

Secondary Sonic Boom Ray Path Prediction using sBOOM

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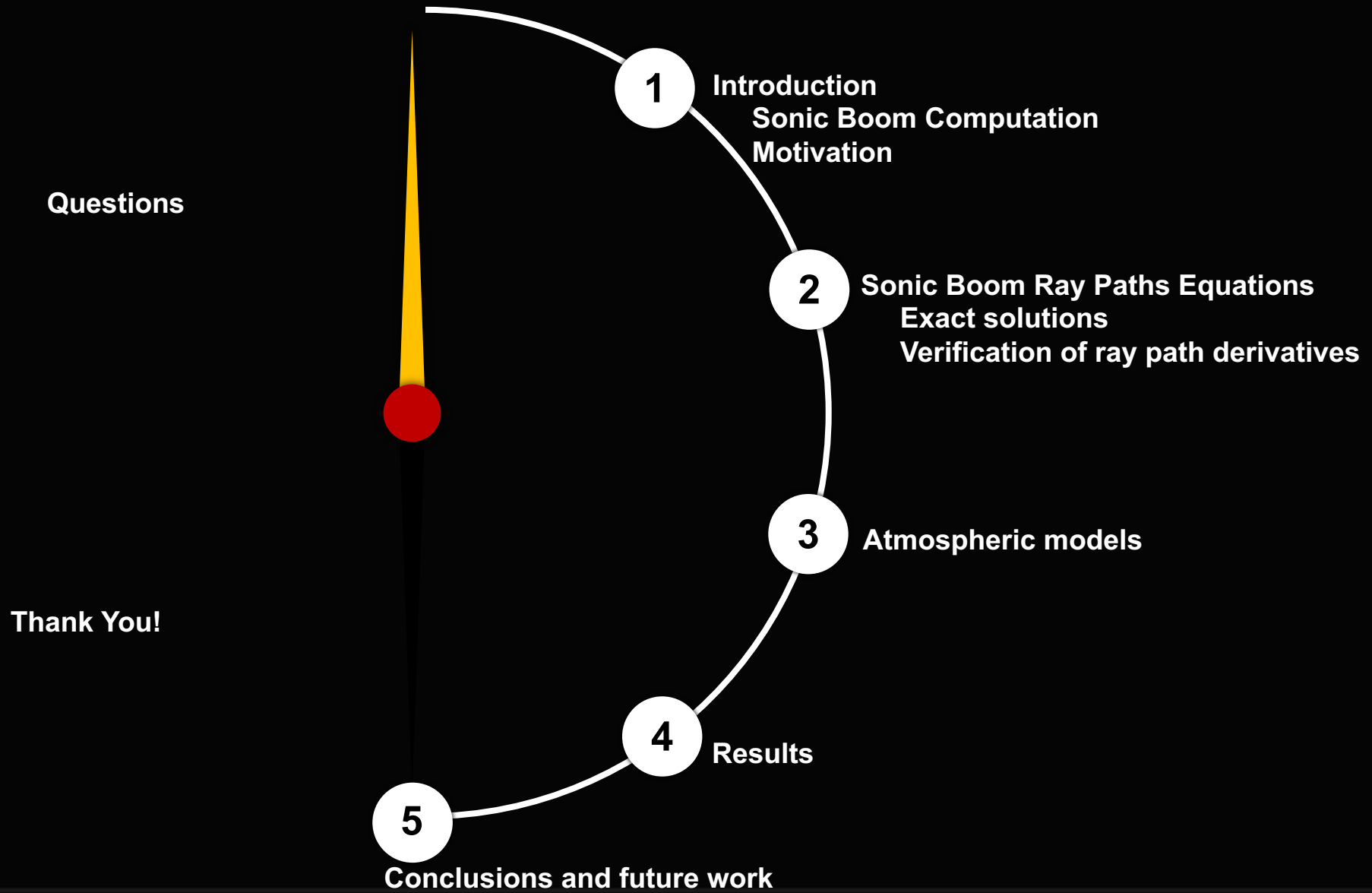
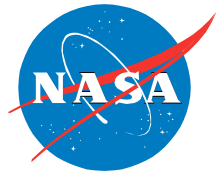
January 3, 2022



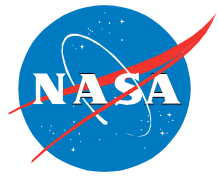
Session: SPSN-01/APA-09, Supersonic Design and Analysis I

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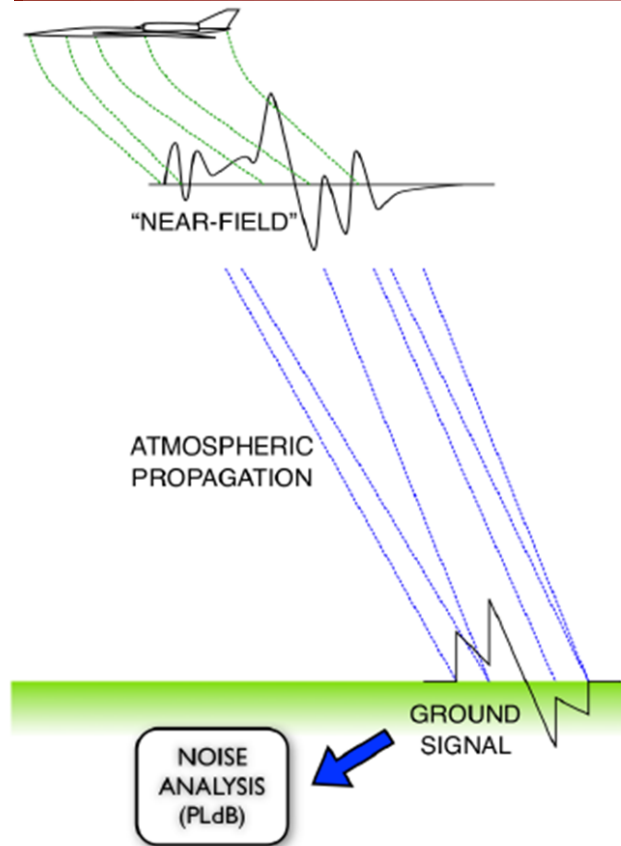
Outline



