

Uncertainty Quantification of CFD Model Assumptions Against Sonic Boom Noise Prediction of a Commercial Supersonic Transport

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This paper presents the results of uncertainty modeling of sonic boom noise generation from commercial supersonic transport considering the Spalart-Allmaras (SA) turbulence modeling parameters as well as Mach number, angle of attack and altitude. Sample generation and analysis for this uncertainty model was performed by UQPCE, which is a software package developed at the NASA Langley Research Center. To build the uncertainty model, 42 cases of sonic boom noise calculation were performed. Computation of the ground noise can be briefly summarized in two steps. First, the near field pressure waveforms are sampled from CFD calculation using the NASA Langley's FUN3D solver. Second, this information is passed to an atmospheric propagation code, sBOOM, which solves an augmented Burger's equation and simulates how the near field waveforms will change while passing through the atmosphere. The ground signature is further processed to obtain the perceived loudness, PLdB. Having a high spatial resolution near the shock wave in the CFD calculation is critical in sonic boom noise prediction. Because the variation in the input parameters for the current uncertainty quantification (UQ) study is likely to lead to change in shock location, angle and strength, the grid adaptation for shock capturing is independently applied for each condition. The final mesh used in the CFD calculation consists of approximately 420 million cells. The pressure signatures are sampled at three, four and five body lengths away from the aircraft to make sure the three dimensional effects around the aircraft are resolved. The results of the UQ analysis shows that within the three aleatory variables, the angle of attack had the most impact against ground noise, followed by the altitude and the Mach number. Between the two SA model parameters, the Kármán constant (κ) was significantly more important than the turbulent Prandtl number (σ), but these two parameters were only marginally significant in the overall prediction variance in ground noise. The UQ procedure explained in this paper can be widely applied to other model parameters.

Nomenclature

CST	= Commercial Supersonic Technology	RANS	= Reynolds-averaged Navier-Stokes
ECS	= Environmental Control System	T_t	= Total Temperature
H	= Altitude	T_∞	= Freestream Static Temperature
L	= Aircraft Length	UQ	= Uncertainty Quantification
M	= Mach Number	α	= Angle of Attack
PLdB	= Perceived Loudness	β	= $\sqrt{M^2 - 1}$
P_s	= Static Pressure	κ	= Kármán constant
P_t	= Total Pressure	σ	= Turbulent Prandtl number
R	= Offset of the sensor away from the aircraft		

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I. Introduction

THE capability of predicting sonic boom noise is essential to the process of designing and certifying supersonic aircraft. For this reason, the effects of uncertainty in the flight condition, model parameters, boundary conditions, etc. against the ground noise prediction has been an active field of research. Understanding the importance of each input parameter in a quantitative manner will not only allow the research and development work to concentrate on aspects that has the most impact against the end result, but also meaningful in designing a robust certification procedure.

Previous work by West and Phillips [1] showed the applicability and significant computational savings of the multi fidelity method for efficient and accurate uncertainty quantification using commercial supersonic transport as one of their case studies. Their high-fidelity model considered turbulence model closure coefficients (σ and κ) for the Spalart-Allmaras (SA) model as UQ inputs. While this study considered M and α as near field aleatory parameters, the effect of altitude, along with other atmospheric parameters, were taken into account in the atmospheric propagation part of their study. The results showed that within the near field parameters, α had the strongest effect against PLdB values and while some impact of σ was observed, κ did not have a meaningful influence against the ground noise.

Phillips, Heath and Schmidt [2] studied the potential ground noise variations due to propulsion uncertainties, using a low-boom concept vehicle. Results from this research indicated that including the propulsion system in system-level uncertainty analysis can significantly increase the estimated confidence intervals for ground noise. The near field aleatory input parameters studied in this work were α , M , H and T_∞ . Although the sensitivity against PLdB values showed dependency to the atmosphere selection (Standard [3, 4] vs Green Bay [5]), it was found that T_∞ had the least impact.

Schaefer et al.[6] performed an UQ study to identify the sensitivity of the SA turbulence model in RANS codes due to uncertainty in the values of model coefficients, namely σ , κ , c_{v1} , c_{w3} , c_{t1} , c_{t2} , c_{b1} , c_{b2} and c_{w2} , following the notation described in section II.B. While their work is not directly targeting supersonic applications, they presented expressions of three of the SA model parameters by σ and one additional model parameter by κ . Incorporating these relations may potentially show difference compared to the SA model work by West and Phillips [1].

