

Hypersonic Boundary-Layer Instabilities over Ogive-Cylinder Models

Anton Scholten,^{*}

North Carolina State University, Raleigh, NC 27695, USA

Pedro Paredes,[†]

National Institute of Aerospace, Hampton, VA 23666, USA

J. Luke Hill,[‡]

Air Force Institute of Technology, WPAFB, OH 45433, USA

Matthew Borg,[§]

Air Force Research Laboratory, WPAFB, OH 45433, USA

Joseph S. Jewell,[¶]

Purdue University, West Lafayette, IN, 47907

Meelan M. Choudhari,^{||}

NASA Langley Research Center, Hampton, VA 23681, USA

Computational investigations of a tangent ogive-cylinder geometry with varying forebodies at zero degrees angle of attack are presented. The model geometry and conditions are selected to match experiments conducted in the Air Force Research Laboratory (AFRL) Mach 6 Ludwig Tube. Five forebodies of interest were selected for the computational studies herein: two sharp ogives, two blunt ogives, and one hemispherical forebody. Computations are performed at a freestream unit Reynolds number of $7.01 \times 10^6 \text{ m}^{-1}$. Each sharp and blunt tip ogive had a 14 and 28 degree version for the tip angles providing a 4 caliber and 2 caliber ogive forebody, respectively. A cylindrical section follows the forebody, resulting in a meter long model matching the experiments. The laminar flow solutions are analyzed. The boundary-layer-edge properties and the velocity and temperature profiles are compared across streamwise locations aft of the ogive-cylinder junction. The blunt forebodies induce an entropy layer that envelopes the boundary-layer profiles. Modal stability analysis identifies most amplified frequencies corresponding to Mack's second modes that agree with experimental results for the sharp forebodies. Similar to the experimental measurements based on wall-mounted pressure sensors, no unstable modes are found for the blunt models. Nonmodal analysis revealed a broadband set of disturbances present for the blunter forebodies, in agreement with experimental observations. Flow perturbation contours of most amplified planar and oblique disturbances are shown to qualitatively match wind tunnel schlieren images, with a switch from rope-like to elongated structures, i.e., from high frequency Mack's second modes to low frequency Mack's first modes, as the forebody angle is increased for the sharp tip, and from boundary-layer to entropy-layer disturbances as the bluntness is increased.

Nomenclature

^{*}Graduate Student, Department of Mechanical and Aerospace Engineering. AIAA Student Member

[†]Senior Research Engineer, Computational AeroSciences Branch, NASA LaRC. AIAA Senior Member

[‡]Graduate Student, Department of Aeronautics and Astronautics. AIAA Student Member

[§]Senior Aerospace Engineer, Aerospace Systems Directorate. AIAA Associate Fellow

[¶]Assistant Professor, School of Aeronautics and Astronautics. AIAA Associate Fellow

^{||}Senior Research Scientist, Computational AeroSciences Branch. AIAA Fellow

C	=	Caliber, forebody length to cylinder diameter ratio
h_t	=	total enthalpy [$\text{kg m}^2 \text{s}^{-2}$]
h_ξ	=	streamwise metric factor
h_ζ	=	spanwise metric factor [m]
m	=	azimuthal wavenumber [rad^{-1}]
M	=	Mach number
N	=	Logarithmic amplification factor
$\hat{\mathbf{q}}$	=	vector of amplitude variables
$\check{\mathbf{q}}$	=	vector of disturbance function variables
$\bar{\mathbf{q}}$	=	vector of base flow variables
$\tilde{\mathbf{q}}$	=	vector of perturbation variables
Re_∞	=	freestream unit Reynolds number [m^{-1}]
r_b	=	local radius of axisymmetric body at the axial station of interest [m]
r_n	=	nose radius [m]
R	=	cylinder radius [m]
T	=	temperature [K]
T_w	=	wall temperature [K]
(u, v, w)	=	streamwise, wall-normal, and azimuthal velocity components [m s^{-1}]
(x, y, z)	=	Cartesian coordinates
x_j	=	ogive-cylinder junction location [m]
α	=	streamwise wavenumber [m^{-1}]
δ_h	=	boundary-layer thickness [m]
κ	=	streamwise curvature [m^{-1}]
ω	=	disturbance angular frequency [s^{-1}]
ρ	=	density [kg m^{-3}]
(ξ, η, ζ)	=	streamwise, wall-normal, and azimuthal coordinates
Superscripts		
*	=	dimensional value
Subscript		
∞	=	freestream value

I. Introduction

Due to the impact of laminar to turbulent transition on the skin friction and the surface heating on aerospace vehicles, the design of such vehicles requires an adequate understanding of the underlying mechanisms leading to transition. For hypersonic vehicles, in particular, these effects are significantly amplified, and yet, the exact cause behind a number of transition scenarios in the hypersonic speed regime remains unexplained at this point.

The wind tunnel experiments of Kendall [1] showed the second Mack mode as being the most dominant instability for hypersonic boundary layers. Subsequent experimental and numerical studies confirmed that the modal growth of Mack-mode instabilities (second-mode waves) is most likely responsible for laminar-turbulent transition on sharp axisymmetric cones at zero degrees angle of attack [2]. Furthermore, Stetson's work [3] has demonstrated the effects of nosetip bluntness on the formation of an entropy layer that may extend well downstream of the nosetip region of axisymmetric cones. With an increasing nosetip radius, the onset of transition is moved downstream. This is supported by the increasing entropy layer having a stabilizing effect on the growth of the Mack-mode instabilities, thus delaying transition. However, experiments have demonstrated a slowdown in the downstream movement of the transition location, and an eventual reversal at nosetip radii greater than some critical value of nosetip bluntness. This reversal conflicts with the predictions of linear stability theory. The available linear stability analyses [4–7] agree that transition reversal cannot be predicted by only considering modal instability amplification, because Mack's second mode disturbances are strongly stabilized by the presence of the entropy layer, and the first mode, commonly responsible for the onset of transition at supersonic boundary-layer edge conditions, is not destabilized. Additionally, the modal amplification of entropy-layer modes is marginal as reported by Refs. [8, 9].

Because of the failure of the modal instability theory to predict the transition within the swallowing distance of the entropy layer, nonmodal growth was proposed by Paredes et al. [10] as the potential basis for a physics-based model for the experimentally observed onset of transition. Results indicate that stationary disturbances that are initiated in

the vicinity of the nosetip can undergo relatively significant nonmodal amplification that increases with the nosetip bluntness. This finding may not provide a definitive link between transient growth and the onset of transition, but is qualitatively consistent with the experimental observations that transition during the reversal regime on large bluntness configurations was highly sensitive to wall roughness, and additionally, was dominated by disturbances originating near the nosetip. However, for moderately-large bluntness values, transition can occur significantly downstream from the nosetip, i.e., over the frustum of the cone [3]. For these moderately blunt conditions, experiments by Stetson [3] and Jewell et al. [11] indicated that uncontrolled nosetip roughness did not influence the transition onset location, suggesting that unsteady freestream disturbances may play a role in initiating frustum transition over moderately-large bluntness cones. Computational analysis by Refs. [10, 12, 13] has shown significant nonmodal growth of both planar and oblique traveling disturbances that peak within the entropy layer and above the boundary-layer edge. The linear, nonmodal amplification of the Mack's energy norm associated with the nonstationary disturbances was lower than that for the three-dimensional stationary disturbances, but increased as the nosetip bluntness was increased [13]. Therefore, Paredes et al. [13] suggested that nonstationary nonmodal traveling disturbances that peak within the entropy layer could lead to transition onset in the absence of appreciable modal instability amplification. Furthermore, Paredes et al. [14] demonstrated that, even though the linear nonmodal disturbances are primarily concentrated outside the boundary layer, their nonlinear interaction can generate stationary streaks that penetrate and amplify within the boundary layer, eventually inducing the onset of transition via the breakdown of these streaks.

While the previous studies focused on axisymmetric cones with blunt, hemispherical nosetips, ogival nosetips are also used in hypersonic vehicles because of their improved aerodynamic performance in terms of drag and heating. Available experimental studies of laminar-turbulent transition on hypersonic configurations with ogival forebodies are scarce. Greenwood [15] and Greenwood and Schneider [16] extended the work to cone-ogive-cylinder geometries by performing experimental studies of the instabilities within the entropy layer in the Boeing/AFOSR Mach 6 Quiet Tunnel (BAM6QT) of Purdue University using a combination of hot-wire anemometry and wall-surface pressure measurements. Their work showed growth of disturbances initially peaking outside the boundary-layer edge and then merging downstream alongside the boundary layer, suggesting these were likely entropy-layer instabilities. Recent experimental measurements by Hill [17] and Hill et al. [18] at the Air Force Research Laboratory Mach 6 Ludwig Tube have focused on the instabilities over a series of sharp and blunt ogive-cylinder geometries. Interchangeable forebodies followed by a cylindrical section to make the model one meter long were used to enable instability measurements past the entropy-layer swallowing location. The ogive forebodies were designed specifically to control the shock curvature, which in turn alters the entropy layer structure and strength, both playing a crucial role in external instability growth [15, 19]. A subset of the geometries of interest for the current computational study are shown in Fig. 1. For all forebodies, three kinds of boundary-layer disturbances were observed. Rope-like boundary-layer disturbances (Fig. 2(a)) were primarily observed for the sharp tipped 4 and 2 caliber ogive forebodies. They were found within the boundary-layer thickness and were identified as planar Mack's second mode instabilities. For these sharp tips, coexisting elongated instabilities (Fig. 2(b)) were observed to randomly alternate with the rope-like disturbances. Those became more dominant as the nosetip angle was increased from 14° to 28° (i.e., 4 and 2 caliber ogive forebodies) and were the only observed disturbances for the 56° nosetip (1 caliber ogive forebody). For blunter tips, wisp-like disturbances located above the boundary-layer edge (Fig. 2(c)) were randomly found with undetermined wavelengths and frequencies. Hill et al. [18] also used focused laser differential interferometry (FLDI) measurements to understand the switch from higher frequency disturbances in the form of rope-like structures to the lower frequency disturbances in the form of elongated structures for the forebody shapes. Figure 3 shows the contours of power spectral density versus wall-normal distance at a streamwise location of approximately 0.5 m from the nosetip for the sharp 4 and 2 caliber ogive forebody models. The higher frequency peak centered around 60 kHz for the sharp 4 caliber ogive model that was associated with the rope-like structures decreases in amplitude for the sharp 2 caliber ogive model and the lower frequency disturbances between 5 and 25 kHz associated with the elongated structures becomes dominant.

