ROBUST CISLUNAR TRAJECTORY OPTIMIZATION VIA MIDCOURSE CORRECTION AND OPTICAL NAVIGATION SCHEDULING

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This paper presents a new approach to optimal trajectory design that considers uncertainties in the system, referred to herein as robust trajectory optimization. This approach assumes an existing reference trajectory and optimizes the locations of midcourse correction burns and utilization of onboard navigation sensors to minimize dispersions in $\Delta v$ or final position. Navigation errors, maneuver execution errors, orbit insertion errors, and environmental modeling errors are considered. The application in this paper is cislunar flight with the goal of injecting into a Near-Rectilinear Halo Orbit for rendezvous with a target vehicle. Two complementary optimization problems are proposed. One problem minimizes the total $\Delta v$ dispersion subject to a final position dispersion constraint. The other problem minimizes the final position dispersion subject to a total $\Delta v$ dispersion constraint. The results from each optimization problem are shown for a complete mission profile.

INTRODUCTION

During the trajectory design process, a mission profile is created to satisfy specific mission objectives and constraints while optimizing performance metrics such as propellant mass or mission duration. Once this task is accomplished, the next natural question is whether the derived trajectory is robust to system uncertainties such as navigation errors, maneuver execution errors, environmental modeling errors, and initial condition uncertainties. A common approach for improving robustness is through the use of midcourse correction (MCC) maneuvers, which are designed to drive the spacecraft back to the nominal trajectory. Incorporating MCCs requires an understanding of how closed-loop system performance is affected by both system uncertainties and guidance, navigation, and control (GN&C) algorithms. These considerations are typically outside the scope of traditional trajectory design and optimization.

This paper proposes a non-traditional robust trajectory optimization approach for cislunar missions. The approach involves embedding linear covariance (LinCov) analysis within an optimization problem to determine the MCC locations and utilization of navigation sensors that minimize dispersions in either $\Delta v$ or final position. The LinCov analysis considers integrated closed-loop system uncertainties caused by errors in navigation, maneuver execution, environmental models, and initial conditions.
conditions. Feasibility of the proposed approach has been previously demonstrated for rendezvous
and proximity operations,\textsuperscript{1,2} and LinCov has shown to be a useful tool for many other applications
including atmospheric entry, planetary approach, attitude determination and control, relative pose
estimation, powered lunar descent, and satellite pointing systems.\textsuperscript{3–9}

LinCov relies upon a precomputed reference trajectory, and the cislunar mission profile con-
sidered in this paper is shown in Figure 1. It begins at Trans-Lunar injection (TLI) and ends at
Near-Rectilinear Halo Orbit Rendezvous (NRR). Red dots denote nominal maneuvers and yellow
dots denote MCCs.

The robust trajectory optimization approach is used to formulate two complementary optimiza-
tion problems. One problem minimizes the total MCC $\Delta v$ dispersion subject to a constraint on the
final position dispersion. The other problem minimizes the final position dispersion subject to a
constraint on the total MCC $\Delta v$ dispersion. For both problems, the optimization variables are the
times of each MCC and the Optical Navigation (OpNav) viewing target during measurement passes
that precede each maneuver. It is assumed that on-board trajectory estimation is performed exclu-
sively with an OpNav system. These optimization problems are solved using a genetic algorithm
(GA).

The remainder of this paper is organized as follows. First, a description of the LinCov analysis,
optimization problems, and GA is provided. Next, a relatively simple example is presented to
illustrate the robust trajectory optimization approach. The optimization approach is then applied to
the end-to-end mission profile spanning TLI to NRR. Last, final remarks are given in the conclusion.
**APPROACH**

**Linear Covariance Analysis**

LinCov relies upon a state vector that contains a set of true state dispersions and navigation state dispersions. This state vector is formed by first defining a true state and navigation state. The true state is comprised of the spacecraft’s inertial position and velocity, as well as exponentially correlated random variables (ECRVs, also known as 1st-order Markov processes) that represent the true attitude dispersion, maneuver execution errors (scale factor, misalignment, and bias), star camera misalignment, OpNav pointing errors, and OpNav measurement biases. The navigation state contains the same elements as the true state except for maneuver execution errors. This LinCov analysis also incorporates the effect of noise on measurements, maneuvers, and environment models; however, noise is not considered a state variable.