The present paper contains the results of uncertainty modeling of sonic boom noise generation from commercial supersonic transport considering the SA turbulence modeling parameters (σ and κ), along with M , α and H . Some of the noticeable differences compared to previous studies aside from using an updated model geometry are: 1) The creation of the run-matrix that consists of the above parameters and the analysis of the results are performed by UQPCE, which is briefly described in section III. 2) As in the case of Phillips et al.[2], all 42 runs were computed with an independently grid-adapted, fully-turbulent calculation using the NASA Langley's FUN3D solver. This method is similar to the highest level of fidelity that West and Phillips [1] presented but the computational difficulty was mitigated through customized scripts for FUN3D to allow precise control over the computational resource being requested during the grid-adaptation procedure. 3) The expressions of the SA model parameters presented by Schaefer et al. [6] are being incorporated to examine its impact against ground noise calculation.

The distribution and ranges of uncertain input parameter used in this work are kept similar to previous studies for comparison purpose and the UQ procedure explained in this paper can be widely applied to other parameters. The next two sections will discuss the method used in the ground noise computation and how the computed ground noises are analyzed.

II. Computational Setup

A. Geometry and Boundary Conditions

Figure 1 shows the schematic of the computational domain used in the CFD calculation. At the *Freestream* (upstream) boundary, Mach number and static reference conditions are specified. Riemann invariant condition was specified at the *Near Field* boundary and supersonic extrapolation was applied at the downstream, *Extrapolate* boundary. Aircraft surfaces were assigned a viscous no-slip condition. Figure 2 shows the location of where the aircraft inlet and outlet boundary conditions are specified and Table 1 lists the assigned quantity. For the current UQ study, the properties listed in Table 1 remains constant for all input conditions.

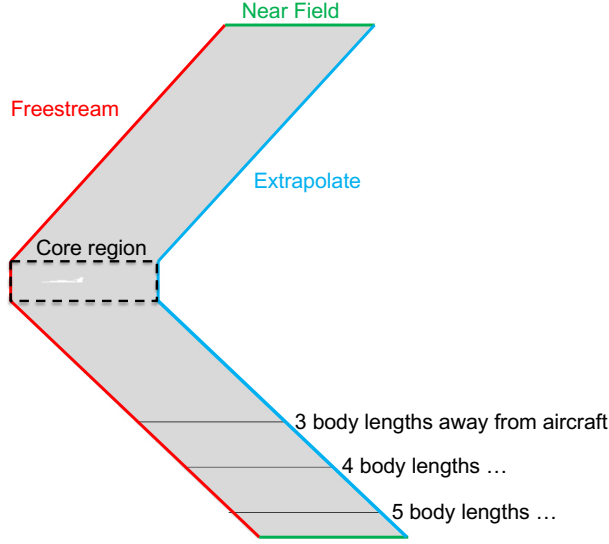


Fig. 1 Computational domain

Table 1 Aircraft inlets and outlets boundary conditions

Location	Quantity Specified
Engine inlet (fan face)	Mass flowrate
Nozzle	P_t and T_t
Engine bay cooling exhausts	P_t and T_t
ECS inlets	Mach number
ECS exhaust	P_t and T_t
Engine bay vents	P_s

The computational domain is initially limited to the cylindrical core region as denoted in Fig.1. Adjoint-based grid adaptation [7] is applied to the core region to fully resolve the shock, expansion, and shear flow characteristics from the vehicle. Based on a convergence study at the mean condition, the grid adaptation is performed three times. This shock-adapted core mesh is extended by the *inflate* [8, 9] tool to sample the pressure signatures sufficiently away from the aircraft so that the three dimensional effects are resolved. The pressure sampling is performed at three, four and five body lengths away from the aircraft to make sure the UQ results are consistent. It should be noted that the extended mesh created by the refine package is Mach aligned and angle of attack adjusted according to the UQ input parameter. Figure 3 shows an example pressure field obtained from the CFD calculation. The near field pressure signatures sampled from this pressure field are shown in Fig.4 for different sampling locations. Building the UQ models using the pressure signatures extracted from three and four body lengths away from the aircraft provided qualitatively identical results. Because the sampling location is closest to the previous work by Phillips et al. [2], the UQ results presented in this paper were obtained from pressure signatures extracted from three body lengths away from the aircraft.