The present paper investigates the effect of the forebody geometry on the modal and nonmodal instability characteristics of the boundary layer over the ogive-cylinder models that were tested in the Air Force Research Laboratory Mach 6 Ludwig Tube by Hill [17] and Hill et al. [18]. The methodologies used to study the modal and nonmodal disturbances are summarized in Section II. Then, the laminar boundary-layer flow over the selected configurations, and modal and nonmodal instability analysis results are presented in Section III. Concluding remarks are given in Section IV.

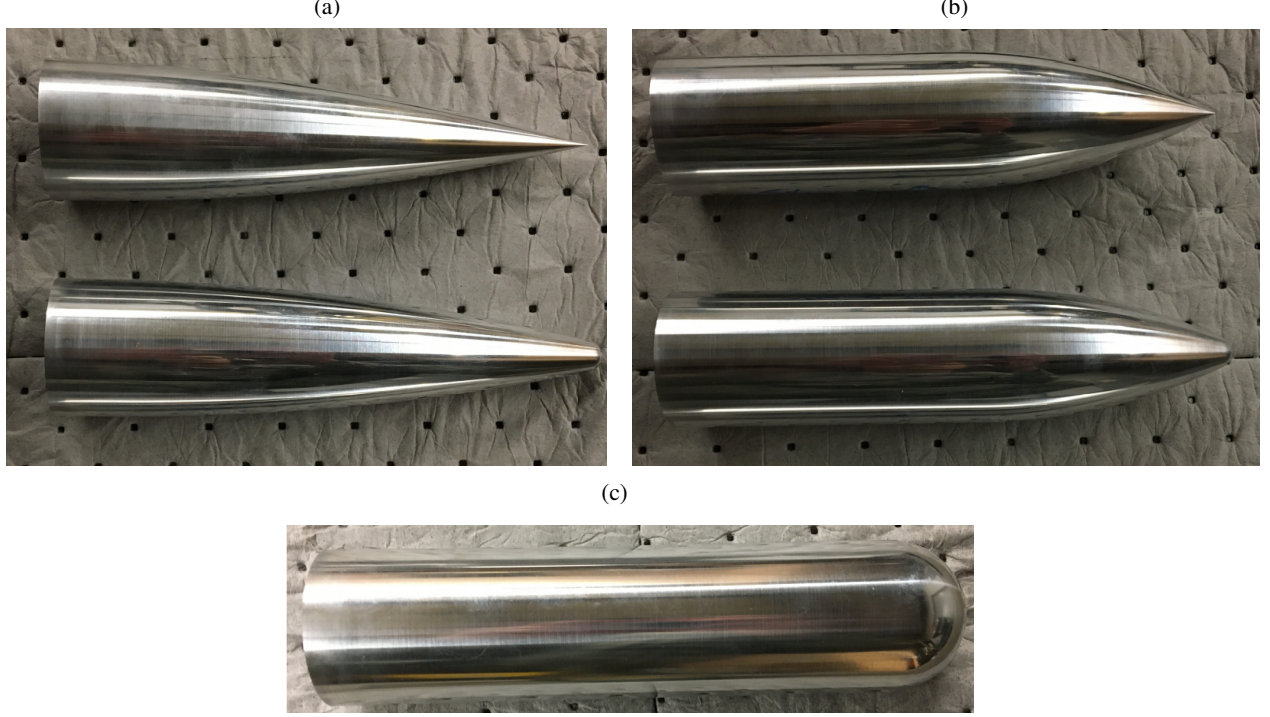


Fig. 1 Experimental ogive forebodies used by Hill [17] (a) 14 degree (4 caliber) sharp and 1/4R (blunt), (b) 28 degree (2 caliber) sharp and 1/4R (blunt), and (c) hemisphere (0.5 caliber) with a cylinder base radius, $R = 31.1$ mm.

II. Theory

In this section, we outline the methodology used for the analysis of disturbance amplification over the configurations of interest. Following the work of Refs. [10, 13], we use the harmonic linearized Navier-Stokes equations (HLNSE) and the linear parabolized stability equations (PSE) frameworks to investigate both modal and nonmodal disturbances.

A. Governing Equations for Modal and Nonmodal Disturbances

The present work is focused on the boundary layers over axisymmetric bodies in a hypersonic flow. The freestream conditions and geometries are selected to match selected configurations from the experiments conducted by Hill et al. [18] with interchangeable sharp and blunt ogival forebodies followed by a cylinder. The Cartesian coordinates are represented by (x, y, z) . For this problem, the computational coordinates are defined as an orthogonal, body-fitted coordinate system, with (ξ, η, ζ) denoting the streamwise, wall-normal, and azimuthal coordinates, respectively, and (u, v, w) representing the corresponding velocity components. Density and temperature are denoted by ρ and T , respectively. The streamwise and azimuthal metric factors are defined as

$$h_\xi = 1 + \kappa\eta, \quad (1)$$

$$h_\zeta = r_b + \eta \cos(\theta), \quad (2)$$

where κ denotes the streamwise curvature, r_b is the local radius, and θ is the local half-angle along the axisymmetric surface, i.e., $\sin(\theta) = dr_b/d\xi$.

The vector of flow variables $\mathbf{q}(\xi, \eta, \zeta, t) = (\rho, u, v, w, T)^T$ is decomposed into a vector of stationary basic state variables $\bar{\mathbf{q}}(\xi, \eta, \zeta) = (\bar{\rho}, \bar{u}, \bar{v}, \bar{w}, \bar{T})^T$ and a vector of perturbation variables $\tilde{\mathbf{q}}(\xi, \eta, \zeta, t) = (\tilde{\rho}, \tilde{u}, \tilde{v}, \tilde{w}, \tilde{T})^T$. For axisymmetric geometries at zero degrees angle of attack, the basic state variables are independent of the azimuthal coordinate and the linear perturbations can be assumed to be harmonic in time and in the azimuthal direction, which lead to the following expression for the perturbations,

$$\tilde{\mathbf{q}}(\xi, \eta, \zeta, t) = \check{\mathbf{q}}(\xi, \eta) \exp [i (m\zeta - \omega t)] + c.c., \quad (3)$$

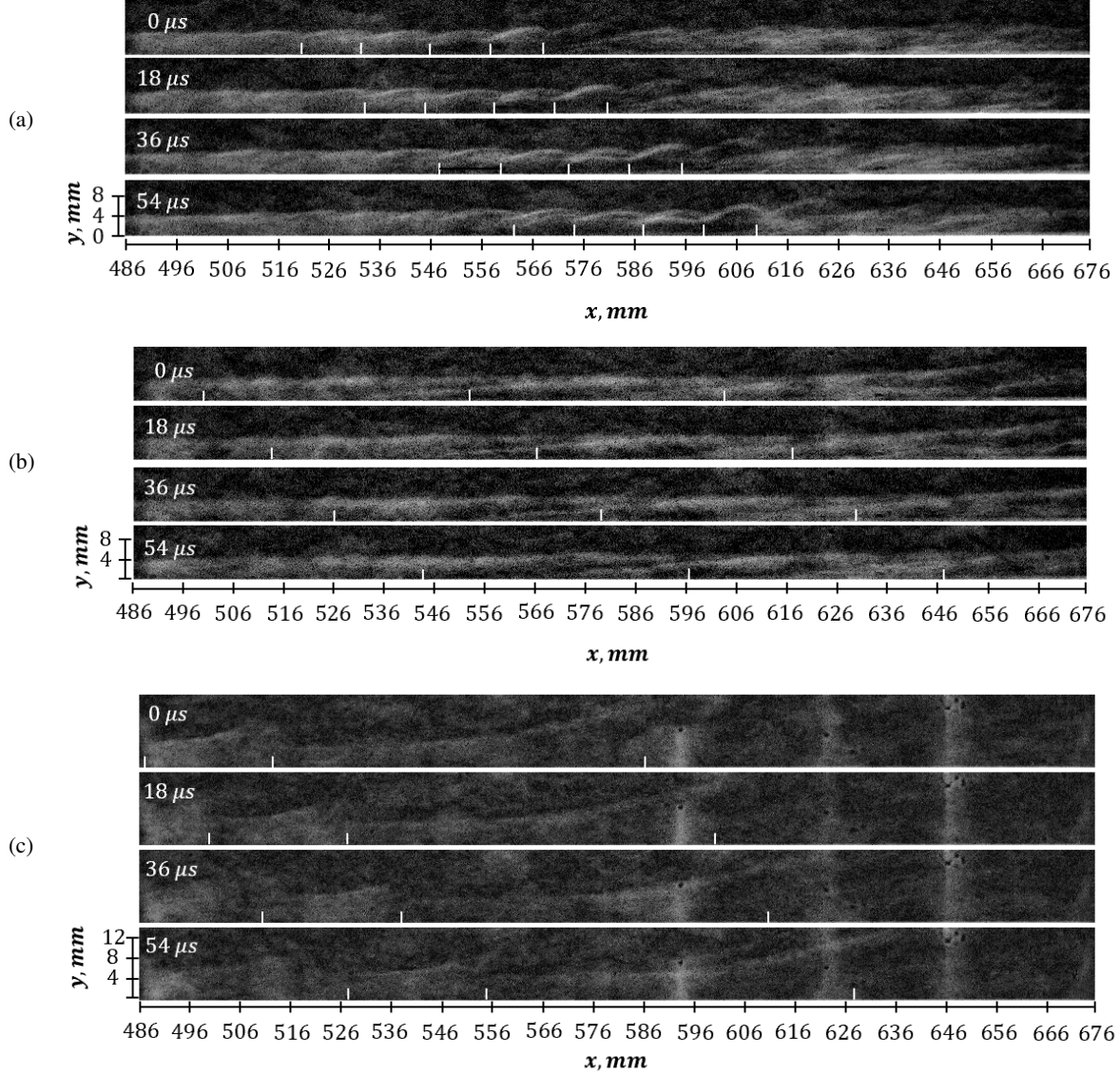


Fig. 2 Experimental schlieren images of the (a) rope-like disturbances visible for the sharp 4 caliber ogive forebody at $Re_\infty = 5.7 \times 10^6 \text{ m}^{-1}$, (b) elongated disturbances visible for the sharp 2 caliber ogive forebody at $Re_\infty = 5.7 \times 10^6 \text{ m}^{-1}$, and (c) wisp-like disturbances visible for the 1/4R (blunt) 4 caliber forebody at $Re_\infty = 9.8 \times 10^6 \text{ m}^{-1}$.

where $\check{\mathbf{q}}(\xi, \eta) = (\check{\rho}, \check{u}, \check{v}, \check{w}, \check{T})^T$ is the vector of disturbance functions, m is the azimuthal wavenumber, and ω is the angular frequency. The disturbance functions $\check{\mathbf{q}}(\xi, \eta)$ satisfy the HLNSE [10], in which linear operators depend on the basic state variables and parameters, and on the angular frequency and azimuthal wavenumber of the perturbation.