LinCov focuses on the propagation, update, and correction of the dispersion covariance, $C_a$, and the navigation filter covariance, $P$. $C_a$ is formed through the true and navigation state dispersions. $P$ is the covariance associated with navigation states estimated by an extended Kalman filter. The discrete-time propagation equations for $C_a$ and $P$ are

$$
C_a(t_{k+1}) = \Phi_a(t_{k+1}, t_k)C_a(t_k)\Phi_a^T(t_{k+1}, t_k) + G_a Q_{D,a} G_a^T
$$

$$
P(t_{k+1}) = \Phi_n(t_{k+1}, t_k)P(t_k)\Phi_n^T(t_{k+1}, t_k) + G_n Q_{D,n} G_n^T
$$

$\Phi_a$ and $\Phi_n$ are state transition matrices. They describe the linearized perturbation dynamics about the reference trajectory. $G_a$ and $G_n$ are used to map environmental uncertainties, described by $Q_{D,a}$ and $Q_{D,n}$, into $C_a$ and $P$.

The update equations for $C_a$ and $P$ at a measurement time $t_i$ are

$$
C_a^+(t_i) = A_a C_a^-(t_i)A_a^T + B_a R^j(t_i)B_a^T
$$

$$
P^+(t_i) = [I - K^j(t_i)H^j] P^-(t_i) [I - K^j(t_i)H^j]^T + K^j(t_i)R^j(t_i)K^j(t_i)^T
$$

where the Kalman gain is written as

$$
K^j(t_i) = P(t_i)(H^j)^T[H^j P^-(t_i) (H^j)^T + R^j(t_i)]^{-1}
$$

The superscript ‘$j$’ denotes the $j$th measurement type. For OpNav measurements, ‘$j$’ also distinguishes whether the Earth or Moon is the viewing target. $H$ is the measurement geometry matrix and $R$ is the measurement noise matrix. $A_a$ and $B_a$ are used to map the effects of measurements and their associated noise to the navigation state dispersions.

The correction equations for $C_a$ and $P$ at a maneuver time $t_m$ are

$$
C_a^+(t_m) = M_a C_a^-(t_m)M_a^T + N_a Q_{w,a} N_a^T
$$

$$
P^+(t_m) = [I + M_n] P^-(t_m) [I + M_n]^T + N_n Q_{w,n} N_n^T
$$

$M_a$ and $M_n$ contain the control partials associated with a linearized two-impulse targeting algorithm. $N_a$ and $N_n$ are used to map the effects of actuator noise, described by $Q_{w,a}^{act}$ and $Q_{w,n}^{act}$, into $C_a$ and $P$. 
Optimization Problem Formulation

The key metrics investigated in this paper are the standard deviations of the dispersions on $\Delta v$ and final position. Both of these quantities are extracted from the dispersion covariance, $C_a$. For brevity, the remainder of the paper will use the word ‘dispersion’ to refer to the standard deviation of the dispersion.

There is an inherent tradeoff between the dispersions in $\Delta v$ and final position (this is discussed in more detail after Figure 4). As a result, two complementary optimization problems are formulated. They are referred to as problem 1 and problem 2. Problem 1 minimizes the total $\Delta v$ dispersion across all midcourse correction maneuvers subject to a constraint on the position dispersion at the final time. The position dispersion constraint is implemented as a penalty function; when the constraint is violated, a large constant $\kappa$ is added to the value of the objective function.

Problem 1: minimize $\sum_{m=1}^{N} \sigma_{\delta\Delta v}(t_m) + \kappa \text{sgn}(\max(0, \sigma_{\delta r}(t_f) - \sigma_{\delta r}^{req}))$ \hspace{1cm} (8)

Problem 2 minimizes the position dispersion at the final time subject to a constraint on the total $\Delta v$ dispersion across all midcourse correction maneuvers. Like problem 1, the $\Delta v$ dispersion constraint is implemented as a penalty function.

Problem 2: minimize $\sigma_{\delta r}(t_f) + \kappa \text{sgn}(\max(0, \sum_{m=1}^{N} \sigma_{\delta\Delta v}(t_m) - \sigma_{\delta\Delta v}^{req}))$ \hspace{1cm} (9)

LinCov Embedded in a Genetic Optimization Algorithm

Due to their complex nature, both optimization problems are solved using a GA. During each GA iteration, candidate values of the optimization variables are passed to the LinCov simulation, which is then evaluated to determine the values of the cost and penalty functions. This interaction is shown in Figure 2.