B. Spalart–Allmaras (SA) Turbulence Model

Two parameters related to Spalart–Allmaras turbulence model are modified as part of the current UQ study, namely the Kármán constant (κ) and the turbulent Prandtl number (σ) that governs diffusion. The full description of the model can be found in the publication by Spalart and Allmaras [10] and the notation used in this paper follows the NASA Langley Research Center Turbulence Modeling Resource (TMR) webpage [11], which is the same notation used by Schaefer et al.[6].

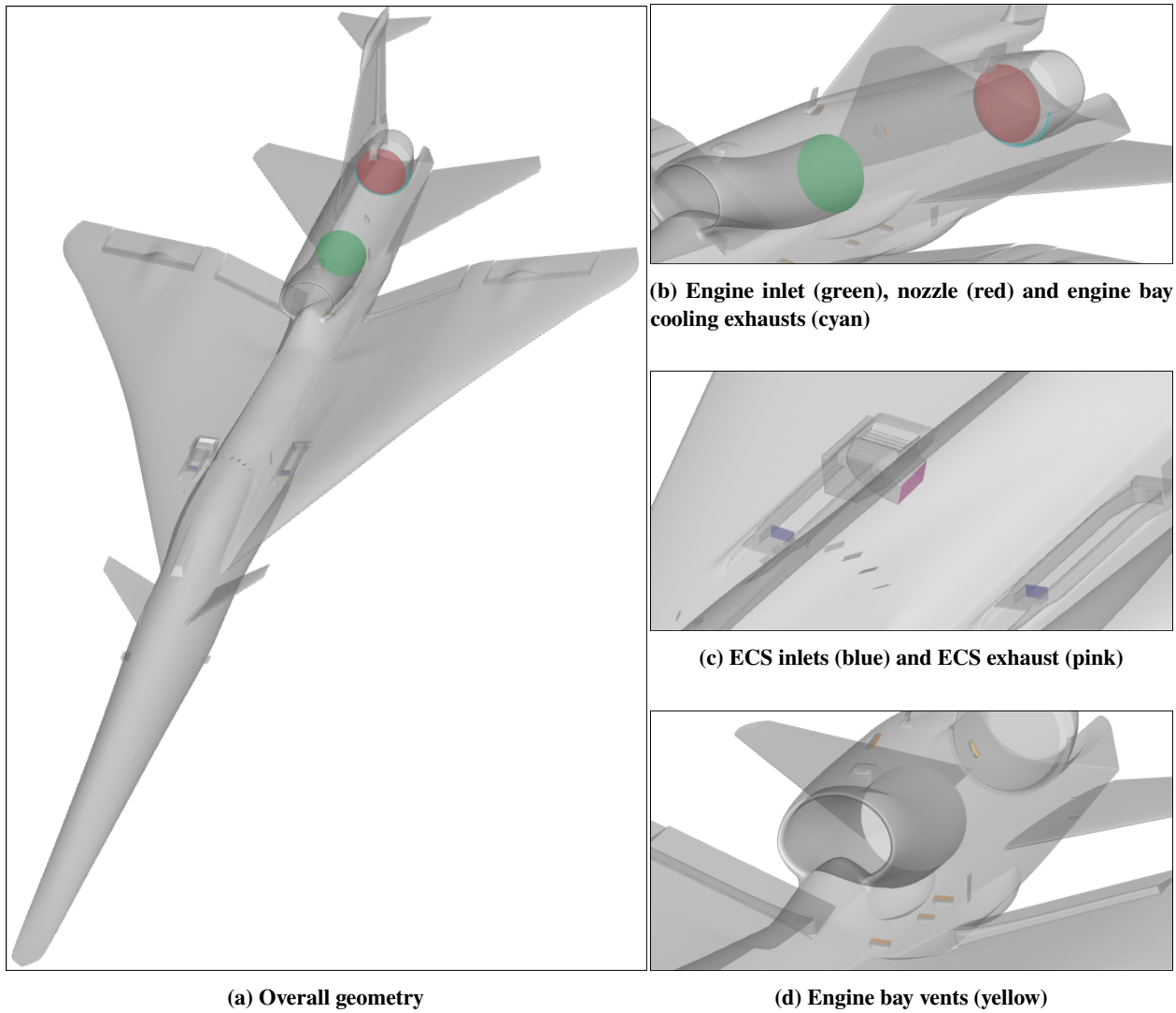


Fig. 2 Location of aircraft inlet and outlet boundary conditions

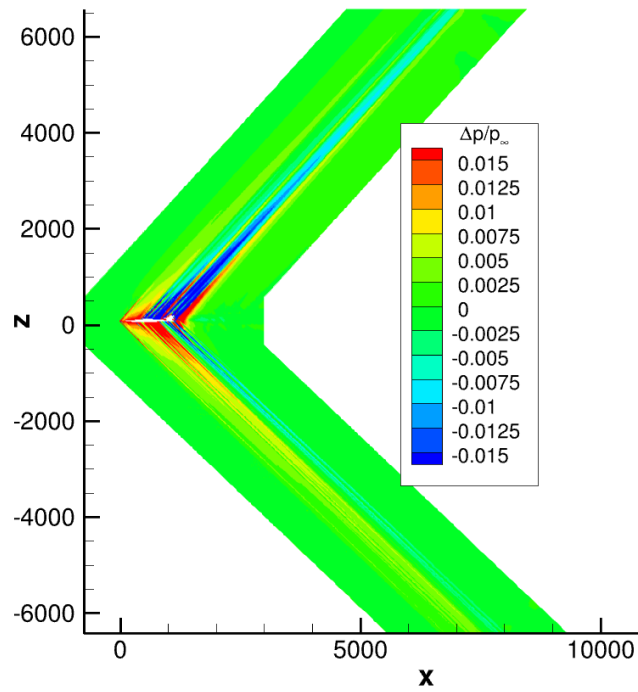


Fig. 3 Example pressure contour: $(M, \alpha, H, \kappa, \sigma) = (1.3990, 2.0621, 53705, 0.3836, 0.6563)$

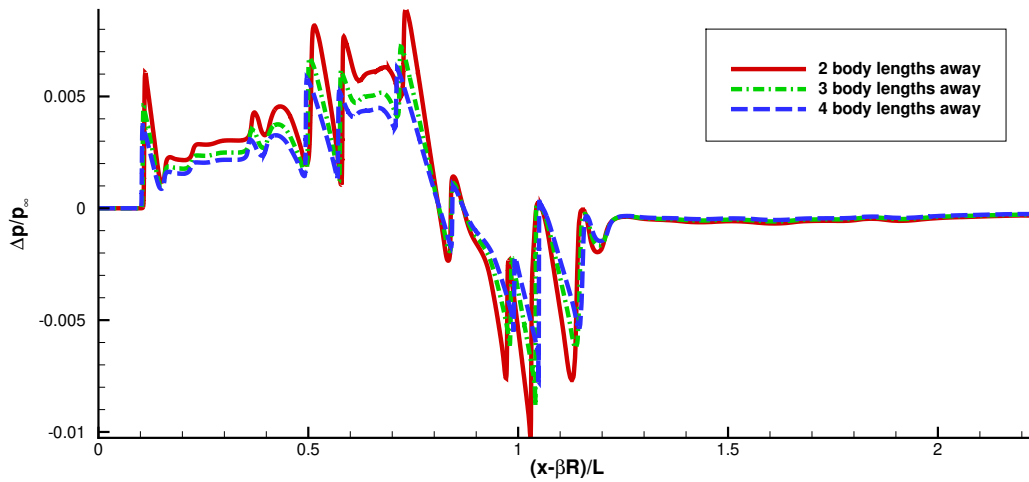


Fig. 4 Near field pressure signature sampled at different locations (data obtained from case shown in Fig.3)

Similar to the previous work by Schaefer et al.[6] , the SA coefficients listed below are calculated using the values of σ and κ :

$$c_{v1} = 7.1 + 37.5(\kappa - 0.41) \quad (1)$$

$$c_{w2} = -1.5672\sigma^3 + 4.1858\sigma^2 - 4.4125\sigma + 1.8478 \quad (2)$$

$$c_{b1} = -0.0291\sigma^3 + 0.0909\sigma^2 - 0.1095\sigma + 0.1768 \quad (3)$$

$$c_{b2} = -0.1828\sigma^3 + 0.5309\sigma^2 - 0.2943\sigma + 0.6357 \quad (4)$$

$$c_{t3} = 0.0 \quad (5)$$

It should be noted that the work by Schaefer et al.[6] had c_{w3} as one of the UQ parameters in their Reduced Dimensionality Analyses (RDA) model while in the current work, c_{w3} is fixed at its standard value of 2.0 because this parameter did not present a strong effect in their UQ work. FUN3D has the capability of adjusting these model parameters through its input file.

