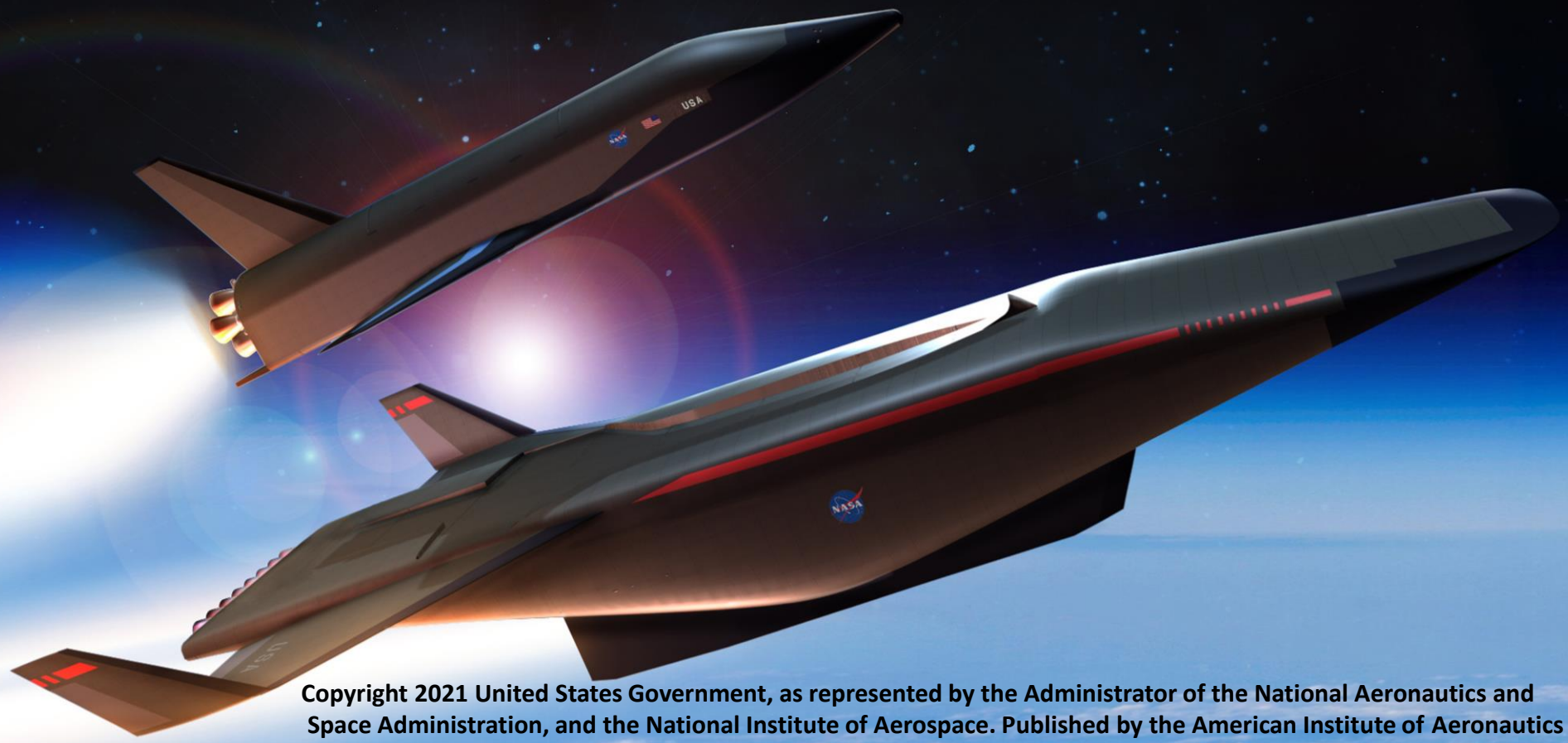
F-ANG+: A 3-D Augmented-Stencil Face-Averaged Nodal-Gradient Cell-Centered Finite-Volume Method for Hypersonic Flows

