travel upward into the atmosphere and reflect back to hit the ground or primary booms reflected off the ground that reflect back to impinge the ground again are referred to as secondary or indirect sonic booms. Because of the much longer ray paths compared to primary boom carpets, secondary booms generally have lower over-pressure amplitudes. However, they are known to cause rumble or adverse building responses due to lower fundamental frequencies that align with structural resonant frequencies. Because of the much longer propagation distances, secondary boom ray paths and ground signatures heavily depend on the prevailing atmospheric conditions, particularly the stratospheric wind profiles.

The primary objectives for this paper are to predict secondary boom ray paths including user defined number of reflections off the ground, develop exact expressions for the ray paths and ray tube areas under standard windless conditions, investigate pressure scaling due to atmospheric stratification and ray tube area changes, and lay the ground work to be able to predict secondary sonic boom ground signatures. The ray path equations used in this study are coupled differential equations [18, 19] listed in Eqs. (1) and (2). In these equations,  $t$  is the propagation time along a ray. Other relevant parameters are defined in Eqs. (3) and (4). In these equations,  $c$  is the speed of sound,  $\vec{w}$  is the wind speed vector,  $\rho$  is the atmospheric density,  $\vec{q}$  is the slowness vector, and  $D$  is the Doppler shift induced by the prevailing winds. The coupled equations are solved to second order numerical accuracy to determine ray paths. Higher order accuracy is easily achieved using Runge-Kutta schemes. However, higher order was deemed unnecessary for the objectives of this paper. Ray tube areas will be calculated from the Jacobian of the ray path based on past literature [19, 20], where ray path equations are differentiated with respect to the azimuthal angle and time of emission of the disturbance from the vehicle.

$$\frac{d\vec{x}}{dt} = \vec{w} + c^2 \vec{q} D \quad (1)$$

$$\frac{d\vec{q}}{dt} = - \left[ \frac{\nabla c}{cD} + \sum_{i=1}^3 q_i \nabla w_i \right] \quad (2)$$

$$\vec{q} = \frac{\hat{n}}{c + \vec{w} \cdot \hat{n}} \quad (3)$$

$$D = 1 + \frac{\vec{w} \cdot \hat{n}}{c} \quad (4)$$

The rest of the paper is organized as follows. Section II derives exact closed-form solutions to the ray path equations and partial derivatives of relevant quantities under the assumption of windless atmospheres. Section III uses complex variables to verify the accuracy of the computed analytical derivatives. Section IV shows the results for the secondary ray paths as well as several observations pertaining to secondary boom propagation. Finally, section V provides the conclusions of this paper and lists certain future work ideas.

## II. Exact Solutions to the Ray Path Equations

For the sake of verifying the ray paths obtained via numerical integration, exact expressions under the assumption of standard atmosphere are derived in this section. This allows comparison of results obtained using numerical schemes with the exact expressions in the absence of winds. When winds are absent,  $D$  from Eq. (4) becomes unity and Eqs. (1), (2), and (3) transform into Eqs. (5), (6), and (7), respectively.

$$\frac{d\vec{x}}{dt} = c^2 \vec{q} \quad (5)$$

$$\frac{d\vec{q}}{dt} = -\frac{\nabla c}{c} \quad (6)$$

$$\vec{q} = \frac{\hat{n}}{c} \quad (7)$$

The standard atmosphere is composed of temperature lapse rate layers of either zero or some predefined constant. Assuming a piece-wise linear interpolation of the temperature, a constant temperature lapse rate between adjacent points is a valid assumption and thus the equations presented here are applicable to a general atmosphere as well. Since speed of sound depends on the temperature, which in turn depends on the altitude, a series of simple manipulations are used to obtain the exact expressions for the solution to the ray path equations. Equation (6) shows that the variation of the slowness vector is dependent on the gradient operator of the speed of sound. However, the speed of sound varies as a function of the altitude (Z-coordinate) alone. Therefore, the X- and Y-components of the slowness vector ( $q_x, q_y$ ) remain constant throughout the ray path propagation. The Z-component of the slowness vector has a dependence on the gradient of the speed of sound. The gradient of the speed of sound, shown in Eq. (9), is derived from the expression for speed of sound given in Eq. (8). In these expressions,  $L$  is the atmospheric lapse rate, which is defined as the rate at which atmospheric temperature changes with altitude.

$$c = \sqrt{\gamma R(T_a - LZ)} \quad (8)$$

$$\frac{dc}{dz} = -\frac{\gamma RL}{2c} \quad (9)$$

Substituting Eq. (9) into Eq. (6) and performing integration followed by simplification leads to an exact expression for the Z-component of the slowness vector, as shown in Eq. (10). Note that for the primary booms, the sign function has a negative value whereas for secondary booms it takes on a positive unity value to begin the propagation. In the case of zero lapse rate, Eq. (10) simplifies to  $q_z = \text{sign} \frac{n_z}{c_a}$ .

$$q_z = \text{sign} \sqrt{\frac{1}{c^2} - \frac{1-n_z^2}{c_a^2}} \quad (10)$$

To obtain the exact ray path equations, the ray path equations may be transformed to be in terms of the ray path length rather than the time coordinate. Eq. (5) for the Z-component transforms into Eq. (11) using  $ds = v_r dt$ , where  $v_r$  is the magnitude of the ray velocity given by the expression  $v_r = c_0 \hat{n} + \vec{w}$ . In this expression, winds are assumed to be absent, allowing the magnitude of the ray velocity to simply equal the speed of sound.

$$\frac{dz}{ds} = cq_z \quad (11)$$

Using the exact expression for  $q_z$  from Eq. (10), substituting into Eq. (11), performing the integration, and imposing the boundary conditions results in Eq. (12) for the case of constant lapse rate.

$$s = s_a + \frac{2c_a^2}{(1-n_z^2)\gamma RL} (q_z c - q_{z,a} c_a) \quad (12)$$

For the simplified case of zero lapse rate, the ray path length expression is much simpler as given in Eq. (13).

$$s = s_a + \frac{z - z_a}{n_z} \quad (13)$$

The individual components of the exact ray paths can be computed by multiplying the ray path length by the unit normal vector along each direction. Therefore,  $X = n_x s$ ,  $Y = n_y s$  and  $Z = n_z s$ . Although the components of the slowness vector,  $q_x$  and  $q_y$ , are constant throughout the length of the propagation, the unit vectors  $n_x$  and  $n_y$  are continuously changing due to the change in the speed of sound as the disturbance propagates to the ground. As long as the assumption of no winds is maintained, these exact expressions are equally valid for both the primary booms and the secondary booms.