C. Pressure Signature Propagation to Ground

The pressure signatures obtained from the CFD calculation is passed to an atmospheric propagation code, sBOOM[12], which solves an augmented Burger’s equation and simulates how the near field waveforms will change while passing through the atmosphere. Among the input parameters, Mach number and altitude varies with the UQ parameters. The quantity of interest in the uncertainty modeling was perceived loudness, PLdB, so that the end result will coincide with one of the CST project’s technical challenges defined as: *Tools and technologies enabling the design of supersonic aircraft that reduce sonic boom noise to 80 PLdB validated as ready for application in a flight demonstrator* [13]. The sampling frequency of sBOOM was set to 200 kHz for all cases.

III. Uncertainty Modeling

All of the uncertainty modeling and analysis contained in this research was performed with the Aeronautics Systems Analysis Branch’s (ASAB) in-house uncertainty code, Uncertainty Quantification with Polynomial Chaos Expansion, UQPCE [14]. UQPCE is an open source, python based research code for use in parametric, non-deterministic computational analysis and design. UQPCE utilizes a non-intrusive polynomial chaos expansion surrogate modeling technique to efficiently estimate uncertainties for computational analyses. The software allows the user to perform an automated uncertainty analysis for any given computational code without requiring modification to the source. UQPCE estimates sensitivities, confidence intervals and other model statistics which can be useful in the conceptual design and analysis of flight vehicles. This software was developed for the ASAB within the Systems Analysis and Concepts Directorate (SACD) at the NASA Langley to study potential impacts of uncertainties on the prediction of ground noise generated from commercial supersonic aircraft concepts.

IV. Results

Table 2 shows the list of uncertain input variables studied in this research and the parameters provided to UQPCE for sample set creation. The mean values of the aleatory variables are close to the cruise condition of the commercial

Table 2 List of uncertain input variables and resulting Sobol indices

	Variable	Mean	Distribution	Range	Sobol index
	Mach number	1.4	normal	Std.Dev.=0.0021	0.059
Aleatory	Angle of attack(deg.)	2.05	normal	Std.Dev.=0.1025	0.789
	Altitude(ft.)	53800	normal	Std.Dev.=269	0.129
Epistemic	κ	0.4	uniform	[0.38,0.42]	0.021
	σ	0.8	uniform	[0.60,1.00]	0.002

supersonic transport and the distribution and range are derived from previous work by Phillips et al.[2]. The mean values of the epistemic variables (the SA model parameters) are widely used default settings and the ranges are set to

match the work by Schaefer et al.[6] for comparison. Different from the aleatory variables where a normal distribution was specified, the epistemic variables are given a uniform distribution.

A histogram of the computed loudness values are shown in Fig.5. The loudness ranged between 79.36 PLdB and

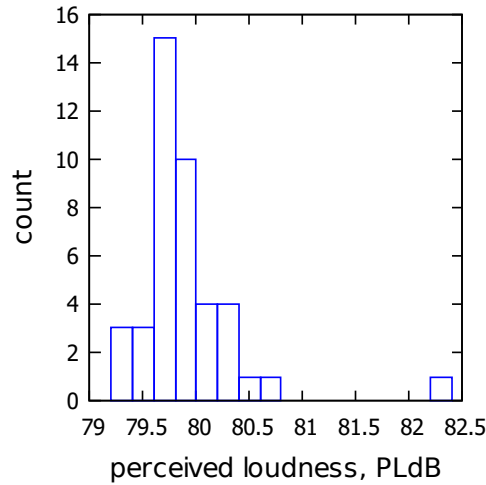


Fig. 5 Distribution of computed loudness with bin-width of 0.2 PLdB

82.39 PLdB with the average of 79.91 PLdB. It can be seen from Fig.5 that the maximum loudness value is rather anomalous because the difference against the second loudest condition is more than 1.7 PLdB. The angle of attack of this condition is considerably large compared to all other conditions and it is likely that this led to the high PLdB value. It should be noted that the actual aircraft is capable of changing its geometry and engine conditions depending on the flight situation but these settings are fixed for the current calculation.

Figures 6 and 7 shows the relationship between the uncertain input parameters and the resulting loudness values for the aleatory and the epistemic variables, respectively.