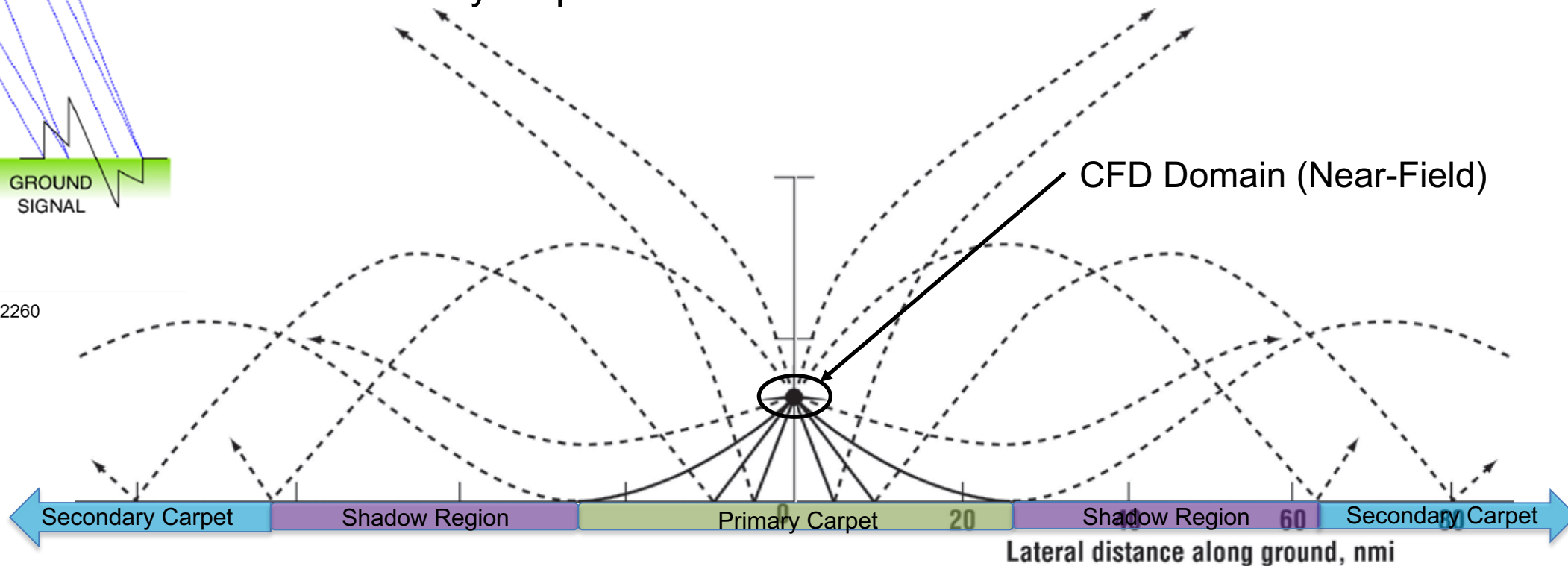
Sonic Boom Computation and Ray Paths



- Sonic boom computation involves
 - Near-field pressures from CFD
 - Atmospheric propagation to the ground
- Aircraft disturbances propagated along ray paths
 - Rays go in all directions
 - Can separate impact into primary carpet, shadow region and secondary carpet



Wintzer AIAA 2015-2260



Investigation of secondary booms and associated ray paths

- Improve functionality to allow computation of secondary booms
- Develop analytical closed-form expressions for the ray paths

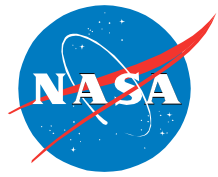
Computation of ray tube areas using Jacobians

- sBOOM currently uses four rays to compute ray tube areas
- In some cases with wind gusts, could potentially lead to erroneous ray tube area computation
- Compute ray tube areas using a single ray

Investigation of noise in loudness gradients due to assumptions of atmospheric parameters

- Typically, atmospheric temperature modeled using piece-wise linear interpolation
- Could lead to numerical noise in loudness gradients

Sonic Boom Ray Path Equations



- Ray paths dictated by coupled differential equations
- Under windless conditions, equations can be simplified
 - Equally applicable to both primary and secondary booms

$$\frac{d\vec{x}}{dt} = \vec{w} + c^2 \vec{q} D$$

$$\frac{d\vec{q}}{dt} = - \left[\frac{\nabla c}{cD} + \sum_{i=1}^3 q_i \nabla w_i \right]$$

$$\vec{q} = \frac{\hat{n}}{c + \vec{w} \cdot \hat{n}}$$

$$D = 1 + \frac{\vec{w} \cdot \hat{n}}{c}$$

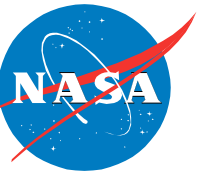
Zero Winds

$$\frac{d\vec{x}}{dt} = c^2 \vec{q}$$

$$\frac{d\vec{q}}{dt} = - \frac{\nabla c}{c}$$

$$\vec{q} = \frac{\hat{n}}{c}$$

Analytical Solutions to Ray Paths under Standard Atmosphere



- Vertical component of the slowness vector is a function of speed of sound and initial Z-component of the ray normal

$$q_z = \text{sign} \sqrt{\frac{1}{c^2} - \frac{1-n_z^2}{c_a^2}}$$

- Analytical exact expression for ray path length under constant lapse rate

$$s = s_a + \frac{2c_a^2}{(1-n_z^2)\gamma RL} (q_z c - q_{z,a} c_a)$$

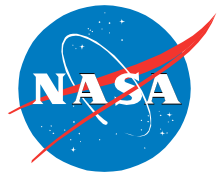
For zero lapse rate

$$s = s_a + \frac{z-z_a}{n_z}$$

- Components of the ray path

$$X = n_x s \quad Y = n_y s \quad Z = n_z s$$

Analytical Solutions to Ray Paths (Contd.)



- Ray tube area defined by the ray Jacobian

$$J = \begin{vmatrix} \frac{\partial X}{\partial s} & \frac{\partial Y}{\partial s} & \frac{\partial Z}{\partial s} \\ \frac{\partial X}{\partial \phi} & \frac{\partial Y}{\partial \phi} & \frac{\partial Z}{\partial \phi} \\ \frac{\partial X}{\partial t_a} & \frac{\partial Y}{\partial t_a} & \frac{\partial Z}{\partial t_a} \end{vmatrix} \xrightarrow[\text{X direction}]{\text{Level flight along}} J = V \left[\frac{\partial Y}{\partial s} \frac{\partial Z}{\partial \phi} - \frac{\partial Y}{\partial \phi} \frac{\partial Z}{\partial s} \right]$$

- Exact expressions for the partial derivatives

$$\frac{\partial q_z}{\partial \phi} = \frac{q_{z,a} \frac{\partial q_{z,a}}{\partial \phi}}{q_z}$$

$$\frac{\partial s}{\partial \phi} = \frac{\partial s_a}{\partial \phi} + \frac{2c_a^2}{\gamma R L n_h^4} \left[n_h^2 \left(c \frac{\partial q_z}{\partial \phi} - c_a \frac{\partial q_{z,a}}{\partial \phi} \right) + 2(q_z c - q_{z,a} c_a) n_z c_a \frac{\partial q_{z,a}}{\partial \phi} \right]$$