In addition to computing the ray path equations, sonic boom propagation requires calculation of the ray tube areas. Many sonic boom prediction codes use four rays that are azimuthally and temporally separated to compute the ray tube areas. Using four rays to determine ray tube areas poses certain difficulties which are exacerbated in cases and atmospheres where temperature and wind gradients are strong. The method described below leverages past work [20] where ray tube areas are computed using a single ray by differentiating the ray paths with respect to the azimuthal angle

and time of emission from the vehicle such as the Jacobian, as described in Eq. (14). Additionally, assuming level flight along the X- direction causes  $\frac{\partial Y}{\partial t_a}$  and  $\frac{\partial Z}{\partial t_a}$  to be identically zero and  $\frac{\partial X}{\partial t_a}$  simplifies to the velocity of the aircraft,  $V$ . Under these assumptions, the Jacobian can be simplified to the expression shown in Eq. (15).

$$J = \begin{bmatrix} \frac{\partial X}{\partial s} & \frac{\partial Y}{\partial s} & \frac{\partial Z}{\partial s} \\ \frac{\partial X}{\partial \phi} & \frac{\partial Y}{\partial \phi} & \frac{\partial Z}{\partial \phi} \\ \frac{\partial X}{\partial t_a} & \frac{\partial Y}{\partial t_a} & \frac{\partial Z}{\partial t_a} \end{bmatrix} \quad (14)$$

$$J = V \begin{bmatrix} \frac{\partial Y}{\partial s} \frac{\partial Z}{\partial \phi} - \frac{\partial Y}{\partial \phi} \frac{\partial Z}{\partial s} \end{bmatrix} \quad (15)$$

The exact expressions for the partial derivatives can be obtained by differentiating the equations above. After differentiation and simplification, Eqs. (16), (17), and (18) show the partial derivatives of  $q_z$ ,  $s$ , and the coordinates of the ray path locations, respectively. In these equations, the subscript "a" refers to the values corresponding to the interface location between atmospheric layers within which temperature is assumed to be linear and  $n_h$  represents the horizontal component of the unit vector normal to the wavefront. These can be substituted into Eq. (15) to obtain the Jacobian that determines the ray tube area.

$$\begin{aligned} \frac{\partial q_z}{\partial \phi} &= \frac{\partial q_{z,a}}{\partial \phi} && \text{for zero lapse rate} \\ \frac{\partial q_z}{\partial \phi} &= \frac{q_{z,a} \frac{\partial q_{z,a}}{\partial \phi}}{q_z} && \text{for constant lapse rate} \end{aligned} \quad (16)$$

$$\begin{aligned} \frac{\partial s}{\partial \phi} &= \frac{\partial s_a}{\partial \phi} - \frac{(s-s_a) \frac{\partial q_z}{\partial \phi}}{q_z} && \text{for zero lapse rate} \\ \frac{\partial s}{\partial \phi} &= \frac{\partial s_a}{\partial \phi} + \frac{2c_a^2}{\gamma R L n_h^4} \left[ n_h^2 (c \frac{\partial q_z}{\partial \phi} - c_a \frac{\partial q_{z,a}}{\partial \phi}) + 2(q_z c - q_{z,a} c_a) n_z c_a \frac{\partial q_{z,a}}{\partial \phi} \right] && \text{for constant lapse rate} \end{aligned} \quad (17)$$

$$\begin{aligned} \frac{\partial X}{\partial \phi} &= \frac{\partial X_a}{\partial \phi} + n_x \frac{\partial s}{\partial \phi} - n_x \frac{\partial s_a}{\partial \phi} \\ \frac{\partial Y}{\partial \phi} &= \frac{\partial Y_a}{\partial \phi} + (s - s_a) \frac{\partial n_y}{\partial \phi} + n_y \frac{\partial s}{\partial \phi} - n_y \frac{\partial s_a}{\partial \phi} \\ \frac{\partial Z}{\partial \phi} &= \frac{\partial Z_a}{\partial \phi} + (s - s_a) \frac{\partial n_z}{\partial \phi} + n_z \frac{\partial s}{\partial \phi} - n_z \frac{\partial s_a}{\partial \phi} \end{aligned} \quad (18)$$

### III. Verification of the Ray Path Derivatives

To verify that the partial derivative values expressed above are correct, a complex variable version of the ray path integration procedure was developed. The complex variable approach [21, 22] has been applied in several other gradient verifications. The main advantage of the complex variable method is that true second-order accuracy is achieved by selecting step sizes without incurring subtractive cancellation errors typically present in real-valued finite differences. Tables 1-3 show the sensitivity values for each of ray path length,  $s$ , Y- component of the ray path, and Z- component of the ray path, respectively. The sensitivity values were computed using analytical expressions derived for the partial derivatives under the assumption of no winds and compared against values obtained using the complex version of the ray path integration for arbitrarily selected altitudes along the propagation path. It can be seen that the values of the gradients match up to 14 decimal places, thus verifying that the sensitivity values are sufficiently accurate.

