Path Planning: Differential Dynamic Programming and Model Predictive Path Integral Control on VTOL Aircraft

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AIAA SciTech Forum and Exposition 2022, San Diego, CA

Outline

- Motivation: Challenge of trajectory planning for urban air mobility (UAM) vehicles
- Trajectory Planning
  - Problem Statement
  - Differential Dynamic Programming (DDP) theory
  - Model Predictive Path Integral Control (MPPI) theory
- Experiments:
  - Sample Results with interesting observations
- Conclusions and Ongoing Work

Lift + Cruise

First application of planning algorithms, used extensively in robotics, to UAM class vehicles

Motivation: Challenge of Urban Air Mobility VTOL vehicles

- Future urban air mobility (UAM) necessitates high levels of automation (due to cost, airspace congestion, pilot experience, novel aircraft etc.)

- Trajectory replanning is required capability to enable UAM (obstacles, traffic, aircraft contingencies, weather, change of mission etc.)

- Trajectory replanning for UAM class vehicles extremely challenging
  - Highly complex nonlinear system: large number of propulsors, novel configurations, aero-propulsive interactions
  - Three phases of flight: hover, transition, and cruise
  - Dynamic complexity suggests large model uncertainty
  - Overactuated vehicles: multiple control solutions
  - Example aircraft (Lift + Cruise): 8 lifting rotors, 4 aero surfaces, 1 pusher propeller

Optimal Trajectory Problem

Problem: Find optimal control (and states) to achieve a desired end state while minimizing cost

State Vector

$$\mathbf{x}_t \in \mathbb{R}^n$$

Discrete system nonlinear dynamics

$$\mathbf{x}_{t+1} = \mathbf{f}(\mathbf{x}_t, \mathbf{u}_t)$$

Control Vector

$$\mathbf{u}_t \in \mathbb{R}^m$$

Cost Function

$$\mathcal{J}(\mathbf{U}) = \sum_{t=1}^{T-1} \mathcal{L}(\mathbf{x}_t, \mathbf{u}_t) + \phi(\mathbf{x}_T)$$

Running Cost

$$\mathcal{L}(\mathbf{x}_t, \mathbf{u}_t)$$

Terminal Cost

$$\phi(\mathbf{x}_T)$$

State Trajectory

$$\mathbf{X} := \{\mathbf{x}_1, \ldots, \mathbf{x}_T\}$$

Control Trajectory

$$\mathbf{U} := \{\mathbf{u}_1, \ldots \mathbf{u}_{T-1}\}$$

Finite Time

$$T \in \mathbb{N}^+$$

Differential Dynamic Programming

DDP: Given nominal trajectory, use linear (or quadratic) approx. of system nonlinear dynamics and quadratic approx. of cost to yield updates to optimal controls that quadratically converge

Value Function (cost-to-go)

Truncated Control Sequence

$$\mathcal{J}_i(\mathbf{x}_i, \mathbf{U}_i) := \sum_{t=i}^{T-1} \mathcal{L}(\mathbf{x}_t, \mathbf{u}_t) + \phi(\mathbf{x}_T) \quad \mathbf{U}_i := \{\mathbf{u}_i, \ldots, \mathbf{u}_{T-1}\}$$

Bellman's Principle of Optimality: find overall optimal control as sequence minimization for each truncated control sequence backwards in time

$$V(\mathbf{x}_i) = \min_{\mathbf{u}_i} \Big[\underbrace{\mathcal{L}(\mathbf{x}_i, \mathbf{u}_i) + V(\mathbf{x}_{i+1})}_{Q(\mathbf{x}_i, \mathbf{u}_i)} \Big]$$