The PSE approximation to the HLNSE is based on isolating the rapid phase variations in the streamwise direction via the disturbance ansatz

$$\check{\mathbf{q}}(\xi, \eta) = \hat{\mathbf{q}}(\xi, \eta) \exp \left[i \int_{\xi_0}^{\xi} \alpha(\xi') d\xi' \right], \quad (4)$$

where the unknown, streamwise varying wavenumber $\alpha(\xi)$ is determined in the course of the solution by imposing an additional constraint

$$\int_{\eta} \hat{\mathbf{q}}^* \frac{\partial \hat{\mathbf{q}}}{\partial \xi} h_{\xi} h_{\zeta} d\eta d\zeta = 0, \quad (5)$$

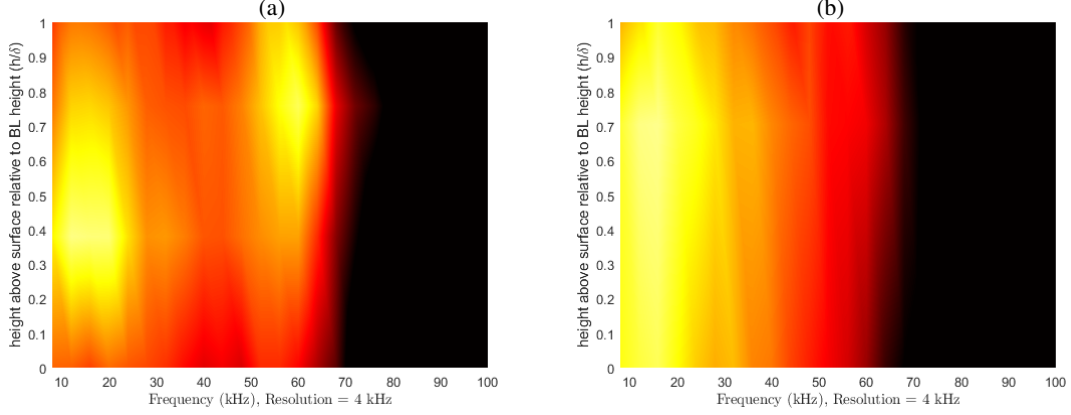


Fig. 3 Power spectral density calculated from FLDI vs different normalized heights above the surface for the sharp (a) 4 caliber and (b) 2 caliber ogive forebodies at $x = 0.5$ m (reproduced from Hill et al. [18]).

which ensures the amplitude functions $\hat{\mathbf{q}}(\xi, \eta) = (\hat{\rho}, \hat{u}, \hat{v}, \hat{w}, \hat{T})^T$ vary slowly in the streamwise direction in comparison with the phase term $\exp \left[i \int_{\xi_0}^{\xi} \alpha(\xi') d\xi' \right]$. Substituting Eq. (4) into the HLNSE and invoking scale separation between the streamwise coordinate and the other two directions yields the PSE.

As discussed in Ref. [10], Eq. (5) constrains the PSE disturbances having a unique, local wavenumber at any streamwise location and, therefore, the PSE are likely to encounter difficulties when the perturbation field includes multiple disturbances with disparate variation along the streamwise direction. Optimal growth perturbations are known to include a superposition of multiple eigenmodes of the quasiparallel disturbance equations, and therefore, the HLNSE are used in the present work to study the nonmodal disturbances.

The onset of laminar-turbulent transition is estimated using the logarithmic amplification ratio, the so-called N -factor, relative to the lower bound location ξ_I where the disturbance first becomes unstable,

$$N_\phi = - \int_{\xi_I}^{\xi} \alpha_i(\xi') d\xi' + \ln [\hat{\phi}(\xi) / \hat{\phi}(\xi_I)], \quad (6)$$

where $\hat{\phi}$ denotes an amplitude norm of $\hat{\mathbf{q}}$, i.e., the wall-pressure magnitude or the disturbance energy norm that is defined as

$$\hat{E}(\xi) = \frac{1}{L_\zeta} \int_{\eta} \hat{\mathbf{q}}(\xi, \eta, \zeta)^H \mathbf{M}_E \hat{\mathbf{q}}(\xi, \eta, \zeta) h_\xi h_\zeta d\eta, \quad (7)$$

where $\mathbf{M}_E$ is the energy weight matrix and the superscript H denotes conjugate transpose. The positive-definite energy norm used here was derived by Chu [20] and used by Mack [21] and Hanifi et al. [22] for linear stability theory. This energy norm is defined as

$$\mathbf{M}_E = \text{diag} \left[\frac{\bar{T}(\xi, \eta)}{\gamma \bar{\rho}(\xi, \eta) M^2}, \bar{\rho}(\xi, \eta), \bar{\rho}(\xi, \eta), \bar{\rho}(\xi, \eta), \frac{\bar{p}(\xi, \eta)}{\gamma(\gamma - 1) \bar{T}(\xi, \eta) M^2} \right]. \quad (8)$$

For the nonmodal analysis, the HLNSE are solved. The optimal initial disturbance, $\tilde{\mathbf{q}}_0$, is defined as the initial or inflow condition at ξ_0 that maximizes the objective function, J , which is defined as a measure of disturbance growth over a specified interval $[\xi_0, \xi_1]$. The definition used in the present study correspond to the outlet energy gain [23, 24] that is defined as

$$G_E = \frac{E(\xi_1)}{E(\xi_0)}. \quad (9)$$

The variational formulation of the problem to determine the maximum of the objective functional $J = G_E$ leads to an optimality system [25], which is solved in an iterative manner, starting from a random solution at ξ_0 that must satisfy the boundary conditions. The HLNSE, $\mathbf{L}\tilde{\mathbf{q}} = 0$, are used to integrate $\tilde{\mathbf{q}}$ up to ξ_1 , where the final optimality condition is used to obtain the initial condition for the backward adjoint equations integration, $\mathbf{L}^\dagger \tilde{\mathbf{q}}^\dagger = 0$. At ξ_0 , the adjoint solution is used to calculate the new initial condition for the forward integration with the initial optimality condition. The iterative procedure finishes when the value of J has converged up to a certain tolerance, which was set to 10^{-4} in the present computations.

III. Results

This section begins with the presentation of the laminar flow solution over the selected ogive-cylinder models at zero degrees angle of attack in a Mach 6.1 free stream from the experiments of Hill et al. [18]. The basic state description is followed by the modal instability characteristics of all selected geometries. Direct comparisons with experimental measurements are shown, and motivate nonmodal stability analysis for blunter tips. Nonmodal results are compared to modal ones and to experimental measurements. Finally, flow perturbation contours for the most amplified planar and oblique disturbances are shown.

A. Laminar Boundary-Layer Flow

The present configurations are selected to match the experiments conducted by Hill et al. [18] in the low-enthalpy Air Force Research Laboratory (AFRL) Mach-6 Ludwieg Tube. The working fluid is air, and effects of chemistry and molecular vibration are negligible. The experimental measurements were taken with six PCB Piezotronics pressure sensors, six Kulite® pressure transducers, high speed schlieren imagery, and focused laser differential interferometry (FLDI). The geometric details of the selected models for the present computational study are outlined in Table 1.

Table 1 Geometrical details of investigated ogive-cylinder geometries. The cylinder base radius is $R^* = 31.1$ mm.

Geometry	Caliber	Ogive half-angle [°]	Nosetip bluntness, r_n^* [mm]	Ogive-cylinder junction, x_j^* [m]
Sharp 4C	4	14	0.1 mm	0.252436
Sharp 2C	2	28	0.1 mm	0.124251
Blunt 4C	4	14	$1/4R$	0.225867
Blunt 2C	2	28	$1/4R$	0.114791
Hemisphere	0.5	-	R	0.0309591

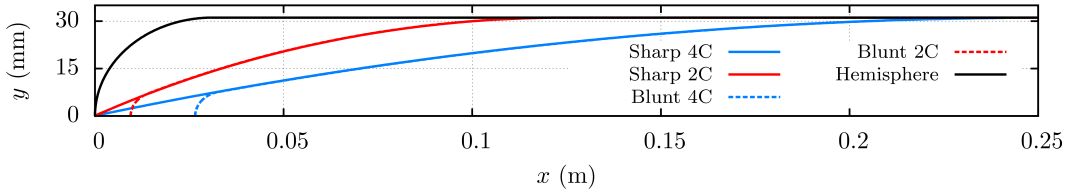


Fig. 4 Profiles of selected forebodies listed in Table 1.

The laminar basic states over the ogive-cylinder geometries are computed for the selected freestream conditions by using a second-order accurate algorithm as implemented in the finite-volume compressible Navier-Stokes flow solver VULCAN-CFD* [26]. The VULCAN-CFD solution is based on the full Navier-Stokes equations and uses the solver's built-in capability to iteratively adapt the computational grid to the bow shock. The Keyes' law for air [27] is used to calculate the dynamic viscosity as a function of temperature. The freestream conditions are selected to replicate those of the experiments. Table 2 includes the selected two representative conditions. An isothermal wall with $T_w^* = 300$ K is used. The computational grid is composed of $1,000 \times 600$ points with enough points clustered next to the model surface to resolve the thickness of the boundary layer.