**Table 1 Comparison of the numerical and complex variable gradients for the ray path length  $s$**

| Altitude | Analytical                    | Complex Variable              |
|----------|-------------------------------|-------------------------------|
| 16750.28 | -1.6109468083896800294497e+02 | -1.6109468083896802308005e+02 |
| 32306.33 | -9.3862283704080709867412e+03 | -9.3862283704080566301212e+03 |
| 113104.4 | -6.1014530179353605490178e+04 | -6.1014530179353638488371e+04 |
| 123344.0 | 9.5338714347898785490543e+04  | 9.5338714347898779413814e+04  |

**Table 2 Comparison of the numerical and complex variable gradients for the ray path Y-component**

| Altitude  | Analytical                    | Complex Variable              |
|-----------|-------------------------------|-------------------------------|
| 17429.45  | -1.0987928942403650580673e+03 | -1.0987928942403651351863e+03 |
| 28557.17  | -1.3884146618263532218407e+04 | -1.3884146618263542503242e+04 |
| 62446.77  | -6.2999001410738070262596e+04 | -6.2999001410738075578080e+04 |
| 117382.29 | -1.1544402686696227465291e+05 | -1.1544402686696228325281e+05 |

**Table 3 Comparison of the numerical and complex variable gradients for the ray path Z-component**

| Altitude | Analytical                    | Complex Variable              |
|----------|-------------------------------|-------------------------------|
| 33502.49 | -3.1616524619558163067268e+02 | -3.1616524619557429314525e+02 |
| 68772.98 | 3.4785656352445130323758e+02  | 3.4785656352445772655048e+02  |
| 91977.74 | 1.9903066677734432232683e+03  | 1.9903066677734325305608e+03  |
| 122141.4 | -2.5847904454752573656151e+04 | -2.5847904454752605168318e+04 |

#### IV. Results

Two atmospheric profiles are chosen for the results shown in this paper. The first is based on the standard atmospheric profile as defined in US Standard Atmosphere, 1976 [23]. The second is a canonical atmosphere used in the NCPA propagation code [24]. Figure 1 shows the temperature profiles used in both models up to an altitude of 150 km. Although the standard atmospheric profile has zero winds, the canonical atmospheric profile assumes an X-wind as plotted in the figure, whereas the Y-wind (lateral) is assumed to be zero.

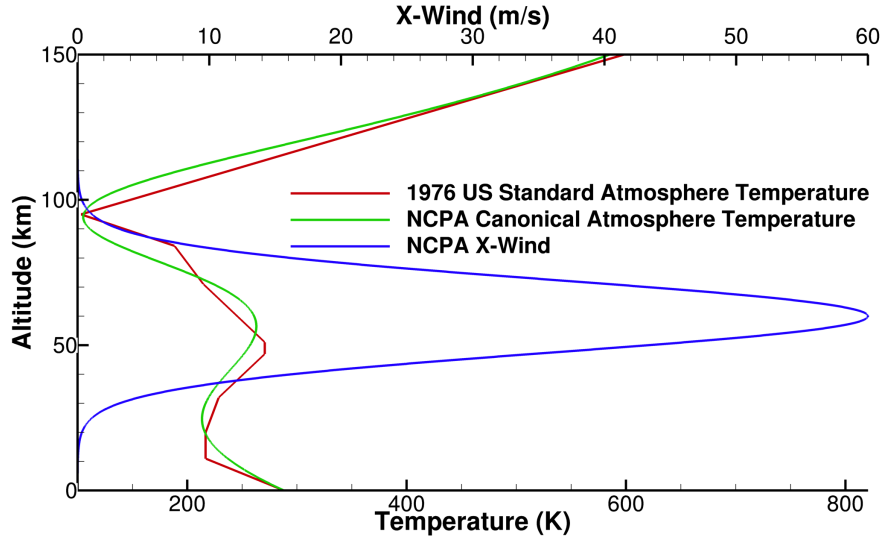


**Fig. 1 Atmospheric models.**

Secondary booms result in a much larger sonic boom carpet with disturbances initially propagating upward and then turning back toward the ground due to the temperature gradients in the upper atmosphere. Figure 2 depicts the ray paths emanating from an aircraft cruising at 16,459.2 m and a Mach number of 1.4, resulting in a secondary boom carpet starting around a lateral distance of  $\approx 60$  km and extending beyond  $\approx 300$  km from the flight path. The primary boom carpet typically ends at a fraction of that distance ( $\approx 20$  km for the case depicted). Also plotted are some ray paths that

are reflected off the ground, propagate upward to eventually turn back at higher altitudes resulting in secondary booms at larger lateral distances from the flight path. The solid lines represent the ray paths corresponding to the standard atmospheric profile and the dashed lines correspond to the NCPA canonical atmosphere. Ray paths corresponding to three different azimuthal angles contributing to the secondary boom carpet,  $\phi = 150, 170$ , and  $180$  degrees, are shown along with a couple of azimuthal angles ( $\phi = 70$  and  $120$  degrees) where the disturbances do not reach the ground. Additionally, the ray path corresponding to a ray close to the edge of the primary boom carpet ( $\phi = 40$  degrees) is also shown. It is interesting to note that higher azimuthal angles (e.g.,  $\phi = 170$  degrees) correspond to the onset of secondary booms whereas shallower angles (e.g.,  $\phi = 120$  degrees) cause the booms to travel much further laterally without ever reaching the ground. The disturbance associated with an azimuthal angle of  $180$  degrees goes directly upward and has a zero lateral range because the Y-component of wind is zero for both the standard atmosphere and the NCPA atmosphere.