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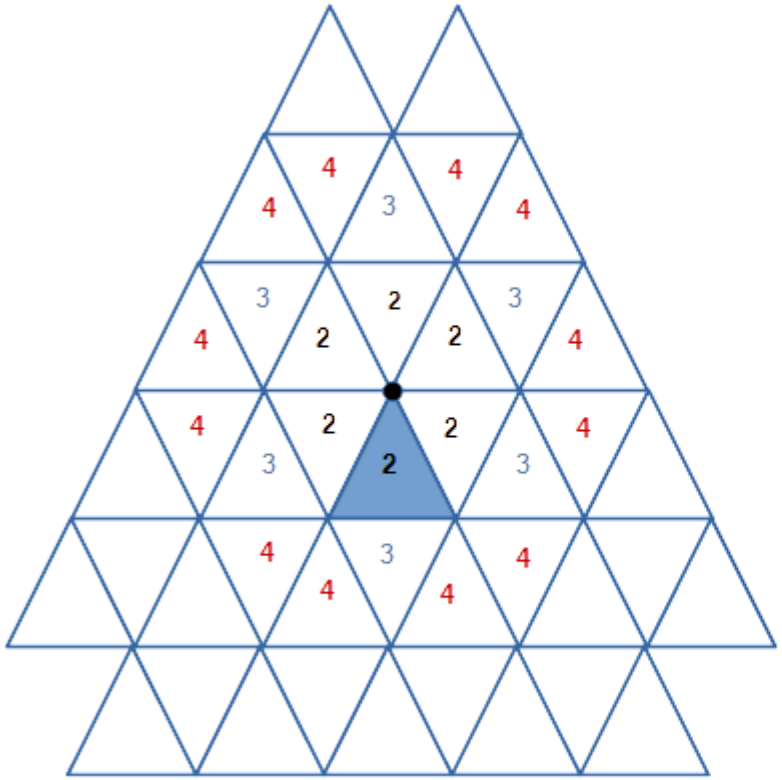
Motivation

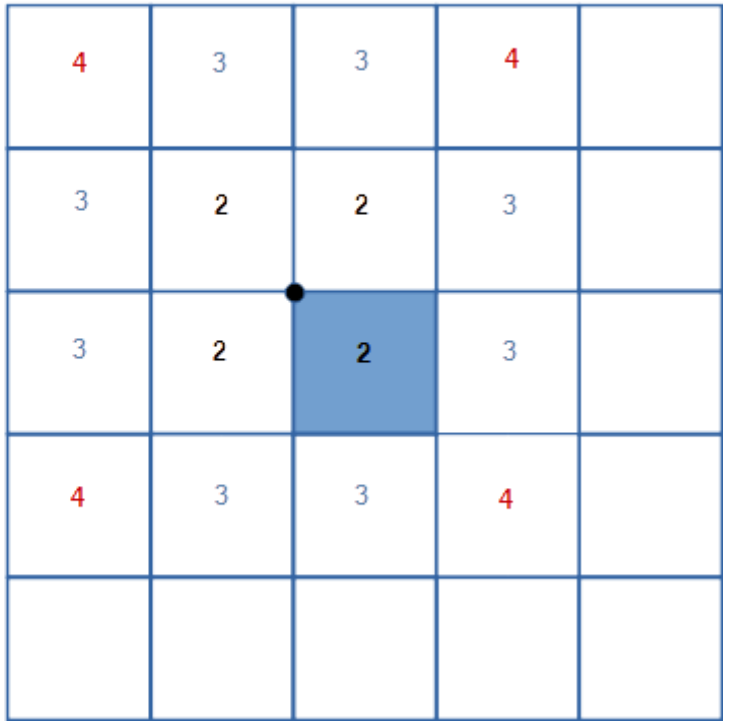
For a 3-D Cell-Centered Finite-Volume Flow Solver

- **We are interested in WLSQ gradient approaches that work well when using tetrahedral grids typically produced when using state-of-the-art unstructured grid adaptation algorithms.**
  - **Park, M.A., Kleb, B., Jones, W.T., Krakos, J.A., Michal, T., Loseille, A, Haimes, R. and Dannenhoffer, J.F. III, "Geometry Modeling for Unstructured Mesh Adaptation," AIAA Paper 2019-2946, June, 2019.**
- **We also seek a stable and robust WLSQ gradient approach that can still reduce the cost and storage of the gradient construction by extending the following work to 3-D.**
  - **Nishikawa, H. and White , J.A., "A Simplified FANG Cell-Centered Finite-Volume Method and Comparison with Other Methods for Trouble-Prone Grids," AIAA Paper 2021-2720, July, 2021.**