Table 2 Flow conditions used for each ogive-cylinder geometry.

u_∞ [m/s]	ρ_∞ [kg/m ³]	T_∞ [K]	T_0 [K]	P_0 [kPa]	M_∞	Re_∞ [m ⁻¹]
931.65	0.0308	58.04	490	896.3	6.1	7.01×10^6

Figure 5 shows the Mach number contours of the five geometries detailed in Table 1 for flow conditions given in Table 2. The evolution of boundary-layer thickness and edge Mach number for the five geometries are displayed on

*visit <http://vulcan-cfd.larc.nasa.gov> for further information about the VULCAN-CFD solver

figure 6. The boundary-layer edge, δ_h , is defined as the wall-normal position where $h_t/h_{t,\infty} = 0.995$, with h_t denoting the total enthalpy, i.e., $h_t = h + 0.5(\bar{u}^2 + \bar{v}^2 + \bar{w}^2)$, where $h = c_p \bar{T}$ is the static enthalpy. The edge Mach number values are extracted at a wall-normal distance equal to $1.5\delta_h$ to obtain more realistic values due to the influence of the entropy layer on the boundary-layer profile. The effect of the increase of both, the ogive half-angle and the nosetip bluntness, is to increase the boundary-layer thickness along the model length. Increasing the half-angle reduced the edge Mach number, while the bluntness introduced a monotonic increase of the edge Mach number toward the sharp value. The Mach number contours of the hemispherical model of Fig. 5(e) show a large region of reduced Mach number above the boundary-layer edge due to the entropy layer induced by the curved shock. A thinner, but still quite evident, entropy layer is also formed in the solutions over the blunt forebodies with nosetip radii equal to $1/4R$.

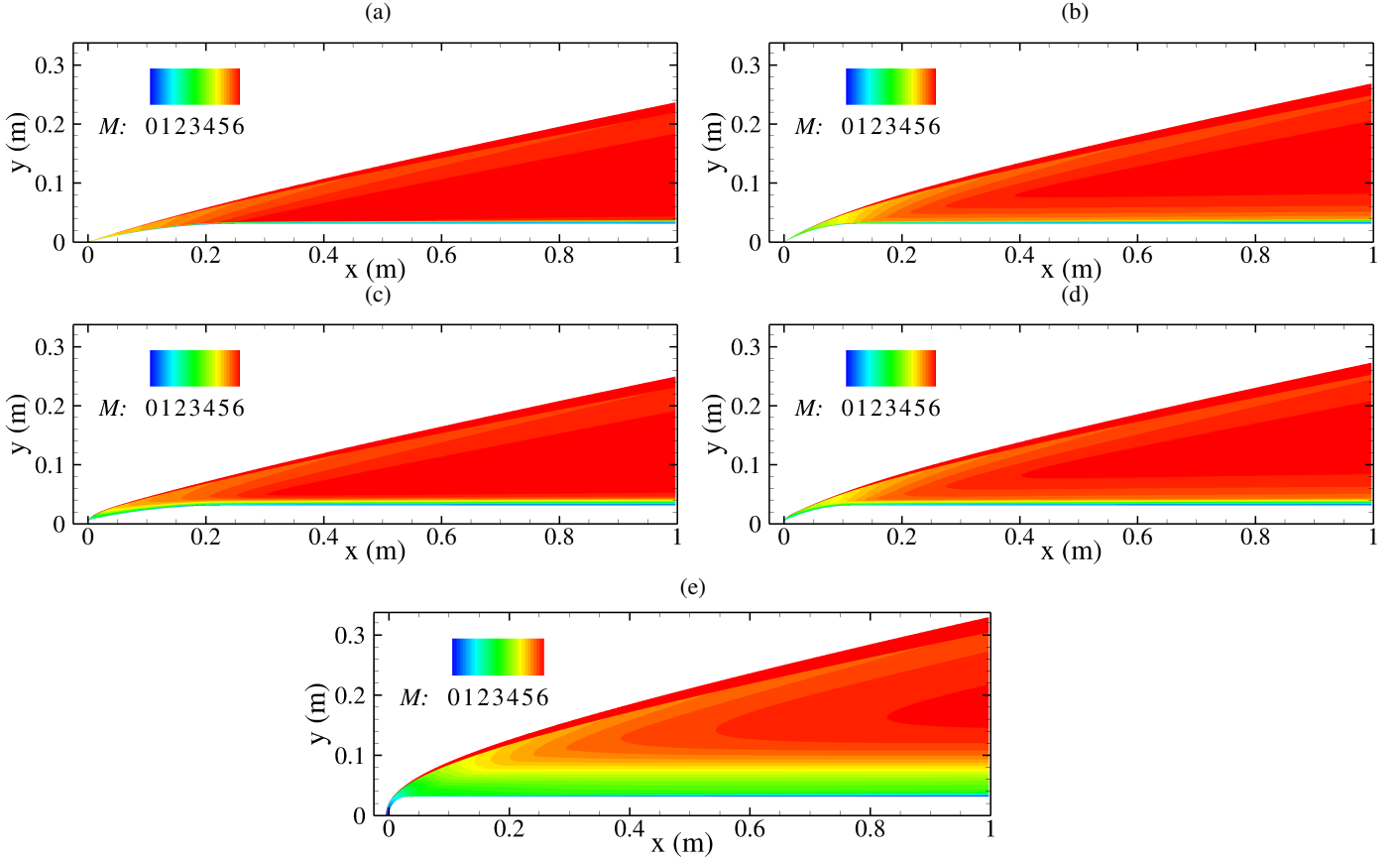


Fig. 5 Mach contours for the (a) sharp and (b) blunt 4 caliber ogive-cylinder, (c) sharp and (d) blunt 2 caliber ogive-cylinder and the (e) hemisphere-cylinder forebody geometries.

Figures 7 and 8 show the streamwise velocity and temperature profiles, respectively, for the five models. To help delineate the influence of the forebody shape, the profiles on the various configurations of interest are compared at the junction between the forebody and the cylinder (x_j) and at three additional, downstream locations ($x = x_j + 0.2$ m, $x_j + 0.4$ m, and $x_j + 0.6$ m). The effect of stronger entropy layer induced by the blunt nosetips is to reduce the streamwise velocity values and increase the temperature values up to the edge of the entropy layer, beyond which the boundary-layer profiles corresponding to the sharp and blunt models nearly overlap with each other.

B. Modal instability analysis

The PSE are used to study the modal amplification of the boundary-layer instability waves. The streamwise evolution of the logarithmic amplification ratio (N -factor curves) of planar waves ($m = 0$) over both sharp ogives is shown in Fig. 9. The N -factor is chosen with respect to the wall-pressure disturbance to allow a meaningful comparison with the experimental data taken with wall-mounted PCB and Kulite® pressure sensors. Although the instability waves begin to amplify farther downstream for the sharp 4 caliber ogive forebody (Fig. 9(a)) in comparison with the sharp 2 caliber

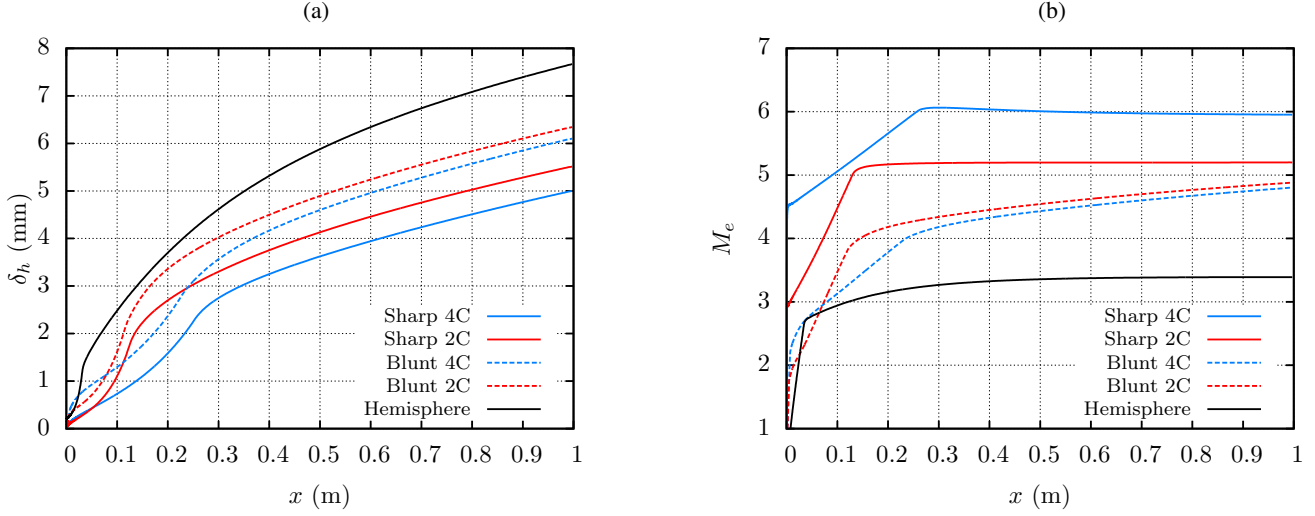


Fig. 6 Axial evolution of (a) boundary-layer thickness and (b) edge Mach number compared across different forebodies.