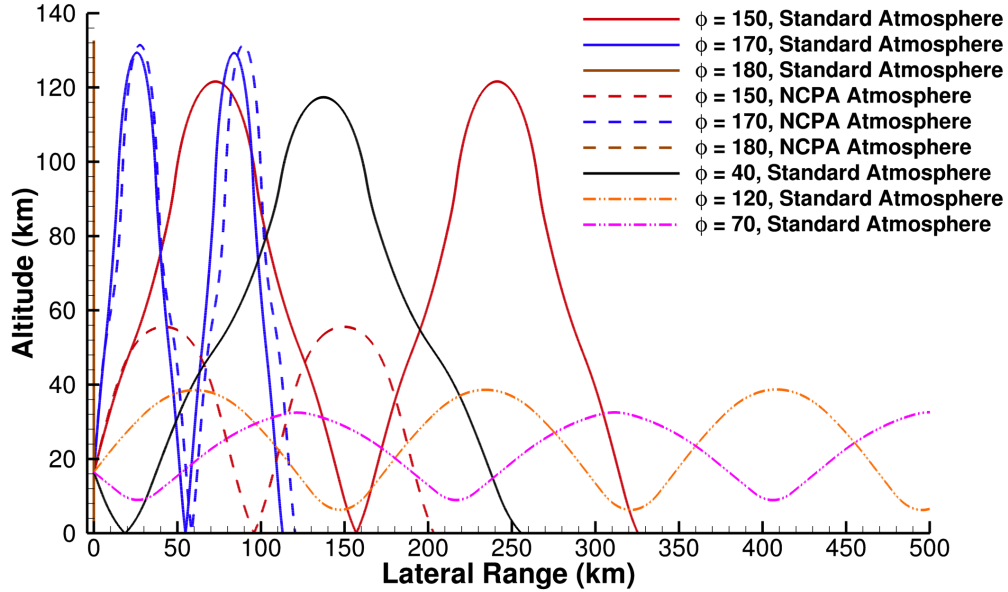


**Fig. 2 Secondary boom ray paths - Lateral Carpet.**

Figure 3 shows the ray paths for the same azimuthal angles as in Fig. 2, except now the longitudinal range is shown. The longitudinal ranges are much larger as expected as it is the direction of the aircraft movement. Interestingly, the onset of secondary boom, for the cases plotted, begins for an azimuthal angle of  $150$  degrees, whereas this corresponded to an azimuthal angle of  $\phi = 170$  degrees for the lateral range.

Atmospheric temperature profile and prevailing winds play a significant role in the location of the secondary booms. This is apparent when looking at Fig. 4, where the vertical component of the slowness vector is plotted as a function of the altitude. At azimuthal angle of  $160$  degrees, the magnitude of  $q_z$  follows a similar trend for both the standard and the NCPA atmosphere. However, as the azimuthal angle reduces to  $152$  degrees, there is an appreciable pinch in  $q_z$  to push it closer to zero around an altitude of  $\approx 60$  km due to differences in temperature and wind profiles between the atmospheres. At  $150$  degrees,  $q_z$  crosses over into the negative values before it has a chance to increase again. A negative value for the slowness vector signifies a downward movement of the ray path, while a positive value implies that the disturbance along the ray path is moving upward and away from the ground. Figure 5 shows the lateral ray paths corresponding to the NCPA atmosphere for azimuthal angles of  $\phi = 150, 151$  and  $152$  degrees respectively. Also included in the figure is the lateral ray path corresponding to the standard atmosphere for an azimuthal angle of  $\phi = 150$  degrees. The ray path corresponding to an azimuthal angle of  $152$  degrees follows the same pattern as for azimuthal angles of  $\phi = 170$  and  $160$  degrees, in that the shape is similar (see Fig. 2) to the ray path from the standard atmosphere except the lateral range is larger because of winds. Due to the change in  $q_z$  as depicted in Fig. 4, the ray path length quickly changes and the lateral range decreases as shown in Fig. 5 for azimuthal angles of  $\phi = 150$ , and  $151$  degrees.

Computation of the ray paths is just the first step that is needed for the prediction of secondary sonic booms. The calculation of the ray tube area is needed to account for the pressure scaling, a manifestation of the conservation of energy, based on the Blokhintzev invariant defined in Eq. (19). In this equation, the values with subscripts correspond to the previous point along the ray paths whereas values without subscripts are representative of the current location

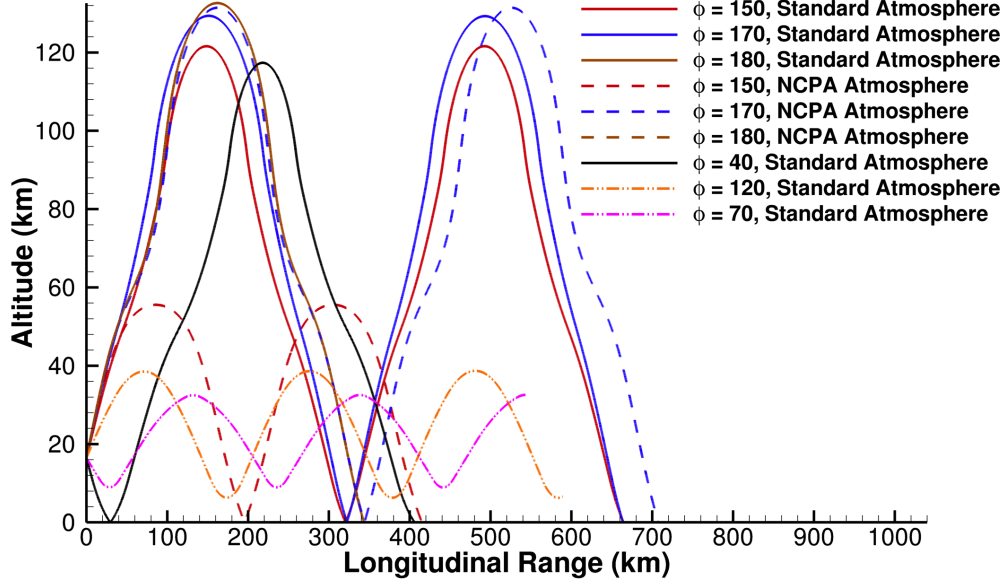


Fig. 3 Secondary boom ray paths - Longitudinal Carpet.