F-ANG+ Uses a Stencil Hierarchy: F-ANG, F-ANG+*fn*, and F-ANG+*nn*

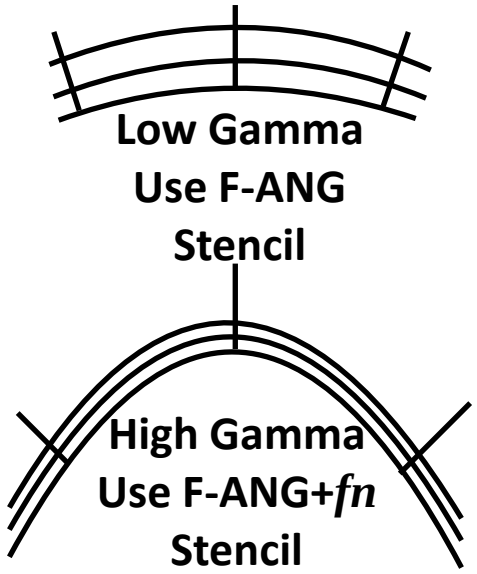
Cells labeled 2 are F-ANG stencil cells
Cells labeled 2 and 3 are F-ANG+*fn* stencil cells
Cells labeled 2, 3, and 4 are F-ANG+*nn* stencil cells





Gradient Construction and Use

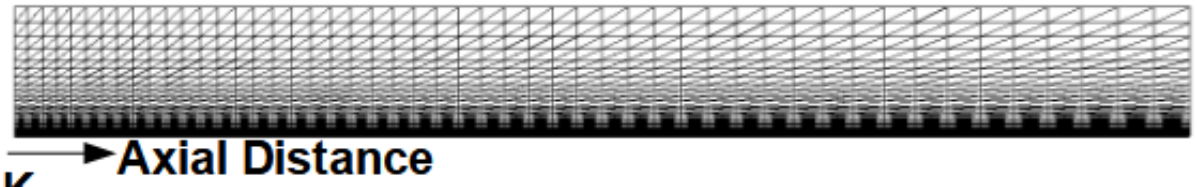
- We introduce a stencil hierarchy based on the F-ANG, F-ANG+*fn*, and FANG+*nn* stencils.
- Use the stencil hierarchy to define a stencil at each node based on the answers to the following questions:
  1) Is the stencil big enough to be well posed for the WLSQ?
  2) Is the stencil stable in the WLSQ for the local cell topology?
     - F-ANG is stable for tets but unstable for other cell topologies.
     - F-ANG+*fn* is stable for all cell topologies.
  3) When using tets, is the stencil stable for the local grid aspect ratio and curvature, i.e., Gamma?
- Compute the inviscid and viscous WLSQ gradients at each node using:
  1) Linear weighted least squares and/or
  2) Quadratic least squares.
- Average the nodal WLSQ gradients to:
  1) Cell-face centroids for use in the inviscid and viscous fluxes.
  2) Cell centroids for use in source terms.
- Compute the fluxes, source terms and form the residual.



Hypersonic Laminar Flat Plate Structured Grid Resolution Study

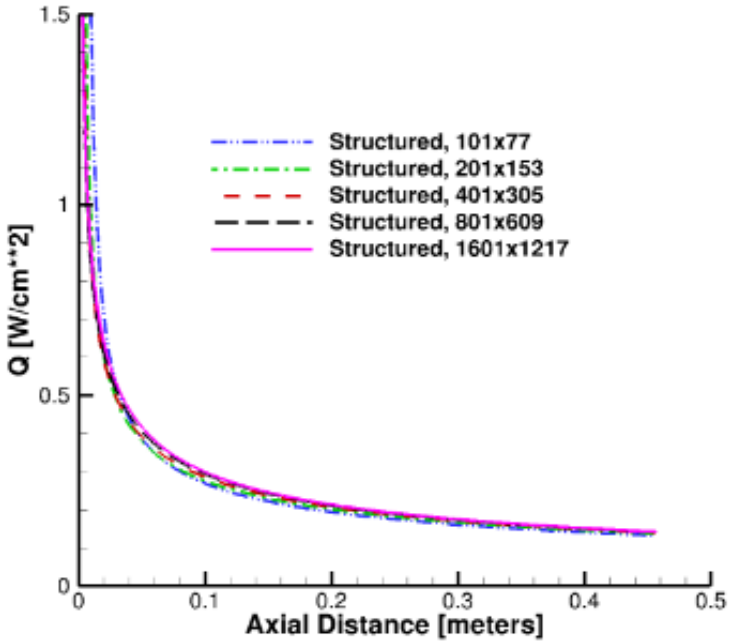
Mach number = 6.0
Thermally perfect air
Pressure = 2,100.98 Pa
Temperature = 63.1 K
Re(unit) = 26,396,219.0/m
Wall Temperature = 335.83 K
Plate Length = 0.4572 meters