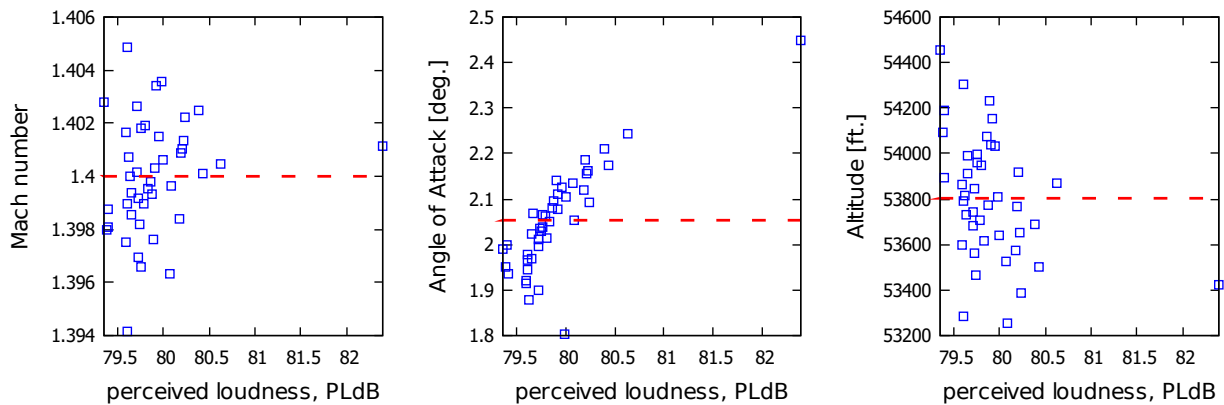


Fig. 6 Relationship between aleatory variables and loudness levels: The red line indicates the mean.

The trend of increasing angle of attack resulting in a larger loudness values can be observed. It should be taken into account that Fig.6 and 7 are showing one variable vs response for a six dimensional space. In order to retrieve deeper insight, the computed loudness values were provided to UQPCE.

The far-right column of Table 2 lists the Sobol indices of each variable. The result of the UQ analysis shows that within the three aleatory variables, the angle of attack had the most impact against ground noise, followed by the altitude and the Mach number. This result is qualitatively consistent with previous work by West and Phillips [1] where the geometry and aircraft inlets and outlets conditions were fixed. It should be noted that later work by Phillips

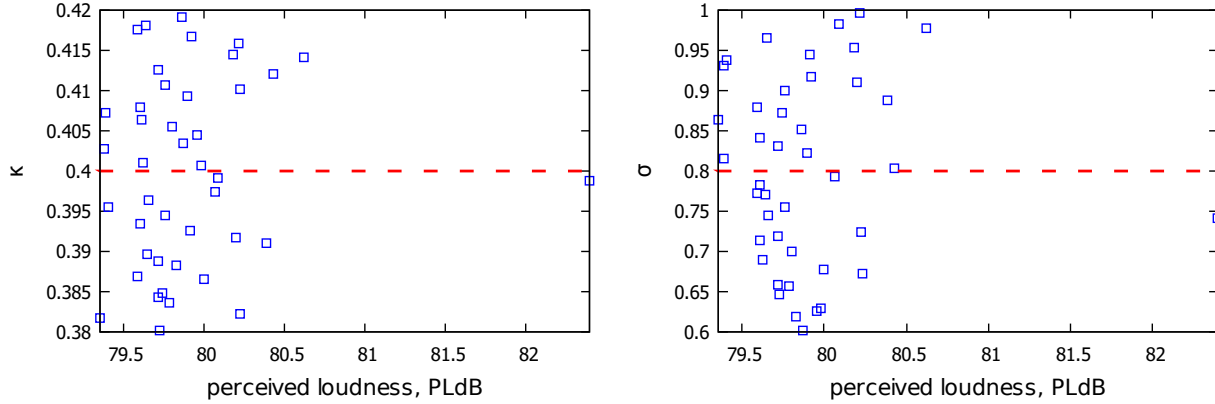


Fig. 7 Relationship between epistemic variables and loudness levels: The red line indicates the mean.

et al. [2] showed that by taking into account of the propulsion uncertainties, including the variable nozzle geometry, made the uncertainties associated to M , α and H to be more comparable. Between the two SA model parameters, κ was significantly more important than σ but these two parameters were only marginally significant in the overall prediction variance in ground noise. This result differs from previous work by West and Phillips [1] where the κ did not show meaningful contribution against ground noise. This is likely to be the result of incorporating the SA parameter expressions by Schaefer et al. [6].