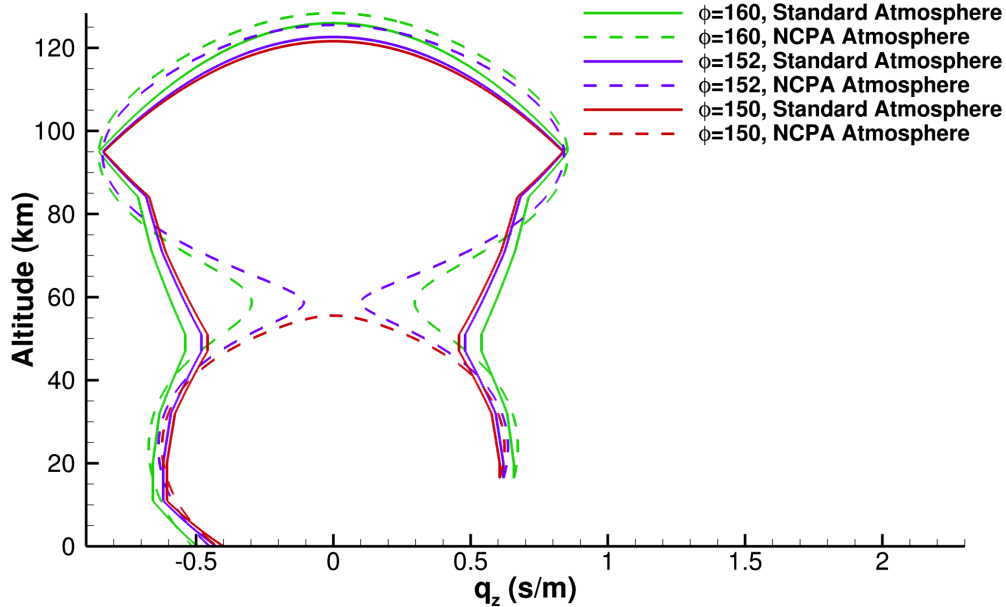


Fig. 4 Variation of the vertical component of the slowness vector.

along the ray path.

$$B = \sqrt{\frac{\rho c A_0}{\rho_0 c_0 A}} \quad (19)$$

The computation of the ray tube area is based on the Jacobian defined in Eq. (15). This requires the computation of the partial derivatives of the ray paths with respect to the ray path length,  $s$ , and azimuthal angle,  $\phi$ . The exact expressions for these partial derivatives are listed in Section II. The partial derivatives are verified to match their complex variable counterparts to the desired accuracy, thus assuring that the computed ray tube areas are accurate.

Figure 6 depicts the variation of Z-coordinate, vertical component of the slowness vector,  $q_z$ , ray path length,  $s$ , and ray tube area,  $A$ , as a function of the longitudinal range for the primary booms in a standard windless atmosphere at an arbitrarily chosen azimuthal angle of 30 degrees. For the primary booms, the ray path length is linear in the isothermal

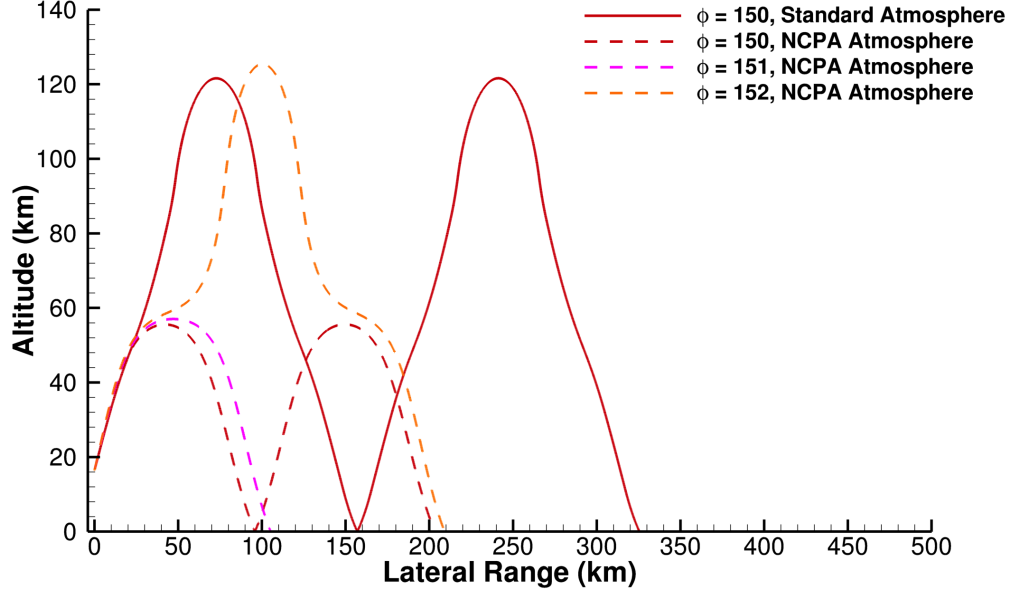


Fig. 5 Secondary boom ray paths around  $\phi = 150$  degrees.

initial atmospheric layer followed by a slight curvature as the disturbance approaches the ground. The slowness vector is initially constant in the isothermal portion of the atmosphere, followed by a gradual increase (remaining negative) as the disturbance reaches the ground. Finally, the ray tube area shows a gradual increase as the disturbance propagates away from the source.

