NDARC
NASA Design and Analysis of Rotorcraft

Theory

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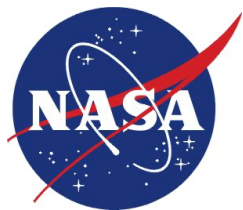- CONTRACTOR REPORT. Scientific and technical findings by NASA-sponsored contractors and grantees.

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Contents

Introduction

The NASA Design and Analysis of Rotorcraft (NDARC) software is an aircraft system analysis tool that supports both conceptual design efforts and technology impact assessments. The principal tasks are to design (or size) a rotorcraft to meet specified requirements, including vertical takeoff and landing (VTOL) operation, and then analyze the performance of the aircraft for a set of conditions. For broad and lasting utility, it is important that the code have the capability to model general rotorcraft configurations, and estimate the performance and weights of advanced rotor concepts. The architecture of the NDARC code accommodates configuration flexibility, a hierarchy of models, and ultimately multidisciplinary design, analysis, and optimization. Initially the software is implemented with low-fidelity models, typically appropriate for the conceptual design environment.

An NDARC job consists of one or more cases, each case optionally performing design and analysis tasks. The design task involves sizing the rotorcraft to satisfy specified design conditions and missions. The analysis tasks can include off-design mission performance calculation, flight performance calculation for point operating conditions, and generation of subsystem or component performance maps. For analysis tasks, the aircraft description can come from the sizing task, from a previous case or a previous NDARC job, or be independently generated (typically the description of an existing aircraft).

The aircraft consists of a set of components, including fuselage, rotors, wings, tails, and propulsion. For each component, attributes such as performance, drag, and weight can be calculated; and the aircraft attributes are obtained from the sum of the component attributes. Description and analysis of conventional rotorcraft configurations is facilitated, while retaining the capability to model novel and advanced concepts. Specific rotorcraft configurations considered are single-main-rotor and tail-rotor helicopter, tandem helicopter, coaxial helicopter, and tiltrotor. The architecture of the code accommodates addition of new or higher-fidelity attribute models for a component, as well as addition of new components.

1–1   Background

The definition and development of NDARC requirements benefited substantially from the experiences and computer codes of the preliminary design team of the U.S. Army Aeroflightdynamics Directorate (AFDD) at Ames Research Center.

In the early 1970s, the codes SSP-1 and SSP-2 were developed by the Systems Research Integration Office (SRIO, in St. Louis) of the U.S. Army Air Mobility Research and Development Laboratory. SSP-1 performed preliminary design to meet specified mission requirements, and SSP-2 estimated the performance for known geometry and engine characteristics, both for single-main-rotor helicopters (ref. 1). Although similar tools were in use in the rotorcraft community, these computer programs were independently developed, to meet the requirements of government analysis. The Advanced Systems Research Office (ASRO, at Ames Research Center) of USAAMRDL produced in 1974 two Preliminary

Systems Design Engineering (PSDE) studies (refs. 2 and 3) using SSP-1 and SSP-2. These two codes were combined into one code called PSDE by Ronald Shinn.

The MIT Flight Transportation Laboratory created design programs for helicopters (ref. 4) and tiltrotors (ref. 5). Michael Scully, who wrote the helicopter design program and was significantly involved in the development of the tiltrotor design program, joined ASRO in 1975, and ideas from the MIT programs began to be reflected in the continuing development of PSDE. An assessment of design trade-offs for the Advanced Scout Helicopter (ASH) used a highly modified version of PSDE (ref. 6).

A DoD Joint Study Group was formed in April 1975 to perform an Interservice Helicopter Commonality Study (HELCOM) for the Director of Defense Research and Engineering. The final HELCOM study report was published in March 1976 (ref. 7). A result of this study was an assessment by ASRO that PSDE needed substantial development, including better mathematical models and better technical substantiation, more flexible mission analysis, and improved productivity for both design and analysis tasks. Thus began an evolutionary improvement of the code, eventually named RASH (after the developer Ronald A. Shinn, as a consequence of the computer system identification of output by the first four characters of the user name). RASH included improvements in flight performance modeling, output depth, mission analysis, parametric weight estimation, design sensitivity studies, off-design cases, and coding style. The code was still only for single-main-rotor helicopters.

In the early 1980s, tool development evolved in two separate directions with the Preliminary Design Team at ASRO. RASH was developed into the HELO (or PDPAC) code, for conventional and compound single-main-rotor helicopters. With the addition of conversion models and wing weight estimation methods (refs. 8 and 9), RASH became the TR code, for tiltrotor aircraft. The JVX Joint Technology Assessment of 1982 utilized the HELO and TR codes. A special version called PDABC, including a weight estimation model for lift-offset rotors (ref. 10), was used to analyze the Advancing Blade Concept. The JVX JTA report (ref. 11) documented the methodology implemented in these codes.

Work in support of the LHX program from 1983 on led to a requirement for maneuver analysis of helicopters and tiltrotors, implemented in the MPP (Maneuver Performance Program) code by John Davis. The core aircraft model in MPP was similar to that in TR and HELO, but the trim strategy in particular was new. A design code does not require extensive maneuver analysis capability, but MPP had an impact on the design code development, with the MPP performance and trim methods incorporated into TR87. The sizing analysis of TR88 and the aircraft flight model from MPP were combined into the VAMP (VSTOL Design and Maneuver Program) code. VAMP combined the capability to analyze helicopters and tiltrotors in a single tool, although the capability of HELO to analyze compound helicopters was not replicated.

In the early 1990s, the RC (RotorCraft) code emerged from the evolution of VAMP, with John Preston as the lead developer (refs. 12 and 13). Some maneuver analysis capabilities from MPP were added, and the analysis capability extended to helicopters. The models were confirmed by comparison with results from TR and HELO. RC was operational by 1994, although HELO and TR continued to be used into the mid-1990s. RC97 was a major version, unifying the tiltrotor and helicopter analyses. The RC code introduced new features and capabilities, and productivity enhancements, as well as coding standards and software configuration control. Special versions of RC were routinely produced to meet the unique requirements of individual projects (such as ref. 14).

NASA, with support from the U.S. Army, in 2005 conducted the design and in-depth analysis of rotorcraft configurations that could satisfy the Vehicle Systems Program technology goals (ref. 15). These

technology goals and accompanying mission were intended to identify enabling technology for civil application of heavy-lift rotorcraft. The emphasis was on efficient cruise and hover, efficient structures, and low noise. The mission specified was to carry 120 passengers for 1200 nm, at a speed of 350 knots and 30,000 ft altitude. The configurations investigated were a Large Civil Tiltrotor (LCTR), a Large Civil Tandem Compound (LCTC), and a Large Advancing Blade Concept (LABC). The results of the NASA Heavy Lift Rotorcraft Systems Investigation subsequently helped define the content and direction of the Subsonic Rotary Wing project in the NASA Fundamental Aeronautics program. The design tool used was the AFDD RC code. This investigation was an example of the role of a rotorcraft sizing code within NASA. The investigation also illustrated the difficulties involved in adapting or modifying RC for configurations other than conventional helicopters and tiltrotors, supporting the requirement for a new tool.

1–2   Requirements

Out of this history, the development of NDARC began in early 2007. NDARC is entirely new software, built on a new architecture for the design and analysis of rotorcraft. From the RC theoretical basis, the parametric weight equations and the Referred Parameter Turboshaft Engine Model were used with only minor changes. Use was also made of the RC component aerodynamic models and rotor performance model. The current users of RC, informed by past and recent applications, contributed significantly to the requirements definition.

The principal tasks are to design (size) rotorcraft to meet specified requirements, and then analyze the performance of the aircraft for a set of flight conditions and missions. Multiple design requirements, from specific flight conditions and various missions, must be used in the sizing task. The aircraft performance analysis must cover the entire spectrum of aircraft capabilities, and allow general and flexible definition of conditions and missions.

For government applications and to support research, it is important to have the capability to model general rotorcraft configurations, including estimates of the performance and weights of advanced rotor concepts. In such an environment, software extensions and modifications are routinely required to meet the unique requirements of individual projects, including introduction of special weight and performance models for particular concepts.

Thus the code architecture must accommodate configuration flexibility and alternate models, including a hierarchy of model fidelity. Although initially implemented with low-fidelity models, typical of the conceptual design environment, ultimately the architecture must allow multidisciplinary design, analysis, and optimization. The component performance and engine models must cover all operating conditions. The software design and architecture must facilitate extension and modification of the software.

Complete and thorough documentation of the theory and its software implementation is essential, to support development and maintenance, and to enable effective use and modification. Most of the history described above supports this requirement by the difficulties encountered in the absence of good documentation. Documentation of the methodology was often prompted only by the need to substantiate conclusions of major technology assessments, and occasionally by the introduction of new users and developers. For a new software implementation of a new architectures, documentation is required from the beginning of the development.
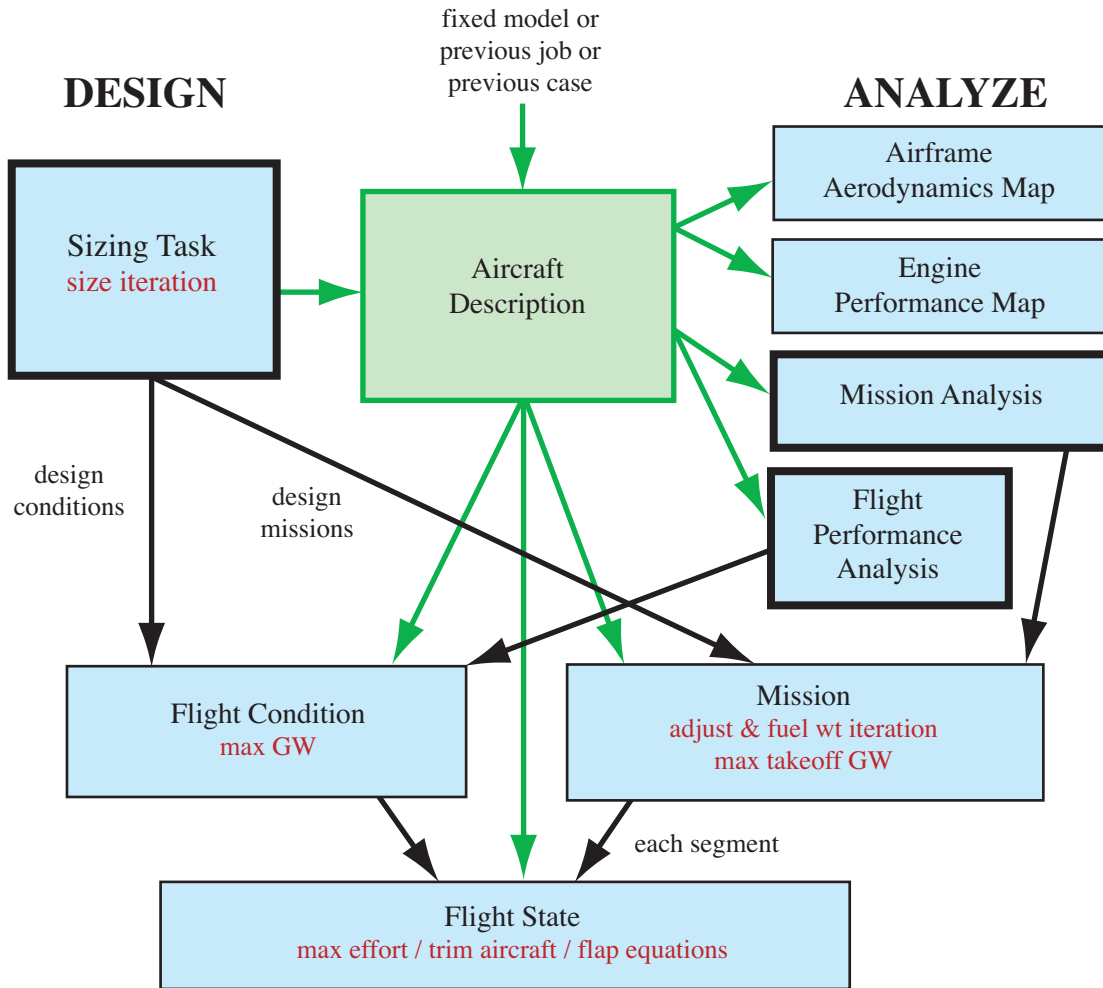
fixed model or
previous job or
previous case

DESIGN   **ANALYZE**

Sizing Task
size iteration

Aircraft
Description

Airframe
Aerodynamics Map

Engine
Performance Map

Mission Analysis

design
conditions

design
missions

Flight
Performance
Analysis

Flight Condition
max GW

Mission
adjust & fuel wt iteration
max takeoff GW

each segment

Flight State
max effort / trim aircraft / flap equations

Figure 1-1. Outline of NDARC tasks.

1–3   Overview

The NDARC code performs design and analysis tasks. The design task involves sizing the rotorcraft to satisfy specified design conditions and missions. The analysis tasks can include off-design mission performance analysis, flight performance calculation for point operating conditions, and generation of subsystem or component performance maps. Figure 1-1 illustrates the tasks. The principal tasks (sizing, mission analysis, and flight performance analysis) are shown in the figure as boxes with heavy borders. Heavy black arrows show control of subordinate tasks.

The aircraft description (fig. 1-1) consists of all the information, input and derived, that defines the aircraft. The aircraft consists of a set of components, including fuselage, rotors, wings, tails, and propulsion. This information can be the result of the sizing task; can come entirely from input, for a fixed model; or can come from the sizing task in a previous case or previous job. The aircraft description information is available to all tasks and all solutions (indicated by light green arrows).

The sizing task determines the dimensions, power, and weight of a rotorcraft that can perform a specified set of design conditions and missions. The aircraft size is characterized by parameters such as design gross weight, weight empty, rotor radius, and engine power available. The relationships between dimensions, power, and weight generally require an iterative solution. From the design flight conditions and missions, the task can determine the total engine power or the rotor radius (or both power and radius can be fixed), as well as the design gross weight, maximum takeoff weight, drive system torque limit, and fuel tank capacity. For each propulsion group, the engine power or the rotor radius can be sized.

Missions are defined for the sizing task and for the mission performance analysis. A mission consists of a number of mission segments, for which time, distance, and fuel burn are evaluated. For the sizing task, certain missions are designated to be used for engine sizing, for design gross weight calculations, for transmission sizing, and for fuel tank sizing. The mission parameters include mission takeoff gross weight and useful load. For specified takeoff fuel weight with adjustable segments, the mission time or distance is adjusted so the fuel required for the mission (burned plus reserve) equals the takeoff fuel weight. The mission iteration is on fuel weight or energy.

Flight conditions are specified for the sizing task and for the flight performance analysis. For the sizing task, certain flight conditions are designated to be used for engine sizing, for design gross weight calculations, for transmission sizing, for maximum takeoff weight calculations, and for antitorque or auxiliary-thrust rotor sizing. The flight condition parameters include gross weight and useful load.

For flight conditions and mission takeoff, the gross weight can be maximized, such that the power required equals the power available.

A flight state is defined for each mission segment and each flight condition. The aircraft performance can be analyzed for the specified state, or a maximum effort performance can be identified. The maximum effort is specified in terms of a quantity such as best endurance or best range, and a variable such as speed, rate of climb, or altitude. The aircraft must be trimmed, by solving for the controls and motion that produce equilibrium in the specified flight state. Different trim solution definitions are required for various flight states. Evaluating the rotor hub forces may require solution of the blade flap equations of motion.

1–4   Terminology

The following terminology is introduced as part of the development of the NDARC theory and software. Relationships among these terms are reflected in figure 1-1.

a) Job: An NDARC job consists of one or more cases.

b) Case: Each case performs design and/or analysis tasks. The analysis tasks can include off-design mission performance calculation, flight performance calculation for point operating conditions, and generation of airframe aerodynamics or engine performance maps.

c) Design Task: Size rotorcraft to satisfy specified set of design flight conditions and/or design missions. Key aircraft design variables are adjusted until all criteria are met. The resulting aircraft description can be basis for the mission analysis and flight performance analysis tasks.

d) Mission Analysis Task: Calculate aircraft performance for one off-design mission.

e) Flight Performance Analysis Task: Calculate aircraft performance for point operating condition.

f) Mission: Ordered set of mission segments, for which time, distance, and fuel burn are evaluated. Gross weight and useful load are specified for the beginning of the mission, and adjusted for fuel burn and useful load changes at each segment. Missions are defined for the sizing task and for the mission performance analysis.

g) Flight Condition: Point operating condition, with specified gross weight and useful load. Flight conditions are specified for the sizing task and for the flight performance analysis.

h) Flight State: Aircraft flight condition, part of definition of each flight condition and each mission segment. Flight state solution involves rotor blade motion, aircraft trim, and perhaps a maximum-effort calculation.

i) Component: The aircraft consists of a set of components, including fuselage, rotors, wings, tails, and propulsion. For each component, attributes such as performance, drag, and weight are calculated.

j) Propulsion: A propulsion group is a set of components and engine groups, connected by a drive system. An engine group consists of one or more engines of a specific type. The components define the power required. The engine groups define the power available. A jet group consists of one or more systems that produce a force on the aircraft. A charge group consists of systems that generate energy for the aircraft. Fuel tank systems are associated with the engine groups, jet groups, and charge groups. Fuel quantity is measured as either weight or energy.

1–5   Analysis Units

The code can use either English or SI units for input, output, and internal calculations. A consistent mass-length-time-temperature system is used, except for weight and power:

	length	mass	time	temperature	weight	power
English:	foot	slug	second	°F	pound	horsepower
SI:	meter	kilogram	second	°C	kilogram	kiloWatt

Weight in the design description is actually mass, with pounds converted to slugs using the reference gravitational acceleration (9.80665 m/sec^2). Gravitational force is the product of the mass and the actual acceleration due to gravity. In addition, the default units for flight conditions and missions are: speed in knots, time in minutes, distance in nautical miles, and rate of climb in feet-per-minute. The user can specify alternate units for these and other quantities.

1–6   Outline of Report

This document provides a complete description of the NDARC theoretical basis and architecture. Chapters 3–5 describe the tasks and solution procedures, the cost model is described in chapter 6, the emissions model is described in chapter 7, and chapters 8–20 present the models for the aircraft and its components. The propulsion system models are described in chapters 15–20. The engine, jet, and charger models are described in chapters 21–28; and the weight models in chapters 29and 31. The accompanying NDARC Input Manual describes the use of the code.

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Nomenclature

The nomenclature for geometry and rotations employs the following conventions. A vector x is a column matrix of three elements, measuring the vector relative to a particular basis (or axes, or frame). The basis is indicated as follows:

a) x^A is a vector measured in axes A;

b) $x^{EF/A}$ is a vector from point F to point E, measured in axes A.

A rotation matrix C is a three-by-three matrix that transforms vectors from one basis to another:

c) C^{BA} transforms vectors from basis A to basis B, so $x^B = C^{BA} x^A$.

The matrix C^{BA} defines the orientation of basis B relative to basis A, so it also may be viewed as rotating the axes from A to B. For a vector u, a cross-product matrix $\widetilde{u}$ is defined as follows:

$$\widetilde{u} = \begin{bmatrix} 0 & -u_3 & u_2 \\ u_3 & 0 & -u_1 \\ -u_2 & u_1 & 0 \end{bmatrix}$$

such that $\widetilde{u}v$ is equivalent to the vector cross-product $\mathbf{u} \times \mathbf{v}$. The cross-product matrix enters the relationship between angular velocity and the time derivative of a rotation matrix:

$$\dot{C}^{AB} = -\widetilde{\omega}^{AB/A} C^{AB} = C^{AB} \widetilde{\omega}^{BA/B}$$

(the Poisson equations). For rotation by an angle α about the $x, y,$ or z axis ($1, 2,$ or 3 axis), the following notation is used:

$$X_\alpha = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & -\sin\alpha & \cos\alpha \end{bmatrix}$$

$$Y_\alpha = \begin{bmatrix} \cos\alpha & 0 & -\sin\alpha \\ 0 & 1 & 0 \\ \sin\alpha & 0 & \cos\alpha \end{bmatrix}$$

$$Z_\alpha = \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Thus for example, $C^{BA} = X_\phi Y_\theta Z_\psi$ means that the axes B are located relative to the axes A by first rotating by angle ψ about the z-axis, then by angle θ about the y-axis, and finally by angle ϕ about the x-axis.

Acronyms

AFDD	U.S. Army Aeroflightdynamics Directorate
ASM	available seat mile
CAS	calibrated airspeed
CG	charge group
CPI	consumer price index
CTM	Cost Too Much (cost model)
EG	engine group
GW	gross weight
IAS	indicated airspeed
IGE	in ground effect
IRP	intermediate rated power
IRS	infrared suppressor
ISA	International Standard Atmosphere
ISO	International Organization for Standardization
JG	jet group
MCP	maximum continuous power
MCT	maximum continuous thrust
MJ	Mega-Joule
MRP	maximum rated power
NDARC	NASA Design and Analysis of Rotorcraft
OEI	one engine inoperative
OGE	out of ground effect
PG	propulsion group
RPJEM	referred parameter jet engine model
RPTEM	referred parameter turboshaft engine model
SDGW	structural design gross weight
SI	Systéme International d'Unités (International System of Units)
SLS	sea level standard
TAS	true airspeed
WMTO	maximum takeoff weight

Weights

W_D	design gross weight
W_E	empty weight
W_{MTO}	maximum takeoff weight
W_{SD}	structural design gross weight
W_G	gross weight, $W_G = W_E + W_{UL} = W_O + W_{\text{pay}} + W_{\text{fuel}}$
W_O	operating weight, $W_O = W_E + W_{FUL}$
W_{UL}	useful load, $W_{UL} = W_{FUL} + W_{\text{pay}} + W_{\text{fuel}}$
W_{pay}	payload
W_{fuel}	fuel weight
W_{FUL}	fixed useful load
W_{burn}	mission fuel burn
W_{vib}	vibration control weight
W_{cont}	contingency weight
χ	technology factor

Fuel Tanks

$W_{\text{fuel}-\text{cap}}$	fuel capacity, maximum usable fuel weight
$E_{\text{fuel}-\text{cap}}$	fuel capacity, maximum usable fuel energy
$V_{\text{fuel}-\text{cap}}$	fuel capacity, volume
N_{auxtank}	number of auxiliary fuel tanks
$W_{\text{aux}-\text{cap}}$	auxiliary fuel tank capacity (weight)
$E_{\text{aux}-\text{cap}}$	auxiliary fuel tank capacity (energy)

Power

P_{reqPG}	power required, propulsion group; $P_{\text{comp}} + P_{\text{xmsn}} + P_{\text{acc}}$
P_{reqEG}	power required, engine group
P_{reqCG}	power required, charge group
P_{avPG}	power available, propulsion group; $\min(\sum f_P P_{avEG}, (\Omega_{\text{prim}}/\Omega_{\text{ref}})P_{DS\text{limit}})$
P_{avEG}	power available, engine group; $(N_{\text{eng}} - N_{\text{inop}})P_{av}$
P_{avCG}	power available, charge group; $(N_{\text{chrg}} - N_{\text{inop}})P_{av}$
P_{comp}	component power required
P_{xmsn}	transmission losses
P_{acc}	accessory power
N_{inop}	number of inoperative systems, engine group or jet group or charge group
$P_{DS\text{limit}}$	drive system torque limit (specified as power limit at reference rotor speed)
$P_{ES\text{limit}}$	engine shaft limit
$P_{RS\text{limit}}$	rotor shaft limit

Engine

P_{eng}	sea level static power available per engine at specified takeoff rating
N_{eng}	number of engines in engine group
P_{av}	power available, installed; $\min(P_a - P_{\text{loss}}, P_{\text{mech}})$
P_a	power available, uninstalled
P_{req}	power required, installed; $P_q - P_{\text{loss}}$
P_q	power required, uninstalled
P_{loss}	installation losses
P_{mech}	mechanical power limit
SP	specific power, $P/\dot{m}$ (conventional units)
sfc	specific fuel consumption, $\dot{w}/P$ (conventional units)
$\dot{m}$	mass flow (conventional units)
$\dot{w}$	fuel flow (conventional units)
$\dot{E}$	energy flow
F_N	net jet thrust
D_{aux}	momentum drag
N	specification turbine speed
SW	specific weight, P/W

Jet

T_{eng}	sea level static thrust available per jet at specified takeoff rating
N_{jet}	number of jets in jet group
T_{avCG}	thrust available, jet group; $(N_{\text{jet}} - N_{\text{inop}})T_{av}$
T_{av}	thrust available, installed; $\min(T_a\eta, T_{\text{mech}})$
T_a	thrust available, uninstalled
T_{reqJG}	thrust required, jet group
T_{req}	thrust required, installed; $T_q\eta$
T_q	thrust required, uninstalled
η	installation losses (efficiency)
T_{mech}	mechanical thrust limit
ST	specific thrust, $T/\dot{m}$ (conventional units)
sfc	specific fuel consumption, $\dot{w}/T$ (conventional units)
$\dot{m}$	mass flow (conventional units)
$\dot{w}$	fuel flow (conventional units)
D_{aux}	momentum drag
SW	specific weight, T/W

Charger

P_{chrg}	sea level static power available per charger at specified takeoff rating
N_{chrg}	number of chargers in charge group
η	installation losses (efficiency)

Tip Speed and Rotation

$V_{\text{tip-ref}}$	reference tip speed, propulsion group primary rotor; each drive state
r	gear ratio; $\Omega_{\text{dep}}/\Omega_{\text{prim}}$ for rotor, $\Omega_{\text{spec}}/\Omega_{\text{prim}}$ for engine
Ω_{prim}	primary rotor rotational speed, $\Omega = V_{\text{tip-ref}}/R$
Ω_{dep}	dependent rotor rotational speed, $\Omega = V_{\text{tip-ref}}/R$
Ω_{spec}	specification engine turbine speed
N_{spec}	specification engine turbine speed (rpm)

Mission

T	mission segment time
D	mission segment distance
dR	mission segment range contribution
E	endurance
R	range
$\dot{w}$	fuel flow

Environment

g	gravitational acceleration
h	altitude
c_s	speed of sound
ρ	density
ν	kinematic viscosity
μ	viscosity
T	temperature, °R or °K
τ	temperature, °F or °C
V_w	wind speed

Axis Systems

I	inertial
F	aircraft
A	component aerodynamic
B	component
V	velocity

Geometry

SL, BL, WL	fixed input position (station line, buttline, waterline) positive aft, right, up; arbitrary origin
$x/L, y/L, z/L$	scaled input position; positive aft, right, up; origin at reference point
L	reference length (fuselage length, rotor radius, or wing span)
x, y, z	calculated position, aircraft axes; positive forward, right, down; origin at reference point for geometry, origin at center of gravity for motion and loads
z^F	component position vector, in aircraft axes, relative reference point
ℓ	length
S_{wet}	wetted area

Motion

ϕ_F, θ_F, ψ_F roll, pitch, yaw angles; orientation airframe axes F relative inertial axes
$\dot{\psi}_F$ turn rate
θ_V, ψ_V climb, sideslip angles; orientation velocity axes V relative inertial axes
v_{AC}^F aircraft velocity
ω_{AC}^F aircraft angular velocity
a_{AC}^F aircraft linear acceleration
n load factor
V aircraft velocity magnitude
V_h horizontal velocity
V_f forward velocity
V_s sideward velocity
V_c climb velocity
V_{cal} calibrated airspeed
V_{ind} indicated airspeed

Aerodynamics and Loads

v component velocity relative air (including interference)
q dynamic pressure, $\frac{1}{2}\rho|v|^2$
α angle of attack, component axes B relative aerodynamic axes A
β sideslip angle, component axes B relative aerodynamic axes A
ℓ_f ratio flap chord to airfoil chord, c_f/c
δ_f flap deflection
F force
M moment
D, Y, L aerodynamic drag, side, lift forces (component aerodynamic axes A)
M_x, M_y, M_z aerodynamic roll, pitch, yaw moments (component aerodynamic axes A)
c_d, c_ℓ section drag, lift coefficients
C_D, C_Y, C_L component drag, side, lift force coefficients
C_ℓ, C_M, C_N component roll, pitch, yaw moment coefficients
D/q drag area, SC_D (S = reference area of component)

Aircraft

DL   disk loading, W_D/A_{ref}
A_{ref} reference rotor area, $\sum f_A A$; typically projected area of lifting rotors
WL   wing loading, W_D/S_{ref}
S_{ref} reference wing area, $\sum S$; sum area all wings
c_{AC} aircraft control
T control matrix
c component control, $c = STc_{AC} + c_0$
α_{tilt} tilt control variable
M aircraft hover figure of merit, $W\sqrt{W/2\rho A_{\mathrm{ref}}}/P$
D_e aircraft effective drag, P/V
L/D_e aircraft effective lift-to-drag ratio, WV/P

Rotor

W/A	disk loading, $W = f_W W_D$
C_W/σ	design blade loading, $W/\rho A V_{\text{tip}}^2 \sigma$ (V_{tip} = hover tip speed)
R	blade radius
A	disk area
σ	solidity (ratio blade area to disk area)
T_{design}	design thrust of antitorque or auxiliary-thrust rotor
r	direction of rotation (1 for counter-clockwise, -1 for clockwise)
r	blade span coordinate
ψ	blade azimuth coordinate
μ	advance ratio
λ	inflow ratio
V_{tip}	tip speed
M_{tip}	tip Mach number
M_{at}	advancing tip Mach number
ν	blade flap frequency (per-rev)
γ	blade Lock number
C_T/σ	thrust coefficient divided by solidity, $T/\rho A (\Omega R)^2 \sigma$
β_c, β_s	longitudinal, lateral flapping (tip-path plane tilt relative shaft)
$\theta_{0.75}$	blade collective pitch angle (at 75% radius)
θ_c, θ_s	lateral, longitudinal blade pitch angle)
H, Y, T	drag, side, thrust force on hub (shaft axes)
M_x, M_y	roll, pitch moment on hub
Q	shaft torque
P_i, P_t, P_o, P_p	induced, interference, profile, parasite power
κ	induced power factor, $P_i = \kappa P_{\text{ideal}}$
$c_{d\text{mean}}$	profile power mean drag coefficient, $C_{Po} = (\sigma/8) c_{d\text{mean}} F_P$
M	rotor hover figure of merit, $T f_D v / P$
L/D_e	rotor effective lift-to-drag ratio, $V L / (P_i + P_o)$
η	propulsive efficiency, $P_p/P = -XV/P$

Wing

W/S	wing loading, $W = f_W W_D$
S	area
b	span
c	chord, S/b
$A\!R$	aspect ratio, b^2/S

Tasks

The NDARC code performs design and analysis tasks. The design task involves sizing the rotorcraft to satisfy specified design conditions and missions. The analysis tasks can include mission performance analysis, flight performance calculation for point operating conditions, and generation of subsystem or component performance maps.

3–1   Size Aircraft for Design Conditions and Missions

3-1.1   Sizing Method

The sizing task determines the dimensions, power, and weight of a rotorcraft that can perform a specified set of design conditions and missions. The aircraft size is characterized by parameters such as design gross weight (W_D) or weight empty (W_E), rotor radius (R), and engine power available (P_{eng}). The relationships between dimensions, power, and weight generally require an iterative solution. From the design flight conditions and missions, the task can determine the total engine power or the rotor radius (or both power and radius can be fixed), as well as the design gross weight, maximum takeoff weight, drive system torque limit, and fuel tank capacity. For each propulsion group, the engine power or the rotor radius can be sized:

> a) Engine power: Determine P_{eng}, for fixed R. The engine power is the maximum of the power required for all designated sizing flight conditions and sizing missions (typically including vertical flight, forward flight, and one-engine inoperative). Hence the engine power is changed by the ratio $\max(P_{reqPG}/P_{avPG})$ (excluding flight states for which zero power margin is calculated, such as maximum gross weight or maximum effort). This approach is the one most commonly used for the sizing task.

> b) Rotor radius: Determine R for input P_{eng}. The maximum power required for all designated sizing flight conditions and sizing missions is calculated, and then the rotor radius determined such that the power required equals the input power available. The change in radius is estimated as $R = R_{\mathrm{old}}\sqrt{P_{reqPG}/P_{avPG}}$ (excluding flight states for which zero power margin is calculated, such as maximum gross weight or maximum effort). For multi-rotor aircraft, the radius can be fixed rather than sized for some rotors.

Alternatively, P_{eng} and R can be input rather than sized. For each engine group that consumes power, the design power can be sized:

> Determine P_{eng}. The design power is the maximum of the power required for all designated sizing flight conditions and sizing missions. Hence the design power

is changed by the ratio $\max(P_{reqEG}/P_{avEG})$ (excluding flight states for which zero power margin is calculated).

For each jet group, the design thrust can be sized:

Determine T_{jet}. The design thrust is the maximum of the thrust required for all designated sizing flight conditions and sizing missions. Hence the design thrust is changed by the ratio $\max(T_{reqJG}/T_{avJG})$ (excluding flight states for which zero thrust margin is calculated).

For each charge group, the design power can be sized:

Determine P_{chrg}. The design power is the maximum of the power required for all designated sizing flight conditions and sizing missions. Hence the design power is changed by the ratio $\max(P_{reqCG}/P_{avCG})$ (excluding flight states for which zero power margin is calculated).

Aircraft parameters can be determined by a subset of the design conditions and missions:

a) Design gross weight W_D: maximum gross weight from designated conditions and missions (for which gross weight is not fixed).
b) Maximum takeoff gross weight W_{MTO}: maximum gross weight from designated conditions (for which gross weight is not fixed).
c) Drive system torque limit $P_{DS\mathrm{limit}}$: maximum torque from designated conditions and missions (for each propulsion group; specified as power limit at reference rotor speed).
d) Fuel tank capacity: maximum fuel weight $W_{\mathrm{fuel-cap}}$ or energy $E_{\mathrm{fuel-cap}}$ from designated missions (without auxiliary tanks).
e) Antitorque or auxiliary thrust rotor design thrust T_{design}: maximum rotor thrust from designated conditions and missions.

Alternatively, these parameters can be fixed at input values. The design gross weight (W_D) can be fixed. The weight empty can be fixed or scaled with W_D, which is implemented by changing the contingency weight.

A successive substitution method is used for the sizing iteration, with an input tolerance ϵ. Relaxation is applied to P_{eng} or R, T_{jet}, P_{chrg}, W_D, W_{MTO}, $P_{DS\mathrm{limit}}$, $W_{\mathrm{fuel-cap}}$ or $E_{\mathrm{fuel-cap}}$, and T_{design}. Convergence is tested in terms of these parameters, and the aircraft weight empty W_E. Two successive substitution loops are used. The outer loop is an iteration on performance: engine power or rotor radius, jet thrust, charger power. The inner loop is an iteration on parameters: W_D, W_{MTO}, $P_{DS\mathrm{limit}}$, $W_{\mathrm{fuel-cap}}$ or $E_{\mathrm{fuel-cap}}$, and T_{design}. Either loop can be absent, depending on the definition of the size task.

For each flight condition and each mission, the gross weight and useful load are specified. The gross weight can be input, maximized, or fallout. For flight conditions, the payload or fuel weight can be specified, and the other calculated; or both payload and fuel weight specified, with gross weight fallout. For missions, the payload or fuel weight can be specified, the other fallout, and then time or distance of mission segments adjusted; or fuel weight calculated from mission, and payload fallout; or both payload and fuel weight specified (or payload specified and fuel weight calculated from mission), with gross weight fallout. For each flight condition and mission segment, the following checks are performed:

a) The power required does not exceed the power available: $P_{reqPG} \leq (1 + \epsilon)P_{avPG}$ (for each propulsion group).

b) The torque required does not exceed the drive system limit: $P_{reqPG}/\Omega \leq (1 + \epsilon)P_{DS\text{limit}}/\Omega_{\text{prim}}$ (for each propulsion group). Rotor shaft torque and engine shaft torque are also checked.

c) The jet thrust required does not exceed the thrust available: $T_{reqJG} \leq (1+\epsilon)T_{avJG}$ (for each jet group).

d) The charger power required does not exceed the power available: $P_{reqCG} \leq (1 + \epsilon)P_{avCG}$ (for each charge group).

e) The fuel weight does not exceed the fuel capacity: $W_{\text{fuel}} \leq (1 + \epsilon)(W_{\text{fuel}-\text{cap}} + \sum N_{\text{auxtank}}W_{\text{aux}-\text{cap}})$ (including auxiliary tanks).

These checks are performed using an input tolerance ϵ.

Sizing flight conditions typically include takeoff (hover or specified vertical rate of climb), one-engine inoperative, cruise or dash, perhaps transmission, and perhaps mission midpoint hover. Sizing missions typically include a design mission and a mission to determine fuel tank capacity.

3-1.2 Component Sizing

3-1.2.1 Propulsion System

The engine size is described by the power P_{eng}, which is the sea-level static power available per engine at a specified takeoff rating. The number of engines N_{eng} is specified for each engine group.

If the sizing task determines the engine power for a propulsion group, the power P_{eng} of at least one engine group is found (including the first engine group). The total power required is $P_{PG} = r \sum N_{\text{eng}}P_{\text{eng}}$, where $r = \max(P_{reqPG}/P_{avPG})$. The sized power is $P_{\text{sized}} = P_{PG} - \sum_{\text{fixed}} N_{\text{eng}}P_{\text{eng}}$, where the sum is over the engine groups for which the power is fixed. Then the sized engine power is $P_{\text{eng}} = f_n P_{\text{sized}}/N_{\text{eng}}$ for the n-th engine group (with f_n an input ratio and $f_1 = \sum_{n \neq 1,\text{sized}} f_n$ for the first group).

The jet size is described by the thrust T_{jet}, which is the sea-level static thrust available per jet at a specified takeoff rating. The number of jets N_{jet} is specified for each jet group. If the sizing task determines the jet thrust for a jet group, the thrust T_{jet} is scaled by the factor $r = \max(T_{reqJG}/T_{avJG})$.

The charger size is described by the power P_{chrg}, which is the sea-level static power available. The number of systems N_{chrg} is specified for each charge group. If the sizing task determines the charger power for a charge group, the power P_{chrg} is scaled by the factor $r = \max(P_{reqCG}/P_{avCG})$.

3-1.2.2 Main Rotor

The main-rotor size is defined by the radius R or disk loading W/A, thrust-weighted solidity σ, hover tip speed V_{tip}, and blade loading $C_W/\sigma = W/\rho A V_{\text{tip}}^2 \sigma$. With more than one main-rotor, the disk loading and blade loading are obtained from an input fraction of design gross weight, $W = f_W W_D$. The air density ρ for C_W/σ is obtained from a specified takeoff condition.

If the rotor radius is fixed for the sizing task, three of (R or W/A), C_W/σ, V_{tip}, and σ are input, and the other parameters are derived. Optionally the radius can be calculated from a specified ratio to the radius of another rotor.

If the sizing task determines the rotor radius (R and W/A), then two of C_W/σ, V_{tip}, and σ are input,

and the other parameter is derived. The radius can be sized for just a subset of the rotors, with fixed radius for the others. The radii of all sized rotors are changed by the same factor.

3-1.2.3 Antitorque or Auxiliary Thrust Rotor

For antitorque and auxiliary thrust rotors, three of (R or W/A), C_W/σ, V_{tip}, and σ are input, and the other parameters are derived. Optionally the radius can be calculated from a specified ratio to the radius of another rotor. Optionally the radius can be scaled with the main-rotor radius. The disk loading and blade loading are based on $f_T T_{\text{design}}$, where f_T is an input factor and T_{design} is the maximum thrust from designated design conditions and missions.

3-1.2.4 Wing

The wing size is defined by the wing area S or wing loading W/S, span (perhaps calculated from other geometry), chord, and aspect ratio. With more than one wing, the wing loading is obtained from an input fraction of design gross weight, $W = f_W W_D$.

Two of the following parameters are input: area (or wing loading), span, chord, and aspect ratio; the other parameters are derived. Optionally the span can be calculated from the rotor radius, fuselage width, and clearance (typically used for tiltrotors). Optionally the span can be calculated from a specified ratio to the span of another wing.

3-1.2.5 Fuel Tank

The fuel tank capacity $W_{\text{fuel}-\text{cap}}$ (maximum usable fuel weight) or $E_{\text{fuel}-\text{cap}}$ (maximum usable fuel energy) is determined from designated sizing missions. The maximum mission fuel required, $W_{\text{fuel}-\text{miss}}$ or $E_{\text{fuel}-\text{miss}}$ (excluding reserves and any fuel in auxiliary tanks, and ignoring any refueling), gives

$$W_{\text{fuel}-\text{cap}} = \max(f_{\text{fuel}-\text{cap}}W_{\text{fuel}-\text{miss}}, W_{\text{fuel}-\text{miss}} + W_{\text{reserve}})$$
$$E_{\text{fuel}-\text{cap}} = \max(f_{\text{fuel}-\text{cap}}E_{\text{fuel}-\text{miss}}, E_{\text{fuel}-\text{miss}} + E_{\text{reserve}})$$

or from mission fuel plus reserves

$$W_{\text{fuel}-\text{cap}} = d_{\text{fuel}-\text{cap}} + f_{\text{fuel}-\text{cap}}(W_{\text{fuel}-\text{miss}} + W_{\text{reserve}})$$
$$E_{\text{fuel}-\text{cap}} = d_{\text{fuel}-\text{cap}} + f_{\text{fuel}-\text{cap}}(E_{\text{fuel}-\text{miss}} + E_{\text{reserve}})$$

or from maximum fuel quantity in the tank during the mission

$$W_{\text{fuel}-\text{cap}} = d_{\text{fuel}-\text{cap}} + f_{\text{fuel}-\text{cap}}W_{\text{fuel}-\text{max}}$$
$$E_{\text{fuel}-\text{cap}} = d_{\text{fuel}-\text{cap}} + f_{\text{fuel}-\text{cap}}E_{\text{fuel}-\text{max}}$$

where $f_{\text{fuel}-\text{cap}} \geq 0$ is an input factor and $d_{\text{fuel}-\text{cap}} \geq 0$ is an input increment. Optionally the maximum mission battery discharge power gives P_{cap}, from which $E_{\text{fuel}-\text{cap}} = \max(E_{\text{fuel}-\text{cap}}, (e_{\text{tank}}/\pi_{\text{tank}})P_{\text{cap}})$ (MJ from kW). Alternatively, the fuel tank capacity $W_{\text{fuel}-\text{cap}}$ or $E_{\text{fuel}-\text{cap}}$ can be input.

3-1.2.6 Weights

The weight empty W_E can be calculated, or input, or scaled with design gross weight: $W_E = d_{WE} + f_{WE}W_D$. Fixed or scaled weight empty is implemented by adjusting the contingency weight.

The structural design gross weight W_{SD} and maximum takeoff weight W_{MTO} can be input, or specified as an increment d plus a fraction f of a weight W:

$$W_{SD} = d_{SDGW} + f_{SDGW}W = \begin{cases} d_{SDGW} + f_{SDGW}W_D \\ d_{SDGW} + f_{SDGW}(W_D - W_{\text{fuel}} + f_{\text{fuel}}W_{\text{fuel-cap}}) \\ d_{SDGW} + f_{SDGW}W_{MTO} \end{cases}$$

$$W_{MTO} = d_{WMTO} + f_{WMTO}W = \begin{cases} d_{WMTO} + f_{WMTO}W_D \\ d_{WMTO} + f_{WMTO}(W_D - W_{\text{fuel}} + W_{\text{fuel-cap}}) \\ d_{WMTO} + f_{WMTO}W_{SD} \end{cases}$$

This convention allows the weights to be input directly ($f = 0$), or scaled with a design weight. For W_{SD}, W is the design gross weight W_D, or W_D adjusted for a specified fuel state (input fraction of fuel capacity), or the maximum takeoff weight W_{MTO}. Alternatively, W_{SD} can be calculated as the gross weight at a designated sizing flight condition. For W_{MTO}, W is the design gross weight W_D, or W_D adjusted for maximum fuel capacity, or the structural design gross weight W_{SD}. Alternatively, W_{MTO} can be calculated as the maximum gross weight possible at a designated sizing flight condition.

3-1.2.7   Drive System Limit

The drive system limit is defined as a power limit, $P_{DS\text{limit}}$. The limit is properly a torque limit, $Q_{DS\text{limit}} = P_{DS\text{limit}}/\Omega_{\text{ref}}$, but is expressed as a power limit for clarity. The drive system limit can be specified as follows (with f_{limit} an input factor):

    a) Input $P_{DS\text{limit}}$.
    b) From the engine takeoff power limit, $P_{DS\text{limit}} = f_{\text{limit}} \sum N_{\text{eng}}P_{\text{eng}}$ (summed over all engine groups).
    c) From the power available at the transmission sizing conditions and missions, $P_{DS\text{limit}} = f_{\text{limit}}(\Omega_{\text{ref}}/\Omega_{\text{prim}}) \sum N_{\text{eng}}P_{av}$ (largest of all conditions and segments).
    d) From the power required at the transmission sizing conditions and missions, $P_{DS\text{limit}} = f_{\text{limit}}(\Omega_{\text{ref}}/\Omega_{\text{prim}}) \sum N_{\text{eng}}P_{req}$ (largest of all conditions and segments).

The drive system limit is a limit on the entire propulsion system. To account for differences in the distribution of power through the drive system, limits are also used for the torque of each rotor shaft ($P_{RS\text{limit}}$) and of each engine group ($P_{ES\text{limit}}$). The engine shaft limit is calculated as for the drive system limit, without the sum over engine groups. The rotor shaft limit is either input or calculated from the rotor power required at the transmission sizing flight conditions. The power limit is associated with a reference rotational speed, and when applied the limit is scaled with the rotational speed of the flight state. The rotation speed for the drive system limit $P_{DS\text{limit}}$ is the hover speed of the primary rotor of the propulsion group (for the first drive state). The rotation speed for the engine shaft limit $P_{ES\text{limit}}$ is the corresponding engine turbine speed. The rotation speed for the rotor shaft limit $P_{RS\text{limit}}$ is the corresponding speed of that rotor.

The drive system limits can be specified for several levels, analogous to engine ratings. The limit $P_{DS\text{limit}}$ is associated with the maximum continuous rating (MCQ or MCP). An alternate rating changes the torque limit by the factor x. Typically $x > 1$ for ratings associated with short duration operation. The torque limit is calculated from $Q_{\text{limit}} = Q/x$ for the flight condition or mission segment. The torque limit is applied as $Q = xQ_{\text{limit}}$.

3–2   Mission Analysis

For the mission analysis, the fuel weight or payload weight is calculated. Power required, torque (drive system, engine shaft, and rotor shaft), and fuel weight are then verified to be within limits. Missions can be fixed or adjustable.

3–3   Flight Performance Analysis

For each performance flight condition, the power required is calculated or maximum gross weight is calculated. Power required, torque (drive system, engine shaft, and rotor shaft), and fuel weight are then verified to be within limits.

3–4   Maps

3-4.1   Engine Performance

The engine performance can be calculated for a specified range of power, altitude, and speed.

3-4.2   Airframe Aerodynamics

The airframe aerodynamic loads can be calculated for a specified range of angle of attack, sideslip angle, and control angles. The aerodynamic analysis evaluates the component lift, drag, and moments given the velocity. The aircraft velocity is $v_{AC}^F = C^{FA}(v\,0\,0)^T$; interference velocity from the rotors is not considered. From the angle of attack α and sideslip angle β, the transformation from wind axes to airframe axes is $C^{FA} = Y_\alpha Z_{-\beta}$ (optionally $C^{FA} = Z_{-\beta}Y_\alpha$ can be used, for better behavior in sideward flight). The loads are summed in the airframe axes (with and without tail loads), and then the wind axis loads are:

$$F^A = \begin{pmatrix} -D \\ Y \\ -L \end{pmatrix} = C^{AF}F^F \qquad\qquad M^A = \begin{pmatrix} M_x \\ M_y \\ M_z \end{pmatrix} = C^{AF}M^F$$

The center of action for the total loads is the fuselage location z_{fuse}. The ratio of the loads to dynamic pressure is required, so a nominal velocity $v = 100$ (ft/sec or m/sec) and sea level standard density are used.

Operation

4–1 Flight Condition

Flight conditions are specified for the sizing task and for the flight performance analysis. For each condition, a flight state is also defined. For the sizing task, certain flight conditions are designated for engine sizing, design gross weight calculations, transmission sizing, maximum takeoff weight calculations, or rotor thrust sizing. The flight condition parameters include gross weight and useful load. The gross weight can be specified as follows, consistent with the sizing method:

a) Design gross weight, W_D (calculated or input).

b) Structural design gross weight, W_{SD}, or maximum takeoff weight, W_{MTO} (which may depend on W_D).

c) Function of W_D: $W = d + fW_D$ (with d an input weight and f an input factor).

d) Function of W_{SD} ($W = d + fW_{SD}$); or function of W_{MTO} ($W = d + fW_{MTO}$).

e) Input W.

f) Gross weight from specified mission segment or flight condition; or function of source ($W = d + fW_{\text{source}}$).

g) Gross weight maximized, such that power required equals specified power: $P_{reqPG} = fP_{avPG} + d$, with d an input power and f an input factor; in general, $\min((fP_{avPG} + d) - P_{reqPG}) = 0$, minimum over all propulsion groups; default $d = 0$ and $f = 1$ gives zero power margin, $\min(P_{avPG} - P_{reqPG}) = 0$.

h) Gross weight maximized, such that thrust required equals specified thrust: $T_{reqJG} = fT_{avJG} + d$, with d an input thrust and f an input factor; in general, $\min((fT_{avJG} + d) - T_{reqJG}) = 0$, minimum over all jet groups; default $d = 0$ and $f = 1$ gives zero thrust margin, $\min(T_{avJG} - T_{reqJG}) = 0$.

i) Gross weight maximized, such that transmission torque equals limit: zero torque margin, $\min(P_{\text{limit}} - P_{req}) = 0$ (mininum over all propulsion groups, engine groups, and rotors).

j) Gross weight maximized, such that power required equals specified power, or thrust required equals specified thrust, or transmission torque equals limit (most restrictive).

k) Gross weight fallout from input payload and fuel weights: $W_G = W_O + W_{\text{pay}} + W_{\text{fuel}}$.

Only the last five options are available for W_D design conditions in the sizing task. Optionally the maximized gross weight can be limited. The gross weight can be obtained from a mission segment only for the sizing task. Optionally the altitude can be obtained from the specified mission segment or flight condition. The secant method or the method of false position is used to solve for the maximum gross

weight. A tolerance ϵ and a perturbation Δ are specified.

The useful load can be specified as follows, consistent with the sizing method and the gross weight specification.

       a) Input payload weight W_{pay}, fuel weight fallout: $W_{\text{fuel}} = W_G - W_O - W_{\text{pay}}$.
       b) Input fuel weight W_{fuel}, payload weight fallout: $W_{\text{pay}} = W_G - W_O - W_{\text{fuel}}$.
       c) Input payload and fuel weights, gross weight fallout (must match gross weight option): $W_G = W_O + W_{\text{pay}} + W_{\text{fuel}}$.

The input fuel weight is $W_{\text{fuel}} = \min(d_{\text{fuel}} + f_{\text{fuel}} W_{\text{fuel-cap}}, W_{\text{fuel-cap}}) + \sum N_{\text{auxtank}} W_{\text{aux-cap}}$. For fallout fuel weight, N_{auxtank} is changed (optionally only increased). If the auxiliary tank weight is greater than the increment in fuel weight needed, then the fallout fuel weight $W_{\text{fuel}} = W_G - W_O - W_{\text{pay}}$ can not be achieved; in such a case, the fuel weight is capped at the maximum fuel capacity and the payload weight changed instead. The fixed useful load can have increments, including crew weight increment; equipment weight increment; and installed folding, wing, wing extension, and other kits. These increments are reflected in the fallout weight. If the motive device burns energy not weight, then the fuel weight is zero and the input fuel energy is $E_{\text{fuel}} = \min(d_{\text{fuel}} + f_{\text{fuel}} E_{\text{fuel-cap}}, E_{\text{fuel-cap}}) + \sum N_{\text{auxtank}} E_{\text{aux-cap}}$.

4–2 Mission

Missions are defined for the sizing task and for the mission performance analysis. A mission consists of a specified number of mission segments. A flight state is defined for each mission segment. For the sizing task, certain missions are designated for engine sizing, design gross weight calculations, transmission sizing, or fuel tank sizing. The mission parameters include mission takeoff gross weight and useful load. The gross weight can be specified as follows, consistent with the sizing method:

       a) Design gross weight, W_D (calculated or input).
       b) Structural design gross weight, W_{SD}, or maximum takeoff weight, W_{MTO} (which may depend on W_D).
       c) Function of W_D: $W = d + fW_D$ (with d an input weight and f an input factor).
       d) Function of W_{SD}, $W = d + fW_{SD}$; or function of W_{MTO}, $W = d + fW_{MTO}$.
       e) Input W.
       f) Gross weight maximized at specified mission segments, such that power required equals specified power: $P_{reqPG} = fP_{avPG} + d$, with d an input power and f an input factor; in general, $\min((fP_{avPG} + d) - P_{reqPG}) = 0$, minimum over all propulsion groups; default $d = 0$ and $f = 1$ gives zero power margin, $\min(P_{avPG} - P_{reqPG}) = 0$.
       g) Gross weight maximized at specified mission segments, such that thrust required equals specified thrust: $T_{reqJG} = fT_{avJG} + d$, with d an input power and f an input factor; in general, $\min((fT_{avJG} + d) - T_{reqJG}) = 0$, minimum over all jet groups; default $d = 0$ and $f = 1$ gives zero thrust margin, $\min(T_{avJG} - T_{reqJG}) = 0$.
       h) Gross weight maximized at specified mission segments, such that transmission torque equals limit: zero torque margin, $\min(P_{\text{limit}} - P_{req}) = 0$ (mininum over all propulsion groups, engine groups, and rotors).
       i) Gross weight maximized at specified mission segments, such that power required equals specified power, or thrust required equals specified thrust, or transmission torque equals limit (most restrictive).

j) Gross weight fallout from input initial payload and fuel weights: $W_G = W_O + W_{\mathrm{pay}} + W_{\mathrm{fuel}}$.

k) Gross weight fallout from input initial payload weight and calculated mission fuel weight: $W_G = W_O + W_{\mathrm{pay}} + W_{\mathrm{fuel}}$.

If maximum gross weight is specified for more than one mission segment, then the minimum takeoff gross weight increment is used; so the power or torque margin is zero for the critical segment and positive for other designated segments. Only the last six options are available for W_D design conditions in the sizing task. Optionally the maximized gross weight can be limited. The secant method or the method of false position is used to solve for the maximum gross weight. A tolerance ϵ and a perturbation Δ are specified.