Lastly, we will examine the near field pressure signatures of selected cases that led to the spread in perceived loudness levels. Table 3 lists the set of UQ parameters that provided the minimum ground noise (id=24), median ground noise excluding the case resulted in the largest ground noise (id=27), maximum ground noise (id=39) and the 2nd largest ground noise (id=10). The condition that led to the 2nd largest ground noise is included in the comparison because the maximum ground noise was given by a condition with a considerably large angle of attack compared to other cases.

Table 3 Selected cases from uncertainty model creation

id	Mach number	Angle of attack (deg.)	Altitude (ft.)	κ	σ	Loudness (PLdB)
24	1.4028	1.9893	54455	0.3818	0.8635	79.361
27	1.3990	2.0621	53705	0.3836	0.6563	79.786
10	1.4004	2.2422	53869	0.4141	0.9782	80.624
39	1.4011	2.4490	53422	0.3988	0.7418	82.392

Figure 8 compares the near field pressure signatures of the four cases in Table 3. It can be seen from Fig.8 that despite the resulting perceived loudness levels having a spread of 3 PLdB, the difference in the intermediate pressure signatures are marginal. The portion of the signatures that are different ($0.6 < (x - \beta R)/L < 0.8$) is driving the ground noise differences and the location of these shock structures is traceable back to the wing [15]. This result indicates that the ground noise prediction is highly sensitive to the accuracy of the CFD calculation and for this reason, choice of model parameters, along with other theoretical/numerical settings must be examined in detail.

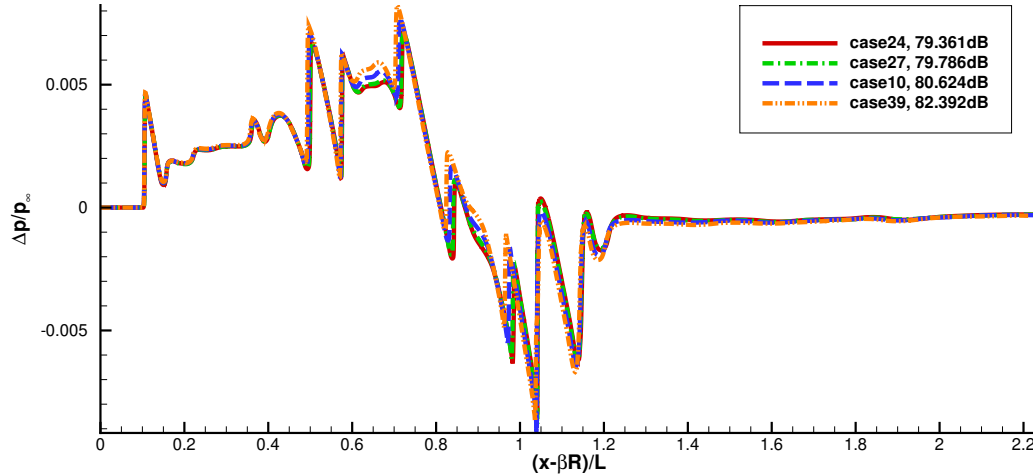


Fig. 8 Comparison of near field pressure signatures obtained by UQ parameter sets that provided the largest, 2nd largest, medium and smallest level of ground noise (PLdB)

V. Summary

Uncertainty modeling of sonic boom noise generation from commercial supersonic transport considering the Spalart-Allmaras (SA) turbulence modeling parameters as well as Mach number, angle of attack and altitude was completed. The result of the UQ analysis shows that within the three aleatory variables, the angle of attack had the most impact against ground noise, followed by the altitude and the Mach number. Between the two SA model parameters, the Kármán constant (κ) was significantly more important than the turbulent Prandtl number (σ), but these two parameters were only marginally significant in the overall prediction variance in ground noise. While the importance of each parameter may be affected when the full vehicle trim is considered, the overall procedure, especially utilizing UQPCE for sample generation and analysis, can be widely applied to other parameters to assess their effect against sonic boom noise generation. Comparison of near field pressure signatures indicated that the ground noise prediction is highly sensitive to the accuracy of the CFD calculation and for this reason, choice of model parameters, along with other theoretical/numerical settings must be further examined.

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