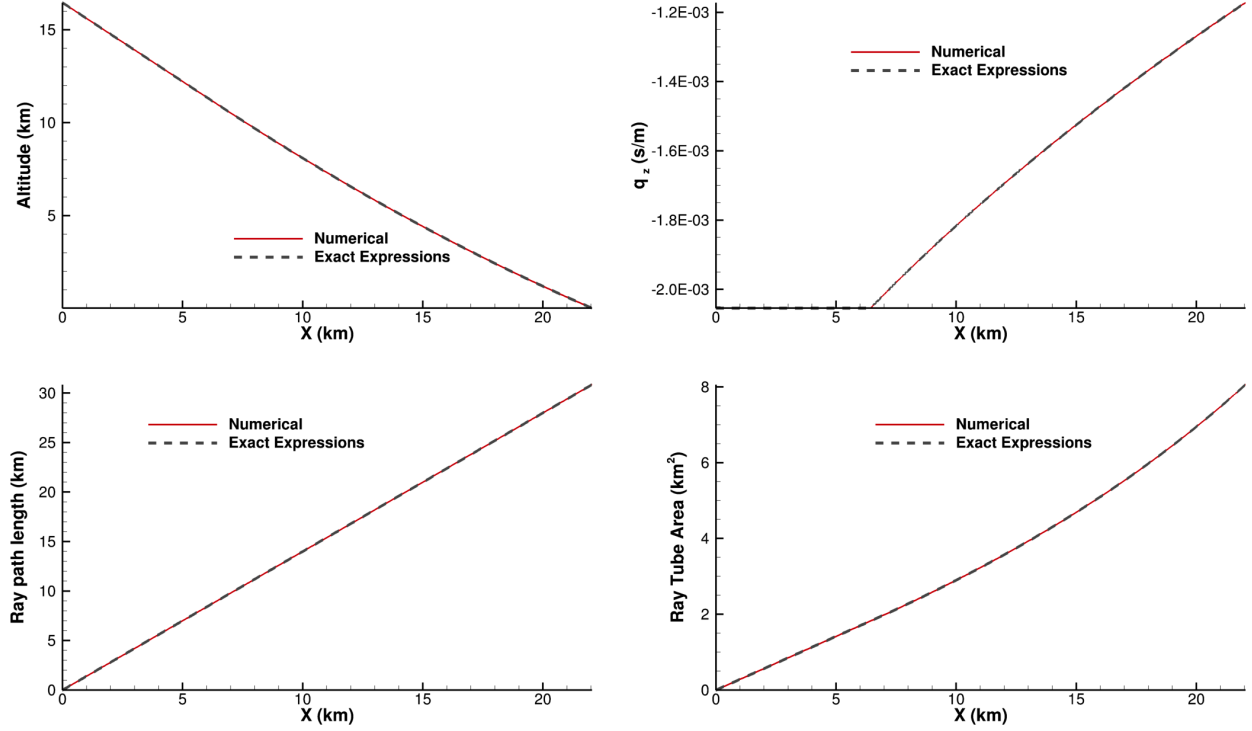


Fig. 6 Metrics calculated for primary booms under standard atmosphere using numerical and exact analytical expressions.

Figure 7 depicts the Z-coordinate and the vertical component of the slowness vector as a function of longitudinal range corresponding to an azimuthal angle 160 degrees when using the standard atmosphere. The Z-coordinate plot shows that the rays reach an altitude of  $\approx 125$  km before turning back to hit the ground at a downstream (longitudinal) distance of  $\approx 250$  km. The vertical component of the slowness vector depicts a couple of inflection points that signify a change of sign in its slope. The location where the slowness vector changes sign from positive to negative corresponds to the peak location of the secondary boom ray path. The key take-away from this figure is that numerical scheme used accurately predicts relevant quantities of interest as shown by the excellent comparison with exact expressions.

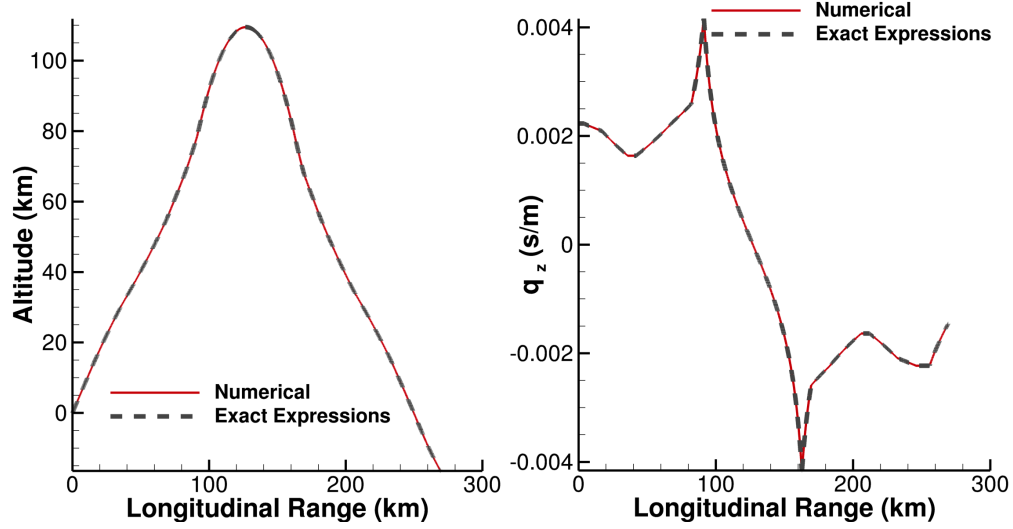


**Fig. 7** Longitudinal range and  $q_z$  calculated for secondary booms under standard atmosphere using numerical and exact analytical expressions.

Figure 8 shows the evolution of the Z-coordinate and the vertical component of the slowness vector corresponding to the NCPA canonical atmosphere and an azimuthal angle 160 degrees. Both quantities are qualitatively similar to those obtained using the standard atmospheric profile.