Constant Lapse Rate

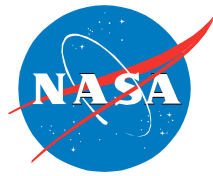
$$\frac{\partial q_z}{\partial \phi} = \frac{\partial q_{z,a}}{\partial \phi}$$

$$\frac{\partial s}{\partial \phi} = \frac{\partial s_a}{\partial \phi} - \frac{(s - s_a) \frac{\partial q_z}{\partial \phi}}{q_z}$$

Zero Lapse Rate

$$\begin{aligned} \frac{\partial X}{\partial \phi} &= \frac{\partial X_a}{\partial \phi} + n_x \frac{\partial s}{\partial \phi} - n_x \frac{\partial s_a}{\partial \phi} \\ \frac{\partial Y}{\partial \phi} &= \frac{\partial Y_a}{\partial \phi} + (s - s_a) \frac{\partial n_y}{\partial \phi} + n_y \frac{\partial s}{\partial \phi} - n_y \frac{\partial s_a}{\partial \phi} \\ \frac{\partial Z}{\partial \phi} &= \frac{\partial Z_a}{\partial \phi} + (s - s_a) \frac{\partial n_z}{\partial \phi} + n_z \frac{\partial s}{\partial \phi} - n_z \frac{\partial s_a}{\partial \phi} \end{aligned}$$

Verification of Ray Path Derivatives



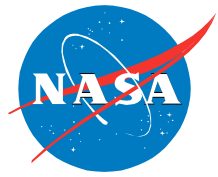
- Complex version of ray path equations developed to verify analytically computed derivatives
 - No subtractive cancellation errors, true second-order accuracy achieved
- Analytical and complex derivatives match well

	Altitude	Analytical	Complex Variable
$\frac{\partial s}{\partial \phi}$	16750.28	-1.6109468083896800294497e+02	-1.6109468083896802308005e+02
	32306.33	-9.3862283704080709867412e+03	-9.3862283704080566301212e+03
	113104.4	-6.1014530179353605490178e+04	-6.1014530179353638488371e+04
	123344.0	9.5338714347898785490543e+04	9.5338714347898779413814e+04

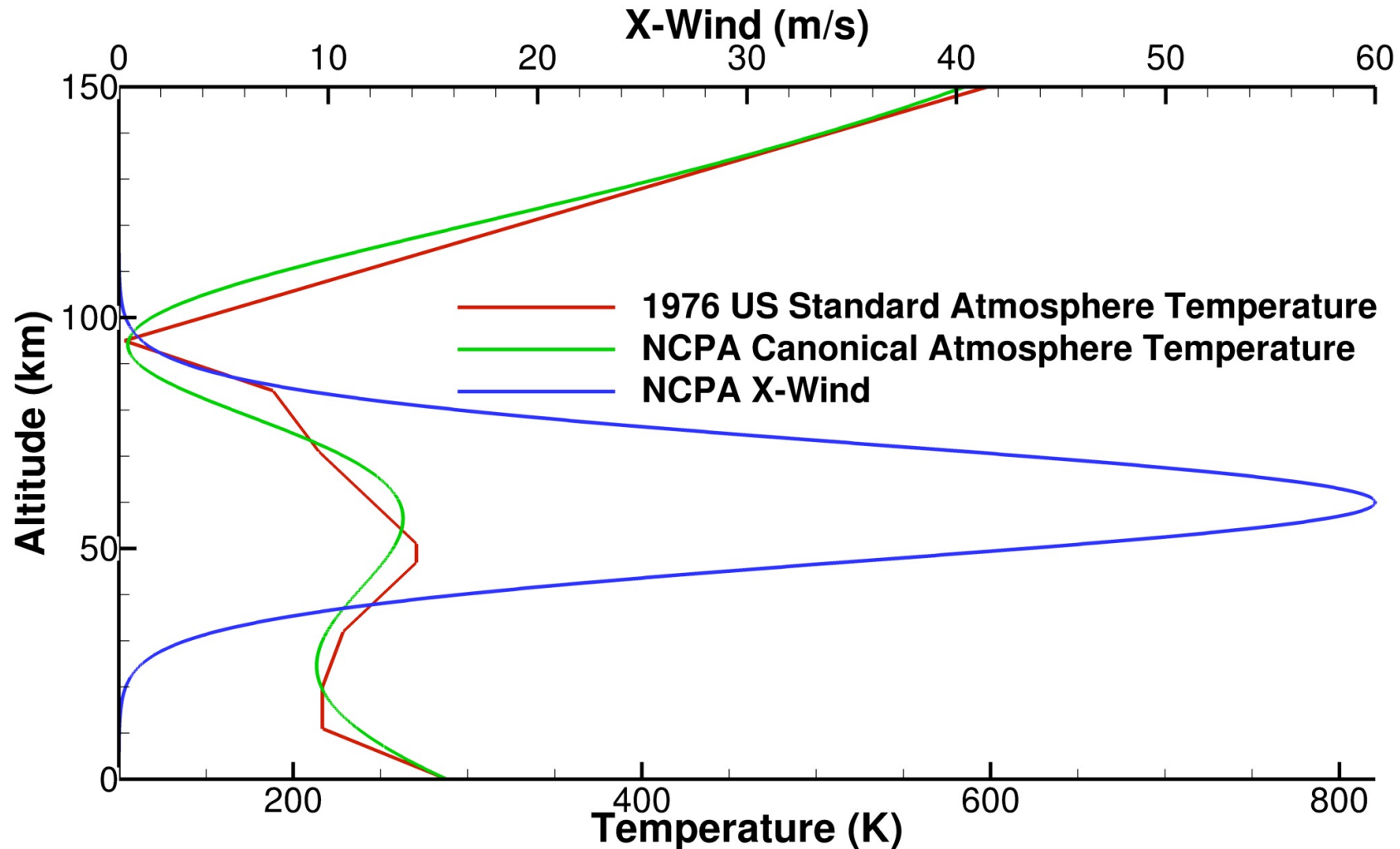
	Altitude	Analytical	Complex Variable
$\frac{\partial Y}{\partial \phi}$	17429.45	-1.0987928942403650580673e+03	-1.0987928942403651351863e+03
	28557.17	-1.3884146618263532218407e+04	-1.3884146618263542503242e+04
	62446.77	-6.2999001410738070262596e+04	-6.2999001410738075578080e+04
	117382.29	-1.1544402686696227465291e+05	-1.1544402686696228325281e+05