The GA refines its search for an optimal solution through a process inspired by natural selection. Each generation of new candidate solutions is influenced by the performance of candidate solutions from previous iterations. Ultimately this process provides a way of efficiently searching for an optimal solution.
ILLUSTRATIVE EXAMPLE

This section presents a simple example to illustrate the new robust trajectory design technique. The NRI to NRR flight segment is used for the example (see Figure 1). It is assumed that two MCCs (OM-1 and OM-2) occur within this segment. The MCCs are intended to guide the vehicle back to the nominal trajectory. However, due to initial dispersions, navigation errors, maneuver execution errors, and environmental modeling uncertainties, the vehicle remains slightly dispersed from the nominal trajectory at the time of NRR. Here the goal is to solve problem 1: determine the times of OM-1 and OM-2 which minimize the total MCC $\Delta v$ dispersion while satisfying a final position dispersion constraint. Figure 3 illustrates this scenario.

For simplicity, it is assumed that simultaneous OpNav measurements to the Earth and Moon are available. It is also assumed that OpNav measurement passes last 2 hours long and terminate 30 minutes before each maneuver. Tables 1, 2, and 3 in the Appendix list the LinCov parameters used to generate the results shown in the remainder of this section.

Since this example involves only two optimization parameters, contour plots of the objective and constraint functions can be generated by sweeping through the set of feasible times for OM-1 and OM-2. This is shown in Figure 4.
These plots illustrate how the timing of MCCs results in a fundamental tradeoff between dispersions in $\Delta v$ and final position. $\Delta v$ dispersions are directly related to maneuver execution errors, some of which are proportional to the commanded $\Delta v$. The commanded $\Delta v$ relies upon a two-impulse targeting scheme and depends on the time between maneuvers and the estimated trajectory dispersions. Position dispersions are driven by initial dispersions, environmental modeling uncertainties, and maneuver execution errors. In general, position dispersions grow until an MCC occurs. These factors contribute to the following trends: early MCCs result in dispersions that are lower in $\Delta v$ but higher in final position, and later MCCs result in dispersions that are higher in $\Delta v$ but lower in final position. The optimal solution strikes a balance between these competing trends.

Figure 5 shows how the solution can be visualized by superimposing the constraint function contours onto the objective function contours. The final position dispersion constraint is chosen as 10 km. The optimal solution is one that has the lowest $\Delta v$ dispersion while remaining along or above the orange contour line which denotes the 10 km position dispersion boundary. This problem was also solved used the GA, and its solution is marked by the black circle. This shows that the GA solution is consistent with the solution through visual inspection.
END-TO-END MISSION ANALYSIS

In this section the new robust trajectory design technique is used to determine the optimal MCC locations and OpNav viewing target for a complete end-to-end mission spanning from TLI to NRR. The nominal mission profile is shown in Figure 1. The problem setup and non-optimal baseline results are presented first. Then the solutions to problems 1 and 2 are presented.

Problem Setup

This section covers the key modeling parameters within the LinCov analysis. It describes error models for measurements and maneuvers, initial condition uncertainties, and other operational assumptions. Referenced tables are listed in the Appendix.

OpNav measurements consist of the apparent centroid location and apparent angular diameter of a target celestial body. OpNav measurement errors are modeled as a combination of a bias (modeled as an ECRV) and noise. Star tracker measurement errors are modeled as a misalignment (modeled as an ECRV) and noise. The specific camera parameters used in this analysis are not given as this information is currently proprietary.

Thruster errors are modeled as a combination of a misalignment, scale factor, bias, and noise. The misalignment, scale factor, and bias are modeled as ECRVs. These error sources contribute to errors in the \( \Delta v \) vectors which represent impulsive maneuvers. Table 4 lists each error model component and its corresponding value for the RCS and main thrusters. It is assumed that RCS thrusters are used for MCCs while main thrusters are used for the nominal maneuvers which occur at TLI, OPF, and NRI.

Table 5 lists the initial dispersions and navigation errors for the vehicle’s translational and rotational states. Position and velocity uncertainties are expressed in a UVW reference frame, where the U-axis points in the radial direction, V-axis points in the downrange direction, and W-axis points in the crossrange direction (analogous to a local vertical, local horizontal frame). The initial dispersions are amplified by errors in the TLI maneuver. As a correction procedure, a fixed trim burn is performed immediately after TLI. The time of this maneuver is not considered in the optimization process.