The useful load can be specified as follows, consistent with the sizing method and the gross weight specification:

a) Input initial payload weight W_{pay}, fuel weight fallout: $W_{\mathrm{fuel}} = W_G - W_O - W_{\mathrm{pay}}$.

b) Input fuel weight W_{fuel}, initial payload weight fallout: $W_{\mathrm{pay}} = W_G - W_O - W_{\mathrm{fuel}}$.

c) Calculated mission fuel weight, initial payload weight fallout: $W_{\mathrm{pay}} = W_G - W_O - W_{\mathrm{fuel}}$.

d) Input payload and fuel weights, takeoff gross weight fallout (must match gross weight option): $W_G = W_O + W_{\mathrm{pay}} + W_{\mathrm{fuel}}$.

e) Input payload weight and calculated mission fuel weight, takeoff gross weight fallout (must match gross weight option): $W_G = W_O + W_{\mathrm{pay}} + W_{\mathrm{fuel}}$.

The input fuel weight is $W_{\mathrm{fuel}} = \min(d_{\mathrm{fuel}} + f_{\mathrm{fuel}} W_{\mathrm{fuel-cap}}, W_{\mathrm{fuel-cap}}) + \sum N_{\mathrm{auxtank}} W_{\mathrm{aux-cap}}$; if the fuel weight is fallout, then this is the initial value for the mission iteration. If the fuel weight is not calculated from the mission, then the mission is changed. The fixed useful load can have increments, including installed folding kits; other increments are specified for individual mission segments. If the motive device burns energy not weight, then the fuel weight is zero and the input fuel energy is $E_{\mathrm{fuel}} = \min(d_{\mathrm{fuel}} + f_{\mathrm{fuel}} E_{\mathrm{fuel-cap}}, E_{\mathrm{fuel-cap}}) + \sum N_{\mathrm{auxtank}} E_{\mathrm{aux-cap}}$.

The takeoff gross weight is evaluated at the start of the mission, perhaps maximized for zero power margin at a specified mission segment (either takeoff conditions or midpoint). Then the aircraft is flown for all segments. For calculated mission fuel weight, the fuel weight at takeoff is set equal to the fuel required for the mission (burned plus reserve; less added by refuel). For specified takeoff fuel weight with adjustable segments, the mission time or distance is adjusted so the fuel required for the mission (burned plus reserve) equals the takeoff fuel weight. The mission iteration is thus on mission fuel weight or energy. Range credit segments (defined below) can also require an iteration. A successive substitution method is used if an iteration is required, with a tolerance ϵ specified. The iteration to maximize takeoff gross weight could be an outer loop around the mission iteration, but instead it is executed as part of the mission iteration. At the specified mission segment, the gross weight is maximized for zero power margin, and the resulting gross weight increment added to the takeoff gross weight for the next mission iteration. Thus takeoff gross weight is also a variable of the mission iteration.

Each mission consists of a specified number of mission segments. The following segment types can be specified:

a) Taxi or warm-up (fuel burned but no distance added to range).

b) Distance: fly segment for specified distance (calculate time).

c) Time: fly segment for specified time (calculate distance).

d) Hold: fly segment for specified time (loiter, so fuel burned but no distance added to range).

e) Climb: climb or descend from present altitude to next segment altitude (calculate time and distance).

f) Spiral: climb or descend from present altitude to next segment altitude (fuel burned but no distance added to range).

g) Fuel: use or replace specified fuel amount (calculate time and distance).

h) Burn: use or replace specified fuel amount (calculate time but no distance added to range).

For each mission segment a payload weight can be specified; or a payload weight change can be specified, as an increment from the initial payload or as a fraction of the initial payload. If the payload is calculated from the number of passengers, then for each mission segment a change in the number of passengers can be specified.

The number of auxiliary fuel tanks can change with each mission segment: N_{auxtank} is changed based on the fuel weight (optionally only increased relative to the input number at takeoff, optionally fixed during mission). For input fuel weight, N_{auxtank} is specified at takeoff. For fallout fuel weight, the takeoff fuel weight is changed for the auxiliary fuel tank weight given N_{auxtank} (fixed $W_G - W_{\mathrm{pay}} = W_O + W_{\mathrm{fuel}}$). If the auxiliary tank weight is greater than the increment in fuel weight needed, then the fallout fuel weight $W_{\mathrm{fuel}} = W_G - W_O - W_{\mathrm{pay}}$ can not be achieved; in such a case, the fuel weight is capped at the maximum fuel capacity and the takeoff payload weight changed instead. For fuel tank design missions, N_{auxtank} and fuel tank capacity is determined from W_{fuel}. Optionally the aircraft can refuel (either on the ground or in the air) at the start of a mission segment, by either filling all tanks to capacity or adding a specified fuel weight. Optionally fuel can be dropped at the start of a mission segment. The fuel weight or energy should be constrained to be non-negative for all mission segments. The fixed useful load can have changes, including crew weight increment, equipment weight increment, and installed wing extension and other kits.

For calculation of the time or distance in a mission segment, a headwind or tailwind can be specified. The wind velocity is a linear function of altitude h: $V_w = \pm(\max(0, d_{\mathrm{wind}} + f_{\mathrm{wind}}h))$, with the plus sign for a headwind and the minus sign for a tailwind. The wind speed is V_w, and the ground speed is $V_g = V_h - V_w$.

Mission fuel reserves can be specified in several ways for each mission. Fuel reserves can be defined in terms of specific mission segments, for example 200 miles plus 20 minutes at speed for best endurance. Fuel reserves can be an input fraction of the fuel burned by all (except reserve) mission segments, so $W_{\mathrm{fuel}} = (1 + f_{\mathrm{res}})W_{\mathrm{burn}}$ or $E_{\mathrm{fuel}} = (1 + f_{\mathrm{res}})E_{\mathrm{burn}}$. Fuel reserves can be an input fraction of the fuel capacity, so $W_{\mathrm{fuel}} = W_{\mathrm{burn}} + f_{\mathrm{res}}W_{\mathrm{fuel-cap}}$ or $E_{\mathrm{fuel}} = E_{\mathrm{burn}} + f_{\mathrm{res}}E_{\mathrm{fuel-cap}}$. If more than one criterion for reserve fuel is specified, the maximum reserve is used. Time and distance in reserve segments are not included in endurance and range.

To facilitate specification of range, range calculated for a group of segments (typically climb and descent segments) can be credited to a designated distance segment. For mission analysis, missions can be fixed or adjustable. In an adjustable mission, the fuel is input, so the time or distance in specified segments is adjusted based on the calculated fuel burned. If more than one segment is adjusted, all must be distance or all must be time or hold. Each segment can have only one special designation: reserve,

adjustable, or range credit.

A segment with a large distance, time, or altitude change can be split into several segments, for more accurate calculation of the performance and fuel burned. The number of segments n can be input, or calculated from an input increment Δ: $n = [x/\Delta] + 1$, where the brackets indicate integer truncation, and x is the total distance, time, or altitude change. Then the change for each split segment is $\Delta = x/n$.

Table 4-1 summarizes the time T, distance D, and range dR calculations for each segment. The segment fuel burned is $dW_{\text{burn}} = T\dot{w}$, where $\dot{w}$ is the fuel flow; and $dE_{\text{burn}} = T\dot{E}$, where $\dot{E}$ is the energy flow. The horizontal velocity is V_h, and the vertical velocity (climb or descent) is V_c. The altitude at the start of the segment is h, and at the end of the segment (start of next segment) h_{end}. The air distance is calculated from the time and speed (D/V_h), without the wind speed.

To use or replace fuel (weight or energy), the increment is specified in terms of the capacity $(dW_{\text{burn}} = d_{\text{tank}} + f_{\text{tank}} W_{\text{fuel-cap}})$ or the current fuel $(dW_{\text{burn}} = d_{\text{tank}} + f_{\text{tank}} W_{\text{fuel}})$. Alternatively, a target fuel is specified and the increment calculated from the current fuel. The tank is charged if the energy rate is negative. The segment time T is the minimum of $dW_{\text{burn}}/\dot{w}$ or $dE_{\text{burn}}/\dot{E}$ for all fuel tank systems.

In an adjusted mission, the distances or times are changed at the end of the mission such that the sum of the fuel burned increments will equal the difference between takeoff fuel weight (plus any added fuel) and the calculated mission fuel: $\sum dW_{\text{burn}} = \sum \dot{w}\, dT = \sum \dot{w}\, dD/(V_h - V_w) = \Delta W_{\text{fuel}}$. The increments are apportioned among the adjusted segments by the factor f, determined from the ratio of the input distances or times: $dD = f\Delta D$ or $dT = f\Delta T$. Hence $\Delta D = \Delta W_{\text{fuel}}/\left(\sum f\dot{w}/(V_h - V_w)\right)$ or $\Delta T = \Delta W_{\text{fuel}}/\left(\sum f\dot{w}\right)$. The approach is similar if fuel energy is used, not weight. For a segment that is a source of range credit, the range increment is set to zero and the distance D is added to D_{other} of the destination segment. For the destination segment, the range contribution remains fixed at the input value, but the time and hence fuel burned are calculated from $(\text{dist} - D_{\text{other}})$. It is necessary to separately accumulate D_{other} from earlier segments and D_{other} from later segments; D_{other} from later segments are estimated initially from the last iteration. At the end of the mission, the times and fuel burned are recalculated for all range credit destination segments.

The segment time, distance, and fuel burned are evaluated by integrating over the segment duration. This integration can be performed by using the horizontal velocity, climb velocity, and fuel flow obtained for the flight state with the gross weight and altitude at the start of the segment; or at the middle of the segment; or the average of the segment start and segment end values (trapezoidal integration). The gross weight at the segment middle equals the gross weight at the segment start, less half the segment fuel burned (obtained from the previous mission iteration). The gross weight at the segment end equals the gross weight at the segment start, less the segment fuel burned. With trapezoidal integration, for the output the flight state is finally evaluated at the segment middle.

The mission endurance (block time), range, and fuel burned are $E = \sum T$, $R = \sum dR$, $W_{\text{burn}} = \sum dW_{\text{burn}}$ (sum over all non-reserve segments). The reserve fuel from mission segments is $W_{\text{res}} = \sum dW_{\text{burn}}$ (sum over all reserve segments). Optionally the reserve fuel is the maximum of that from mission segments and the fraction $f_{\text{res}} W_{\text{burn}}$, or the fraction $f_{\text{res}} W_{\text{fuel-cap}}$. The calculated mission fuel is then $W_{\text{fuel}} = W_{\text{burn}} + W_{\text{res}}$.

Table 4-1. Mission segment calculations.

segment kind	time T	distance D	range dR
taxi	time	0	D
distance	$D/(V_h - V_w)$	dist	D
time	time	$T(V_h - V_w)$	D
hold	time	0	D
climb	$(h - h_{\text{end}})/V_c$	$T(V_h - V_w)$	D
spiral	$(h - h_{\text{end}})/V_c$	0	D
fuel	$dW_{\text{burn}}/\dot{w}$ or $dE_{\text{burn}}/\dot{E}$	$T(V_h - V_w)$	D
burn	$dW_{\text{burn}}/\dot{w}$ or $dE_{\text{burn}}/\dot{E}$	0	D
range credit			
source	T	D	0
destination	$D/(V_h - V_w)$	$\text{dist} - D_{\text{other}}$	dist
adjusted			
distance	$T + dD/(V_h - V_w)$	$D + dD = D + f\Delta D$	D_{new}
time	$T + dT = T + f\Delta T$	$D + dT(V_h - V_w)$	D_{new}
hold	$T + dT = T + f\Delta T$	0	D_{new}

The average fuel flow is W_{burn}/E (lb/hr) and the average energy flow is E_{burn}/E (MJ/hr or kW), where E is the block time. The average specific range is R/W_{burn} (nm/lb or nm/kg) and R/E_{burn} (nm/MJ or nm/kWh), where R is the range. A fuel efficiency measure for the mission is the product of the payload and range, divided by the fuel weight: $e = W_{\text{pay}}R/W_{\text{burn}}$ (ton-nm/lb or ton-nm/kg) or $e = W_{\text{pay}}R/E_{\text{burn}}$ (ton-nm/MJ or ton-nm/kWh). A productivity measure for the mission is $p = W_{\text{pay}}V/W_O$ (ton-kt/lb or ton-kt/kg), where W_O is the operating weight and V the block speed; or $p = W_{\text{pay}}V/W_{\text{burn}}$ (ton-kt/lb or ton-kt/kg), or $p = W_{\text{pay}}V/E_{\text{burn}}$ (ton-kt/MJ or ton-kt/kWh). The Bréguet range equation $R = RF\ln(W_0/W_1)$ is obtained by integrating $dR = -RF(dW/W)$ for constant range factor

$$RF = \frac{L/D_e}{\text{sfc}} = \frac{WV/P}{\text{sfc}}$$

Constant RF implies operation at constant $L/D_e = WV/P$. The endurance $E = EF\ln(W_0/W_1)$ is obtained by integrating $dE = -EF(dW/W)$ for constant endurance factor $EF = (W/P)/(\text{sfc})$. Constant EF implies operation at constant W/P (not constant $C_L^{3/2}/C_D$ as for an airplane). It follows that overall range and endurance factors can be calculated from the mission performance:

$$RF = \frac{R}{\ln W_0/W_1}$$

$$EF = \frac{E}{\ln W_0/W_1}$$

where $W_0 = W_{to}$ is the takeoff weight, and $W_1 = W_{to} - W_{\text{burn}}$. If energy is burned, not weight, $dR = (V/\dot{E})dE = RF\,dE$ and $dE = (1/\dot{E})dE = EF\,dE$ integrate to $R = RF\,E_{\text{burn}}$ and $E = EF\,E_{\text{burn}}$; and the overall range and endurance factors can be calculated from the mission performance: $RF = R/E_{\text{burn}}$ and $EF = E/E_{\text{burn}}$. In addition, the efficiency metrics can be based on the equivalent fuel burned $E_{\text{burn}}/e_{\text{ref}}$.
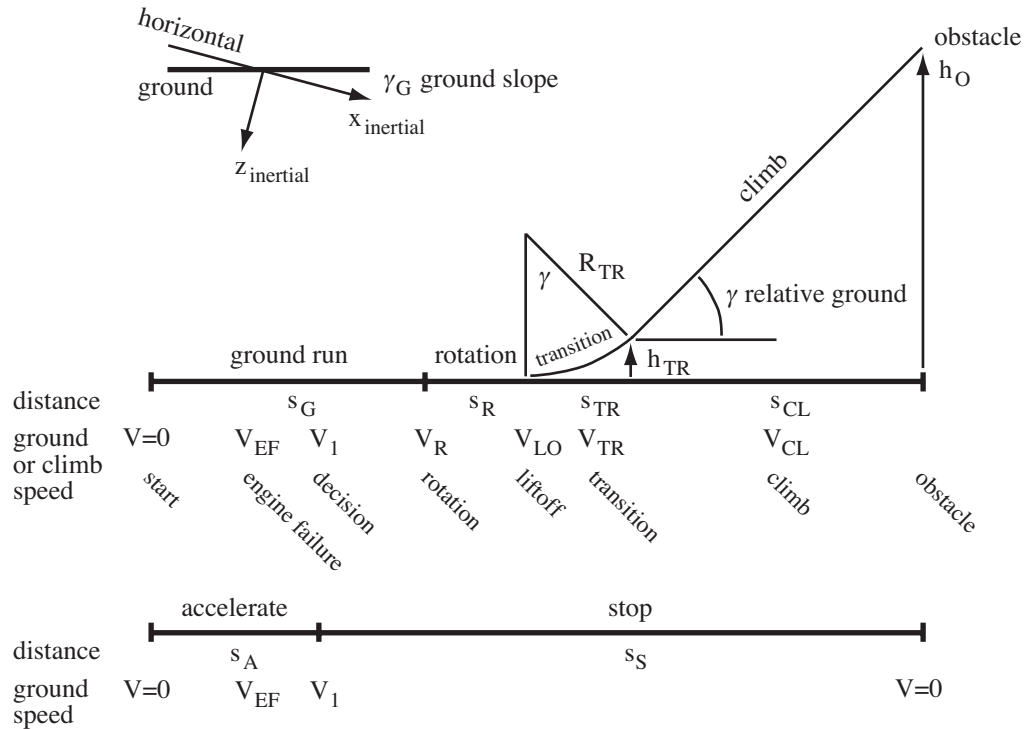
Figure 4-1. Takeoff distance and accelerate-stop distance elements.

4–3   Takeoff Distance

The takeoff distance can be calculated, either as ground run plus climb to clear an obstacle or accelerate-stop distance in case of engine failure. The obstacle height h_o is typically 35 ft for commercial transport aircraft, or 50 ft for military aircraft and general aviation. This calculation allows determination of the balanced field length: engine failure at critical speed, such that the distance to clear the obstacle equals the distance to stop. Landing and VTOL takeoff calculations are not implemented, as these are best solved as an optimal control problem.

The takeoff distance consists of a ground run, from zero ground speed to liftoff speed V_{LO}, perhaps including engine failure at speed V_{EF}; then rotation, transition, and climb; or decelerate to stop. Figure 4-1 describes the elements of the takeoff distance and the accelerate-stop distance, with the associated speeds. The ground is at angle γ_G relative to the horizontal (inertial axes), with γ_G positive for takeoff up hill. The takeoff profile is defined in terms of ground speed or climb speed, input as calibrated airspeed (CAS). The aircraft speed relative to the air is obtained from the ground speed, wind, and ground slope. The aircraft acceleration as a function of ground speed is integrated to obtain the ground distance, as well as the time, height, and fuel burned. Usually the speed increases from the start to liftoff (or engine failure), but the calculated acceleration depends on the flight state specification. The analysis checks for consistency of the input velocity and the calculated acceleration (on the ground), and for consistency of the input height and input or calculated climb angle (during climb).

The takeoff profile consists of a set of mission segments. The first segment is at the start of the takeoff, $V = 0$. Subsequent segments correspond to the ends of the integration intervals. The last

segment has the aircraft at the required obstacle height, or stopped on the ground. The mission can consist of just one takeoff, more than one takeoff, or both takeoff and non-takeoff segments. Takeoff segments contribute to the mission fuel burned, but do not contribute to the mission time, distance, or range. The takeoff distance calculation is performed for a set of adjacent segments, the first segment specified as the takeoff start, and the last segment identified as before a non-takeoff segment or before another takeoff start. The takeoff distance is calculated if a liftoff segment (with V_{LO}) is specified; otherwise the accelerate-stop distance is calculated. Table 4-2 summarizes the mission segments for takeoff calculations. There can be only one liftoff, engine failure, rotation, and transition segment (or none). The engine failure segment must occur before the liftoff segment. Rotation and transition segments must occur after liftoff. All ground run segments must be before liftoff, and all climb segments must be after liftoff. Takeoff segments (except start, rotation, and transition) can be split, in terms of height for climb and in terms of velocity for other segments. Splitting the takeoff or engine failure segment produces additional ground run segments. Separately defining multiple ground run, climb, or brake segments allows configuration variation during the takeoff.

Each takeoff segment requires that the flight state specify the appropriate configuration, trim option, and maximum effort. In particular, the number of inoperative engines for a segment is part of the flight state specification, regardless of whether or not an engine failure segment is defined. The engine failure segment (if present) serves to implement a delay in decision after failure: for a time t_1 after engine failure, the engine rating, power fraction, and friction of the engine failure segment are used (so the engine failure segment corresponds to conditions before failure). The number of inoperative engines specified must be consistent with the presence of the engine failure segment. The takeoff is assumed to occur at fixed altitude (so the maximum-effort variable can not be altitude). The flight state velocity specification is superseded by the ground or climb speed input for the takeoff segment. The flight state specification of height above ground level is superseded by the height input for the takeoff segment.

The ground distance, time, height, and fuel burned are calculated for each takeoff segment. The takeoff distance or accelerate-stop distance is the sum of the ground distance of all segments. Takeoff segments do not contribute to mission time, distance, or range.

Table 4-2. Mission segments for takeoff calculation.

takeoff distance		accelerate-stop distance	
start	$V = 0$	start	$V = 0$
ground run	V	ground run	V
engine failure	V_{EF}	engine failure	V_{EF}
ground run	V	brake	V
liftoff	V_{LO}	brake	$V = 0$
rotation	V_R		
transition	V_{TR}		
climb, to h	V_{CL}		
climb, to h_o	V_{CL}		

4-3.1   Ground Run

The takeoff starts at zero ground speed and accelerates to liftoff ground speed V_{LO} (input as CAS). Possibly an engine failure speed $V_{EF} < V_{LO}$ is specified. Start, liftoff, and engine failure segments

designate events, but otherwise are analyzed as ground run segments. The decision speed V_1 is t_1 seconds after engine failure (typically $t_1 = 1$ to 2 sec). Up to t_1 after engine failure, conditions of the engine failure segment are used (so the engine failure segment corresponds to conditions before failure). The aircraft acceleration is obtained from the thrust minus drag ($T - D$ in airplane notation), plus a friction force proportional to the weight on wheels ($W - L$ in airplane notation):

$$Ma = T - D - \mu(W - L) = \sum F_x - \mu \sum F_z$$

from the force components in ground axes (rotated by the ground slope angle γ_G from inertial axes). Table 4-3 gives typical values of the friction coefficient μ. The velocity of the aircraft relative to the air is obtained from the ground velocity V, wind velocity V_w (assumed parallel to the ground here), and the ground slope: $V_h = (V + V_w)\cos\gamma_G$ and $V_c = (V + V_w)\sin\gamma_G$. The takeoff configuration is specified, including atmosphere, in ground effect, gear down, power rating, nacelle tilt, flap setting, and number of inoperative engines. An appropriate trim option is specified, typically fixed attitude with longitudinal force trimmed using collective, for a given longitudinal acceleration. Perhaps the net aircraft yaw moment is trimmed with pedal. The maximum-effort condition is specified: maximum longitudinal acceleration (ground axes) for zero power margin. The aircraft acceleration as a function of ground speed is integrated to obtain the segment time, ground distance, height, and fuel burned:

$$t_G = \int dt = \int \frac{dt}{dv}\, dv = \int \frac{dv}{a} = \sum_{\text{seg}} \frac{1}{2}\left(\frac{1}{a_2} + \frac{1}{a_1}\right)(v_2 - v_1) = \sum_{\text{seg}} \Delta t$$

$$s_G = \int v\, dt = \int v\, \frac{dt}{dv}\, dv = \int \frac{v\, dv}{a} = \int \frac{d(v^2)}{2a} = \sum_{\text{seg}} \frac{1}{2}\left(\frac{1}{2a_2} + \frac{1}{2a_1}\right)(v_2^2 - v_1^2) = \sum_{\text{seg}}\left(\frac{v_2 + v_1}{2}\right)\Delta t$$

$$h_G = 0$$

$$w_G = \int \dot{w}_f\, dt = \sum_{\text{seg}}\left(\frac{\dot{w}_{f2} + \dot{w}_{f1}}{2}\right)\Delta t$$

Trapezoidal integration is used; each segment corresponds to the end of an integration integral.

Table 4-3. Typical friction coefficient μ.

surface	rolling	braking
dry and hard	0.03–0.05	0.30–0.50
grass	0.08	0.20
ice	0.02	0.06–0.10

4-3.2 Brake

After engine failure, the aircraft can decelerate to a stop. The operating engines are at idle. Reverse thrust is not permitted for the accelerate-stop distance calculation. The braking configuration is specified. Typically no trim option is executed; rather the aircraft has fixed attitude with controls for zero rotor thrust (such as zero collective and pedal). The aircraft acceleration as a function of ground speed is integrated, as for ground run.

4-3.3   Rotation

Rotation occurs at speed V_R; usually $V_R = V_{LO}$ is used. The duration t_R is specified, then $s_R = V_R t_R$, $h_R = 0$, and $w_R = \dot{w}_f t_R$ are the ground distance, height, and fuel burned. Typically $t_R = 1$ to 3 sec.

4-3.4   Transition

Transition from liftoff to climb is modeled as a constant load factor pull-up to the specified climb angle γ, at speed V_{TR}. Usually $V_{TR} = V_{LO}$ is used, and typically $n_{TR} \cong 1.2$. From the load factor $n_{TR} = 1 + V_{TR}^2/gR_{TR}$, the flight path radius is $R_{TR} = V_{TR}^2/(g(n_{TR}-1))$ and the pitch rate is $\dot{\theta} = V_{TR}/R_{TR}$. Then

$$t_{TR} = \gamma/\dot{\theta} = \gamma \frac{R_{TR}}{V_{TR}} = \gamma \frac{V_{TR}}{g(n_{TR} - 1)}$$

$$s_{TR} = R_{TR} \sin \gamma$$

$$h_{TR} = R_{TR}(1 - \cos \gamma)$$

$$w_{TR} = \dot{w}_f t_{TR}$$

are the time, ground distance, height, and fuel burned.

4-3.5   Climb

Climb occurs at an angle γ relative to the ground and air speed V_{CL}, from the transition height h_{TR} to the obstacle height h_o (perhaps in several climb segments). The climb configuration is specified, including atmosphere, in ground effect, gear down or retracted, power rating, nacelle tilt, flap setting, and number of inoperative engines. An appropriate trim option is specified, typically aircraft force and moment trimmed with attitude and controls. The climb angle and air speed can be fixed or a maximum-effort condition can be specified. The maximum-effort options are fixed air speed and maximum rate of climb for zero power margin; or airspeed for best climb rate or best climb angle with maximum rate of climb for zero power margin. Not implemented is a maximum-effort calculation of maximum flight path acceleration for zero power margin, for specified climb angle; this calculation would require integration of the acceleration as a function of flight speed. For the climb segment, the input V_{CL} is the magnitude of the aircraft velocity relative to the air, and the climb angle relative to the horizon is $\theta_V = \gamma + \gamma_G$. Hence from the maximum-effort calculation, the climb angle relative to the ground is $\gamma = \theta_V - \gamma_G$ and the ground speed is $V_{\text{ground}} = V_{CL} \cos \gamma - V_w$ (from the wind speed V_w). Then

$$t_{CL} = s_{CL}/V_{\text{ground}}$$

$$s_{CL} = (h - h_{\text{last}})/\tan \gamma$$

$$h_{CL} = h$$

$$w_{CL} = \dot{w}_f t_{CL}$$

are the time, ground distance, height, and fuel burned.

4–4   Flight State

A flight state is defined for each flight condition (sizing task design conditions and flight performance analysis), and for each mission segment. The following parameters are required:

a) Speed: flight speed and vertical rate of climb, with the following options:

1) Specify horizontal speed (or forward speed or velocity magnitude), rate of climb (or climb angle), and sideslip angle.

2) Hover or vertical flight (input vertical rate of climb; climb angle 0 or ±90 deg).

3) Left or right sideward flight (input velocity and rate of climb; sideslip angle ±90 deg).

4) Rearward flight (input velocity and rate of climb; sideslip angle 180 deg).

b) Aircraft motion:

1) Pitch and roll angles (Aircraft values or flight state input; initial values for trim variables, fixed otherwise).

2) Turn, pull-up, or linear acceleration.

c) Altitude: For mission segment, optionally input, or from last mission segment; climb segment end altitude from next segment.

d) Atmosphere:

1) Standard day, polar day, tropical day, or hot day at specified altitude.
2) Standard day, polar day, or tropical day plus temperature increment.
3) Standard day, polar day, or tropical day and specified temperature.
4) Input density and temperature.
5) Input density, speed of sound, and viscosity.

e) Headwind or tailwind, with the wind velocity a linear function of altitude.

f) Height of landing gear above ground level. Landing gear state (extended or retracted).

g) Aircraft control state: input, or conversion schedule.

h) Aircraft control values (Aircraft values or flight state input; initial values for trim variables, fixed otherwise).

i) Aircraft center-of-gravity position (increment or input value).

For each propulsion group, the following parameters are required:

i) Drive system state.

j) Rotor tip speed for primary rotor:

1) Input.
2) Reference.
3) Conversion schedule or function speed.
4) Default for hover, cruise, maneuver, one engine inoperative (OEI), or transmission sizing condition.
5) From input rotor advance ratio μ, or rotor advancing tip Mach number M_{at}.

For each engine group (which is associated with a propulsion group):

k) Number of inoperative engines.

l) Infrared suppressor state: off (hot exhaust) or on (suppressed exhaust).

m) Engine rating, fraction of rated engine power available, and drive system rating.

For each jet group:

> n) Number of inoperative jets.
> o) Jet rating, fraction of rated thrust available.

For each charge group:

> p) Number of inoperative chargers.
> q) Charger rating, fraction of rated power available.

Aircraft and rotor performance parameters for each flight state:

> r) Aircraft drag: forward flight drag increment, accounting for payload aerodynamics.
> s) Rotor performance: induced power factor κ and profile power mean c_d.

The aircraft trim state and trim targets are also specified.

The aircraft performance can be analyzed for the specified state, or a maximum-effort performance can be identified. For the maximum effort, a quantity and variable are specified. The available maximum-effort quantities include:

> a) Best endurance: maximum $1/\dot{w}$.
> b) Best range: 99% maximum $V/\dot{w}$ (high side), or low side, or 100%; flight speed or ground speed.
> c) Best climb or descent rate: maximum V_z or $1/P$.
> d) Best climb or descent angle: maximum V_z/V_h or V/P.
> e) Ceiling: maximum altitude.
> f) Power limit: zero power margin, $\min(P_{avPG} - P_{reqPG}) = 0$ (minimum over all propulsion groups).
> g) Torque limit: zero torque margin, $\min(Q_{\text{limit}} - Q_{req}) = 0$ (minimum over all propulsion groups, engine groups, and rotors; Q_{limit} expressed as power at reference rotation speed).
> j) Jet thrust limit: zero thrust margin, $\min(T_{avJG} - T_{reqJG}) = 0$ (minimum over all jet groups).
> i) Power limit or torque limit or thrust limit: most restrictive.
> j) Battery limit: zero power margin, $\min(P_{\max} - |\dot{E}_{\text{batt}}|) = 0$ (minimum over all fuel tanks).
> k) Rotor stall: zero rotor thrust margin, $(C_T/\sigma)_{\max} - |C_T/\sigma| = 0$ (for designated rotor, steady or transient or equation limit).
> l) Wing stall: zero wing lift margin, $C_{L\max} - C_L = 0$ (for designated wing).

Here $\dot{w}$ is the aircraft fuel flow (or equivalent fuel flow $\dot{E}/e_{\text{ref}}$), and P is the aircraft power. The available maximum-effort variables include:

> a) Horizontal velocity V_h or vertical rate of climb V_z (times an input factor).
> b) Aircraft sideslip angle.
> c) Altitude.
> d) Aircraft angular rate, $\dot{\theta}$ (pull-up) or $\dot{\psi}$ (turn).
> e) Aircraft linear acceleration (airframe, inertial, or ground axes).

f) Aircraft controls.

g) Aircraft orientation, θ (pitch) or ϕ (roll).

h) Propulsion group tip speed or engine speed.

If the variable is velocity, first the velocity is found for the specified maximum effort; then the performance is evaluated at that velocity times an input factor. For endurance, range, or climb, the slope of the quantity to be maximized must be zero; hence in all cases the target is zero. The slope of the quantity is evaluated by first-order backward difference. For the range, first the variable is found such that $V/\dot{w}$ is maximized (slope zero), then the variable is found such that $V/\dot{w}$ equals 99% of that maximum. Two maximum-effort quantity/variable pairs can be specified, and solved in nested iterations. The secant method or the method of false position is used to solve for the maximum effort. The task of finding maximum endurance, range, or climb is usually solved using the golden-section or curve-fit method. A tolerance ϵ and a perturbation Δ are specified.

A headwind or tailwind can be specified. The wind velocity is a linear function of altitude h: $V_w = \pm(\max(0, d_{\text{wind}} + f_{\text{wind}}h))$, with the plus sign for a headwind and the minus sign for a tailwind. The wind speed is V_w, and the ground speed is $V_g = V_h - V_w$.

Given the gross weight and useful load (from the flight condition or mission specification), the performance is calculated for this flight state. The calculated state information includes weight, speed and velocity orientation (climb and sideslip), aircraft Euler angles, rotor tip speeds, and aircraft controls.

A number of performance metrics are calculated for each flight state. The aircraft effective drag is $D_e = P/V$, hence the effective lift-to-drag ratio is $L/D_e = WV/P$. For these metrics, the aircraft power is the sum of the engine group power and jet group propulsive power: $P = P_{req} + VT_{\text{jet}}$. The specific fuel consumption is $\text{sfc} = \dot{w}/P$, and the energy efficiency is $\eta = P/\dot{E}$. The specific range is the ratio of the speed to the fuel flow: $V/\dot{w}$ (nm/lb or nm/kg) or $V/\dot{E}$ (nm/MJ or nm/kWh). The specific endurance is $1/\dot{w}$ (sec/lb or sec/kg) or $1/\dot{E}$ (sec/MJ or sec/kWh). From the Bréguet range equation, it follows that the range for which the fuel equals 1% of the gross weight is

$$R_{1\%GW} = \frac{L/D_e}{\text{sfc}} \ln\left(\frac{1}{.99}\right)$$

(not used for energy). A fuel efficiency measure is the product of the payload and specific range: $e = W_{\text{pay}}(V/\dot{w})$ (ton-nm/lb or ton-nm/kg). A productivity measure is $p = W_{\text{pay}}V/W_O$ (ton-kt/lb or ton-kt/kg), where W_O is the operating weight. If fuel energy is used, not weight, these performance metrics are based on the equivalent $\dot{w} = \dot{E}/e_{\text{ref}}$.

The aircraft weight statement defines the fixed useful load and operating weight for the design configuration. For each flight state, the fixed useful load may be different from the design configuration, because of changes in auxiliary fuel tank weight or kit weights or increments in crew or equipment weights. Thus the fixed useful load weight is calculated for the flight state; and from it the useful load weight and operating weight. The gross weight, payload weight, and usable fuel weight (in standard and auxiliary tanks) completes the weight information for the flight state.

4–5 Environment and Atmosphere

The aerodynamic environment is defined by the speed of sound c_s, density ρ, and kinematic viscosity $\nu = \mu/\rho$ of the air (or other fluid). These quantities can be obtained from the standard day (International

Standard Atmosphere), or input directly. Polar day, tropical day, and hot day atmospheres can also be used. The following options are implemented:

> a) Input the altitude h_{geom} and a temperature increment ΔT. Calculate the temperature from altitude and pressure from temperature for the standard day (or polar day, tropical day, hot day), add ΔT, and then calculate the density from the equation of state for a perfect gas. Calculate the speed of sound and viscosity from the temperature.

> b) Input the pressure altitude h_{geom} and the temperature τ (°F or °C). Calculate the pressure (from temperature vs. altitude) for the standard day (or polar day, tropical day, hot day), and then calculate the density from the equation of state for a perfect gas. Calculate the speed of sound and viscosity from the temperature.

> c) Input the density ρ and the temperature τ (°F or °C). Calculate the speed of sound and viscosity from the temperature.

> d) Input the density ρ, sound speed c_s, and viscosity μ. Calculate the temperature from the sound speed.

> e) For a fluid that is not air on earth, input the density ρ, sound speed c_s, and viscosity μ.

Here h_{geom} is the geometric altitude above mean sea level. The sources of the atmosphere descriptions are references 1–6.

The gravitational acceleration g can have the standard value or an input value.

The International Standard Atmosphere (ISA) is a model for the variation with altitude of pressure, temperature, density, and viscosity, published as International Standard ISO 2533 by the International Organization for Standardization (ISO) (ref. 1). The ISA is intended for use in calculations and design of flying vehicles, to present the test results of flying vehicles and their components under identical conditions, and to allow unification in the field of development and calibration of instruments. The ISA is defined up to 80 km geopotential altitude and is identical to the ICAO Standard Atmosphere up to 32 km.

Dry air is modeled in the ISA as a perfect gas with a mean molecular weight, and hence a gas constant R, defined by adopted values for sea level pressure, temperature, and density (p_0, T_0, ρ_0). The speed of sound at sea level c_{s0} is defined by an adopted value for the ratio of specific heats γ. The variation of temperature with geopotential altitude is defined by adopted values for vertical temperature gradients (lapse rates, L_b) and altitudes (h_b). The variation of pressure with geopotential altitude is further defined by an adopted value for the standard acceleration of free fall (g). The variation of dynamic viscosity μ with temperature is defined by adopted values for Sutherland's empirical coefficients β and S.

The required parameters are given in table 4-4, including the acceleration produced by gravity, g. The temperature T is in °K, while τ is °C (perhaps input as °F); $T = T_{\text{zero}} + \tau$. The gas constant is $R = p_0/\rho_0 T_0$, and μ_0 is actually obtained from S and β. The standard atmosphere is defined in SI units. Although table 4-4 gives values in both SI and English units, all the calculations for the aerodynamic environment are performed in SI units. As required, the results are converted to English units using the exact conversion factors for length and force.

The ISA consists of a series of altitude ranges with constant lapse rate L_b (linear temperature change with altitude). Thus at altitude h_g, the standard day temperature is

$$T_{\text{std}} = T_b + L_b(h_g - h_b)$$

for $h_g > h_b$. The altitude ranges and lapse rates are given in table 4-5. Note that h_0 is sea level, and h_1 is the boundary between the troposphere and the stratosphere. This altitude h_g is the geopotential height, calculated assuming constant acceleration due to gravity. The geometric height h is calculated using an inverse square law for gravity. Hence $h_g = rh/(r + h)$, where r is the nominal radius of the Earth. The standard day pressure is obtained from hydrostatic equilibrium ($dp = -\rho g\, dh_g$) and the equation of state for a perfect gas ($p = \rho RT$, so $dp/p = -(g/RT)dh_g$). In isothermal regions ($L_b = 0$) the standard day pressure is

$$\frac{p_{\text{std}}}{p_b} = e^{-(g/RT)(h_g - h_b)}$$

and in gradient regions ($L_b \neq 0$)

$$\frac{p_{\text{std}}}{p_b} = \left(\frac{T}{T_b}\right)^{-g/RL_b}$$

where p_b is the pressure at h_b, obtained from these equations by working up from sea level. Let $T_0, p_0, \rho_0,$ c_{s0}, μ_0 be the temperature, pressure, density, sound speed, and viscosity at sea level standard conditions. Then the density, sound speed, and viscosity are obtained from

$$\rho = \rho_0 \left(\frac{p}{p_0}\right)\left(\frac{T}{T_0}\right)^{-1}$$

$$c_s = c_{s0}\left(\frac{T}{T_0}\right)^{1/2}$$

$$\mu = \mu_0 \left(\frac{(T/T_0)^{3/2}}{\alpha(T/T_0) + 1 - \alpha}\right)$$

where $\mu_0 = \beta T_0^{3/2}/(T_0 + S)$ and $\alpha = T_0/(T_0 + S)$. For the cases using input temperature, $T = T_{\text{zero}} + \tau$. The density altitude and pressure altitude are calculated for reference. From the density and the standard day (troposphere only), the density altitude is:

$$h_d = \frac{T_0}{|L_0|}\left(1 - \left(\frac{\rho}{\rho_0}\right)^{1/(g/R|L_0|-1)}\right)$$

From the pressure $p = \rho RT$ and the standard day,

$$h_p = \frac{T_0}{|L_0|}\left(1 - \left(\frac{p}{p_0}\right)^{1/(g/R|L_0|)}\right) = \frac{T_0}{|L_0|}\left(1 - \left(\frac{\rho}{\rho_0}\frac{T}{T_0}\right)^{1/(g/R|L_0|)}\right)$$

is the pressure altitude.

The polar and tropical days are atmospheric models that describe realistic profiles of extremes of temperature and density, needed to calculate performance in near-worst-case conditions (ref. 6). These atmospheres are hydrodynamically balanced and can be used in calculations involving engine performance and aerodynamic characteristics, including calculations of true vertical velocity (ref. 4). Tables of air properties for the polar and tropical days are given in MIL-STD-3013A, based on MIL-STD-210A. The break points in the lapse rates are evident in MIL-STD-210A, in terms of degrees C

as a function of altitude in ft (geopotential altitude in MIL-C-5011B). The pressure ratio at sea level is $\delta = 30.268/29.92 = 1.0116$ for the polar day, and $\delta = 1$ for the tropical day. The altitude ranges and lapse rates are given in table 4-6 for the polar day and in table 4-7 for the tropical day. Figure 4-2 compares the temperature profiles. The air properties for the polar and tropical days are calculated from h_b, L_b, and T_b using the equations of hydrostatic equilibrium, as for the standard day. The primary data for the polar and tropical days are the altitude in ft and temperature in °C; h_b in km and L_b are derived using the exact conversion factor for length.

The conditions of the standard day, polar day, and tropical day are applicable to free air conditions. Temperatures close to the surface of the earth, even at high elevations, can be considerably higher than those for free air. The hot day is a model of ground-level atmospheric conditions, to be used for takeoff and other ground operations at elevations up to 15,000 ft (ref. 6). The hot day properties are statistically sampled and are not hydrodynamically balanced, hence should be used for approximately constant altitude conditions (ref. 4).

The origins of the hot day are in ref. 2, which has $103°$F for sea level, and a lapse rate $-3.7°$F per 1000 ft (geometric) to 40,000 ft. The pressure in the table is from the equations for equilibrium, which gives the pressure altitude; these data are not used further. According to MIL-STD-210A (paragraph 3.1.1), the pressure (hence pressure altitude) as a function of altitude was obtained from statistics. Up to 15,000 ft, the ratio of geometric altitude to pressure altitude is 1.050. The -3.7 geometric lapse rate plus the mapping of geometric to pressure altitude gave °F vs. pressure altitude, rounded to 1 decimal place. Then °C was calculated from °F, rounded to 1 decimal place. The pressure ratio δ followed from the pressure altitude. The geometric altitude (to 35,000 ft) in MIL-STD-210A was calculated from the temperature and the -3.7 lapse rate, rounded to 100s. MIL-C-5011B has the same data as MIL-STD-210A, but does not give geometric altitude, and truncates the table at 15,000 ft. MIL-STD-3013A took °C (already rounded to 1 decimal place) vs. pressure altitude (ft) from MIL-STD-210A as the temperature profile up to 15,000 ft, and calculated °F (2 decimal places, so no more loss of information) from °C. The geopotential altitude was calculated from °C and $-3.7°$F lapse rate, rounded to 100s; but this altitude information is not meaningful, since the atmosphere is not in equilibrium; only the pressure altitude is used. Thus the hot day model has a sea level temperature of $39.4°$C ($102.92°$F). Curve fitting the data to 15,000 ft gives a lapse rate of $-7.065°$C per 1000 m = $-2.1534°$C per 1000 ft (table 4-8).

Given the pressure altitude h, the hot day temperature and pressure ratio are

$$T_{\text{hot}} = T_b + L_b(h - h_b)$$

$$\frac{p_{\text{hot}}}{p_0} = \left(1 + \frac{L_0}{T_0}h\right)^{-g/RL_b}$$

using the standard day $L_0 = -6.5$. A constant lapse rate fits the tabular data for the hot day atmosphere to only about $0.1°$C. Thus the tabular data can be used directly instead (table 4-9, from MIL-C-5011B), with linear interpolation to the specified pressure altitude. MIL-STD-3013A has the same data, but only for altitudes a multiple of 1000 ft.

4–6   References

1) International Organization for Standardization. "Standard Atmosphere." ISO 2533-1975(E), May 1975.

2) Theiss, E.C. "Proposed Standard Cold and Hot Atmospheres for Aeronautical Design." Wright Air Development Center, United States Air Force, Technical Memorandum Report WCSE 141, June 1952.

3) Department of Defense Military Specification. "Climatic Extremes for Military Equipment." MIL-STD-210A, August 1957.

4) Department of Defense Military Specification. "Charts: Standard Aircraft Characteristics and Performance, Piloted Aircraft (Fixed Wing). Appendix IC, Atmospheric Tables." MIL-C-005011B(USAF), June 1977.

5) Department of Defense Military Specification. "Climatic Information to Determine Design and Test Requirements for Military Systems and Equipment." MIL-STD-210C, January 1987.

6) Department of Defense Military Specification. "Glossary of Definitions, Ground Rules, and Mission Profiles to Define Air Vehicle Performance Capability." MIL-STD-3013A, September 2008.
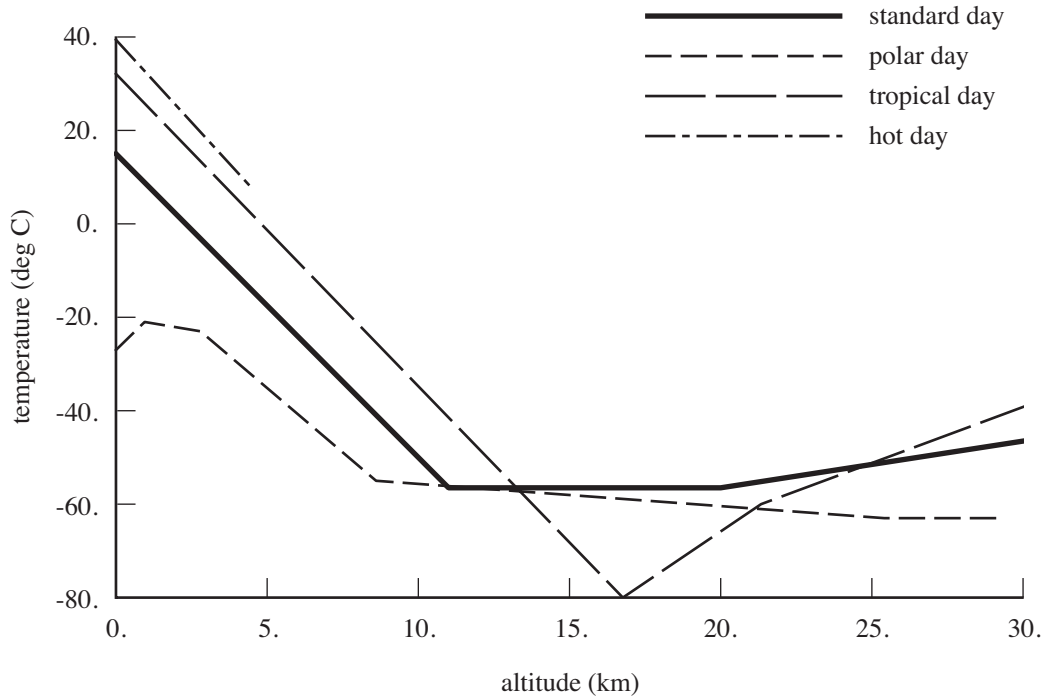
Figure 4-2. Temperature as a function of altitude.

Table 4-4. Constants adopted for calculation of the ISA.

parameter	SI units	English units
units h	m	ft
units τ	°C	°F
m per ft		0.3048
kg per lbm		0.45359237
T_0	288.15 °K	518.67 °R
T_{zero}	273.15 °K	459.67 °R
p_0	101325.0 N/m^2	2116.22 lb/ft^2
ρ_0	1.225 kg/m^3	0.002377 slug/ft^3
c_{s0}	340.294 m/sec	1116.45 ft/sec
μ_0	1.7894E-5 kg/m-sec	3.7372E-7 slug/ft-sec
S	110.4 °K	
β	1.458E-6	
γ	1.4	
g	9.80665 m/sec^2	32.17405 ft/sec^2
r	6356766 m	20855531 ft

Table 4-5. Temperatures and vertical temperature gradients: standard day.

level		base altitude h_b	lapse rate L_b	temperature T_b		
		km	°K/km	°K	°C	°F
	troposphere	–2	–6.5	301.15	28	82.4
0	troposphere	0	–6.5	288.15	15	59
1	tropopause	11	0	216.65	–56.5	–69.7
2	stratosphere	20	+1.0	216.65	–56.5	–69.7
3	stratosphere	32	+2.8	228.65	–44.5	–48.1
4	stratopause	47	0	270.65	–2.5	27.5
5	mesosphere	51	–2.8	270.65	–2.5	27.5
6	mesosphere	71	–2.0	214.65	–58.5	–73.3
7	mesopause	80	0	196.65	–76.5	–105.7

Table 4-6. Temperatures and vertical temperature gradients: polar day.

base altitude h_b		lapse rate L_b	temperature T_b			pressure ratio
ft	km *	°K/km *	°K	°C	°F *	$\delta = p/p_0$
0	0	6.326	246.15	–27	–16.6	30.268/29.92
3111.871	0.948	–1.083	252.15	–21	–5.8	
9172.604	2.796	–5.511	250.15	–23	–9.4	
28224.543	8.603	–0.476	218.15	–55	–67.0	
83363.393	25.409	0	210.15	–63	–81.4	

* derived

Table 4-7. Temperatures and vertical temperature gradients: tropical day.

base altitude h_b		lapse rate L_b	temperature T_b			pressure ratio
ft	km *	°K/km *	°K	°C	°F *	$\delta = p/p_0$
0	0	–6.687	305.25	32.1	89.78	1
55000	16.764	4.374	193.15	–80	–112.00	
70000	21.336	2.401	213.15	–60	–76.00	
100745	30.707		235.65	–37.5	–35.50	

* derived

Table 4-8. Temperatures and vertical temperature gradients: hot day.

base altitude h_b		lapse rate L_b	temperature T_b			pressure ratio
ft	km *	°K/km	°K	°C	°F *	$\delta = p/p_0$
0	0	–7.065	312.55	39.4	102.92	1
15000	4.572		280.25 *	7.10 *	44.78	

* derived

Table 4-9. Temperature table for hot day (from MIL-C-5011B).

pressure altitude		temperature		
ft	km *	°K	°C	°F *
0	0	312.55	39.4	102.92
500	0.152	311.55	38.4	101.12
1000	0.305	310.45	37.3	99.14
1500	0.457	309.45	36.3	97.34
2000	0.610	308.35	35.2	95.36
2500	0.762	307.25	34.1	93.38
3000	0.914	306.25	33.1	91.58
3500	1.067	305.15	32.0	89.60
4000	1.219	304.05	30.9	87.62
4500	1.372	302.95	29.8	85.64
5000	1.524	301.85	28.7	83.66
5500	1.676	300.75	27.6	81.68
6000	1.829	299.65	26.5	79.70
6500	1.981	298.55	25.4	77.72
7000	2.134	297.45	24.3	75.74
7500	2.286	296.35	23.2	73.76
8000	2.438	295.25	22.1	71.78
8500	2.591	294.15	21.0	69.80
9000	2.743	293.05	19.9	67.82
9500	2.896	291.95	18.8	65.84
10000	3.048	290.85	17.7	63.86
10500	3.200	289.85	16.7	62.06
11000	3.353	288.85	15.7	60.26
11500	3.505	287.75	14.6	58.28
12000	3.658	286.75	13.6	56.48
12500	3.810	285.65	12.5	54.50
13000	3.962	284.55	11.4	52.52
13500	4.115	283.55	10.4	50.72
14000	4.267	282.45	9.3	48.74
14500	4.420	281.35	8.2	46.76
15000	4.572	280.35	7.2	44.96

* derived

Chapter 5

Solution Procedures

The NDARC code performs design and analysis tasks. The design task involves sizing the rotorcraft to satisfy specified design conditions and missions. The analysis tasks can include off-design mission performance analysis, flight performance calculation for point operating conditions, and generation of subsystem or component performance maps. Figure 5-1 illustrates the tasks. The principal tasks (sizing, mission analysis, and flight performance analysis) are shown in the figure as boxes with dark borders. Dark black arrows show control of subordinate tasks.

The aircraft description (fig. 5-1) consists of all the information, input and derived, that defines the aircraft. The aircraft consists of a set of components, including fuselage, rotors, wings, tails, and propulsion. This information can be the result of the sizing task; can come entirely from input, for a fixed model; or can come from the sizing task in a previous case or previous job. The aircraft description information is available to all tasks and all solutions (indicated by light green arrows).

Missions are defined for the sizing task and for the mission performance analysis. A mission consists of a specified number of mission segments, for which time, distance, and fuel burn are evaluated. For specified takeoff fuel weight with adjustable segments, the mission time or distance is adjusted so the fuel required for the mission (burned plus reserve) equals the takeoff fuel weight. The mission iteration is on fuel weight or energy.

Flight conditions are specified for the sizing task and for the flight performance analysis.

For flight conditions and mission takeoff, the gross weight can be maximized such that the power required equals the power available.

A flight state is defined for each mission segment and each flight condition. The aircraft performance can be analyzed for the specified state, or a maximum-effort performance can be identified. The maximum effort is specified in terms of a quantity such as best endurance or best range, and a variable such as speed, rate of climb, or altitude. The aircraft must be trimmed, by solving for the controls and motion that produce equilibrium in the specified flight state. Different trim solution definitions are required for various flight states. Evaluating the rotor hub forces may require solution of the blade flap equations of motion.

The sizing task is described in more detail in chapter 3. The flight condition, mission, and flight state calculations are described in chapter 4. The solution of the blade flap equations of motion is described in chapter 11. The present chapter provides details of the solution procedures implemented for each iteration of the analysis.