Differential Dynamic Programming

- Given: Initial state x_0, nominal control trajectory $\mathbf{u}_{0:T-1}$
- Repeat until convergence:
  - Forward pass:
    - Forward simulate dynamics from x_0 using $\mathbf{u}_{0:T-1}$ to get state trajectory $\mathbf{x}_{0:T}$
    - Compute derivatives of dynamics $f, f_x, f_u, f_{xx}, f_{xu}, f_{ux}, f_{uu}$ at each time t
    - Compute costs and derivatives $\ell, \ell_x, \ell_u, \ell_{xx}, \ell_{xu}, \ell_{ux}, \ell_{uu}$ at each time t
  - Backward pass:
    - Compute quadratic value function expansion $Q, Q_x, Q_u, Q_{xx}, Q_{xu}, Q_{ux}, Q_{uu}$ at each time t
    - Compute feedforward and feedback gains k and K at each time t
    - Update control $u_t \leftarrow u_t + \alpha k_t + K_t \delta x_t$
      - $\alpha \in [0, 1]$ is a learning rate that is tuned
      - We perform line search on α to heuristically find the best learning rate

Model Predictive Path Integral Control

Stochastic sampling algorithm to generate optimal trajectories

- Iterative algorithm: equivalent to Hamilton-Jacobi-Bellman equation
- Generalized (nonlinear) cost criteria (not just quadratic)
- Parallel processing (GPU) of sampled (noisy) controls and propagates all trajectories simultaneously
- Parallel computation enables real-time trajectory planning
- Control law update exponentially weighted average of sampled controls

Model Predictive Path Integral (cont)

Given

$$\mathbf{x}_{t+1} = \mathbf{f}(\mathbf{x}_t, \mathbf{v}_t)$$

Nonlinear Dynamics

$$\mathbf{v}_t \sim \mathcal{N}(\mathbf{u}_t, \Sigma)$$
$$\Sigma \in \mathbb{R}^{m \times m}$$

Gaussian Controls Distribution

Cost Formulation

$$\mathcal{L}(\mathbf{x}_t, \mathbf{u}_t) := q(\mathbf{x}_t) + \frac{\lambda}{2}\mathbf{u}_t^{\top}\Sigma^{-1}\mathbf{u}_t$$

$$\mathcal{J}(\mathbf{U}) = \sum_{t=1}^{T-1} \mathcal{L}(\mathbf{x}_t, \mathbf{u}_t) + \phi(\mathbf{x}_T)$$

