Time-Varying Reference Model Modification of Adaptive Control with Multi-Objective Performance Optimization

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This paper presents an adaptive control approach that modifies a reference model in order to satisfy a multi-objective optimization of two performance signals: a linear signal and a quadratic signal. The time-varying reference model modification is accomplished by the real-time solutions of the time-varying Riccati and Sylvester equations coupled with the least-squares parameter estimation of the sensitivities of the performance signals to be minimized. The effectiveness of the proposed method is to be demonstrated by a flight control application of maneuver load alleviation and drag minimization.

I. Introduction

In this work, we develop a multi-objective performance-based model-reference adaptive control (MRAC) with the goal of providing adaptation while seeking to optimize two performance signals for an uncertain system. The plant is assumed to have sufficient control redundancy to achieve the objective of performance optimization in addition to the usual MRAC objective of maintaining tracking performance in the presence of matched uncertainty.

The performance signals are available from sensor measurements but their sensitivities with respect to the state and control variables are assumed to be unknown. Two types of signals are considered: a signal with linear state and control sensitivities and a signal with linear quadratic state and control sensitivities. Least-squares parameter estimation is designed to estimate the performance metric sensitivities. The performance optimization is cast as a multi-objective gradient optimization problem. The gradient of the Hamiltonian function is computed in real-time from the estimates of the sensitivities. This results in time-varying modified Riccati and Sylvester equations. The performance optimizing controller is then augmented to the reference model. This results in a time-varying modified reference model in order for MRAC to achieve simultaneously the original tracking performance objective as well as the performance optimization objective.

Reference model modification in model-reference adaptive control has been investigated extensively by many researchers but time-varying modification is generally not considered in these investigations. One common approach has been considered is to include a linear time-invariant (LTI) tracking error feedback term in the reference model which is sometimes referred to as a closed-loop reference model developed by Stepanyan, Lavretsky, and Gibson.[1–3] This modification can improve transient performance of a closed-loop adaptive control system. A similar approach is the frequency-limited adaptive control developed by Yucelen et al. whereby the reference model includes a feedback term that contains the difference between the tracking error and its filtered version.[4] Another well known approach is the pseudo-control hedging method developed by Johnson and Calise which augments to the reference model a hedging control term to account for the difference between the command signal sent to and the output of the actuator which effectively modifies the reference command signal in the presence of control saturation.[5] Command governor-based adaptive control is another modification method similar to the pseudo-control hedging method developed by Yucelen and Johnson whereby the reference command signal is augmented by a command governor output signal generated by a command governor LTI model with the tracking error as the input signal.[11] In all of these methods, the modified reference model remains LTI.

Multi-objective optimization in the context of MRAC has been addressed by a number of researchers. One approach is the bi-objective and multi-objective adaptive control method based on the optimal control modification

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developed by Nguyen which seeks to minimize both the tracking error and the predictor error for systems with input uncertainty and unmatched uncertainty.[7,8] Another approach developed by Abdollahi and Chowdhary is the adaptive-optimal multi-objective optimization which utilizes Gaussian Process (GP)-based adaptive control to track a reference model obtained from a model predictive control for the reference command shaping.[9] Kim and Hovakimyan consider the problem of optimal trade-off between robustness and performance of the $\mathscr{L}_1$ adaptive controller by means of an multi-criteria optimization search for the design parameters of the $\mathscr{L}_1$ filter.[10] The performance optimization in these studies relates to control-relevant metrics that quantify the performance and robustness of a controller.

In the proposed approach, a number of new contributions are made. The performance optimization is not restricted to control-relevant metrics but can deal with physics-relevant metrics that impose physical constraints on a plant. The gradient optimization leads to a set of modified time-varying Riccati and Sylvester equations whereby the standard weighting matrices Q and R are modified to incorporate the estimates of the state and control sensitivities. In addition, the reference command signal is designed to be generated from a reference command model in order to allow the performance optimizing controller to be obtained explicitly from the gradient optimization in a feedback form. The resultant time-varying modification of the reference model as a consequence of the performance optimization objective necessitates a revision to the standard MRAC whereby the usual constant weighting matrix P in the adaptive law obtained from solving the Lyapunov equation is replaced by the time-varying weighting matrix R computed from the modified Riccati equation.

In the previous work, a single objective is used for the optimization of the performance signal.[11,12] In this study, the optimization involves two performance signals. The resulting performance optimizing controller utilizes the solution of a modified Riccati equation. A simulation of maneuver load alleviation and drag minimization control for an aircraft using the proposed method is planned to demonstrate its effectiveness.

II. Performance Optimizing Adaptive Control

Consider a plant model

$$\dot{x} = Ax + B\left[u + \Theta^{*\top}\Phi(x)\right] \tag{1}$$

subject to $x(0) = x_0$, where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the control vector, $\Theta^* \in \mathbb{R}^{l\times m}$ is an unknown constant matrix that represents a matched uncertainty parameter, $\Phi(x) \in \mathbb{R}^l$ is a known regressor function, and $A \in \mathbb{R}^{n\times n}$ and $B \in \mathbb{R}^{n\times m}$ are known matrices. The plant is considered to be a full-state system with all the available states for control. Therefore, the pair (A, B) is assumed to be controllable.

Suppose the following signals are to be minimized

$$y_p = Cx + Du \tag{2}$$

$$z_p = \|Ex + Fu\|^2 \tag{3}$$

where $y_p(t) \in \mathbb{R}^k$ with $k \leq n$ and $z_p(t) \in \mathbb{R}$ are available from measurements, and $C \in \mathbb{R}^{k\times n}$, $D \in \mathbb{R}^{k\times m}$, $E \in \mathbb{R}^{1\times n}$, and $F \in \mathbb{R}^{1\times m}$ are unknown matrices. These signals are referred to as the performance signals of the plant. The goal is to design an adaptive controller that tracks a reference model as well as optimizes the performance signals $y_p(t)$ and $z_p(t)$.

To address the signal optimization, we first introduce a reference command model of the form

$$\dot{z} = A_r z + z_r \tag{4}$$

$$r = C_r z \tag{5}$$

subject to $z_r(0) = z_{r_0}$, where $z(t) \in \mathbb{R}^p$, $p \geq q$, $z_r \in \mathbb{R}^p$ is a constant command signal, $A_r \in \mathbb{R}^{p\times p}$ is a stable matrix and $C_r \in \mathbb{R}^{q\times p}$.

We consider the following controller:

$$u = u_{nom} + u_{ad} + u_p \tag{6}$$

The nominal controller $u_{nom}(t)$ is defined as

$$u_{nom} = K_x x + K_r r \tag{7}$$

The adaptive controller $u_{ad}(t)$ is designed to cancel the matched uncertainty as

$$u_{ad} = -\Theta^\top(t)\Phi(x) \tag{8}$$

The performance optimizing controller $u_p(t)$ is designed to minimize the performance signals $y_p(t)$ and $z_p(t)$ and in so doing modify the reference model to meet the control objective. The closed-loop plant then becomes

$$\dot{x} = A_m x + B_m r + B u_p + B u_{ad} + B\Theta^{*\top}\Phi(x) \tag{9}$$

where $A_m = A + BK_x$ which is Hurwitz and $B_m = BK_r$.

The reference model is established as the ideal closed-loop plant with $u_{ad}(t) = -\Theta^{*\top}\Phi(x)$, thus resulting in

$$\dot{x}_m = A_m x_m + B_m r + B u_p \tag{10}$$

The last term modifies the standard reference model in order to minimize the performance signals $y_p(t)$ and $z_p(t)$.