The nested iteration loops involved in the solution process are indicated by the subtitles in the boxes of figure 5-1, and illustrated in more detail in figure 5-2. The flight state solution involves up
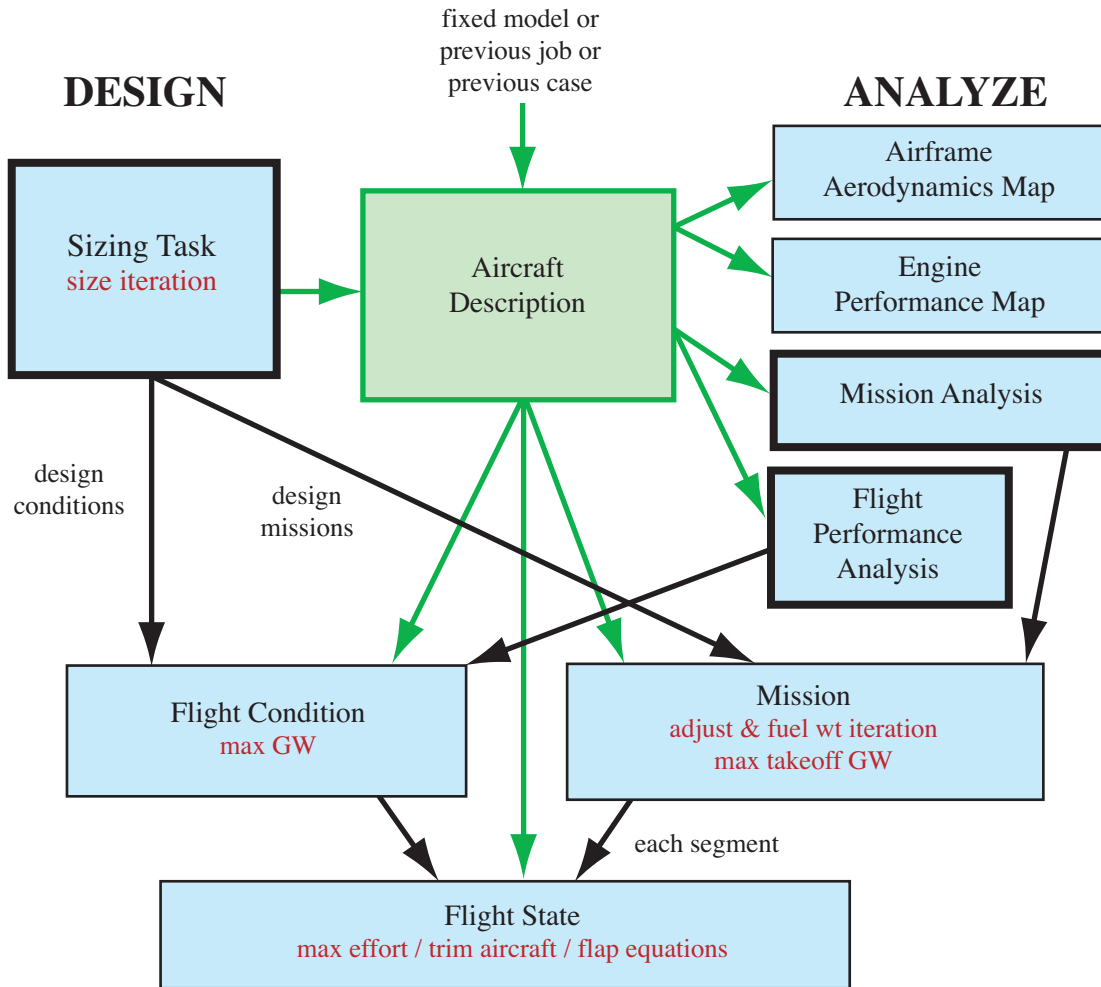
Figure 5-1. Outline of NDARC tasks.

to three loops. The innermost loop is the solution of the blade flap equations of motion, needed for an accurate evaluation of the rotor hub forces. The next loop is the trim solution, which is required for most flight states. The flight state optionally has one or two maximum-effort iterations. The flight state solution is executed for each flight condition and for each mission segment. A flight condition solution or any mission segment solution can optionally maximize the aircraft gross weight. The mission usually requires an iterative solution, for fuel weight or for adjustable segment time or distance. Thus each flight condition solution involves up to four nested iterations: maximum gross weight (outer), maximum effort, trim, and blade motion (inner). Each mission solution involves up to five nested iterations: mission (outer), and then for each segment maximum gross weight, maximum effort, trim, and blade motion (inner). Finally, the design task introduces a sizing iteration, which is the outermost loop of the process.

Optionally the size, mission, trim, and/or rotor iterations can be implemented in external solution procedures. Then the internal iterations are suppressed, and the analysis calculates residuals for use by the external methods. The maximum gross weight and maximum effort iterations do not involve residuals, so for an external solution the internal calculations are not defined, and the internal iterations do not exist.
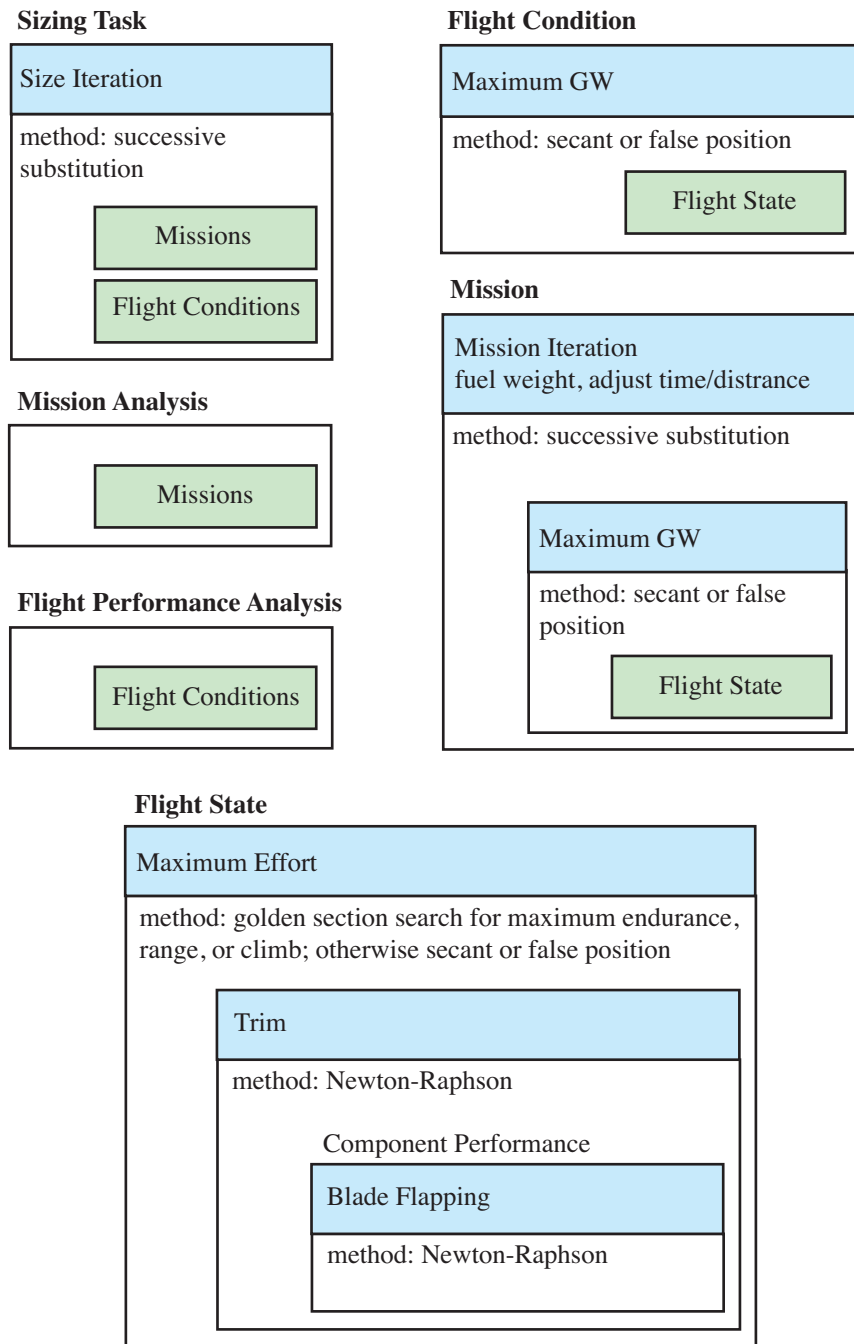
Sizing Task

Size Iteration
method: successive substitution

       | Missions |
       | Flight Conditions |

Mission Analysis

       | Missions |

Flight Performance Analysis

       | Flight Conditions |

Flight Condition

Maximum GW
method: secant or false position

       | Flight State |

Mission

Mission Iteration fuel weight, adjust time/distrance
method: successive substitution

    | Maximum GW |
    | method: secant or false position |
       | Flight State |

Flight State

Maximum Effort
method: golden section search for maximum endurance, range, or climb; otherwise secant or false position

    | Trim |
    | method: Newton-Raphson |

       Component Performance

       | Blade Flapping |
       | method: Newton-Raphson |

Figure 5-2. Design and analysis tasks, with nested loops and solution methods.

5–1 Iterative Solution Tasks

5-1.1 Tolerance and Perturbation

For each solution procedure, a tolerance ϵ and a perturbation Δ may be required. Single values are specified for the task and then scaled for each element tested or perturbed.

The scaling is based on a reference weight W (design gross weight, or derived from aircraft $C_T/\sigma = 0.07$), a reference length L (fuselage length, rotor radius, or wing span), and a reference power P (aircraft installed power, or derived from $P = W\sqrt{W/2\rho A}$). Then the force reference is $F = W$, the moment reference is $M = WL/10$, and the angle reference is $A = 1$ deg. The velocity reference is $V = 400$ knots. The angular velocity reference is $\Omega = V/L$ (in deg/sec). The coefficient reference is $C = 0.6$ for wings and $C = 0.1$ for rotors. Altitude scale is $H = 10000$ ft. Acceleration scale is $G = g$ (acceleration due to gravity). The range scale is $X = 100$ nm. These scaling variables are referred to in the subsections that follow, and in tables 5-1 to 5-4.

5-1.2 Size Aircraft

The sizing task determines the dimensions, power, and weight of a rotorcraft that can perform a specified set of design conditions and missions. The aircraft size is characterized by parameters such as design gross weight, weight empty, rotor radius, and engine power available. The relationships between dimensions, power, and weight generally require an iterative solution. From the design flight conditions and missions, the task can determine the total engine power or the rotor radius (or both power and radius can be fixed), as well as the design gross weight, maximum takeoff weight, drive system torque limit, and fuel tank capacity. For each propulsion group, the engine power or the rotor radius can be sized.

A successive substitution method is used for the sizing iteration, with an input tolerance ϵ. Relaxation is applied to P_{eng} or R, T_{jet}, P_{chrg}, W_D, W_{MTO}, $P_{DS\text{limit}}$, $W_{\text{fuel-cap}}$ or $E_{\text{fuel-cap}}$, and T_{design}. Two successive substitution loops are used. The outer loop is an iteration on performance: engine power or rotor radius, jet thrust, charger power. The inner loop is an iteration on parameters: W_D, W_{MTO}, $P_{DS\text{limit}}$, $W_{\text{fuel-cap}}$ or $E_{\text{fuel-cap}}$, and T_{design}. Either loop can be absent, depending on the definition of the size task. Convergence is tested in terms of these parameters, and the aircraft weight empty W_E. The tolerance is $0.1P\epsilon$ for engine power and drive system limit; $0.01W\epsilon$ for gross weight, maximum takeoff weight, fuel weight, jet thrust, and design rotor thrust; and $0.1L\epsilon$ for rotor radius.

5-1.3 Mission

Missions consist of a specified number of segments, for which time, distance, and fuel burn are evaluated. For calculated mission fuel weight, the fuel weight at takeoff is adjusted to equal the fuel required for the mission (burned plus reserve). For specified takeoff fuel weight with adjustable segments, the mission time or distance is adjusted so the fuel required for the mission (burned plus reserve) equals the takeoff fuel weight. The mission iteration is thus on fuel weight or energy. Range credit segments can also require an iteration.

A successive substitution method is used if an iteration is required, with a tolerance ϵ specified. The principal iteration variable is takeoff fuel weight, for which the tolerance is $0.01W\epsilon$. For calculated mission fuel weight, the relaxation is applied to the mission fuel value used to update the takeoff fuel weight. For specified takeoff fuel weight, the relationship is applied to the fuel weight increment used to adjust the mission segments. The tolerance for the distance flown in range credit segments is $X\epsilon$. The relaxation is applied to the distance flown in the destination segments for range credit.

5-1.4   Maximum Gross Weight

Flight conditions are specified for the sizing task and for the flight performance analysis. Mission takeoff conditions are specified for the sizing task and for the mission analysis. Optionally for flight conditions and mission takeoff, the gross weight can be maximized, such that the power required equals the power available, $\min(P_{avPG} - P_{reqPG}) = 0$ (zero power margin, minimum over all propulsion groups); or such that the power required equals an input power, $\min((d + fP_{avPG}) - P_{reqPG}) = 0$ (minimum over all propulsion groups, with d an input power and f an input factor; this convention allows the power to be input directly, $f = 0$, or scaled with power available). Similarly, the gross weight can be maximized for zero jet thrust margin, or zero torque margin.

The secant method or the method of false position is used to solve for the maximum gross weight. A tolerance ϵ and a perturbation Δ are specified. The variable is gross weight, with initial increment of $W\Delta$, and tolerance of $0.01W\epsilon$. Note that the convergence test is applied to the magnitude of the gross weight increment.

5-1.5   Maximum Effort

The aircraft performance can be analyzed for the specified state or a maximum-effort performance can be identified. The secant method or the method of false position is used to solve for the maximum effort. The task of finding maximum endurance, range, or climb is usually solved using the golden-section or curve-fit method. A tolerance ϵ and a perturbation Δ are specified.

A quantity and variable are specified for the maximum-effort calculation. Tables 5-1 and 5-2 summarize the available choices, with the tolerance and initial increment used for the variables. Note that the convergence test is applied to the magnitude of the variable increment. Optionally two quantity/variable pairs can be specified, solved in nested iterations. The two variables must be unique. The two variables can maximize the same quantity (endurance, range, or climb). If the variable is velocity, first the velocity is found for the specified maximum effort; the performance is then evaluated at that velocity times an input factor. For endurance, range, or climb, the slope of the quantity to be maximized must be zero; hence in all cases the target is zero. The slope of the quantity is evaluated by first-order backward difference. For the range, first the variable is found such that $V/\dot{w}$ is maximized (slope zero), and then the variable is found such that $V/\dot{w}$ equals 99% of that maximum; for the latter the variable perturbation is increased by a factor of 4 to ensure that the solution is found on the correct side of the maximum.

Table 5-1. Maximum-effort solution.

maximum-effort variable		initial increment	tolerance
horizontal velocity	V_h	$0.1V\Delta$	$0.1V\epsilon$
vertical rate of climb	V_z	$0.1V\Delta$	$0.1V\epsilon$
aircraft velocity	β (sideslip)	$100A\Delta$	$100A\epsilon$
altitude		$H\Delta$	$H\epsilon$
aircraft angular rate	$\dot{\theta}$ (pullup), $\dot{\psi}$ (turn)	$\Omega\Delta$	$\Omega\epsilon$
aircraft linear acceleration	a_x, a_y, a_z	$G\Delta$	$G\epsilon$
aircraft control	angle	$100A\Delta$	$100A\epsilon$
aircraft orientation	θ (pitch), ϕ (roll)	$100A\Delta$	$100A\epsilon$
propulsion group tip speed	V_{tip}	$V\Delta$	$V\epsilon$
propulsion group engine speed	N_{spec}	$10\Omega\Delta$	$10\Omega\epsilon$

Table 5-2. Maximum-effort solution.

maximum-effort quantity		
best endurance	maximum $1/\dot{w}$	
best range	99% maximum $V/\dot{w}$	high or low side, or 100%
best climb or descent rate	maximum V_z or $1/P$	
best climb or descent angle	maximum V_z/V or V/P	
ceiling	maximum altitude	
power limit	power margin, $\min(P_{avPG} - P_{reqPG}) = 0$	over all propulsion groups
torque limit	torque margin, $\min(Q_{\text{limit}} - Q_{req}) = 0$	over all limits
thrust limit	thrust margin, $\min(T_{avJG} - T_{reqJG}) = 0$	over all jet groups
power, torque, thrust limit	power, torque, thrust margin	most restrictive
battery limit	power margin, $\min(P_{\max} - |\dot{E}_{\text{batt}}|) = 0$	over all fuel tanks
rotor stall	thrust margin, $(C_T/\sigma)_{\max} - |C_T/\sigma| = 0$	for designated rotor
wing stall	lift margin, $C_{L\max} - C_L = 0$	for designated wing

Table 5-3. Trim solution.

trim quantity		target	tolerance
aircraft total force	x, y, z components	0	$F\epsilon$
aircraft total moment	x, y, z components	0	$M\epsilon$
aircraft load factor	x, y, z components	Flight State	ϵ
propulsion group power		Flight State	$P\epsilon$
power margin	$P_{avPG} - P_{reqPG}$	Flight State	$P\epsilon$
torque margin	$P_{DS\text{limit}} - P_{reqPG}$	Flight State	$P\epsilon$
engine group power		Flight State	$P\epsilon$
power margin	$P_{avEG} - P_{reqEG}$	Flight State	$P\epsilon$
jet group thrust		Flight State	$F\epsilon$
thrust margin	$T_{avJG} - T_{reqJG}$	Flight State	$F\epsilon$
charge group power		Flight State	$P\epsilon$
charge power margin	$P_{avCG} - P_{reqCG}$	Flight State	$P\epsilon$
fuel tank energy flow	$\dot{E}_{\text{batt}}$	Flight State	$P\epsilon$
battery power margin	$P_{\max} - |\dot{E}_{\text{batt}}|$	Flight State	$P\epsilon$
rotor force	lift, vertical, propulsive	Flight State, component schedule	$F\epsilon$
rotor thrust	C_T/σ	Flight State, component schedule	$C\epsilon$
rotor thrust margin	$(C_T/\sigma)_{\max} - |C_T/\sigma|$	Flight State	$C\epsilon$
rotor flapping	β_c, β_s	Flight State	$A\epsilon$
rotor hub moment	x (roll), y (pitch)	Flight State	$M\epsilon$
rotor torque		Flight State	$M\epsilon$
wing force	lift	Flight State, component schedule	$F\epsilon$
wing lift coefficient	C_L	Flight State, component schedule	$C\epsilon$
wing lift margin	$C_{L\max} - C_L$	Flight State	$C\epsilon$
tail force	lift	Flight State	$F\epsilon$

Table 5-4. Trim solution.

trim variable		perturbation
aircraft control	angle	$100A\Delta$
aircraft orientation	θ (pitch), ϕ (roll)	$100A\Delta$
aircraft velocity	V_h (horizontal velocity)	$V\Delta$
aircraft velocity	V_z (vertical velocity)	$V\Delta$
aircraft velocity	β (sideslip)	$100A\Delta$
aircraft angular rate	$\dot{\theta}$ (pullup), $\dot{\psi}$ (turn)	$\Omega\Delta$
propulsion group tip speed	V_{tip}	$20V\Delta$
propulsion group engine speed	N_{spec}	$10\Omega\Delta$

5-1.6   Trim

The aircraft trim operation solves for the controls and motion that produce equilibrium in the specified flight state. A Newton–Raphson method is used for trim. The derivative matrix is obtained by numerical perturbation. A tolerance ϵ and a perturbation Δ are specified.

Different trim solution definitions are required for various flight states. Therefore one or more trim states are defined for the analysis, and the appropriate trim state selected for each flight state of a performance condition or mission segment. For each trim state, the trim quantities, trim variables, and targets are specified. Tables 5-3 and 5-4 summarize the available choices, with the tolerances and perturbations used.

5-1.7   Rotor Flap Equations

Evaluating the rotor hub forces may require solution of the flap equations $E(v) = 0$. For tip-path plane command, the thrust and flapping are known, so $v = (\theta_{0.75}\, \theta_c\, \theta_s)^T$. For no-feathering plane command, the thrust and cyclic pitch are known, so $v = (\theta_{0.75}\, \beta_c\, \beta_s)^T$. A Newton–Raphson solution method is used: from $E(v_{n+1}) \cong E(v_n) + (dE/dv)(v_{n+1} - v_n) = 0$, the iterative solution is

$$v_{n+1} = v_n - C\, E(v_n)$$

where $C = f(dE/dv)^{-1}$, including the relaxation factor f. The derivative matrix for axial flow can be used. Alternatively, the derivative matrix dE/dv can be obtained by numerical perturbation. Convergence of the Newton–Raphson iteration is tested in terms of $|E| < \epsilon$ for each equation, where ϵ is an input tolerance.

5–2   Theory

The analysis uses several methods to solve nonlinear algebraic equations. Such equations may be written in two forms: (a) fixed point $x = G(x)$, and (b) zero point $f(x) = 0$; where x, G, and f are vectors. The analysis provides operations that implement the function G or f. Solution procedures appropriate for the zero point form can be applied to equations in fixed point form, by defining $f(x) = x - G(x)$. In this context, f can be considered the iteration error.

Efficient and convergent methods are required to find the solution $x = \alpha$ of these equations. Note that $f'(\alpha) = 0$ or $G'(\alpha) = 1$ means that α is a higher-order root. For nonlinear problems, the method will

```
successive substitution iteration
    save:  x_old = x
    evaluate  x
    relax:  x = λx + (1 − λ)x_old
    test convergence:  error = ‖x − x_old‖ ≤ λtolerance × weight
```

<div align="center">Figure 5-3. Outline of successive substitution method.</div>

be iterative: $x_{n+1} = F(x_n)$. The operation F depends on the solution method. The solution error is:

$$\epsilon_{n+1} = \alpha - x_{n+1} = F(\alpha) - F(x_n) = (\alpha - x_n)F'(\xi_n) \cong \epsilon_n F'(\alpha)$$

Thus the iteration will converge if F is not too sensitive to errors in x: $|F'(\alpha)| < 1$ for scalar x. For x a vector, the criterion is that all the eigenvalues of the derivative matrix $\partial F/\partial x$ have magnitude less than one. The equations in this section are generally written for scalar x; the extension to vector x is straightforward. Convergence is linear for F' nonzero, quadratic for $F' = 0$. Iterative methods have a relaxation factor (and other parameters) to improve convergence, and a tolerance to measure convergence.

The following subsections describe the solution methods used for the various iterations, as shown in figure 5-2.

5-2.1   Successive Substitution Method

The successive substitution method (with relaxation) is an example of a fixed point solution. A direct iteration is simply $x_{n+1} = G(x_n)$, but $|G'| > 1$ for many practical problems. A relaxed iteration uses $F = (1 - \lambda)x + \lambda G$:

$$x_{n+1} = (1 - \lambda)x_n + \lambda G(x_n) = x_n - \lambda f(x_n)$$

with relaxation factor λ. The convergence criterion is then

$$|F'(\alpha)| = |1 - \lambda + \lambda G'| < 1$$

so a value of λ can be found to ensure convergence for any finite G'. Specifically, the iteration converges if the magnitude of λ is less than the magnitude of $2/(1 - G') = 2/f'$ (and λ has the same sign as $1 - G' = f'$). Quadratic convergence ($F' = 0$) is obtained with $\lambda = 1/(1 - G') = 1/f'$. Over-relaxation ($\lambda > 1$) can be used if $|G'| < 1$. Since the correct solution $x = \alpha$ is not known, convergence must be tested by comparing the values of two successive iterations:

$$\text{error} = \|x_{n+1} - x_n\| \leq \text{tolerance}$$

where the error is some norm of the difference between iterations (typically absolute value for scalar x). Note that the effect of the relaxation factor is to reduce the difference between iterations:

$$x_{n+1} - x_n = \lambda\big(G(x_n) - x_n\big)$$

Hence the convergence test is applied to $(x_{n+1} - x_n)/\lambda$, in order to maintain the definition of tolerance independent of relaxation. The corresponding residual is $(x_{n+1} - x_n)$. The process for the successive substitution method is shown in figure 5-3.

```
initialize
   evaluate h
   test convergence:  error = |h_j − h_targetj| ≤ tolerance × weight_j
   initialize derivative matrix D to input matrix
   calculate gain matrix: C = λD⁻¹
iteration
   identify derivative matrix
      optional perturbation identification
         perturb each element of x: δx_i = Δ × weight_i
         evaluate h
         calculate D
         calculate gain matrix: C = λD⁻¹
   increment solution: δx = −C(h − h_target)
   evaluate h
   test convergence:  error = |h_j − h_targetj| ≤ tolerance × weight_j
```

Figure 5-4. Outline of Newton–Raphson method.

5-2.2   Newton–Raphson Method

The Newton–Raphson method (with relaxation and identification) is an example of a zero point solution. The Taylor series expansion of $f(x) = 0$ leads to the iteration operator $F = x − f/f'$:

$$x_{n+1} = x_n − [f'(x_n)]^{-1} f(x_n)$$

which gives quadratic convergence. The behavior of this iteration depends on the accuracy of the derivative f'. Here it is assumed that the analysis can evaluate directly f, but not f'. It is necessary to evaluate f' by numerical perturbation of f, and for efficiency the derivatives may not be evaluated for each x_n. These approximations compromise the convergence of the method, so a relaxation factor λ is introduced to compensate. Hence a modified Newton–Raphson iteration is used, $F = x − Cf$:

$$x_{n+1} = x_n − Cf(x_n) = x_n − \lambda D^{-1}f(x_n)$$

where the derivative matrix D is an estimate of f'. The convergence criterion is then

$$|F'(\alpha)| = |1 − Cf'| = |1 − \lambda D^{-1}f'| < 1$$

since $f(\alpha) = 0$. The iteration converges if the magnitude of λ is less than the magnitude of $2D/f'$ (and λ has the same sign as D/f'). Quadratic convergence is obtained with $\lambda = D/f'$ (which would require λ to change during the iteration however). The Newton–Raphson method ideally uses the local derivative in the gain factor, $C = 1/f'$, so has quadratic convergence:

$$F'(\alpha) = \frac{ff''}{f'^2} = 0$$

since $f(\alpha) = 0$ (if $f' \neq 0$ and f'' is finite; if $f' = 0$, then there is a multiple root, $F' = 1/2$, and the convergence is only linear). A relaxation factor is still useful, since the convergence is only quadratic sufficiently close to the solution. A Newton–Raphson method has good convergence when x is sufficiently close to the solution, but frequently has difficulty converging elsewhere. Hence the initial

```
initialize
    evaluate f₀ at x₀,  f₁ at  x₁ = x₀ + Δx,  f₂ at  x₂ = x₁ + Δx
iteration
    calculate derivative f'
        secant: from f₀ and f₁
        false position: from f₀, and f₁ or f₂ (opposite sign from f₀)
    calculate gain: C = λ/f'
    increment solution: δx = -Cf
    shift: f₂ = f₁,  f₁ = f₀
    evaluate f
    test convergence
```

Figure 5-5. Outline of secant method or method of false position.

estimate x_0 that starts the iteration is an important parameter affecting convergence. Convergence of the solution for x may be tested in terms of the required value (zero) for f:

$$\text{error} = \|f\| \leq \text{tolerance}$$

where the error is some norm of f (typically absolute value for scalar f). The corresponding residual is f.

The derivative matrix D is obtained by an identification process. The perturbation identification can be performed at the beginning of the iteration, and optionally every M_{PID} iterations thereafter. The derivative matrix is calculated from a one-step finite-difference expression (first order). Each element x_i of the vector x is perturbed, one at a time, giving the i-th column of D:

$$D = \begin{bmatrix} \cdots & \dfrac{\partial f}{\partial x_i} & \cdots \end{bmatrix} = \begin{bmatrix} \cdots & \dfrac{f(x_i + \delta x_i) - f(x_i)}{\delta x_i} & \cdots \end{bmatrix}$$

Alternatively, a two-step finite-difference expression (second order) can be used:

$$D = \begin{bmatrix} \cdots & \dfrac{\partial f}{\partial x_i} & \cdots \end{bmatrix} = \begin{bmatrix} \cdots & \dfrac{f(x_i + \delta x_i) - f(x_i - \delta x_i)}{2\delta x_i} & \cdots \end{bmatrix}$$

With this procedure, the accuracy of D (hence convergence) can be affected by both the magnitude and sign of the perturbation (only the magnitude for a two-step difference).

The process for the Newton–Raphson method is shown in figure 5-4. A problem specified as $h(x) = h_{\text{target}}$ becomes a zero point problem with $f = h - h_{\text{target}}$. A successive substitution problem, $x = G(x)$, becomes a zero point problem with $f = x - G$. At the beginning of the solution, x has an initial value. The perturbation identification can optionally never be performed (so an input matrix is required), be performed at the beginning of the iteration, or be performed at the beginning and every M_{PID} iterations thereafter.

5-2.3   Secant Method

The secant method (with relaxation) is developed from the Newton–Raphson method. The modified Newton–Raphson iteration is:

$$x_{n+1} = x_n - Cf(x_n) = x_n - \lambda D^{-1}f(x_n)$$

```
initialize
    evaluate f₀ at x₀,  f₁ at x₁ = x₀ + Δx,  f₂ at x₂ = x₁ + Δx
bracket maximum: while not f₁ ≥ f₀, f₂
    if f₂ > f₀, then x₃ = x₂ + (x₂ − x₁);  1,2,3 → 0,1,2
    if f₀ > f₂, then x₃ = x₀ − (x₁ − x₀);  3,0,1 → 0,1,2
iteration (search)
    if x₂ − x₁ > x₁ − x₀, then x₃ = x₁ + W(x₂ − x₁)
        if f₃ < f₁, then 0,1,3 → 0,1,2
        if f₃ > f₁, then 1,3,2 → 0,1,2
    if x₁ − x₀ > x₂ − x₁, then x₃ = x₁ − W(x₁ − x₀)
        if f₃ < f₁, then 3,1,2 → 0,1,2
        if f₃ > f₁, then 0,3,1 → 0,1,2
    test convergence
```

Figure 5-6. Outline of golden-section search.

where the derivative matrix D is an estimate of f'. In the secant method, the derivative of f is evaluated numerically at each step:

$$f'(x_n) \cong \frac{f(x_n) - f(x_{n-1})}{x_n - x_{n-1}}$$

It can be shown that then the error reduces during the iteration according to:

$$|\epsilon_{n+1}| \cong |f''/2f'| \, |\epsilon_n| \, |\epsilon_{n-1}| \cong |f''/2f'|^{.62} |\epsilon_n|^{1.62}$$

which is slower than the quadratic convergence of the Newton–Raphson method (ϵ_n^2), but still better than linear convergence. In practical problems, whether the iteration converges at all is often more important than the rate of convergence. Limiting the maximum amplitude of the derivative estimate may also be appropriate. Note that with $f = x - G(x)$, the derivative f' is dimensionless, so a universal limit (say maximum $|f'| = 0.3$) can be specified. A limit on the maximum increment of x (as a fraction of the x value) can also be imposed. The process for the secant method is shown in figure 5-5.

5-2.4   Method of False Position

The method of false position is a derivative of the secant method, based on calculating the derivative with values that bracket the solution. The iteration starts with values of x_0 and x_1 such that $f(x_0)$ and $f(x_1)$ have opposite signs. Then the derivative f' and new estimate x_{n+1} are

$$f'(x_n) \cong \frac{f(x_n) - f(x_k)}{x_n - x_k}$$
$$x_{n+1} = x_n - \lambda D^{-1} f(x_n)$$

using $k = n - 1$ or $k = n - 2$ such that $f(x_n)$ and $f(x_k)$ have opposite signs. The convergence is slower (roughly linear) than for the secant method, but by keeping the solution bracketed convergence is guaranteed. The process for the method of false position is shown in figure 5-5.

5-2.5   Golden-Section Search

The golden-section search method can be used to find the solution x that maximizes $f(x)$. The problem of maximizing $f(x)$ can be attacked by applying the secant method or method of false position to

```
initialize
    evaluate  f₀ at  x₀,  f₁ at  x₁ = x₀ + Δx,  f₂ at  x₂ = x₁ + Δx
bracket maximum: while not  f₁ ≥ f₀, f₂
    if  f₂ > f₀,  then  x₃ = x₂ + (x₂ − x₁);  1,2,3  →  0,1,2
    if  f₀ > f₂,  then  x₃ = x₀ − (x₁ − x₀);  3,0,1  →  0,1,2
    f_max = f₁
curve fit
    f_max = f₁,  x_max = x₁
    evaluate  f for  x = x_max + nΔx and  x = x_max − nΔx
    least-squared error solution for polynomial coefficients
    solve polynomial for  x at peak  f
```

Figure 5-7. Outline of curve-fit method.

the derivative $f'(x) = 0$, but that approach is often not satisfactory as it depends on numerical evaluation of the second derivative. The golden-section search method begins with a set of three values $x_0 < x_1 < x_2$ and the corresponding functions f_0, f_1, f_2. The x value is incremented until the maximum is bracketed, $f_1 \geq f_0, f_2$. Then a new value x_3 is selected in the interval x_0 to x_2; f_3 evaluated; and the new set of $x_0 < x_1 < x_2$ determined such that the maximum is still bracketed. The new value x_3 is a fraction $W = (3 - \sqrt{5})/2 \cong 0.38197$ from x_1 into the largest segment. The process for the golden-section search is shown in figure 5-6.

5-2.6   Curve-Fit Method

The curve-fit method can be used to find the solution x that maximizes $f(x)$, by fitting the solution to a polynomial. If the function f is flat around the maximum and the inner loop tolerances are not tight enough, the golden-section search can become erratic, particularly for best range and best endurance calculations. Curve fitting the evaluated points and then solving the curve for the maximum has the potential to improve the behavior. The curve-fit method begins with a set of three values $x_0 < x_1 < x_2$ and the corresponding functions f_0, f_1, f_2. The x value is incremented until the maximum is bracketed, $f_1 \geq f_0, f_2$, giving a course maximum f_{max} at x_{max}. Next a set of x and f values are generated by incrementing x above and below x_{max}, until $f < r_{fit} f_{max}$ is found (typically $r_{fit} = 0.98$ for best range). This set of points is fit to the cubic polynomial $f = c_3 z^3 + c_2 z^2 + c_1 z + c_0$, $z = x/x_{max} - 1$ (or to a quadradic polynomial). Let $c^T = (c_0 \, c_1 \, c_2 \, c_3)$ and $\xi^T = (1 \, z \, z^2 \, z^3)$. Then the least-squared-error solution for the coefficients is

$$c = \left(\sum_i \xi_i \xi_i^T \right)^{-1} \left(\sum_i f_i \xi_i \right)$$

where the sums are over the set of points to be fit. For a quadratic polynomial fit, the solution is then

$$x = x_{max} \left(1 - c_1/2c_2 \pm \sqrt{1 - r} \, \sqrt{(c_1/2c_2)^2 - c_0/c_2} \right)$$

where $r = 1$ for the maximum, or $r = 0.99$ for 99% best range. For a cubic polynomial fit, the maximum is at

$$z = -\frac{c_2}{3c_3} \left(1 - \sqrt{1 - \frac{3c_3 c_1}{c_2^2}} \right) \cong -\frac{c_1}{2c_2} \left(1 + \frac{1}{4} \frac{3c_3 c_1}{c_2^2} \right)$$

It is simplest to search the cubic for the peak (z where $df/dz = 0$), and then if necessary search for the 99% range point ($f = 0.99 f_{peak}$) The process for the golden-section search is shown in figure 5-7.

Cost

Costs are estimated using statistical models based on historical aircraft price and maintenance cost data, with appropriate factors to account for technology impact and inflation. The aircraft purchase price (C_{AC}, in dollars) covers airframe, mission equipment package (MEP), and flight control electronics (FCE) costs. The direct operating cost (DOC, in cents per available seat mile (ASM)) is the sum of maintenance cost (C_{maint}, in dollars per flight hour), flight crew salary and expenses, fuel and oil cost, depreciation, insurance cost, and finance cost.

Two models are implemented: the Harris-Scully CTM cost model, which covers aircraft purchase price, maintenance cost, and direct operating cost; and the Scott rotorcraft cost model, which gives the aircraft unit flyaway cost in terms of component costs.

Inflation factors can be input, or internal factors used. Table 6-1 gives the internal inflation factors for DoD (ref. 1) and CPI (ref. 2). For years beyond the data in the table, optionally the inflation factor is extrapolated based on the last yearly increase.

6–1   Harris-Scully CTM Rotorcraft Cost Model

The Harris-Scully CTM rotorcraft cost model (refs. 3–6) gives an estimate of aircraft purchase price, maintenance cost, and direct operating cost. The model was developed for shaft-driven helicopters and turboprop aircraft.

6-1.1   Aircraft Purchase Price

Aircraft purchase price is estimated from the statistical relationship of Harris and Scully (ref. 3, updated in 2001), based on a 1994 database of mostly civil aircraft (plus EH-101, UH-60L, CH-47D, CH-53E, and MV-22). The model starts with a function of aircraft weight and power; has several complexity factors; a factor for rotorcraft or turboprop aircraft; and a country or industry factor (specifically U.S. military). The model includes (as $/lb) separate calculations of a composite construction increment (increase or decrease), mission equipment package cost, and flight control electronics cost. The model accounts for inflation and includes an overall technology factor. With these equations, the purchase price is predicted within 20% for 96% of 128 rotorcraft (figs. 6-1 and 6-2), implying a standard deviation of 10%. For the five military aircraft, price is predicted within $\pm 10\%$ or less.

The basic statistical relationship for airframe purchase price is:

$$c_{AF} = 739.91\, K_{ET} K_{EN} K_{LG} K_R\, W_{AF}^{1.0619} (P/W_{AF})^{0.5887} N_{\text{blade}}^{0.1465}$$

with $W_{AF} = W_E + \Delta W_{\text{kit}} - W_{\text{MEP}} - W_{\text{FCE}} - W_{\text{batt}}$, including airframe kits ΔW_{kit} (the wing and wing extension kits, and optionally the folding kit); and P is the sum of the rated takeoff power of all engines.

This relationship is based on aircraft with turboshaft engines, so a factor f_{motor} is included in the sum for electrical engines (motors and generators). If the aircraft shaft power is zero (as for jet aircraft), $P/W_{AF} = 0.25$ is used. The configuration factor $K_{\text{config}} = K_{ET}K_{EN}K_{LG}K_R$ has the factors:

$K_{ET} =$	1.0 for turbine aircraft	0.557 for piston aircraft
$K_{EN} =$	1.0 for multi-engine aircraft	0.736 for single-engine aircraft
$K_{LG} =$	1.0 for retractable landing gear	0.884 for fixed landing gear
$K_R =$	1.0 for single main-rotor	1.057 for twin main-rotors, 1.117 for four main-rotors

The number of blades and the configuration factor are essentially measures of complexity. In particular, retractable/fixed landing gear is a surrogate for general complexity.

The term $C_{\text{comp}} = r_{\text{comp}}W_{\text{comp}}$ accounts for additional costs for composite construction (negative for cost savings); W_{comp} is the composite structure weight, obtained as an input fraction of the component weight, with separate fractions for body, tail, pylon, and wing weight. The MEP, FCE, and battery costs are obtained from input cost-per-weight factors: $C_{\text{MEP}} = r_{\text{MEP}}W_{\text{MEP}}$, $C_{\text{FCE}} = r_{\text{FCE}}W_{\text{FCE}}$, and $C_{\text{batt}} = r_{\text{batt}}E_{\text{fuel-cap}}$.

The statistical cost equation for c_{AF} is based on 1994 dollars and current technology levels. Including an inflation factor F_i and technology factor χ_{AF} gives the purchase price C_{AC}:

$$C_{AC} = \chi_{AF}(F_i c_{AF}) + C_{\text{comp}} + C_{\text{MEP}} + C_{\text{FCE}} + C_{\text{batt}}$$

In addition to technology, χ accounts for calibration and industry factors; for example, $\chi_{AF} = 0.87$ for U.S. Military (ref. 3). This equation also estimates turboprop airliner purchase price by setting $N_{\text{rotor}} = N_{\text{blade}} = 1$ and using the additional factor 0.8754 (pressurized) or 0.7646 (unpressurized). The purchase price in \$/lb or \$/kg is

$$r_{AF} = (\chi_{AF}(F_i c_{AF}))/W_{AF}$$
$$r_{AC} = C_{AC}/(W_E + \Delta W_{\text{kit}})$$

for the airframe and the aircraft. Parameters are defined in table 6-2, including units as used in these equations.

6-1.2   Maintenance Cost

Total maintenance cost per hour (dollars per flight hour) is estimated from the statistical relationship of Harris and Scully (ref. 3, updated in 2001). The maintenance cost per hour is:

$$c_{\text{maint}} = 0.49885\, W_E^{0.3746} P^{0.4635}$$

This equation is based on 1994 dollars and current technology levels. Including an inflation factor F_i and technology factor χ_{maint} gives the maintenance cost per flight hour C_{maint}:

$$C_{\text{maint}} = \chi_{\text{maint}}(F_i c_{\text{maint}}) + C_{\text{batt-maint}}$$

Parameters are defined in table 6-2, including units as used in these equations.

Alternatively, the maintenance cost c_{maint} can be calculated from separate estimates of labor, parts (airframe, engine, and avionics), engine overhaul, and major periodic maintenance costs. The equations

for these maintenance cost components are from Harris (ref. 6), based on a 2011 civil database. The contributions to the dollars per flight hour are:

$$C_{\text{labor}} = r_{\text{labor}}(\text{MMH/FH})$$
$$c_{\text{parts}} = M_{\text{parts}}(C_{AC}/10^6)^{0.68}$$
$$c_{\text{engine}} = M_{\text{engine}}P^{0.67}$$
$$c_{\text{major}} = M_{\text{major}}(C_{AC}/10^6)$$
$$c_{\text{batt-maint}} = M_{\text{batt}}E_{\text{fuel-cap}}$$

where r_{labor} is the maintenance labor rate (dollars per hour). The battery maintenance cost includes replacement. The maintenance-man-hours per flight-hour is estimated from

$$\text{MMH/FH} = M_{\text{labor}}W_E^{0.78}$$

or specified directly. These equations are based on 2011 dollars and current technology levels. For current best practice (bottom of data), the constants are $M_{\text{labor}} = 0.0017$, $M_{\text{parts}} = 34$, $M_{\text{engine}} = 1.45$, $M_{\text{major}} = 18$; while for current average practice $M_{\text{labor}} = 0.0027$, $M_{\text{parts}} = 56$, $M_{\text{engine}} = 1.74$, $M_{\text{major}} = 28$. Finally, the maintenance cost per flight hour is

$$C_{\text{maint}} = \chi_{\text{maint}}F_i(c_{\text{parts}} + c_{\text{engine}} + c_{\text{major}}) + C_{\text{labor}} + C_{\text{batt-maint}}$$

including an inflation factor F_i and technology factor χ_{maint}.

6-1.3   Direct Operating Cost

Given specifics of the operation (including available block hours, non-flight time, spares fraction, and financial numbers), the direct operating cost is calculated for each mission. The direct operating cost includes maintenance, fuel, crew, depreciation, insurance, finance, and emission trading scheme (ETS) costs. The contributions to the yearly operating cost are:

$$C_{\text{fuel}} = \big(G_{\text{fuel}}(W_{\text{fuel}}/\rho_{\text{fuel}}) + G_{\text{energy}}E_{\text{fuel}}\big)N_{\text{dep}}$$
$$C_{\text{crew}} = 2.84F_iK_{\text{crew}}W_{MTO}^{0.4}B$$
$$C_{\text{dep}} = C_{AC}\,\frac{1+S}{D}\,(1-V)$$
$$C_{\text{ins}} = K_{\text{ins}}\,C_{AC}$$
$$C_{\text{fin}} = C_{AC}\,\frac{1+S}{D}\,\frac{2L+1}{4}\,\frac{i}{100}$$
$$C_{\text{ETS}} = K_{\text{ETS}}W_{CO_2}N_{\text{dep}}$$

There is a separate credit (G_{credit}, perhaps zero) for mission energy generation ($E_{\text{fuel}} < 0$). The crew factor $K_{\text{crew}} = 1$ corresponds to low-cost, domestic airlines (1994 dollars). The insurance factor $K_{\text{ins}} = 0.0056$ corresponds to 2005 airline operations (ref. 5). The fuel burn W_{fuel} and E_{fuel}, block time T_{miss}, block range R_{miss}, and CO_2 weight W_{CO_2} are obtained for a designated mission. The number of departures per year is $N_{\text{dep}} = B/T_{\text{miss}}$. The flight time per trip is $T_{\text{trip}} = T_{\text{miss}} - T_{NF}$. The flight hours per year are $T_F = T_{\text{trip}}N_{\text{dep}}$. Alternatively, the sum of the crew, insurance, and depreciation costs can be estimated from the purchase price:

$$C_{\text{crew}} + C_{\text{dep}} + C_{\text{ins}} = K_{\text{cdi}}\big(175000 + 29.8(C_{AC}/1000)\big)$$

where K_{cdi} is a calibration factor (ref. 6). The yearly operating cost C_{OP} and DOC (cents per available seat mile) are then:

$$C_{OP} = T_F C_{\mathrm{maint}} + C_{\mathrm{fuel}} + C_{\mathrm{crew}} + C_{\mathrm{dep}} + C_{\mathrm{ins}} + C_{\mathrm{fin}} + C_{\mathrm{ETS}}$$
$$\mathrm{DOC} = 100\, C_{OP}/\mathrm{ASM}$$

where the available seat miles per year are $\mathrm{ASM} = 1.1508 N_{\mathrm{pass}} R_{\mathrm{miss}} N_{\mathrm{dep}}$ (range in nm). The trip operating cost is $C_{\mathrm{trip}} = C_{OP}/N_{\mathrm{dep}}$. The passenger operating cost is $C_{\mathrm{pass}} = C_{\mathrm{trip}}/(N_{\mathrm{pass}}\, \mathrm{LoadFactor}/100)$.

6–2   Scott Rotorcraft Cost Model

The Scott rotorcraft cost model (refs. 7–8) gives an estimate of aircraft unit flyaway cost in terms of component costs. The model was developed for shaft-driven helicopters and turboprop aircraft.

For the Scott high-fidelity prediction approach, fully-parametric equations were developed for the components which represent the greatest joint interest among design and cost concerns. Components such as the landing gear, fuel system, electrical group, and furnishings which typically do not contribute major cost-driving effects to the overall cost of most aircraft received lower analytical priority. These components were assessed using a combination of semi-parametric methods based on either the costs of related components or analogies based on percentages or cost-per-weight trends.

The equations are for the N_p-th unit production cost of a component in a delivery lot of N_q units ($N_p - 1$ units already produced prior to the current lot). The unit production cost represents the recurring price the customer pays for the materials and labor required by the component. The integration and assembly of the components along with the systems engineering and the profit and fees charged by the manufacturer on a recurring basis with each lot are applied to the entire set of components by separate relations. The only exception to this rule in the high-fidelity model is the estimate of the total procurement cost of engines. In keeping with the practice observed more frequently in aircraft procurement, engines are modeled as items which are procured separately and integrated into the vehicle as government-furnished equipment (GFE).

Both the high and low-fidelity methods predict the unit flyaway price c_{FA}. The lower fidelity Harris-Scully model predicts this quantity directly. The higher fidelity component-based model predicts unit flyaway price by summing the costs of the prime equipment production, integration/assembly and systems engineering, and profit:

$$c_{\mathrm{FA}} = c_{\mathrm{pq}} + c_{\mathrm{int+SE}} + c_{\mathrm{profit}}$$

The flyaway cost estimate c_{FA} is based on 2018 dollars and current technology levels. Including an inflation factor F_i and technology factor χ_{comp} gives the purchase price C_{AC}:

$$C_{AC} = \chi_{\mathrm{comp}}(F_i c_{FA}) + C_{\mathrm{comp}} + C_{\mathrm{MEP}} + C_{\mathrm{FCE}} + C_{\mathrm{batt}}$$

where MEP and FCE costs are here included in the component cost estimates, but increments for composite construction cost and battery cost may still be required. For the component level model, the predicted prime equipment cost is the sum of the parametric and semi-parametric analysis components:

$$c_{\mathrm{pq}} = c_{\mathrm{param}} + c_{\mathrm{semiparam}}$$
$$c_{\mathrm{param}} = c_{\mathrm{wing}} + c_{\mathrm{rotor}} + c_{\mathrm{fuse}} + c_{\mathrm{eng}} + c_{\mathrm{prop}} + c_{\mathrm{xmsn}} + c_{\mathrm{av}}$$
$$c_{\mathrm{semiparam}} = c_{\mathrm{emp+nac+LG}} + c_{\mathrm{elec}} + c_{\mathrm{env}} + c_{\mathrm{FC+Inst+Hyd}} + c_{\mathrm{aux+fuelsys+propsys}} + c_{\mathrm{arm+furn+LH}}$$

The parametric cost elements are calculated first, since several of the semi-parametric equations depend on the parametric estimates.

In terms of overall performance, the high-fidelity model predicts the unit flyaway price with average error of 9.3% for military aircraft and 36.0% for civil helicopters. The Harris-Scully model's average accuracy was 39.8% for military aircraft and 21.1% for civil helicopters.

The component cost estimating relationships follow. Each equation estimates the unit price of the component for the N_p-th production aircraft in thousands of FY18 US Dollars, where N_q is the number of units delivered in the production lot containing the N_p-th unit (and $N_p - 1$ is the number of units previously produced). Thus $N_p = N_q = 1$ gives the component costs for the first aircraft produced. Parameters are defined in table 6-3, including units as used in these equations.

Wing

$$c_{\text{wing}} = 1.4683\, W_{\text{wing}}^{0.5176} \left(\frac{f_{\text{wing}} W_{MTO}}{S_w} \right)^{0.9963} A_w^{0.5478} \sigma_{\text{wing}}^{-0.2742} (Y - 1986)^{-0.3050} N_q^{-0.0813} N_p^{-0.1125}$$

where $\sigma_{\text{wing}} = W_{\text{wing}}/S_w$ is the wing weight per planform area; and f_{wing} is the design lift-share fraction of the wing in cruise ($f_{\text{wing}} = 1.0$ for fully wing-borne forward flight).

Rotor

$$c_{\text{rotor}} = 191.67\, W_{\text{rotor}}^{0.7127} N_{\text{blade}}^{1.5546} A_{\text{blade}}^{-1.3931} f_{\text{MRhub}}^{2.4423} (Y - 1968)^{0.0944} N_q^{-0.0130} N_p^{-0.1572}$$

where W_{rotor} is the weight of all main and tail rotors; and $f_{\text{MRhub}} = W_{\text{hub}}/(W_{\text{hub}} + W_{\text{blade}})$ is the hub's fraction of the main rotor weight.

Fuselage

$$c_{\text{fuse}} = 19.968\, W_{\text{fuse}}^{0.5232} f_{\text{sec}}^{-1.2330} \sigma_{\text{fuse}}^{-0.1131} H_{\text{fuse}} (Y - 1986)^{0.2788} N_q^{-0.0367} N_p^{-0.1054}$$

where $\sigma_{\text{fuse}} = W_{\text{fuse}}/(2\pi r_{\text{fuse}} L_{\text{fuse}})$ is the fuselage weight per wetted area; $f_{\text{sec}} = W_{\text{sec}}/W_{\text{fuse}}$ is the fraction of secondary fuselage weight, $W_{\text{fuse}} = W_{\text{prim}} + W_{\text{sec}}$; and $H_{\text{fuse}} = \kappa_{\text{boom}} \kappa_{\text{dev}}$:

$$\kappa_{\text{boom}} = \begin{cases} 0.6290 & \text{fuselage includes tailboom} \\ 1.0 & \text{no tailboom} \end{cases}$$

$$\kappa_{\text{dev}} = \begin{cases} 1.4282 & \text{early LRIP of clean sheet design} \\ 1.0 & \text{otherwise} \end{cases}$$

Turboshaft Engine
Full unit procurement cost of turboshaft engines for VTOL applications:

$$c_{\text{eng}} = 0.0056921\, P_{\text{eng}}^{0.8152} SP^{0.8304} Pr_{\text{avg}}^{0.7557} \text{TBO}_{\text{eng}}^{0.3657} H_{\text{eng}} (Y - 1955)^{-0.2475} N_p^{-0.0759}$$

where $Pr_{\text{avg}} = \text{OPR}^{1/N_{st}}$ is the stage-averaged engine compressor pressure ratio (OPR the compressor overall pressure ratio, and N_{st} the number of compressor stages); and $H_{\text{eng}} = \kappa_{\text{mar}} \kappa_{\text{FADEC}}$:

$$\kappa_{\text{mar}} = \begin{cases} 1.1644 & \text{marinized engine} \\ 1.0 & \text{non-marinized engine} \end{cases}$$

$$\kappa_{\text{FADEC}} = \begin{cases} 0.7298 & \text{FADEC equipped engine} \\ 1.0 & \text{non-FADEC equipped} \end{cases}$$

For non-rotorcraft/VTOL applications, the equations for turboprops and propellers are used.

Turboprop Engine

Unit cost of one turboprop engine or high performance aviation reciprocating engine:

$$c_{\text{eng}} = 1.1786\, P_{\text{eng}}^{0.9465}$$

Electrical Engine

Unit cost of one motor or generator:

$$c_{\text{eng}} = K_{\text{motor}} P_{\text{eng}}^{X_{\text{motor}}}$$

Propeller

Unit cost of one propeller:

$$c_{\text{prop}} = 0.0209\, P_{\text{prop}}^{1.1432}$$

Drive

$$c_{\text{xmsn}} = 1.2329\, W_{\text{xmsn}}^{0.4309} N_{gb}^{2.2692} r_{\text{xmsn}}^{0.2745} H_{\text{xmsn}} (Y - 1968)^{-0.1566} N_q^{-0.0501} N_p^{-0.0281}$$

where $r_{\text{xmsn}} = \Omega_{\text{eng}}/\Omega_{\text{MR}}$ is the overall reduction ratio of the transmission system; and $H_{\text{xmsn}} = \kappa_{\text{rg}}\kappa_{\text{mar}}$

$$\kappa_{\text{rg}} = \begin{cases} 0.3241 & \text{engine group includes reduction gearbox} \\ 1.0 & \text{transmission input direct from engine shaft} \end{cases}$$

$$\kappa_{\text{mar}} = \begin{cases} 2.0203 & \text{marinized transmission system} \\ 1.0 & \text{non-marinized transmission} \end{cases}$$

Avionics

Unit cost of base avionics package, which includes an automatic flight control system (afcs), communication and navigation systems, vehicle management and HUMS systems (if installed), but not specialized mission equipment such as survivability, imaging, or fire control:

$$c_{\text{av}} = 0.0269\, W_{\text{av}}^{1.6050} f_{\text{afcs}}^{0.4948} H_{\text{av}} (Y - 1970)^{0.1141}$$

where $W_{\text{av}} = W_{\text{afcs}} + W_{\text{comm}} + W_{\text{nav}} + W_{\text{vms}} + W_{\text{HUMS}}$ is the weight of the base avionics package; $f_{\text{afcs}} = W_{\text{afcs}}/W_{\text{av}}$ is the automatic flight control system's fraction of the total base avionics package; and $H_{\text{av}} = \kappa_{\text{dev}}\kappa_{\text{UAV}}$:

$$\kappa_{\text{dev}} = \begin{cases} 5.1690 & \text{early LRIP of new avionics package} \\ 1.0 & \text{otherwise} \end{cases}$$

$$\kappa_{\text{UAV}} = \begin{cases} 3.3230 & \text{unmanned medium to long endurance aircraft} \\ 1.0 & \text{otherwise} \end{cases}$$

Empennage, Nacelle, and Landing Gear

The smaller components of airframe structure are estimated together. Adjustment would likely be warranted for non-conventional tails, nacelle, or air induction configurations. The cost is: Fixed landing gear:

$$c_{\text{emp+nac+LG}} = 1.8870\, W_{\text{emp+nac+LG}}^{0.6596} \kappa_{\text{ret}}$$

where

$$\kappa_{\rm ret} = \begin{cases} 2.6786 & \text{retractable landing gear} \\ 1.0 & \text{fixed landing gear} \end{cases}$$

Auxiliary Power System, Fuel, and Propulsion Systems

The cost of the auxiliary power group, fuel system, exhaust system, and propulsion controls and accessories are considered together.

$$\rho_{\rm aux+fuelsys+propsys} = \frac{c_{\rm aux+fuelsys+propsys}}{W_{\rm aux+fuelsys+propsys}} = 0.0034379\, W_{\rm propsys}^{0.9630}$$

where $W_{\rm propsys}$ is the weight of the engine exhaust, engine starting and control systems, and the propulsion accessories.