Unstructured Triangular Grid
Created From a 51x39 Node Quadrilateral Grid

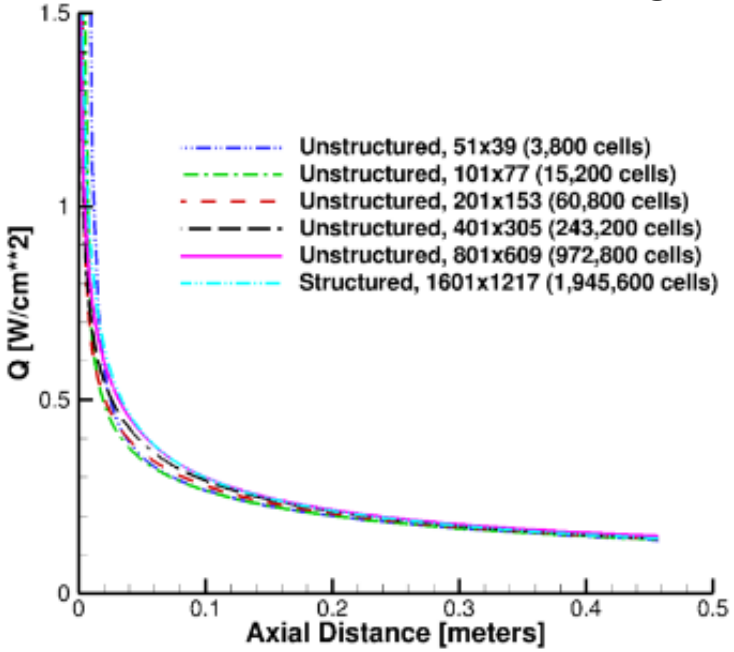


$|L_2|$ of the residual of all solutions
Converged >= 8 order of magnitude

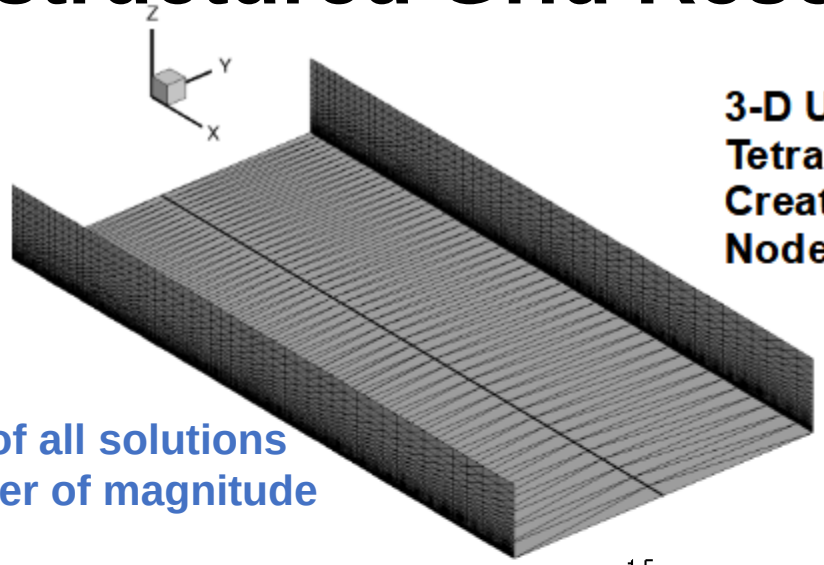
2-D Structured Grids