The performance signals to be minimized due to the performance optimizing controller $u_p(t)$ is defined as

$$y_p = Cx + Du_p \tag{11}$$

$$z_p = \left\| Ex + Fu_p \right\|^2 \tag{12}$$

Let $\hat{y}_p(t)$ and $\hat{z}_p(t)$ be the estimates of $y_p(t)$ and $z_p(t)$, respectively

$$\hat{y}_p = \hat{C}x + \hat{D}u_p \tag{13}$$

$$\hat{z}_p = \left\| \hat{E}x + \hat{F}u_p \right\|^2 \tag{14}$$

where $\hat{C}(t)$, $\hat{D}(t)$, $\hat{E}(t)$, and $\hat{F}(t)$ are the estimates of C, D, E, and F, respectively. The estimation is a least-squares gradient method that minimizes the following cost function

$$J = \frac{1}{2}e_y^\top e_y + \frac{1}{2}e_z^\top e_z \tag{15}$$

where

$$e_y = \hat{y}_p - y_p = \tilde{C}x + \tilde{D}u_p \tag{16}$$

$$e_z = \sqrt{\hat{z}_p} - \sqrt{z_p} = \tilde{E}x + \tilde{F}u_p \tag{17}$$

where $\tilde{C}(t) = \hat{C}(t) - C$, $\tilde{D}(t) = \hat{D}(t) - D$, $\tilde{E}(t) = \hat{E}(t) - E$, and $\tilde{F}(t) = \hat{F}(t) - F$ are the estimation errors.

This results in the least-squares gradient adaptive laws

$$\dot{\hat{C}}^\top = -\Gamma_C \frac{\partial J}{\partial \tilde{C}} = -\Gamma_C x e_y^\top \tag{18}$$

$$\dot{\hat{D}}^\top = -\Gamma_D \frac{\partial J}{\partial \tilde{D}} = -\Gamma_D u_p e_y^\top \tag{19}$$

$$\dot{\hat{E}}^\top = -\Gamma_E \frac{\partial J}{\partial \tilde{E}} = -\Gamma_E x \varepsilon_z^\top \tag{20}$$

$$\dot{\hat{F}}^\top = -\frac{1}{2}\Gamma_F \frac{\partial J}{\partial \tilde{F}} = -\Gamma_F u_p e_z^\top \tag{21}$$

where $\Gamma_C > 0 \in \mathbb{R}^{n\times n}$, $\Gamma_D > 0 \in \mathbb{R}^{m\times m} > 0$, $\Gamma_E > 0 \in \mathbb{R}^{n\times n}$, and $\Gamma_F > 0 \in \mathbb{R}^{m\times m}$ are positive definite adaptation rate matrices.

To design the performance optimizing controller $u_p(t)$, we consider the ideal closed-loop plant with $u_{ad}^*(t) = -\Theta^{*\top}\Phi(x)$

$$\dot{x} = A_m x + B_m r + B u_p \tag{22}$$

We now cast this problem as a multi-objective optimization of the ideal plant using the following infinite-time-horizon cost function:

$$J = \frac{1}{2}\int_0^{t_f\to\infty} \left(\hat{y}_p^\top Q \hat{y}_p + q\hat{z}_p + u_p^\top R u_p \right) dt \tag{23}$$

where $Q > 0 \in \mathbb{R}^{k\times k}$, $q > 0 \in \mathbb{R}$, and $R > 0 \in \mathbb{R}^{m\times m}$ are positive definite weighting matrices, subject to the plant dynamics in Eq. (22). The Hamiltonian function is defined as

$$H = \frac{1}{2}\hat{y}_p^\top Q\hat{y}_p + \frac{1}{2}q\hat{z}_p + \frac{1}{2}u_p^\top Ru_p + \mu^\top (A_m x + B_m r + Bu_p) \tag{24}$$

where $\mu(t) \in \mathbb{R}^n$ is an adjoint vector which acts to enforce the dynamic constraint imposed by the plant.

Then, the necessary conditions are obtained as

$$\dot{\mu} = -\frac{\partial H}{\partial x}^\top = -\left(\hat{C}^\top Q\hat{C} + \hat{E}^\top q\hat{E}\right)x - \left(\hat{C}^\top Q\hat{D} + \hat{E}^\top q\hat{F}\right)u_p - A_m^\top \mu \tag{25}$$

subject to the transversality condition $\mu\left(t_f\right) = 0$, and

$$\frac{\partial H}{\partial u_p}^\top = \left(\hat{D}^\top Q\hat{C} + \hat{F}^\top q\hat{E}\right)x + \left(R + \hat{D}^\top Q\hat{D} + \hat{F}^\top q\hat{F}\right)u_p + B^\top \mu = 0 \tag{26}$$

The optimal control $u_p(t)$ is then obtained as

$$u_p = -\left(R + \hat{D}^\top Q\hat{D} + \hat{F}^\top q\hat{F}\right)^{-1}\left[B^\top \mu + \left(\hat{D}^\top Q\hat{C} + \hat{F}^\top q\hat{E}\right)x\right] \tag{27}$$

Using the assumed solution of

$$\mu = Wx + Vz + Uz_r \tag{28}$$

we obtain the following equations:

$$\dot{W} + W\bar{A} + \bar{A}^\top W - WB\bar{R}^{-1}B^\top W + \bar{Q} = 0 \tag{29}$$

$$\dot{V} + \left(\bar{A}^\top - WB\bar{R}^{-1}B^\top\right)V + VA_r + WB_mC_r = 0 \tag{30}$$

$$\dot{U} + \left(\bar{A}^\top - WB\bar{R}^{-1}B^\top\right)U + V = 0 \tag{31}$$

subject to the transversality conditions $W\left(t_f\right) = 0$, $V\left(t_f\right) = 0$, and $U\left(t_f\right) = 0$, where

$$\bar{A} = A_m - B\bar{R}^{-1}\left(\hat{D}^\top Q\hat{C} + \hat{F}^\top q\hat{E}\right) \tag{32}$$

$$\bar{Q} = \left(\hat{C}^\top Q\hat{C} + \hat{E}^\top q\hat{E}\right) - \left(\hat{C}^\top Q\hat{D} + \hat{E}^\top q\hat{F}\right)\bar{R}^{-1}\left(\hat{D}^\top Q\hat{C} + \hat{F}^\top q\hat{E}\right) \tag{33}$$

$$\bar{R} = R + \hat{D}^\top Q\hat{D} + \hat{F}^\top q\hat{F} \tag{34}$$

Q is chosen such that $\bar{Q} > 0$. Equation (29) is recognized as the time-varying differential Riccati equation with the time-varying matrix $\bar{A}(t)$ and time-varying positive definite weighting matrices $\bar{Q}(t)$ and $\bar{R}(t)$ which are updated at each time step as $\hat{C}(t)$, $\hat{D}(t)$, $\hat{E}(t)$, and $\hat{F}(t)$ are computed from the least-squares gradient adaptive laws. The existence of the solution of a time-varying differential Riccati equation depends on the properties of the time-varying matrices $\bar{A}(t)$, $\bar{Q}(t)$, and $\bar{R}(t)$ according to the following theorem:

Theorem 1: Let $\bar{A}(t) = \bar{A}^* + \delta_{\bar{A}}(t)$, $\bar{Q}(t) = \bar{Q}^* + \delta_{\bar{Q}}(t) > 0$, and $\bar{R}(t) = \bar{R}^* + \delta_{\bar{R}}(t) > 0$ where $\bar{A}^*$, $\bar{Q}^*$, and $\bar{R}^*$ are some constant matrices. If $\delta_{\bar{A}}(t)$, $\delta_{\bar{Q}}(t)$, and $\delta_{\bar{R}}(t)$ are at least piecewise continuous for all $t \in [0,\infty)$; $\lim_{t\to\infty}\delta_{\bar{A}}(t) = 0$, $\lim_{t\to\infty}\delta_{\bar{Q}}(t) = 0$, and $\lim_{t\to\infty}\delta_{\bar{R}}(t) = 0$; and furthermore, $\dot{\delta}_{\bar{A}}(t)$, $\dot{\delta}_{\bar{Q}}(t)$, and $\dot{\delta}_{\bar{R}}(t)$ are uniformly continuous; then $\bar{A}(t)$, $\bar{Q}(t)$, and $\bar{R}(t)$ tend to their constant solutions $\bar{A}^*$, $\bar{Q}^*$, and $\bar{R}^*$, respectively, as $t \to \infty$. Consequently, the solution of the time-varying differential Riccati equation exists and also tends to its constant solution in the limit as $t_f \to \infty$.