Flight Controls, Instruments, and Hydraulic Systems

The automatic flight control system is considered part of the avionics group, not the flight control group. Flight controls weight in this group denotes the weight of the system controls and the cockpit controls, $W_{FC} = W_{FC,Sys} + W_{FC,CC}$:

$$c_{\rm FC+Inst+hyd} = 1.8557(W_{FC,Sys} + W_{FC,CC} + W_{\rm inst} + W_{\rm hyd})^{0.84757}$$

Electrical

The cost of the electrical group is estimated from the cost of the avionics and environmental groups:

$$\rho_{\rm elec} = \frac{c_{\rm elec}}{W_{\rm elec}} = 0.01503\, (c_{\rm av} + c_{\rm env})^{0.6353}$$

Environmental

The environmental group includes the anti-ice and air conditioning groups. The environmental group is estimated as a fraction of the parametric prime equipment components:

$$c_{\rm env} = f_{\rm env}(c_{\rm wing} + c_{\rm fuse} + c_{\rm rotor} + c_{\rm drive} + c_{\rm eng} + c_{\rm av})$$

where $f_{\rm env}$ is 0.5% to 3.0% for transport aircraft.

Armament, Furnishings, Load and Handling

The armament provisions, furnishings, and load and handling groups are estimated as a fraction of the fuselage cost:

$$c_{\rm arm+furn+LH} = f_{\rm arm+furn+LH}\, c_{\rm fuse}$$

where $f_{\rm arm+furn+LH} \approx 0.12$ for conventional transport aircraft.

Integration and Assembly, Systems Engineering, and Profit

Based on the observed history of U.S. government aircraft procurement, the costs of integration and assembly, systems engineering, and prime manufacturers' profit are estimated as a function of the production cost of the prime equipment $c_{\rm pq}$. Note that the engine group cost is included as a component of prime equipment, and already includes the cost of engine assembly and profit.

$$c_{\rm int+SE} = 7.8271\, c_{\rm pq}^{0.6897}$$

$$c_{\rm profit} = 5.2003\, (c_{\rm pq} - c_{\rm eng})^{0.6749}$$

For commercial and civil aircraft, integration and assembly, systems engineering, and profit are collectively estimated as a percentage of prime equipment:

$$c_{\text{int}+\text{SE}} + c_{\text{profit}} = f_{\text{int}+\text{SE}+\text{profit}} \, c_{\text{pq}}$$

where $f_{\text{int}+\text{SE}+\text{profit}} \approx 0.25$.

6–3   References

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2) "Consumer Price Index." U.S. Department of Labor, Bureau of Labor Statistics, 2020. All Urban Consumers (CPI-U), U.S. city average, All items.

3) Harris, F.D., and Scully, M.P. "Rotorcraft Cost Too Much." Journal of the American Helicopter Society, Vol. 43, No. 1, January 1998.

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5) Coy, J.J. "Cost Analysis for Large Civil Transport Rotorcraft." American Helicopter Society Vertical Lift Aircraft Design Conference, San Francisco, California, January 2006.

6) Harris, F.D. "Introduction to Autogyros, Helicopters, and Other V/STOL Aircraft." NASA SP 2011-215959, volume 2, 2012.

7) Scott, R. "A Design-Centric Evaluation of Multi-Fidelity Cost Modeling Approaches." Forty-Fourth European Rotorcraft Forum, Delft, The Netherlands, September 2018.

8) Scott, R. and Vegh, J.M. "Progress Toward a New Conceptual Assessment Tool for Aircraft Cost." Vertical Flight Society Annual Forum, October 2020.

Table 6-1. DoD and CPI inflation factors.

year	inflation factors DoD	CPI	year	inflation factors DoD	CPI	year	inflation factors DoD	CPI
1911			1951	17.02	17.54	1991	93.99	91.90
1912			1952	16.26	17.88	1992	96.21	94.67
1913		6.68	1953	16.31	18.02	1993	98.16	97.50
1914		6.75	1954	16.02	18.15	1994	100.00	100.00
1915		6.82	1955	17.29	18.08	1995	101.70	102.83
1916		7.35	1956	17.58	18.35	1996	103.18	105.87
1917		8.64	1957	18.17	18.96	1997	104.32	108.30
1918		10.19	1958	18.02	19.50	1998	105.40	109.99
1919		11.67	1959	18.11	19.64	1999	106.81	112.42
1920		13.50	1960	18.34	19.97	2000	108.39	116.19
1921		12.08	1961	18.31	20.18	2001	109.92	119.50
1922		11.34	1962	18.84	20.38	2002	111.62	121.39
1923		11.54	1963	19.09	20.65	2003	113.95	124.16
1924		11.54	1964	19.76	20.92	2004	116.92	127.46
1925		11.81	1965	20.36	21.26	2005	120.10	131.78
1926		11.94	1966	21.97	21.86	2006	123.06	136.03
1927		11.74	1967	22.77	22.54	2007	125.56	139.91
1928		11.54	1968	24.03	23.48	2008	127.43	145.28
1929		11.54	1969	25.10	24.76	2009	128.95	144.76
1930		11.27	1970	25.76	26.18	2010	131.66	147.14
1931		10.26	1971	27.24	27.33	2011	133.84	151.78
1932		9.24	1972	28.98	28.21	2012	136.07	154.92
1933		8.77	1973	31.34	29.96	2013	138.34	157.19
1934		9.04	1974	34.13	33.27	2014	139.54	159.74
1935		9.24	1975	38.10	36.30	2015	141.40	159.93
1936		9.38	1976	42.14	38.39	2016	144.23	161.28
1937		9.72	1977	43.75	40.89	2017	147.05	164.68
1938		9.51	1978	47.82	43.99	2018	150.01	167.82
1939		9.38	1979	53.02	48.99	2019	152.98	171.66
1940		9.45	1980	58.52	55.60	2020	156.04	174.00
1941		9.92	1981	63.80	61.34	2021*	159.16	
1942		11.00	1982	68.35	65.11	2022*	162.34	
1943		11.67	1983	71.92	67.21	2023*	165.59	
1944		11.88	1984	74.57	70.11	2024*	168.90	
1945		12.15	1985	76.86	72.60	2025*	172.27	
1946		13.16	1986	79.19	73.95			
1947		15.05	1987	81.87	76.65			
1948		16.26	1988	85.01	79.82			
1949		16.06	1989	88.19	83.67			
1950	15.37	16.26	1990	91.30	88.19			

* projected

Table 6-2. Cost model parameters.

parameter	definition	units
W_E	weight empty	lb
W_{MTO}	maximum takeoff weight	lb
N_{blade}	number of blades per rotor	
P	rated takeoff power (all engines)	hp
f_{motor}	weighting factor for electrical engines	
W_{MEP}	fixed useful load weight, mission equipment package	lb or kg
W_{FCE}	fixed useful load weight, flight control electronics	lb or kg
W_{batt}	battery weight	lb or kg
$E_{\text{fuel}-\text{cap}}$	battery capacity	MJ
r_{MEP}	cost factor, mission equipment package	\$/lb or \$/kg
r_{FCE}	cost factor, flight control electronics	\$/lb or \$/kg
r_{batt}	cost factor, battery	\$/MJ or \$/kWh
r_{comp}	additional cost for composite construction	\$/lb or \$/kg
F_i	inflation factor, relative 1994	
W_{fuel}	mission fuel burned	lb or kg
E_{fuel}	mission energy used	MJ
T_{miss}	mission time	hr
R_{miss}	mission range	nm
W_{CO_2}	mission CO_2 emissions weight	kg
G_{fuel}	fuel cost	\$/gallon or \$/liter
G_{energy}	energy cost	\$/MJ or \$/kWh
G_{credit}	energy credit	\$/MJ or \$/kWh
B	available block hours	hr
S	spares per aircraft (fraction purchase price)	
D	depreciation period	yr
V	residual value (fraction)	
L	loan period	yr
i	interest rate	%
T_{NF}	non-flight time per trip	hr
N_{pass}	number of passengers	
ρ_{fuel}	fuel density (weight per volume)	lb/gal or kg/liter
N_{dep}	number of departures per year	
K_{crew}	crew factor	
K_{ins}	insurance cost fraction	
T_F	flight hours per year	
LoadFactor	passenger load factor	%

Table 6-3. Scott cost model parameters.

parameter	definition	units
c	cost	$K
ρ	cost per weight	$K/lb
Y	fiscal year of procurement	
N_p	production unit corresponding to the estimated cost	
N_q	number of units delivered in production lot containing N_p-th aircraft	
W	component weight	lb
Wing		
A_w	wing aspect ratio	
S_w	wing planform area	ft^2
f_{wing}	design lift-share fraction of wing in cruise	
W_{MTO}	maximum takeoff weight	lb
Rotor		
A_{blade}	rotor blade aspect ratio	
N_{blade}	rumber of blades per main rotor	
f_{MRhub}	hub fraction of the main rotor weight	
Fuselage		
L_{fuse}	fuselage length	ft
r_{fuse}	fuselage half-width at point of maximum fuselage diameter	ft
f_{sec}	fraction of secondary fuselage weight	
Engine and Propeller		
P_{eng}	engine maximum rated power (MRP)	hp
P_{prop}	propeller shaft maximum rated power	hp
$SP = P_{\text{eng}}/\dot{m}$	engine specific power	hp/lb/sec
Pr_{avg}	stage-averaged engine compressor pressure ratio	
TBO_{eng}	engine time between overhaul	flight hours
K_{motor}	electrical engine cost factor	
X_{motor}	electrical engine cost exponent	
Drive		
N_{gb}	drive system number of gearboxes	
Ω_{eng}	engine output shaft design speed	rpm
Ω_{MR}	main rotor shaft design speed	rpm
Avionics		
f_{afcs}	weight fraction of automatic flight control system in avionics weight	
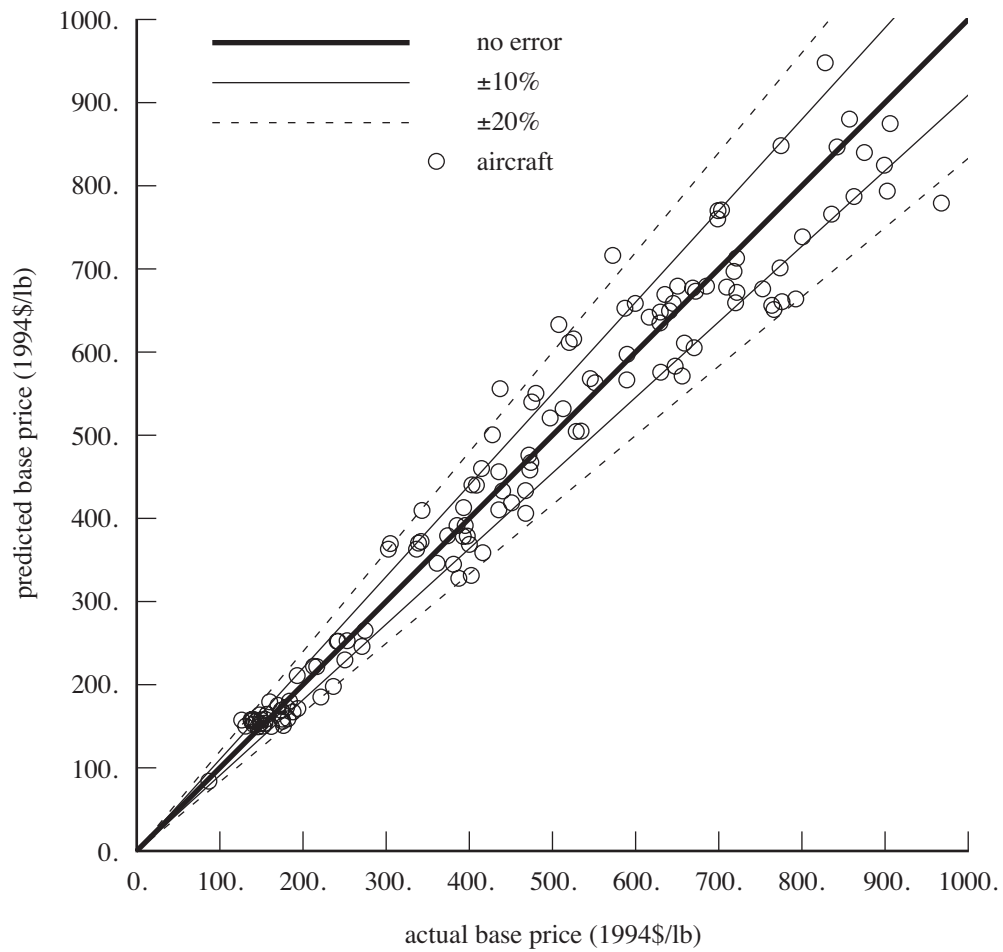
Figure 6-1. Statistical estimation of rotorcraft purchase price ($/lb).
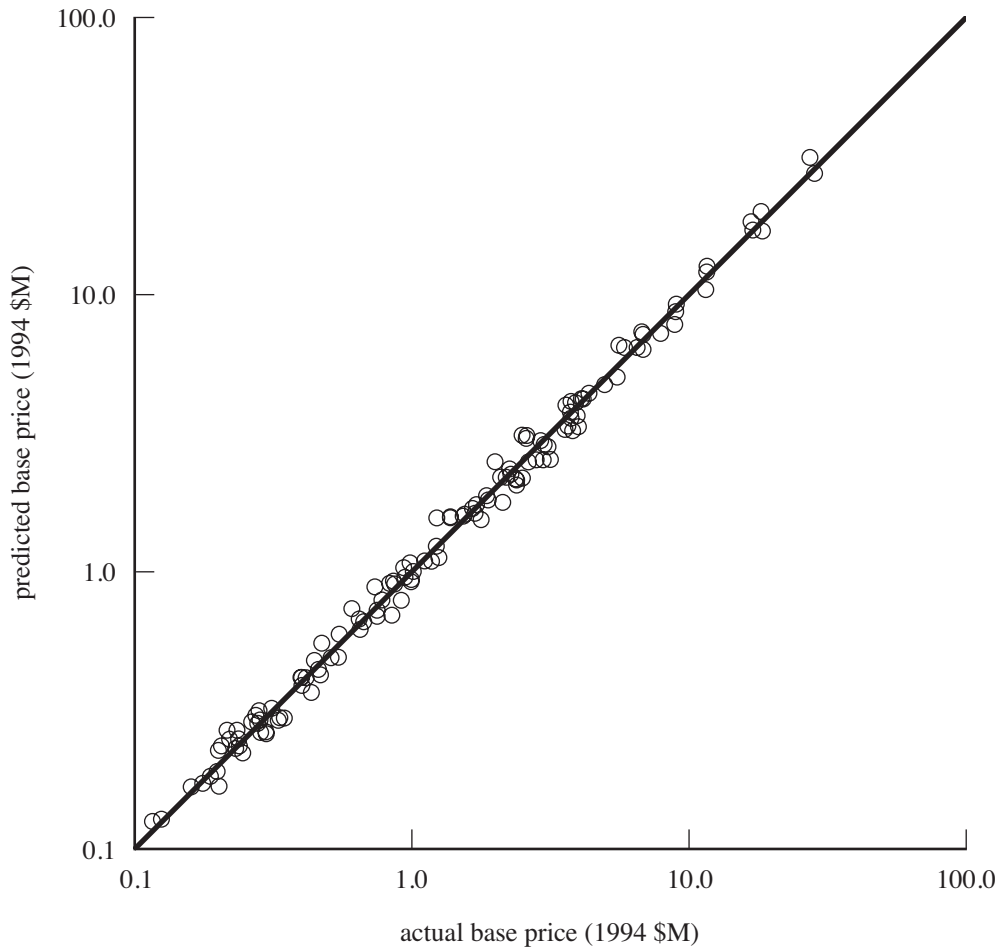
Figure 6-2. Statistical estimation of rotorcraft purchase price ($M).

Emissions

Evaluating the environmental impact of an aircraft begins with calculating the engine emissions for the missions flown. It is best to use metrics that account for all relevant aircraft emissions, although metrics based on a single species can be useful. From the engine emissions, the atmospheric concentrations are evaluated, then radiative forcing (RF). Radiative forcing is a measure of the amount of heat trapped in the atmosphere by a particular pollutant, and is expressed in terms of trapped energy per unit area (W/m^2). Finally, climate changes and the resulting impacts and damages can be determined. The steps in this causal chain become increasingly important and increasingly uncertain. Any metric that is used to evaluate new aircraft concepts must balance uncertainty with relevance. The present environmental impact models were developed by Russell (ref. 1), based on the work of Dallara, Kroo, and Waitz (ref. 2).

The important aircraft emission species are the direct greenhouse gas carbon dioxide (CO_2), nitrogen oxides (NO_x), water vapor (H_2O), aerosols of soot and sulphate (SO_4), and aviation-induced cloudiness (AIC). Carbon dioxide has a long lifetime relative to chemical processes in the atmosphere, hence aviation CO_2 impacts do not vary with altitude and can be calculated as for other sources. NO_x emissions cause changes in RF indirectly through chemical processes in the atmosphere. Increases in NO_x lead to increases in atmospheric ozone by a short lifetime process (O_{3S}), a warming effect, and over long timescales reductions in the greenhouse gas methane (CH_4), a cooling effect. The methane reduction has a secondary effect of reducing ozone (O_{3L}), a cooling effect. Water vapor, short-lived ozone, soot, and sulfate aerosols have lifetimes much shorter than one year, hence have impacts for only a short time after emission. Aviation-induced cloudiness (AIC), including the effects of linear contrails and altered cirrus cloudiness, have a short-lived warming effect. Total radiative forcing from aircraft can be several times greater than forcing due to CO_2 alone, depending on altitude and AIC conditions. The impact of NO_x emissions is similar in magnitude to that of CO_2, but there is greater uncertainty in the NO_x RF values. Estimates of the RF due to induced cirrus are particularly uncertain. The uncertainty of the models and parameters for the various emission species is discussed in reference 2.

The environmental impact of the aircraft operation is estimated in terms of two metrics: emissions trading scheme (ETS) credits, and average temperature response (ATR). The impact of energy used is also modeled.

7–1 Emissions Trading Scheme Credits

The emissions trading scheme (ETS) is the European Union system to curb the effects of greenhouse gas emissions on global climate change, by limiting the amount of carbon dioxide that can be produced by large polluters such as energy and industrial installations. The ETS was extended to the aviation sector in early 2012. Under the ETS, each member nation has an emission cap that is used to allocate allowable carbon emissions to their industrial operators. Operators that do not use their entire allocation

can sell their unused "carbon credits" on the open market, while operators exceeding their allocation must purchase credits on the market. The price of carbon credits has varied widely, ranging from below 10 to above 30 Euro/ton CO_2. The metric used here is the weight of CO_2 produced:

$$W_{CO_2} = K_{\text{fuel}} W_{\text{burn}} + K_{\text{energy}} E_{\text{burn}}$$

per mission. Typically $K_{\text{fuel}} = EI_{CO_2} \cong 3.75$ (lb/lb or kg/kg) and $K_{\text{energy}} \cong 0.14$ (kg/MJ or kg/kWh). Different aircraft designs or operations can be compared in terms of W_{CO_2}. Accounting for the influence on operating cost requires the price of carbon credits.

K_{fuel} depends on the combustion process and the chemical composition of the fuel (ref. 3). For combustion of conventional jet fuel, $K_{\text{fuel}} = 3.16\,\text{kg/kg} = 74\,\text{g/MJ} = 266\,\text{g/kWh}$ (using 42.8 MJ/kg). Accounting for emissions from production of the fuel (principally carbon dioxide, methane, and nitrogen oxides), gives an equivalent CO_2 value perhaps 20% larger: $K_{\text{fuel}} = 3.75\,\text{kg/kg} = 88\,\text{g/MJ} = 315\,\text{g/kWh}$. For combustion of alternative jet fuels, typically $K_{\text{fuel}} = 3.01\,\text{kg/kg} = 70\,\text{g/MJ} = 263\,\text{g/kWh}$. Accounting for biomass credit (CO_2 absorbed during biomass growth), the equivalent CO_2 value might be 50% lower than for conventional fuel (ref. 3).

K_{energy} depends on the energy source. Table 7-1 (from ref. 4) gives typical values of equivalent CO_2 for electricity from various sources. Worldwide average values for equivalent CO_2 range from 6 to 1100 g/kWh. The current average for the United States is about $K_{\text{energy}} = 500\,\text{g/kWh} = 139\,\text{g/MJ}$. Optionally there is no emissions credit for energy generation ($E_{\text{burn}} < 0$).

Table 7-1. Electricity green-house-gas intensity (equivalent CO_2, g/kWh).

source	median	range
coal	860	780-1100
gas	500	400-600
small wind turbine	35	20-50
nuclear	22	10-30
large wind turbine	20	15-25
hydroelectric	18	10-20

7–2   Average Temperature Response

The average temperature response (ATR) quantifies the lifetime global mean temperature change caused by operation of an aircraft, as a measure of climate change (ref. 2). ATR can be used with a number of different climate models, but simple linear climate models are appropriate for the conceptual design of rotorcraft. While atmospheric processes are nonlinear, the globally averaged responses to small perturbations (in magnitude similar to commercial aircraft emissions) of CO_2, NO_x, and AIC perturbations are nearly linear (ref. 2). The ATR metric is based on the radiative forcing (RF) generated by each emission species. Radiative forcing measures the net imbalance of incoming and outgoing energies in the Earth-atmosphere system caused by a perturbation. RF is an instantaneous measure that quantifies the change in energy that produces changes in climate properties, including temperature. Many climate change metrics, such as Global Warming Potentials, rely on RF, but do not specifically target emissions due to aviation. The total RF for all emitted pollutants is used to calculate the global temperature response. The use of an altitude-sensitive climate model captures the effects of aircraft operating conditions. In addition, ATR includes parameters such as usage rates and operating lifetime of the aircraft to determine the total climate impact.

The average temperature response is calculated by integrating the temperature change during and after H years of sustained operation of an aircraft:

$$\text{ATR} = \frac{1}{H} \int_0^\infty \Delta T_H(t)\, w(t)\, dt$$

ATR has units of temperature. The metric can also be used in relative terms, by dividing by the value for a baseline design. $\Delta T_H(t)$ is the global mean temperature change resulting from the operation of the given design, where emissions are assumed to be constant for the first H years and zero thereafter. This temperature change is a function of the radiative forcing caused by the emission of a number of different pollutant species. There are multiple models available with varying levels of fidelity for calculating ΔT_H. Here linear climate models and functions are used, based on reference 2. One limitation of this model is that the RF of NO_x and AIC is based on data that only goes down to 17500 ft. Below this altitude the effects of NO_x and AIC are assumed constant.

There are climate effects from perturbations remaining in the Earth-atmosphere system after the aircraft operating lifetime has ended. Hence ΔT_H increases for time up to H, and thereafter decays, but is not zero. The weighting function $w(t)$ allows discounting of temperature change effects in the years following H, so that long-term effects such as CO_2 warming do not dominate ATR. The function

$$w(t) = \begin{cases} 1 & t \le H \\ \dfrac{1}{(1+r)^{t-H}} & H < t \le t_{\max} \\ 0 & t > t_{\max} \end{cases}$$

has unit weighting during the operating lifetime, and exponential devaluation thereafter, in terms of the discount rate r (typically $r = 2$ to 5%). A rate of zero means that post-operation impacts are equally important compared with impacts during operating years; a rate of infinity means that post-operation impacts have no importance; and a positive, finite rate means that the temperature change each post-operation year is less important than the temperature change experienced the previous year (by the factor $1/(1+r)$).

Also contributing to the ATR metric is aviation induced cloudiness (AIC), which includes both contrails and aviation-induced cirrus clouds. Following the methodology of reference 2, the impact of AIC is assumed to be a function of cruise altitude and distance, and does not account for changes in water vapor emissions or exhaust temperature or atmospheric conditions. The consequence of these assumptions is that for a given rotorcraft mission, the radiative forcing due to AIC may be greatly overestimated. Also, strategies are currently being developed to reduce the effects of AIC by rerouting around areas in the atmosphere prone to aircraft contrails, so future aircraft may have significantly reduced climate impacts due to AIC. Since there is a high degree of uncertainty in radiative forcing due to AIC, calculations of ATR should be made both with and without its effects.

Design for minimum ATR depends on the time horizon, discount rate, operating altitude and speed, and the engine technology. If there is no discounting of long-term effects ($r = 0$), radiative forcing due to CO_2 dominates the ATR, favoring a higher cruise altitude. When long term effects are included and engine technology for low NO_x is assumed, ATR can be generally minimized by designing for low fuel burn. If only short-term impacts are considered ($r = \infty$), then NO_x emissions have a greater impact, and a lower cruise altitude results in decreased climate impact, despite increased fuel burn and CO_2 emissions. The inclusion of AIC in the metric calculations also has a large impact on the design, and favors a low cruise altitude.

7-2.1   Aircraft Mission

ATR is calculated assuming constant emissions during the first H years of operation and zero emissions thereafter. For a single operating mission, ΔT_H is the sum over all mission segments, times a utilization rate of U missions per year. The information for each mission segment includes: fuel burned W_{burn}, energy used E_{burn}, altitude h, air distance flown d, engine fuel flow $\dot{w}$, and operating condition (M, θ, δ).

7-2.2   Engine Emissions

Engine emissions can be quantified in terms of an emissions index (EI, lb/lb or kg/kg) that relates the amount of a pollutant species to the amount of fuel burned. For some emission species, such as CO_2 and water vapor, the EI is nearly constant, assuming ideal or near-ideal fuel combustion. For others, particularly NO_x, the EI varies widely throughout the operating range of the engine. The emission of species i is thus obtained from the fuel consumption summed over all mission segments:

$$e_i = \sum_{\text{seg}} EI_i W_{\text{burn}}$$

The time-varying annual emissions E is found by summing over all operations conducted in a year. Assuming a single operating mission, $E_i = Ue_i$, where U is the aircraft utilization rate (number of missions per year).

For each fuel, EI is required for CO_2, H_2O, SO_4, soot, and NO_x. Optionally there is no emissions credit for energy generation ($E_{\text{burn}} < 0$).

7-2.3   Turboshaft Engine Emissions

For carbon dioxide, water vapor, sulfates, and soot the emissions indices are assumed to be constant. The EI of CO_2, H_2O, or SO_4 depends solely on the composition of the fuel. The EI for soot can vary with engine operating condition, but because soot comprises a small fraction of total climate impacts (typically less than 5%), this index can also be assumed constant. The values used for the constant EI emissions are given in table 7-2 (from ref. 2).

Table 7-2. Emission indices for constant EI species.

species	emission index EI (lb/lb or kg/kg)		
	jet fuel	methane	hydrogen
CO_2	3.16	2.75	0
H_2O	1.26	2.25	8.94
SO_4	2.0×10^{-4}		
soot	4.0×10^{-5}		

The EI for oxides of nitrogen (including NO and NO_2, collectively called NO_x) depends on operating conditions, including engine throttle setting, Mach number, and altitude (temperature and pressure). Hence EI_{NO_x} must be evaluated for each mission segment. Engines are required to comply with NO_x emissions regulations during landing and takeoff. The International Civil Aviation Organization (ICAO) measures and publishes certification data relating fuel flow, thrust, and EI_{NO_x} at four sea-level static throttle settings that simulate taxi, takeoff, climb, and approach operating conditions. However, ICAO

does not measure NO_x at cruise conditions, because these emissions are not currently regulated. There-
fore, a model is needed to compute EI_{NO_x} at specific flight conditions including cruise. Fuel flow
correlation methods have been developed for this purpose. While there is a large amount of published
turbofan NO_x emissions data and established methods for estimating variation with altitude, there is
little public information for turboshaft engines. Russell (ref. 1) developed a method to estimate EI_{NO_x}
based on engine performance information from the NDARC engine models, ICAO turbofan emissions
data, and the German Aerospace Center (DLR) fuel flow method described in reference 5. The DLR
semi-empirical method models the relationship between engine throttle setting and EI_{NO_x} at varying
ambient temperature and pressure conditions. The DLR method has been shown to predict EI_{NO_x} to
within approximately 10% of measurements at typical cruise conditions.

The emission index for NO_x is calculated from the fuel flow $\dot{w}$ as follows. The fuel flow is corrected
to referred conditions at the inlet: $\dot{w}_{corr} = \dot{w}/\delta_t\sqrt{\theta_t})$, where the temperature and pressure ratios are

$$\theta_t = \left(1 + \frac{\gamma-1}{2}M^2\right)\theta = \left(1 + 0.2M^2\right)\theta$$

$$\delta_t = \left(1 + \frac{\gamma-1}{2}M^2\right)^{\frac{\gamma}{\gamma-1}}\delta = \left(1 + 0.2M^2\right)^{3.5}\delta$$

(for ratio of specific heats $\gamma = 1.4$). The engine model gives the fuel flow $\dot{w}$ as a function of $P_{req} = f_P P_{to}$
at SLS static conditions, which can be interpolated to find f_P such that $\dot{w} = \dot{w}_{corr}$. The corrected emission
index is scaled to SLS takeoff power, hence

$$EI_{NO_x corr} = K_{EI0} + K_{EI1}f_P$$

The constants depend on the engine design and technology. Typically $1000K_{EI0} = 3.6739$ and $1000K_{EI1}$
$= 7.48$ for a low-emissions engine (based on CF34); $1000K_{EI0} = 2.4392$ and $1000K_{EI1} = 23.26$ for a
high-emissions engine (based on HTF7000). Finally, the emission index is

$$EI_{NO_x} = EI_{NO_x corr}\theta_t^3\delta_t^{0.4}F_H$$

with the humidity correction factor $F_H = \exp\left((6.29 - e^{-0.000143(h-12900)})/53.2\right)$, altitude h in ft.

There are two assumptions in this method. The first assumption is that the variation of EI_{NO_x} for
a turboshaft is the same as that of a turbofan engine. Since both are gas turbine engines, employing the
same thermal cycle, this assumption should be valid. The second assumption is that the RPTEM engine
model is producing variations in fuel flow with speed and altitude in a manner consistent with the DLR
fuel flow method. Since both the fuel flow method and the engine model in NDARC are based on real
engine data, this assumption should be valid as well. Two different engines are used as references. The
GE CF34-3B and the Honeywell HTF7000 have fuel flows similar to what is calculated for a turboshaft
engine. Both engines are relatively modern, high bypass ratio, small turbofans. The HTF7000 and CF34
also represent the upper and lower bounds on NO_x emissions for this category of engine, so they should
bracket the expected quantity of NO_x emissions for a turboshaft engine. Compared to the differences
between the EI_{NO_x} values of the two reference engines, the speed and altitude corrections are relatively
small.

An alternative approach is to calculate the emission index for NO_x from the engine power required.
Reference 6 gives the parametric relation

$$EI_{NO_x} = K_{EIs}\left(\frac{P_{req}}{1000}\right)^{0.5677}$$

with P_{req} in hp, and $1000K_{EIs} = 4$ to 14, depending on the engine design and technology.

With more than one turboshaft engine group using fuel from a tank, the emission index for a segment is weighted by the fuel flow, hence by the fuel burned: $EI_{NO_x} = \sum EI\,\dot{w}/\sum\dot{w}$ (sum over engine groups).

7-2.4   Energy Emissions

The environmental impact of energy used is quantified in terms of constants K (kg/MJ or kg/kWh) that relate the amount of a pollutant species to the amount of energy burned. These constants depend on the source of the energy, and are required for CO_2, H_2O, SO_4, soot, and NO_x. The emission of species i is thus obtained from the energy consumption summed over all mission segments:

$$e_i = \sum_{seg} K_i E_{burn}$$

The time-varying annual emissions are $E_i = Ue_i$, which is added to the engine emissions. In the climate model, the altitude corresponding to these emissions is zero.

7-2.5   Climate Model and Radiative Forcing

A climate model is required to determine the temperature change resulting from the emissions produced by an aircraft operation. The simplest approach (ref. 2) is to use a linear climate model, so the time-varying radiative forcing of each species is obtained from the emissions and a climate impulse response function. Although derived from the globally averaged results of some complex, three-dimensional, global climate model, a linear model simplifies the physics and chemistry of aircraft-induced climate change, and captures first-order effects. The climate model used, developed in reference 2 specifically for aircraft operations, includes the altitude variation for NO_x and AIC impacts. Different models are required for long-lived gases, short-lived pollutants, and AIC. Models for RF from all emissions except CO_2, which has a very long atmospheric lifetime, depend on the assumed geographical distribution of emissions. The models used are based on average impacts from fleetwide routing in a single year within the last decade, quantifying the average forcing caused by emissions spatially distributed according to routing similar to current traffic.

For long-lived emission species i (CO_2, CH_4, O_{3L}), the radiative forcing RF_i is obtained by integrating the product of the E_i (kg/yr) and the impulse response G_i (W/m^2/kg):

$$RF_i(t) = s_i(h) \int_0^t G_i(t-\tau)E_i(\tau)\,d\tau$$

$$= U\sum_{seg} s_i(h)\Big(EI_i W_{burn} + K_i E_{burn}\Big)A_i \int_0^t (G_i(t-\tau)/A_i)u(\tau)\,d\tau$$

$$= U\sum_{seg} s_i(h)a_i A_i X_i(t)$$

where $a_i = (EI_i W_{burn} + K_i E_{burn})$, $u(t)$ is the emission time history (unit amplitude), and for $t > 0$

$$G_{CO_2}(t) = A_{CO_2}\left(1 + \Sigma_{j=1}^3 \alpha_{cj}\left(e^{-t/\tau_{cj}} - 1\right)\right)$$

$$G_{CH_4}(t) = A_{CH_4}e^{-t/\tau_n}$$

$$G_{O_{3L}}(t) = A_{O_{3L}}e^{-t/\tau_n}$$

EI_{NO_x} is used for CH_4, O_{3L}. The parameters are given in table 7-3. The forcing factors $s_i(h)$ are given as a function of altitude in table 7-4; $s = 1$ for carbon dioxide.

For short-lived emission species (H_2O, O_{3S}, soot, SO_4), RF is assumed to be proportional to the emissions:

$$RF_i(t) = s_i(h) \, (RF_{\text{ref}}/E_{\text{ref}})_i \, E_i(t)$$

$$= U \sum_{\text{seg}} s_i(h) \Big(EI_i W_{\text{burn}} + K_i E_{\text{burn}} \Big) (RF_{\text{ref}}/E_{\text{ref}})_i \, u(t)$$

$$= U \sum_{\text{seg}} s_i(h) a_i A_i X_i(t)$$

where $A_i = RF_{\text{ref}}/E_{\text{ref}}$ and $X_i(t) = u(t)$. EI_{NO_x} is used for O_{3S}. The parameters $(RF_{\text{ref}}/E_{\text{ref}})$ are given in table 7-3. The forcing factor $s = 1$ except for O_{3S}, which is given in table 7-4.

Aviation-induced cloudiness (AIC) effects are assumed to be proportional to the distance traveled:

$$RF_{\text{AIC}}(t) = s_{\text{AIC}}(h) \, f_{\text{AIC}} \, (RF_{\text{ref}}/L_{\text{ref}})_{\text{AIC}} \, L(t)$$

$$= U \sum_{\text{seg}} s_{\text{AIC}}(h) df_{\text{AIC}} \, (RF_{\text{ref}}/L_{\text{ref}})_{\text{AIC}} \, u(t)$$

$$= U \sum_{\text{seg}} s_{\text{AIC}}(h) a_{\text{AIC}} A_{\text{AIC}} X_{\text{AIC}}(t)$$

where $L = U \sum_{\text{seg}} d$ is the stage length flown per year, $a_{\text{AIC}} = d$; $A_{\text{AIC}} = f_{\text{AIC}} RF_{\text{ref}}/L_{\text{ref}}$ and $X_{\text{AIC}}(t) = u(t)$. The parameter $(RF_{\text{ref}}/L_{\text{ref}})$ is given in table 7-3, and the forcing factor s_{AIC} in table 7-4; f_{AIC} is an input correction factor.

These forcing factors (table 7-4) characterize globally and annually averaged RFs based on current flight routes. Forcing factors, particularly for AIC impacts, are expected to vary with time of day, season, and earth latitude, effects which are averaged in this model.

The normalized RF for each species is based on the RF that would result from a doubling of carbon dioxide:

$$RF_i^*(t) = f_i \frac{RF_i(t)}{RF_{2xCO_2}} = U \sum_{\text{seg}} s_i(h) a_i A_i \frac{f_i}{RF_{2xCO_2}} X_i(t)$$

for CO_2, CH_4, O_{3L}, O_{3S}, H_2O, soot, SO_4, and AIC. The efficacy factors f_i are given in table 7-3; $f_{CO_2} = 1$. From the total normalized RF and a climate impulse response function, the time-varying global mean temperature change is evaluated:

$$\Delta T(t) = \int_0^t G_T(t - \tau) \sum_i RF_i^*(\tau) \, d\tau$$

$$= \sum_i U \sum_{\text{seg}} s_i(h) a_i A_i \frac{f_i}{RF_{2xCO_2}} S \int_0^t (G_T(t - \tau)/S) X_i(\tau) \, d\tau$$

$$= \sum_i U \sum_{\text{seg}} s_i(h) a_i A_i \frac{f_i}{RF_{2xCO_2}} S Y_i(t)$$

where

$$G_T(t) = S \left(\frac{\alpha_t}{\tau_{t1}} e^{-t/\tau_{t1}} + \frac{1 - \alpha_t}{\tau_{t2}} e^{-t/\tau_{t2}} \right)$$

The time constants in G_T describe the thermal response of the earth system to an energy perturbation, and S is the steady-state temperature change produced by a constant annual forcing of RF_{2xCO_2}. The parameters are given in table 7-3.

The corresponding weight emitted for each species is $W_i = \sum_{\text{seg}}(EI_i W_{\text{burn}} + K_i E_{\text{burn}}) = \sum_{\text{seg}} a_i$ per mission ($W_i = HU \sum_{\text{seg}} a_i$ over the operating duration H).

7-2.6   ATR Evaluation

The average temperature response (ATR) is calculated from $\Delta T = \Delta T_H$, assuming constant emissions during the first H years of operation and zero emissions thereafter. Hence the shape function $u(t) = 1$ for $0 < t < H$, and zero at other times. For a single operating mission, ΔT_H is the sum over all mission segments (each giving fuel and energy burned and distance traveled), with a utilization rate of U missions per year. The radiative forcing factors are:

$$X_{CO_2}(t) = \int_0^t \left(1 + \Sigma_j \alpha_{cj} \left(e^{-(t-\tau)/\tau_{cj}} - 1\right)\right) u(\tau)\, d\tau$$

$$= \begin{cases} t(1 - \Sigma_j \alpha_{cj}) + \Sigma_j \alpha_{cj} \tau_{cj} \left(1 - e^{-t/\tau_{cj}}\right) & t < H \\ H(1 - \Sigma_j \alpha_{cj}) + \Sigma_j \alpha_{cj} \tau_{cj} \left(e^{-(t-H)/\tau_{cj}} - e^{-t/\tau_{cj}}\right) & t > H \end{cases}$$

$$X_{CH_4}(t) = X_{O_{3L}}(t) = \int_0^t e^{-(t-\tau)/\tau_n} u(\tau)\, d\tau$$

$$= \begin{cases} \tau_n \left(1 - e^{-t/\tau_n}\right) & t < H \\ \tau_n \left(e^{-(t-H)/\tau_n} - e^{-t/\tau_n}\right) & t > H \end{cases}$$

and $X_i(t) = u(t)$ for the other species. The temperature change factors $Y_i(t) = \int_0^t (G_T(t-\tau)/S) X_i(\tau)\, d\tau$ are:

$$Y_{CO_2}(t) = \begin{cases} \begin{aligned} & \alpha_t \left[\left(t - \tau_{t1}(1 - e^{-t/\tau_{t1}})\right)(1 - \Sigma_j \alpha_{cj}) \right. \\ & \left. + \Sigma_j \alpha_{cj} \tau_{cj} \left\{1 + \frac{\tau_{t1}}{\tau_{cj} - \tau_{t1}} e^{-t/\tau_{t1}} - \frac{\tau_{cj}}{\tau_{cj} - \tau_{t1}} e^{-t/\tau_{cj}}\right\}\right] \\ & + (1 - \alpha_t)\left[\left(t - \tau_{t2}(1 - e^{-t/\tau_{t2}})\right)(1 - \Sigma_j \alpha_{cj}) \right. \\ & \left. + \Sigma_j \alpha_{cj} \tau_{cj} \left\{1 + \frac{\tau_{t2}}{\tau_{cj} - \tau_{t2}} e^{-t/\tau_{t2}} - \frac{\tau_{cj}}{\tau_{cj} - \tau_{t2}} e^{-t/\tau_{cj}}\right\}\right] \end{aligned} & t < H \\[2em] \begin{aligned} & \alpha_t \left[\left(H - \tau_{t1}(e^{-(t-H)/\tau_{t1}} - e^{-t/\tau_{t1}})\right)(1 - \Sigma_j \alpha_{cj}) \right. \\ & + \Sigma_j \alpha_{cj} \tau_{cj} \left\{-\frac{\tau_{t1}}{\tau_{cj} - \tau_{t1}} \left(e^{-(t-H)/\tau_{t1}} - e^{-t/\tau_{t1}}\right) \right. \\ & \left.\left. + \frac{\tau_{cj}}{\tau_{cj} - \tau_{t1}} \left(e^{-(t-H)/\tau_{cj}} - e^{-t/\tau_{cj}}\right)\right\}\right] \\ & + (1 - \alpha_t)\left[\left(H - \tau_{t2}(e^{-(t-H)/\tau_{t2}} - e^{-t/\tau_{t2}})\right)(1 - \Sigma_j \alpha_{cj}) \right. \\ & + \Sigma_j \alpha_{cj} \tau_{cj} \left\{-\frac{\tau_{t2}}{\tau_{cj} - \tau_{t2}} \left(e^{-(t-H)/\tau_{t2}} - e^{-t/\tau_{t2}}\right) \right. \\ & \left.\left. + \frac{\tau_{cj}}{\tau_{cj} - \tau_{t2}} \left(e^{-(t-H)/\tau_{cj}} - e^{-t/\tau_{cj}}\right)\right\}\right] \end{aligned} & t > H \end{cases}$$

$$Y_{\text{CH}_4}(t) = Y_{\text{O}_{3L}}(t) = \begin{cases} \alpha_t \tau_n \left[1 + \dfrac{\tau_{t1}}{\tau_n - \tau_{t1}} e^{-t/\tau_{t1}} - \dfrac{\tau_n}{\tau_n - \tau_{t1}} e^{-t/\tau_n} \right] \\ \qquad + (1 - \alpha_t)\tau_n \left[1 + \dfrac{\tau_{t2}}{\tau_n - \tau_{t2}} e^{-t/\tau_{t2}} - \dfrac{\tau_n}{\tau_n - \tau_{t2}} e^{-t/\tau_n} \right] \qquad t < H \\[2em] \alpha_t \tau_n \left[-\dfrac{\tau_{t1}}{\tau_n - \tau_{t1}} \left(e^{-(t-H)/\tau_{t1}} - e^{-t/\tau_{t1}} \right) \right. \\ \qquad \left. + \dfrac{\tau_n}{\tau_n - \tau_{t1}} \left(e^{-(t-H)/\tau_n} - e^{-t/\tau_n} \right) \right] \\ \qquad + (1 - \alpha_t)\tau_n \left[-\dfrac{\tau_{t2}}{\tau_n - \tau_{t2}} \left(e^{-(t-H)/\tau_{t2}} - e^{-t/\tau_{t2}} \right) \right. \qquad t > H \\ \qquad \left. + \dfrac{\tau_n}{\tau_n - \tau_{t2}} \left(e^{-(t-H)/\tau_n} - e^{-t/\tau_n} \right) \right] \end{cases}$$

and for the other species

$$Y_i(t) = \begin{cases} \alpha_t \left[1 - e^{-t/\tau_{t1}} \right] + (1 - \alpha_t)\left[1 - e^{-t/\tau_{t2}} \right] & t < H \\[1em] \alpha_t \left[e^{-(t-H)/\tau_{t1}} - e^{-t/\tau_{t1}} \right] + (1 - \alpha_t)\left[e^{-(t-H)/\tau_{t2}} - e^{-t/\tau_{t2}} \right] & t > H \end{cases}$$

Then the average temperature response is

$$\begin{aligned} \text{ATR} &= \frac{1}{H} \int_0^{t_{\max}} \Delta T_H(t)\, w(t)\, dt \\ &= \sum_i U \sum_{\text{seg}} s_i(h) a_i A_i \frac{f_i}{RF_{2x\text{CO}_2}} S \frac{1}{H} \int_0^{t_{\max}} Y_i(t) w(t)\, dt \\ &= \sum_i U \sum_{\text{seg}} s_i(h) a_i A_i \frac{f_i}{RF_{2x\text{CO}_2}} S Z_i \\ &= \sum_i \text{ATR}_i \end{aligned}$$

Given H and r, Z_i is evaluated by numerical integration. For the special cases of $r = 0$ (no discounting) and $r = \infty$ (no effects beyond H):

$$
Z_{\mathrm{CO_2}} = \begin{cases}
\begin{aligned}
&\left(\frac{H}{2} - \alpha_t\tau_{t1} - (1-\alpha_t)\tau_{t2}\right)(1 - \Sigma_j\alpha_{cj}) + \Sigma_j\alpha_{cj}\tau_{cj} \\
&\quad + \alpha_t\left[\tau_{t1}\frac{\tau_{t1}}{H}\left(1 - e^{-H/\tau_{t1}}\right)(1 - \Sigma_j\alpha_{cj})\right.\\
&\qquad \left.+\Sigma_j\alpha_{cj}\tau_{cj}\left\{\frac{\tau_{t1}}{\tau_{cj} - \tau_{t1}}\frac{\tau_{t1}}{H}\left(1 - e^{-H/\tau_{t1}}\right) - \frac{\tau_{cj}}{\tau_{cj} - \tau_{t1}}\frac{\tau_{cj}}{H}\left(1 - e^{-H/\tau_{cj}}\right)\right\}\right]\\
&\quad + (1-\alpha_t)\left[\tau_{t2}\frac{\tau_{t2}}{H}\left(1 - e^{-H/\tau_{t2}}\right)(1 - \Sigma_j\alpha_{cj})\right.\\
&\qquad \left.+\Sigma_j\alpha_{cj}\tau_{cj}\left\{\frac{\tau_{t2}}{\tau_{cj} - \tau_{t2}}\frac{\tau_{t2}}{H}\left(1 - e^{-H/\tau_{t2}}\right) - \frac{\tau_{cj}}{\tau_{cj} - \tau_{t2}}\frac{\tau_{cj}}{H}\left(1 - e^{-H/\tau_{cj}}\right)\right\}\right]
\end{aligned} & r = \infty \\[2em]
\begin{aligned}
&\left(t_{\max} - \frac{H}{2} - \alpha_t\tau_{t1} - (1-\alpha_t)\tau_{t2}\right)(1 - \Sigma_j\alpha_{cj}) + \Sigma_j\alpha_{cj}\tau_{cj} \\
&\quad + \alpha_t\left[\tau_{t1}\frac{\tau_{t1}}{H}\left(e^{-(t_{\max}-H)/\tau_{t1}} - e^{-t_{\max}/\tau_{t1}}\right)(1 - \Sigma_j\alpha_{cj})\right.\\
&\qquad +\Sigma_j\alpha_{cj}\tau_{cj}\left\{\frac{\tau_{t1}}{\tau_{cj} - \tau_{t1}}\frac{\tau_{t1}}{H}\left(e^{-(t_{\max}-H)/\tau_{t1}} - e^{-t_{\max}/\tau_{t1}}\right)\right.\\
&\qquad\qquad \left.\left.- \frac{\tau_{cj}}{\tau_{cj} - \tau_{t1}}\frac{\tau_{cj}}{H}\left(e^{-(t_{\max}-H)/\tau_{cj}} - e^{-t_{\max}/\tau_{cj}}\right)\right\}\right]\\
&\quad + (1-\alpha_t)\left[\tau_{t2}\frac{\tau_{t2}}{H}\left(e^{-(t_{\max}-H)/\tau_{t2}} - e^{-t_{\max}/\tau_{t2}}\right)(1 - \Sigma_j\alpha_{cj})\right.\\
&\qquad +\Sigma_j\alpha_{cj}\tau_{cj}\left\{\frac{\tau_{t2}}{\tau_{cj} - \tau_{t2}}\frac{\tau_{t2}}{H}\left(e^{-(t_{\max}-H)/\tau_{t2}} - e^{-t_{\max}/\tau_{t2}}\right)\right.\\
&\qquad\qquad \left.\left.- \frac{\tau_{cj}}{\tau_{cj} - \tau_{t2}}\frac{\tau_{cj}}{H}\left(e^{-(t_{\max}-H)/\tau_{cj}} - e^{-t_{\max}/\tau_{cj}}\right)\right\}\right]
\end{aligned} & r = 0
\end{cases}
$$

$$
Z_{\mathrm{CH_4}} = Z_{\mathrm{O_{3L}}} = \begin{cases}
\begin{aligned}
&\tau_n + \alpha_t\tau_n\left[\frac{\tau_{t1}}{\tau_n - \tau_{t1}}\frac{\tau_{t1}}{H}\left(1 - e^{-H/\tau_{t1}}\right) - \frac{\tau_n}{\tau_n - \tau_{t1}}\frac{\tau_n}{H}\left(1 - e^{-H/\tau_n}\right)\right]\\
&\quad + (1-\alpha_t)\tau_n\left[\frac{\tau_{t2}}{\tau_n - \tau_{t2}}\frac{\tau_{t2}}{H}\left(1 - e^{-H/\tau_{t2}}\right) - \frac{\tau_n}{\tau_n - \tau_{t2}}\frac{\tau_n}{H}\left(1 - e^{-H/\tau_n}\right)\right]
\end{aligned} & r = \infty \\[2em]
\begin{aligned}
&\tau_n + \alpha_t\tau_n\left[\frac{\tau_{t1}}{\tau_n - \tau_{t1}}\frac{\tau_{t1}}{H}\left(e^{-(t_{\max}-H)/\tau_{t1}} - e^{-t_{\max}/\tau_{t1}}\right)\right.\\
&\qquad\qquad \left.- \frac{\tau_n}{\tau_n - \tau_{t1}}\frac{\tau_n}{H}\left(e^{-(t_{\max}-H)/\tau_n} - e^{-t_{\max}/\tau_n}\right)\right]\\
&\quad + (1-\alpha_t)\tau_n\left[\frac{\tau_{t2}}{\tau_n - \tau_{t2}}\frac{\tau_{t2}}{H}\left(e^{-(t_{\max}-H)/\tau_{t2}} - e^{-t_{\max}/\tau_{t2}}\right)\right.\\
&\qquad\qquad \left.- \frac{\tau_n}{\tau_n - \tau_{t2}}\frac{\tau_n}{H}\left(e^{-(t_{\max}-H)/\tau_n} - e^{-t_{\max}/\tau_n}\right)\right]
\end{aligned} & r = 0
\end{cases}
$$

and for the other species:

$$
Z_i = \begin{cases}
1 + \alpha_t\left[-\frac{\tau_{t1}}{H}\left(1 - e^{-H/\tau_{t1}}\right)\right] + (1-\alpha_t)\left[-\frac{\tau_{t2}}{H}\left(1 - e^{-H/\tau_{t2}}\right)\right] & r = \infty \\[1.5em]
\begin{aligned}
&1 + \alpha_t\left[-\frac{\tau_{t1}}{H}\left(e^{-(t_{\max}-H)/\tau_{t1}} - e^{-t_{\max}/\tau_{t1}}\right)\right]\\
&\quad + (1-\alpha_t)\left[-\frac{\tau_{t2}}{H}\left(e^{-(t_{\max}-H)/\tau_{t2}} - e^{-t_{\max}/\tau_{t2}}\right)\right]
\end{aligned} & r = 0
\end{cases}
$$

Since $\Sigma_j \alpha_{cj} \neq 1$, the radiative forcing for CO_2 is finite at long times ($X_{CO_2} \to H(1 - \Sigma_j \alpha_{cj})$), and CO_2 dominates the temperature response. It is necessary to introduce $t_{\max}$ so that Z_{CO_2} (and ATR) is finite in the absence of discounting ($r = 0$).

7–3   References

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6) Rindlisbacher, T. "Guidance on the Determination of Helicopter Emissions." Federal Office of Civil Aviation, Swiss Confederation, March 2009.

Table 7-3. RF factor s for NO_x impacts and AIC.

altitude (ft)	AIC	CH_4, O_{3L}	O_{3S}
<17500	0.000	0.868	0.470
17500	0.029	0.868	0.470
19500	0.000	0.924	0.557
21500	0.000	0.959	0.620
23500	0.171	0.965	0.711
25500	0.399	0.947	0.713
27500	0.798	0.933	0.809
29500	1.254	0.930	0.931
31500	1.710	0.942	1.009
33500	2.109	0.977	1.131
35500	1.824	1.136	1.431
37500	1.539	1.213	1.630
39500	0.969	1.204	1.802
41500	0.798	1.201	1.937

Table 7-4. Radiative forcing models.

CO_2 parameters		
A_{CO_2}	1.80×10^{-15}	(W/m²)/(kg CO_2)
α_{c1}	0.259	
α_{c2}	0.338	
α_{c3}	0.186	
τ_{c1}	172.9	yr
τ_{c2}	18.51	yr
τ_{c3}	1.186	yr
NO_x parameters		
A_{CH_4}	-5.16×10^{-13}	(W/m²)/(kg NO_x)
$A_{O_{3L}}$	-1.21×10^{-13}	(W/m²)/(kg NO_x)
τ_n	12.0	yr
f_{CH_4}	1.18	
f_{O_3}	1.37	
$(RF_{\text{ref}}/E_{\text{ref}})_{O_{3S}}$	1.01×10^{-11}	(W/m²)/(kg NO_x)
H_2O, SO_4, soot, and AIC parameters		
f_{H_2O}	1.14	
f_{SO_4}	0.9	
f_{soot}	0.7	
f_{AIC}	0.59	
$(RF_{\text{ref}}/E_{\text{ref}})_{H_2O}$	7.43×10^{-15}	(W/m²)/(kg H_2O)
$(RF_{\text{ref}}/E_{\text{ref}})_{SO_4}$	-1.00×10^{-10}	(W/m²)/(kg SO_4)
$(RF_{\text{ref}}/E_{\text{ref}})_{\text{soot}}$	5.00×10^{-10}	(W/m²)/(kg soot)
$(RF_{\text{ref}}/L_{\text{ref}})_{AIC}$	2.21×10^{-12}	(W/m²)/nm
Temperature change model parameters		
RF_{2xCO_2}	3.70	W/m²
S	3.0	K
α_t	0.595	
τ_{t1}	8.4	yr
τ_{t2}	409.5	yr

Chapter 8

Aircraft

The aircraft consists of a set of components, including rotors, wings, tails, fuselage, and propulsion. For each component, attributes such as performance, drag, and weight can be calculated. The aircraft attributes are obtained from the sum of the component attributes. Description and analysis of conventional rotorcraft configurations is facilitated, while retaining the capability to model novel and advanced concepts. Specific rotorcraft configurations considered include: single-main-rotor and tail-rotor helicopter, tandem helicopter, coaxial helicopter, tiltrotor, compound helicopter, multicopter, and airplane.