	Altitude	Analytical	Complex Variable
$\frac{\partial Z}{\partial \phi}$	33502.49	-3.1616524619558163067268e+02	-3.1616524619557429314525e+02
	68772.98	3.4785656352445130323758e+02	3.4785656352445772655048e+02
	91977.74	1.9903066677734432232683e+03	1.9903066677734325305608e+03
	122141.4	-2.5847904454752573656151e+04	-2.5847904454752605168318e+04

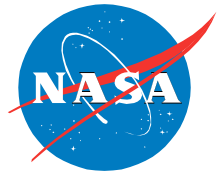
Atmospheres Used



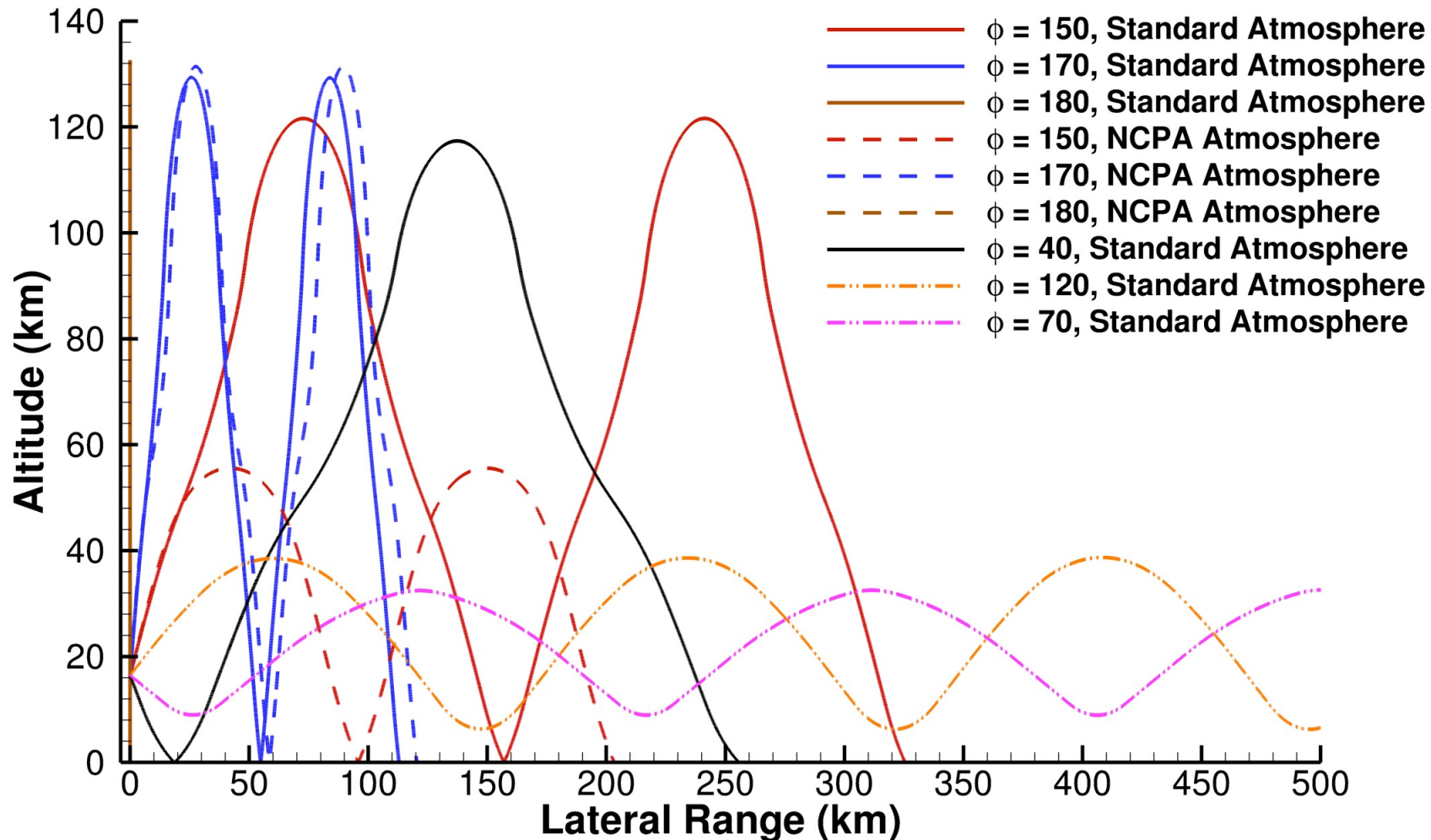
- Couple of atmospheres used
 - Standard atmosphere (No Winds)
 - NCPA canonical atmosphere