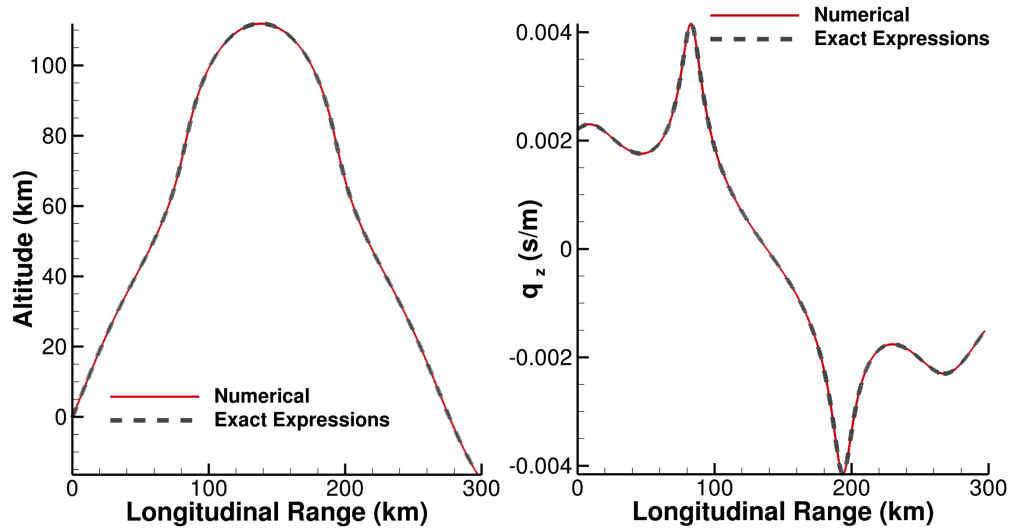


**Fig. 8** Longitudinal range and  $q_z$  calculated for secondary booms under windless NCPA canonical atmosphere using numerical and exact analytical expressions.

Figure 9 depicts the comparison of the ray tube areas computed under standard and NCPA canonical atmospheres, respectively. Under both atmospheres, the ray tube area increases within each atmospheric layer with a discontinuous

jump at each interface between the layers. In the case of NCPA atmosphere, there are multiple jumps, showing up as numerical noise in the ray tube area computation. Although non-physical, this is a direct manifestation of the specification of the atmospheric layers that are separated by discontinuous temperature lapse rate jumps. This is not immediately apparent for primary booms, but the discontinuity, though small, is magnified for secondary booms. The reason for these jumps is further discussed in the following paragraphs. Additionally, unlike for primary booms, the ray tube area significantly decreases as it approaches the highest altitude in its propagation path, and then starts to increase again as the ray turns back toward the ground. In some instances with different temperature gradients and wind conditions, there may be situations where the ray tube area approaches zero along the secondary boom ray paths. When this happens, traditional ray theory can no longer account for the sonic boom prediction, and there should be a hand-off to a different mathematical model and numerical implementation that considers diffraction effects, such as the non-linear Tricomi equation. Although extremely important, those special cases are not discussed in this paper and are left for future work.

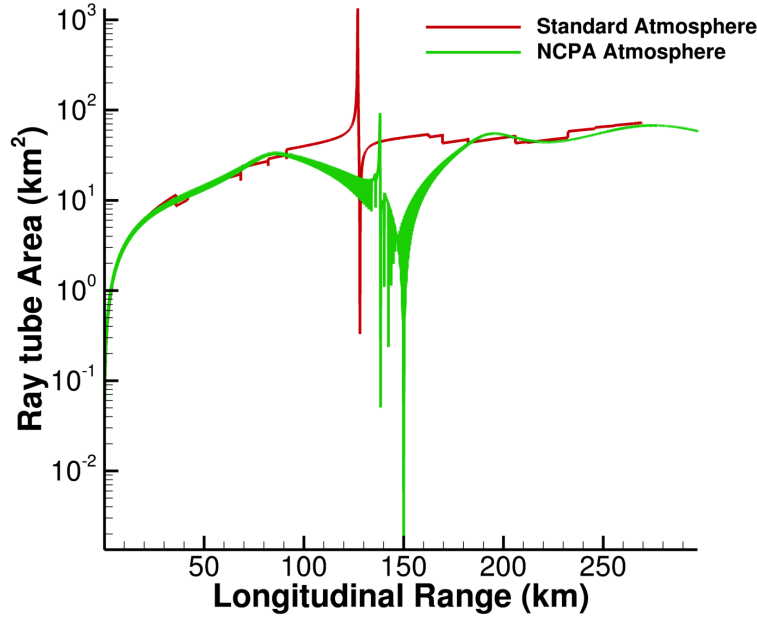


**Fig. 9 Comparison of ray tube areas for secondary booms using standard and NCPA canonical atmospheres.**

Once the ray tube areas are obtained, the Blokhintzev invariant may be computed using Eq. (19). For the computations shown in this paper, the hydrostatic equation is used to compute the atmospheric pressures from the temperature profile, and the state equation is used to determine the density values. Figure 10 shows the Blokhintzev invariant values computed for the standard and the canonical atmospheres, respectively. This plot shows a saw-tooth pattern of the scaling factor where a slope discontinuity arises at each of the atmospheric interface locations. This is a direct manifestation of the slope discontinuity in the temperature profiles as describe below. Even though the NCPA temperature profile appears continuous in Fig. 1, during numerical implementation it is approximated by piece-wise linear interpolation. While this approximation does not have an impact on the ray paths, speed of sound or other relevant metrics, it has a significant impact of the computed partial derivatives of the ray paths.

Figure 11(a) shows the computed temperature lapse rates for the NCPA atmosphere. Visually, this curve seems to be continuous in both values and slope. However, the zoomed-in plot shown in Fig. 11(b) shows that the lapse rates are only continuous in values (hence only C0 continuous) and not in their derivatives (not C1 continuous). This causes the partial derivatives of the ray paths to have jump discontinuities at the atmospheric interface locations and translates to jumps in the Blokhintzev pressure scaling term.