The following components form the aircraft:

a) Systems: The systems component contains weight information (fixed useful load, vibration, contingency, and systems and equipment) for the aircraft.

b) Fuselage: There is one fuselage for the aircraft.

c) Landing Gear: There is one landing gear for the aircraft.

d) Rotors: The aircraft can have one or more rotors, or no rotors. In addition to main-rotors, the component can model tail-rotors, propellers, proprotors, and ducted fans.

e) Wings: The aircraft can have one or more wings, or no wings.

f) Tails: The aircraft can have one or more horizontal or vertical tail surfaces, or no tails.

g) Fuel Tanks: There are one or more fuel tank systems for the aircraft. Fuel tank systems are associated with the engine groups, jet groups, and charge groups. Fuel quantity is measured as either weight or energy. There can be one or more sizes of auxiliary fuel tanks.

h) Propulsion Groups: The aircraft can have one or more propulsion groups, or none. Each propulsion group is a set of components (rotors) and engine groups, connected by a drive system. The components define the power required. The engine groups define the power available.

i) Engine Groups: An engine group consists of one or more engines of a specific type. An engine group transfers power by shaft torque, so it is associated with a propulsion group. For each engine type an engine model is defined. The engine model describes a particular engine, used in one or more engine groups.

j) Jet Groups: The aircraft can have one or more jet groups, or none. A jet group produces a force on the aircraft. A jet model describes a particular jet, used in one or more jet groups.

k) Charge Groups: The aircraft can have one or more charge groups, or none. A charge group generates energy for the aircraft. A charge model describes a particular charger, used in one or more charge groups.

8–1   Loading

The aircraft disk loading is the ratio of the design gross weight and a reference rotor area: $DL = W_D/A_{\text{ref}}$. The reference area is a sum of specified fractions of the rotor areas, $A_{\text{ref}} = \sum f_A A$ (typically the projected area of the lifting rotors). The disk loading of a rotor is the ratio of a specified fraction of the design gross weight and the rotor area:

$$(DL)_{\text{rotor}} = \frac{T}{A} = \frac{f_W W_D}{A} = \frac{f_W}{A/A_{\text{ref}}} \frac{W_D}{A_{\text{ref}}}$$

where probably $\sum_{\text{rotor}} f_W = 1$, and the lifting rotors are all rotors not designated antitorque or auxiliary-thrust. If there are N lifting rotors of the same area, with no overlap, then $f_A = 1$, $A_{\text{ref}} = NA$, $f_W = A/A_{\text{ref}} = 1/N$, and $(DL)_{\text{rotor}} = DL$. For rotors designated antitorque or auxiliary-thrust, the disk loading is calculated from the design rotor thrust: $(DL)_{\text{rotor}} = T_{\text{design}}/A$.

For coaxial rotors, the default reference area is the area of one rotor: $f_A = 1/2$, $A_{\text{ref}} = A$, $f_W = 1/2$, and $(DL)_{\text{rotor}} = 1/2 DL$. For tandem rotors, the default reference area is the projected area: $A_{\text{ref}} = (2 - m)A$, where mA is the overlap area ($m = 0$ for no overlap, $m = 1$ for coaxial). Then $f_A = \frac{2-m}{2}$, $f_W = 1/2$, and $(DL)_{\text{rotor}} = \frac{2-m}{2} DL$. Optionally, the reference area for tandem rotors can be total rotor area instead: $A_{\text{ref}} = 2A$.

The aircraft wing loading is the ratio of the design gross weight and a reference wing area: $WL = W_D/S_{\text{ref}}$. The reference area is a sum of the wing areas, $S_{\text{ref}} = \sum S$. The wing loading of an individual wing is the ratio of a specified fraction of the design gross weight and the wing area:

$$(WL)_{\text{wing}} = \frac{W}{S} = \frac{f_W W_D}{S} = \frac{f_W}{S/S_{\text{ref}}} \frac{W_D}{S_{\text{ref}}}$$

where probably $\sum_{\text{wing}} f_W = 1$. If there are N wings of the same area, then $f_W = S/S_{\text{ref}} = 1/N$, and $(WL)_{\text{wing}} = WL$.

The aircraft power loading is the ratio of the design gross weight and the total installed takeoff power: $W/P = W_D/\sum N_{\text{eng}} P\text{eng}$, where the sum is over all engine groups.

8–2   Controls

A set of aircraft controls c_{AC} are defined, and these aircraft controls are connected to the component controls. The connection to the component control c is typically of the form $c = STc_{AC} + c_0$, where T is an input matrix and c_0 the component control for zero aircraft control. The connection (matrix T) is defined for a specified number of control system states (allowing change of control configuration with flight state). The factor S is available for internal scaling of the matrix. The control state and initial control values are specified for each flight state. Figure 8-1 illustrates the control relationships.

Typical (default) aircraft controls are the pilot's controls: collective stick, lateral and longitudinal cyclic sticks, pedal, and tilt. Units and sign convention of the pilot's controls are contained in the matrix T. For the single-main-rotor and tail-rotor configuration, it is often convenient for the collective and cyclic stick motion to equal the collective and cyclic pitch input of the main-rotor, and the pedal motion to equal the collective pitch input of the tail-rotor. The aircraft controls should be scaled to approximately the same amplitude, by appropriate definition of the matrix T and scale factor S.

These aircraft controls are available for trim of the aircraft. Any aircraft controls not selected for trim will remain fixed at the values specified for the flight state. Thus by defining additional aircraft controls, component controls can be specified as required for a flight state.
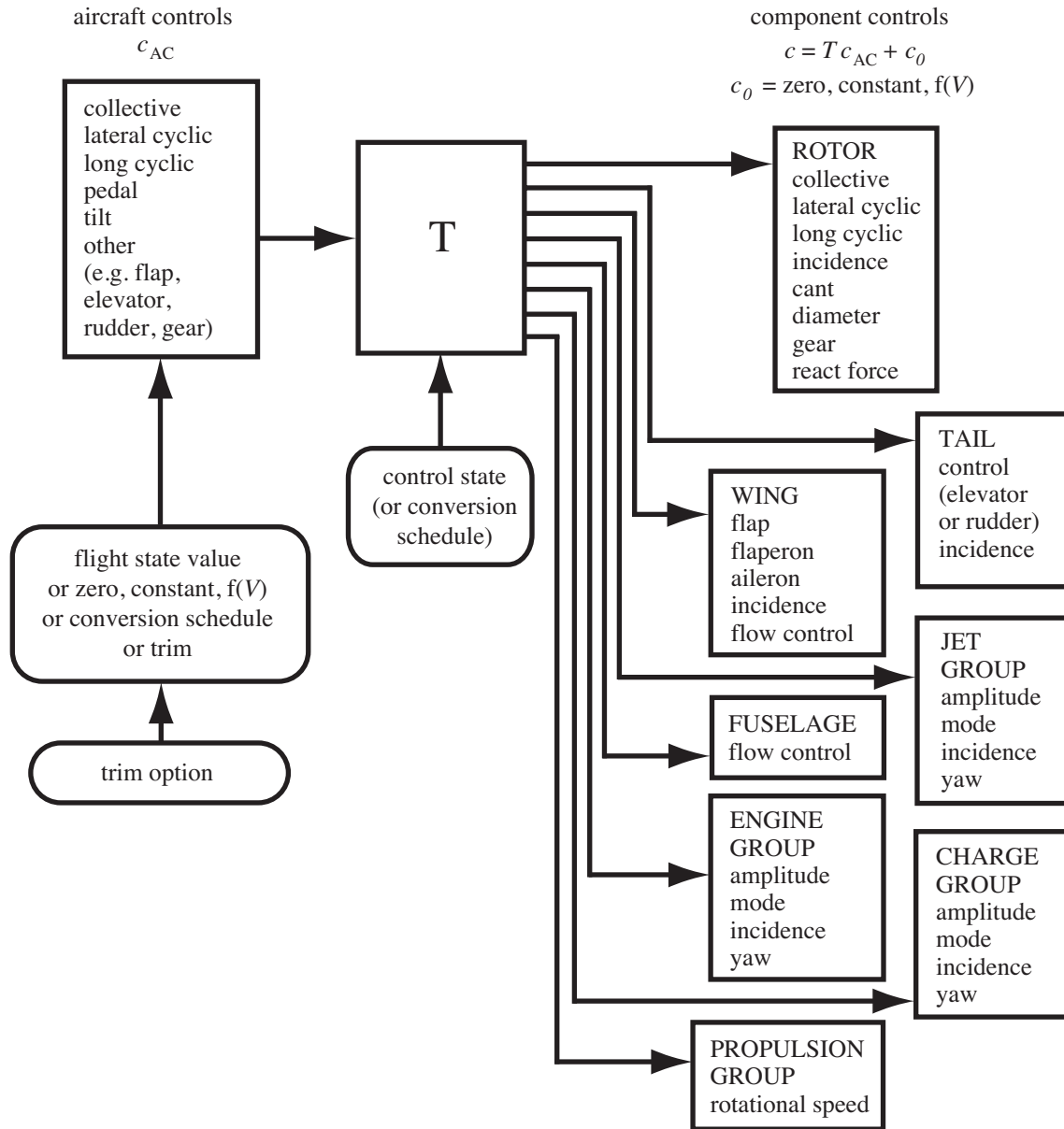
aircraft controls
c_{AC}

component controls
$c = T c_{AC} + c_0$
c_0 = zero, constant, f(V)

Figure 8-1. Aircraft and component controls.

Each aircraft control variable c_{AC} can be zero, constant, or a function of flight speed (piecewise linear input). The flight state input can override this value of the aircraft control. The input value is an initial value if the control is a trim variable.

Each component control variable c_0 (value for zero aircraft control) can be zero, constant, or a function of flight speed (piecewise linear input). Optionally the use of c_0 can be suppressed for a flight state. The component control from aircraft control ($T c_{AC}$) is a fixed value, or a function of speed, or a linear function of another control (perhaps a trim variable).

The tilt control variable α_{tilt} is intended for nacelle tilt angle or conversion control, particularly for tiltrotors. The convention is $\alpha_{\text{tilt}} = 0$ for cruise, and $\alpha_{\text{tilt}} = 90$ deg for helicopter mode. If α_{tilt} exists as a control, it can be zero, constant, or a function of flight speed (piecewise linear input).

An optional control conversion schedule is defined in terms of conversion speeds: hover and helicopter mode for speeds below $V_{C\text{hover}}$, cruise mode for speeds above $V_{C\text{cruise}}$, and conversion mode between. The nacelle angle is $\alpha_{\text{tilt}} = 90$ in helicopter mode, $\alpha_{\text{tilt}} = 0$ in airplane mode, and it varies linearly with speed in conversion mode. The tip speed is $V_{\text{tip-hover}}$ in helicopter and conversion mode, and $V_{\text{tip-cruise}}$ in airplane mode. Control states and drive system states are defined for helicopter, cruise, and conversion mode flight. The flight state specifies the nacelle tilt angle, tip speeds, control state, and drive system state, including the option to obtain any or all of these quantities from the conversion schedule.

The flight speed used for control scheduling is usually the calibrated airspeed (CAS), hence variation with dynamic pressure. Velocity schedules are used for conversion, controls and motion, rotor tip speed, landing gear retraction, trim targets, and drive system ratings. Optionally these velocity schedules use indicated airspeed V_{ind}, calibrated airspeed V_{cal}, or true airspeed V.

The control matrices T can be defined based on the configuration. Let c_{AC0}, c_{ACc}, c_{ACs}, c_{ACp} be the pilot's controls (collective, lateral cyclic, longitudinal cyclic, and pedal). For the helicopter, the first rotor is the main-rotor and the second rotor is the tail-rotor; then

$$
\begin{pmatrix} T_{\text{Mcoll}} \\ T_{\text{Mlat}} \\ T_{\text{Mlng}} \\ T_{\text{Tcoll}} \end{pmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -r & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -r \end{bmatrix} \begin{pmatrix} c_{AC0} \\ c_{ACc} \\ c_{ACs} \\ c_{ACp} \end{pmatrix}
$$

where r is the main-rotor direction of rotation ($r = 1$ for counter-clockwise rotation, $r = -1$ for clockwise rotation). For the tandem configuration, the first rotor is the front rotor and the second rotor is the rear rotor; then

$$
\begin{pmatrix} T_{\text{Fcoll}} \\ T_{\text{Flat}} \\ T_{\text{Rcoll}} \\ T_{\text{Rlat}} \end{pmatrix} = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & -r_F & 0 & -r_F \\ 1 & 0 & 1 & 0 \\ 0 & -r_R & 0 & r_R \end{bmatrix} \begin{pmatrix} c_{AC0} \\ c_{ACc} \\ c_{ACs} \\ c_{ACp} \end{pmatrix}
$$

For the coaxial configuration:

$$
\begin{pmatrix} T_{1\text{coll}} \\ T_{1\text{lat}} \\ T_{1\text{lng}} \\ T_{2\text{coll}} \\ T_{2\text{lat}} \\ T_{2\text{lng}} \end{pmatrix} = \begin{bmatrix} 1 & 0 & 0 & r_1 \\ 0 & -r_1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & r_2 \\ 0 & -r_2 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix} \begin{pmatrix} c_{AC0} \\ c_{ACc} \\ c_{ACs} \\ c_{ACp} \end{pmatrix}
$$

For the tiltrotor, the first rotor is the right rotor and the second rotor is the left rotor; then

$$
\begin{pmatrix} T_{\text{Rcoll}} \\ T_{\text{Rlng}} \\ T_{\text{Lcoll}} \\ T_{\text{Llng}} \\ T_{\text{ail}} \\ T_{\text{elev}} \\ T_{\text{rud}} \end{pmatrix} = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & -1 & -1 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} c_{AC0} \\ c_{ACc} \\ c_{ACs} \\ c_{ACp} \end{pmatrix}
$$

with cyclic stick and pedal connected to rotor controls only for helicopter mode. For the multicopter configuration:

$$T_{i\text{coll}} = c_{AC0} - c_{ACc} \sin \psi_i + c_{ACs} \cos \psi_i + c_{ACp} r_i$$

where ψ_i is the angle of the i-th rotor, measured clockwise from the forward longitudinal axis. Alternative multicopter control strategies use rotor rotational speed instead of collective, or cyclic for yaw control.

The sign conventions for the pilot's controls are collective stick positive up, lateral cyclic stick positive right, longitudinal cyclic stick positive forward, and pedal positive nose right. The rotor controls are a positive Fourier series, with azimuth measured in the direction of rotation.

8–3   Trim

The aircraft trim operation solves for the controls and motion that produce equilibrium in the specified flight state. In steady flight (including hover, level flight, climb and descent, and turns), equilibrium implies zero net force and moment on the aircraft. In general, there can be additional quantities that at equilibrium must equal target values. In practice, the trim solution can deal with a subset of these quantities. Usually it is at least necessary to achieve equilibrium in the aircraft lift and drag forces, as well as in yaw moment for torque balance. The basic purpose of the trim solution is to determine the component states, including aircraft drag and rotor thrust, sufficient to evaluate the aircraft performance.

Different trim solution definitions are required for various flight states. Therefore one or more trim states are defined for the analysis, and the appropriate trim state selected for each flight state of a performance condition or mission segment. For each trim state, the trim quantities, trim variables, and targets are specified. The available trim quantities include:

> aircraft total force and moment; aircraft load factor;
> propulsion group power;
> power margin $P_{avPG} - P_{reqPG}$; torque margin $P_{DS\text{limit}} - P_{reqPG}$;
> engine group power; power margin $P_{avEG} - P_{reqEG}$; momentum margin $F_G - F_{Greq}$;
> jet group thrust; thrust margin $T_{avJG} - T_{reqJG}$; momentum margin $F_G - F_{Greq}$;
> charge group power; charge power margin $P_{avCG} - P_{reqCG}$;
> fuel tank energy flow $\dot{E}_{\text{batt}}$; battery power margin, $P_{\max} - |\dot{E}_{\text{batt}}|$;
> rotor force (lift, vertical, or propulsive); rotor shaft power;
> rotor thrust C_T/σ; rotor thrust margin $(C_T/\sigma)_{\max} - |C_T/\sigma|$;
> rotor flapping β_c, β_s; rotor hub moment, roll and pitch; rotor torque;
> wing force; wing lift coefficient C_L; wing lift margin $C_{L\max} - C_L$;
> tail force.

Targets for aircraft total force and total moment (including inertial loads in turns) are always zero. The available trim variables include:

> aircraft controls;
> aircraft orientation, θ (pitch), ϕ (roll);
> aircraft horizontal velocity V_h;
> aircraft vertical rate of climb V_c; aircraft sideslip angle;
> aircraft angular rate, $\dot{\theta}$ (pullup), $\dot{\psi}$ (turn);
> propulsion group tip speed or engine speed.

axes for description of   aircraft coordinate
aircraft geometry   system
(arbitrary reference)   (origin at CG)
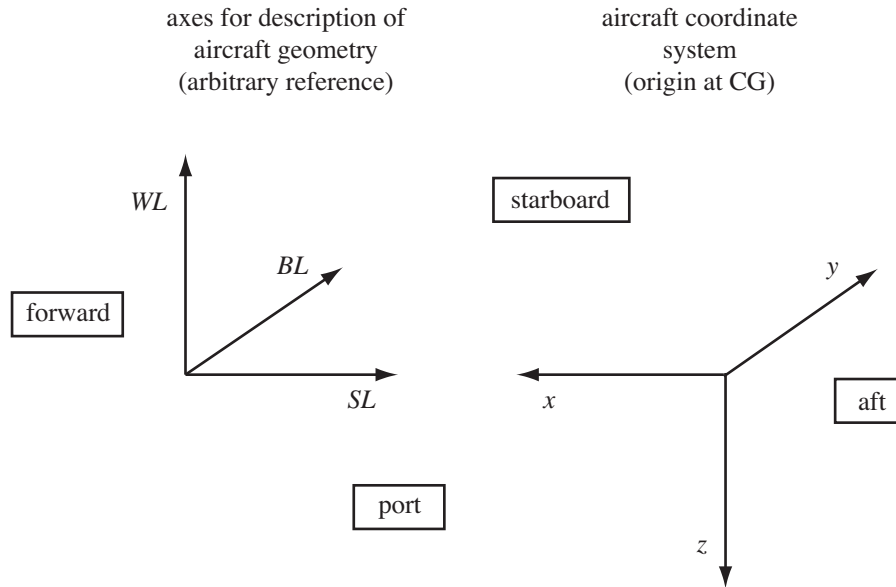
Figure 8-2. Aircraft geometry.

The aircraft orientation variables are the Euler angles of the body axes relative to inertial axes. The aircraft controls (appropriately scaled) are connected to the component controls.

A Newton–Raphson method is used for trim. The derivative matrix is obtained by numerical perturbation. A tolerance ϵ and a perturbation Δ are specified.

8–4   Geometry

The aircraft coordinate system has the x-axis forward, y-axis to the right, and z-axis down, measured from the center of gravity (fig. 8-2). These aircraft axes are body axes (x is not aligned with the wind), the orientation determined by the convention used for the input geometry. The center of gravity is the appropriate origin for describing the motion of the aircraft, and summing the forces and moments acting on the aircraft.

Layout of the geometry is typically in terms of station line (SL, positive aft), buttline (BL, positive right), and waterline (WL, positive up), measured relative to some arbitrary origin (fig. 8-2). The x-y-z axes are parallel to the SL-BL-WL directions. One or more locations are defined for each component of the aircraft. Each component will at least have a location that is the point where component forces and moments act on the aircraft. Each location is input in fixed or scaled form. The fixed form input is SL/BL/WL (dimensional). The scaled form input is x/L (positive aft), y/L (positive right), and z/L (positive up), based on a reference length L, from a reference point. The reference length is the rotor radius or wing span of a designated component, or the fuselage length. The reference point can optionally be input, or the location (hub) of a designated rotor, or the location (center of action) of a designated wing component, or the location (center of action) of the fuselage, or the location of the center of gravity. Fixed input can be used for the entire aircraft, or just for certain components.

From this fixed or scaled input and the current aircraft size, the actual geometry (x, y, z) can be

calculated for each location. There are also options to calculate geometry from other parameters (such as tiltrotor span from rotor radius and clearance). This calculated geometry has the sign convention of the aircraft axes (x positive forward, y positive right, z positive down), but has the origin at the reference point (which may or may not be the center of gravity). All input uses the same sign convention; all internal calculations use the same sign conventions. Table 8-1 summarizes the conventions.

The location of the aircraft center of gravity is specified for a baseline configuration. With tilting rotors, this location is in helicopter mode. For each flight state the aircraft center of gravity is calculated, from the baseline location plus any shift due to nacelle tilt, plus an input center-of-gravity increment. The aircraft center-of-gravity can also be shifted by wing tilt. Alternatively, the aircraft center-of-gravity location for the flight state can be input. Any change of the center-of-gravity position with fuel burn during a mission is not automatically calculated, but could be accounted for using the flight state input.

The aircraft operating length and width are calculated from the component positions and dimensions: $\ell_{\text{total}} = x_{\max} - x_{\min}$ and $w_{\text{total}} = y_{\max} - y_{\min}$, where the maximum and minimum dimensions are for the fuselage and all rotors, wings, and tails. The corresponding footprint area is then $S_{\text{total}} = \ell_{\text{total}} w_{\text{total}}$.

Table 8-1. Geometry conventions.

	layout	scaled input	calculated	motion and loads
origin	arbitrary	reference point	reference point	center of gravity
x	SL (+ aft)	x/L (+ aft)	x (+ forward)	x (+ forward)
y	BL (+ right)	y/L (+ right)	y (+ right)	y (+ right)
z	WL (+ up)	z/L (+ up)	z (+ down)	z (+ down)

8–5   Aircraft Motion

The aircraft velocity and orientation are defined by the following parameters: flight speed V; turn rate; orientation of the body frame relative to inertial axes (Euler angles); and orientation of the velocity frame relative to inertial axes (flight path angles). Aircraft conventions are followed for the direction and orientation of axes: the z-axis is down, the x-axis forward, and the y-axis to the right; and a yaw-pitch-roll sequence is used for the Euler angles. However, the airframe axes are body axes (fixed to the airframe, regardless of the flight direction) rather than wind axes (which have the x-axis in the direction of the flight speed). The orientation of the body frame F relative to inertial axes I is defined by yaw, pitch, and roll Euler angles, which are rotations about the z, y, and x axes, respectively:

$$C^{FI} = X_{\phi_F} Y_{\theta_F} Z_{\psi_F}$$

So yaw is positive to the right, pitch is positive nose up, and roll is positive to the right. The flight path is specified by the velocity V, in the positive x-axis direction of the velocity axes. The orientation of the velocity axes V relative to inertial axes I is defined by yaw (sideslip) and pitch (climb) angles:

$$C^{VI} = Y_{\theta_V} Z_{\psi_V} Z_{\psi_F}$$

Sideslip is positive for the aircraft moving to the right, and climb is positive for the aircraft moving up. Then

$$C^{FV} = C^{FI} C^{IV} = X_{\phi_F} Y_{\theta_F} Z_{-\psi_V} Y_{-\theta_V}$$

In straight flight, all these angles and matrices are constant. In turning flight at a constant yaw rate, the yaw angle is $\psi_F = \dot{\psi}_F t$; the turn radius is $R_T = V_h / \dot{\psi}_F$; and the nominal bank angle and load factor are $\tan \phi_F = \sqrt{n^2 - 1} = \dot{\psi}_F V_h / g$. Then the forward, sideward, and climb velocities are:

$$V_f = V \cos \theta_V \cos \psi_V = V_h \cos \psi_V$$
$$V_s = V \cos \theta_V \sin \psi_V = V_h \sin \psi_V$$
$$V_c = V \sin \theta_V = V_h \tan \theta_V$$

where $V_h = V \cos \theta_V$ is the horizontal velocity component. The velocity components in airframe axes are $v_{AC}^F = v^{FI/F} = C^{FV} (V\,0\,0)^T$ (aircraft velocity relative to the air). The calibrated airspeed is calculated from the true airspeed V:

$$V_{\text{cal}} = V \sqrt{\sigma} \sqrt{ \frac{ \left[\delta((1 + 0.2M^2)^{7/2} - 1) + 1 \right]^{2/7} - 1 }{ 0.2 M^2 \delta } } \cong V \sqrt{\sigma} \left[1 + \frac{1}{8}(1 - \delta)M^2 + \frac{3}{640}(1 - 10\delta + 9\delta^2)M^4 \right]$$

where $\sigma = \rho / \rho_0$ is the density ratio, $\delta = p / p_0$ is the pressure ratio, and M is the Mach number. The indicated airspeed V_{ind} is obtained from calibrated airspeed using a specified conversion table. The aircraft angular velocity is

$$\omega_{AC}^F = \omega^{FI/F} = R \begin{pmatrix} \dot{\phi}_F \\ \dot{\theta}_F \\ \dot{\psi}_F \end{pmatrix} = \begin{bmatrix} 1 & 0 & -\sin\theta_F \\ 0 & \cos\phi_F & \sin\phi_F \cos\theta_F \\ 0 & -\sin\phi_F & \cos\phi_F \cos\theta_F \end{bmatrix} \begin{pmatrix} \dot{\phi}_F \\ \dot{\theta}_F \\ \dot{\psi}_F \end{pmatrix}$$

For steady state flight, $\dot{\theta}_F = \dot{\phi}_F = 0$; $\dot{\psi}_F$ is nonzero in a turn.

A headwind or tailwind can be specified. The wind velocity is a linear function of altitude h: $V_w = \pm(\max(0, d_{\text{wind}} + f_{\text{wind}} h))$, with the plus sign for a headwind and the minus sign for a tailwind. For example, California-to-Hawaii 85th percentile winter quartile headwind profile is $V_w = 9.59 + 0.00149h$ (with altitude h in ft). The wind speed is V_w, and the ground speed is $V_g = V_h - V_w$.

Accelerated flight is also considered, in terms of linear acceleration $a_{AC}^F = \dot{v}^{FI/F} = g n_L$ and pitch rate $\dot{\theta}_F$. The nominal pullup load factor is $n = 1 + \dot{\theta}_F V_h / g$. For accelerated flight, the instantaneous equilibrium of the forces and moments on the aircraft is evaluated, for specified acceleration and angular velocity; the equations of motion are not integrated to define a maneuver. Note that the fuselage and wing aerodynamic models do not include all roll and yaw moment terms needed for general unsteady flight (notably derivatives L_v, L_p, L_r, N_v, N_p, N_r).

The aircraft pitch and roll angles are available for trim of the aircraft. Any motion not selected for trim will remain fixed at the values specified for the flight state. The pitch and roll angles each can be zero, constant, or a function of flight speed (piecewise linear input). The flight state input can override this value of the aircraft motion. The input value is an initial value if the motion is a trim variable.

The aircraft never-exceed speed V_{NE} (knots TAS) can be specified by a combination of tabular, stall, compressibility, and Mach number limits. The tabular limit $V_{\text{NE}t}$ is a function of density altitude h_d and weight ratio $r_W = W_G / W_D$, and perhaps temperature τ (°C). The stall limit is evaluated for each rotor from the steady or transient or equation definition of the rotor thrust capability, $(C_T / \sigma)_{\max}$ as a function of advance ratio μ. Given C_T / σ, the steady or transient function is interpolated to μ; or the equation $(C_T / \sigma)_{\max} = K_0 - K_1 \mu^2$ is solved for $\mu = \sqrt{(K_0 - |C_T/\sigma|)/K_1}$; then $V_{\text{NE}s} = \mu V_{\text{tip}}$. The compressibility limit, typically obtained from flight test, is the indicated airspeed (knots) as a function of density altitude h_d, temperature τ (°C), main rotor tip speed V_{tip}, and weight ratio $r_W = W_G / W_D$:

$$V_{\text{NE}c} = C_1 - C_2 h_d + C_3 \tau - C_4 V_{\text{tip}} - C_5 r_W$$

The Mach number limit is evaluated for each rotor from a specified advancing tip Mach number: $V_{\mathrm{NE}m} = M_{at}c_s - V_{\mathrm{tip}}$. The aircraft never-exceed speed is the minimum of the active limits: $V_{\mathrm{NE}} = \min(V_{\mathrm{NE}t}, V_{\mathrm{NE}s}, V_{\mathrm{NE}c}, V_{\mathrm{NE}m})$. Finally, minimum and maximum speeds can be specified, so $V_{\mathrm{NE}} = \max(V_{\min}, \min(V_{\max}, V_{\mathrm{NE}}))$; where $V_{\min}$ and $V_{\max}$ are each the minimum of true airspeed and indicated airspeed values (expressed as true airspeed).

8–6   Loads and Performance

For each component, the power required and the net forces and moments acting on the aircraft can be calculated. The aerodynamic forces F and moments M are typically calculated in wind axes and then resolved into body axes (x, y, z), relative to the origin of the body axes (the aircraft center of gravity). The power and loads of all components are summed to obtain the aircraft power and loads. Typically the trim solution drives the net forces and moments on the aircraft to zero.

The aircraft equations of motion, in body axes F with origin at the aircraft center of gravity, are the equations of force and moment equilibrium:

$$m(\dot{v}^{FI/F} + \widetilde{\omega}^{FI/F} v^{FI/F}) = F^F + F_{\mathrm{grav}}^F$$
$$I^F \dot{\omega}^{FI/F} + \widetilde{\omega}^{FI/F} I^F \omega^{FI/F} = M^F$$

where $m = W/g$ is the aircraft mass; the gravitational force is $F_{\mathrm{grav}}^F = mC^{FI}g^I = mC^{FI}(0\,0\,g)^T$; and the moment of inertia matrix is

$$I^F = \begin{bmatrix} I_{xx} & -I_{xy} & -I_{xz} \\ -I_{yx} & I_{yy} & -I_{yz} \\ -I_{zx} & -I_{zy} & I_{zz} \end{bmatrix}$$

For steady flight, $\dot{\omega}^{FI/F} = \dot{v}^{FI/F} = 0$, and $\omega^{FI/F} = R(0\,0\,\dot{\psi}_F)^T$ is nonzero only in turns. For accelerated flight, $\dot{v}^{FI/F}$ can be nonzero, and $\omega^{FI/F} = R(0\,\dot{\theta}_F\,\dot{\psi}_F)^T$. The equations of motion are thus

$$m(a_{AC}^F + \widetilde{\omega}_{AC}^F v_{AC}^F) = F^F + F_{\mathrm{grav}}^F$$
$$\widetilde{\omega}_{AC}^F I^F \omega_{AC}^F = M^F$$

The body axis load factor is $n = (C^{FI}g^I - (a_{AC}^F + \widetilde{\omega}_{AC}^F v_{AC}^F))/g$. The a_{AC}^F term is absent for steady flight. The forces and moments are the sum of loads from all components of the aircraft:

$$F^F = F_{\mathrm{fus}}^F + \sum F_{\mathrm{rotor}}^F + \sum F_{\mathrm{wing}}^F + \sum F_{\mathrm{tail}}^F + F_{\mathrm{tank}}^F + \sum F_{\mathrm{engine}}^F + \sum F_{\mathrm{jet}}^F + \sum F_{\mathrm{charge}}^F$$
$$M^F = M_{\mathrm{fus}}^F + \sum M_{\mathrm{rotor}}^F + \sum M_{\mathrm{wing}}^F + \sum M_{\mathrm{tail}}^F + M_{\mathrm{tank}}^F + \sum M_{\mathrm{engine}}^F + \sum M_{\mathrm{jet}}^F + \sum M_{\mathrm{charge}}^F$$

Forces and moments in inertial axes are also of interest ($F^I = C^{IF}F^F$ and $M^I = C^{IF}M^F$). A particular component can have more than one source of loads; for example, the rotor component produces hub forces and moments, but also includes hub and pylon drag. The equations of motion are $E_f = F^F + F_{\mathrm{grav}}^F - F_{\mathrm{inertial}}^F = 0$ and $E_m = M^F - M_{\mathrm{inertial}}^F = 0$.

The component power required P_{comp} is evaluated for all components (rotors, motors, and compressors) of the propulsion group. The total power required for the propulsion group P_{reqPG} is obtained by adding the transmission losses and accessory power. The power required for the propulsion group must be distributed to the engine groups. The fuel flow is calculated from the engine power, jet thrust, charger power, and equipment power required. The total fuel flow is the sum from all components of the aircraft: $\dot{w} = \sum \dot{w}_{reqEG} + \sum \dot{w}_{reqJG} + \sum \dot{w}_{reqCG} + \dot{w}_{\mathrm{eq}}$.

8–7   Aerodynamics

Each component has a position z^F in aircraft axes F, relative to the reference point; and orientation of component axes B relative to aircraft axes given by the rotation matrix C^{BF}. It is expected that the component axes are (roughly) x forward and z down (or in negative lift direction). The aerodynamic model must be consistent with the convention for component orientation. Acting at the component are interference velocities v_{int}^F (velocity of air, in F axes), from all other components. Then the total component velocity relative to the air is

$$v^F = v_{AC}^F + \widetilde{\omega}_{AC}^F \Delta z^F - \sum v_{\text{int}}^F$$

where $\Delta z^F = z^F - z_{cg}^F$. Then $v^B = C^{BF} v^F$ is the velocity in component axes. The aerodynamic environment is defined in the component axes: velocity magnitude $v = |v^B|$, dynamic pressure $q = \frac{1}{2}\rho v^2$, angle of attack α, and sideslip angle β. The angle of attack and sideslip angle provide the transformation between airframe axes and velocity axes:

$$C^{BA} = Y_\alpha Z_{-\beta}$$

This is the conventional aircraft definition, corresponding to yaw-then-pitch of the airframe axes relative to the velocity vector. By definition, the velocity is along the x-axis in the A axes, $v^B = C^{BA}(v\,0\,0)^T$; from which the angle of attack and sideslip in terms of the components of v^B are obtained:

$$\alpha = \tan^{-1} v_3^B / v_1^B$$
$$\beta = \sin^{-1} v_2^B / |v^B|$$

This definition is not well behaved for $v_1^B = 0$ (it gives $\alpha = 90\,\text{sign}(v_3^B)$), so for sideward flight a pitch-then-yaw definition can be useful: $C^{BA} = Z_{-\beta} Y_\alpha$. Then

$$\alpha = \sin^{-1} v_3^B / |v^B|$$
$$\beta = \tan^{-1} v_2^B / v_1^B$$

which gives $\beta = 90\,\text{sign}(v_2^B)$ for $v_1^B = 0$.

The component aerodynamic model may include coefficient values for sideward flight, but not have equations for a continuous variation of the coefficients with sideslip angle. For such cases, sideward flight is defined as $|\beta| = 80$ to 100 degrees.

From v, q, α, and β, the aerodynamic model calculates the component force and moment, in wind axes acting at z^F:

$$F^A = \begin{pmatrix} -D \\ Y \\ -L \end{pmatrix} \qquad\qquad M^A = \begin{pmatrix} M_x \\ M_y \\ M_z \end{pmatrix}$$

where D, Y, and L are the drag, side force, and lift; M_x, M_y, and M_z are the roll, pitch, and yaw moments. The aerodynamic loads in aircraft axes acting at the center of gravity are then:

$$F^F = C^{FB} C^{BA} F^A$$
$$M^F = C^{FB} C^{BA} M^A + \widetilde{\Delta z^F} F^F$$

where $\Delta z^F = z^F - z_{cg}^F$. In hover and low speed, the download is calculated: $F_z^I = k^T (C^{IF} F^F)$, the downward component of the aerodynamic force in inertial axes. Download can be expressed as a fraction

of the total rotor vertical force or as a fraction of gross weight. The aerodynamic model also calculates the interference velocities caused by this component at all other components: $v_{\text{int}}^F = C^{FB} v_{\text{int}}^B$.

Equations for the aerodynamics models are defined for all angles in radians. Input values of angles are, however, in degrees.

The aircraft neutral point is calculated from the airframe aerodynamics with all controls set to zero. The neutral point is here defined as the longitudinal position about which the derivative of the pitch moment with lift is zero. Hence $SL_{na} = SL_{cg} - \Delta M / \Delta L$, with the change in lift and moment calculated from the loads at angles of attack of 0 and 5 deg.

8–8   Compressible Aerodynamics

The aerodynamic coefficients of lifting surfaces (wing and empennage) can be corrected for compressibility effects. Based on Prandtl-Glauert, the incompressible lift-curve slope is multiplied by the factor

$$F_c = \begin{cases} \left[\dfrac{1}{\sqrt{1-M^2}} \right] & M < M_{\text{div}} \\[3mm] \left[\dfrac{1-M}{(1-M_{\text{div}})\sqrt{1-M_{\text{div}}^2}} \right] & M_{\text{div}} < M < M_{\text{div}} + .1 \\[3mm] \left[\dfrac{1-M}{(1-M_{\text{div}})\sqrt{1-M_{\text{div}}^2}} + \dfrac{M-M_{\text{div}}-.1}{1-M_{\text{div}}-.1} \right] & M_{\text{div}} + .1 < M < 1 \\[3mm] 1. & M > 1 \end{cases}$$

Typically the lift-divergence Mach number $M_{\text{div}} \cong 0.75$, about 0.1 less than the drag-divergence Mach number. A compressible drag increment is modeled following ref. 1 (fig. 6):

$$C_{Dc} = C_{Dcc}\left(6.5x^2 + 28x^4 + 2870x^8 \right)$$

where $x = (M/M_{cc} - 0.7)$ (with M/M_{cc} restricted to the range 0.7 to 1.1) and $C_{Dcc} \cong 0.0011$ is the drag increment at $M = M_{cc}$. The critical Mach number depends on the lift coefficient:

$$M_{cc} = M_{cc0} - M_{cc1}\left(0.03C_L + 0.19C_L^2 + 0.78C_L^3 \right)$$

with typically $M_{cc0} \cong 0.74$, $M_{cc1} \cong 0.31$, and C_L restricted to the range 0 to 1.

8–9   Trailing-Edge Flaps

The lifting surfaces have controls in the form of trailing-edge flaps: flap, flaperon, and aileron for wings; elevator or rudder for tails. The aerodynamic loads generated by flap deflection δ_f (radians) are estimated based on two-dimensional aerodynamic data (as summarized in refs. 2 and 3). Let $\ell_f = c_f/c$ be the ratio of the flap chord to the wing chord. The lift coefficient is $c_\ell = c_{\ell\alpha}(\alpha + \tau\eta\delta_f)$, where $\eta \cong 0.85 - 0.43\delta_f$ is an empirical correction for viscous effects (ref. 2, eq. 3.54 and fig. 3.36). Thin airfoil theory gives

$$\tau = 1 - \frac{\theta_f - \sin\theta_f}{\pi} \cong \left(\sin(\frac{\pi}{2}\ell_f) \right)^n$$

with $\theta_f = \cos^{-1}(2\ell_f - 1)$ (ref. 2, eq. 3.56 and fig. 3.35; ref. 3, eq. 5.40). The last expression is an approximation that is a good fit to the thin airfoil theory result for $n = \frac{1}{2}$, and a good approximation including the effects of real flow for $n = \frac{2}{3}$ (ref. 3, fig. 5.18); the last expression with $n = \frac{2}{3}$ is used

here. The increase of maximum lift coefficient caused by flap deflection is less than the increase in lift coefficient, so the stall angle of attack is decreased. Approximately

$$\frac{\Delta c_{\ell \max}}{\Delta c_\ell} \cong (1 - \ell_f)(1 + \ell_f - 5\ell_f^2 + 3\ell_f^3)$$

(ref. 2, fig. 3.37). Thin airfoil theory gives the moment coefficient increment about the quarter chord:

$$\Delta c_m = -0.85\left((1 - \ell_f)\sin\theta_f\right)\delta_f = -0.85\left((1 - \ell_f)2\sqrt{(1 - \ell_f)\ell_f}\right)\delta_f$$

(ref. 2, eq. 3.57; ref. 3, eq. 5.41); with the factor of 0.85 accounting for real flow effects (ref. 3, fig. 5.19). The drag increment is estimated using

$$\Delta C_D = 0.9\,\ell_f^{1.38}\,\frac{S_f}{S}\,\sin^2\delta_f$$

for slotted flaps (ref. 2, eq. 3.51). In summary, the section load increments are:

$$\Delta c_\ell = c_{\ell\alpha}\,\frac{c_f}{c}\,L_f\eta_f\delta_f$$

$$\Delta c_{\ell\max} = X_f\,\Delta c_\ell$$

$$\Delta c_m = \frac{c_f}{c}\,M_f\delta_f$$

The decrease in angle of attack for maximum lift is

$$\Delta\alpha_{\max} = -\frac{\Delta c_\ell - \Delta c_{\ell\max}}{c_{\ell\alpha}} = -(1 - X_f)\frac{\Delta c_\ell}{c_{\ell\alpha}}$$

The coefficients

$$\eta_f = 0.85 - 0.43|\delta_f| = \eta_0 - \eta_1|\delta_f|$$

$$L_f = \frac{1}{\ell_f}\left(\sin(\frac{\pi}{2}\ell_f)\right)^{2/3}$$

$$X_f = (1 - \ell_f)(1 + \ell_f - 5\ell_f^2 + 3\ell_f^3)$$

$$M_f = -0.85\frac{1}{\ell_f}\left((1 - \ell_f)2\sqrt{(1 - \ell_f)\ell_f}\right)$$

$$D_f = 0.9\,\ell_f^{1.38}$$

follow from these equations.

For three-dimensional aerodynamic loads, these two-dimensional coefficients are corrected by using the three-dimensional lift-curve slope, and multiplying by the ratio of flap span to wing span b_f/b. Then the wing load increments caused by flap deflection, in terms of coefficients based on the wing area, are:

$$\Delta C_L = \frac{S_f}{S}\,C_{L\alpha}L_f\eta_f\delta_f K_\ell$$

$$\Delta C_M = \frac{S_f}{S}\,M_f\delta_f K_m$$

$$\Delta C_D = \frac{S_f}{S}\,D_f\sin^2\delta_f K_d$$

$$\Delta C_{L\max} = X_f\,(\Delta C_L/K_\ell)K_x$$

$$\Delta\alpha_{\max} = -(K_\ell - X_f K_x)\frac{\Delta C_L/K_\ell}{C_{L\alpha}}$$

where S_f/S is the ratio of flap area to wing area, and factors K have been introduced to calibrate or correct the loads.

8–10   Flow Control

The wing or fuselage can have flow control, depending on a momentum coefficient $C_\mu = \dot{m}V_j/qS$, where $\dot{m}V_j$ is the momentum flux, q is the free stream dynamic pressure, and S the reference area (wing planform area or fuselage wetted area). The mass flux is $\dot{m} = \rho_j V_j S_{\text{slot}}$, from the jet density and velocity and the slot area, hence $C_\mu = 2(V_j/V)^2(S_{\text{slot}}/S)$. Aerodynamic load increments are produced by the momentum flux, modelled as:

$$\Delta C_L = C_{L\alpha}(L_{\mu s}\sqrt{C_\mu} + L_{\mu 1}C_\mu + L_{\mu 2}C_\mu^2)$$
$$\Delta C_M = M_\mu C_\mu$$
$$\Delta C_D = D_\mu C_\mu$$

$$\Delta C_{L\max} = X_\mu C_{L\mu}$$
$$\Delta \alpha_{\max} = -(1 - X_\mu)\frac{\Delta C_L}{C_{L\alpha}}$$

To account for loss of effectiveness at high levels of blowing, these load increments are evaluated at an effective momentum coefficient of $C_{\mu\text{eff}} = \min(C_\mu, C_{\mu\text{limit}})$, using an input limit $C_{\mu\text{limit}}$. The direct reaction force of the momentum flux is included in the lift and drag changes.

8–11   Drag

Each component can contribute drag to the aircraft. A fixed drag can be specified as a drag area D/q; or the drag can be scaled, specified as a drag coefficient C_D based on an appropriate area S. There may also be other ways to define a scaled drag value. For fixed drag, the coefficient is $C_D = (D/q)/S$ (the aerodynamic model is formulated in terms of drag coefficient). For scaled drag, the drag area is $D/q = SC_D$. For all components, the drag $(D/q)_{\text{comp}}$ or $C_{D\text{comp}}$ is defined for forward flight or cruise; typically this is the minimum drag value. For some components, the vertical drag $((D/q)_{V\text{comp}}$ or $C_{DV\text{comp}})$ or sideward drag $((D/q)_{S\text{comp}}$ or $C_{DS\text{comp}})$ is defined. For some components, the aerodynamic model includes drag due to lift, angle of attack, or stall.

Table 8-2 summarizes the component contributions to drag, and the corresponding reference areas. If no reference area is indicated, then the input is only drag area D/q. An appropriate drag reference area is defined for each component, and either input or calculated. Wetted area is calculated for each component, even if it is not the reference area. The component wetted areas are summed to obtain the aircraft wetted area. Some of the weight models also require the wetted area. The component drag contributions must be consistent. In particular, a rotor with a spinner (such as on a tiltrotor aircraft) would likely not have hub drag. The pylon is the rotor support and the nacelle is the engine support. The drag model for a tiltrotor aircraft with tilting engines would use the pylon drag (and no nacelle drag), since the pylon is connected to the rotor shaft axes; with non-tilting engines it would use the nacelle drag as well.

Optionally the aircraft drag can be fixed. The quantity specified is the sum (over all components) of the drag area D/q (minimum drag, excluding drag due to lift and angle of attack), without accounting for interference effects on dynamic pressure. The input parameter can be D/q; or the drag can be scaled, specified as a drag coefficient based on the rotor disk area, so $D/q = A_{\text{ref}}C_D$ (where A_{ref} is the reference rotor disk area); or the drag can be estimated based on the gross weight, $D/q = k(W_{MTO}/1000)^{2/3}$ (where W_{MTO} is the maximum takeoff gross weight; units of k are ft^2/k-lb$^{2/3}$ or m^2/Mg$^{2/3}$). Based on historical data, the drag coefficient $C_D = 0.02$ for old helicopters, $C_D = 0.008$ for current low-drag helicopters. Based on historical data, $k = 9$ for old helicopters, $k = 2.5$ for current low-drag helicopters, $k = 1.6$ for current tiltrotors, and $k = 1.4$ for turboprop aircraft (English units). If the aircraft drag is input, then the fuselage contingency drag is adjusted so the total aircraft D/q equals the input value.

Table 8-2. Component contributions to drag.

component	drag contribution	reference area
fuselage	fuselage fuselage vertical fittings and fixtures rotor-body interference contingency (aircraft) payload increment (flight state)	fuselage wetted area fuselage projected area fuselage wetted area fuselage wetted area — —
landing gear	landing gear	—
rotor	hub, hub vertical duct, duct vertical pylon, pylon vertical spinner	rotor disk area duct wetted area pylon wetted area spinner wetted area
wing	wing, wing vertical wing-body interference	wing planform area wing planform area
tail	tail, tail vertical	tail planform area
fuel tank	auxiliary tank (flight state)	—
engine	nacelle, nacelle vertical momentum drag	nacelle wetted area —
jet	nacelle, nacelle vertical momentum drag	nacelle wetted area —
charger	nacelle, nacelle vertical momentum drag	nacelle wetted area —

Optionally the aircraft vertical drag (download fraction) can be fixed. The quantity specified is the sum over all components of the vertical drag area $(D/q)_V$. The input parameter can be $(D/q)_V$, or $k = (D/q)_V/A_{ref}$ (where A_{ref} is reference rotor disk area). Approximating the dynamic pressure in the wake as $q = \frac{1}{2}\rho(2v_h)^2 = T/A_{ref}$, the download fraction is $DL/T = q(D/q)_V/T = k$. If the aircraft vertical drag is input, then the fuselage contingency vertical drag is adjusted so the total aircraft $(D/q)_V$ equals the input value.

The nominal drag areas of the components and the aircraft are part of the aircraft description and are used when the aircraft drag is fixed. The nominal drag area is calculated for low-speed helicopter flight, for high-speed cruise flight, and for vertical flight. An incidence angle i is specified for the rotors, wings, and nacelles, to be used solely to calculate the nominal helicopter and vertical drag areas. The convention is that $i = 0$ if the component does not tilt. Table 8-3 summarizes the contributions to the nominal drag areas, with D for the drag in normal flow and D_V for the drag in vertical flow. While vertical drag parameters are part of the aerodynamic model for the hub, duct, pylon, and nacelle, aerodynamic interference at the rotor and at the propulsion components is not considered, so these terms do not contribute to download. In the context of download, only the fuselage, wing, tail, and contingency contribute to the nominal vertical drag.

From the input and the current aircraft size, the drag areas D/q and coefficients C_D are calculated. The aerodynamic analysis is usually in terms of coefficients. If the aircraft drag is fixed for the aircraft

model, then the fuselage contingency drag is set:

$$(D/q)_{\text{cont}} = (D/q)_{\text{fixed}} - \sum (D/q)_{\text{comp}}$$

and similarly for fixed vertical drag. Note that this adjustment ignores changes caused by interference in the dynamic pressure and the velocity direction, which will affect the actual component drag.

The component aerodynamic model calculates the drag, typically from a drag coefficient C_D, a reference area, and the air velocity of the component. The drag force is then $D = \sum q_{\text{comp}} S_{\text{ref}} C_D$, where the dynamic pressure q_{comp} includes interference. From the aerodynamic forces and moments in wind axes, the total force and moment in body axes (F^F and M^F) are calculated. For reference, the aircraft total drag and total drag area are

$$D_{AC} = \sum e_d^T F_{\text{aero}}^F$$
$$(D/q)_{AC} = D_{AC}/q$$

where the aircraft velocity (without interference) gives the direction $e_d = -v_{AC}^F/|v_{AC}^F|$ and dynamic pressure $q = \tfrac{1}{2}\rho|v_{AC}^F|^2$; and F_{aero}^F is the component aerodynamic force. An overall skin friction drag coefficient is then $C_{D\,AC} = (D/q)_{AC\text{wet}}/S_{AC}$, based on the aircraft wetted area $S_{AC} = \sum S_{\text{wet}}$ and excluding drag terms not associated with skin friction (specifically landing gear, rotor hub, and contingency drag).

Table 8-3. Component contributions to nominal drag area.

component	drag contribution	cruise	helicopter	vertical
fuselage	fuselage	D	D	D_V
	fittings and fixtures	D	D	D
	rotor-body int	D	D	D
landing gear	landing gear	D	D	0
	retractable	0	D	0
rotor	hub	D	$D\cos^2 i + D_V \sin^2 i$	0
	duct	D	$D\cos^2 i + D_V \sin^2 i$	0
	pylon	D	$D\cos^2 i + D_V \sin^2 i$	0
	spinner	D	D	0
wing	wing	D	$D\cos^2 i + D_V \sin^2 i$	$D\sin^2 i + D_V \cos^2 i$
	wing-body int	D	D	D
tail	horizontal tail	D	D	$D_V \cos^2 \phi$
tail	vertical tail	D	D	$D_V \sin^2 \phi$
engine	nacelle	D	$D\cos^2 i + D_V \sin^2 i$	0
jet	nacelle	D	$D\cos^2 i + D_V \sin^2 i$	0
charger	nacelle	D	$D\cos^2 i + D_V \sin^2 i$	0
	contingency	D	D	D_V

8–12   Performance Metrics

The following performance metrics are calculated for the aircraft. The aircraft hover figure of merit is $M = W\sqrt{W/2\rho A_{\text{ref}}}/P$. The aircraft effective drag is $D_e = P/V$, hence the effective lift-to-drag ratio

is $L/D_e = WV/P$. For these metrics, the aircraft power is the sum of the engine group power, jet group propulsive power, and charge group power: $P = P_{req} + VT_{\text{jet}} + P_{\text{chrg}}$. The aircraft power loading is W/P (lb/hp or kg/kW). Isolated rotor performance metrics are described in Chapter 12.

8–13   Weights

The design gross weight W_D is a principal parameter defining the aircraft, either input or determined by the sizing task from designated design conditions and missions. Typically W_D is the takeoff weight for the primary design mission. The design gross weight is used by the analysis to calculate the rotor disk loading and blade loading, wing loading, power loading, and thrust loading; to obtain aircraft moments of inertia from the radii of gyration; for tolerance and perturbation scales of the solution procedures; optionally to define structural design gross weight and maximum takeoff weight; and optionally to specify the gross weight for missions and flight conditions.

The aircraft weight statement defines the empty weight, fixed useful load, and operating weight for the design configuration. The aircraft weight statement is the sum of the weight statements for all the aircraft components, the component weight determined by input or by parametric calculations with technology factors. The definitions of the weight terms are as follows:

$$\begin{aligned}
\text{gross weight} \qquad & W_G = W_E + W_{UL} = W_O + W_{\text{pay}} + W_{\text{fuel}} \\
\text{operating weight} \qquad & W_O = W_E + W_{FUL} \\
\text{useful load} \qquad & W_{UL} = W_{FUL} + W_{\text{pay}} + W_{\text{fuel}}
\end{aligned}$$

where W_E is the weight empty; W_{FUL} the fixed useful load; W_{pay} the payload weight; and W_{fuel} the usable fuel weight. Aircraft weight definitions are given in SAWE RP7D (ref. 4), including:

> Payload is any item which is being transported and is directly related to the purpose of the flight as opposed to items that are necessary for the flight operation. Payload can include, but is not limited to, passengers, cargo, passenger baggage, ammo, internal and external stores, and fuel which is to be delivered to another aircraft or site. Payload may or may not be expended in flight.

> Operating weight is the sum of aircraft weight empty and operating items. Operating weight is equivalent to takeoff gross weight less usable fuel, payload, and any item to be expended in flight.

> Weight empty is an engineering term which is defined as the weight of the complete aircraft as defined in the aircraft specifications, dry, clean, and empty except for fluids in closed systems such as a hydraulic system.

The weight empty consists of structure, propulsion group, systems and equipment, vibration, and contingency weights. The weight empty W_E can be calculated, or input, or scaled with design gross weight: $W_E = d_{WE} + f_{WE}W_D$. Fixed or scaled weight empty is implemented by adjusting the contingency weight so W_E equals the required value. If the design gross weight is input, then the payload or fuel weight must be fallout.