2-D Unstructured Triangular Grids

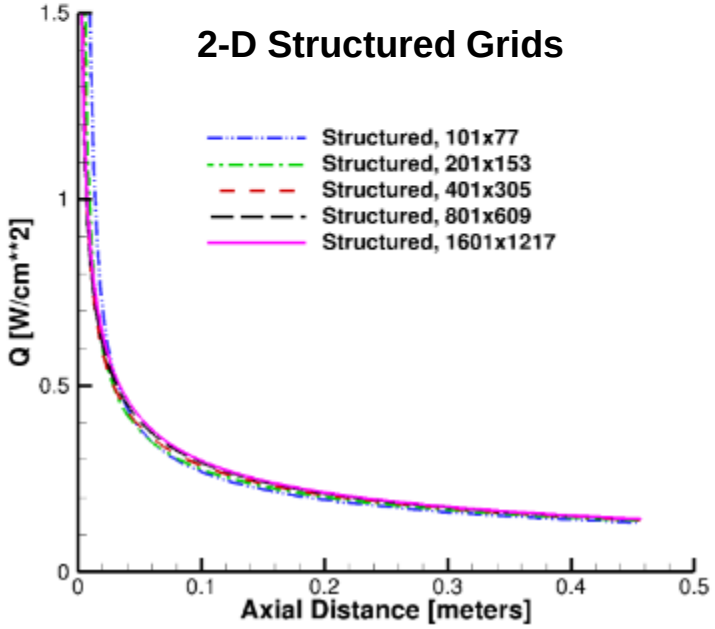


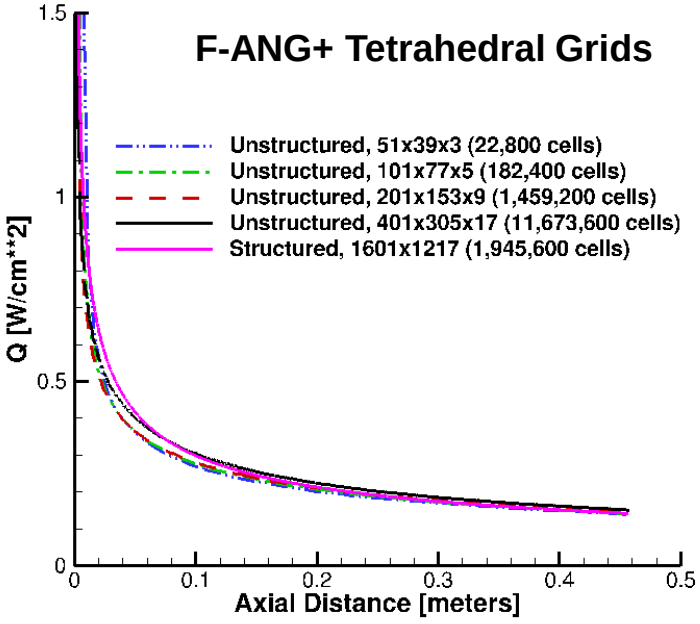
Hypersonic Laminar Flat Plate Unstructured Grid Resolution Study



3-D Unstructured Tetrahedral (Tet) Grid Created From 51x35x3 Node Hexahedral Grid