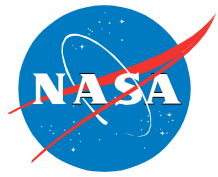
Results – Ray Paths



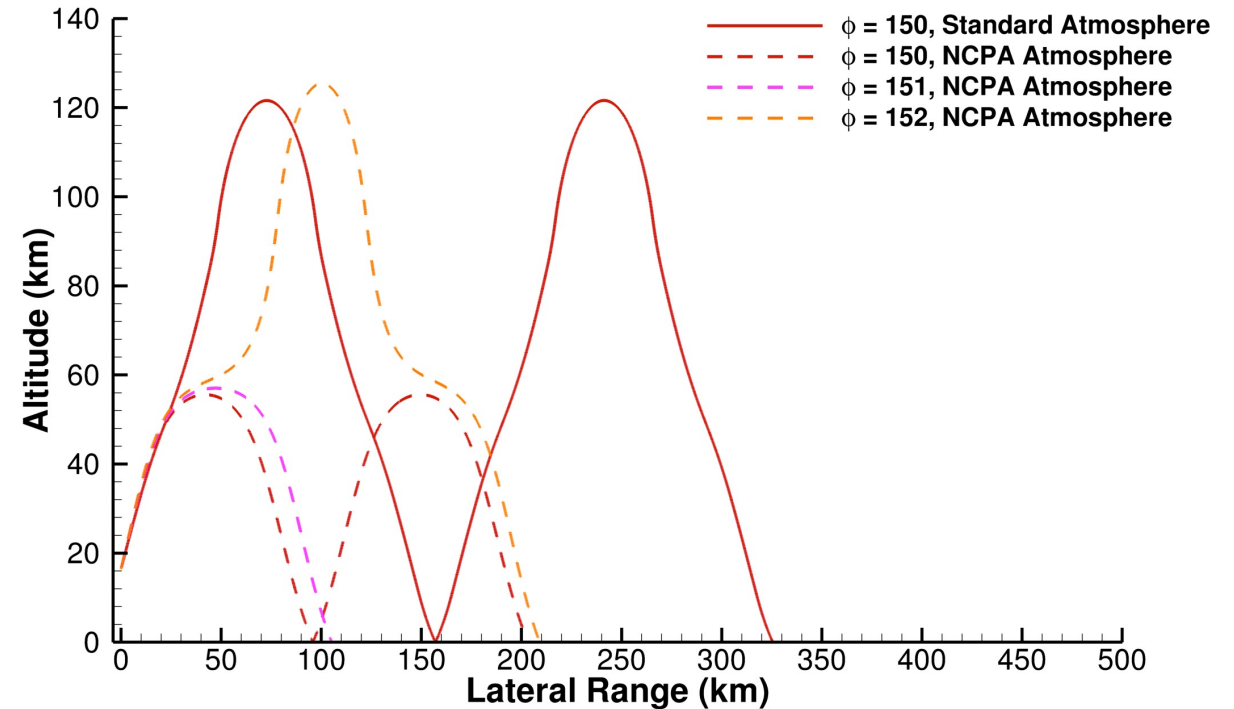
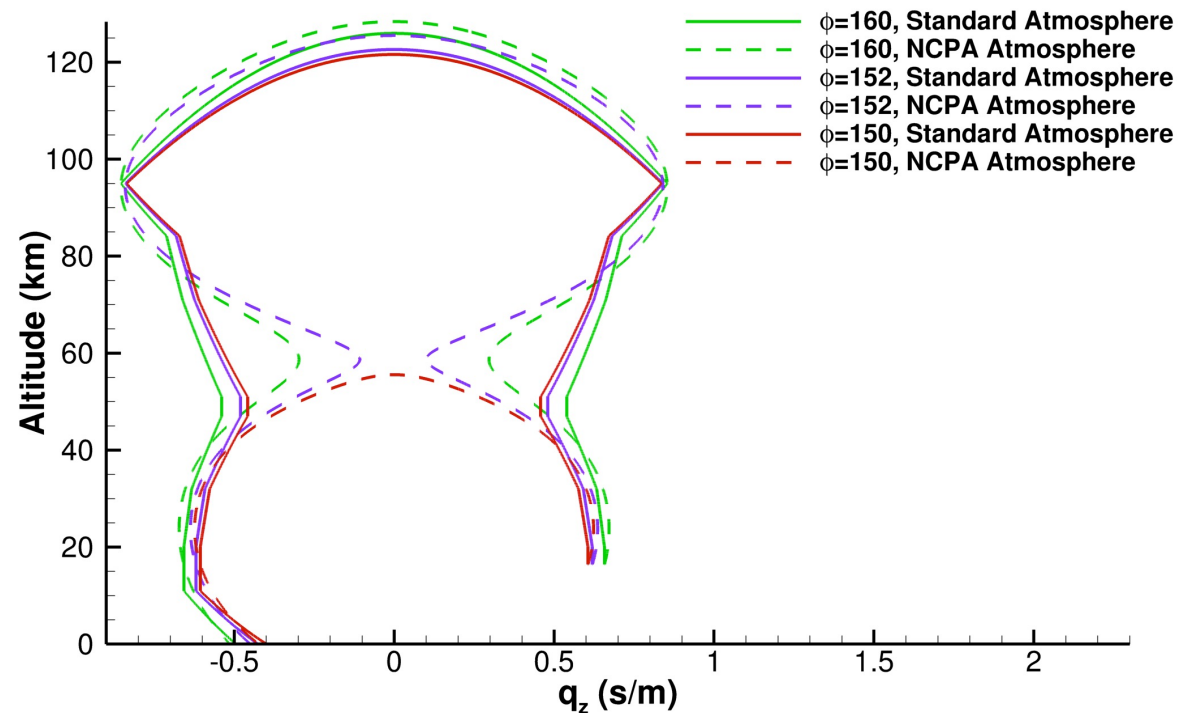
- Ray paths show significant variation due to winds
 - Can model multiple bounces
- Disturbances at certain azimuthal angles do not reach the ground



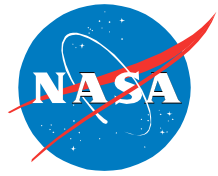
Results – Ray Paths (Contd.)



- Vertical component of the slowness vector determines the highest altitude reached for secondary booms
 - For NCPA atmospheres, the change happens around $\phi=150^\circ$
- Ray paths much different for $\phi=151^\circ$ and $\phi=152^\circ$ under NCPA atmosphere