Table 6 describes various operational constraints and assumptions. The OpNav measurement pass duration is notionally 2 hours long. However, this duration has lower priority than the constraint on the time between the last OpNav measurement and a maneuver. In the process of searching for an optimal solution, if consecutive maneuvers are placed less than 2 hours apart, then the measurement pass is reduced to ensure the 30 minute gap is enforced. Likewise, maneuvers are constrained to occur at least 30 minutes apart. In the event where maneuvers are separated by exactly 30 minutes, no measurements are taken in between them. The OpNav field of view (FOV) constraint value is dependent on camera properties. If the apparent angular diameter of the target celestial body exceeds the FOV constraint, no OpNav measurements are taken.
Baseline Results

Figure 6 shows the position dispersion time history, OpNav scheduling, position estimation error time history, and MCC $\Delta v$ dispersions using the nominal MCC locations and two-camera OpNav system. The vertical lines denote maneuver locations; solid lines indicate nominal maneuvers and dashed lines indicate MCCs.

![Figure 6](image)

Figure 6: Position dispersion time history, OpNav scheduling, position estimation error time history, and MCC $\Delta v$ dispersions using the baseline MCC locations and two-camera OpNav system.

The two-camera OpNav system assumes that measurements of the Earth and Moon can be taken simultaneously. However, due to FOV constraints, there are instances where only one target celestial body can be measured. This is shown on the OpNav schedule plot. The FOV constraint is violated as the vehicle remains close to the Earth before OTC-1 and as it passes near the Moon around the time of OPF.

As indicated in Figure 6, the 3-$\sigma$ position dispersion at NRR is 18.4 km. The total 3-$\sigma$ $\Delta v$ dispersion is 35.4 m/s. These values are used as the constraints in problems 1 and 2.
Minimize $\Delta v$ Dispersion

The objective of this problem is to determine the MCC times and OpNav viewing targets which minimize the total MCC $\Delta v$ dispersion while ensuring the final position dispersion does not exceed 18.4 km. There are eight MCCs scheduled, and their grouping is consistent with the nominal trajectory: four between TLI and OPF, 2 between OPF and NRI, and 2 between NRI and NRR. Figure 7 shows the resulting position dispersion time history, OpNav scheduling, position estimation error time history, and MCC $\Delta v$ dispersions.

![Diagram showing position dispersion, OpNav schedule, position estimation error, and MCC $\Delta v$ dispersions.]

Figure 7: Position dispersion time history, OpNav scheduling, position estimation error time history, and MCC $\Delta v$ dispersions for the optimal solution to problem 1.

There is a noticeable shift in the MCCs from their baseline locations. OTC-3 through OTC-6 occur earlier which leads to a drop in $\Delta v$ dispersions. Early placement of OTC-6 allows the position dispersion to grow larger before NRI, but OM-1 and OM-2 are arranged to drive it back below the constraint at NRR. Overall, these results show that through optimally selecting the MCC times and OpNav viewing target, the baseline position dispersion can be met with a single-camera OpNav system while also reducing the $\Delta v$ dispersion by over 40%.
Minimize Position Dispersion

The objective of this problem is to determine the MCC times and OpNav viewing targets which minimize the final position dispersion while ensuring the total MCC $\Delta v$ dispersion does not exceed 35.4 m/s. There are eight MCCs scheduled, and their grouping is consistent with the nominal trajectory. Figure 8 shows the resulting position dispersion time history, OpNav scheduling, position estimation error time history, and MCC $\Delta v$ dispersions.

Like Figure 7, there is a noticeable shift in the MCCs from their baseline locations. OTC-5 and OTC-6 are moved near OPF leading to lower $\Delta v$ dispersions. In contrast to the solution to problem 1, the position dispersion after OTC-6 does not grow as rapidly because some OpNav measurements are made before OTC-5. OM-1 and OM-2 are moved near NRR to reduce the final position dispersion. Overall, these results show that through optimally selecting the MCC times and OpNav viewing target, the baseline MCC $\Delta v$ dispersion can be met with a single-camera OpNav system while also reducing the final position dispersion by nearly 25%.