Proof: The Hamiltonian system for a regulator design with $V(t) = U(t) = 0$ is defined as

$$\begin{bmatrix} \dot{x} \\ \dot{\mu} \end{bmatrix} = \begin{bmatrix} \bar{A} & -B\bar{R}^{-1}B \\ -\bar{Q} & -\bar{A}^\top \end{bmatrix}\begin{bmatrix} x \\ \mu \end{bmatrix} = H\begin{bmatrix} x \\ \mu \end{bmatrix} \tag{35}$$

where $H(t)$ is the time-varying Hamiltonian matrix. The solution of Eq. (29) depends continuously on the time-varying stable eigenvalues of $H(t)$ in frozen time. For a first-order SISO system with $H(t)$ a 2×2 matrix, it can be shown that $\|\lambda(H(t))\|^2 \le \left\|\bar{A}^* + \delta_{\bar{A}}(t)\right\|^2 + \|B\|^2 \left\|\left(\bar{R}^* + \delta_{\bar{R}}(t)\right)^{-1}\right\| \left\|\left(\bar{Q}^* + \delta_Q(t)\right)\right\|$. Therefore, $\delta_{\bar{A}}(t)$, $\delta_{\bar{Q}}(t)$, and $\delta_{\bar{R}}(t)$must be at least piecewise continuous and uniformly bounded for all $t \in [0,\infty)$.[13] Let $\Lambda(t) = \operatorname{diag}(-\Lambda_H(t), \Lambda_H(t))$ be the eigenvalue matrix where $\Lambda_H(t)$ is the diagonal matrix of positive real eigenvalues of $H(t)$. Then, the transition matrix $\Phi(t,t_0)$ is defined as

$$\frac{d\Phi}{dt} = -\Lambda_H \Phi \tag{36}$$

where $\Phi(t_0,t_0) = I$. Then, $\Phi(t,t_0)$ is exponentially stable if there exists ε such that $\left\|\dot{\Lambda}_H(t)\right\| \le \varepsilon$.[13] This implies that $\dot{\delta}_{\bar{A}}(t)$, $\dot{\delta}_{\bar{Q}}(t)$, and $\dot{\delta}_{\bar{R}}(t)$ are bounded for all $t \in [0,\infty)$. Moreover, we require $\bar{A}(t) \to \bar{A}^*$, $\bar{R}(t) \to \bar{R}^*$, and $\bar{Q}(t) \to \bar{Q}^*$ uniformly as $t \to \infty$. This implies $\lim_{t\to\infty} \delta_{\bar{A}}(t) = 0$, $\lim_{t\to\infty} \delta_{\bar{Q}}(t) = 0$, and $\lim_{t\to\infty} \delta_{\bar{R}}(t) = 0$. Furthermore, $\dot{\bar{A}}(t) \to 0$, $\dot{\bar{R}}(t) \to 0$, $\dot{\bar{Q}}(t) \to 0$ which require $\lim_{t\to\infty} \dot{\delta}_{\bar{A}}(t) = 0$, $\lim_{t\to\infty} \dot{\delta}_{\bar{Q}}(t) = 0$, and $\lim_{t\to\infty} \dot{\delta}_{\bar{R}}(t) = 0$. Then, $\dot{\delta}_{\bar{A}}(t)$, $\dot{\delta}_{\bar{Q}}(t)$, and $\dot{\delta}_{\bar{R}}(t)$ are uniformly continuous according to the Barbalat's lemma. The solution of $W(t)$ is given by[14]

$$W = (X_{21} + X_{22}S)(X_{11} + X_{12}S)^{-1} \tag{37}$$

where

$$S = \Phi(t_f - t, 0)\, S(t_f)\, \Phi(t_f - t, 0) \tag{38}$$

with $X_{ij}(t)$, $i,j = 1,2$ as the right eigenvector partitioned matrices such that $X(t)\Lambda(t)X^{-1}(t) = H(t)$, and $S(t_f) = -X_{22}^{-1}(t_f)X_{21}(t_f)$. Let $W(t) = W^* + \delta_W(t)$ be the solution of the time-varying Riccati equation. Then,

$$\dot{\delta}_W + (W^* + \delta_W)\left(\bar{A}^* + \delta_{\bar{A}}\right) + \left(\bar{A}^* + \delta_{\bar{A}}\right)^\top (W^* + \delta_W) - (W^* + \delta_W) B \left(\bar{R}^* + \delta_{\bar{R}}\right)^{-1} B^\top (W^* + \delta_W) + \left(\bar{Q}^* + \delta_{\bar{Q}}\right) = 0 \tag{39}$$

Separating terms and taking the limit as $t \to \infty$ yield

$$W^*\bar{A}^* + \bar{A}^{*\top} W^* - W^* B \bar{R}^{*-1} B^\top W^* + \bar{Q}^* = 0 \tag{40}$$

$$\dot{\delta}_W + \delta_W \bar{A}^* + \bar{A}^{*\top}\delta_W - \delta_W B\bar{R}^{*-1}B^\top \delta_W - \delta_W B\bar{R}^{*-1}B^\top W^* - W^* B\bar{R}^{*-1}B^\top \delta_W = 0 \tag{41}$$

Now transforming Eq. (41) into the time-to-go variable $\tau = t_f - t$ gives

$$-\frac{d\delta_W}{d\tau} + \delta_W \bar{A}^* + \bar{A}^{*\top}\delta_W - \delta_W B\bar{R}^{*-1}B^\top \delta_W - \delta_W B\bar{R}^{*-1}B^\top W^* - W^* B\bar{R}^{*-1}B^\top \delta_W = 0 \tag{42}$$

subject to $\delta_W(\tau = 0) = 0$ since $W(t_f) = 0$. The solution of Eq. (41) yields $\delta_W = 0$ and $\dot{\delta}_W = 0$ as $t_f \to \infty$. Thus, $W(t) \to W^*$ as $t_f \to \infty$ which implies $\dot{W}(t) \to 0$. Therefore, $W(t)$ can be computed by the time-varying algebraic Riccati equation.

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Equation (30) is recognized as the time-varying differential Sylvester equation. The existence of the solution of Eq. (30) can be established from Theorem 1. If $W(t) \to \bar{W}^*$, then the closed-loop state transition matrix $\bar{A}(t) - B\bar{R}^{-1}(t)B^\top W(t)$ tends to a constant Hurwitz matrix $A_c = \bar{A}^* - B\bar{R}^{*-1}B^\top W^*$. Equations (30) is transformed into the time-to-go variable as

$$\frac{dV^*}{d\tau} = A_c^\top V^* + V^* A_r + W B_m C_r \tag{43}$$

subject to $V^*(\tau = 0) = 0$. Since A_r is a stable matrix, and A_r can be chosen such that $\lambda_i(A_c) + \lambda_j(A_r) \ne 0$ for all i and j, then it follows that $V^*(t)$ is bounded as $t_f \to \infty$ and tends to a constant solution of the following algebraic Sylvester equation:

$$A_c^\top V^* + V^* A_r + W^* B_m C_r = 0 \tag{44}$$

Equation (31) is transformed into the time-to-go variable as

$$\frac{dU^*}{d\tau} = A_c^\top U^* + V^* \tag{45}$$

subject to $U^*(\tau = 0) = 0$. Since $V^*(t)$ exists, $U^*(t)$ is bounded as $t_f \to \infty$ and tends to the following solution:

$$U^* = -A_c^{-\top} V^* \tag{46}$$

Thus, the solutions of Eqs. (29), (30), and (31) are computed from their corresponding time-varying algebraic equations. The performance optimizing controller u_p is then expressed as

$$u_p = \bar{K}_x(t)x + \bar{K}_z(t)z + \bar{K}_{z_r}(t)z_r \tag{47}$$

where

$$\bar{K}_x = -\bar{R}^{-1}\left(B^\top W + \hat{D}^\top Q\hat{C} + \hat{F}^\top q\hat{E}\right) \tag{48}$$

$$\bar{K}_z = -\bar{R}^{-1}B^\top V \tag{49}$$

$$\bar{K}_{z_r} = \bar{R}^{-1}B^\top A_c^{-\top}V \tag{50}$$

In lieu of the reference command model, the optimal control solution can also be formulated by using an alternative expression of the adjoint solution as

$$\mu = Wx + T \tag{51}$$

where T is the solution of

$$T = Vz + Uz_r \tag{52}$$

This results in the the following differential equation:

$$\dot{T} + \left(\bar{A}^\top - WB_p\bar{R}^{-1}B_p^\top\right)T + WB_m r = 0 \tag{53}$$

subject to $T\left(t_f\right) = 0$. It can be shown that $T^*(t)$ is bounded as $t_f \to \infty$ and tends to

$$T^* = -A_c^{-\top}W^*B_m r - \sum_{n=1}^{\infty}(-1)^n\left(A_c^\top\right)^{-n-1}W^*B_m r^{(n)} \tag{54}$$

Note that the series solution of $T^*(t)$ is not a closed-form solution and depends on the time derivatives of $r(t)$. This illustrates the advantage of the reference command model.