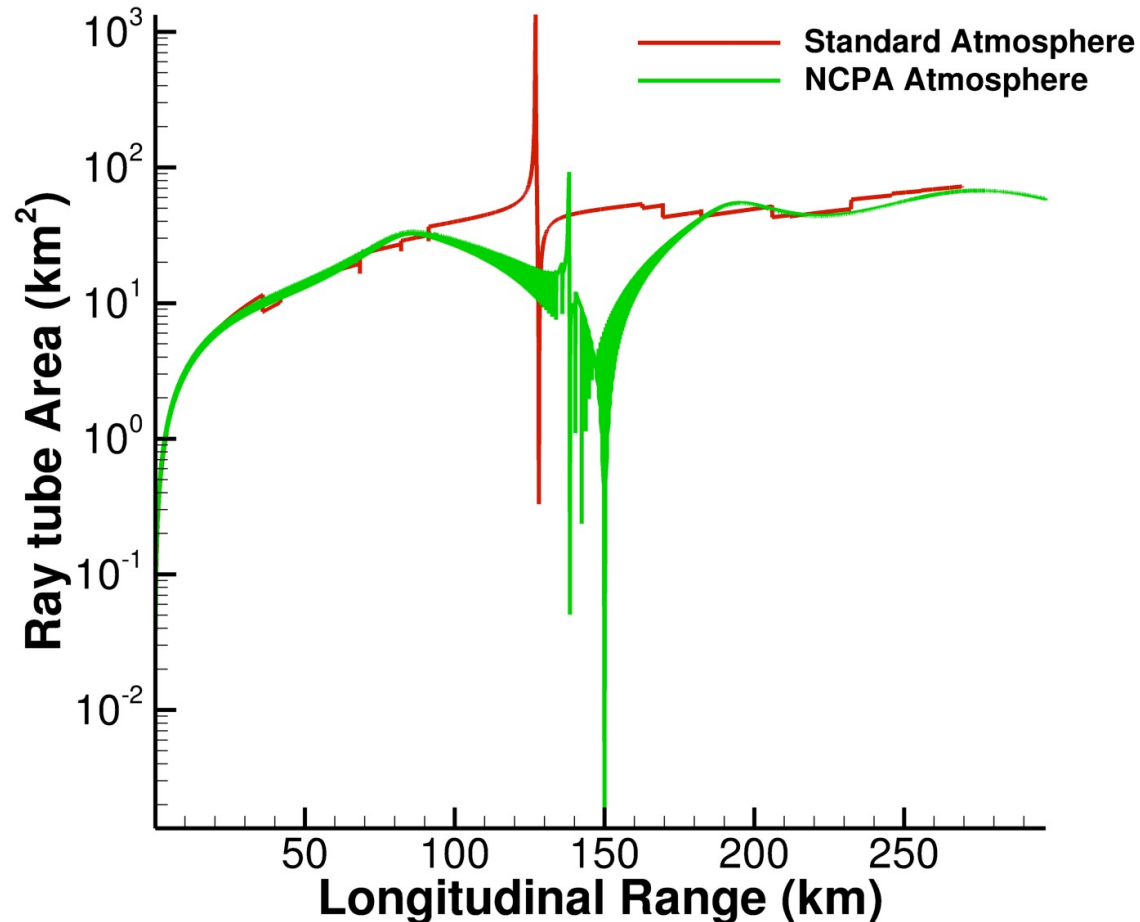


Results – Ray Tube Areas

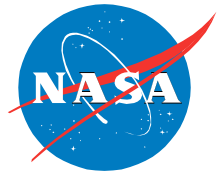


- Ray tube area significantly decreases as the highest altitude is reached
 - Potential situations when this goes to zero
 - Numerical noise introduced at the atmospheric layer interfaces

Propagation Parameters
 $M = 1.4$, Altitude = 16459.2 m
 $\phi = 160^\circ$

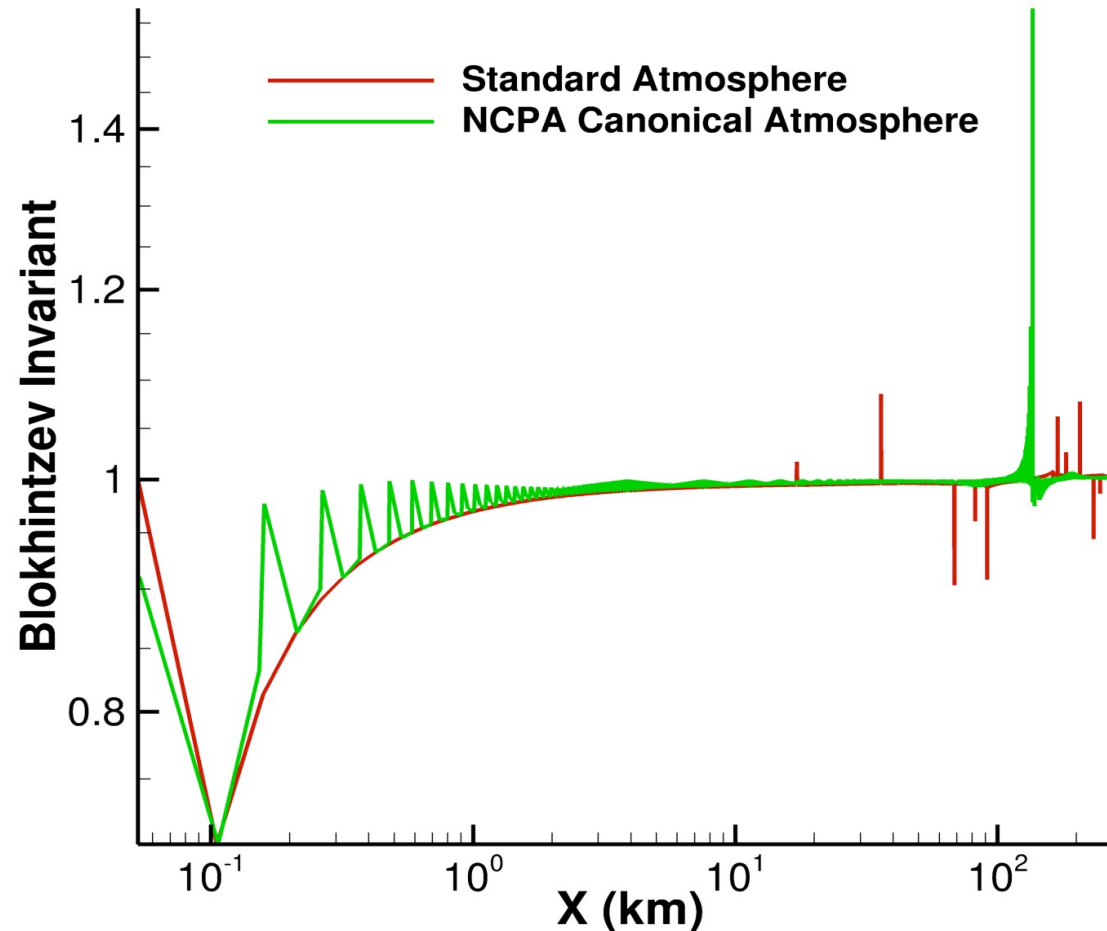


Results – Blokhintzev Invariant, Secondary Booms



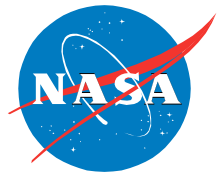
- Blokhintzev invariant for pressure scaling
 - Manifestation of conservation of energy
- Hydrostatic equation used to compute pressures
- State equation used to compute densities