The structural design gross weight W_{SD} and maximum takeoff weight W_{MTO} can be input, or

specified as an increment d plus a fraction f of a weight W:

$$W_{SD} = d_{SDGW} + f_{SDGW}W = \begin{cases} d_{SDGW} + f_{SDGW}W_D \\ d_{SDGW} + f_{SDGW}(W_D - W_{\text{fuel}} + f_{\text{fuel}}W_{\text{fuel-cap}}) \\ d_{SDGW} + f_{SDGW}W_{MTO} \end{cases}$$

$$W_{MTO} = d_{WMTO} + f_{WMTO}W = \begin{cases} d_{WMTO} + f_{WMTO}W_D \\ d_{WMTO} + f_{WMTO}(W_D - W_{\text{fuel}} + W_{\text{fuel-cap}}) \\ d_{WMTO} + f_{WMTO}W_{SD} \end{cases}$$

This convention allows the weights to be input directly ($f = 0$), or scaled with a design weight. For W_{SD}, W is the design gross weight W_D, or W_D adjusted for a specified fuel state (input fraction of fuel capacity), or the maximum takeoff weight W_{MTO}. Alternatively, W_{SD} can be calculated as the gross weight at a designated sizing flight condition. The structural design gross weight is used in the weight estimation. For W_{MTO}, W is the design gross weight W_D, or W_D adjusted for maximum fuel capacity, or the structural design gross weight W_{SD}. Alternatively, W_{MTO} can be calculated as the maximum gross weight possible at a designated sizing flight condition. The maximum takeoff weight is used in the cost model, in the scaled aircraft and hub drag, and in the weight estimation.

The design ultimate load factor n_{zult} at the structural design gross weight W_{SD} is specified, in particular for use in the component weight estimates. The structural design gross weight W_{SD} and design ultimate load factor n_{zult} are used for the fuselage, rotor, and wing weight estimations. The maximum takeoff weight W_{MTO} is used for the cost and drag (scaled aircraft and hub), and for the weights (system, fuselage, landing gear, and engine group).

The gross weight W_G is specified for each flight condition and mission, perhaps in terms of the design gross weight W_D. For a each flight state, the fixed useful load may be different from the design configuration because of changes in auxiliary fuel tank weight, or kit weights, or increments in crew or equipment weights. Thus the fixed useful load weight is calculated for the flight state; and from it the useful load weight and operating weight are calculated. The gross weight, payload weight, and usable fuel weight (in standard and auxiliary tanks) complete the weight information for the flight state.

For each weight group, fixed (input) weights can be specified; or weight increments dW added to the results of the parametric weight model. The parametric weight model includes technology factors χ. Thus typically a component or element weight is obtained from $W = \chi W_{\text{model}} + dW$. Weight of individual elements in a group can be fixed by using dW and setting the corresponding technology factor $\chi = 0$. With $\chi \neq 0$, the increment dW can account for something not included in the parametric model.

For scaled weights of all components, the AFDD weight models are implemented. The user can incorporate custom weight models as well.

The operating weight is composed of scaled and fixed weights, so the design gross weight can be written $W_D = W_O + W_{\text{pay}} + W_{\text{fuel}} = W_{O\text{fixed}} + W_{O\text{scaled}} + W_{\text{pay}} + W_{\text{fuel}}$. The growth factor is the change in gross weight due to a change in payload:

$$\frac{\partial W_D}{\partial W_{\text{pay}}} = 1 + \frac{\partial W_{O\text{scaled}}}{\partial W_{\text{pay}}} + \frac{\partial W_{\text{fuel}}}{\partial W_{\text{pay}}} = 1 + \left(\frac{\partial W_{O\text{scaled}}}{\partial W_D} + \frac{\partial W_{\text{fuel}}}{\partial W_D} \right) \frac{\partial W_D}{\partial W_{\text{pay}}}$$

$$\cong 1 + \left(\frac{W_{O\text{scaled}}}{W_D} + \frac{W_{\text{fuel}}}{W_D} \right) \frac{\partial W_D}{\partial W_{\text{pay}}} = \frac{1}{1 - \phi_{O\text{scaled}} - \phi_{\text{fuel}}}$$

in terms of the weight fractions $\phi = W/W_D$.

The aircraft operating weight can be divided into core vehicle weight and military load. The core vehicle weight is the weight in minimum airworthy state, with the aircraft capable of normal flight throughout the envelope, but not mission capable. Military load is the sum of fixed useful load and military features in weight empty. Thus

$$\text{weight empty} = \text{core vehicle weight} + \text{military features}$$

$$\text{military load} = \text{fixed useful load} + \text{military features in weight empty}$$

$$\text{operating weight} = W_E + W_{FUL} = \text{core vehicle weight} + \text{military load}$$

In terms of the weight breakdown used here, military features in weight empty consist of folding weight (wing, rotor, tail, fuselage terms), crashworthiness weight (fuselage, landing gear terms), marinization weight (fuselage), rotor brake (drive system), avionics group (mission equipment), armament group, furnishings and equipment group, anti-icing group (including electrical group term), and load and handling group.

8–14   Weight Statement

Aircraft weight information is stored in a data structure that follows SAWE RP8A Group Weight Statement format (ref. 5), as outlined in figure 8-3. The asterisks designate extensions of RP8A for the purposes of this analysis. Typically only the lowest elements of the hierarchy are specified; higher elements are obtained by summation. Fixed (input) weight elements are identified in the data structure. A weight statement data structure exists for each component. The aircraft weight statement is the sum of the structures from all components.

8–15   References

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2) McCormick, B.W. *Aerodynamics, Aeronautics, and Flight Mechanics*. New York: John Wiley & Sons, Second Edition, 1995.

3) Kuethe, A.M., and Chow, C.-Y. *Foundations of Aerodynamics*. New York: John Wiley & Sons, Fifth Edition, 1998.

4) "Mass Properties Management and Control for Military Aircraft, Revision D." Society of Allied Weight Engineers, Recommended Practice Number 7, May 2004.

5) "Weight and Balance Data Reporting Forms for Aircraft (including Rotorcraft), Revision A." Society of Allied Weight Engineers, Recommended Practice Number 8, June 1997.

```
WEIGHT EMPTY
    STRUCTURE
        wing group
            basic structure
            secondary structure
                fairings (*), fittings (*), fold/tilt (*)
            control surfaces
        rotor group
            blade assembly
            hub & hinge
                basic (*), fairing/spinner (*), blade fold (*), shaft (*)
            rotor support structure (*), duct (*)
        empennage group
            horizontal tail (*)
                basic (*), fold (*)
            vertical tail (*)
                basic (*), fold (*)
            tail rotor (*)
                blades, hub & hinge, rotor supports, rotor/fan duct
        fuselage group
            basic (*)
            wing & rotor fold/retraction (*)
            tail fold/tilt (*)
            marinization (*)
            pressurization (*)
            crashworthiness (*)
        alighting gear group
            basic (*), retraction (*), crashworthiness (*)
        engine section or nacelle group
            engine support (*), engine cowling (*), pylon support (*)
        air induction group
    PROPULSION GROUP
        engine system
            engine
            exhaust system
            accessories (*)
        propeller/fan installation
            blades (*), hub & hinge (*), rotor supports (*), rotor/fan duct (*)
        fuel system
            tanks and support
            plumbing
        drive system
            gear boxes
            transmission drive
            rotor shaft
            rotor brake (*)
            clutch (*)
            gas drive
```

Figure 8-3a. Weight statement (* indicates extension of RP8A).

SYSTEMS AND EQUIPMENT
   flight controls group
      cockpit controls
      automatic flight control system
      system controls
         fixed wing systems
            non-boosted (*), boost mechanisms (*)
         rotary wing systems
            non-boosted (*), boost mechanisms (*), boosted (*)
         conversion systems
            non-boosted (*), boost mechanisms (*)
   auxiliary power group
   instruments group
   hydraulic group
      fixed wing (*), rotary wing (*), conversion (*)
      equipment (*)
   pneumatic group
   electrical group
      aircraft (*), anti-icing (*)
   avionics group (mission equipment)
   armament group
      armament provisions (*), armor (*)
   furnishings & equipment group
   environmental control group
   anti-icing group
   load & handling group
VIBRATION (*)
CONTINGENCY
FIXED USEFUL LOAD
   crew
   fluids (oil, unusable fuel) (*)
   auxiliary fuel tanks
   other fixed useful load (*)
   equipment increment (*)
   folding kit (*)
   wing extension kit (*)
   wing kit (*)
   other kit (*)
PAYLOAD
USABLE FUEL
   standard tanks (*)
   auxiliary tanks (*)

OPERATING WEIGHT = weight empty + fixed useful load
USEFUL LOAD = fixed useful load + payload + usable fuel

GROSS WEIGHT = weight empty + useful load
GROSS WEIGHT = operating weight + payload + usable fuel

Figure 8-3b. Weight statement (* indicates extension of RP8A).

Systems

The systems component contains weight information (fixed useful load, vibration, contingency, and systems and equipment).

9–1   Weights

The weight empty consists of structure, propulsion group, systems and equipment, vibration, and contingency weights. The vibration control weight can be input, or specified as a fraction of weight empty: $W_{\text{vib}} = f_{\text{vib}}W_E$. The contingency weight can be input, or specified as a fraction of weight empty: $W_{\text{cont}} = f_{\text{cont}}W_E$. However, if the weight empty is input, then the contingency weight is adjusted so W_E equals the required value. The weights of all components are evaluated and summed, producing the aircraft weight empty less vibration and contingency weight, W_X. Then:

> a) Fixed or scaled weight empty: W_{vib} input or $W_{\text{vib}} = f_{\text{vib}}W_E$, $W_{\text{cont}} = W_E - (W_X + W_{\text{vib}})$.
> b) Both fractional: $W_E = W_X/(1 - f_{\text{vib}} - f_{\text{cont}})$, $W_{\text{vib}} = f_{\text{vib}}W_E$, $W_{\text{cont}} = f_{\text{cont}}W_E$.
> c) Only vibration weight fractional: W_{cont} input, $W_E = (W_X + W_{\text{cont}})/(1 - f_{\text{vib}})$, $W_{\text{vib}} = f_{\text{vib}}W_E$.
> d) Only contingency weight fractional: W_{vib} input, $W_E = (W_X + W_{\text{vib}})/(1 - f_{\text{cont}})$, $W_{\text{cont}} = f_{\text{cont}}W_E$.
> e) Both input: W_{vib} and W_{cont} input, $W_E = W_X + W_{\text{vib}} + W_{\text{cont}}$.

Finally, the operating weight $W_O = W_E + W_{FUL}$ is recalculated.

Systems and equipment includes the following fixed (input) weights: auxiliary power group, instruments group, pneumatic group, electrical group (aircraft), avionics group (mission equipment), armament group (armor and armament provisions), furnishings and equipment group, environmental control group, and load and handling group. Systems and equipment includes the following scaled weights: flight controls group, hydraulic group, electrical group (anti-icing), and anti-icing group.

Flight controls group includes the following fixed (input) weights: cockpit controls and automatic flight control system. Flight controls group includes the following scaled weights: fixed wing systems, rotary wing systems, and conversion or thrust vectoring systems. Rotary wing flight control weights can be calculated for the entire aircraft (using rotor parameters such as chord and tip speed for a designated rotor), an approach that is consistent with parametric weight equations developed for conventional two-rotor configurations. Alternatively, rotary wing flight control weights can be calculated separately for each rotor and then summed. The fixed wing flight controls and the conversion controls can be absent.

The fixed useful load W_{FUL} consists of crew (W_{crew}), trapped fluids (oil and unusable fuel, W_{trap}), auxiliary fuel tanks (W_{auxtank}), equipment increment, kits (folding, wing, wing extension, other), and

other fixed useful load ($W_{FULother}$). W_{crew}, W_{trap}, and $W_{FULother}$ are input. For a each flight state, the fixed useful load may be different from the design configuration because of changes in auxiliary fuel tank weight, kit weight, and crew or equpment weight increments.

Folding weights can be calculated in several weight groups, including wing, rotor, empennage, and fuselage. These weights are the total weights for folding and the impact of folding on the group. A fraction f_{foldkit} of these weights can be in a kit, hence optionally removable. Thus, of the total folding weight, the fraction f_{foldkit} is a kit weight in the fixed useful load of the weight statement, while the remainder is kept in the component group weight.

9–2   Detailed Weight Definition

Figure 9-1 presents a more detailed description of the weights for the systems and equipment, and the useful load. The input information for a group can be just the total weight (ΔW in the following equations), or can include all the terms.

The electrical group consists of power supply, power conversion, power distribution and controls, lights and signal devices, and equipment supports; plus a term associated with the anti-icing group. The total weight is

$$W_{\text{electrical}} = W_{\text{elect aircraft}} + W_{DI\text{elect}}$$
$$W_{\text{elect aircraft}} = W_{\text{supply}} + W_{\text{conv}} + W_{\text{distrib}} + W_{\text{lights}} + W_{\text{support}} + \Delta W_{\text{electrical}}$$

The avionics group consists of equipment and installation; here the equipment weights include installation. The equipment items are communications, navigation, identification, control and display, aircraft survivability, and mission system equipment; plus armament electronics. The total weight is

$$W_{\text{MEQ}} = W_{\text{com}} + W_{\text{nav}} + W_{\text{ident}} + W_{\text{display}} + W_{\text{survive}} + W_{\text{mission}} + \Delta W_{\text{MEQ}} + W_{\text{arm elect}}$$

The armament group consists of armament provisions (gun provisions, turret systems, expendable weapons provisions) and armor. Armament electronics weight $W_{\text{arm elect}}$ (such as targeting, sights, radar) is part of the avionics group.

$$W_{\text{arm prov}} = W_{\text{gun}} + W_{\text{turret}} + W_{\text{expend}} + \Delta W_{\text{arm prov}}$$
$$W_{\text{armor}} = U_{\text{armor floor}} S_{\text{cabin floor}} + U_{\text{armor wall}} S_{\text{cabin wall}} + U_{\text{armor crew}} N_{\text{crew seat}} + \Delta W_{\text{armor}}$$

Here $U_{\text{armor floor}}$, $U_{\text{armor wall}}$ are the armor weights per surface area; $S_{\text{cabin floor}}$ and $S_{\text{cabin wall}}$ are the cabin floor and wall areas; $U_{\text{armor crew}}$ is the armor weight per crew; and $N_{\text{crew seat}}$ is number of crew seats.

The furnishings and equipment group consists of accommodation for personnel, miscellaneous equipment, furnishings, and emergency equipment. Accommodation for personnel consists of seats, miscellaneous accommodation (including galleys, toilets), and the oxygen system. Miscellaneous equipment includes cockpit displays. Furnishings includes floor covering, trim, partitions, crash padding, and acoustic and thermal insulation; but excludes vibration absorbers. Emergency equipment consists of fire detection and extinguishing, and other emergency equipment (including first aid, survival kit, and life

raft). The total weight is

$$W_{\text{furnish}} = W_{\text{accom}} + W_{\text{misc}} + W_{\text{furn}} + W_{\text{emerg}} + \Delta W_{\text{furnish}}$$

$$W_{\text{accom}} = (U_{\text{seat crew}} + U_{\text{accom crew}} + U_{\text{ox crew}})N_{\text{crew seat}}$$
$$+ (U_{\text{seat pass}} + U_{\text{accom pass}} + U_{\text{ox pass}})N_{\text{pass seat}}$$

$$W_{\text{furn}} = W_{\text{trim}} + U_{\text{insulation}}S_{\text{cabin}}$$

$$W_{\text{emerg}} = W_{\text{fire}} + W_{\text{other}}$$

Here $U_{\text{zz crew}}$, $U_{\text{zz pass}}$ are the weights per crew and passenger; $N_{\text{crew seat}}$, $N_{\text{pass seat}}$ are the number of crew and passenger seats; $U_{\text{insulation}}$ is the acoustic and thermal insulation weight per area; and S_{cabin} is the total cabin surface area. The load and handling group consists of aircraft handling and load handling (cargo handling, hoist, external load provisions). The total weight is

$$W_{\text{load}} = W_{\text{aircraft}} + W_{\text{cargo}} + \Delta W_{\text{load}}$$

$$W_{\text{cargo}} = U_{\text{handling}}S_{\text{cabin floor}} + W_{\text{hoist}} + W_{\text{ext prov}}$$

Here U_{handling} is the cargo handling weight per cabin floor area, and $S_{\text{cabin floor}}$ is the cabin floor area.

The crew weight is

$$W_{\text{crew}} = U_{\text{crew}}N_{\text{crew}} + \Delta W_{\text{crew}}$$

Here U_{crew} is the weight per crew, and N_{crew} is the number of crew. A crew weight increment for a flight condition or mission is given by δN_{crew} and δW_{crew}. Other fixed useful load consists of various categories, such as baggage, gun installations, weapons provisions, aircraft survivability equipment (chaff, flares), survival kits, life rafts, and oxygen.

An equipment increment can be defined for a flight condition or mission, in terms of $\delta N_{\text{crew seat}}$, $\delta N_{\text{pass seat}}$, and δW_{equip}:

$$W_{\text{equip inc}} = U_{\text{crew seat inc}}\,\delta N_{\text{crew seat}} + U_{\text{pass seat inc}}\,\delta N_{\text{pass seat}} + \delta W_{\text{equip}}$$

The default weights per crew and passenger seats are $U_{\text{crew seat inc}} = U_{\text{seat crew}} + U_{\text{accom crew}} + U_{\text{ox crew}} + U_{\text{armor crew}}$ and $U_{\text{pass seat inc}} = U_{\text{seat pass}} + U_{\text{accom pass}} + U_{\text{ox pass}}$.

The payload consists of passengers or troops, cargo (internal and external), ammunition, and weapons:

$$W_{\text{payload}} = U_{\text{pass}}N_{\text{pass}} + W_{\text{cargo}} + W_{\text{ext load}} + W_{\text{ammo}} + W_{\text{weapons}} + \Delta W_{\text{payload}}$$

Here U_{pass} is the weight per passenger, and N_{pass} is the number of passengers. A payload increment is defined by $\Delta W_{\text{payload}}$. For fallout payload, the value of $\Delta W_{\text{payload}}$ is adjusted.

```
WEIGHT EMPTY
    SYSTEMS AND EQUIPMENT
        electrical group
            aircraft
                power supply
                power conversion
                power distribution and controls
                lights and signal devices
                equipment supports
            anti-icing
        avionics group (mission equipment)
            equipment
            installation
        armament group
            armament provisions
            armor
        furnishings & equipment group
            accommodation for personnel
                seats
                miscellaneous accommodation
                oxygen system
            miscellaneous equipment
            furnishings
            emergency equipment
                fire detection and extinguishing
                other emergency equipment
        load & handling group
            aircraft handling
            load handling
USEFUL LOAD
    FIXED USEFUL LOAD
        crew
        other fixed useful load
            various categories
        equipment increment
    PAYLOAD
        passengers/troops
        cargo
        ammunition
        weapons
```

Figure 9-1. Details of weight descriptions (based on RP8A).

Chapter 10

Fuselage

There is one fuselage component for the aircraft.

<u>10–1   Geometry</u>

The fuselage length ℓ_{fus} can be input or calculated. The calculated length depends on the longitudinal positions of all components. Let $x_{\max}$ and $x_{\min}$ be the maximum (forward) and minimum (aft) position of all rotors, wings, and tails. Then the calculated fuselage length is

$$\ell_{\text{fus}} = \ell_{\text{nose}} + (x_{\max} - x_{\min}) + \ell_{\text{aft}}$$

The nose length ℓ_{nose} (distance forward of hub) and aft length ℓ_{aft} (distance aft of hub) are input, or calculated as $\ell_{\text{nose}} = f_{\text{nose}}L$ and $\ell_{\text{aft}} = f_{\text{aft}}L$. Typically $f_{\text{aft}} = 0$ or negative for the main-rotor and tail-rotor configuration, and $f_{\text{aft}} = 0.75$ for the coaxial configuration. The fuselage width w_{fus} is input. The reference length L is the rotor radius or wing span of a designated component, or the input fuselage length.

The fuselage wetted area S_{wet} (reference area for drag coefficients) and projected area S_{proj} (reference area for vertical drag) are input (excluding or including the tail boom terms); or calculated from the nose length:
$$S_{\text{wet}} = f_{\text{wet}} \left(2\ell_{\text{nose}}h_{\text{fus}} + 2\ell_{\text{nose}}w_{\text{fus}} + 2h_{\text{fus}}w_{\text{fus}}\right) + C_{\text{boom}}L$$
$$S_{\text{proj}} = f_{\text{proj}} \left(\ell_{\text{nose}}w_{\text{fus}}\right) + w_{\text{boom}}L$$
using input fuselage height h_{fus}, and factors f_{wet} and f_{proj}; or calculated from the fuselage length:

$$S_{\text{wet}} = f_{\text{wet}} \left(2\ell_{\text{fus}}h_{\text{fus}} + 2\ell_{\text{fus}}w_{\text{fus}} + 2h_{\text{fus}}w_{\text{fus}}\right) + C_{\text{boom}}L$$
$$S_{\text{proj}} = f_{\text{proj}} \left(\ell_{\text{fus}}w_{\text{fus}}\right) + w_{\text{boom}}L$$

Using the nose length and the tail boom area is probably best for a single-main-rotor and tail-rotor helicopter. Here C_{boom} is the effective tail boom circumference (boom wetted area divided by reference length), and w_{boom} is the effective tail boom width (boom vertical area divided by reference length). Alternatively, the wetted area and projected area can be estimated based on the gross weight:

$$S_{\text{wet}} = k_{\text{wet}}(W_D/1000)^{2/3}$$
$$S_{\text{proj}} = k_{\text{proj}}(W_D/1000)^{2/3}$$

where W_D is the design gross weight; units of k are ft²/k-lb$^{2/3}$ or m²/Mg$^{2/3}$.

Cabin areas are required for weight estimates: total cabin surface area S_{cabin} for acoustic and thermal insulation weight (furnishings and equipment group); cabin floor area $S_{\text{cabin floor}}$ for armor and

cargo handling weights; and cabin wall area $S_{\text{cabin wall}}$ for armor weight. These areas are input, or calculated from the fuselage dimensions:

$$S_{\text{cabin}} = f_{\text{cabin}} \left(2\ell_{\text{fus}}h_{\text{fus}} + 2\ell_{\text{fus}}w_{\text{fus}}\right)$$

$$S_{\text{cabin floor}} = f_{\text{floor}} \left(\ell_{\text{fus}}w_{\text{fus}}\right)$$

$$S_{\text{cabin wall}} = f_{\text{wall}} \left(2\ell_{\text{fus}}h_{\text{fus}}\right)$$

with typically $f \cong 0.6$.

The fuselage contribution to the aircraft operating length is $x_{\text{fus}} + f_{\text{ref}}\ell_{\text{fus}}$ (forward) and $x_{\text{fus}} - (1 - f_{\text{ref}})\ell_{\text{fus}}$ (aft). Here f_{ref} is the position of the fuselage aerodynamic reference location aft of the nose, as a fraction of the fuselage length. If the fuselage length is input, then f_{ref} is input; if the fuselage length is calculated, then $f_{\text{ref}} = (x_{\text{max}} + \ell_{\text{nose}} - x_{\text{fus}})/\ell_{\text{fus}}$.

10–2   Control and Loads

The fuselage has a position z^F, where the aerodynamic forces act; and the component axes are aligned with the aircraft axes, $C^{BF} = I$.

Optionally the fuselage can have flow control. The momentum coefficient C_μ is a control, allowing it to be connected to an aircraft control and thus set (or trimmed) for each flight state.

10–3   Aerodynamics

The aerodynamic velocity of the fuselage relative to the air, including interference, is calculated in component axes, v^B. The angle of attack α_{fus}, sideslip angle β_{fus} (hence C^{BA}), and dynamic pressure q are calculated from v^B. The reference area for the fuselage forward flight drag is the fuselage wetted area S_{wet}, which is input or calculated as described previously. The reference area for the fuselage vertical drag is the fuselage projected area S_{proj}, which is input or calculated as described previously.

The momentum coefficient $C_\mu = \dot{m}V_j/qS$ is a control of the fuselage, where $\dot{m}V_j$ is the momentum flux, q is the free stream dynamic pressure, and S the wetted area. The momentum flux (gross force) required is supplied by the compressor or jet: $F_{Greq} = \dot{m}V_j = qSC_\mu$. The engine group or jet group performance includes the efficiency of the flow control ducts and nozzle. The compressor model also gives the engine group power required to produce F_{Greq} (summed over all components with flow control). From the momentum coefficient, the increments of lift coefficient, maximum lift angle, moment coefficient, and drag coefficient are evaluated: $\Delta C_{L\mu}$, $\Delta\alpha_{\text{max}\mu}$, $\Delta C_{M\mu}$, $\Delta C_{D\mu}$.

10-3.1   Drag

The drag area or drag coefficient is defined for forward flight, vertical flight, and sideward flight. In addition, the forward flight drag area or drag coefficient is defined for fixtures and fittings, and for rotor-body interference. The effective angle of attack is $\alpha_e = \alpha_{\text{fus}} - \alpha_{D\text{min}}$, where $\alpha_{D\text{min}}$ is the angle of minimum drag; in reverse flow ($|\alpha_e| > 90$), $\alpha_e \leftarrow \alpha_e - 180\,\text{sign}\alpha_e$. For angles of attack less than a transition angle α_t, the drag coefficient equals the forward flight (minimum) drag C_{D0}, plus an angle of attack term and the flow control increment. Thus if $|\alpha_e| \leq \alpha_t$

$$C_D = C_{D0}\left(1 + K_d|\alpha_e|^{X_d}\right) + \Delta C_{D\mu}$$

and otherwise
$$C_{Dt} = C_{D0}\left(1 + K_d|\alpha_t|^{X_d}\right) + \Delta C_{D\mu}$$
$$C_D = C_{Dt} + \left(\frac{S_{\text{proj}}}{S_{\text{wet}}}C_{DV} - C_{Dt}\right)\sin\left(\frac{\pi}{2}\frac{|\alpha_e| - \alpha_t}{\pi/2 - \alpha_t}\right)$$

and similarly for the transition of payload drag $(D/q)_{\text{pay}}$ and contingency drag $(D/q)_{\text{cont}}$. Optionally there might be no angle-of-attack variation at low angles ($K_d = 0$), or quadratic variation ($X_d = 2$). With an input transition angle, there will be a jump in the slope of the drag coefficient at α_t. For a smooth transition, the transition angle that matches slopes as well as coefficients is found by solving

$$\left(\frac{2X_d}{\pi} - 1\right)\alpha_t^{X_d} - X_d\alpha_t^{X_d - 1} + \frac{(S_{\text{proj}}/S_{\text{wet}})C_{DV} - C_{D0} - \Delta C_{D\mu}}{K_d C_{D0}} = 0$$

This calculation of the transition angle is only implemented with quadratic variation, for which

$$\alpha_t = \frac{1}{a}\left(1 + \sqrt{1 - a\frac{(S_{\text{proj}}/S_{\text{wet}})C_{DV} - C_{D0} - \Delta C_{D\mu}}{K_d C_{D0}}}\right)$$

with $a = (4/\pi) - 1$; α_t is however required to be between 15 and 45 deg. For sideward flight ($v_x^B = 0$) the drag is obtained using $\phi_v = \tan^{-1}(-v_z^B/v_y^B)$ to interpolate between sideward and vertical coefficients:

$$C_D = C_{DS}\cos^2\phi_v + \frac{S_{\text{proj}}}{S_{\text{wet}}}C_{DV}\sin^2\phi_v$$

Then the drag force is

$$D = qS_{\text{wet}}\left(C_D + C_{D\text{fit}} + \sum C_{D\text{rb}}\right) + q\left((D/q)_{\text{pay}} + (D/q)_{\text{cont}}\right)$$

including drag coefficient for fixtures and fittings $C_{D\text{fit}}$ and rotor-body interference $C_{D\text{rb}}$ (summed over all rotors); drag area of the payload (specified for flight state); and contingency drag area.

10-3.2   Lift and Pitch Moment

The fuselage lift and pitch moment are defined in fixed form (L/q and M/q), or scaled form (C_L and C_M, based on the fuselage wetted area and fuselage length). The effective angle of attack is $\alpha_e = \alpha_{\text{fus}} - \alpha_{zl}$, where α_{zl} is the angle of zero lift; in reverse flow ($|\alpha_e| > 90$), $\alpha_e \leftarrow \alpha_e - 180\,\text{sign}\alpha_e$. Let $A_{\max} = \alpha_{\max} + \Delta\alpha_{\max\mu}$ be the angle-of-attack increment (above or below zero lift angle) for maximum lift, including the change from flow control. If $|\alpha_e| \leq A_{\max}$

$$C_L = C_{L\alpha}\alpha_e + \Delta C_{L\mu}$$
$$C_M = C_{M0} + C_{M\alpha}\alpha_e + \Delta C_{M\mu}$$

and otherwise
$$C_L = C_{L\alpha}A_{\max}\text{sign}\alpha_e\left(\frac{\pi/2 - |\alpha_e|}{\pi/2 - |A_{\max}|}\right)$$
$$C_M = (C_{M0} + C_{M\alpha}A_{\max}\text{sign}\alpha_e)\left(\frac{\pi/2 - |\alpha_e|}{\pi/2 - |A_{\max}|}\right)$$

for zero lift and moment at 90 deg angle of attack. In sideward flight, these coefficients are zero. Then $L = qS_{\text{wet}}C_L$ and $M = qS_{\text{wet}}\ell_{\text{fus}}C_M$ are the lift and pitch moment.

10-3.3 Side Force and Yaw Moment

The fuselage side force and yaw moment are defined in fixed form (Y/q and N/q), or scaled form (C_Y and C_N, based on the fuselage wetted area and fuselage length). The effective sideslip angle is $\beta_e = \beta_{\text{fus}} - \beta_{zy}$, where β_{zy} is the angle of zero side force; in reverse flow ($|\beta_e| > 90$), $\beta_e \leftarrow \beta_e - 180\,\text{sign}\beta_e$. Let $\beta_{\max}$ be the sideslip angle increment (above or below zero side force angle) for maximum side force. If $|\beta_e| \leq \beta_{\max}$

$$C_Y = C_{Y\beta}\beta_e$$

$$C_N = C_{N0} + C_{N\beta}\beta_e$$

and otherwise

$$C_Y = C_{Y\beta}\beta_{\max}\text{sign}\beta_e \left(\frac{\pi/2 - |\beta_e|}{\pi/2 - |\beta_{\max}|} \right)$$

$$C_N = (C_{N0} + C_{N\beta}\beta_{\max}\text{sign}\beta_e) \left(\frac{\pi/2 - |\beta_e|}{\pi/2 - |\beta_{\max}|} \right)$$

for zero side force and yaw moment at 90 deg sideslip angle. Then $Y = qS_{\text{wet}}C_Y$ and $N = qS_{\text{wet}}\ell_{\text{fus}}C_N$ are the side force and yaw moment. The roll moment is zero.

10–4   Weights

The fuselage group consists of the basic structure; wing and rotor fold/retraction; tail fold/tilt; and marinization, pressurization, and crashworthiness structure.

Chapter 11

Landing Gear

There is one landing gear component for the aircraft. The landing gear can be located on the body or on the wing. The landing gear can be fixed or retractable; a gear retraction speed is specified (CAS), or the landing gear state can be specified in the flight state.

11–1 Geometry

The landing gear has a position z^F, where the aerodynamic forces act. The component axes are aligned with the aircraft axes, $C^{BF} = I$. The landing gear has no control variables. The height of the bottom of the landing gear above ground level, h_{LG}, is specified in the flight state. The landing gear position z^F is a distance d_{LG} above the bottom of the gear.

11-1.1 Drag

The drag area is specified for landing gear extended, $(D/q)_{LG}$. The velocity relative to the air at z^F gives the drag direction $e_d = -v^F/|v^F|$ and dynamic pressure $q = \frac{1}{2}\rho|v^F|^2$ (no interference). Then

$$F^F = e_d\, q(D/q)_{LG}$$

is the total drag force.

11–2 Weights

The alighting gear group consists of basic structure, retraction, and crashworthiness structure.

Rotor

The aircraft can have one or more rotors, or no rotors. In addition to main-rotors, the rotor component can model tail-rotors, propellers, proprotors, ducted fans, thrust vectoring rotors, and auxiliary-thrust rotors. The principal configuration designation (main-rotor, tail-rotor, or propeller) is identified for each rotor component, and in particular determines where the weights are put in the weight statement (summarized in table 12-1). Each configuration can possibly have a separate performance or weight model, which is separately specified. Antitorque rotors and auxiliary-thrust rotors can be identified, for special sizing options. Other configuration features are variable diameter and ducted fan, and reaction drive. Optionally the rotor can stoppable, hence for each flight state operated stopped, stopped and stowed, or stopped as wing.

Table 12-1. Principal configuration designation.

configuration	weight statement	weight model	performance model
main-rotor	rotor group	rotor	rotor
tail-rotor	empennage group	tail-rotor	rotor
propeller	propulsion group	rotor, aux thrust	rotor

Multi-rotor systems (such as coaxial or tandem configuration) are modeled as a set of separate rotors, in order to accommodate the description of the position, orientation, controls, and loads. Optionally the location of the center of the rotor system can be specified and the rotor locations calculated based on input separation parameters. The performance calculation for twin and multiple rotor systems can include the mutual influence of the induced velocity on the power.

The main-rotor size is defined by the radius R or disk loading W/A, thrust-weighted solidity σ, hover tip speed V_{tip}, and blade loading $C_W/\sigma = W/\rho A V_{\text{tip}}^2 \sigma$. With more than one main-rotor, the disk loading and blade loading are obtained from an input fraction of design gross weight, $W = f_W W_D$. The air density ρ for C_W/σ is obtained from a specified takeoff condition. If the rotor radius is fixed for the sizing task, three of (R or W/A), C_W/σ, V_{tip}, and σ are input, and the other parameters are derived. Optionally the radius can be calculated from a specified ratio to the radius of another rotor. If the sizing task determines the rotor radius (R and W/A), then two of C_W/σ, V_{tip}, and σ are input, and the other parameter is derived. The radius can be sized for just a subset of the rotors, with fixed radius for the others.

For antitorque and auxiliary-thrust rotors, three of (R or W/A), C_W/σ, V_{tip}, and σ are input, and the other parameters are derived. Optionally the radius can be calculated from a specified ratio to the
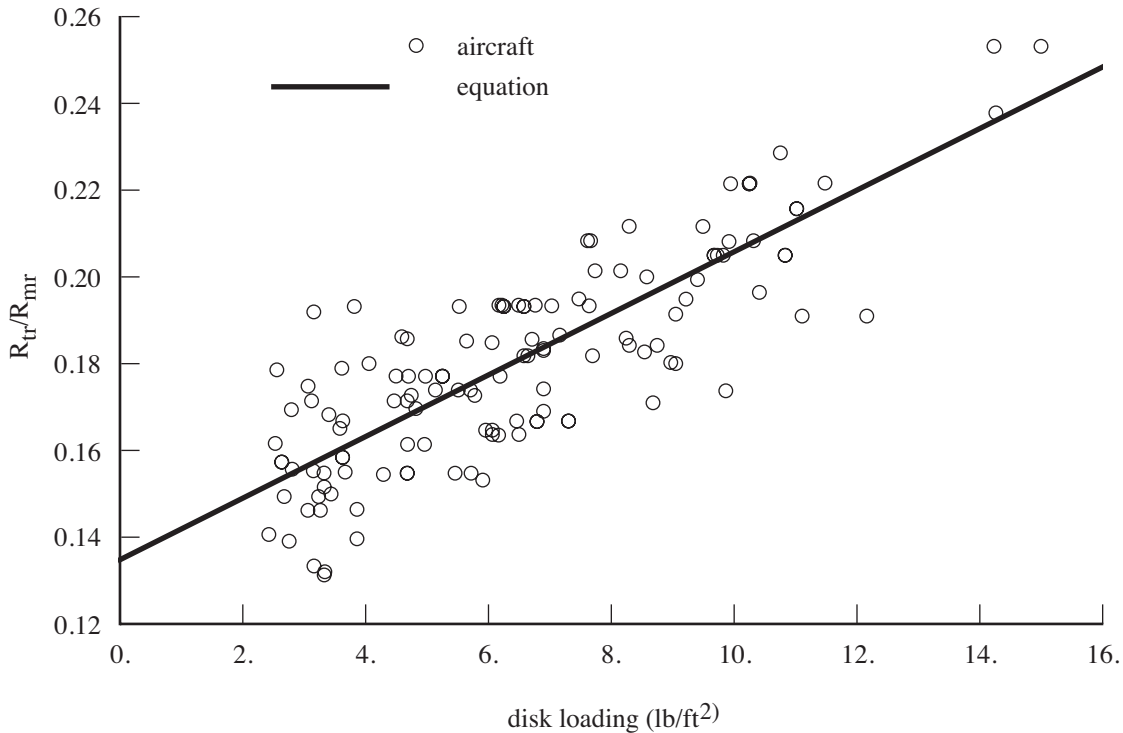
Figure 12-1. Tail-rotor radius scaling.

radius of another rotor. The disk loading and blade loading are based on fT, where f is an input factor and T is the maximum thrust from designated design conditions. Optionally the tail-rotor radius can be scaled with the main-rotor radius: $R = fR_{\mathrm{mr}}(0.1348 + 0.0071W/A)$, where f is an input factor and the units of disk loading W/A are lb/ft^2. Figure 12-1 shows the basis for this scaling.

12–1 Drive System

The drive system defines gear ratios for all the components it connects. The gear ratio is the ratio of the component rotational speed to that of the primary rotor. There is one primary rotor per propulsion group (for which the reference tip speed is specified); other components are dependent (for which a gear ratio is specified). There can be more than one drive system state, in order to model a multiple-speed or variable-speed transmission. Each drive system state corresponds to a set of gear ratios.

For the primary rotor, a reference tip speed $V_{\mathrm{tip-ref}}$ is defined for each drive system state. By convention, the "hover tip speed" refers to the reference tip speed for drive state #1. If the sizing task changes the hover tip speed, then the ratios of the reference tip speeds at different engine states are kept constant. By convention, the gear ratio of the primary rotor is $r = 1$. For dependent rotors, either the gear ratio is specified (for each drive system state) or a tip speed is specified and the gear ratio is calculated ($r = \Omega_{\mathrm{dep}}/\Omega_{\mathrm{prim}}$, $\Omega = V_{\mathrm{tip-ref}}/R$). For the engine group, either the gear ratio is specified (for each drive system state) or the gear ratio is calculated from the specification engine turbine speed $\Omega_{\mathrm{spec}} = (2\pi/60)N_{\mathrm{spec}}$ and the reference tip speed of the primary rotor ($r = \Omega_{\mathrm{spec}}/\Omega_{\mathrm{prim}}$, $\Omega_{\mathrm{prim}} = V_{\mathrm{tip-ref}}/R$). The latter option means the specification engine turbine speed N_{spec} corresponds

to $V_{\text{tip}-\text{ref}}$ for all drive system states. To determine the gear ratios, the reference tip speed and radius are used, corresponding to hover.

The flight state specifies the tip speed of the primary rotor and the drive system state, for each propulsion group. The drive system state defines the gear ratio for dependent rotors and the engine groups. From the rotor radius, the rotational speed of the primary rotor is obtained ($\Omega_{\text{prim}} = V_{\text{tip}}/R$); from the gear ratios, the rotational speed of dependent rotors ($\Omega_{\text{dep}} = r\Omega_{\text{prim}}$) and the engine groups ($N = (60/2\pi)r_{\text{eng}}\Omega_{\text{prim}}$) are obtained; and from the rotor radius, the tip speed of the dependent rotor ($V_{\text{tip}} = \Omega_{\text{dep}}R$) is obtained. The flight state specification of the tip speed can be an input value, the reference tip speed, a function of flight speed or a conversion schedule, or one of several default values. These relationships between tip speed and rotational speed use the actual radius of the rotors in the flight state, which for a variable-diameter rotor may not be the same as the reference, hover radius.

A designated drive system state can have a variable speed (variable gear ratio) transmission, by introducing a factor f_{gear} on the gear ratio when the speeds of the dependent rotors and engines are evaluated. The factor f_{gear} is a component control, which can be connected to an aircraft control and thus set for each flight state.

An optional conversion schedule is defined in terms of two speeds: hover and helicopter mode for speeds below $V_{C\text{hover}}$, cruise mode for speeds above $V_{C\text{cruise}}$, and conversion mode for speeds between $V_{C\text{hover}}$ and $V_{C\text{cruise}}$. The tip speed is $V_{\text{tip}-\text{hover}}$ in helicopter and conversion mode, and $V_{\text{tip}-\text{cruise}}$ in airplane mode. Drive system states are defined for helicopter, cruise, and conversion mode flight. The flight state specifies the nacelle tilt angle, tip speeds, control state, and drive system state, including the option to obtain any or all of these quantities from the conversion schedule.

Several default values of the tip speed are defined for use by the flight state, including cruise, maneuver, one-engine inoperative, drive system limit conditions, and a function of flight speed (piecewise linear input). Optionally these default values can be input as a fraction of the hover tip speed. Optionally the tip speed can be calculated from $\mu = V/V_{\text{tip}}$, so $V_{\text{tip}} = V/\mu$; or from $M_{at} = M_{\text{tip}}\sqrt{(1+\mu)^2 + \mu_z^2}$, so $V_{\text{tip}} = \sqrt{(c_s M_{at})^2 - V_z^2} - V$. Optionally the tip speed can be the minimum of the input value or that for M_{at}.

The sizing task might change the hover tip speed (reference tip speed for drive system state #1), the reference tip speed of a dependent rotor, a rotor radius, or the specification engine turbine speed N_{spec}. In such cases the gear ratios and other parameters are recalculated. Note that it is not consistent to change the reference tip speed of a dependent rotor if the gear ratio is a fixed input.

An increment on the primary rotor rotational speed (or primary engine group, if there are no rotors) is a control variable of the propulsion group.

12–2   Geometry

The rotor rotation direction is described by the parameter r: $r = 1$ for counter-clockwise rotation and $r = -1$ for clockwise rotation (as viewed from the positive thrust side of the rotor).

The rotor solidity and blade mean chord are related by $\sigma = Nc/\pi R$; usually thrust-weighted values are used, but geometric values are also required by the analysis. The geometric mean chord is the average of the chord over the rotor blade span r, from root cutout r_c to tip: $c_g = \int c\,dr / \int dr$. The thrust-weighted chord is the average of the chord over the rotor blade span, from root cutout to tip, weighted by

r^2: $c_t = \int cr^2 dr / \int r^2 dr$. The ratio of thrust-weighted and geometric chords is $f = c_t/c_g = \sigma_t/\sigma_g$ (equal to $f = c_{.75R}/c_{.50R}$ for linear taper and no root cutout). The blade chord distribution can be specified in terms of $f = c_t/c_g$; linear taper t; or a general chord distribution $c(r) = c_{\text{ref}}\hat{c}(r)$, where c_{ref} is the thrust-weighted chord c_t. The thrust-weighted solidity σ_t is a design parameter. The geometric solidity is evaluated from $\sigma_g = \sigma_t/f$ as required. The chord distribution $\hat{c}(r)$ is used in the blade element theory evaluation of the rotor flapping and forces.

The linear taper ratio is $t = c_1/c_0$, where c_1 is the tip chord and c_0 is the chord extrapolated to the center of rotation ($r = 0$); so the chord is $c = c_0(1 - (1 - t)r)$. It follows that

$$f = \frac{c_t}{c_g} = \frac{1 - \frac{3}{4}a(1 - t)}{1 - \frac{1}{2}b(1 - t)}$$

where $a = (1 - r_c^4)/(1 - r_c^3) = 1 + r_c^3/(1 + r_c + r_c^2)$ and $b = 1 + r_c$; which can be inverted for the (equivalent linear) taper from f

$$t = \frac{f(4 - 2b) - 4 + 3a}{3a - 2fb}$$

Given the taper t, f is evaluated and $\hat{c}(r) = 1 + (r - 0.75)\frac{4(t-1)}{1+3t}$. Given the chord ratio f, the equivalent taper t is evaluated and $\hat{c}(r) = 1 + (r - 0.75)\frac{4(t-1)}{1+3t}$. Given the chord distribution $\hat{c}(r)$, integration gives c_g, c_t, and $f = c_t/c_g$, and the equivalent taper t is evaluated.

The rotor hub is at position z_{hub}^F. Optionally, a component of the position can be calculated, superseding the location input. The calculated geometry depends on the configuration. For a coaxial rotor, the rotor separation is $s = |k^T C^{SF}(z_{\text{hub1}}^F - z_{\text{hub2}}^F)/(2R)|$ (fraction rotor diameter), or the hub locations are calculated from the input separation s, and the input location midway between the hubs:

$$z_{\text{hub}}^F = z_{\text{center}}^F \pm C^{FS}\begin{pmatrix} 0 \\ 0 \\ sR \end{pmatrix}$$

For a tandem rotor, the rotor longitudinal overlap is $o = \Delta\ell/(2R) = 1 - \ell/(2R)$ (fraction rotor diameter), or the hub locations are calculated from the input overlap o, and the input location midway between the hubs:

$$x_{\text{hub}} = x_{\text{center}} \pm R(1 - o)$$

For a tail-rotor, the longitudinal position can be calculated from the main-rotor radius R, tail-rotor radius R_{tr}, and tail-rotor/main-rotor clearance d_{tr}:

$$x_{\text{hub}tr} = x_{\text{hub}mr} - (R_{mr} + d_{tr} + R_{tr})$$

For a multicopter, the longitudinal and lateral position can be calculated from the rotor radius R, the scaled arm length ℓ_i of the i-th rotor, and the angle ψ_i (measured clockwise from the forward longitudinal axis):

$$x_{\text{hub}} = x_{\text{center}} + R\ell_i \cos\psi_i$$

$$y_{\text{hub}} = y_{\text{center}} + R\ell_i \sin\psi_i$$

For a tiltrotor, the lateral position can be calculated from the rotor radius R (cruise value for variable-diameter rotor), fuselage/rotor clearance d_{fus}, and fuselage width w_{fus}:

$$y_{\text{hub}} = \pm (fR + d_{\text{fus}} + \tfrac{1}{2}w_{\text{fus}})$$

with the pivot, pylon, and nacelle center-of-gravity lateral positions adjusted to keep the same relative position to the hub. The calculated clearance between the rotor and fuselage is $d_{\text{fus}} = |y_{\text{hub}}| - (R + \frac{1}{2}w_{\text{fus}})$. Alternatively, for a tiltrotor the lateral position can be calculated from the wing span, $y_{\text{hub}} = \pm b/2$, so the rotors are at the wing tips, or from a designated wing panel edge, $y_{\text{hub}} = \pm \eta_p(b/2)$.

For twin rotors (tandem, side-by-side, or coaxial), the overlap is $o = \Delta \ell/(2R) = 1 - \ell/(2R)$ (fraction of diameter; 0 for no overlap and 1 for coaxial), where the hub-to-hub separation is $\ell = [(x_{\text{hub1}} - x_{\text{hub2}})^2 + (y_{\text{hub1}} - y_{\text{hub2}})^2]^{1/2}$ ($\ell = 2R$ for no overlap and $\ell = 0$ for coaxial). The overlap area is mA, with A the area of one rotor disk and

$$ m = \frac{2}{\pi} \left[\cos^{-1}(\ell/2R) - (\ell/2R)\sqrt{1 - (\ell/2R)^2} \right] $$

The vertical separation is $s = |z_{\text{hub1}} - z_{\text{hub2}}|/(2R)$.

The reference areas for the component drag coefficients are the rotor disk area $A = \pi R^2$ (for hub drag), pylon wetted area S_{pylon}, duct wetted area $2S_{\text{duct}}$, and spinner wetted area S_{spin}. The pylon wetted area is input, or calculated from the drive system (gear box and rotor shaft) weight, or from the drive system plus engine system (engine, exhaust, and accessories) weight:

$$ S_{\text{pylon}} = k\left(w/N_{\text{rotor}}\right)^{2/3} $$

where $w = W_{gbrs}$ or $w = W_{gbrs} + \sum W_{ES}$ and the units of k are ft²/lb²/³ or m²/kg²/³. The pylon area is included in the aircraft wetted area if the pylon drag coefficient is nonzero. The duct wetted area is twice the duct area S_{duct}. The duct area is input, or calculated from the rotor circumference and the duct length $\ell_{\text{duct}} = kR$:

$$ S_{\text{duct}} = (2\pi R)\ell_{\text{duct}} $$

The spinner wetted area is input, or calculated from the spinner frontal area:

$$ S_{\text{spin}} = k(\pi R_{\text{spin}}^2) $$

where R_{spin} is the spinner radius, which is specified as a fraction of the rotor radius.

The rotor contribution to the aircraft operating length and width is calculated from the locus of the rotor disk: $z_{\text{disk}} = z_{\text{hub}} + RC^{FS}(\cos \psi \ \sin \psi \ 0)^T$. The longitudinal distance from the hub position is $\Delta x = R(a\cos\psi + b\sin\psi)$, so the maximum distance is $\Delta x = \pm R\sqrt{a^2 + b^2}$. The lateral distance from the hub position is $\Delta y = R(c\cos\psi + d\sin\psi)$, so the maximum distance is $\Delta y = \pm R\sqrt{c^2 + d^2}$.

12–3   Control and Loads

The rotor controls consist of collective, lateral cyclic, longitudinal cyclic, and perhaps shaft incidence (tilt) and cant angles. Rotor cyclic control can be defined in terms of tip-path plane or no-feathering plane command. The collective control variable is the rotor thrust amplitude or the collective pitch angle.

The relationship between tip-path plane tilt and hub moment is $M = \frac{N}{2}I_b\Omega^2(\nu^2 - 1)\beta = K_{\text{hub}}\beta$, where N is the number of blades, Ω the rotor speed, and ν the dimensionless fundamental flap frequency. The flap moment of inertia I_b is obtained from the Lock number: $\gamma = \rho acR^4/I_b$, for sea level standard (SLS) density ρ, lift-curve slope $a = 5.7$, and thrust-weighted chord (or from the blade weight, or from an autorotation index). The flap frequency and Lock number are specified for hover radius and rotational speed. The flap frequency and hub stiffness are required for the radius and rotational speed of the flight

state. For a hingeless rotor, the blade flap spring is $K_{\text{flap}} = I_b\Omega^2(\nu^2 - 1)$, obtained from the hover quantities; then $K_{\text{hub}} = \frac{N}{2}K_{\text{flap}}$ and

$$\nu^2 = 1 + \frac{K_{\text{flap}}}{I_b\Omega^2}$$

For an articulated rotor, the hinge offset is $e = Rx/(1+x)$, $x = \frac{2}{3}(\nu^2 - 1)$ from the hover quantities; then

$$\nu^2 = 1 + \frac{3}{2}\frac{e/R}{1 - e/R}$$

and $K_{\text{hub}} = \frac{N}{2}I_b\Omega^2(\nu^2 - 1)$, using I_b from γ (and scaled with R for a variable diameter rotor) and Ω for the flight state.

Optionally the rotor can have a variable diameter. The rotor diameter is treated as a control, allowing it to be connected to an aircraft control and thus set for each flight state. The basic variation can be specified based on the conversion schedule, or input as a function of flight speed (piecewise linear input). For the conversion schedule, the rotor radius is R_{hover} for speeds below $V_{C\text{hover}}$, $R_{\text{cruise}} = fR_{\text{hover}}$ for speeds above $V_{C\text{cruise}}$, and linear with flight speed in conversion mode. During the diameter change, the chord, chord radial distribution, and blade weight are assumed fixed; hence solidity scales as $\sigma \sim 1/R$, blade flap moment of inertia as $I_b \sim R^2$, and Lock number as $\gamma \sim R^2$.

Optionally the rotor can have reaction drive. The reaction jet force F_{react} is a control, allowing it to be connected to an aircraft control and thus set for each flight state, including trim of propulsion group power.

12-3.1   Control Variables

The collective control variable is direct command of rotor thrust magnitude T or C_T/σ (in shaft axes), from which the collective pitch angle can be calculated; or rotor collective pitch angle $\theta_{0.75}$, from which the thrust and inflow can be calculated.

Shaft tilt control variables are incidence (tilt) and cant angles, acting at a pivot location.

Tip-path plane command is direct control of the tip-path plane tilt, hence tilt of the thrust vector. This control mode requires calculation of rotor cyclic pitch angles from the flapping. The control variables are longitudinal tilt β_c (positive forward) and lateral tilt β_s (positive toward retreating side). Alternatively, the cyclic control can be specified in terms of hub moment or lift offset, if the blade flap frequency is greater than 1/rev. The relationship between tip-path plane tilt and hub moment is $M = K_{\text{hub}}\beta$, and between moment and lift offset is $M = o(TR)$. Thus the flapping is

$$\begin{pmatrix} \beta_s \\ \beta_c \end{pmatrix} = \frac{1}{K_{\text{hub}}}\begin{pmatrix} rM_x \\ -M_y \end{pmatrix} = \frac{TR}{K_{\text{hub}}}\begin{pmatrix} o_x \\ -o_y \end{pmatrix}$$

for hub moment command or lift offset command, respectively.

No-feathering plane command is control of rotor cyclic pitch angles, usually producing tilt of the thrust vector. This control mode requires calculation of rotor tip-path plane tilt from the cyclic control, including the influence of inflow. The control variables are longitudinal cyclic pitch angle θ_s (positive aft) and lateral cyclic pitch angle θ_c (positive toward retreating side).

12-3.2   Aircraft Controls

Each control can be connected to the aircraft controls c_{AC}: $c = c_0 + STc_{AC}$, with c_0 zero, constant, or a function of flight speed (piecewise linear input). The factor S can be introduced to automatically scale

the collective matrix: $S = a/6 = 1/60$ if the collective control variable is C_T/σ; $S = \rho V_{\text{tip}}^2 A_{\text{blade}}(a/6)$ if the collective control variable is rotor thrust T; $S = 1$ if the collective control variable is pitch angle $\theta_{0.75}$. For cyclic matrices, $S = 1$ with no-feathering plane command, and $S = -1$ for tip-path plane command.