The uniform continuity condition for $\dot{\delta}_{\bar{A}}(t)$, $\dot{\delta}_{\bar{Q}}(t)$, and $\dot{\delta}_{\bar{R}}(t)$ requires the parameter convergence of the estimates $\hat{C}(t)$, $\hat{D}(t)$, $\hat{E}(t)$, and $\hat{F}(t)$ to some constant matrices $\bar{C}^*$, $\bar{D}^*$, $\bar{E}^*$, and $\bar{F}^*$ respectively. Let $\Psi = []$This convergence can be stated in the following theorem:

Theorem 2: The least-squares gradient adaptive laws ensures a parameter convergence of $\hat{C}(t)$, $\hat{D}(t)$, $\hat{E}(t)$, and $\hat{F}(t)$ to some constant solutions $\bar{C}^*$, $\bar{D}^*$, $\bar{E}^*$, and $\bar{F}^*$, respectively.

Proof: Choose a Lyapunov candidate function

$$V = \frac{1}{2}\operatorname{trace}\left(\tilde{C}\Gamma_C^{-1}\tilde{C}^\top + \tilde{D}\Gamma_D^{-1}\tilde{D}^\top + \tilde{E}\Gamma_E^{-1}\tilde{E}^\top + \tilde{F}\Gamma_F^{-1}\tilde{F}^\top\right) \tag{55}$$

for the ideal plant in Eq. (22). Then, $\dot{V}\left(\tilde{C},\tilde{D},\tilde{E},\tilde{F}\right)$ is evaluated as

$$\dot{V} = -e_y^\top\tilde{C}x - e_y^\top\tilde{D}u_p - \varepsilon_z^\top\tilde{E}x - \varepsilon_z^\top\tilde{F}u_p = -e_y^\top e_y - \varepsilon_z^\top e_z \le 0 \tag{56}$$

Thus, $\left\{\tilde{C}(t),\tilde{D}(t),\tilde{E}(t),\tilde{F}(t)\right\} \in \mathscr{L}_2 \cap \mathscr{L}_\infty$, but $\left\{x(t),u_p(t)\right\} \in \mathscr{L}_\infty$. It follow that $\{\bar{K}_x(t),\bar{K}_z(t),\bar{K}_{z_r}\} \in \mathscr{L}_\infty$. Therefore, for the ideal plant, $\dot{x}(t) \in \mathscr{L}_\infty$. We note that $\ddot{V}\left(\tilde{C},\tilde{D},\tilde{E},\tilde{F}\right)$ is bounded since $\dot{x}(t) \in \mathscr{L}_\infty$ and $\dot{u}_p(t) \in \mathscr{L}_\infty$ by the virtue of $\left\{\dot{z}(t),\dot{\bar{K}}_x(t),\dot{\bar{K}}_z(t),\dot{\bar{K}}_{z_r}(t)\right\} \in \mathscr{L}_\infty$. Therefore, $\dot{V}\left(\tilde{C},\tilde{D},\tilde{E},\tilde{F}\right)$ is uniformly continuous. Invoking the Barbalat's lemma, $\dot{V}\left(\tilde{C},\tilde{D},\tilde{E},\tilde{F}\right) \to 0$ as $t \to \infty$. This implies $e_y(t) \to 0$ and $e_z(t) \to 0$ as $t \to \infty$. Since $\ddot{\hat{C}}(t)$, $\ddot{\hat{D}}(t)$, $\ddot{\hat{E}}(t)$, and $\ddot{\hat{F}}(t)$ are bounded, it follows that $\dot{\hat{C}}(t)$, $\dot{\hat{D}}(t)$, $\dot{\hat{E}}(t)$, and $\dot{\hat{F}}(t)$ are also uniformly continuous with $\dot{\hat{C}}(t) \to 0$, $\dot{\hat{D}}(t) \to 0$, $\dot{\hat{E}}(t) \to 0$, and $\dot{\hat{F}}(t) \to 0$ as $t \to \infty$. Let

$$\Delta\hat{C} = \hat{C}(t) - \hat{C}(t-\Delta t) = -\int_{t-\Delta t}^{t}\Gamma_c x e_y^\top d\tau \tag{57}$$

$$\Delta\hat{D} = \hat{D}(t) - \hat{D}(t-\Delta t) = -\int_{t-\Delta t}^{t}\Gamma_D u_p e_y^\top d\tau \tag{58}$$

$$\Delta\hat{E} = \hat{E}(t) - \hat{E}(t-\Delta t) = -\int_{t-\Delta t}^{t} \Gamma_E x e_z^\top d\tau \tag{59}$$

$$\Delta\hat{F} = \hat{F}(t) - \hat{F}(t-\Delta t) = -\int_{t-\Delta t}^{t} \Gamma_F u_p e_z^\top d\tau \tag{60}$$

Then, $\Delta\hat{C}(t) \to 0$, $\Delta\hat{D}(t) \to 0$, $\Delta\hat{E}(t) \to 0$, and $\Delta\hat{F}(t) \to 0$ since $e_y(t) \to 0$ and $e_z(t) \to 0$ as $t \to \infty$. This implies that $\hat{C}(t) \to \bar{C}^*$, $\hat{D}(t) \to \bar{D}^*$, $\hat{E}(t) \to \bar{E}^*$, and $\hat{F}(t) \to \bar{F}^*$ as $t \to \infty$. Therefore, $\bar{A}(t)$, $\bar{Q}(t)$, $\bar{R}(t)$, $\bar{K}_x(t)$, $\bar{K}_z(t)$, and $\bar{K}_{z_r}(t)$ all converge to the constant matrices $\bar{A}^*$, $\bar{Q}^*$, $\bar{R}^*$, $\bar{K}_x^*$, $\bar{K}_z^*$, and $\bar{K}_{z_r}^*$, respectively. Note that $\hat{C}(t)$, $\hat{D}(t)$, $\hat{E}(t)$, and $\hat{F}(t)$ do not necessarily converge to the true unknown matrices C, D, E, and F, respectively. Thus, $\dot{\hat{C}}(t) \to 0$, $\dot{\hat{D}}(t) \to 0$, $\dot{\hat{E}}(t) \to 0$, and $\dot{\hat{F}}(t) \to 0$ do not imply $\tilde{C}(t) \to 0$, $\tilde{D}(t) \to 0$, $\tilde{E}(t) \to 0$, and $\tilde{F}(t) \to 0$ but rather $\Delta\hat{C}(t) \to 0$, $\Delta\hat{D}(t) \to 0$, $\Delta\hat{E}(t) \to 0$, and $\Delta\hat{F}(t) \to 0$ as $t \to \infty$.