Propagation Parameters
 $M = 1.4$, Altitude = 16459.2 m
 $\phi = 160^\circ$

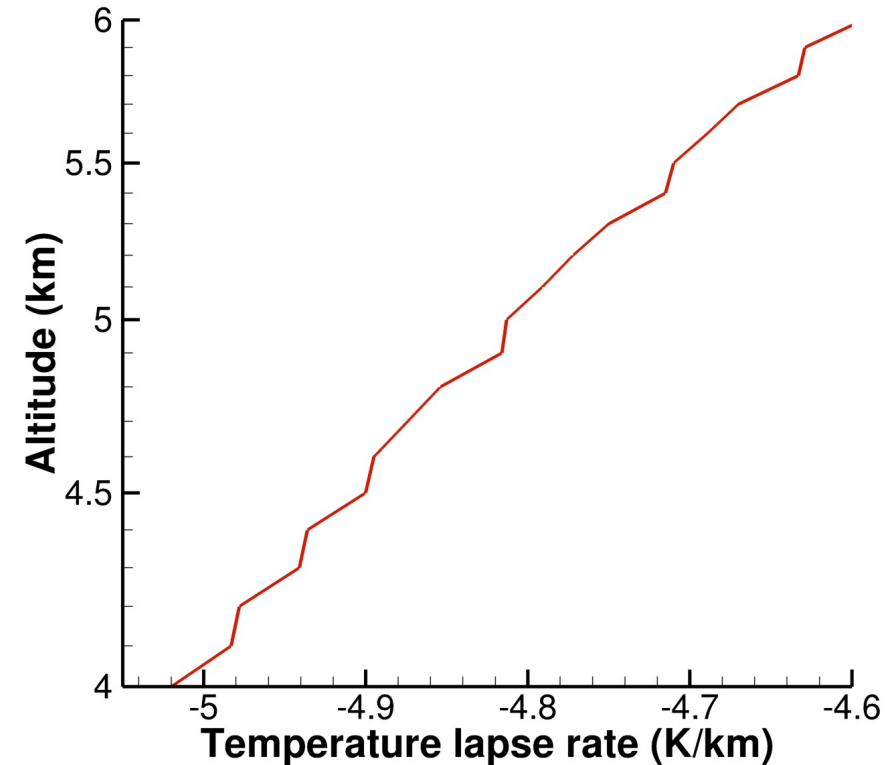
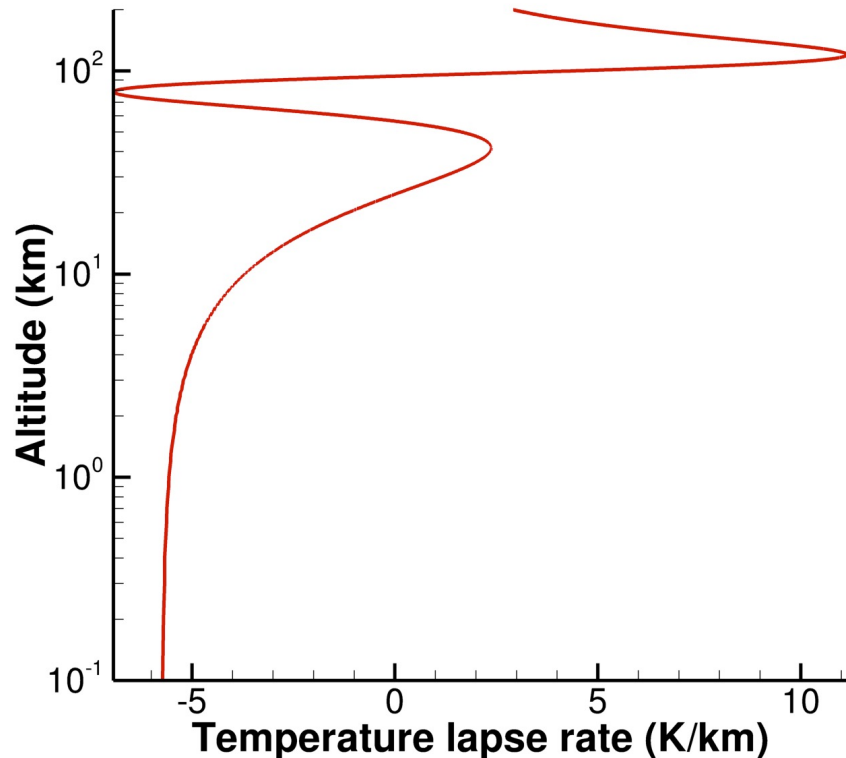


$$B = \sqrt{\frac{\rho c A_0}{\rho_0 c_0 A}}$$

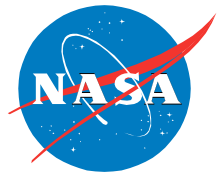
Temperature Lapse Rate



- Atmospheric temperature assumed to be piece-wise linear
- Computed lapse rates appear smooth, but are only C0, not C1
- Minor impact on ray path metrics
- Significant impact on ray path derivatives, ray tube areas, loudness gradients – anything to do with cumulative gradients
 - Significant issue for secondary booms than primary booms because these discontinuities are accumulated over very large distances

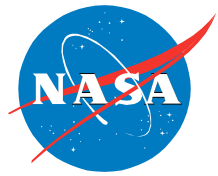


Concluding Remarks



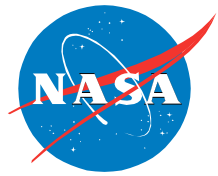
- Atmospheric wind profiles play a much larger role in secondary booms than primary booms due to their long ray paths
- Interpolation assumptions in atmospheric quantities have a magnified effect for secondary booms
- Depending upon the prevailing atmospheric conditions, especially winds, there could be a significant change in the extent of the secondary ray paths between azimuthal angles even when they are close
- Special numerical considerations and solution approaches would have to be pursued when ray tube area approaches zero for secondary booms

Future Work



- At higher altitudes for secondary booms:
 - Model humidity profiles
 - Ensure relaxation and loss mechanisms are properly modeled
 - Research sources that can provide measured winds
- Consideration of special cases when ray tube area could approach zero at higher altitudes
- Investigation of the discontinuities in Blokhintzev invariant
- Computation of secondary booms and associated loudness metrics.

Questions?



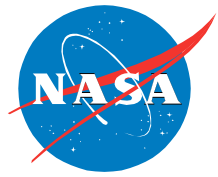
Acknowledgements:

NASA Commercial Supersonic Technology (CST) Project

Joel Lonzaga @ NASA Langley for inspiring derivation of the exact expressions for ray paths