12-3.3   Rotor Axes and Shaft Tilt

The rotor hub is at position z_{hub}^F, where the rotor forces and moments act; the orientation of the rotor shaft axes relative to the aircraft axes is given by the rotation matrix C^{SF}. The pivot is at position z_{pivot}^F. The hub or shaft axes S have origin at the hub node; the z-axis is the shaft, positive in the positive thrust direction; and the x-axis downstream or up. The rotor orientation is specified by selecting a nominal direction in body axes (positive or negative x-, y-, or z-axis) for the positive thrust direction; the other two axes are then the axes of control. For a main-rotor, the nominal direction would be the negative z-axis; for a tail-rotor, it would be the lateral axis (ry-axis, depending on the direction of rotation of the main-rotor); and for a propeller, the nominal direction would be the positive x-axis. This selection defines a rotation matrix W from F to S axes. The hub and pivot axes have a fixed orientation relative to the body axes:

$$\text{hub incidence and cant:} \qquad C^{HF} = U_{\theta_h} V_{\phi_h}$$

$$\text{pivot dihedral, pitch, and sweep:} \qquad C^{PF} = X_{\phi_p} Y_{\theta_p} Z_{\psi_p}$$

where U and V depend on the nominal direction, as described in table 12-2. The shaft control consists of incidence and cant about the pivot axes, from reference angles i_{ref} and c_{ref}:

$$C_{\text{cont}} = U_{i - i_{\text{ref}}} V_{c - c_{\text{ref}}}$$

For a tiltrotor aircraft, one of the aircraft controls is the nacelle angle, with the convention $\alpha_{\text{tilt}} = 0$ for cruise, and $\alpha_{\text{tilt}} = 90$ deg for helicopter mode. The rotor shaft incidence angle is then connected to α_{tilt} by defining the matrix T_i appropriately. For the locations and orientation input in helicopter mode, $i_{\text{ref}} = 90$. Thus the orientation of the shaft axes relative to the body axes is:

$$C^{SF} = W C^{HF} C^{FP} C_{\text{cont}} C^{PF}$$

or just $C^{SF} = W C^{HF}$ with no shaft control. From the pivot location z_{pivot}^F and the hub location for the reference shaft control z_{hub0}^F, the hub location in general is

$$z_{\text{hub}}^F = z_{\text{pivot}}^F + (C^{FP} C_{\text{cont}} C^{PF})^T (z_{\text{hub0}}^F - z_{\text{pivot}}^F) - \Delta z_{\text{hub}}^F \sin(i - i_{\text{ref}})$$

To account for more complex mechanisms than a simple pivot, the increment Δz_{hub}^F is included. Similarly, the pylon location and nacelle center-of-gravity location can be calculated for given shaft control. The shift in the aircraft center of gravity produced by nacelle tilt is

$$W(z_{cg}^F - z_{cg0}^F) = W_{\text{move}}(z_{\text{nac}}^F - z_{\text{nac0}}^F) = W_{\text{move}}(C^{FP} C_{\text{cont}}^T C^{PF} - I)(z_{\text{nac0}}^F - z_{\text{pivot}}^F)$$

where W is the gross weight and W_{move} the weight moved. Table 12-2 summarizes the geometry options.

Table 12-2. Rotor shaft axes.

	nominal thrust z^S-axis		x^S-axis		W	incidence + for T	cant + for T	$U_{\theta_h} V_{\phi_h}$
main-rotor	$-z^F$	up	$-x^F$	aft	Y_{180}	aft	right	$Y_\theta X_\phi$
other	z^F	down	$-x^F$	aft	Z_{180}	aft	right	$Y_{-\theta} X_{-\phi}$
propeller	x^F	forward	$-z^F$	up	Y_{90}	up	right	$Y_\theta Z_\phi$
other	$-x^F$	aft	$-z^F$	up	$Z_{180} Y_{-90}$	up	right	$Y_{-\theta} Z_{-\phi}$
tail-rotor ($r = 1$)	y^F	right	$-x^F$	aft	$Z_{180} X_{-90}$	aft	up	$Z_\theta X_{-\phi}$
tail-rotor ($r = -1$)	$-y^F$	left	$-x^F$	aft	$Z_{180} X_{90}$	aft	up	$Z_{-\theta} X_\phi$

$$Y_{180} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \qquad Z_{180}Y_{-90} = \begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$$

$$Z_{180} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad Z_{180}X_{-90} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$Y_{90} = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \qquad Z_{180}X_{90} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{bmatrix}$$

12-3.4   Hub Loads

The rotor controls give the thrust magnitude and the tip-path plane tilt angles β_c and β_s, either directly or from the collective and cyclic pitch. The forces acting on the hub are the thrust T, drag H, and side force Y (positive in z-, x-, y-axis directions, respectively). The hub pitch and roll moments are proportional to the flap angles. The hub torque is obtained from the shaft power P_{shaft} and rotor speed Ω. The force and moment acting on the hub, in shaft axes, are then:

$$F^S = \begin{pmatrix} H \\ Y \\ T \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ -f_B T \end{pmatrix}$$

$$M^S = \begin{pmatrix} M_x \\ M_y \\ -rQ \end{pmatrix} = \begin{pmatrix} K_{\text{hub}} \left(r\beta_s \right) \\ K_{\text{hub}} \left(-\beta_c \right) \\ -r P_{\text{shaft}}/\Omega \end{pmatrix}$$

The force includes a term proportional to the rotor thrust and an input blockage factor $f_B = \Delta T/T \geq 0$. This term accounts for blockage or download, as an alternative to including the drag of the fuselage or a lifting surface in the aircraft trim. For example, f_B can model the tail-rotor blockage caused by operation near the vertical tail. The rotor loads in aircraft axes acting at the center of gravity are then:

$$F^F = C^{FS} F^S$$
$$M^F = C^{FS} M^S + \widetilde{\Delta z^F} F^F$$

where $\Delta z^F = z^F_{\text{hub}} - z^F_{cg}$.

The wind axis lift L and drag X are calculated from the net rotor hub force F^F and the rotor velocity v^F. The velocity relative to the air gives the propulsive force direction $e_p = v^F/|v^F|$ (no interference) and the velocity magnitude $V = |v^F|$. The drag and lift components of the force are $X = -e_p^T F^F$ and $L = |(I - e_p e_p^T)F^F|$, respectively. Thus $XV = -(v^F)^T F^F$ and $L^2 = |F^F|^2 - |X|^2$. The rotor contribution to vertical force is the z-axis component of the force in inertial axes, $F_V = -k^T C^{IF} F^F$.

12-3.5   Download

Rotor-induced download on the airframe can be modeled in several ways. The first approach is to calculate the aerodynamic force on components (fuselage, wings, and tails), produced by the rotor wake-induced interference velocity. Optionally the aircraft vertical drag $(D/q)_V$ can be fixed. Aircraft trim increases the rotor thrust to counter the download force.

The second approach is to introduce a blockage factor $f_B = \Delta T/T \geq 0$ to produce an additional force $f_B T$ on the rotor hub, opposing the rotor thrust. Again, aircraft trim increases the rotor thrust to counter the blockage force.

The third approach is to use a download factor $f_{DL} = 1/(1 - \Delta T/T) \geq 1$ and evaluate the rotor induced and profile power at the increased thrust $T f_{DL}$. The download will not be reflected in the rotor thrust that trims the aircraft, but rather in higher rotor power for a given thrust.

The blockage factor and download factor for hover, out of ground effect are given by $B = \Delta T/T$ and $DL = \Delta T/T$, respectively. These values are reduced with forward speed and in ground effect: $f_B = B f_\mu f_z$, $f_{DL} = 1/(1 - DL\, f_\mu f_z)$, where

$$f_\mu = (1 - m)(1 - a_{DL} m)$$
$$f_z = (1 - \zeta^2)(1 - b_{DL}\zeta^4)$$

with $m = \mu/\mu_{DL}$ and $\zeta = (z_g/D)_{DL}/(z_g/D)$. The blockage and download are zero for $\mu > \mu_{DL}$ or $z_g/D < (z_g/D)_{DL}$.

12–4   Aerodynamics

The rotor velocity relative to the air is $v^F = v^F_{AC} + \widetilde{\omega}^F_{AC} \Delta z^F$ in aircraft axes. The velocities in shaft axes are

$$v^S = C^{SF} v^F = \Omega R \begin{pmatrix} -\mu_x \\ r\mu_y \\ \mu_z \end{pmatrix} \qquad \omega^S = C^{SF} \omega^F_{AC} = \Omega \begin{pmatrix} r\dot{\alpha}_x \\ \dot{\alpha}_y \\ r\dot{\alpha}_z \end{pmatrix}$$

where ΩR is the rotor tip speed. The advance ratio μ, inflow ratio λ, and shaft angle of attack α are defined as

$$\mu = \sqrt{\mu_x^2 + \mu_y^2}$$
$$\lambda = \lambda_i + \mu_z$$
$$\alpha = \tan^{-1}(\mu_z/\mu)$$

The blade velocity relative to the air has the maximum amplitude (advancing tip velocity) of $\mu_{at} = \sqrt{(1+\mu)^2 + \mu_z^2}$, from which the advancing tip Mach number is $M_{at} = M_{\text{tip}}\mu_{at}$, using the tip Mach number $M_{\text{tip}} = (\Omega R)/c_s$. The rotor thrust coefficient is defined as $C_T = T/\rho A(\Omega R)^2$. The dimensionless ideal induced velocity λ_i is calculated from μ, μ_z, and C_T; then the dimensional velocity is $v_i = \Omega R \lambda_i$. The ideal induced power is then $P_{\text{ideal}} = T v_i$. Note that for these inflow velocities, the subscript "i" denotes "ideal."

12-4.1   Ideal Inflow

The ideal wake-induced velocity is obtained from Glauert's momentum theory:

$$\lambda_i = \frac{C_T}{2\sqrt{\lambda^2 + \mu^2}} = \frac{s\lambda_h^2}{\sqrt{\lambda^2 + \mu^2}}$$

where $\lambda = \lambda_i + \mu_z$, $\lambda_h^2 = |C_T|/2$ (λ_h is always positive), and $s = \text{sign}\, C_T$. This expression is generalized to

$$\lambda_i = \lambda_h \, s \, F(\mu/\lambda_h, s\mu_z/\lambda_h)$$

If μ is zero, the equation for λ_i can be solved analytically. Otherwise, for non-axial flow, the equation is written as follows:

$$\lambda = \frac{s\lambda_h^2}{\sqrt{\lambda^2 + \mu^2}} + \mu_z$$

Using λ instead of λ_i as the independent variable simplifies implementation of the ducted fan model. A Newton–Raphson solution for λ gives:

$$\widehat{\lambda}_{in} = \frac{s\lambda_h^2}{\sqrt{\lambda_n^2 + \mu^2}}$$

$$\lambda_{n+1} = \lambda_n - \frac{\lambda_n - \mu_z - \widehat{\lambda}_{in}}{1 + \widehat{\lambda}_{in}\lambda_n/(\lambda_n^2 + \mu^2)} f$$

A relaxation factor of $f = 0.5$ is used to improve convergence. Three or four iterations are usually sufficient, using

$$\lambda \cong \frac{s\lambda_h^2}{\sqrt{(s\lambda_h + \mu_z)^2 + \mu^2}} + \mu_z$$

to start the solution. To eliminate the singularity of the momentum theory result at ideal autorotation, the expression

$$\lambda = \mu_z \left[\frac{0.373\mu_z^2 + 0.598\mu^2}{\lambda_h^2} - 0.991 \right]$$

is used when

$$1.5\mu^2 + (2s\mu_z + 3\lambda_h)^2 < \lambda_h^2$$

The equation $\lambda = \mu_z(a\mu_z^2 - b\lambda_h^2 + c\mu^2)/\lambda_h^2$ is an approximation for the induced power measured in the turbulent-wake and vortex-ring states. Matching this equation to the axial-flow momentum theory result at $\mu_z = -2\lambda_h$ and $\mu_z = -\lambda_h$ gives $a = \sqrt{5}/6 = 0.3726780$ and $b = (4\sqrt{5} - 3)/6 = 0.9907120$. Then matching to the forward-flight momentum theory result at $(\mu = \lambda_h, \mu_z = -1.5\lambda_h)$ gives $c = 0.5980197$. For axial flow ($\mu = 0$) the solution is:

$$\lambda = \begin{cases} \dfrac{\mu_z}{2} + s\sqrt{\left(\dfrac{\mu_z}{2}\right)^2 + \lambda_h^2} & -\lambda_h < s\mu_z \\[3ex] \mu_z \left[\dfrac{0.373\mu_z^2}{\lambda_h^2} - 0.991 \right] & -2\lambda_h < s\mu_z < -\lambda_h \\[3ex] \dfrac{\mu_z}{2} - s\sqrt{\left(\dfrac{\mu_z}{2}\right)^2 - \lambda_h^2} & s\mu_z < -2\lambda_h \end{cases}$$

Note that λ_i and v_i are the ideal induced velocities; additional factors are required for the wake-induced velocity or induced power calculations.

12-4.1.1 Ducted Fan

Rotor momentum theory can be extended to the case of a ducted fan. Consider a rotor system with disk area A, operating at speed V, with an angle α between V and the disk plane. The induced velocity at the rotor disk is v, and in the far wake $w = f_W v$. The far wake area is $A_\infty = A/f_A$. The axial velocity at the fan is $f_{Vz}V_z$, with f_{Vz} accounting for acceleration or deceleration through the duct. The edgewise velocity at the fan is $f_{Vx}V_x$, with $f_{Vx} = 1.0$ for wing-like behavior, or $f_{Vx} = 0$ for tube-like behavior of the flow. The total thrust (rotor plus duct) is T, and the rotor thrust is $T_{\text{rotor}} = f_T T$. For this model, the duct aerodynamics are defined by the thrust ratio f_T or far wake area ratio f_A, plus the fan velocity ratio

f_V. The mass flux through the rotor disk is $\dot{m} = \rho A U = \rho A_\infty U_\infty$, where U and U_∞ are respectively the total velocity magnitudes at the fan and in the far wake:

$$U^2 = (f_{Vx} V \cos\alpha)^2 + (f_{Vz} V \sin\alpha + v)^2$$
$$U_\infty^2 = (V \cos\alpha)^2 + (V \sin\alpha + w)^2$$

Mass conservation ($f_A = A/A_\infty = U_\infty/U$) relates f_A and f_W. Momentum and energy conservation give

$$T = \dot{m} w = \rho A U_\infty w / f_A = \rho A U f_W v$$
$$P = \frac{1}{2} \dot{m} w \left(2V \sin\alpha + \frac{w}{\eta_D} \right) = T \left(V \sin\alpha + \frac{w}{2\eta_D} \right)$$

where η_D is the duct efficiency, accounting for total pressure loss in flow through the duct. With these expressions, the span of the lifting system in forward flight is assumed equal to the rotor diameter $2R$. Next it is required that the power equals the rotor induced and parasite loss:

$$P = T_{\text{rotor}}(f_{Vz} V \sin\alpha + v) = T f_T (f_{Vz} V \sin\alpha + v)$$

In axial flow, this result can be derived from Bernoulli's equation for the pressure in the wake. In forward flight, any induced drag on the duct is being neglected. From these two expressions for power, $V_z + f_W v / 2\eta_D = f_T (f_{Vz} V_z + v)$ is obtained, relating f_T and f_W. With no duct ($f_T = f_{Vx} = f_{Vz} = \eta_D = 1$), the far wake velocity is always $w = 2v$, hence $f_W = 2$. With an ideal duct ($f_A = f_{Vx} = f_{Vz} = 1$), the far wake velocity is $f_W = 1$. In hover (with or without a duct), $f_W = f_A = 2\eta_D f_T$, and $v = \sqrt{2/f_W} v_h$. The rotor ideal induced power is $P_{\text{ideal}} = T w / 2\eta_D = f_D T v$, introducing the duct factor $f_D = f_W / 2\eta_D$.

For a ducted fan, the thrust C_T is calculated from the total load (rotor plus duct). To define the duct effectiveness, either the thrust ratio $f_T = T_{\text{rotor}}/T$ or the far wake area ratio $f_A = A/A_\infty$ is specified (and the fan velocity ratio f_V). The wake-induced velocity is obtained from the momentum theory result for a ducted fan: $\lambda_h^2 = (f_W \lambda_i/2)\sqrt{(f_{Vx}\mu)^2 + (f_{Vz}\mu_z + \lambda_i)^2}$. If the thrust ratio f_T is specified, this can be written

$$f_{Vz}\mu_z + \lambda_i = \frac{s\lambda_h^2/(f_T \eta_D)}{\sqrt{(f_{Vz}\mu_z + \lambda_i)^2 + (f_{Vx}\mu)^2}} + \frac{\mu_z}{f_T}$$

In this form, λ_i can be determined using the free-rotor expressions given previously: replacing λ_h^2, μ_z, μ, and λ with $\lambda_h^2/f_T \eta_D$, μ_z/f_T, $f_{Vx}\mu$, and $f_{Vz}\mu_z + \lambda_i$, respectively. Then from λ_i the velocity and area ratios are obtained:

$$f_W = 2\eta_D \left(f_T - (1 - f_T f_{Vz})\frac{\mu_z}{\lambda_i} \right)$$

$$f_A = \sqrt{\frac{\mu^2 + (\mu_z + f_W \lambda_i)^2}{(f_{Vx}\mu)^2 + (f_{Vz}\mu_z + \lambda_i)^2}}$$

If instead the area ratio f_A is specified, it is simplest to first solve for the far wake velocity $f_W \lambda_i$:

$$\mu_z + f_W \lambda_i = \frac{s\lambda_h^2 2 f_A}{\sqrt{(\mu_z + f_W \lambda_i)^2 + \mu^2}} + \mu_z$$

In this form, $f_W \lambda_i$ can be determined using the free-rotor expressions given previously: replacing λ_h^2 and λ with $\lambda_h^2 2 f_A$ and $\mu_z + f_W \lambda_i$, respectively. The induced velocity is

$$(f_{Vz}\mu_z + \lambda_i)^2 = \frac{1}{f_A^2}\left[\mu^2 + (\mu_z + f_W \lambda_i)^2 \right] - (f_{Vx}\mu)^2$$

The velocity ratio is $f_W = (f_W \lambda_i)/\lambda_i$, and

$$f_T = \frac{\mu_z + f_W \lambda_i / 2\eta_D}{f_{Vz}\mu_z + \lambda_i}$$

is the thrust ratio. However, physical problems and convergence difficulties are encountered with this approach in descent, if an arbitrary value of f_T is permitted. From the expression for f_T, f_T should approach $1/f_{Vz}$ at high rates of climb or descent. To avoid problems with an arbitrary value of f_T, it is assumed that the input value of f_T defines the velocity ratio $f_W = 2\eta_D f_T$ in descent. So in descent μ_z is not replaced by μ_z/f_T.

12-4.1.2   Ground Effect

The wake-induced velocity is reduced when the rotor disk is in the proximity of the ground plane. Ground effect in hover can be described in terms of the figure of merit $M = (T^{3/2}/\sqrt{2\rho A})/P$ as a function of scaled rotor height above the ground, $z_g/D = z_g/2R$. Usually the test data are given as the ratio of the thrust to OGE thrust, for constant power: $T/T_\infty = (M/M_\infty)^{2/3} = \kappa_g \geq 1$. The effect on power at constant thrust is then $P = P_\infty f_g$, where $f_g = \kappa_g^{-3/2} \leq 1$. Ground effect is generally negligible at heights above $z_g/D = 1.5$ and at forward speeds above $\mu = 3\lambda_h$.

The ground plane is assumed to be perpendicular to the inertial frame z-axis. The ground normal (directed downward) is $k_g^F = C^{FI}k$ in airframe axes, or $k_g^S = C^{SF}k_g^F$ in rotor shaft axes. The height of the landing gear above ground level, h_{LG}, is specified in the flight state. The height of the rotor hub above ground level is then

$$z_g = h_{LG} - (k_g^F)^T (z_{\text{hub}}^F - z_{LG}^F) + d_{LG}$$

where z_{LG}^F is the position of the landing gear in the airframe, and d_{LG} is the distance from the bottom of the gear to the location z_{LG}. From the velocity

$$v^S = \begin{pmatrix} \mu_x \\ -r\mu_y \\ -\lambda \end{pmatrix}$$

the angle ϵ between the ground normal and the rotor wake is evaluated: $\cos \epsilon = (k_g^S)^T v^S/|v^S|$ ($\epsilon = 0$ for hover, $\epsilon = 90$ deg in forward flight). Note that if the rotor shaft is vertical, then $\cos \epsilon = \lambda/\sqrt{\mu^2 + \lambda^2}$ (see ref. 1). The expressions for ground effect in hover are generalized to forward flight by using $(z_g/\cos \epsilon)$ in place of z_g. No ground effect correction is applied if the wake is directed upward ($\cos \epsilon \leq 0$), or if $z_g/\cos \epsilon > 1.5D$. From $z_g/D\cos \epsilon$, the ground effect factor $f_g = \kappa_g^{-3/2}$ is calculated. Then

$$(\lambda_i)_{\text{IGE}} = f_g (\lambda_i)_{\text{OGE}}$$

is the effective ideal induced velocity.

Several empirical ground effect models are implemented: from Cheeseman and Bennett (ref. 1, basic model and using blade-element (BE) theory to incorporate influence of thrust), from Law (ref. 2), from Hayden (ref. 3), a curve fit of the interpolation from Zbrozek (ref. 4), and from University of

Maryland (ref. 5):

$$
f_g = \begin{cases}
\left[1 - \dfrac{1}{(4z_g/R)^2} \right]^{3/2} & \text{Cheeseman and Bennett} \\[3ex]
\left[1 + 1.5\,\dfrac{\sigma a \lambda_i}{4C_T}\,\dfrac{1}{(4z_g/R)^2} \right]^{-3/2} & \text{Cheeseman and Bennett (BE)} \\[3ex]
\left[\dfrac{1.0991 - 0.1042/(z_g/D)}{1 + (C_T/\sigma)(0.2894 - 0.3913/(z_g/D))} \right]^{3/2} & \text{Law} \\[3ex]
\left[0.9926 + \dfrac{0.03794}{(z_g/2R)^2} \right]^{-1} & \text{Hayden} \\[3ex]
\left[0.9122 + \dfrac{0.0544}{(z_g/R)\sqrt{C_T/\sigma}} \right]^{-3/2} & \text{Zbrozek} \\[3ex]
\left[0.146 + 2.090\left(\tfrac{z_g}{R}\right) - 2.068\left(\tfrac{z_g}{R}\right)^2 + 0.932\left(\tfrac{z_g}{R}\right)^3 - 0.157\left(\tfrac{z_g}{R}\right)^4 \right] & \text{University of Maryland}
\end{cases}
$$

These equations break down at small height above the ground, and so are restricted to $z_g/D \geq 0.15$; however, the database for ground effect extends only to about $z/D = 0.3$, except for the Maryland data, which goes to $z/D = 0.1$. Also, $f_g \leq 1$ is required. Figure 12-2 shows $T/T_\infty = \kappa_g = f_g^{-2/3}$ as a function of z/R for these models ($C_T/\sigma = 0.05, 0.10, 0.15$), compared with test data from several sources.

Alternatively, a table can be specified for either $\kappa_g = T/T_\infty$ or $f_g = P/P_\infty$, as a function of C_T/σ and $h/D = z_g/2R$; or as a function of C_T/σ, h/D, and M_{tip}.

The influence of the ground on tiltrotor power required is stronger than for an isolated rotor. This further reduction of power is probably due to a reduction of wing and fuselage download when operating near the ground. For quad tiltrotors, an upload on the airframe has been measured at low heights. These effects can be modeled by using an effective distance above the ground: $z_e = C_g z_g$, with typically $C_g = 0.5$ for a tiltrotor.

12-4.1.3 Inflow Gradient

As a simple approximation to nonuniform induced velocity distribution, a linear variation over the disk is used: $\Delta\lambda = \lambda_x r \cos\psi + \lambda_y r \sin\psi$. There are contributions to $\Delta\lambda$ from forward flight and from hub moments, which influence the relationship between flapping and cyclic. The linear inflow variation caused by forward flight is $\Delta\lambda_f = \lambda_i(\kappa_x r \cos\psi + \kappa_y r \sin\psi)$, where λ_i is the mean inflow. Typically κ_x is positive, and roughly 1 at high speed, and κ_y is smaller in magnitude and negative. Both κ_x and κ_y must be zero in hover. Based on references 6–9, the following models are considered:

$$\text{Coleman and Feingold:} \quad \kappa_{x0} = f_x \frac{15\pi}{32}\tan\chi/2 = f_x \frac{15\pi}{32}\frac{\mu}{\sqrt{\mu^2 + \lambda^2} + |\lambda|}$$

$$\kappa_{y0} = -f_y 2\mu$$

$$\text{White and Blake:} \quad \kappa_{x0} = f_x \sqrt{2}\sin\chi = f_x\sqrt{2}\frac{\mu}{\sqrt{\mu^2 + \lambda^2}}$$

$$\kappa_{y0} = -2f_y\mu$$

where $\tan\chi = |\lambda|/\mu$ is the wake angle. Extending these results to include sideward velocity gives $\kappa_x = (\kappa_{x0}\mu_x + \kappa_{y0}\mu_y)/\mu$ and $\kappa_y = (-\kappa_{x0}\mu_y + \kappa_{y0}\mu_x)/\mu$. For flexibility, the empirical factors f_x and f_y
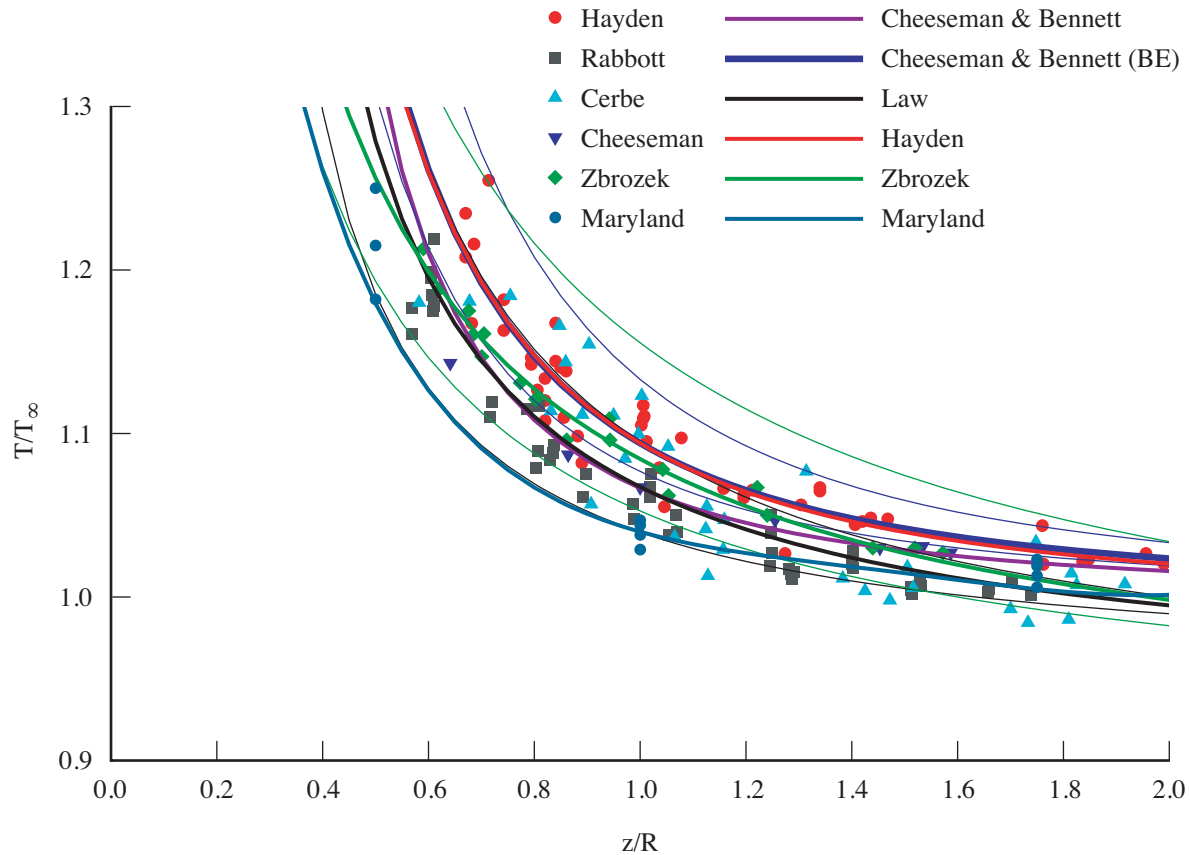
Figure 12-2. Ground effect models (hover).

have been introduced (values of 1.0 give the baseline model). There is also an inflow variation produced by any net aerodynamic moment on the rotor disk, which can be evaluated using a differential form of momentum theory:

$$\Delta\lambda_m = \frac{f_m}{\sqrt{\mu^2 + \lambda^2}} \left(-2C_{My}r\cos\psi + 2C_{Mx}r\sin\psi\right) = \lambda_{xm}r\cos\psi + \lambda_{ym}r\sin\psi$$

including empirical factor f_m. Note that the denominator of the hub moment term is zero for a hovering rotor at zero thrust; so this inflow contribution should not be used for cases of low speed and low thrust.

12-4.2   Rotor Forces

When direct control of the rotor thrust magnitude is used, the rotor collective pitch angle $\theta_{0.75}$ must be calculated from the thrust C_T/σ. If the commanded variable is the collective pitch angle, then it is necessary to calculate the rotor thrust, resulting in more computation, particularly since all quantities depending on the thrust (inflow, induced power factor, and mean drag coefficient) are also unknown. There may be flight states where the commanded thrust can not be produced by the rotor, even with stall neglected in the section aerodynamics. This condition manifests as an inability to solve for the collective pitch given the thrust. In this circumstance the trim method can be changed so the required or specified thrust is an achievable value, or commanded collective pitch control can be used.

   Cyclic control consists of tip-path plane command, requiring calculation of the rotor cyclic pitch angles from the flapping; or no-feathering plane command, requiring calculation of the tip-path plane tilt from the cyclic control angles. The longitudinal tip-path plane tilt is β_c (positive forward) and the lateral tilt is β_s (positive toward retreating side). The longitudinal cyclic pitch angle is θ_s (positive aft), and the lateral cyclic pitch angle is θ_c (positive toward retreating side). Tip-path plane command is appropriate for main-rotors. For rotors with no cyclic pitch, no-feathering plane command must be used.

   The forces acting on the hub are the thrust T, drag H, and side force Y (positive in z-, x-, y-axis directions, respectively). The aerodynamic analysis is conducted for a clockwise rotating rotor, with appropriate sign changes for lateral velocity, flapping, and force. The analysis is conducted in dimensionless form, based on the actual radius and rotational speed of the flight state. The inplane hub forces are produced by tilt of the thrust vector with the tip-path plane, plus forces in the tip-path plane, and profile terms (produced by the blade drag coefficient). The orientation of the tip-path axes relative to the shaft axes is then $C^{PS} = X_{r\beta_s} Y_{-\beta_c}$. Then

$$\begin{pmatrix} C_H \\ C_Y \\ C_T \end{pmatrix} = C^{SP} \begin{pmatrix} 0 \\ 0 \\ C_T/C_{33}^{SP} \end{pmatrix} + \begin{pmatrix} C_{Htpp} \\ rC_{Ytpp} \\ 0 \end{pmatrix} + \begin{pmatrix} C_{Ho} \\ rC_{Yo} \\ 0 \end{pmatrix}$$

The inplane forces relative to the tip-path plane can be neglected, or calculated by blade element theory. Note that with thrust and tip-path plane command and C_{Htpp} and C_{Ytpp} neglected, it is not necessary to solve for the rotor collective and cyclic pitch angles. In general the inplane forces relative to the tip-path plane are not zero, and may be significant, as for a rotor with large flap stiffness. Figures 12-3a and b show, respectively, the tip-path plane tilt and thrust vector tilt with cyclic pitch control (no-feathering plane tilt), as functions of flap stiffness (frequency), for several rotor thrust values. The difference between tip-path plane tilt (fig. 12-3a) and thrust vector tilt (fig. 12-3b) is caused by tilt of the thrust vector relative to the tip-path plane.

   The profile inplane forces can be obtained from simplified equations, or calculated by blade element theory. The simplified method uses:

$$\begin{pmatrix} C_{Ho} \\ C_{Yo} \end{pmatrix} = \frac{\sigma}{8} c_{dmean} F_H \begin{pmatrix} \mu_x/\mu \\ -\mu_y/\mu \end{pmatrix}$$

where the mean drag coefficient c_{dmean} is from the profile power calculation. The function F_H accounts for the increase of the blade section velocity with rotor edgewise and axial speed: $C_{Ho} = \int \frac{1}{2}\sigma c_d U(r \sin\psi + \mu)dr = \int \frac{1}{2}\sigma c_d (u_T^2 + u_R^2 + u_P^2)^{1/2}(r \sin\psi + \mu)dr$; so (from ref. 10)

$$F_H = 4\frac{1}{2\pi} \int_0^{2\pi} \int_0^1 \left((r + \mu \sin\psi)^2 + (\mu \cos\psi)^2 + \mu_z^2\right)^{1/2} (r \sin\psi + \mu) \, dr \, d\psi$$

$$\cong \sqrt{1 + V^2} \left(3\mu + \frac{1}{4}\mu^3 \frac{V^2 - 1}{(1 + V^2)^2}\right) + \left(\mu\mu_z^2 + \frac{3}{4}\mu^3\right) \ln \left[\frac{\sqrt{1 + V^2} + 1}{V}\right]$$

with $V^2 = \mu^2 + \mu_z^2$.

12-4.3   Blade Element Theory

   Blade element theory is the basis for the solution for the collective and cyclic pitch angles (or thrust and flap angles) and evaluation of the rotor inplane hub forces. The section aerodynamics are described
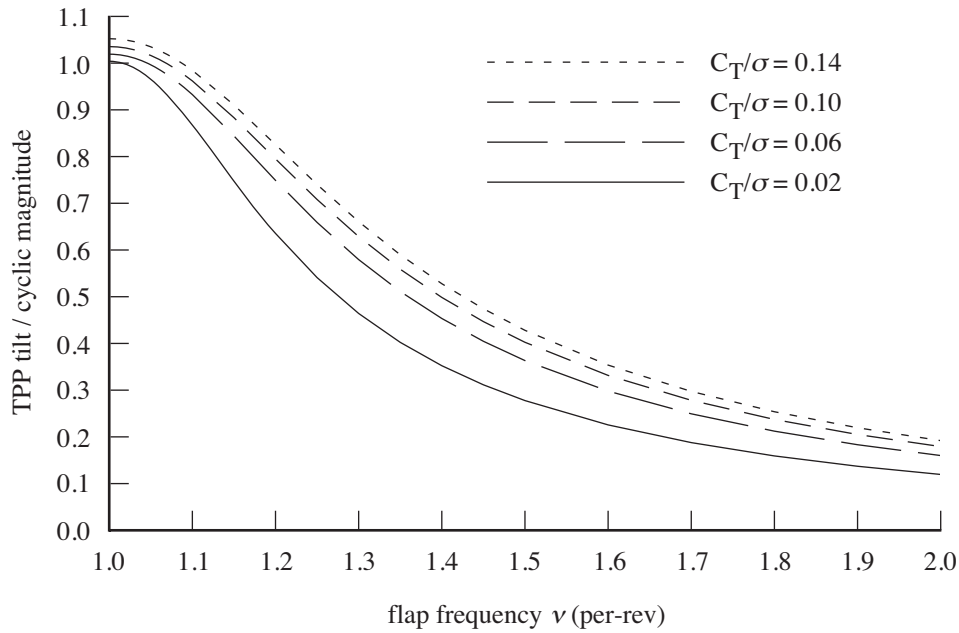
Figure 12-3a. Tip-path plane tilt with cyclic pitch.

by lift varying linearly with angle of attack, $c_\ell = c_{\ell\alpha}\alpha$ (no stall), and a constant mean drag coefficient $c_{d\text{mean}}$ (from the profile power calculation). The analysis is conducted in dimensionless form (based on density ρ, rotor rotational speed Ω, and blade radius R of the flight state). So in the following σ, ν, and γ are for the actual ρ, Ω, and R; and $a = 5.7$ is the lift-curve slope used in the Lock number γ. The blade section aerodynamic environment is described by the three components of velocity, from which the yaw and inflow angles are obtained, and then the angle of attack:

$$u_T = r + \mu_x \sin\psi + \mu_y \cos\psi$$

$$u_R = \mu_x \cos\psi - \mu_y \sin\psi$$

$$u_P = \lambda + r(\dot{\beta} + \dot{\alpha}_x \sin\psi - \dot{\alpha}_y \cos\psi) + u_R\beta$$

$$U^2 = u_T^2 + u_P^2$$

$$\cos\Lambda = U/\sqrt{u_T^2 + u_P^2 + u_R^2}$$

$$\phi = \tan^{-1} u_P/u_T$$

$$\alpha = \theta - \phi$$

In reverse flow ($|\alpha| > 90$), $\alpha \leftarrow \alpha - 180 \,\text{sign}\alpha$, and then $c_\ell = c_{\ell\alpha}\alpha$ still (airfoil tables are not used). The blade pitch consists of collective, cyclic, twist, and pitch-flap coupling terms. The flap motion is rigid rotation about a hinge with no offset, and only coning and once-per-revolution terms are considered:

$$\theta = \theta_{0.75} + \theta_{tw} + \theta_c \cos\psi + \theta_s \sin\psi - K_P\beta$$

$$\beta = \beta_0 + \beta_c \cos\psi + \beta_s \sin\psi$$

where $K_P = \tan\delta_3$. The twist is measured relative to $0.75R$; $\theta_{tw} = \theta_L(r - 0.75)$ for linear twist. The mean inflow is $\lambda_0 = \kappa\lambda_i$, using the induced velocity factor κ from the induced power model. The inflow includes gradients caused by edgewise flight and hub moments:

$$\lambda = \mu_z + \lambda_0(1 + \kappa_x r \cos\psi + \kappa_y r \sin\psi) + \Delta\lambda_m$$

$$= \mu_z + \lambda_0(1 + \kappa_x r \cos\psi + \kappa_y r \sin\psi) + \frac{f_m}{\sqrt{\mu^2 + \lambda^2}} \left(-2C_{My}r \cos\psi + 2C_{Mx}r \sin\psi\right)$$
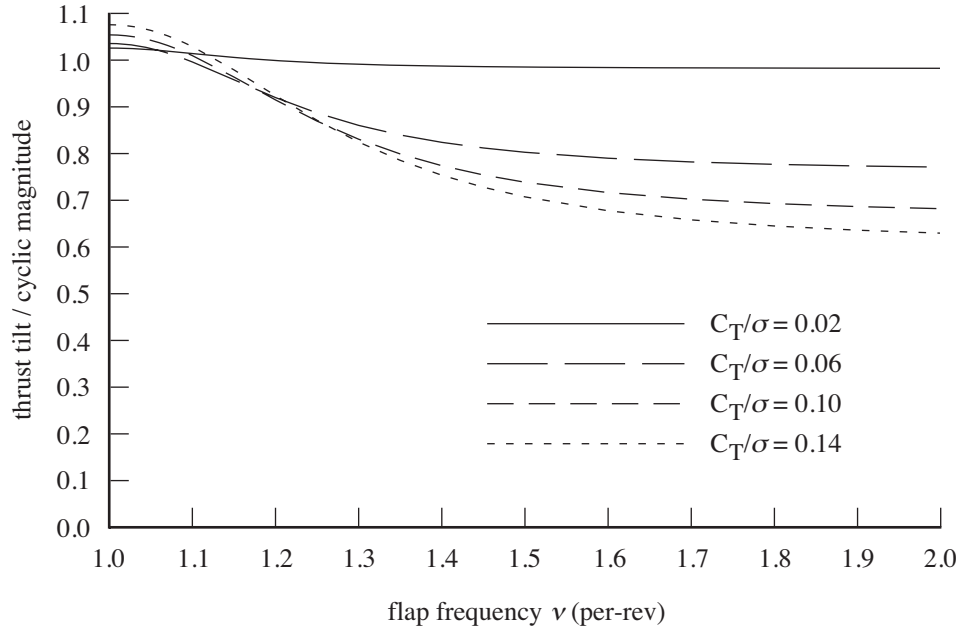
Figure 12-3b. Thrust vector tilt with cyclic pitch.

From the hub moments

$$\begin{pmatrix} -C_{My} \\ C_{Mx} \end{pmatrix} = \frac{\sigma a}{2} \frac{\nu^2 - 1}{\gamma} \begin{pmatrix} \beta_c \\ \beta_s \end{pmatrix}$$

the inflow gradient is

$$\Delta \lambda_m = \left(\frac{f_m}{\sqrt{\mu^2 + \lambda^2}} \frac{\sigma a}{8} \right) \frac{\nu^2 - 1}{\gamma/8} \left(r\beta_c \cos\psi + r\beta_s \sin\psi \right) = K_m \frac{\nu^2 - 1}{\gamma/8} \left(r\beta_c \cos\psi + r\beta_s \sin\psi \right)$$

The constant K_m is associated with a lift-deficiency function:

$$C = \frac{1}{1 + K_m} = \frac{1}{1 + f_m \sigma a / \left(8\sqrt{\mu^2 + \lambda^2} \right)}$$

The blade chord is $c(r) = c_{\text{ref}} \hat{c}(r)$, where c_{ref} is the thrust-weighted chord (chord at $0.75R$ for linear taper). Yawed flow effects increase the section drag coefficient, hence $c_d = c_{d\text{mean}} / \cos\Lambda$. The section forces in velocity axes and shaft axes are

$$L = \frac{1}{2}\rho U^2 c c_\ell \qquad\qquad\qquad F_z = L\cos\phi - D\sin\phi = \frac{1}{2}\rho U c (c_\ell u_T - c_d u_P)$$

$$D = \frac{1}{2}\rho U^2 c c_d \qquad\qquad\qquad F_x = L\sin\phi + D\cos\phi = \frac{1}{2}\rho U c (c_\ell u_P + c_d u_T)$$

$$R = \frac{1}{2}\rho U^2 c c_r = D\tan\Lambda \qquad\qquad F_r = -\beta F_z + R = -\beta F_z + \frac{1}{2}\rho U c c_d u_R$$

These equations for the section environment and section forces are applicable to high inflow (large μ_z),

sideward flight (μ_y), and reverse flow ($u_T < 0$). The total forces on the rotor hub are

$$T = N \int F_z \, dr$$

$$H = N \int F_x \sin\psi + F_r \cos\psi \, dr$$

$$Y = N \int -F_x \cos\psi + F_r \sin\psi \, dr$$

with an average over the rotor azimuth implied, along with the integration over the radius. Lift forces are integrated from the root cutout r_{root} to the tip loss factor B. Drag forces are integrated from the root cutout to the tip.

In coefficient form (forces divided by $\rho A V_{\text{tip}}^2$) the rotor thrust and inplane forces are:

$$C_T = \sigma \int \widehat{F}_z \, dr \qquad\qquad \widehat{F}_z = \frac{1}{2}\hat{c}U(c_\ell u_T - c_d u_P)$$

$$C_H = \sigma \int \widehat{F}_x \sin\psi + \widehat{F}_r \cos\psi \, dr \qquad \widehat{F}_x = \frac{1}{2}\hat{c}U(c_\ell u_P + c_d u_T)$$

$$C_Y = \sigma \int -\widehat{F}_x \cos\psi + \widehat{F}_r \sin\psi \, dr \qquad \widehat{F}_r = -\beta\widehat{F}_z + \frac{1}{2}\hat{c}U c_d u_R$$

(and the sign of C_Y is changed for a clockwise rotating rotor). The terms $\Delta\widehat{F}_x \cong \widehat{F}_z\dot{\beta}$ and $\Delta\widehat{F}_r = -\widehat{F}_z\beta$ produce tilt of the thrust vector with the tip-path plane ($C_H = -C_T\beta_c$ and $C_Y = -C_T\beta_s$), which are accounted for directly. The section drag coefficient c_d produces the profile inplane forces. The approximation $u_P \cong \mu_z$ is consistent with the simplified method (using the function F_H), hence

$$\widehat{F}_{xo} = \frac{1}{2}\hat{c}U_0 c_d u_T \qquad C_{Ho} = \sigma \int \widehat{F}_{xo}\sin\psi + \widehat{F}_{ro}\cos\psi \, dr = \frac{\sigma}{2}\int \hat{c}U_0 c_d(r\sin\psi + \mu_x)\, dr$$

$$\widehat{F}_{ro} = \frac{1}{2}\hat{c}U_0 c_d u_R \qquad C_{Yo} = \sigma \int -\widehat{F}_{xo}\cos\psi + \widehat{F}_{ro}\sin\psi \, dr = -\frac{\sigma}{2}\int \hat{c}U_0 c_d(r\cos\psi + \mu_y)\, dr$$

where $U_0^2 = u_T^2 + \mu_z^2$, and $c_d = c_{d\text{mean}}/\cos\Lambda$. Using blade element theory to evaluate C_{Ho} and C_{Yo} accounts for the planform ($\hat{c}$) and root cutout. Using the function F_H implies a rectangular blade and no root cutout (plus at most a 1% error approximating the exact integration). The remaining terms in the section forces produce the inplane loads relative to the tip-path plane:

$$\widehat{F}_{xi} = \widehat{F}_x - \widehat{F}_z\dot{\beta} - \widehat{F}_{xo} = \frac{1}{2}\hat{c}U c_\ell(u_P - u_T\dot{\beta}) + \frac{1}{2}\hat{c}U c_d((1 - U_0/U)u_T + u_P\dot{\beta})$$

$$\widehat{F}_{ri} = \widehat{F}_r + \widehat{F}_z\beta - \widehat{F}_{ro} = \frac{1}{2}\hat{c}U c_d(1 - U_0/U)u_R$$

$$C_{H\text{tpp}} = \sigma \int \widehat{F}_{xi}\sin\psi + \widehat{F}_{ri}\cos\psi \, dr$$

$$C_{Y\text{tpp}} = \sigma \int -\widehat{F}_{xi}\cos\psi + \widehat{F}_{ri}\sin\psi \, dr$$

(including small profile terms from $U_0 \neq U$).

Evaluating these inplane forces requires the collective and cyclic pitch angles and the flapping motion. The thrust equation must be solved for the rotor collective pitch or the rotor thrust. The relationship between cyclic pitch and flapping is defined by the rotor flap dynamics. The flap motion is

rigid rotation about a central hinge, with a flap frequency $\nu > 1$ for articulated or hingeless rotors. The flapping equation of motion is

$$\ddot{\beta} + \nu^2\beta + 2\dot{\alpha}_y \sin\psi + 2\dot{\alpha}_x \cos\psi = \frac{\gamma}{a}\int \widehat{F}_z r\, dr + (\nu^2 - 1)\beta_p$$

including precone angle β_p; the Lock number $\gamma = \rho a c_{\mathrm{ref}} R^4/I_b$. This equation is solved for the mean (coning) and 1/rev (tip-path plane tilt) flap motion:

$$\nu^2\beta_0 = \frac{\gamma}{a}\int \widehat{F}_z r\, dr + (\nu^2 - 1)\beta_p$$

$$(\nu^2 - 1)\begin{pmatrix}\beta_c \\ \beta_s\end{pmatrix} = \frac{\gamma}{a}\int \widehat{F}_z r\, dr \begin{pmatrix}2\cos\psi \\ 2\sin\psi\end{pmatrix} + \begin{pmatrix}2\dot{\alpha}_x \\ 2\dot{\alpha}_y\end{pmatrix}$$

with an average over the rotor azimuth implied. The solution for the coning is largely decoupled by introducing the thrust:

$$\nu_0^2\beta_0 = \frac{\gamma}{8}\frac{6C_T}{\sigma a} + (\nu_0^2 - 1)\beta_p + \frac{\gamma}{a}\int \widehat{F}_z(r - 3/4)\, dr$$

A separate flap frequency ν_0 is used for coning, in order to model teetering and gimballed rotors. For an articulated rotor, $\beta_p = 0$ should be used.

The thrust and flapping equations of motion that must be solved are:

$$E_t = \frac{6}{a}\int \widehat{F}_z\, dr - \frac{6C_T}{\sigma a}$$

$$\begin{pmatrix}E_c \\ E_s\end{pmatrix} = \frac{8}{a}\int \widehat{F}_z r\, dr \begin{pmatrix}2\cos\psi \\ 2\sin\psi\end{pmatrix} - \frac{\nu^2 - 1}{\gamma/8}\begin{pmatrix}\beta_c \\ \beta_s\end{pmatrix} + \frac{16}{\gamma}\begin{pmatrix}\dot{\alpha}_x \\ \dot{\alpha}_y\end{pmatrix}$$

The solution v such that $E(v) = 0$ is required. The variables are $v = (\theta_{0.75}\,\theta_c\,\theta_s)^T$ for thrust and tip-path plane command; $v = (\theta_{0.75}\,\beta_c\,\beta_s)^T$ for thrust and no-feathering plane command; $v = (C_T/\sigma\,\theta_c\,\theta_s)^T$ for collective pitch and tip-path plane command; $v = (C_T/\sigma\,\beta_c\,\beta_s)^T$ for collective pitch and no-feathering plane command. Note that since $c_\ell = c_{\ell\alpha}\alpha$ is used (no stall), these equations are linear in θ. However, if $\partial T/\partial\theta_{0.75}$ is small, the solution may not produce a reasonable collective for commanded thrust. A Newton–Raphson solution method is used: from $E(v_{n+1}) \cong E(v_n) + (dE/dv)(v_{n+1} - v_n) = 0$, the iterative solution is

$$v_{n+1} = v_n - C\,E(v_n)$$

where $C = f(dE/dv)^{-1}$, including the relaxation factor f. The derivative matrix dE/dv is obtained by numerical perturbation. Convergence of the Newton–Raphson iteration is tested in terms of $|E| < \epsilon$ for each equation, where ϵ is an input tolerance.

12–5 Power

The rotor power consists of induced, profile, parasite, and adjustment terms: $P = P_i + P_o + P_p + P_x$. The parasite power (including climb/descent power for the aircraft) is obtained from the wind axis drag force: $P_p = -XV = (v^F)^T F^F$.

The induced power is calculated from the ideal power: $P_i = \kappa P_{\mathrm{ideal}} = \kappa f_D T v_{\mathrm{ideal}}$. The empirical factor κ accounts for the effects of nonuniform inflow, non-ideal span loading, tip losses, swirl, blockage, and other phenomena that increase the induced power losses ($\kappa > 1$). For a ducted fan, $f_D = f_W/2\eta_D$ is

introduced. The induced power at zero thrust is zero in this model (or accounted for as a profile power increment). If κ is deduced from an independent calculation of induced power, nonzero P_i at low thrust will be reflected in large κ values.

The profile power is calculated from a mean blade drag coefficient: $P_o = \rho A(\Omega R)^3 C_{Po}$, $C_{Po} = (\sigma/8)c_{d\text{mean}}F_P$. The function $F_P(\mu, \mu_z)$ accounts for the increase of the blade section velocity with rotor edgewise and axial speed: $C_{Po} = \int \frac{1}{2}\sigma c_d U^3 dr = \int \frac{1}{2}\sigma c_d (u_T^2 + u_R^2 + u_P^2)^{3/2} dr$; so (from ref. 10)

$$F_P = 4\frac{1}{2\pi}\int_0^{2\pi}\int_0^1 \left((r + \mu\sin\psi)^2 + (\mu\cos\psi)^2 + \mu_z^2\right)^{3/2} dr\, d\psi$$

$$\cong \sqrt{1+V^2}\left(1 + \frac{5}{2}V^2 + \frac{3}{8}\mu^2\frac{4 + 7V^2 + 4V^4}{(1+V^2)^2} - \frac{9}{16}\frac{\mu^4}{1+V^2}\right)$$

$$+ \left(\frac{3}{2}\mu_z^4 + \frac{3}{2}\mu_z^2\mu^2 + \frac{9}{16}\mu^4\right)\ln\left[\frac{\sqrt{1+V^2}+1}{V}\right]$$

with $V^2 = \mu^2 + \mu_z^2$. This expression is exact when $\mu = 0$, and $f_P \sim 4V^3$ for large V.

Two performance methods are implemented, the energy method and the table method. The induced power factor and mean blade drag coefficient are obtained from equations with the energy method, or from tables with the table method. Optionally κ and $c_{d\text{mean}}$ can be specified for each flight state, superseding the performance method values.

Rotor power adjustments (P_x) are associated with wing interference, propulsive force efficiency, and climb efficiency.

12-5.1   Energy Performance Method

12-5.1.1   Induced Power

The induced power is calculated from the ideal power: $P_i = \kappa P_{\text{ideal}} = \kappa f_D T v_{\text{ideal}}$. Reference values of κ are specified for hover, axial cruise (propeller), and edgewise cruise (helicopter): κ_{hover}, κ_{prop}, and κ_{edge}. Two models are implemented: constant model and standard model. The constant model uses $\kappa = \kappa_{\text{hover}}$ if $\mu = \mu_z = 0$; or $\kappa = \kappa_{\text{prop}}$ if $|\mu| < 0.1|\mu_z|$; or $\kappa = \kappa_{\text{edge}}$ otherwise.

The standard model calculates an axial flow factor κ_{axial} from κ_{hover}, κ_{climb}, and κ_{prop}. Let $\Delta = C_T/\sigma - (C_T/\sigma)_{\text{ind}}$. For hover and low-speed axial climb, including a variation with thrust, the inflow factor is

$$\kappa_h = \kappa_{\text{hover}} + k_{h1}\Delta_h + k_{h2}|\Delta_h|^{X_{h2}} + (\kappa_{\text{climb}} - \kappa_{\text{hover}})\frac{2}{\pi}\tan^{-1}\left[((|\mu_z|/\lambda_h)/M_{\text{axial}})^{X_{\text{axial}}}\right]$$

where $|\mu_z|/\lambda_h = M_{\text{axial}}$ is the midpoint of the transition between hover and climb, and X_{axial} is large for a fast transition. Figure 12-4 illustrates κ in hover (with a minimum value). Figure 12-5 shows the behavior of this function for a helicopter in climb ($X_{\text{axial}} = 0.65$). A polynomial describes the variation with axial velocity, scaled so $\kappa = \kappa_h$ at $\mu_z = 0$ and $\kappa = \kappa_p$ at $\mu_z = \mu_{z\text{prop}}$. Including variations with thrust and shaft angle and propulsive force:

$$\kappa_p = \kappa_{\text{prop}} + k_{p1}\Delta_p + k_{p2}|\Delta_p|^{X_{p2}} + k_{p\alpha}\mu_z^2|\mu|^{X_{p\alpha}} + k_{px}\left((|C_T/\sigma|/\mu_z^2)^{X_{px}} - (|C_T/\sigma|_{\text{ind}}/\mu_{z\text{prop}}^2)^{X_{px}}\right)$$

$$S = (\kappa_p - (\kappa_h + k_{a1}\mu_{z\text{prop}}))/(k_{a2}\mu_{z\text{prop}}^2 + k_{a3}|\mu_{z\text{prop}}|^{X_a})$$

$$\kappa_{\text{axial}} = \kappa_h + k_{a1}|\mu_z| + S(k_{a2}\mu_z^2 + k_{a3}|\mu_z|^{X_a})$$
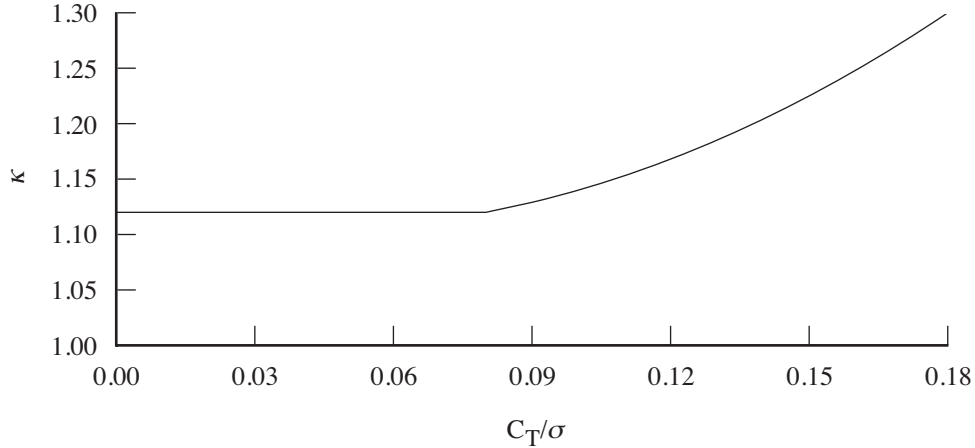
Figure 12-4. Induced power factor for rotor in hover.

with $S = 0$ if $k_{a2} = k_{a3} = 0$ (not scaled); and $\kappa_{\mathrm{axial}} = \kappa_h$ if $\mu_{z\mathrm{prop}} = 0$ or $\mu_z < \mu_{z\mathrm{tran}}$.