$|L_2|$ of the residual of all solutions converged >= 8 order of magnitude



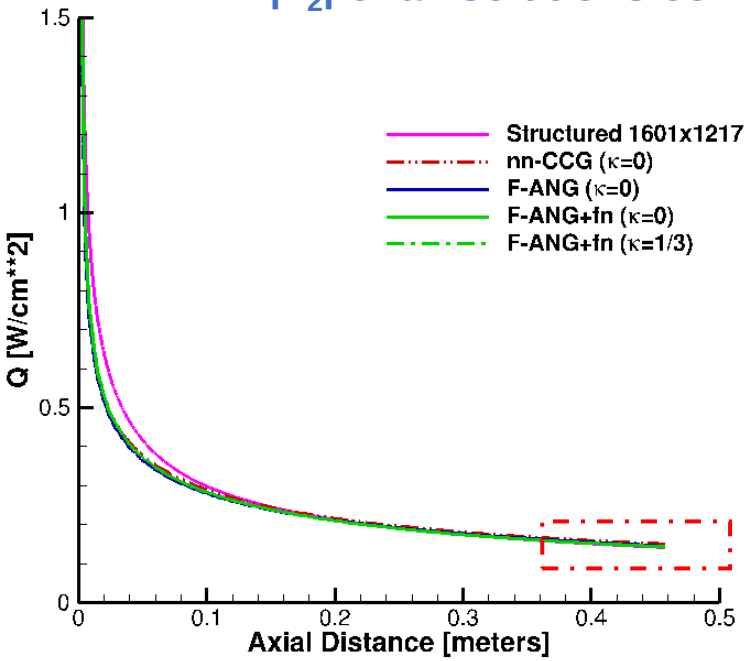


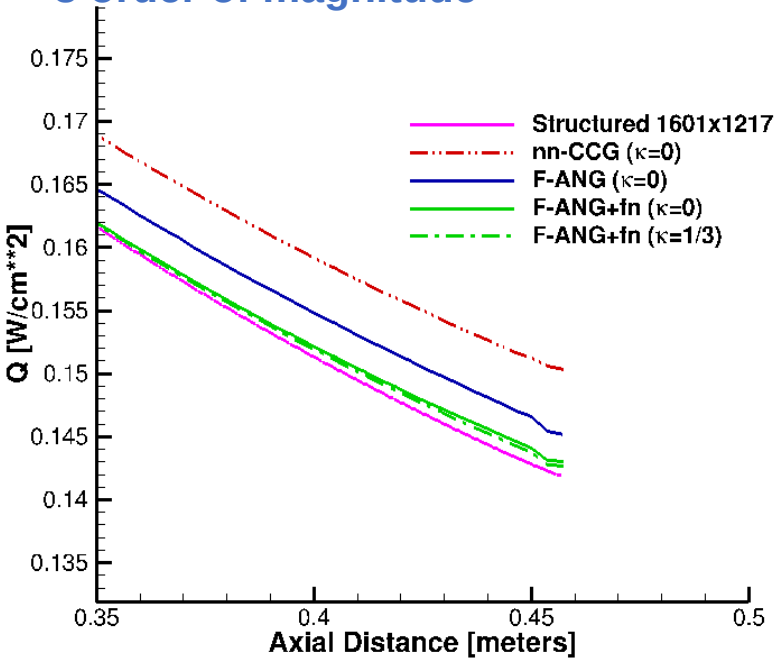
Hypersonic Laminar Flat Plate Unstructured Grid Resolution Study

Comparison of Wall Heat Transfer Distribution Using Several Gradient Stencil Approaches on the 201x153x9 Node Tetrahedral Grid

Gradient Stencil	No. of Stencils	Min. Stencil	Max. Stencil	Mean Stencil	Gradient Cost	Relative Cost
nn-CCG	1,459,200	19	66.9	106	97,620,480	1.000
F-ANG	276,777	6	22.0	46	6,089,094	0.062
F-ANG+*fn*	276,777	9	42.0	91	11,624,634	0.119