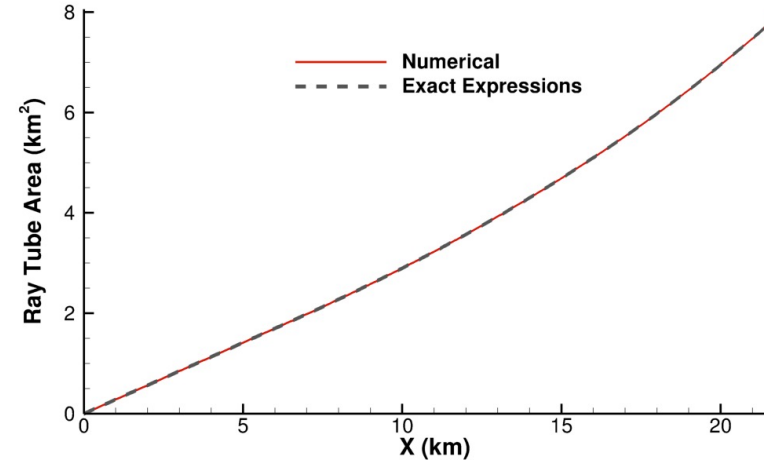
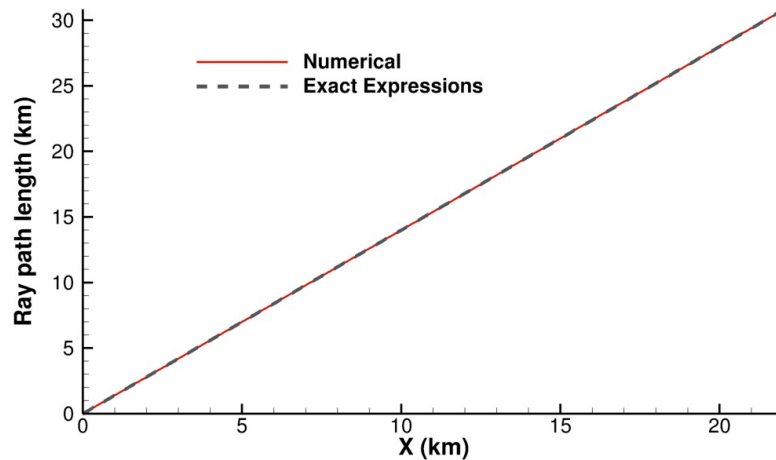
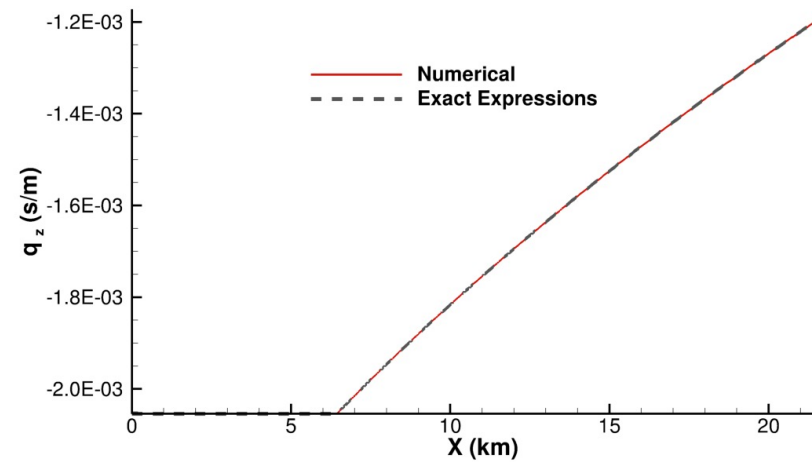
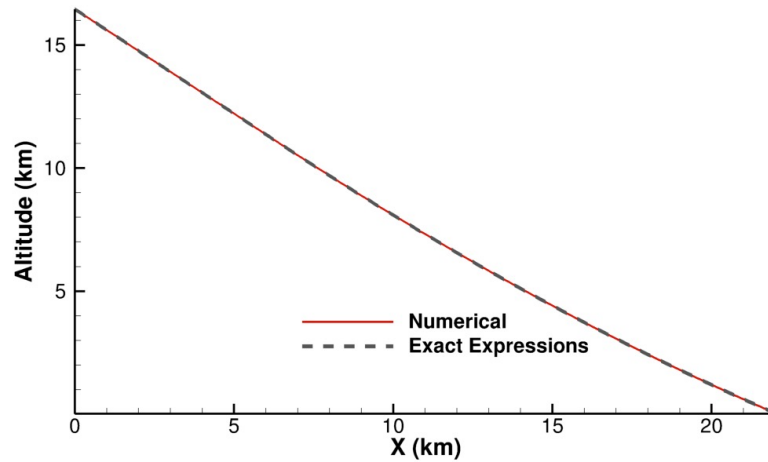


Results – Primary Booms

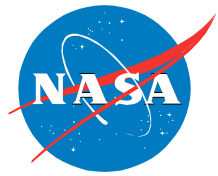


- Primary boom ray paths show linear behavior initially followed by slightly curved ray paths
- Ray tube areas computed using Jacobian
- Excellent comparison on numerical and exact expressions

Propagation Parameters
 $M = 1.4$, Altitude = 16459.2 m
Standard Atmosphere, $\phi = 30^\circ$



sBOOM – Nonlinear Propagation Analysis and Design Tool

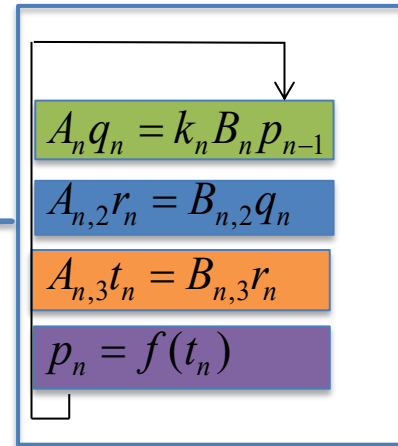


- Follows sonic boom propagation best practices
 - Ignores diffraction (near the carpet edges)
- Ray path equations

$$\frac{dx}{dt} = \bar{w} + c^2 q D \quad \frac{dq}{dt} = - \left[\frac{\nabla c}{cD} + \sum_{i=1}^3 q_i \nabla w_i \right] \quad q = \frac{\bar{n}}{c + \bar{w} \cdot \bar{n}}$$

- Equation modeled: Augmented Burgers Equation

$$\begin{aligned} \frac{\partial p}{\partial \sigma} = & \frac{\beta \Omega}{2 \rho_0 c_0^3} \left(\frac{c_0}{v_g} \right) \frac{\partial p^2}{\partial \tau} & \text{Non-linearity} \\ & + \frac{\partial p}{\partial \sigma} \frac{\partial B}{\partial \sigma} p & \text{Blokhintzev Invariance} \\ & + \frac{\delta \Omega^2}{2 \rho_0 c_0^3} \left(\frac{c_0}{v_g} \right) \frac{\partial^2 p}{\partial \tau^2} & \text{Thermo-viscous effect} \\ & + \sum_v \frac{m_v}{2 c_0} \frac{t_v \Omega^2}{1 + t_v \Omega} \frac{\partial}{\partial \tau} \left(\frac{c_0}{v_g} \right) \frac{\partial^2 p}{\partial \tau^2} & \text{Molecular relaxation effect} \end{aligned}$$



$$B = \sqrt{\frac{\rho c S_0 D_0^2}{\rho_0 c_0 S D^2} \left(\frac{\frac{c v g_0}{D_0}}{\frac{c_0 v_g}{D}} \right)}, v_g = |c \cdot \bar{n} + \bar{w}|, D = 1 + \frac{\bar{w} \cdot \bar{n}}{c}, \Omega = \frac{1}{D}$$

- Burgers equation based propagation validated with acoustic measurements with fairly good agreement