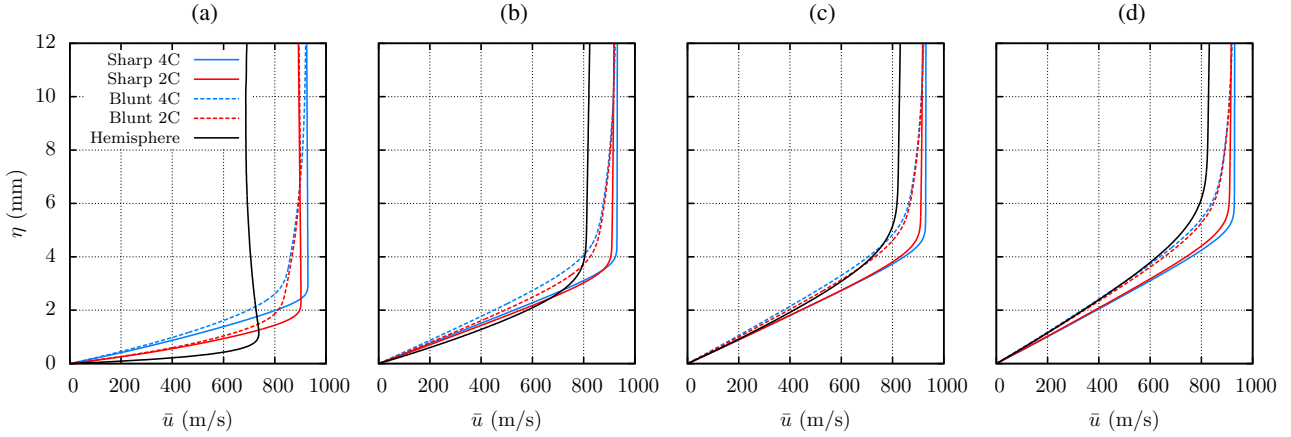


Fig. 7 Streamwise velocity profiles for each geometry at the (a) junction location between the forebody and cylinder $x = x_j$, (b) $x = x_j + 0.2$ m, (c) $x = x_j + 0.4$ m, and (d) $x = x_j + 0.6$ m.

ogive forebody geometry (Fig. 9(b)) because of the longer ogival forebody with convex curvature and favorable pressure gradient, the growth rates are larger along the cylinder for the slender 4 caliber ogive forebody geometry. Consequently, the linear amplification of planar Mack mode waves is reduced by the increase of the ogive angle. As will be presented next, oblique first mode disturbances are also amplified over both sharp ogive forebody models.

Experimental results from both PCB and Kulite[®] sensors are compared to modal linear stability results for the sharp 4 and 2 caliber forebodies in figure 10. The locations of the PCB and Kulite[®] sensors are given in Table 3. Kulite[®] 1 failed prematurely in the experiments and is thus not shown in later comparisons. The PCB data have been high-pass filtered at 15 kHz to remove the sensor's inherent inability to measure lower frequencies. The N -factor spectra correspond to the envelope across azimuthal wavenumber values for each frequency. For each streamwise station, the lower and higher frequency peaks correspond to oblique first mode and planar second mode waves, respectively. The second mode peaks observed between 60 and 130 kHz at the locations of the PCBs 1, 2, 3 and Kulites[®] 2, 3 coincide well with computational measurements for the sharp 4 caliber ogive forebody. Moving downstream, the amplification increases and frequency decreases due to the boundary-layer thickness increasing. The measured spectra become broad for the downstream sensors due to the breakdown to turbulence. For the sharp 2 caliber ogive forebody, the amplification is lower for both experimental and computational results and there is also agreement between planar second mode peaks for PCBs 2, 3 and Kulites[®] 2, 3.

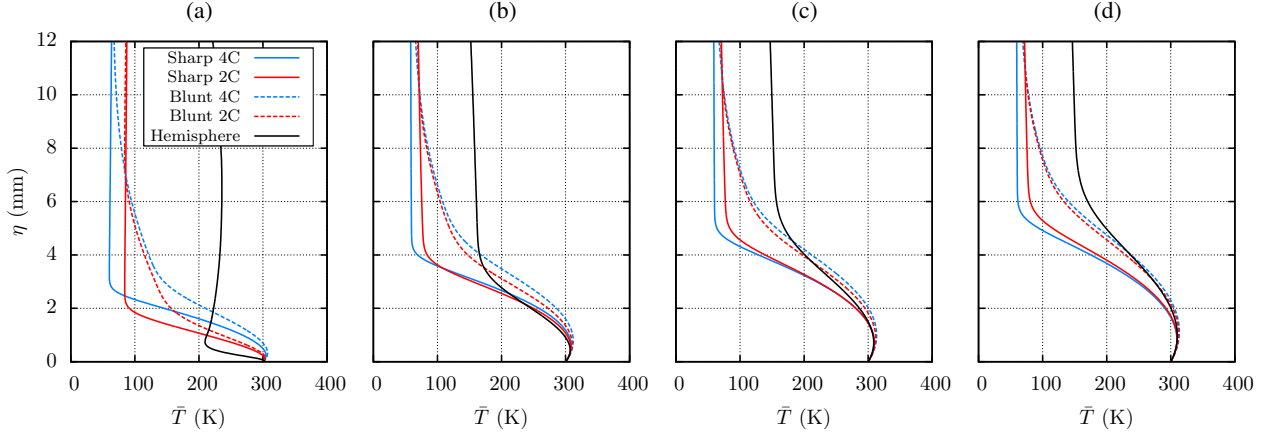


Fig. 8 Temperature profiles for each geometry at (a) the junction location between the forebody and cylinder $x = x_j$, (b) $x = x_j + 0.2$ m, (c) $x = x_j + 0.4$ m, and (d) $x = x_j + 0.6$ m.

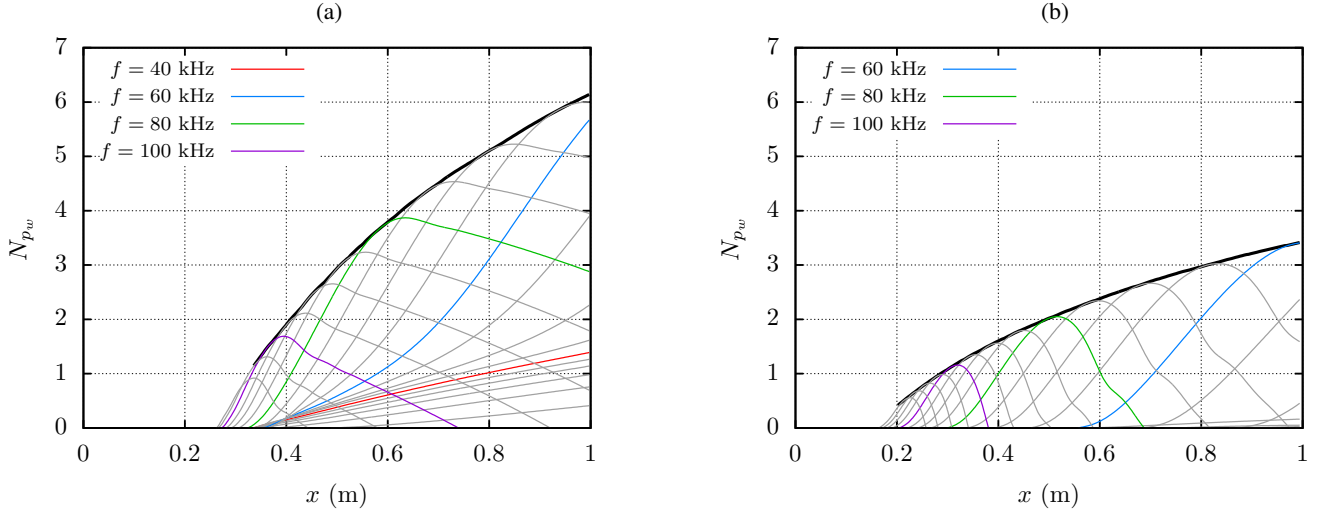


Fig. 9 N -factor curves based on wall-pressure perturbation for planar waves in boundary-layer flows over (a) sharp 4 caliber and (b) sharp 2 caliber ogives. The gray lines correspond to selected disturbance frequencies with a frequency increment of 5 kHz. A subset of those frequencies is highlighted in different colors as shown in the legend. The black line denotes the N -factor envelope for each configuration.

Table 3 PCB and Kulite[®] sensor locations along the ogive-cylinder geometry.

PCB	1	2	3	4	5	6
Distance from tip, x (m)	0.29	0.38	0.54	0.63	0.79	0.87
Kulite [®]	1	2	3	4	5	6
Distance from tip, x (m)	0.34	0.42	0.59	0.67	0.83	0.92

The N -factor values based on the disturbance energy obtained with PSE at each azimuthal wavenumber and frequency are given in Fig. 11 for the sharp 4 and 2 caliber ogive forebodies at the location of the PCB 3. As shown in Fig. 9, the amplification of planar second mode waves decreases as the nosetip angle is increased from 14° to 28° . The lower frequency, oblique first mode disturbances are also amplified over these two sharp configurations. In contrast to the planar second mode, the oblique first modes are more amplified at the PCB 3 location for the 2 caliber ogive than for the 4 caliber ogive. The stabilizing influence of the ogive angle on the second mode waves is consistent with the

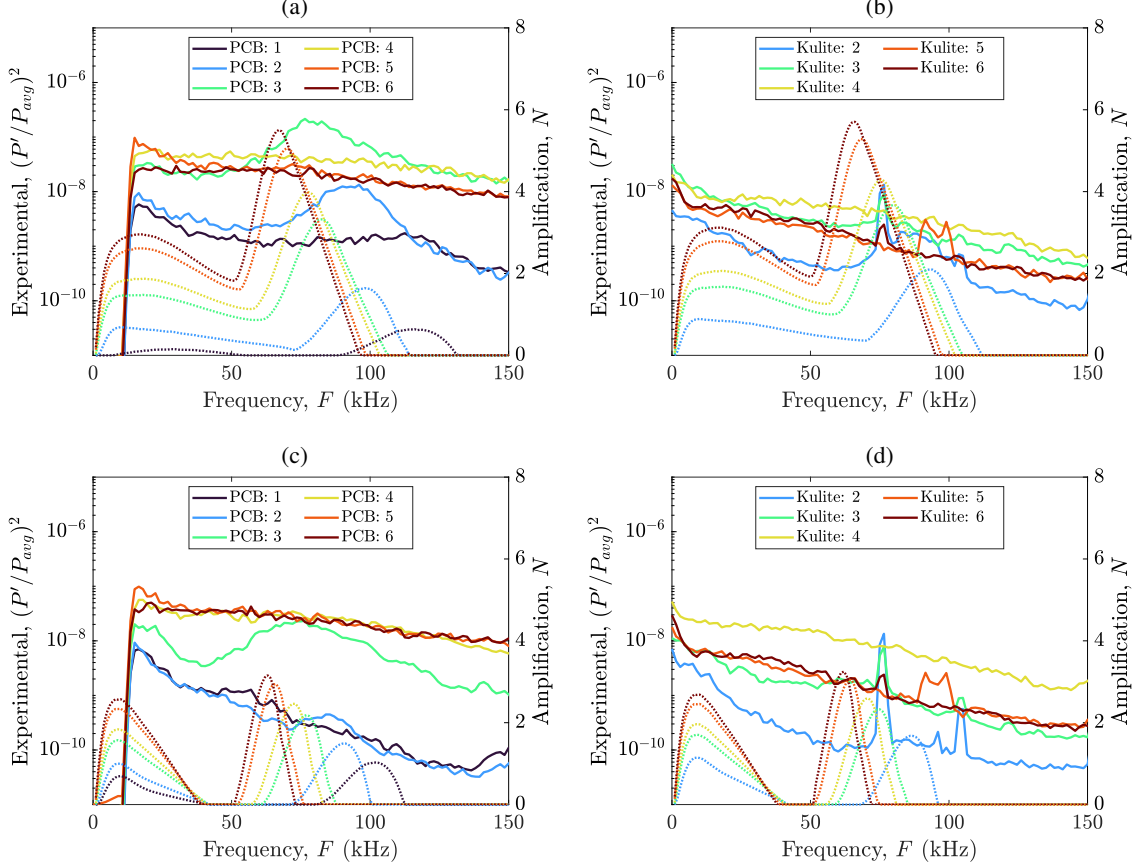


Fig. 10 Interpolated PSE results at the (a) PCB and (b) Kulite® locations compared to experimental results for the sharp 4 caliber ogive forebody and (c) and (d) for the sharp 2 caliber ogive forebody, respectively.