■

The closed-loop plant with the performance optimizing controller $u_p(t)$ now becomes

$$\dot{x} = (A_m + B\bar{K}_x)x + B_m r + B(K_z z + K_{z_r} z_r) + B\left[u_{ad} + \Theta^{*\top}\Phi(x)\right] \tag{61}$$

Then, the performance optimizing reference model is expressed as

$$\dot{x}_m^* = A_m^* x_m^* + B_m r + BK_z^* z + BK_{z_r}^* z_r \tag{62}$$

where $A_m^* = A_m + B\bar{K}_x^*$. Let $\hat{x}_m(t)$ be the estimate of $x_m^*(t)$. Then, the time-varying performance optimizing reference model is given by

$$\dot{\hat{x}}_m = (A_m + B\bar{K}_x)\hat{x}_m + B_m r + BK_z z + BK_{z_r} z_r \tag{63}$$

Thus, $\hat{x}_m(t) \to x_m^*(t)$ as $t \to \infty$. Let $e(t) = \hat{x}_m(t) - x(t)$ be re-defined as the tracking error based on the time-varying modified reference model. Then, the tracking error equation becomes

$$\dot{e} = (A_m + B_p\bar{K}_x)e + B\tilde{\Theta}^\top\Phi(x) \tag{64}$$

Because of the time-varying reference model, the standard MRAC update law in Eq. (12) must be revised to

$$\dot{\Theta} = -\Gamma\Phi(x)e^\top WB \tag{65}$$

where the time-varying weighting matrix $W(t)$ replaces the constant weighting matrix P in Eq. (12). As a further illustration, the MRAC update law using the optimal control modification[15,16] is given by

$$\dot{\Theta} = -\Gamma\Phi(x)\left[e^\top W - \nu\Phi^\top(x)\Theta B^\top W(A_m + B\bar{K}_x)^{-1}\right]B \tag{66}$$

where $\nu > 0$ is a modification parameter. By setting $\nu = 0$, we recover the new MRAC update law in Eq. (65).

The time-varying weighting matrix $W(t)$is obtained from the time-varying algebraic Riccati equation

$$W\bar{A} + \bar{A}^\top W - WB\bar{R}^{-1}B^\top W + \bar{Q} = 0 \tag{67}$$

Equivalently, the time-varying algebraic Riccati equation can be approximated by a stable time-varying differential Riccati equation integrated forward in time afforded by the infinite-time horizon optimal control solution as follows:

$$\varepsilon\dot{W} = W\bar{A} + \bar{A}^\top W - WB\bar{R}^{-1}B^\top W + \bar{Q} \tag{68}$$

with $W(0) = W_0 > 0$ and $0 < \varepsilon < 1$ is a small positive constant. Using the singular perturbation argument, we transform Eq. (68) into the following form in the stretch time variable $\tau = \frac{t}{\varepsilon}$:

$$\frac{dW}{d\tau} = W\bar{A}(\tau) + \bar{A}(\tau)^\top W - WB\bar{R}(\tau)^{-1}B^\top W + \bar{Q}(\tau) \tag{69}$$

with $W(\tau = 0) = W_0$. By letting $\varepsilon \to 0$ in Eq. (68) which corresponds to $\tau = \mathscr{O}\left(\frac{1}{\varepsilon}\right) \to \infty$, the asymptotic outer solution of Eq. (68) is then obtained from Eq. (67). The inner solution of Eq. (68) corresponding to $t = \mathscr{O}(\varepsilon)$ obtained from Eq. (69) gives

$$\frac{dW_i}{d\tau} \approx W_0\bar{A}_0 + \bar{A}_0^\top W_0 - W_0B\bar{R}_0^{-1}B^\top W_0 + \bar{Q}_0 = \sigma \tag{70}$$

where σ is a small-value matrix. The subscript i denotes the inner solution. The subscript 0 denotes the value of a matrix evaluated at $\tau = 0$ which depends on the initial conditions $\hat{C}(0) = C_0$, $\hat{D}(0) = D_0$, $\hat{E}(0) = E_0$, and $\hat{F}(0) = F_0$. It follows that $\left\|\dot{W}_i\right\| = \frac{\|\sigma\|}{\varepsilon}$. Moreover, the outer solution also yields $\dot{W} = 0$ as $t \to \infty$ corresponding to the constant solution W^* when $\hat{C}(t) \to \bar{C}^*$, $\hat{D}(t) \to \bar{D}^*$, $\hat{E}(t) \to \bar{E}^*$, and $\hat{F}(t) \to \bar{F}^*$. The asymptotic solution of $W(t)$ is given by the outer solution which dominates the inner solution. The stability of the optimal control modification adaptive law can now be shown in the following proof:

Theorem 3: The adaptive law 66 is stable and the tracking error is uniformly ultimately bounded.

Proof: Choose a Lyapunov candidate function

$$V\left(e,\tilde{\Theta}\right) = e^\top We + \operatorname{trace}\left(\tilde{\Theta}^\top \Gamma^{-1}\tilde{\Theta}\right) \tag{71}$$

Then, $\dot{V}\left(e,\tilde{\Theta}\right)$ is evaluated as

$$\dot{V}\left(e,\tilde{\Theta}\right) = -e^\top\left(\dot{W} + WA_m + WB\bar{K}_x + A_m^\top W + \bar{K}_x^\top B^\top W\right)e - 2\nu\Phi^\top(x)\Theta B^\top W\left(A_m + B\bar{K}_x\right)^{-1}B\tilde{\Theta}^\top\Phi(x) \tag{72}$$

Using the asymptotic outer solutions of the singularly perturbed differential Riccati equation (68), we get

$$\begin{aligned}\dot{V}\left(e,\tilde{\Theta}\right) = &-e^\top\left(\bar{Q} + WB\bar{R}^{-1}B^\top W\right)e - 2\nu\Phi^\top(x)\Theta B^\top W\left(A_m + B\bar{K}_x\right)^{-1}B\tilde{\Theta}^\top\Phi(x) \le -c_1\|e\|^2\\ &-\nu c_2\|\Phi(x)\|^2\left(\left\|\tilde{\Theta}\right\| - c_3\right)^2 + \nu c_2 c_3^2\|\Phi(x)\|^2\end{aligned} \tag{73}$$

where $c_1 = \inf_t \lambda_{min}\left(\bar{Q}^* + W^*B\bar{R}^{*-1}B^\top W^*\right) > 0$, $c_2 = \inf_t \lambda_{min}\left(B^\top A_m^{*-\top}\left(\bar{Q}^* + W^*B\bar{R}^{*-1}B^\top W^*\right)A_m^{*-1}B\right) > 0$, and $c_3 = \frac{\sup_t\left\|B^\top W^* A_m^{*-1}B\right\|\|\Theta^*\|}{c_2} > 0$. Then, $\|\Phi(x)\| \le \Phi_0$ for some $0 < \nu < \nu_{max}$. Thus, $\dot{V}\left(e,\tilde{\Theta}\right) \le 0$ outside a compact set. Therefore, the closed-loop system with the optimal control modification is uniformly ultimately bounded with the following ultimate bounds:

$$\|e\| \le \sqrt{\frac{\lambda_{max}(W^*)p^2 + \lambda_{max}(\Gamma^{-1})\alpha^2}{\lambda_{min}(W^*)}} \tag{74}$$

$$\left\|\tilde{\Theta}_a\right\| \le \sqrt{\frac{\lambda_{max}(W^*)p^2 + \lambda_{max}(\Gamma^{-1})\alpha^2}{\lambda_{min}(\Gamma^{-1})}} \tag{75}$$

where $p = \sqrt{\frac{\nu c_2 c_3^2 \Phi_0^2}{c_1}}$ and $\alpha = 2c_3$.

When $\nu = 0$, we recover the new MRAC update law. Then, it can be shown that $\ddot{V}\left(e,\tilde{\Theta}\right)$ is bounded. Invoking the Barbalat's lemma, $\dot{V}\left(e,\tilde{\Theta}\right) \to 0$ which implies $e(t) \to 0$ as $t \to \infty$. Thus, asymptotic tracking is achieved with the time-varying reference model modification by the performance optimizing controller for the new MRAC update law.

III. Application

We will first demonstrate an adaptive control design with a single optimization of the performance signal $y_p(t)$ by a maneuver load alleviation application for a flexible wing Generic Transport Model (GTM) equipped with the Variable Camber Continuous Trailing Edge Flap (VCCTEF)[17] as shown in Fig. 1. One of the performance objectives in flight control is to maintain structural load limits on aircraft wings during a maneuver. Aircraft wings are normally designed to meet certain load factors, typically 2.5 times the aircraft weight for a transport aircraft. The availability of the VCCTEF allows the wing lift distribution to be re-distributed for the maneuver load alleviation control and drag minimization which can be viewed as two important performance optimizing flight control objectives for maintaining structural integrity and improving fuel efficiency in passenger transports. A pull-up maneuver is simulated with the VCCTEF deployed for the maneuver load alleviation and drag minimization while the elevator is deployed for the pitch rate control.