For primary booms under standard atmosphere, there is typically only a single interface between the troposphere and the stratosphere where there could be a jump. However, for secondary booms, due to the much longer ray paths, the disturbances pass through multiple atmospheric layers, leading to multiple slope discontinuities in the pressure scaling term. Although this may not have a significant impact on the predicted sonic booms, it could potentially introduce numerical noise into the adjoint sensitivities and loudness gradients. Further investigation is needed to ensure that the

slope discontinuity at the interface of the different atmospheric layers is eliminated. This is unfortunately beyond the scope of the current work and will be pursued as future research.

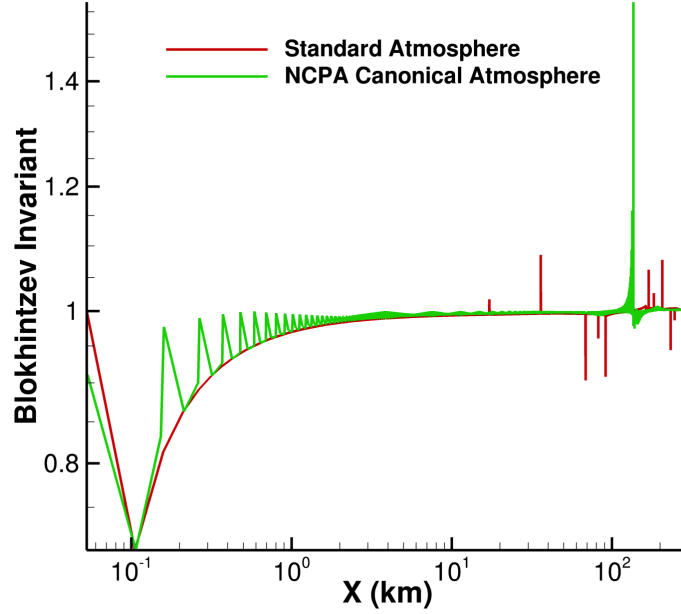


**Fig. 10** Blokhintzev invariant for secondary booms.

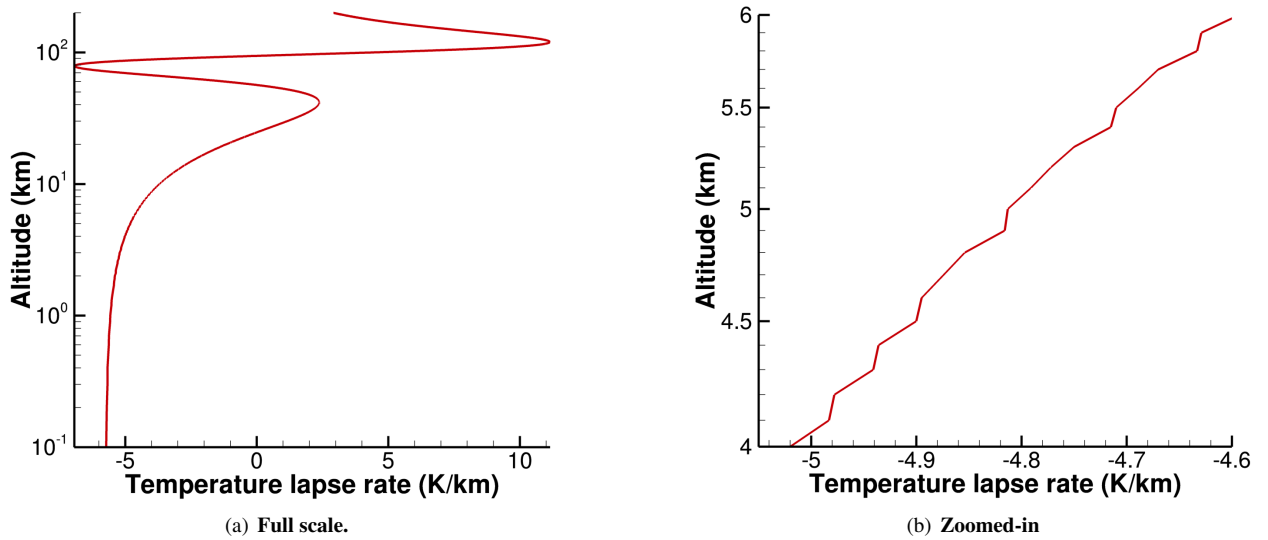


**Fig. 11** NCPA Atmosphere Computed Lapse Rate.

## V. Summary and Concluding Remarks

The ray path equations were solved to predict secondary ray paths with user designed number of reflections off the ground. Exact solutions of the ray path equations and ray tube areas were derived and discussed. These exact equations are equally applicable for primary and secondary booms. The partial derivatives required for the computation of the ray tube area Jacobian were computed with numerical approaches and verified to be accurate using the complex variable approach. Secondary ray paths, ray tube areas and Blokhintzev invariant results were presented for both standard and canonical NCPA atmospheres, and the differences were discussed.

Based on the work presented, the following conclusions can be made.

- 1) Atmospheric wind profiles play a much larger role in secondary booms than primary booms due to their long ray paths
- 2) Interpolation assumptions in atmospheric quantities have a magnified effect for secondary booms
- 3) Depending upon the prevailing atmospheric conditions, especially winds, there could be a significant change in the extent of the secondary ray paths between azimuthal angles even when they are close.
- 4) Special numerical considerations and solution approaches would have to be pursued when ray tube area approaches zero for secondary booms

Future work would include investigation of the discontinuity in the Blokhintzev invariant due to the piece-wise linear interpolation of the temperature profile. Work would also need to be performed to account for prevailing winds and special cases such as the ray tube approaching zero in the upper atmospheres. Most databases did not have measured atmospheric quantities at high altitudes needed for secondary booms. Therefore, future work would need to research into sources that can provide measured winds and other relevant parameters. Multiple runs would need to be completed using measured and realistic winds in geographically diverse locations to document the statistical distributions associated with secondary boom extent, as well as corresponding loudness metrics.

### Acknowledgments

The author wishes to thank NASA's Commercial Supersonic Technology (CST) project for sponsoring this research.

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