Figure 8: Position dispersion time history, OpNav scheduling, position estimation error time history, and MCC $\Delta v$ dispersions for the optimal solution to problem 2.
CONCLUSION

This paper presented a new approach to optimal cislunar trajectory design that considers uncertainties in navigation, maneuver execution, environmental modeling, and initial conditions. Given a nominal trajectory, MCC times and OpNav viewing targets are selected which minimize dispersions in either $\Delta v$ or final position. This process was first illustrated for an individual segment along an outbound cislunar trajectory. End-to-end trajectory analysis was then presented, where it was shown that optimal solutions with a single-camera OpNav system provided better performance than baseline MCC times and a two-camera system.

REFERENCES


APPENDIX

Tables 1, 2, and 3 list relevant LinCov parameters used in the illustrative example. Tables 4, 5, and 6 list relevant LinCov parameters and operational constraints used in the end-to-end analysis.

Table 1: Error sources and values for OpNav and star tracker measurements used in the illustrative example.

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>OpNav centroid location noise (3-$\sigma$)</td>
<td></td>
<td>arcsec</td>
</tr>
<tr>
<td>OpNav planetary diameter noise (3-$\sigma$)</td>
<td></td>
<td>arcsec</td>
</tr>
<tr>
<td>Star tracker misalignment steady-state (3-$\sigma$)</td>
<td></td>
<td>arcsec</td>
</tr>
<tr>
<td>Star tracker noise (3-$\sigma$)</td>
<td></td>
<td>arcsec</td>
</tr>
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Table 2: Thruster error sources and values used in the illustrative example.

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
<th>Units</th>
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</thead>
<tbody>
<tr>
<td>Misalignment steady-state (3-σ)</td>
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<td>mrad</td>
</tr>
<tr>
<td>Misalignment time constant (3-σ)</td>
<td>2.78</td>
<td>hr</td>
</tr>
<tr>
<td>Scale factor steady-state (3-σ)</td>
<td>0.3</td>
<td>%</td>
</tr>
<tr>
<td>Scale factor time constant (3-σ)</td>
<td>2.78</td>
<td>hr</td>
</tr>
<tr>
<td>Bias steady-state (3-σ)</td>
<td>3</td>
<td>mm/s</td>
</tr>
<tr>
<td>Bias time constant</td>
<td>2.78</td>
<td>hr</td>
</tr>
<tr>
<td>Noise (3-σ)</td>
<td>3</td>
<td>mm/s</td>
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</table>

Table 3: Initial dispersions and navigation errors used in the illustrative example.

<table>
<thead>
<tr>
<th>Description</th>
<th>Value (3-σ)</th>
<th>Units</th>
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<tbody>
<tr>
<td>Position dispersion and estimation error (U,V,W)</td>
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<td>km</td>
</tr>
<tr>
<td>Velocity dispersion and estimation error (U,V,W)</td>
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</tr>
<tr>
<td>Attitude dispersion (per axis)</td>
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<td>deg</td>
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Table 4: Error sources and values for RCS and main thrusters used in the end-to-end analysis.

<table>
<thead>
<tr>
<th>Description</th>
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<th>Main Thrusters</th>
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<tr>
<td>Misalignment steady-state (3-σ)</td>
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<td>Misalignment time constant</td>
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</tr>
<tr>
<td>Noise (3-σ)</td>
<td>225</td>
<td>600</td>
<td>mm/s</td>
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Table 5: Initial dispersions and navigation errors used in the end-to-end analysis.

<table>
<thead>
<tr>
<th>Description</th>
<th>Value (3-σ)</th>
<th>Units</th>
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<tbody>
<tr>
<td>Position dispersion (U,V,W)</td>
<td>(10, 20, 20)</td>
<td>km</td>
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<tr>
<td>Velocity dispersion (U,V,W)</td>
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<td>Attitude dispersion (per axis)</td>
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<tr>
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<td>m</td>
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<tr>
<td>Velocity estimation error (U,V,W)</td>
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<td>Attitude estimation error (per axis)</td>
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<td>deg</td>
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Table 6: Operational constraints and assumptions used in the end-to-end analysis.

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<tr>
<td>Notional OpNav measurement pass duration</td>
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<td>Time between last OpNav measurement and maneuver</td>
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<td>min</td>
</tr>
<tr>
<td>Minimum time between maneuvers</td>
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<td>min</td>
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<tr>
<td>OpNav measurement frequency</td>
<td>60</td>
<td>sec</td>
</tr>
<tr>
<td>OpNav FOV constraint</td>
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<td>deg</td>
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