measurements of Hill et al. [18], since they observed that the rope-like instability structures in the schlieren images and the spectral signature of disturbances in the frequency range identified as second Mack modes vanished as the angle of the sharp ogives was increased. However, elongated structures were observed to dominate for the sharp ogives with larger angles. The FLDI measurements of Fig. 3 were used to associate these elongated structures with lower frequencies in the range of 5-25 kHz and find their peak within the boundary layer. Because of the correlation with the frequencies of the oblique first-mode waves, the measured elongated structures can be identified as oblique first-mode waves for the sharp 4 and 2 caliber ogive forebody.

For blunter tips, the stronger entropy layer has a stabilizing effect on modal instabilities as seen by the smoother velocity and temperature profiles previously shown in Figs. 7 and 8. No unstable modes were found for the blunt and hemispherical forebodies. This is in agreement with the experimental results as no distinct peaks were observed in the measured power spectra for these geometries [17]. Because of the absence of modal disturbance amplification in the blunt nosetip models, nonmodal analysis is investigated to provide further understanding of the disturbances present.

C. Nonmodal instability analysis

An alternate representation of the modal contours presented in Fig. 11 for the sharp-tipped forebodies is provided in Fig. 12 with comparison to nonmodal results. The HLNSE are used to compute the optimal disturbance energy gain over the interval from $\xi_0 = 0.1$ m to $\xi_1 = 0.55$ m. The initial location was selected based on calculations with different initial locations that showed a minimal change in disturbance amplification by further reducing the value while allowing for the decay of perturbations far from the wall and below the bow shock. The final location was selected to approximately coincide with the PCB 3 location. The nonmodal disturbance amplification is shown in terms of $0.5 \log G_E$ to allow a direct comparison with the modal N -factor based on total disturbance energy. A good agreement between the locations of planar and oblique peaks is seen between the modal and nonmodal results. For the selected optimization interval, the

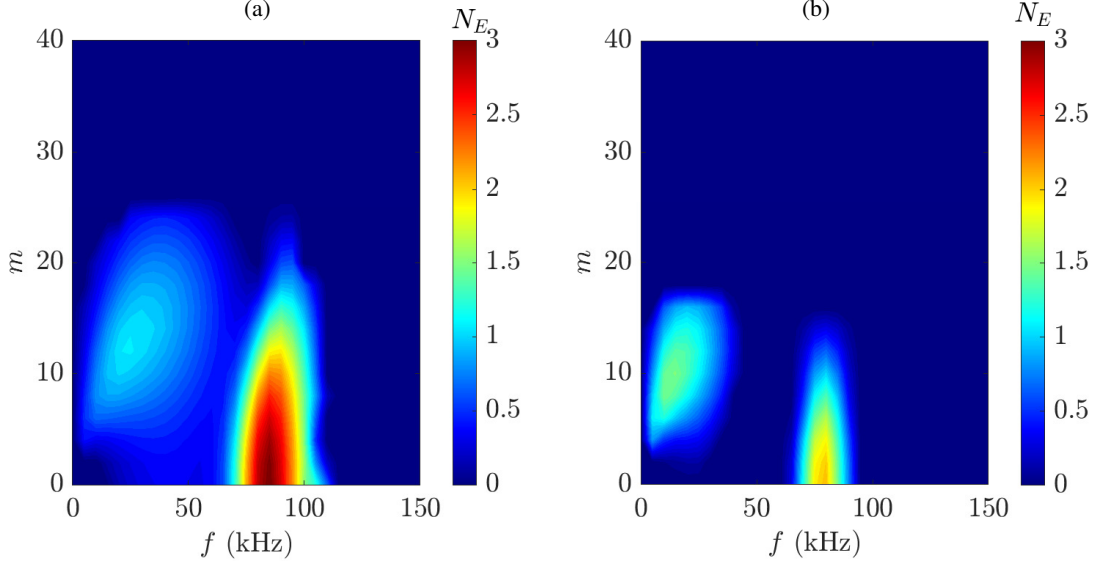


Fig. 11 N -factor contours calculated with PSE for the sharp (a) 4 and (b) 2 caliber ogive forebody.

oblique first mode disturbances experience stronger nonmodal amplification in comparison to the planar second mode disturbances due to the transient growth lift-up effect. For the 2 caliber ogive forebody, nonmodal analysis shows the first mode becoming dominant while modal analysis does not. It is noteworthy that the FLDI measurements of Fig. 3(b) for the sharp tipped 2 caliber ogive forebody also showed stronger disturbances in the first mode frequency range.

Comparing experimental data to nonmodal results shows good agreement for the second mode peaks present in the sharp tips between 60 to 100 kHz (Fig. 13). For the blunter tips, a broader range of amplified frequencies is obtained with no clear peaks, matching experimental results (the peaks in the Kulite[®] data are spurious resonant peaks). Because the nonmodal analysis outflow was selected to match the PCB 3 location, nonmodal results are not shown for downstream PCB and Kulite[®] sensors. Comparisons with the blunt 2 caliber and the hemispherical forebodies showed a similar, broadband, set of disturbances as for the blunt 4 caliber ogive forebody.

Similar to PSE results, the optimal disturbance energy gain at each azimuthal wavenumber and frequency are given in figure 14 for the different forebodies at the PCB 3 location. As the fineness ratio is decreased for the sharp forebodies, a shift in global maximum energy gain from the boundary-layer planar second mode to the a lower frequency oblique first mode is seen (Figs 14(a) and 14(b)). The increase in nosetip bluntness shows a change from the narrow frequency band corresponding to the second mode to a broadband less amplified disturbance. As will be shown next, they correspond to entropy-layer disturbances. The same switch from second Mack mode disturbances to broadband entropy-layer disturbances was found by Paredes et al. [13] for variable bluntness blunt cones at Mach 6. Furthermore, the frequency broadening shown previously for blunter tips is visible for the 1/4R nosetips. A low frequency peak is also observed for both 1/4R blunt ogive forebodies. The amplification of traveling nonmodal disturbances is found to decrease by the further increase of the nosetip bluntness to the hemispherical forebody (Fig. 14(e)). The frequency and azimuthal wavenumber values of maximum planar and oblique optimal disturbance growth are summarized in Table 4.

Contours of representative flow perturbations associated with the maximum overall and planar disturbances are shown for the sharp 4 and 2 caliber, and blunt 4 and 2 caliber ogive forebodies in Figs. 15-17. The boundary-layer and entropy-layer edges are shown as solid and dashed lines, respectively. The entropy-layer edge, δ_S , is defined as the location where the local entropy increment is 0.25 times the entropy increment at the wall ($\Delta S(\xi, \eta = \delta_S) = 0.25\Delta S_{wall}$) [10]. For the sharp 4 caliber ogive forebody, planar second mode waves are visible. These correspond to the rope-like structures observed by Hill et al. [18] primarily for the sharp nosetips. As fineness ratio decreases from 4 calibers to 2 calibers for the sharp models, both planar second mode and oblique first mode become comparable in linear amplification (Fig. 16). This results in the experimentally random observation of rope-like and elongated instabilities as both have similar amplification. Eventually, the spectrum is dominated by the nonmodal, entropy-layer disturbance for the blunt 4 caliber ogive forebody (Fig. 17). These disturbances are found to peak within the entropy-layer and boundary-layer edges. In the experiments, those are referred to as wisp structures and can be seen in Fig. 2(c). Instead, weak nonmodal entropy-layer disturbances were present and the most amplified wave corresponded to the low frequency