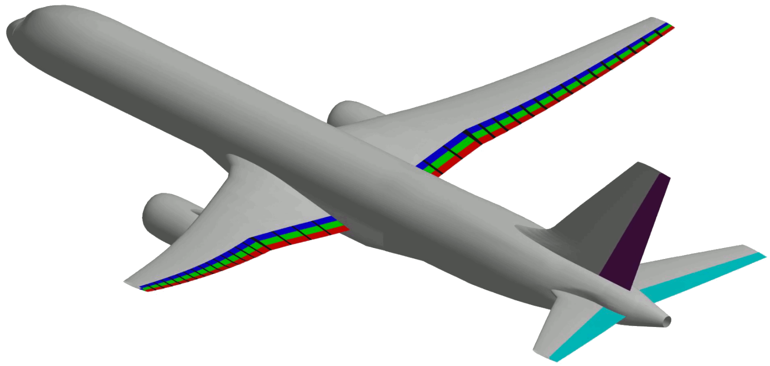


Figure 1. GTM with Variable Camber Continuous Trailing Edge Flap System

The performance optimizing controller $u_p(t)$ comprises 16 individual trailing edge flap segments starting from the Yehudi break, each connected to an adjacent flap by an elastomer transition section as shown in blue in Fig. 1. Due to the stiffness imposed by the elastomer transition section, a constraint on the relative deflection of any two adjacent flaps is imposed on the control input command. A virtual control concept is used whereby the actually flap deflection of each flap is mapped into a mathematically smooth shape function whose coefficients are the virtual control variables. A cubic Chebyshev polynomial is selected as a candidate shape function. Then, the flap deflection of the i-th flap, where $i = 1, 2, \ldots, 16$ starting from the wing root to the wing tip, is expressed as

$$\delta_i = c_0 + c_1 k + c_2\left(2k^2 - 1\right) + c_3\left(4k^3 - 3k\right) \tag{76}$$

where $k = \frac{i-1}{n-1}$, $n = 16$, and $c_j(t)$, $j = 0, 1, 2, 3$ are the virtual control variables.

Let $u_p(t) = c(t) = \begin{bmatrix} c_0(t) & c_1(t) & c_2(t) & c_3(t) \end{bmatrix}^\top \in \mathbb{R}^4$ be a vector of the virtual control inputs. Then, the longitudinal aircraft model is described by

$$\dot{x} = Ax + B_p u_p + B_a\left(u_a + \Theta^{*\top} x\right) \tag{77}$$

where $x(t) = \begin{bmatrix} \frac{\Delta h(t)}{\bar{h}} & \frac{\Delta V(t)}{\bar{V}} & \alpha(t) & q(t) & \theta(t) \end{bmatrix}^\top \in \mathbb{R}^5$ is the aircraft state vector, $u_a(t) = \delta_e(t)$ is the adaptive controller, and $\Theta^* = \begin{bmatrix} 0 & 0 & \theta_\alpha^* & \theta_q^* & 0 \end{bmatrix}^\top \in \mathbb{R}^5$ is a matched uncertainty. The standard aircraft naming convention is used for the incremental altitude $\Delta h(t)$, airspeed $\Delta V(t)$, angle of attack $\alpha(t)$, pitch rate $q(t)$, pitch attitude $\theta(t)$. The matched uncertainty is represented by θ_α^* and θ_q^*. The bar symbol denotes the trim value. We design a pitch attitude controller for the elevator to track the following second-order reference model:

$$\ddot{\theta}_m + 2\zeta\omega_n\dot{\theta}_m + \omega_n^2\theta_m = \omega_n^2 r \tag{78}$$

where ζ is the desired closed-loop damping ratio and ω_n is the desired closed-loop frequency. This reference model is designed to only track a pitch attitude command without any knowledge of the wing root bending moment or the drag force to be minimized during a pitch maneuver. The nominal controller for the elevator is designed as

$$u_{nom} = K_x x + k_r r = k_h\frac{h}{\bar{h}} + k_V\frac{V}{\bar{V}} + k_\alpha\alpha + k_q q + k_\theta\theta + k_r r \tag{79}$$

The initial reference model is designed to be the closed-loop nominal plant with the nominal controller as

$$\dot{x}_m = (A + B_a K_x)x_m + B_a k_r r \tag{80}$$

The nominal controller is augmented by an adaptive controller as

$$u_a = u_{nom} - \theta_\alpha(t)\alpha - \theta_q(t)q \tag{81}$$

where $\theta_\alpha(t)$ and $\theta_q(t)$ are updated from the optimal control modification adaptive law according to Eq. (66).

For the maneuver load alleviation, the performance metric is taken to be the wing root bending moment, denoted as $y_p(t) \in \mathbb{R}$. For the drag minimization, the performance metric is the drag force on the aircraft, denoted as $z_p(t) \in \mathbb{R}$, which can be indirectly measured as a function of the aircraft lift force. We assume that both the wing bending moment and drag measurements are available.

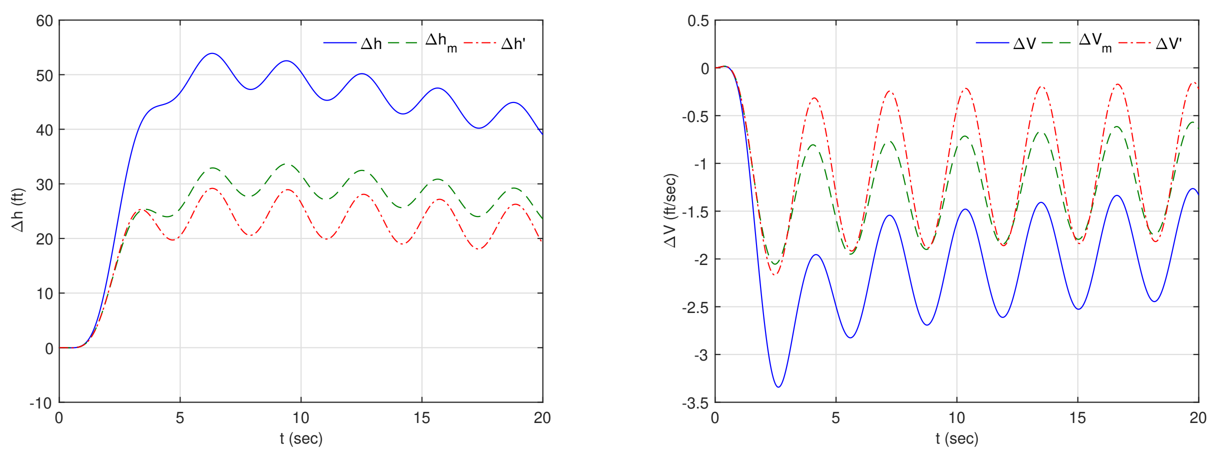
The reference command model is designed as

$$\begin{bmatrix} \dot{z} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} 0 & \omega \\ -\omega & 0 \end{bmatrix} \begin{bmatrix} z \\ r \end{bmatrix} + \begin{bmatrix} 0 \\ \theta_0 \omega \end{bmatrix} \tag{82}$$