Parallel Computation: Dynamics and Cost

$$\mathbf{V}^{(k)} := \{\mathbf{v}_1^{(k)}, \ldots, \mathbf{v}_{T-1}^{(k)}\}$$

$$\mathcal{J}^{(k)} = \mathcal{J}(\mathbf{V}^{(k)})$$

Normalize

$$\eta = \sum_{k=1}^{K} \exp(-\lambda^{-1}(J^{(k)} - \rho))$$

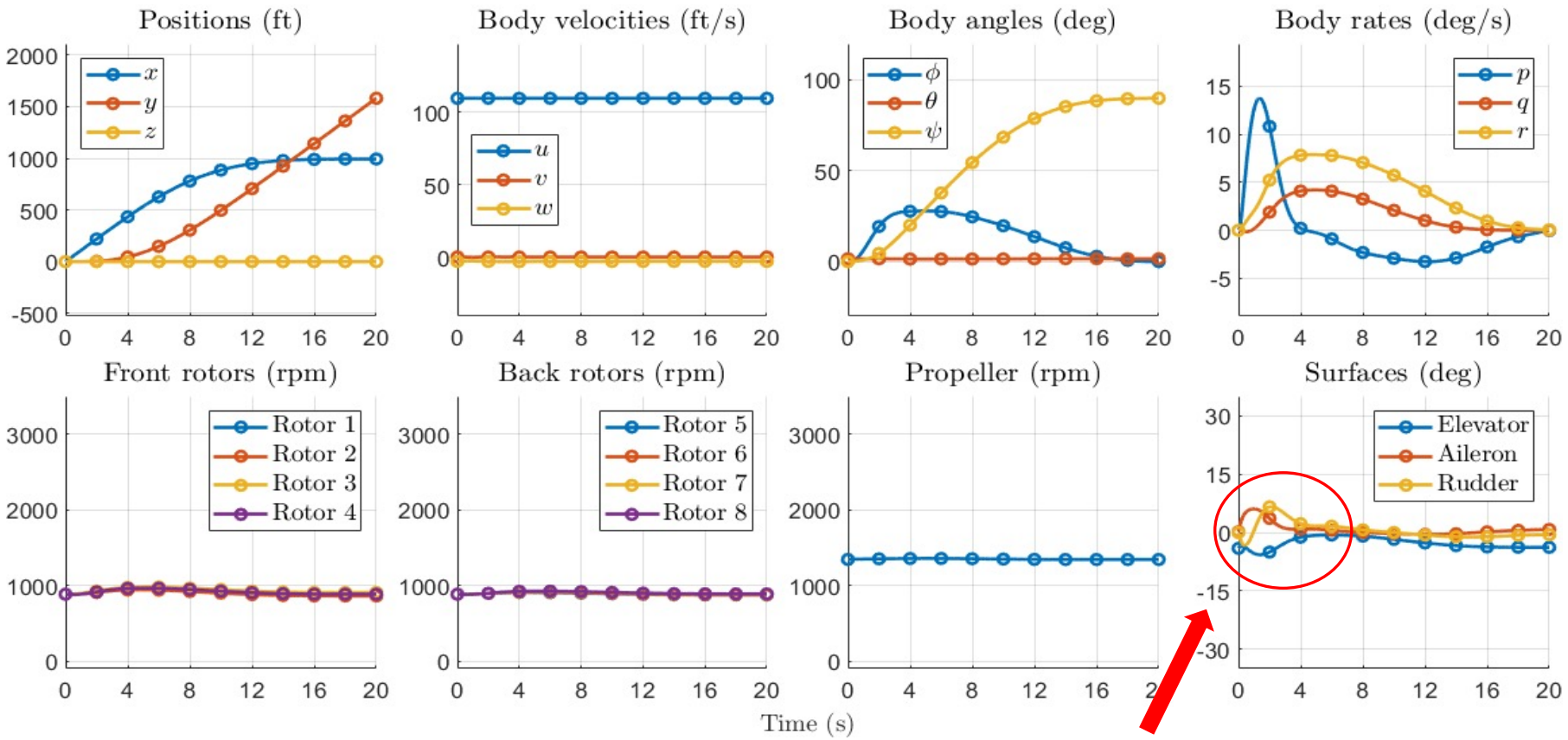
Compute Weights

$$w^{(k)} = \frac{1}{\eta}\exp(-\lambda^{-1}(J^{(k)} - \rho))$$

Compute Optimal Controls

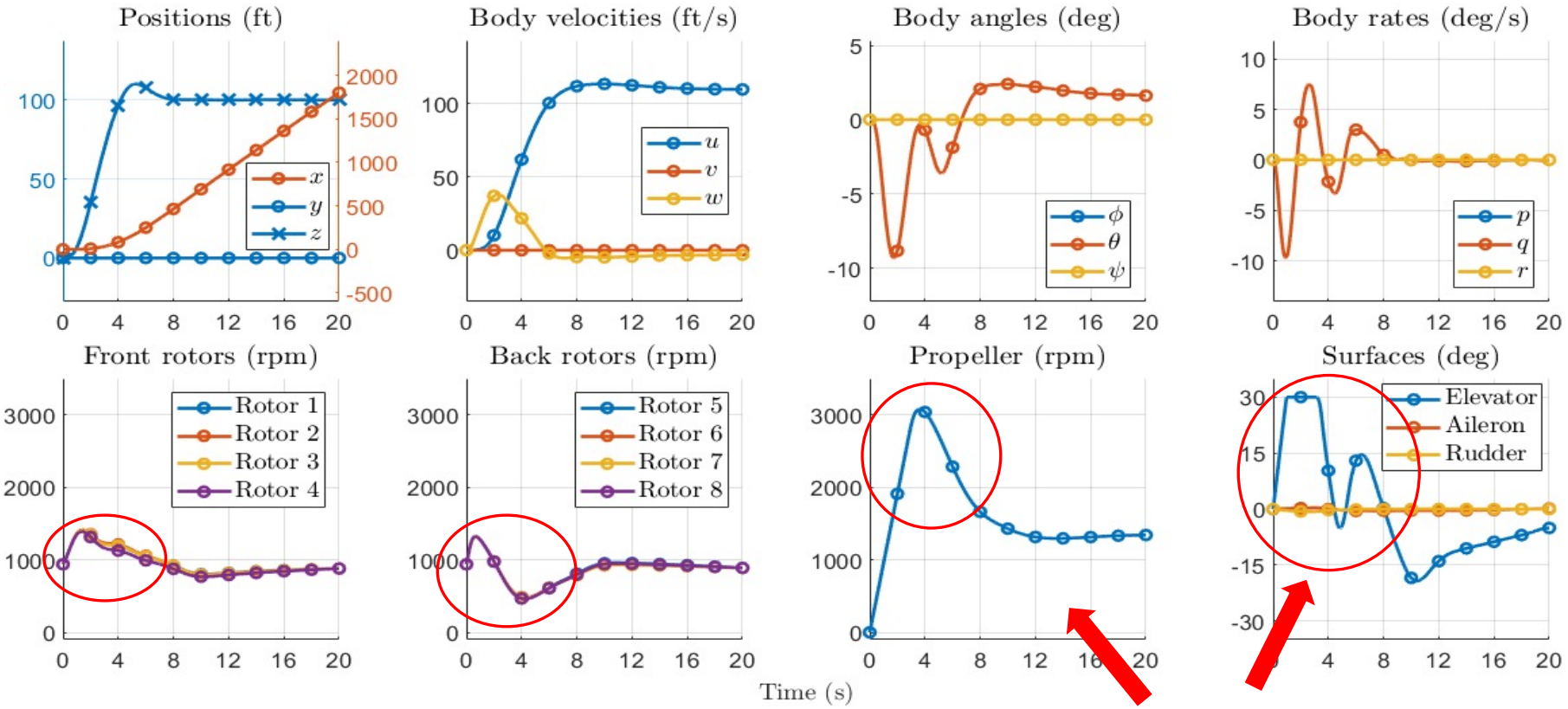
$$\mathbf{u}_t^* = \sum_{k=1}^{K} w^{(k)}\mathbf{v}_t^{(k)}$$