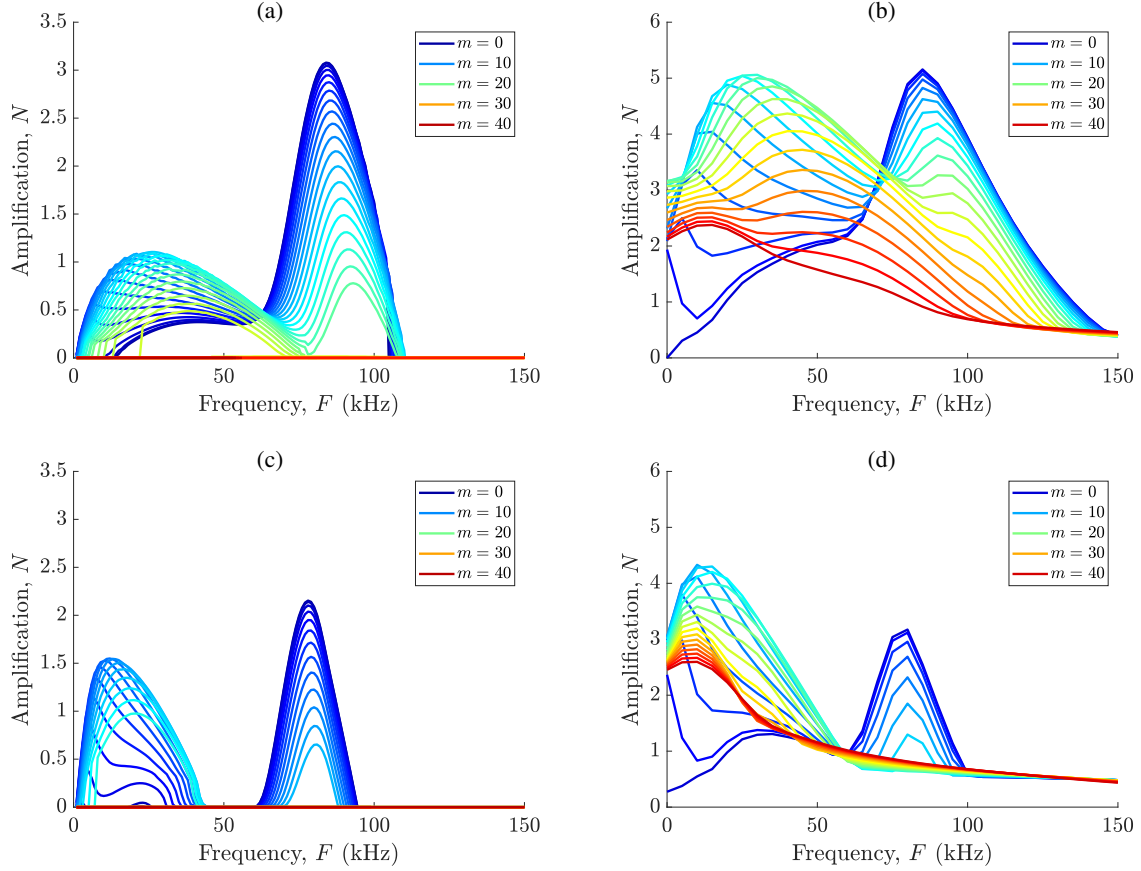


Fig. 12 N-factor curves from (a,c) modal analysis based on PSE and (b,d) nonmodal analysis based on HLNSE for the sharp (a,b) 4 caliber and (c,d) 2 caliber ogive forebodies.

Table 4 Frequency and azimuthal wavenumber corresponding to maximum planar and oblique optimal disturbance energy gain based on the HLNSE for the different forebodies.

Geometry	Planar ($m = 0$)	Oblique	
	f , kHz	f , kHz	m
Sharp 4C	85	30	16
Sharp 2C	80	10	10
Blunt 4C	60	5	10
Blunt 2C	60	5	10
Hemisphere	35	0	14

oblique disturbance. This agrees with experimental observations as only elongated structures were measured for that tip.

IV. Summary and Concluding Remarks

Laminar boundary-layer properties alongside modal and nonmodal stability analysis for ogive-cylinder models used in the experiments conducted by Hill [17] in the low-enthalpy Air Force Research Laboratory (AFRL) Mach-6 Ludwig Tube are presented. Comparisons involving the streamwise-velocity and temperature profiles downstream of the ogive-cylinder junction showed the effect of the ogive angle and bluntness on both sets of profiles. The effect of the ogive angle (or fineness ratio) is to increase the boundary-layer thickness and to reduce the edge Mach number over the cylinder region. The nosetip bluntness induces an entropy layer that envelopes the boundary-layer profiles. The linear

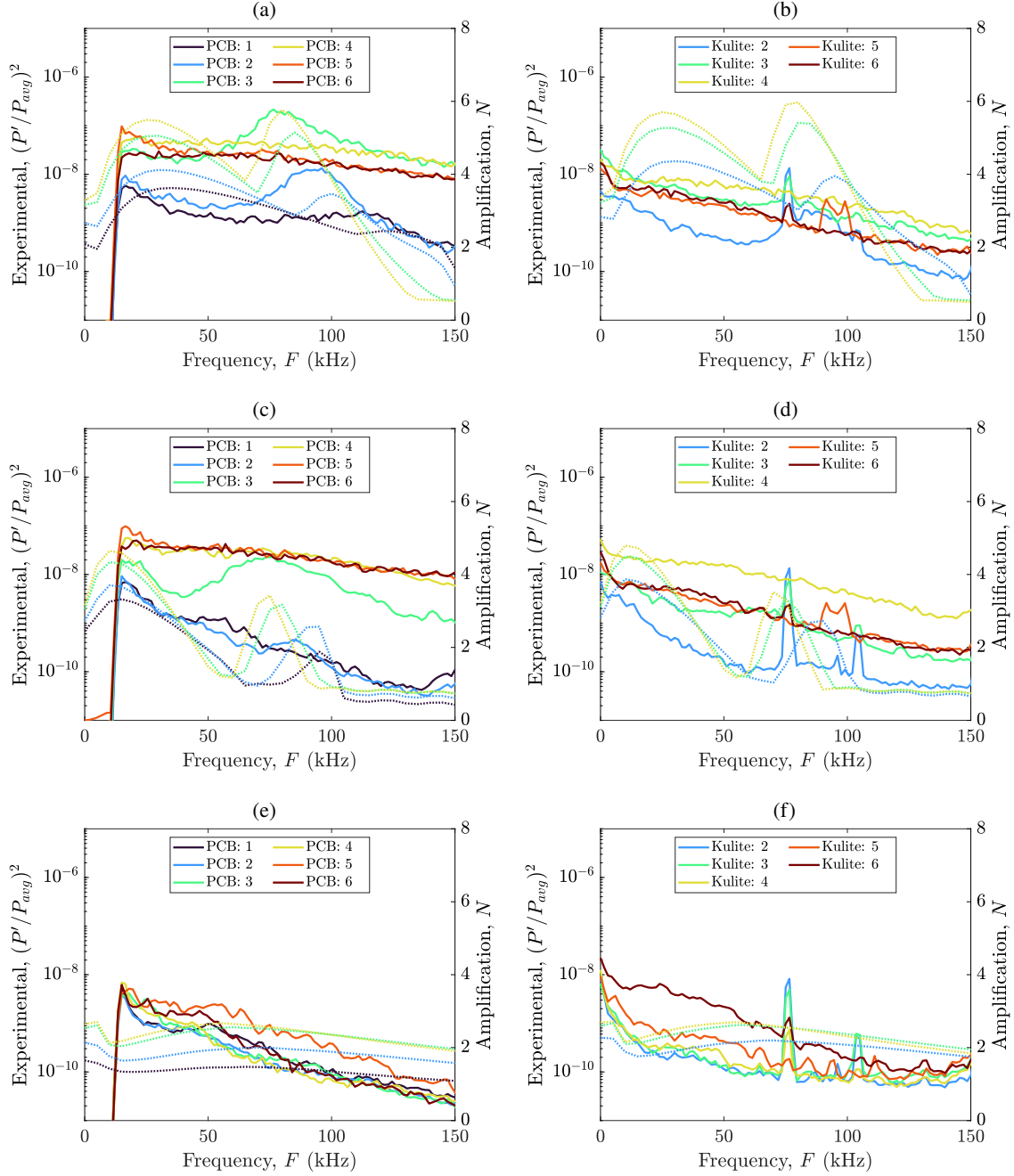


Fig. 13 Interpolated nonmodal results at the (a,c,e) PCB and (b,d,f) Kulite[®] locations compared to experimental results for the (a,b) sharp 4 caliber ogive, (c,d) sharp 2 caliber ogive, and (e,f) blunt 4 caliber ogive forebodies.

stability analysis results for the sharp 4 and 2 caliber ogive forebodies based on the parabolized stability equations show a slightly larger amplification of oblique Mack's first mode waves and a reduced amplification of planar Mack's second mode waves with an increase in the ogive angle (i.e., decrease in fineness ratio). Furthermore, the frequencies associated with the peak N -factor of planar second mode waves along the streamwise direction matched the PCB and Kulite[®] readings from the experiments. The PSE results are found to correlate with the FLDI measurements that capture the reduction in second mode amplification and dominance of lower frequency disturbances in the range of 5 to 25 kHz that matches the oblique first mode frequency bandwidth as the nosetip angle is increased (i.e. fineness ratio is decreased) for the sharp models. The PSE analysis for the blunt ogives with nose radii equal to one quarter of the base radius and