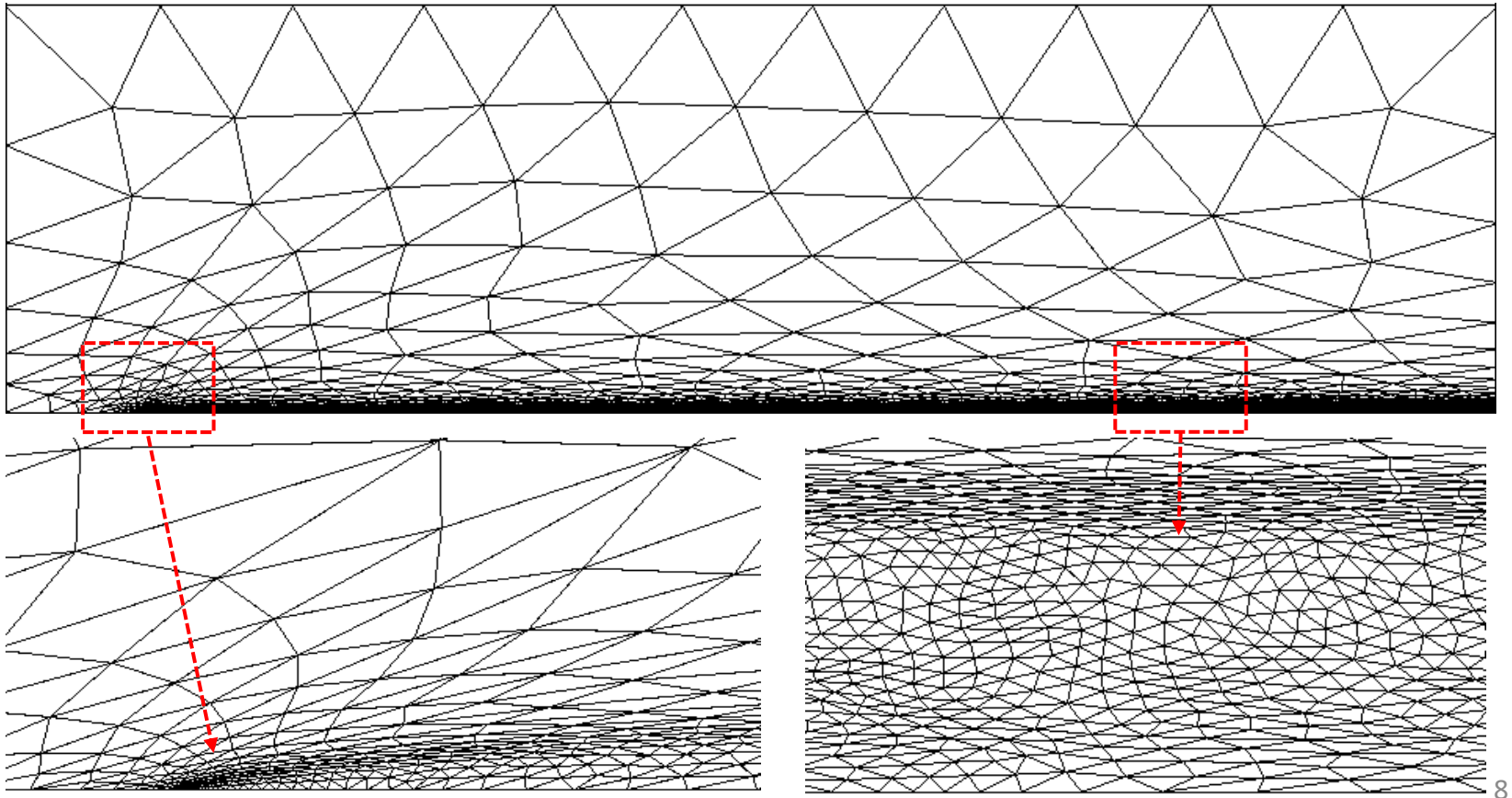
$|L_2|$ of all solutions converged >= 8 order of magnitude





Sketch-to-Solution/*refine* Adapted Tetrahedral Grid on Symmetry Plane Using F-ANG+ and Linear WLSQ

- 29th Grid Containing ~640,000 Tetrahedra
- Adaptation Metric was the Total Enthalpy Hessian
- Grid aspect ratio was limited to be <= 4