Case Studies: DDP 90° Turn



DDP turn using typical aero surfaces / not propulsors

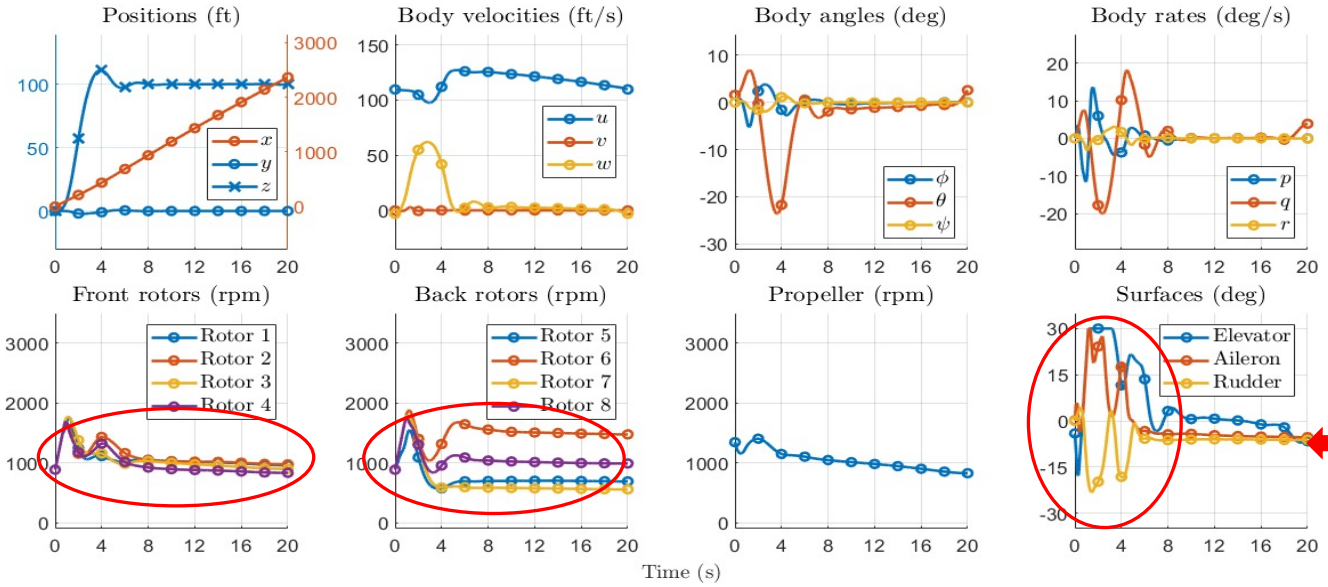
Case Studies: DDP Hover - Transition



Interesting use of differential rotors, pusher and elevator

Case Studies: DDP Transition Altitude Change

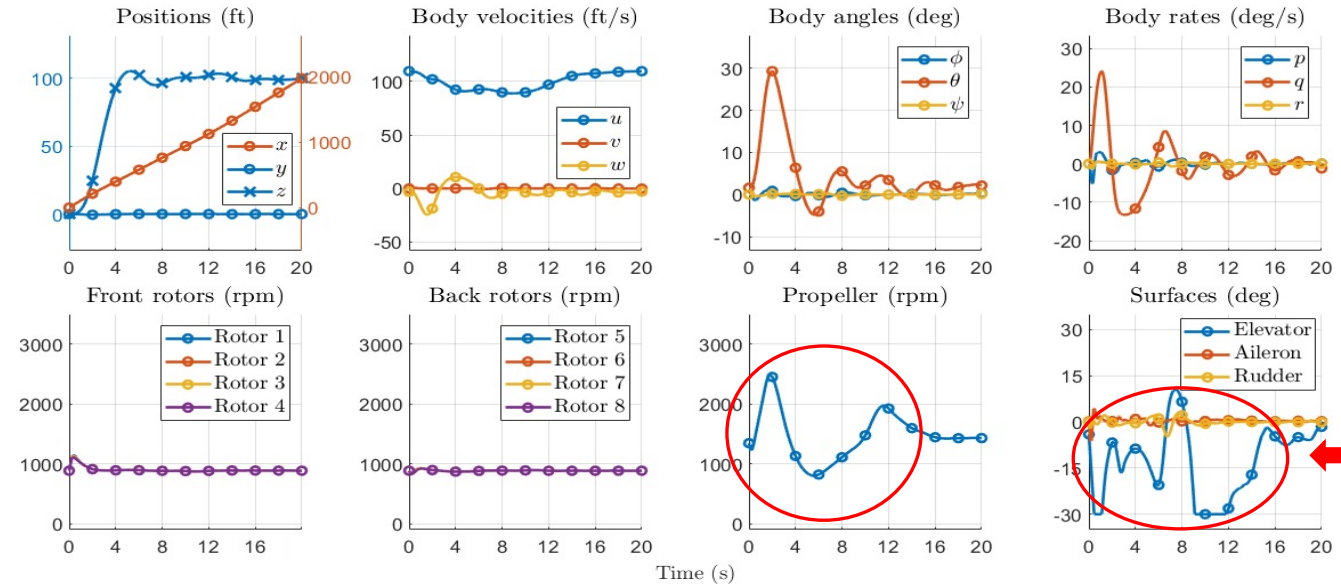
Using Rotors to Ascend



Effects of Overactuated Vehicle

Novel climb: differential rotor and all aero effectors

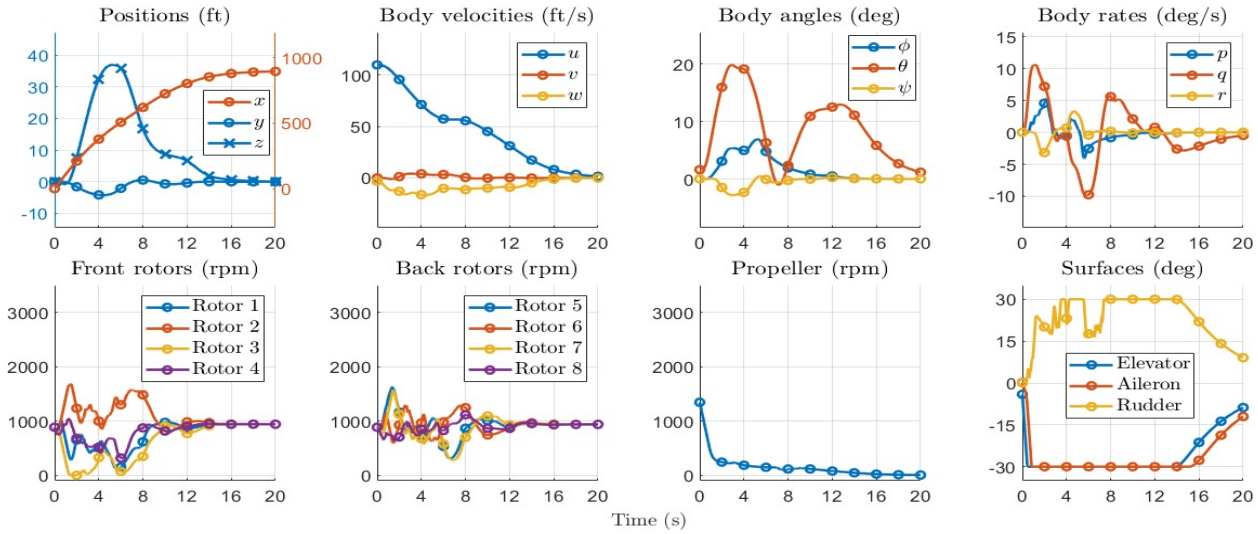
Without Rotors to Ascend



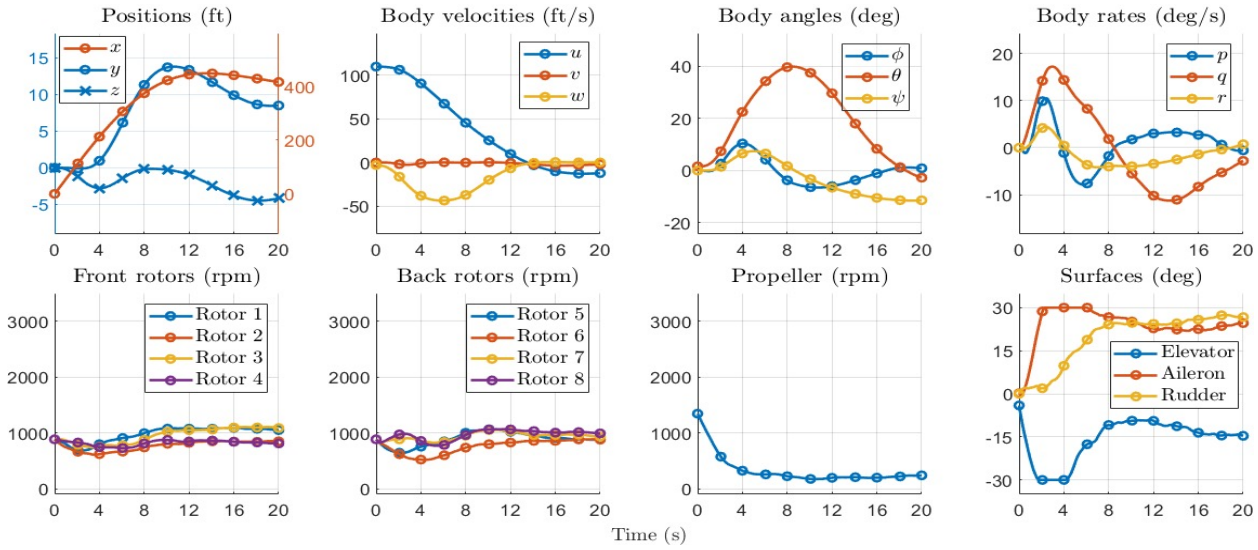