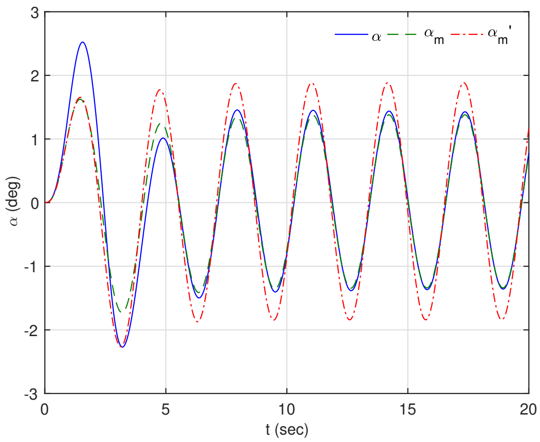
The resulting reference command signal is explicitly obtained as $r(t) = \theta_0 \sin \omega t$ where $\theta_0 = 4^\circ$ and $\omega = 2$ rad/sec. The performance optimizing weighting coefficients for the maneuver load alleviation and drag minimization are selected to be $q_{y_p} = 6 \times 10^{-12}$ and $q_{z_p} = 6 \times 10^{-8}$. The control weighting matrix is selected to be $R = 1000I$. The unknown matrices $\Theta(t)$, $\hat{C}(t)$, $\hat{D}(t)$, $\hat{E}(t)$, and $\hat{F}(t)$ are all initialized with zero values. The adaptation rate matrices are chosen to be $\Gamma = \operatorname{diag}(0, 0, 0.2, 0.5, 0)$, $\Gamma_C = \operatorname{diag}(10, 10000, 2000, 5, 1)$, $\Gamma_D = \operatorname{diag}(300, 400, 300, 500)$, $\Gamma_E = \Gamma_C$, and $\Gamma_F = \Gamma_D$. The optimal control modification adaptive law is implemented with $\nu = 0.01$.

Figures 2 (a)-(e) shows the time histories of the changes in the aircraft altitude $\Delta h(t)$, airspeed $\Delta V(t)$, angle of attack $\alpha(t)$, pitch rate $q(t)$, and pitch attitude $\theta(t)$, respectively. In each plot, three time histories are presented: the solid blue line is the response to performance optimizing adaptive control, the dash green line is the output of the time-varying modified reference model denoted by a symbol with the subscript m, and the dash-dot red line is the response to the standard MRAC with the original linear time-invariant reference model denoted by a prime symbol. The aircraft angle of attack $\alpha(t)$, pitch rate $q(t)$, and pitch attitude $\theta(t)$ all track the time-varying modified reference model very well with the performance optimizing adaptive control. The original reference model is no longer tracked by the performance optimizing adaptive control.

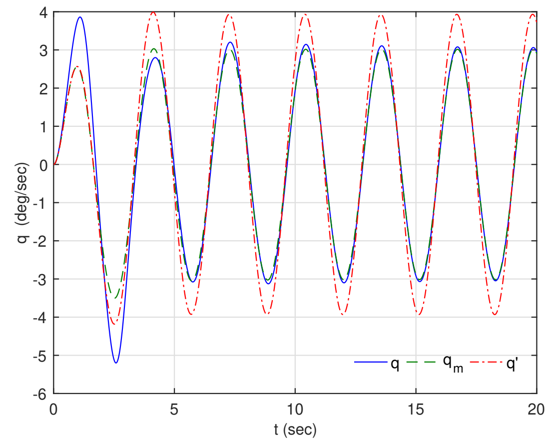
The responses of the wing root bending moment $y_p(t)$ and drag force $z_p(t)$ are shown in Fig. 3(a) and (b), respectively. In each plot, three time histories are presented: the solid blue line is the response to performance optimizing adaptive control, the dash green line is the estimate, denoted by the hat symbol, computed from the least-square gradient adaptive laws, and the dash-dot red line is the response to the standard MRAC with the original reference model.The benefit of the performance optimizing adaptive control can readily be seen in these figures. Without the performance optimization, the standard MRAC with the original reference model produces higher wing root bending moment $y_p(t)$ and drag force $z_p(t)$, shown by the dash-dot red lines, than the performance optimizing adaptive control with time-varying modified reference model, shown by the solid blue and dash green lines. The wing root bending moment is reduced by as much as 41% and the drag force is reduced by as much as 61% due to the performance optimizing adaptive control, as shown in Fig. 3(a) and (b), respectively. The initial drag force produced by the standard MRAC with the original reference model is well over 12,000 lb which is excessive. The estimates $\hat{y}_p(t)$ and $\hat{z}_p(t)$ converge to the measurements $y_p(t)$ and $z_p(t)$ very well.Thus, the effectiveness of the performance optimizing adaptive control is demonstrated.



(a) (b)



(c)



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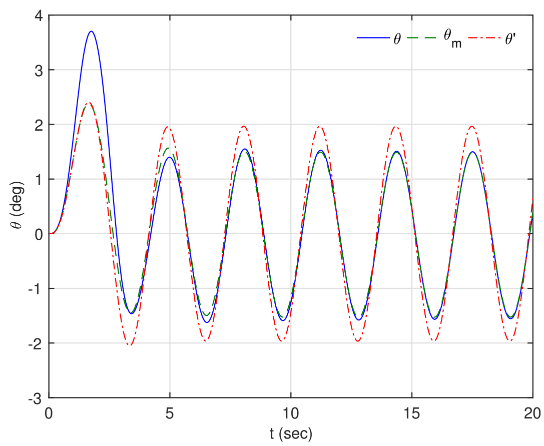
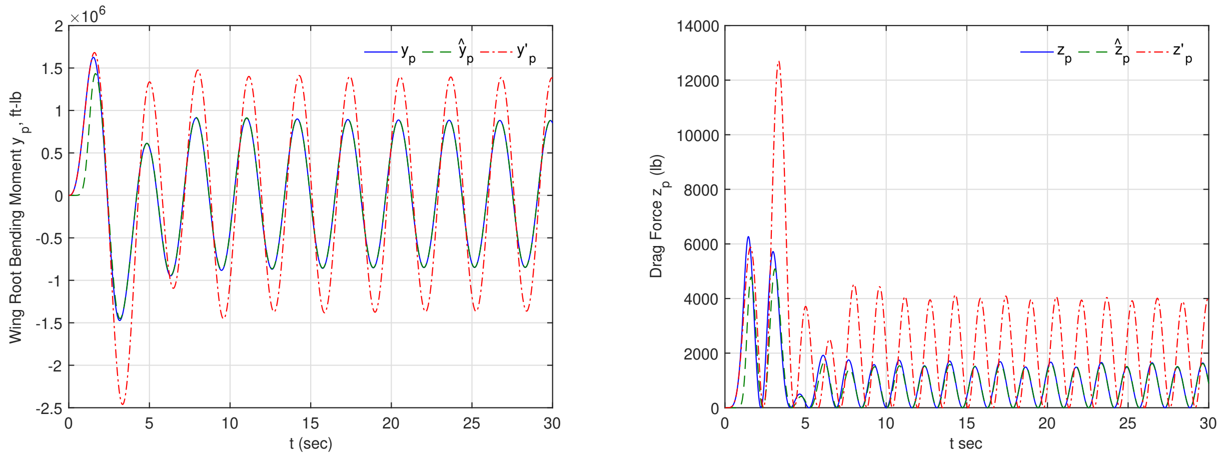


(e)