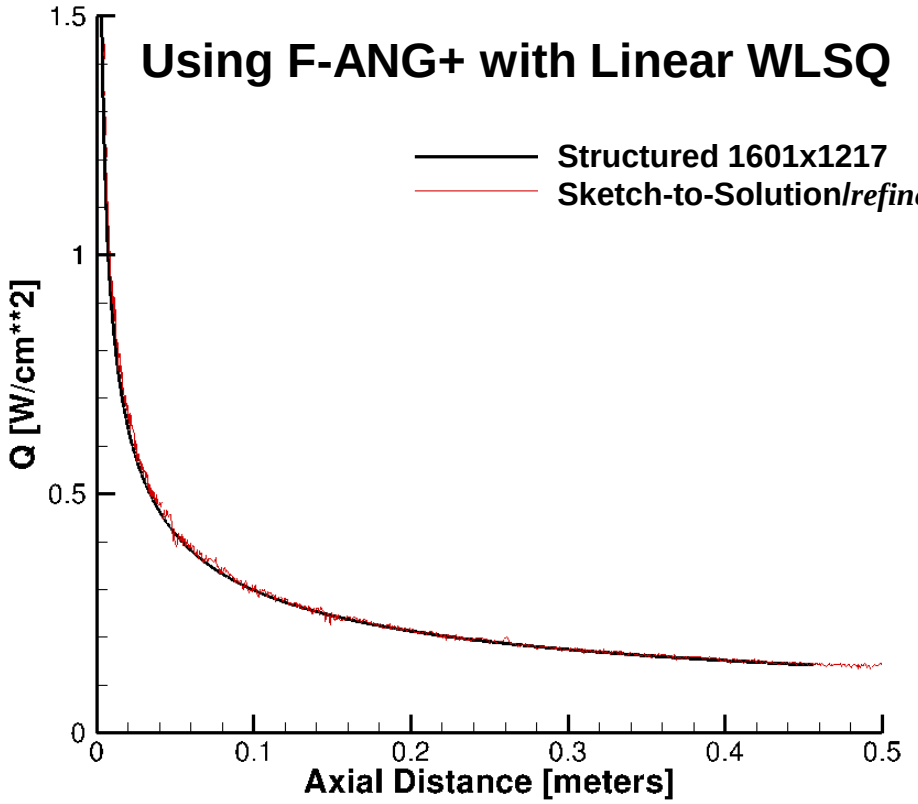


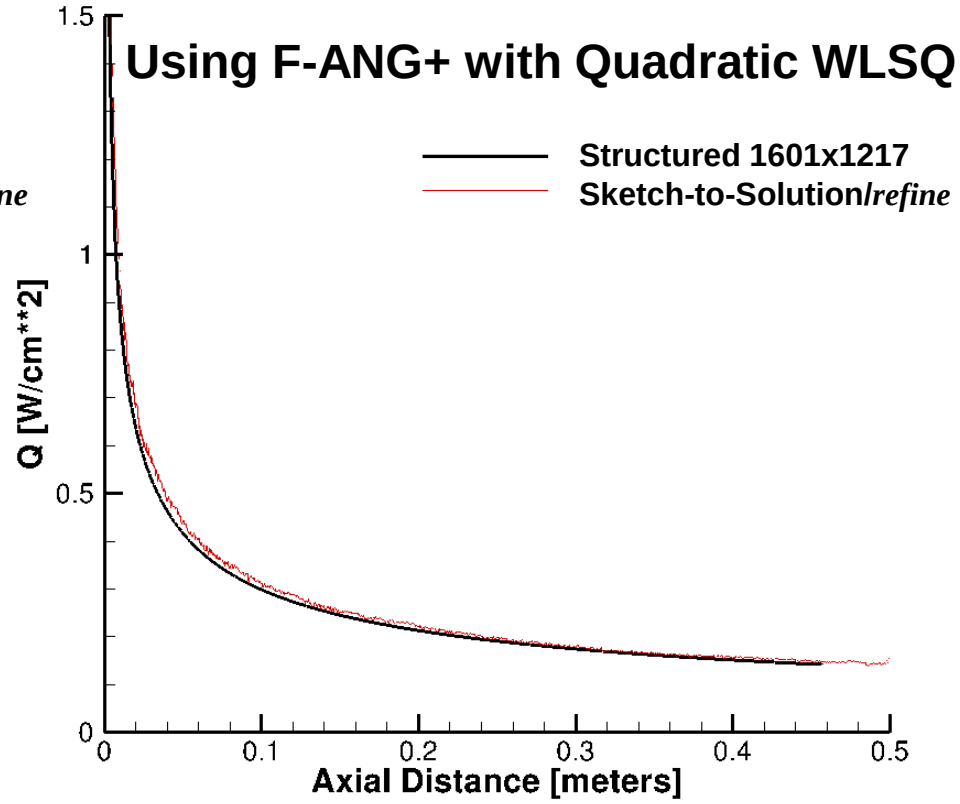
Leading Edge Detail

Boundary Layer Detail

Hypersonic Laminar Flat Plate Sketch-to-Solution/*refine* Tetrahedral Grid Solutions Using F-ANG+

- **Wall Heat Transfer Distribution on the 29th Grid Containing ~640,000 Tetrahedra**
- **Adaptation Metric was the Total Enthalpy Hessian**





Conclusions

- As with F-ANG, the F-ANG+*fn* gradient was found to be less expensive and more accurate than the *nn*-CCG approach.
- The F-ANG+*fn* approach using linear WLSQ gradients gave excellent agreement with 2-D structured grid wall heat transfer distribution results when using the Sketch-to-Solution/*refine* workflow.
- A stable and convergent solution process was demonstrated in the paper for a 3-D turbulent hypersonic flow when using a problematic grid containing tetrahedral, pyramidal, prismatic, and hexahedral cells.
- Work is planned to compute gamma on 3-D grids for use when selecting the node-centered gradient stencil from the stencil hierarchy.



Acknowledgments

This work was supported by the Hypersonic Technology Project