Classic climb: elevator and pusher

Case Studies: Cruise to Hover: DDP & MPPI

DDP



MPPI

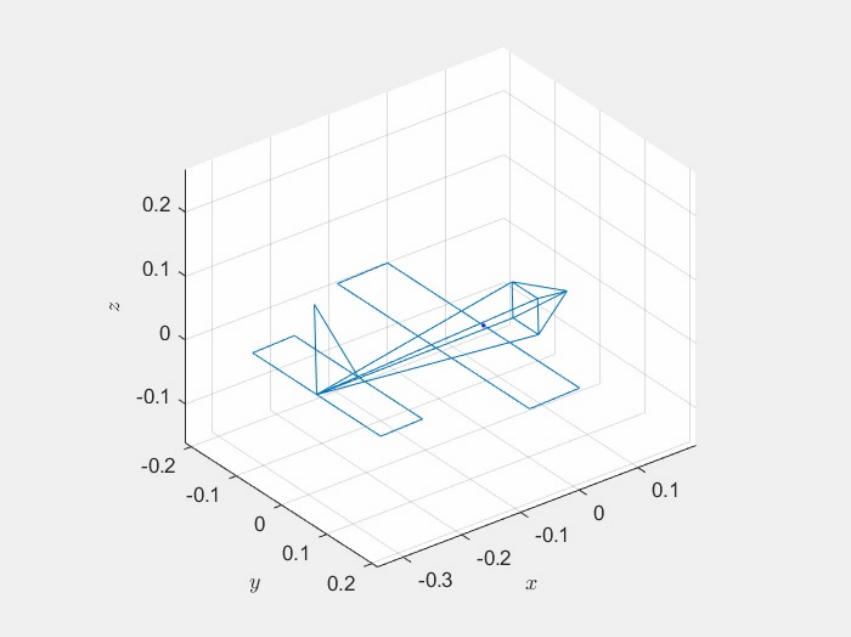


Differences in cost/weightings

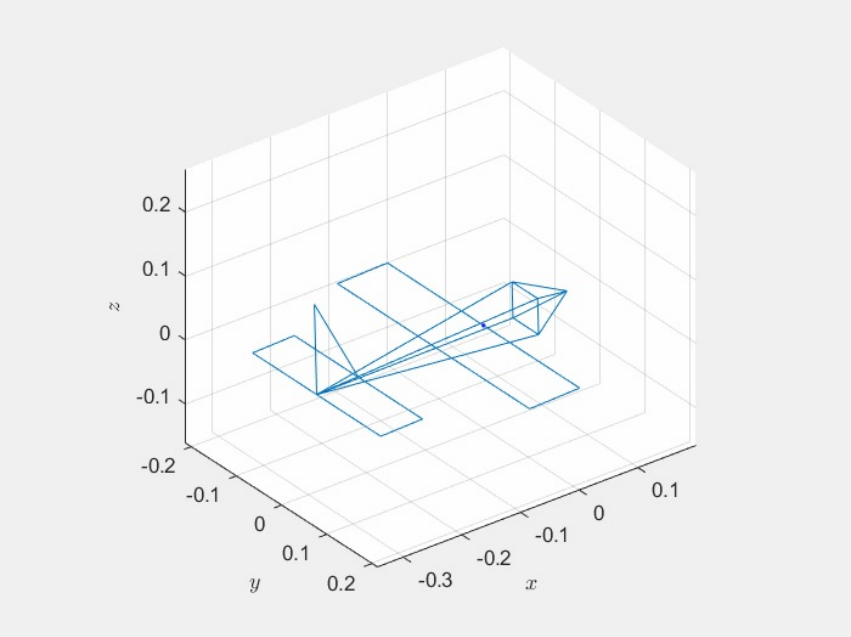
Different use of rotor/propeller & elevator combinations

Case Studies: Cruise to Hover: DDP & MPPI

DDP



MPPI



Conclusions and Ongoing Work

- **Conclusions**
  - First application of planning algorithms, used extensively in robotics, to UAM class vehicles
  - Both DDP and MPPI demonstrated capability to trajectory plan for UAM class vehicles across all flight phases
  - Balancing required between computation time and planning horizon

- **Ongoing / Future Work**
  - Determine minimum vehicle model required to plan (e.g., fewer states, split into longitudinal/lateral models etc..)
  - New parameterized DDP theory developed (paper pending)
  - Enhance research with state constraints and other improvements and additions to these algorithms → allows specification of different mission objectives in trajectory planning and replanning

Questions?

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