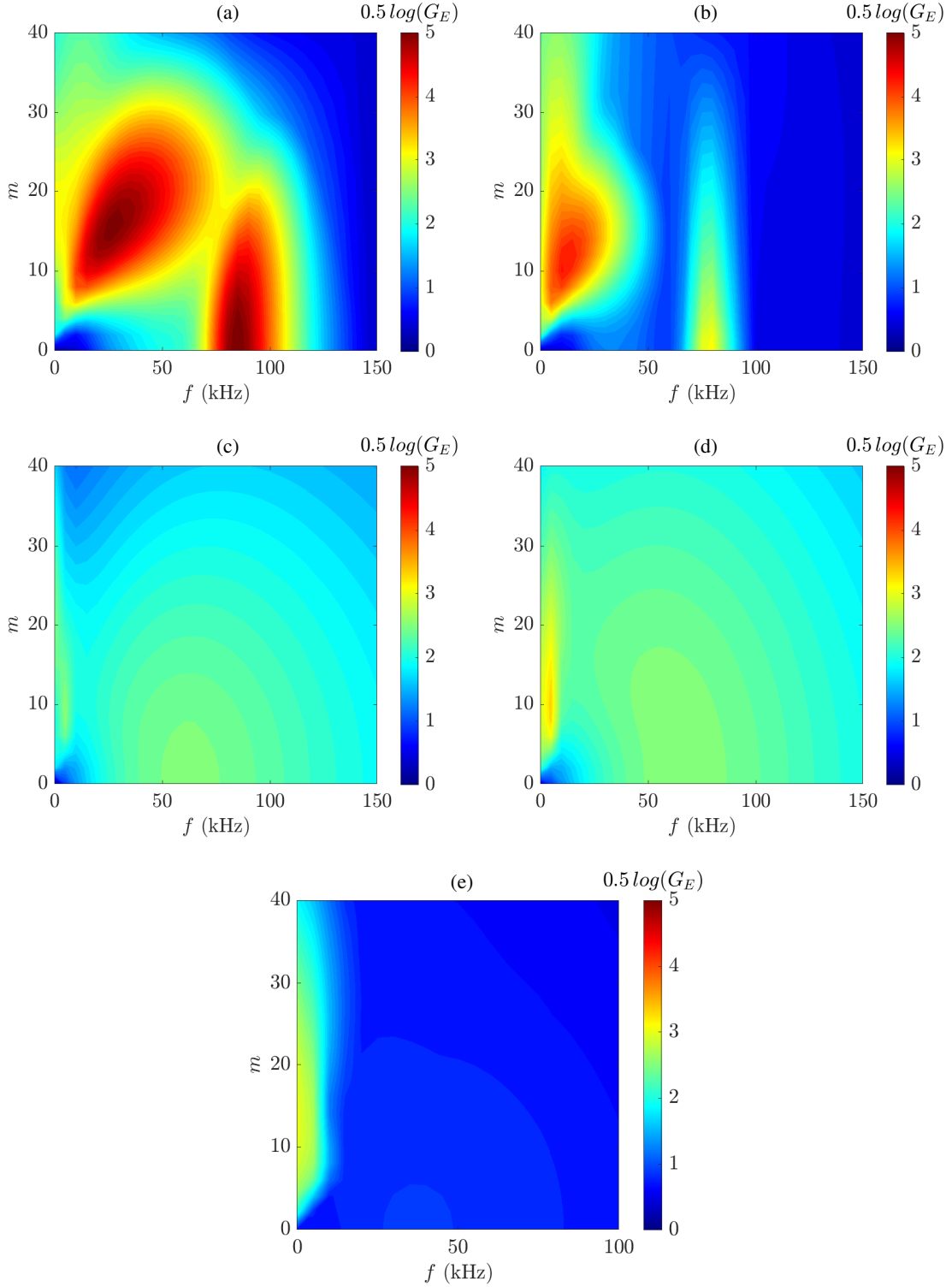


Fig. 14 Contours of logarithmic optimal disturbance energy gain as calculated with HLNSE for the (a) sharp 4 caliber, (b) sharp 2 caliber, (c) blunt 4 caliber, (d) blunt 2 caliber ogives, and (e) hemispherical forebodies.

for the hemispherical forebody models do not display unstable planar or oblique modes, which is in agreement with the wall-pressure disturbance measurements that do not show any distinct peaks in the power spectra. Nonmodal analysis

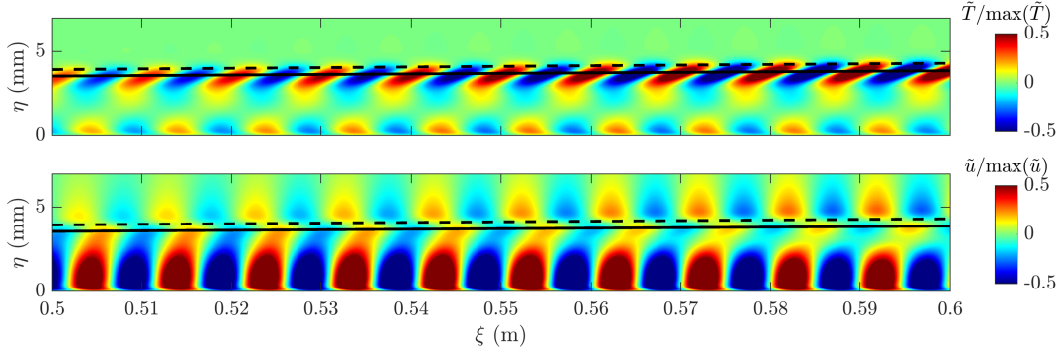


Fig. 15 Contours of flow perturbations for the sharp 4 caliber ogive forebody associated with optimal planar disturbance growth at $f = 85$ kHz. The continuous black line represents the boundary-layer edge, while the dashed black line represents the entropy-layer edge.

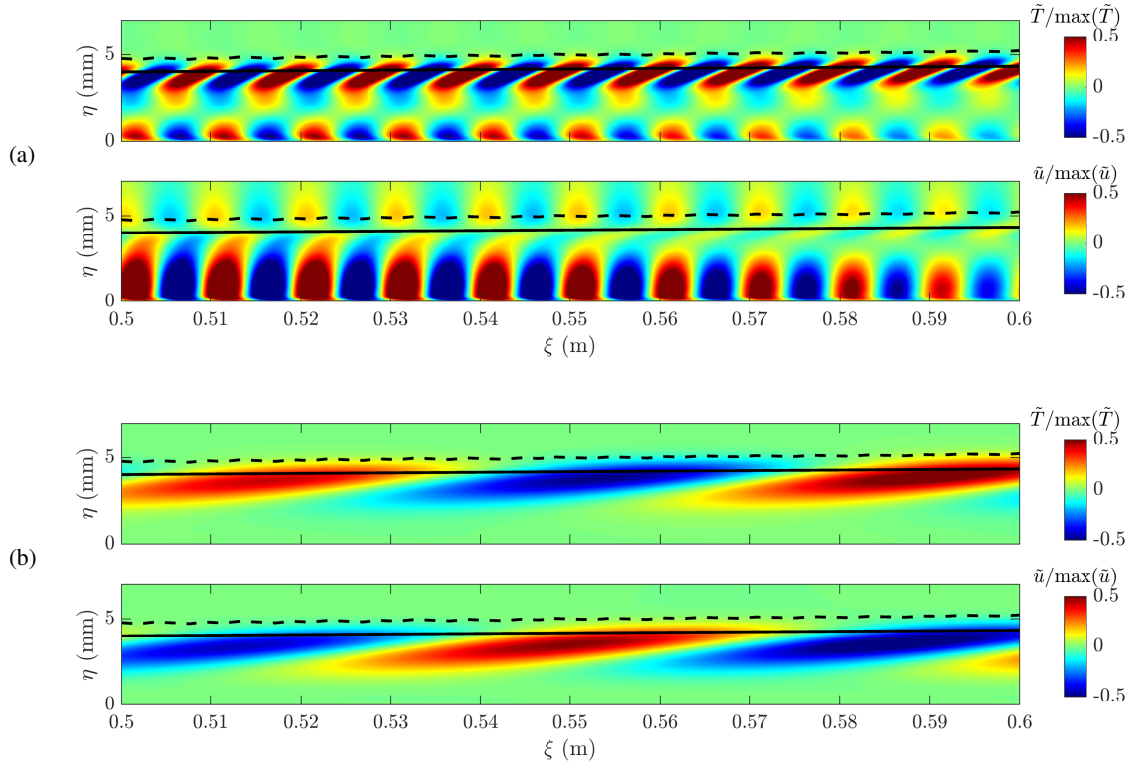


Fig. 16 Contours of flow perturbations for the sharp 2 caliber ogive forebody associated with optimal disturbance growth for (a) planar waves at $f = 80$ kHz and (b) oblique waves at $f = 10$ kHz, $m = 10$. The continuous black line represents the boundary-layer edge, while the dashed black line represents the entropy-layer edge.

using the harmonic linearized Navier-Stokes equations shows higher nonmodal amplification for oblique first-mode disturbances in the sharper tips, as well as a shift in dominance from the planar second mode to the oblique first mode from the 4 to 2 caliber sharp ogive forebodies. For blunter forebodies, nonmodal analysis identifies broadband disturbances, which is in agreement with experiments. Contours of flow perturbations show the experimentally observed structures and the switch from higher frequency second mode disturbances identified as rope-like structures to lower frequency first mode disturbances that are found to correspond to the elongated structures, as well as the switch from

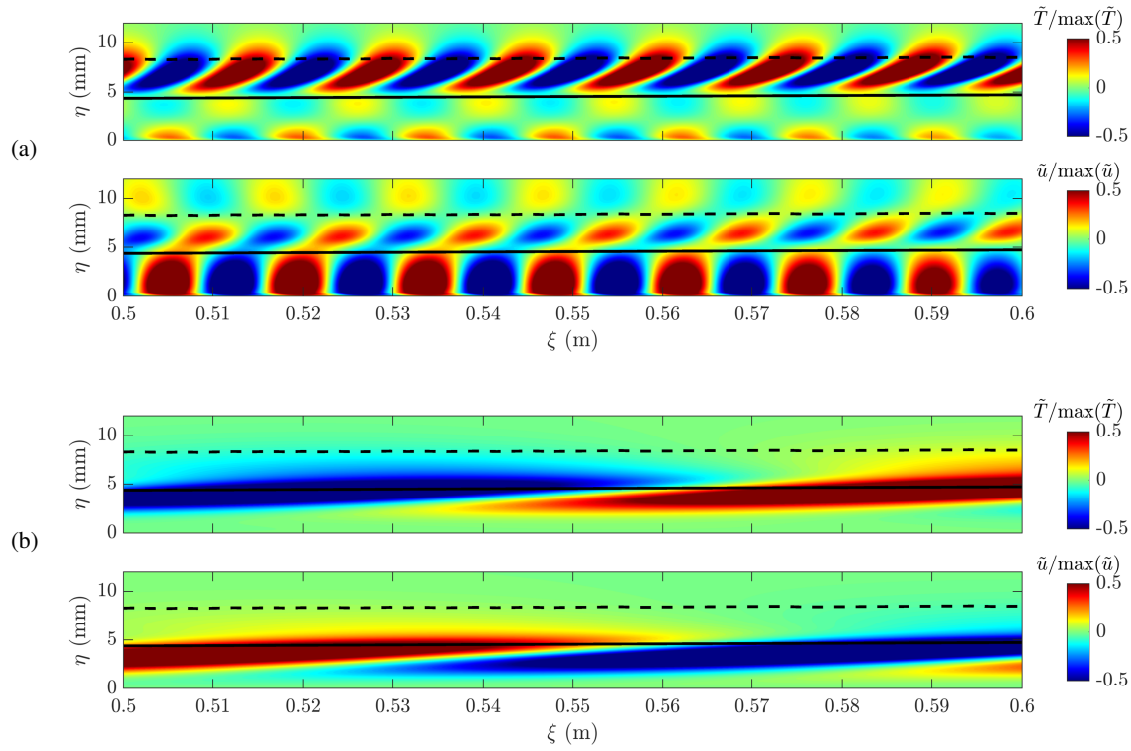


Fig. 17 Contours of flow perturbations for the blunt (1/4R) 4 caliber ogive forebody associated with optimal disturbance growth for (a) planar waves at $f = 60$ kHz and (b) oblique waves at $f = 5$ kHz, $m = 10$. The continuous black line represents the boundary-layer edge, while the dashed black line represents the entropy-layer edge.

boundary-layer to entropy-layer disturbances identified as wisp-like structures for increased bluntness.

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