Figure 2. Aircraft Response to Performance Optimizing Adaptive Control

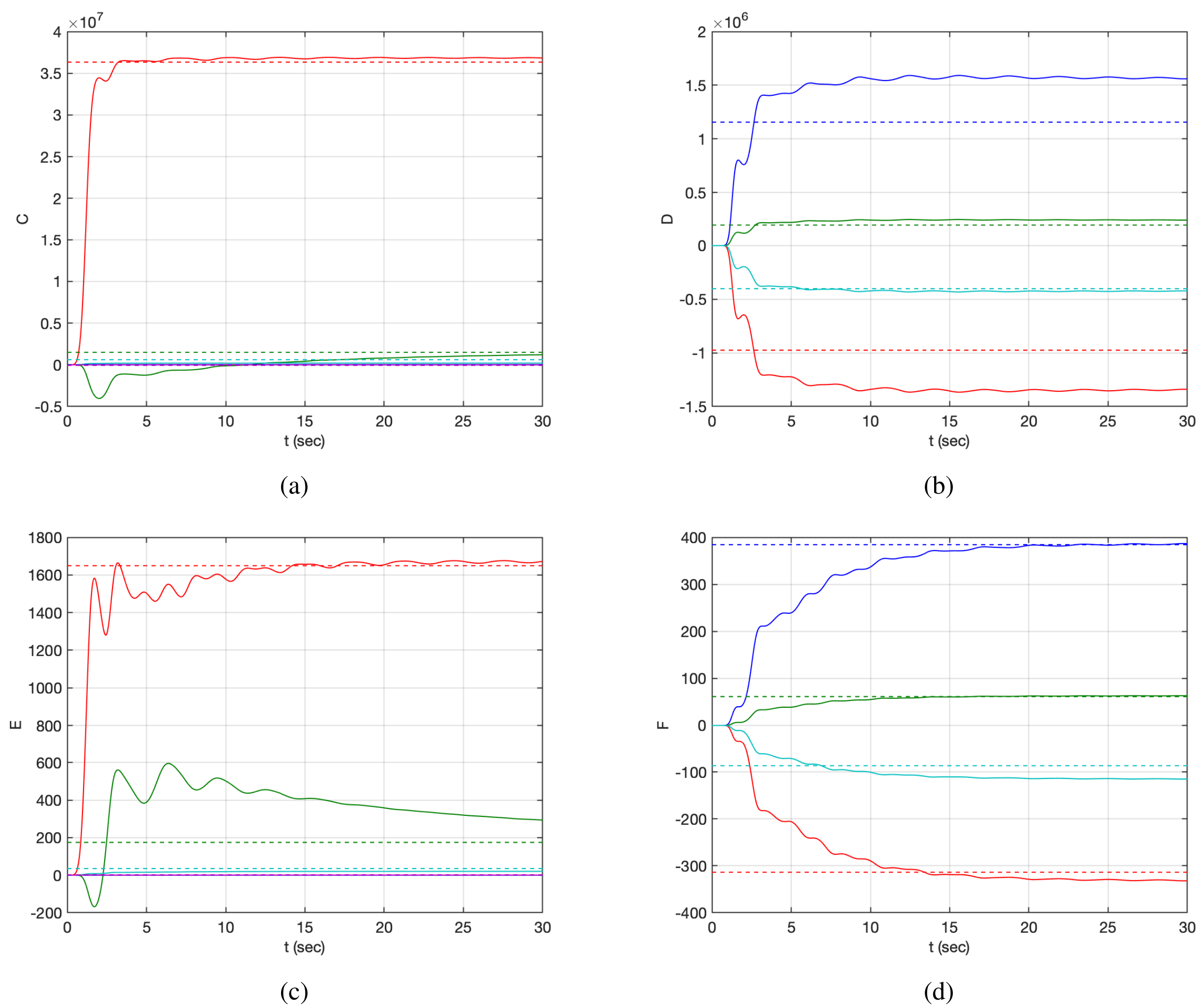
Figures 4(a)-(e) show the time histories of the adaptive parameters $\hat{C}(t)$, $\hat{D}(t)$, $\hat{E}(t)$, $\hat{F}(t)$, and $\Theta(t)$, respectively. The values of adaptive parameters $\hat{C}(t)$ and $\hat{F}(t)$ converge to their steady state values close to the true values. Some of the values of the adaptive parameters $\hat{D}(t)$ and $\hat{E}(t)$ also converge close to their true values. The values of the adaptive parameter $\Theta(t)$ d converge to their true values but seem to converge to some steady state values. The convergence of the adaptive parameters $\Theta(t)$ is affected by the optimal control modification which improves robustness by reducing the initial high frequency oscillations in the signals but only guarantees bounded adaptive parameters. Comparing the convergence of the adaptive parameters $\hat{C}(t)$, $\hat{D}(t)$, $\hat{E}(t)$, and $\hat{F}(t)$ to the convergence of the adaptive parameters $\Theta(t)$, it confirms that in general the least-squares parameter estimation tends to provide better parameter convergence than MRAC.[18]

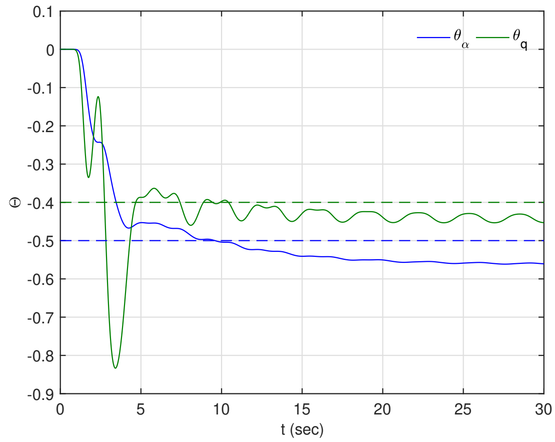
Figures 5(a) and (b) show the control signals of the elevator deflection $\delta_e(t)$ and the VCCTEF deflections $\delta(t)$, respectively. The maximum amplitude of the elevator is 2.5° and the maximum amplitude of the 16 elements of the VCCTEF is 3.5°. All the control signals appear to be well-behaved with no observed high frequency oscillations.



(a) (b)

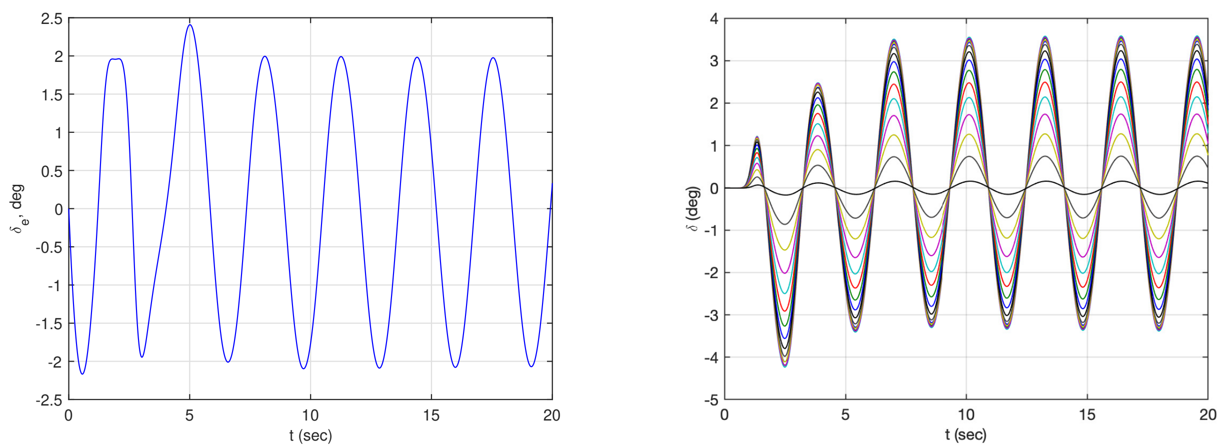
Figure 3. Wing Root Bending Moment and Drag Force Due to Performance Optimizing Adaptive Control





(e)

Figure 4. Adaptive Parameters $\hat{C}(t)$, $\hat{D}(t)$, $\hat{E}(t)$, $\hat{F}(t)$, and $\Theta(t)$



(a) (b)

Figure 5. Elevator Deflection and VCCTEF Deflections

IV. Conclusion

This paper presents a performance optimizing adaptive control method that includes a real-time optimization of two sets of performance metrics in the adaptive law. The adaptive control has two objectives. One objective provides the normal plant adaptation to cancel a matched uncertainty in the plant dynamics in order to track an initial reference model which has no notion of performance optimization. The second objective is dedicated to the performance optimization of the plant. The plant is assumed to have measurements of the performance metrics which has unknown state and control sensitivities. The two sets of performance metrics account for both linear and second-order state and control variations. The performance optimization is formulated as a multi-objective optimal control problem coupled with least-squares adaptive laws for estimating the unknown sensitivities of the performance metrics needed for the optimization. This results in time-varying Riccati equation and Sylvester equation whose solutions provide the control gains to modify the original linear time-invariant reference model into a time-varying reference model. By designing the adaptive control to track the modified time-varying reference model, the objective of performance optimization is met. A flight control simulation of maneuver load alleviation and drag minimization for a passenger transport aircraft demonstrates the effectiveness of the performance optimizing adaptive control in reducing the wing root bending moment by 41% and drag force by 61% during a pitch maneuver.

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