Finally the induced power factor κ is calculated from κ_{axial} and κ_{edge}. A polynomial describes the variation with edgewise advance ratio, scaled so $\kappa = \kappa_{\mathrm{axial}}$ at $\mu = 0$, and $\kappa = \kappa_e$ or $\kappa_e + \kappa_{\mathrm{axial}}$ at $\mu = \mu_{\mathrm{edge}}$. Thus the induced power factor is

$$\kappa_e = f_\alpha f_{\mathrm{off}}(\kappa_{\mathrm{edge}} + k_{t1}\Delta_t + k_{t2}|\Delta_t|^{X_{t2}})$$
$$S = (\kappa_x - k_{e1}\mu_{\mathrm{edge}})/(k_{e2}\mu_{\mathrm{edge}}^2 + k_{e3}|\mu_{\mathrm{edge}}|^{X_e})$$
$$\kappa = \kappa_{\mathrm{axial}} + k_{e1}\mu + S(k_{e2}\mu^2 + k_{e3}\mu^{X_e})$$

where $\kappa_x = \kappa_e - \kappa_{\mathrm{axial}}$ for $\kappa = \kappa_e$ at $\mu = \mu_{\mathrm{edge}}$; or $\kappa_x = \kappa_e$ for $\kappa = \kappa_e + \kappa_{\mathrm{axial}}$ at $\mu = \mu_{\mathrm{edge}}$; with $S = 0$ if $k_{e2} = k_{e3} = 0$ (not scaled); and $\kappa = \kappa_{\mathrm{axial}}$ if $\mu_{\mathrm{edge}} = 0$. The function $f_\alpha = 1 - k_{e\alpha}\mu_z$ accounts for the influence of angle of attack (μ_z/μ) or rotor drag (C_X). The function $f_{\mathrm{off}} = 1 - k_{o1}(1 - e^{-k_{o2}o_x})$ accounts for the influence of lift offset, $o_x = rM_x/TR = (K_{\mathrm{hub}}/TR)\beta_s$. Figure 12-6 illustrates κ in edgewise flight. Minimum and maximum values of the induced power factor, κ_{min} and κ_{max}, are also specified.

12-5.1.2   Profile Power

The profile power is calculated from a mean blade drag coefficient: $C_{Po} = (\sigma/8)c_{d\mathrm{mean}}F_P$. Since the blade mean lift coefficient is $c_\ell \cong 6C_T/\sigma$, the drag coefficient is estimated as a function of blade loading C_T/σ (using thrust-weighted solidity). With separate estimates of the basic, stall, and compressibility drag, the mean drag coefficient is:

$$c_{d\mathrm{mean}} = \chi S\left(c_{d\mathrm{basic}} + c_{d\mathrm{stall}} + c_{d\mathrm{comp}}\right)$$

where χ is a technology factor. The factor $S = (Re_{\mathrm{ref}}/Re)^{X_{\mathrm{Re}}}$ accounts for Reynolds number effects on the drag coefficient; typically $X_{\mathrm{Re}} = 0.2$ for a turbulent boundary layer. Re is based on the thrust-weighted chord, $0.75V_{\mathrm{tip}}$, and the flight state; Re_{ref} corresponds to the input c_d information. The following models are implemented for the basic drag:

a) Array model: The basic drag $c_{d\mathrm{basic}}$ is input as a function of C_T/σ; the array is linearly interpolated.
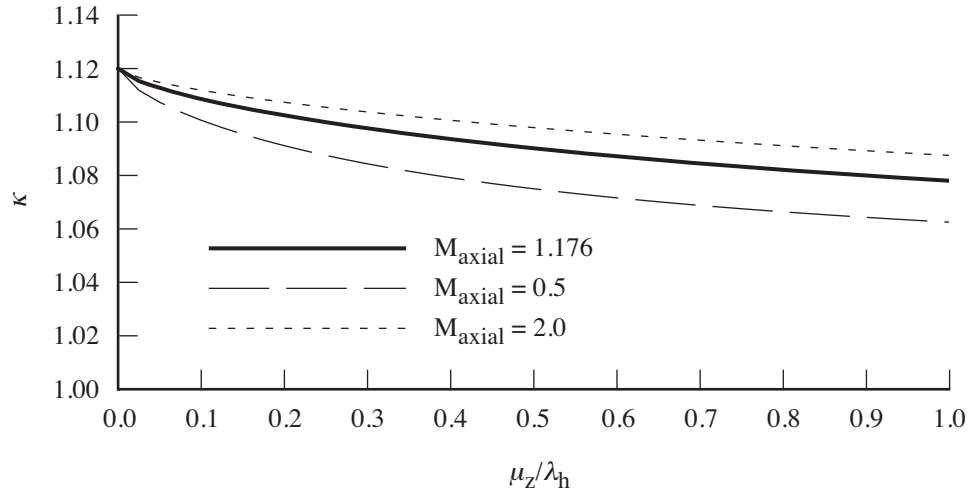
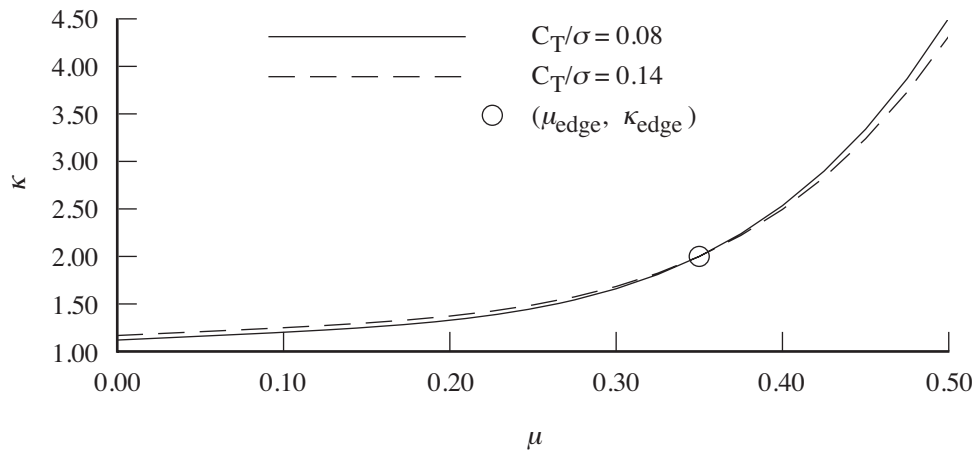Figure 12-5. Induced power factor for rotor in axial flight.

Figure 12-6. Induced power factor for rotor in edgewise flight.

b) Equation model: The basic drag $c_{d\text{basic}}$ is a quadratic function of C_T/σ, plus an additional term allowing faster growth at high (sub-stall) angles of attack. Let $\Delta = |C_T/\sigma - (C_T/\sigma)_{D\text{min}}|$, where $(C_T/\sigma)_{D\text{min}}$ corresponds to the minimum drag and $\Delta_{\text{sep}} = |C_T/\sigma| - (C_T/\sigma)_{\text{sep}}$. Values of the basic drag equation are specified for helicopter (hover and edgewise) and propeller (axial climb and cruise) operation:

$$c_{dh} = d_{0\text{hel}} + d_{1\text{hel}}\Delta + d_{2\text{hel}}\Delta^2 + d_{\text{sep}}\Delta_{\text{sep}}^{X_{\text{sep}}}$$

$$c_{dp} = d_{0\text{prop}} + d_{1\text{prop}}\Delta + d_{2\text{prop}}\Delta^2 + d_{\text{sep}}\Delta_{\text{sep}}^{X_{\text{sep}}} + d_{p\alpha}\mu_z^2|\mu|^{X_{p\alpha}}$$

The separation term is present only if $\Delta_{\text{sep}} > 0$. The helicopter and propeller values are interpolated as a function of μ_z:

$$c_{d\text{basic}} = c_{dh} + (c_{dp} - c_{dh})\frac{2}{\pi}\tan^{-1}(|\mu_z|/\lambda_h) + d_{f1}\mu + d_{f2}\mu^{X_f} + d_{z1}\mu_z + d_{z2}|\mu_z|^{X_z}$$

so $|\mu_z|/\lambda_h = 1$ is the midpoint of the transition. The last terms are the effect of edgewise and axial flow.

The stall drag increment represents the rise of profile power caused by the occurrence of significant stall on the rotor disk. Let $\Delta_s = |C_T/\sigma| - (f_s/f_\alpha f_{\text{off}})(C_T/\sigma)_s$ (f_s is an input factor). The function $f_\alpha = 1 - d_{s\alpha}\mu_z$ accounts for the influence of angle of attack (μ_z/μ) or rotor drag (C_X). The function $f_{\text{off}} = 1 - d_{o1}(1 - e^{-d_{o2}o_x})$ accounts for the influence of lift offset, $o_x = rM_x/TR = (K_{\text{hub}}/TR)\beta_s$. Then $c_{d\text{stall}} = d_{s1}\Delta_s^{X_{s1}} + d_{s2}\Delta_s^{X_{s2}}$ (zero if $\Delta_s \leq 0$). The blade loading at which the stall affects the entire rotor power, $(C_T/\sigma)_s$, is an input function of the velocity ratio $V = \sqrt{\mu^2 + \mu_z^2}$.

The compressibility drag increment depends on the advancing tip Mach number M_{at}, and the tip airfoil thickness-to-chord ratio τ. The following models are implemented:

a) Drag divergence model: Let $\Delta M = M_{at} - M_{dd}$, where M_{dd} is the drag divergence Mach number of the tip section. Then the compressibility increment in the mean drag coefficient is

$$c_{d\text{comp}} = d_{m1}\Delta M + d_{m2}\Delta M^{X_m}$$

(ref. 11). The drag increment is only used if $\Delta M > 0$. M_{dd} is a function of the advancing tip lift coefficient, $c_{\ell(1,90)}$. The advancing tip lift is estimated from $\alpha_{(1,90)} = (\theta_{.75} + 0.25\theta_L + \theta_s - (\lambda - \beta_c)/(1 + \mu)) \cong 1.6(1 - 2.97\mu + 2.21\mu^2)(6C_T/\sigma a) + 0.25\theta_{tw}$ (zero above $\mu = 0.6$). Then the Korn expression (ref. 12) gives M_{dd} for small lift coefficient:

$$M_{dd} = \kappa_A - \kappa|c_\ell| - \tau = M_{dd0} - \kappa|c_\ell|$$

where M_{dd0} is the drag divergence Mach number at zero lift, and typically $\kappa = 0.16$.

b) Similarity model: From transonic small-disturbance theory (refs. 13–14), the scaled wave drag must be a function only of $K_1 = (M_{at}^2 - 1)/[M_{at}^2\tau(1 + \gamma)]^{2/3}$. An approximation for the wave drag increment is

$$\Delta c_d = \frac{\tau^{5/3}}{[M_{at}^2(1 + \gamma)]^{1/3}} D(K_1) = \frac{\tau^{5/3}}{[M_{at}^2(1 + \gamma)]^{1/3}} 1.774(K_1 + 1.674)^{5/2}$$

(constant for $K_1 > -0.2$). Integration of Δc_d over the rotor disk gives the compressibility increment in the profile power. Following Harris, the resulting compressibility increment in the mean drag coefficient is approximately:

$$c_{d\text{comp}} = 1.52f(K_1 + 1)^2[(1 + \mu)\tau]^{5/2}(1 + \gamma)^{1/2}$$

including the input correction factor f; $c_{d\text{comp}}$ is zero for $K_1 < -1$, and constant for $K_1 > -0.2$.

c) Tip Mach number model: From the tip Mach number and thrust coefficient:

$$\Delta C_{Po} = d_{mt}\left(M_{\text{tip}} - M_{\text{tip limit}}\right)^{2.5}\left(C_T - C_{T\text{limit}}\right)^{2.5} - d_{mt}\left(M_{\text{tip ref}} - M_{\text{tip limit}}\right)^{2.5}\left(C_T - C_{T\text{limit}}\right)^{2.5}$$

and then $c_{d\text{comp}} = \Delta C_{Po}/((\sigma/8)F_P)$. The compressibility increment is zero if $M_{\text{tip}} < M_{\text{tip limit}}$ or $C_T < C_{T\text{limit}}$. $M_{\text{tip ref}}$ is the tip Mach number corresponding to the basic profile power model.

Figure 12-7 shows typical stall functions $(C_T/\sigma)_s$ for two rotors with different stall characteristics, designated high-stall and low-stall, resulting from design features such as different airfoils, Figure 12-7 also shows, for reference, typical helicopter rotor steady and transient load limits. Figure 12-8 illustrates the mean drag coefficient in hover, showing c_{dh} with and without the separation term, and the total for the high-stall and low-stall cases. Figure 12-9 illustrates the mean drag coefficient in forward flight,
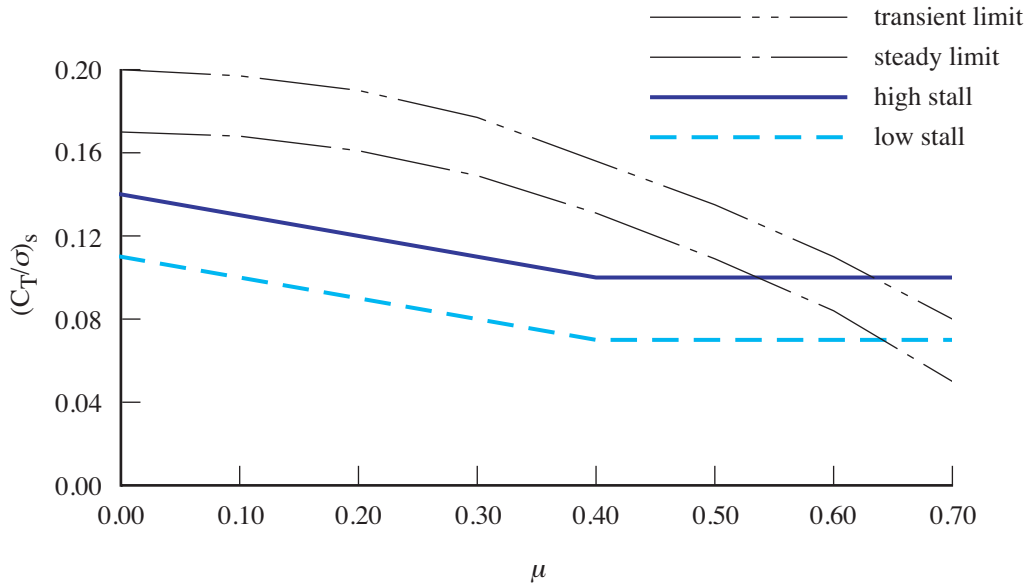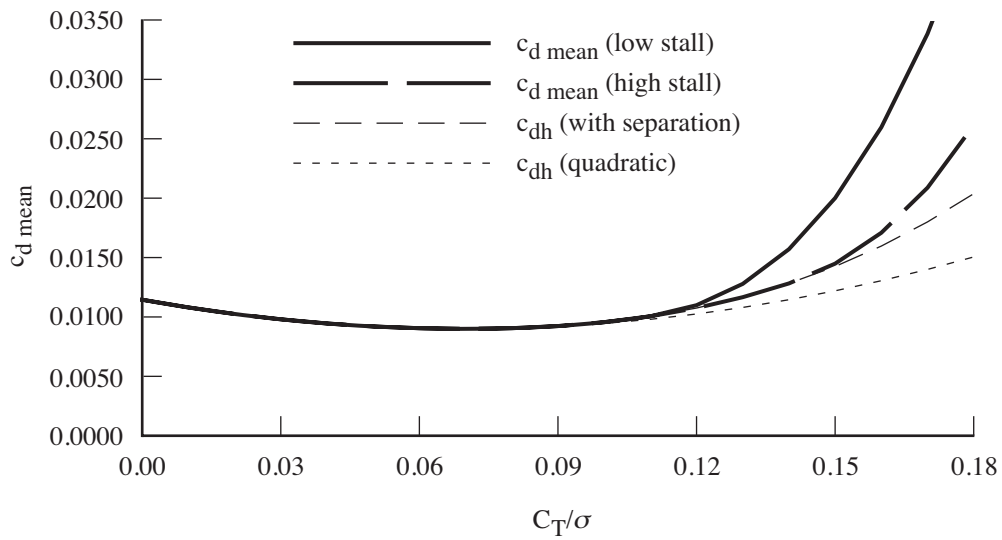
Figure 12-7. Stall function.

Figure 12-8. Mean drag coefficient for rotor in hover.

showing the compressibility term $c_{d\text{comp}}$, and the growth in profile power with C_T/σ and μ as the stall drag increment increases.

Rotor thrust capability $(C_T/\sigma)_{\max}$ as a function of advance ratio μ can be specified for steady and transient operation. Alternatively, the thrust limit can be obtained from an equation:
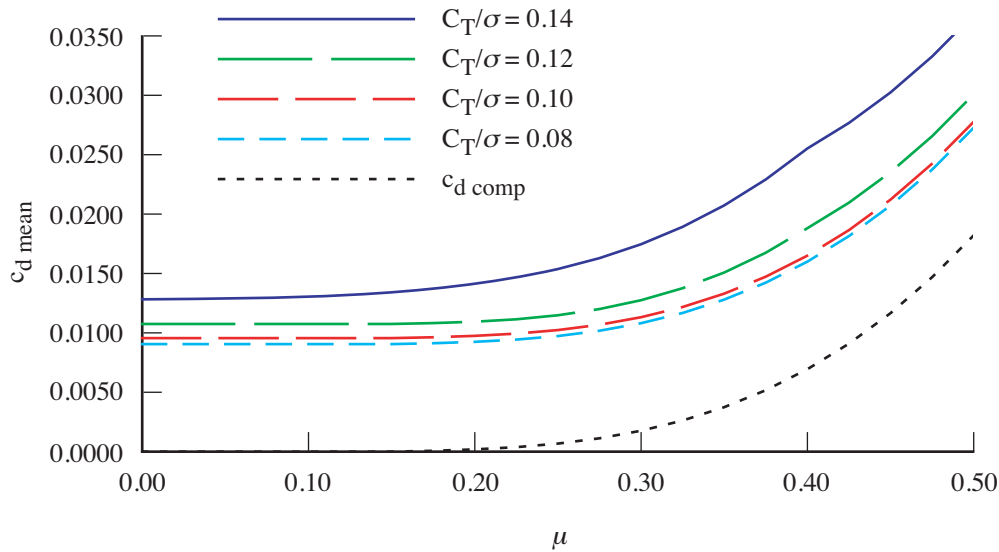
$$(C_T/\sigma)_{\max} = K_0 - K_1\mu^2$$

Figure 12-9a. Mean drag coefficient for rotor in forward flight, high-stall.

For example, $(C_T/\sigma)_{\max} = 0.17 - 0.25\mu^2$ corresponds to the steady limit, and $(C_T/\sigma)_{\max} = 0.20 - 0.25\mu^2$ corresponds to the transient limit. One of these limits can be used to calculate the aircraft never-exceed speed. The rotor thrust margin $(C_T/\sigma)_{\max} - |C_T/\sigma|$ is available for the maximum effort or trim solution.

12-5.1.3   Twin and Multiple Rotors

For twin rotors, the induced power is determined by the induced velocity of the rotor system, not the individual rotors. The induced power is still obtained using $P_i = \kappa P_{\mathrm{ideal}} = \kappa f_D T v_{\mathrm{ideal}}$ for each rotor, but the ideal induced velocity is calculated for an equivalent thrust C_{Te} based on the thrust and geometry of both rotors. The profile power calculation is not changed for twin rotors.

In hover, the twin rotor induced velocity is $v_i = \kappa_{\mathrm{twin}}\sqrt{T/2\rho A_p}$, from the total thrust T and the projected disk area $A_p = (2 - m)A$. The overlap fraction m is calculated from the rotor hub separation ℓ. A correction factor for the twin rotor ideal power is also included. For a coaxial rotor, typically $\kappa_{\mathrm{twin}} \cong 0.90$. So the ideal inflow is calculated for $C_{Te} = (C_{T1} + C_{T2})/(2 - m)$.

In forward flight, the induced velocity $v_i = \kappa_{\mathrm{twin}}T/2\rho AV$ is calculated from the equivalent thrust $T_e = x_1 T_1 + x_2 T_2$, which gives $v_i = \kappa_{\mathrm{twin}}(x_1 v_1 + x_2 v_2) = \kappa_{\mathrm{twin}}(x_1 T_1/2\rho AV + x_2 T_2/2\rho AV)$. The induced velocity of a coaxial rotor is $v_i = \kappa_{\mathrm{twin}}T/(2\rho AV)$, from the total thrust T and a span of $2R$. The ideal inflow is thus calculated for $C_{Te} = C_{T1} + C_{T2}$. The correction factor for ideal induced power (biplane effect) is $\kappa_{\mathrm{twin}} \cong 0.88$ to 0.81 for rotor separations of $0.06D$ to $0.12D$. The induced velocity of side-by-side rotors is $v_i = \kappa_{\mathrm{twin}}T/(2\rho A_e V)$, from the total thrust T and a span of $2R + \ell$, hence $A_e = A(1 + \ell/2R)^2$. The ideal inflow is thus calculated for $C_{Te} = (C_{T1} + C_{T2})/(1 + \ell/2R)^2$. If there is no overlap ($\ell > 2R$), it is assumed that there is no performance impact of interference between the rotors. The induced velocity of tandem rotors is $v_F = \kappa_{\mathrm{twin}}(T_F/(2\rho AV) + x_R T_R/(2\rho AV))$ for the front rotor and $v_R = \kappa_{\mathrm{twin}}(T_R/(2\rho AV) + x_F T_F/(2\rho AV))$ for the rear rotor. For large separation, $x_R \cong 0$ and $x_F \cong 2$; for the coaxial limit $x_R = x_F = 1$ is appropriate. Here $x_R = m$ and $x_F = 2 - m$ is used.
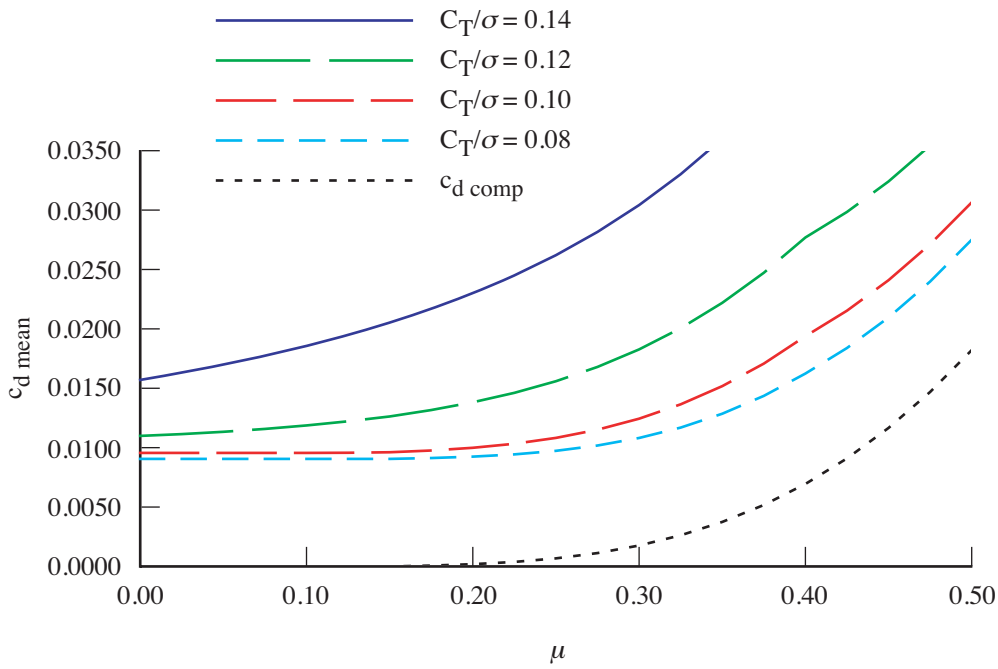
Figure 12-9b. Mean drag coefficient for rotor in forward flight, low-stall.

In summary, the model for twin rotor ideal induced velocity uses $C_{Te} = x_1 C_{T1} + x_2 C_{T2}$ and the correction factor κ_{twin}. In hover, $x_h = 1/(2-m)$; in forward flight of coaxial and tandem rotors, $x_f = 1$ for this rotor and $x_f = m$ or $x_f = 2 - m$ for the other rotor; in forward flight of side-by-side rotors, $x_f = 1/(1 + \ell/2R)^2$ ($x = 1/2$ if there is no overlap, $\ell/2R > 1$). In high-speed axial flight (propeller), $x_p = x_h$ is used. The transition between hover, propeller, and forward flight is accomplished using

$$x_a = \frac{x_p \mu_z^2 + x_h C_a \lambda_h^2}{\mu_z^2 + C_a \lambda_h^2} \qquad x = \frac{x_f \mu^2 + x_a C \lambda_h^2}{\mu^2 + C \lambda_h^2}$$

with typically $C = 1$ to 4. This transition is applied to x for both rotors, and to κ_{twin}.

With a coaxial rotor in hover, the lower rotor acts in the contracted wake of the upper rotor. Momentum theory gives the ideal induced power for coaxial rotors with large vertical separation (ref. 15): $P_u = T_u v_u$, $v_u^2 = T_u/2\rho A$ for the upper rotor; and $P_\ell = (\bar{a}s/\sqrt{\tau})T_\ell v_\ell$, $v_\ell^2 = T_\ell/2\rho A$ for the lower rotor. Here $\tau = T_\ell/T_u$; $\bar{a}$ is the average of the disk loading weighted by the induced velocity, hence a measure of nonuniform loading on the lower rotor ($\bar{a} = 1.05$ to 1.10 typically); and the momentum theory solution is

$$\frac{\bar{a}s}{\sqrt{\tau}} = \frac{1}{2\tau^{3/2}} \left(\sqrt{1 + 4(1+\tau)^2 \bar{a}\tau} - 1 \right)$$

The optimum solution for equal power of the upper and lower rotors is $\bar{a}s\tau = 1$, giving $\tau = T_\ell/T_u \cong 2/3$. Hence for the coaxial rotor in hover the ideal induced velocity is calculated from $C_{Te} = C_{Tu}$ for the upper rotor and from $C_{Te} = (\bar{a}s/\sqrt{\tau})^2 C_{T\ell}$ for the lower rotor, with $\kappa_{\text{twin}} = 1$. Thus $x_h = 1/(2-m) = 1/2$ and the input hover κ_{twin} is not used, unless the coaxial rotor is modeled as a tandem rotor with zero longitudinal separation.

For multiple rotors, the model for ideal induced velocity is extended to use $C_{Te} = \sum x_n C_{Tn}$ (sum over all rotors, scaled with square of tip speed), and the correction factor κ_{twin}. The thrust factors x_n

and correction factor κ_{twin} are input for hover, propeller, and forward flight; with the transition between hover, propeller, and forward flight as above using C and C_a.

12-5.2 Table Performance Method

The induced power is calculated from the ideal power: $P_i = \kappa P_{\text{ideal}} = \kappa f_D T v_{\text{ideal}}$. The induced power factor κ is obtained from an input table (linearly interpolated) that can be a function of up to three independent variables. Optionally κ from the equations can be retained as an increment to the table value.

The profile power is calculated from a mean blade drag coefficient: $P_o = \rho A (\Omega R)^3 C_{Po} = \rho A (\Omega R)^3 \frac{\sigma}{8} c_{d\text{mean}} F_P$. The mean drag coefficient $c_{d\text{mean}}$, or alternatively $c_{d\text{mean}} F_P = 8 C_{Po}/\sigma$, is obtained from an input table (linearly interpolated) that can be a function of up to three independent variables. Optionally $c_{d\text{mean}}$ from the equations can be retained as an increment to the table value.

The independent variables can be velocity magnitude V/V_{tip}, horizontal velocity V_h/V_{tip}, edgewise advance ratio μ (hub plane or tip-path plane), axial velocity ratio μ_z (hub plane or tip-path plane), shaft angle of attack $\alpha = \tan^{-1}(\mu_z/\mu)$ (hub plane or tip-path plane), blade loading C_T/σ, lift offset M_x/TR, tip Mach number M_{tip}, or advancing tip Mach number M_{at}. The thrust value used in the table can include download. The thrust value used in the table can include ground effect (thrust divided by κ_g), or the power result from the table can include ground effect (power multiplied by f_g).

12-5.3 Power Adjustments

Rotor power adjustments are associated with wing interference (P_w), propulsive force efficiency (P_d), and climb efficiency (P_v).

The interference power can be produced by interactions from the wing. The component of the wing interference velocity v_{ind}^F parallel to the rotor force vector F^F (velocity roughly normal to the rotor disk) produces a power change $P_w \cong v_{\text{ind}}^T F$. The component of the interference velocity perpendicular to the rotor force vector (velocity roughly in the plane of the rotor disk) produces interference through the swirl, hence $P_w \propto (V/\Omega R)|v_{\text{ind}}||F|$. Thus the interference power is calculated from

$$P_w = -K_{\text{int}n} v_{\text{ind}}^T F + K_{\text{int}p} \frac{V}{\Omega R} \sqrt{(|v_{\text{ind}}||F|)^2 - (v_{\text{ind}}^T F)^2}$$

Separate interference factors K_{int} are used for the two terms. $K_{\text{int}p}$ is negative for favorable interference.

An effect of propulsive force on the rotor induced and profile power can be included in terms of an efficiency η:

$$P_d = V \Delta D \left(\frac{1}{\eta} - 1 \right)$$

where $\Delta D = q\big(D/q - (D/q)_{\text{ref}}\big)$, and the reference propulsive force $(D/q)_{\text{ref}}$ corresponds to the baseline profile and induced power models. In terms of the rotor wind axis forces, the propulsive force is here $D = -X$ (or $D = -X\cos\alpha_c - L\sin\alpha_c$ in climb, $\tan\alpha_c = V_c/V_h$). The efficiency is a function of rotor speed and blade loading, $\eta(C_T/\sigma, V/V_{\text{tip}})$. This increment is intended for use with the table model for the reference propulsive force.

The main rotor propulsive power $P_p = -XV$ includes the climb power $P_c = W_G V_c$, and the influence of climb on the induced power is contained in v_{ideal} and the appropriate terms of κ. Alternatively, the influence of climb on induced and profile power can be obtained from an efficiency factor f_{climb} as a

function of V_c/v_h: $P - P_{\text{level}} = W_G V_c f_{\text{climb}}$. In high speed flight, climb or descent does not change the induced velocity much, so the net change in induced power due to climb would be $W_G V_c (f_{\text{climb}} - 1)$. In low speed flight or axial climb, the induced velocity is influenced by climb rate. Let the difference between the ideal induced velocity and the level flight induced velocity be $v_{\text{ideal}} - v_{\text{level}} = v_h x$. Then the net change in power due to climb is $P_v = W_G V_c (f_{\text{climb}} - 1) - \kappa T v_h x$. In axial climb

$$v_{\text{ideal}} - v_{\text{level}} = -V_c/2 + \sqrt{(V_c/2)^2 + v_h^2} - v_h = v_h \left(-V_c/v_h 2 + \sqrt{(V_c/v_h 2)^2 + 1} - 1 \right) = v_h x$$

In forward flight, v_{level} is evaluated as for v_{ideal}, but with $\mu_z = 0$. With multiple main rotors, P_v for each rotor is obtained from W_G/N_{rotor}. This climb power increment is intended for use with the table model for level flight power.

12–6   Power Required and Reaction Drive

The total rotor power required is $P_{req} = P_i + P_o + P_p + P_x$. In most helicopter designs the power is delivered to the rotor by a mechanical drive, through the rotor shaft torque. Such designs require a transmission and a means for balancing the main-rotor torque. The shaft power is $P_{\text{shaft}} = P_{req}$, which contributes to the propulsion group power required, P_{reqPG}, and produces a torque on the aircraft.

An alternative is to supply the power by a jet reaction drive of the rotor, using cold or hot air ejected out of the blade tips or trailing edges. Helicopters have also been designed with ram jets on the blade tips, or with jet flaps on the blade trailing edges that use compressed air generated in the fuselage. Since there is no torque reaction between the helicopter and rotor (except for the small bearing friction), no transmission or antitorque device is required, resulting in a considerable weight saving. With a jet reaction drive, the propulsion system is potentially lighter and simpler, although the aerodynamic and thermal efficiency are lower. The helicopter must still have a mechanism for yaw control. With reaction drive the shaft power is $P_{\text{shaft}} = P_{req} - P_{\text{react}}$, where the reaction power P_{react} contributes to the engine group or jet group power required. The reaction drive produces a force F_{react} on the rotor blades at effective radial station r_{react}, so $P_{\text{react}} = \Omega r_{\text{react}} F_{\text{react}}$. Momentum balance gives the total force

$$F = \sum \left[\dot{m} (V_{\text{react}} - \Omega r_{\text{react}}) e_\psi - \dot{m} V e_x \right]$$

where e_ψ and e_x are unit vectors perpendicular to the blade and in the free stream direction; and the sum is over all blades. The average force in the nonrotating frame is the drag of the inlet momentum ($\dot{m}_{\text{react}} V$), which is accounted for in the engine group or jet group model. The mean in the rotating frame gives the total jet force required

$$F_{\text{react}} = \dot{m}_{\text{react}} (V_{\text{react}} - \Omega r_{\text{react}})$$

The net reaction force F_{react} is a control of the rotor component. The gross thrust (momentum flux) required is supplied by the compressor or jet:

$$F_{Greq} = \dot{m}_{\text{react}} V_{\text{react}} = F_{\text{react}} + \dot{m}_{\text{react}} \Omega r_{\text{react}}$$

The compressor model also gives the engine group power needed to produce F_{Greq}. The engine group or jet group performance includes the efficiency of the blade duct and nozzle, perhaps even with tip burning. If the reaction drive is turned off ($F_{\text{react}} = 0$), then the rotor must be trimmed such that $P_{req} = 0$.

12–7   Performance Metrics

Several performance metrics are calculated for each rotor. The induced power factor is $\kappa = P_i/P_{\text{ideal}}$. The rotor mean drag coefficient is $c_d = (8C_{Po}/\sigma)/F_P$, using the function $F(\mu, \mu_z)$ given previously. The rotor effective lift-to-drag ratio is a measure of the induced and profile power: $L/D_e = VL/(P_i + P_o)$. The hover figure of merit is $M = Tf_Dv/P$. The propeller propulsive efficiency is $\eta = P_p/P = -XV/P$. These two metrics can be combined as a momentum efficiency: $\eta_{\text{mom}} = T(V + w/2\eta_D)/P$, where $w/2\eta_D = f_W v/2\eta_D = f_D v$.

12–8   Interference

The rotor can produce aerodynamic interference velocities at the other components (fuselage, wings, and tails). The induced velocity at the rotor disk is κv_i, acting opposite the thrust (z-axis of tip-path plane axes). So $v_{\text{ind}}^P = -k^P \kappa v_i$, and $v_{\text{ind}}^F = C^{FP} v_{\text{ind}}^P$. The total velocity of the rotor disk relative to the air consists of the aircraft velocity and the induced velocity from this rotor: $v_{\text{total}}^F = v^F - v_{\text{ind}}^F$. The direction of the wake axis is thus $e_w^P = -C^{PF} v_{\text{total}}^F/|v_{\text{total}}^F|$ (for zero total velocity, $e_w^P = -k^P$ is used). The angle of the wake axis from the thrust axis is $\chi = \cos^{-1}|(k^P)^T e_w^P|$.

The interference velocity v_{int}^F at each component is proportional to the induced velocity v_{ind}^F (hence is in the same direction), with factors accounting for the stage of wake development and the position of the component relative to the rotor wake. The far wake velocity is $w = f_W v_i$, and the contracted wake area is $A_c = \pi R_c^2 = A/f_A$. The solution for the ideal inflow gives f_W and f_A. For an open rotor, $f_W = 2$. For a ducted rotor, the inflow and wake depend on the wake area ratio f_A, or on the ratio of the rotor thrust to total thrust: $f_T = T_{\text{rotor}}/T$. The corresponding velocity and area ratios at an arbitrary point on the wake axis are f_w and f_a, related by

$$f_a = \sqrt{\frac{\mu^2 + (\mu_z + f_w \lambda_i)^2}{(f_{Vx}\mu)^2 + (f_{Vz}\mu_z + \lambda_i)^2}}$$

Vortex theory for hover gives the variation of the induced velocity with distance z below the rotor disk:

$$v = v(0)\left(1 + \frac{z/R}{\sqrt{1 + (z/R)^2}}\right)$$

With this equation the velocity varies from zero far above the disk to $v = 2v(0)$ far below the disk. To use this expression in edgewise flow and for ducted rotors, the distance z/R is replaced by ζ_w/tR, where ζ_w is the distance along the wake axis, and the parameter t is introduced to adjust the rate of change (t small for faster transition to far wake limit). Hence the velocity inside the wake is $f_w v_i$, where

$$f_w = f_W f_z = \begin{cases} 1 + \dfrac{\zeta_w/tR}{\sqrt{1 + (\zeta_w/tR)^2}} & \zeta_w < 0 \\[3ex] 1 + (f_W - 1)\dfrac{\zeta_w/tR}{\sqrt{1 + (\zeta_w/tR)^2}} & \zeta_w > 0 \end{cases}$$

and the contracted radius is $R_c = R/\sqrt{f_a}$.

The wake is a skewed cylinder, starting at the rotor disk and with the axis oriented by e_w^P. The interference velocity is required at the position z_B^F on a component. Whether this point is inside or outside the wake cylinder is determined by finding its distance from the wake axis, in a plane parallel to

the rotor disk. The position relative to the rotor hub is $\xi_B^P = C^{PF}(z_B^F - z_{\text{hub}}^F)$; the corresponding point on the wake axis is $\xi_A^P = e_w^P \zeta_w$. Requiring ξ_B^P and ξ_A^P have the same z value in the tip-path plane axes gives

$$\zeta_w = \frac{(k^P)^T C^{PF}(z_B^F - z_{\text{hub}}^F)}{(k^P)^T e_w^P}$$

from which f_z, f_w, f_a, and R_c are evaluated. The distance r from the wake axis is then

$$r^2 = \left((i^P)^T(\xi_B^P - \xi_A^P)\right)^2 + \left((j^P)^T(\xi_B^P - \xi_A^P)\right)^2$$

The transition from full velocity inside the wake to zero velocity outside the wake is accomplished in the distance sR_c, using

$$f_r = \begin{cases} 1 & r \leq R_c \\ 1 - (r - R_c)/(sR_c) & \\ 0 & r \geq (1+s)R_c \end{cases}$$

($s = 0$ for an abrupt transition, s large for always in wake).

The interference velocity at the component (at z_B^F) is calculated from the induced velocity v_{ind}^F, the factors $f_W f_z$ accounting for axial development of the wake velocity, the factor f_r accounting for immersion in the wake, and an input empirical factor K_{int}:

$$v_{\text{int}}^F = K_{\text{int}} \, f_W f_z f_r f_t \, v_{\text{ind}}^F$$

An additional factor f_t for twin rotors is included. Optionally the development along the wake axis can be a step function ($f_W f_z = 0$, 1, f_W above the rotor, on the rotor disk, and below the rotor disk, respectively); nominal ($t = 1$); or use an input rate parameter t. Optionally the wake immersion can use the contracted radius R_c or the uncontracted radius R; can be a step function ($s = 0$, so $f_r = 1$ and 0 inside and outside the wake boundary); can be always immersed ($s = \infty$ so $f_r = 1$ always); or can use an input transition distance s. Optionally the interference factor K_{int} can be reduced from an input value at low speed to zero at high speed, with linear variation over a specified speed range.

To account for the extent of the wing or tail area immersed in the rotor wake, the interference velocity is calculated at several points along the span and averaged. The increment in position is $\Delta z_B^F = C^{FB}(0 \, \Delta y \, 0)^T$, $\Delta y = (b/2)(-1 + (2i - 1)/N)$ for $i = 1$ to N; where b is the wing span. The average interference is calculated separately for each wing panel (left and right), by interpolating the interference velocity at N points along the wing to $N/2$ points along the panel span.

For twin main-rotors (tandem, side-by-side, or coaxial), the performance may be calculated for the rotor system, but the interference velocity is still calculated separately for each rotor, based on its disk loading. At the component, the velocities from all rotors are summed, and the total used to calculate the angle of attack and dynamic pressure. This sum must give the interference velocity of the twin rotor system, which requires the correction factor f_t. Consider differential momentum theory to estimate the induced velocity of twin rotors in hover. For the first rotor, the thrust and area in the non-overlap region are $(1-m)T_1$ and $(1-m)A$, hence the induced velocity is $v_1 = \kappa\sqrt{T_1/2\rho A}$; similarly $v_2 = \kappa\sqrt{T_2/2\rho A}$. In the overlap region the thrust and area are $mT_1 + mT_2$ and mA, hence the induced velocity is $v_m = \kappa\sqrt{(T_1 + T_2)/2\rho A}$. So for equal thrust, the velocity in the overlap region (everywhere for the coaxial configuration) is $\sqrt{2}$ larger. The factor K_T is introduced to adjust the overlap velocity: $v_m = \kappa(K_T/\sqrt{2})\sqrt{(T_1 + T_2)/2\rho A}$. The interference velocities are calculated separately for the two

rotors, with the correction factor f_t: $v_{\text{int1}} = f_t \kappa \sqrt{T_1/2\rho A}$ and $v_{\text{int2}} = f_t \kappa \sqrt{T_2/2\rho A}$. The sum $v_{\text{int1}} + v_{\text{int2}}$ must take the required value. Below the non-overlap region, the component is in the wake of only one of the rotors, so the interference velocity from the other rotor is zero, and thus $f_t = 1$. Below the overlap region, the component is in the wake of both rotors, and the sum of the interference velocities equals v_m if

$$f_{th} = \frac{K_T/\sqrt{2}}{\sqrt{\tau_1} + \sqrt{\tau_2}}$$

where $\tau_n = T_n/(T_1 + T_2)$ is the thrust ratio. For equal thrusts, $f_{th} = K_T/2$; or $f_{th} = 1/\sqrt{2}$ for the nominal velocity. The expression $f_t = f_{th}\cos^2\chi + \sin^2\chi$ gives the required correction factor, with $f_t = 1$ in edgewise flight. Optionally the correction for twin rotors can be omitted ($f_t = 1$); nominal ($K_T = \sqrt{2}$); or use an input velocity factor in the overlap region (K_T).

12–9 Stoppable Rotor

The rotor can stoppable, hence for each flight state operated rotating, stopped, stopped and stowed, or stopped as wing.

> a) Stopped: The rotor flapping, forces, and power are zero. Alternate stopped hub drag area or coefficient is used. The stopped blade drag is obtained from the coefficient $C_{D\text{blade}}$, based on the geometric blade area $A_{\text{blade}} = \sigma_g A$.
>
> b) Stopped and stowed: The rotor flapping, forces, and power are zero. Alternate stowed hub drag area or coefficient is used.
>
> c) Stopped as a wing: The rotor flapping, forces, and power are zero. The hub drag is zero. The aerodynamic loads are obtained from a wing model (in other operating modes the loads from this wing are zero). The wing area is the geometric blade area, and the wing span is $2R$ (R for one-bladed rotor). The wing has only one panel. The weight from the wing model is zero.

In rotating mode operation, the basic rotor model gives the flapping, forces, and power.

12–10   Drag

The rotor component includes drag forces acting on the hub and spinner (at z^F_{hub}) and on the pylon (at z^F_{pylon}). The component drag contributions must be consistent. In particular, a rotor with a spinner (such as on a tiltrotor aircraft) would likely not have hub drag. The pylon is the rotor support and the nacelle is the engine support. The drag model for a tiltrotor aircraft with tilting engines would use the pylon drag (and no nacelle drag), since the pylon is connected to the rotor shaft axes; with non-tilting engines it would use the nacelle drag as well.

The body axes for the drag analysis are rotated about the y-axis relative to the rotor shaft axes: $C^{BF} = C^{BS}C^{SF}$, where $C^{BS} = Y_{-\theta_{\text{ref}}}$. The pitch angle θ_{ref} can be input, or the rotation appropriate for a helicopter rotor or a propeller can be specified.

> a) Consider a helicopter rotor, with the shaft axes oriented z-axis up and x-axis downstream. It is appropriate that the angle of attack is $\alpha = 0$ for forward flight and $\alpha = -90$ deg for hover, meaning that the body axes are oriented z-axis down and x-axis forward. Hence $\theta_{\text{ref}} = 180$ deg.
>
> b) Consider a propeller or tiltrotor, with the shaft axes oriented z-axis forward and

x-axis up. It is appropriate that the angle of attack is $\alpha = 0$ in cruise and $\alpha = 90$ deg
for helicopter mode (with a tilting pylon), meaning that the body axes are oriented
z-axis down and *x*-axis forward. Hence $\theta_{\text{ref}} = 90$ deg.

The aerodynamic velocity relative to the air is calculated in component axes, v^B. The angle of attack
α and dynamic pressure q are calculated from v^B. The reference areas for the drag coefficients are the
rotor disk area $A = \pi R^2$ (for hub drag), pylon wetted area S_{pylon}, ducted wetted area $2S_{\text{duct}}$, and spinner
wetted area S_{spin}; these areas are input or calculated as described previously.

The hub drag can be fixed, specified as a drag area D/q; or the drag can be scaled, specified as a
drag coefficient C_D based on the rotor disk area $A = \pi R^2$; or the drag can be estimated based on the
gross weight, using a squared-cubed relationship or a square-root relationship. Based on historical data,
the drag coefficient $C_D = 0.004$ for typical hubs, $C_D = 0.0024$ for current low-drag hubs, and $C_D =
0.0015$ for faired hubs. For the squared-cubed relationship: $(D/q)_{\text{hub}} = k((W_{MTO}/N_{\text{rotor}})/1000)^{2/3}$
(where W_{MTO}/N_{rotor} is the maximum takeoff gross weight per lifting rotor; units of k are ft²/k-lb²/³ or
m²/Mg²/³). Based on historical data, $k = 1.4$ for typical hubs, $k = 0.8$ for current low-drag hubs, and
$k = 0.5$ for faired hubs (English units). For the square-root relationship: $(D/q)_{\text{hub}} = k\sqrt{W_{MTO}/N_{\text{rotor}}}$
(where W_{MTO}/N_{rotor} is the maximum takeoff gross weight per lifting rotor; units of k are ft²/lb¹/² or
m²/kg¹/²); based on historical data (ref. 16), $k = 0.074$ for single rotor helicopters, $k = 0.049$ for tandem
rotor helicopters (probably a blade number effect), $k = 0.038$ for hingeless rotors, and $k = 0.027$ for faired
hubs (English units). To handle multi-rotor aircraft, the scaling weight $w = W_{MTO}/N_{\text{rotor}}$ is calculated
as for disk loading: $w = f_W W_{MTO}$ for main-rotors or $w = fT$ for antitorque and auxiliary-thrust rotors.

The hub vertical drag can be fixed, specified as a drag area D/q; or the drag can be scaled, specified
as a drag coefficient C_D based on the rotor disk area $A = \pi R^2$.

The pylon forward flight drag and vertical drag are specified as drag area or drag coefficient, based
on the pylon wetted area. The duct forward flight drag and vertical drag are specified as drag area or drag
coefficient, based on the duct wetted area. The spinner drag is specified as drag area or drag coefficient,
based on the spinner wetted area. For a stopped rotor, alternate values of the hub drag are used, plus
a stopped blade drag coefficient $C_{D\text{blade}}$ based on the geometric blade area. For a stopped and stowed
rotor, alternate values of the hub drag are used. For a rotor stopped as a wing, the hub drag is zero and
the aerodynamic loads obtained from a wing model.

The drag coefficient for the hub or pylon or duct at angle of attack α is

$$C_D = C_{D0} + (C_{DV} - C_{D0})|\sin\alpha|^{X_d}$$

Optionally the variation can be quadratic ($X_d = 2$). For sideward flight, $C_{D\text{hub}} = C_{D0}$ for the hub,
$C_{D\text{pylon}} = C_{DV}$ for the pylon, and $C_{D\text{duct}} = C_{DV}$ for the duct. Then the total component drag force is

$$D = qAC_{D\text{hub}} + qS_{\text{pylon}}C_{D\text{pylon}} + qS_{\text{duct}}C_{D\text{duct}} + qS_{\text{spin}}C_{D\text{spin}} + qA_{\text{blade}}C_{D\text{blade}}$$

The dynamic pressure and position of the hub are used for the duct and spinner. The force and moment
produced by the drag are

$$F^F = \sum e_d D$$
$$M^F = \sum \widetilde{\Delta z^F} F^F$$

where $\Delta z^F = z^F - z_{cg}^F$ (separate locations are defined for the rotor hub and for the pylon), and e_d is the
drag direction. The velocity relative to the air gives $e_d = -v^F/|v^F|$ (no interference).

12–11   Weights

The rotor configuration determines where the weights occur in the weight statement, as summarized in table 12-3. The rotor group consists of blade assembly, hub and hinge, fairing/spinner, blade fold structure, inter-rotor shaft, rotor support, and duct. The tail-rotor (in empennage group) or the propeller/fan installation (in propulsion group) consists of blade assembly, hub and hinge, rotor support, and duct.

There are separate weight models for main-rotors, tail-rotors, and auxiliary-thrust systems (propellers). The tail-rotor model requires a torque calculated from the drive system rated power and main-rotor rotational speed: $Q = P_{DS\text{limit}}/\Omega_{mr}$. The auxiliary-thrust model requires the design maximum thrust of the propeller. The engine section or nacelle group includes the engine support weight and pylon support weight; these must be consistent with the use of the rotor support structural weight.

Table 12-3. Principal configuration designation.

configuration	weight statement	weight model	performance model
main-rotor	rotor group	rotor	rotor
tail-rotor	empennage group	tail-rotor	rotor
propeller	propulsion group	rotor, aux thrust	rotor

The flap moment of inertia I_b and the Lock number $\gamma = \rho a c R^4/I_b$ are required for the blade motion solution. Several options are implemented to calculate I_b. The Lock number can be specified, and then $I_b = \rho a c R^4/\gamma$ used, independent of the blade weight; this is the only option for the tail-rotor and auxiliary-thrust weight models, which do not give separate blade and hub weight estimates. The moment of inertia I_b can be calculated from the blade weight and the weight distribution. The Lock number can be specified, hence $I_b = \rho a c R^4/\gamma$, and then mass added to the blade to achieve this value. An autorotation index $AI = KE/P = \frac{1}{2}NI_b\Omega^2/P$ can be specified, hence the required I_b, and then mass added to the blade to achieve this value. Reference 17 describes this and other autorotation indices; $AI = KE/P \geq 3$ sec gives good autorotation characteristics for small helicopters.

In order to increase the moment of inertia, a tip weight W_t can be added to each blade at radial station r_t. Thus the total blade weight is $W_b = \chi w_b + dW_b + (1+f)W_tN$ (lb or kg); where w_b is the blade weight estimate, χ the technology factor, dW_b a specified weight increment; and the factor f accounts for the blade weight increase required by the centrifugal force due to W_t. The mass per blade is $M_b = W_b/N$ (slug or kg), or M_{b0} without the tip weight. The blade moment of inertia is

$$I_b = R^2(r_2^2(M_{b0} + fM_t) + r_t^2 M_t) = I_{b0} + R^2(r_t^2 + fr_2^2)M_t$$

where r_2 is the radius of gyration of the distributed mass. Typically $r_2 \cong 0.6$; $r_2 = 1/\sqrt{3} = 0.577$ for uniform mass distribution. If the required moment of inertia I_b is greater than I_{b0}, the tip mass M_t is needed. Additional mass is required inboard to react the centrifugal force increase due to M_t. This additional mass is less effective than M_t at increasing I_b. Assume a fraction a of the blade mass reacts the centrifugal force F, so $\Delta M/M_{b0} = a\Delta F/F_0$. The reference values are $M_{b0} = R \int m\, dr$ and $F_0 = \Omega^2 R^2 \int rm\, dr = \Omega^2 R r_1 M_{b0}$, where $r_1 \cong \frac{1}{2}$. Then

$$\Delta M = a\frac{M_{b0}}{F_0}\Delta F = a\frac{1}{\Omega^2 R r_1}\left(\Omega^2 R r_1 \Delta M + \Omega^2 R r_t M_t\right) = a\left(\Delta M + \frac{r_t}{r_1}M_t\right) = \frac{ar_t/r_1}{1-a}M_t = fM_t$$

With $a \cong \frac{1}{3}$, $f \cong 1$. The tip mass required to produce $\Delta I_b = I_b - I_{b0}$ is $M_t = \Delta I_b/(R^2(r_t^2 + fr_2^2))$, and the total blade weight increment is $\Delta W_b = (1 + f)M_t N$ (lb or kg).

12–12   References

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Wing

The aircraft can have one or more wings, or no wings.

13–1   Geometry

The wing is described by planform area S, span b, mean chord $c = S/b$, and aspect ratio $AR = b^2/S$. These parameters are for the entire wing. The geometry is specified in terms of two of the following parameters: S or wing loading W/S, b (perhaps calculated from other geometry), c, $AR = b^2/S$. With more than one wing, the wing loading is obtained from an input fraction of design gross weight, $W = f_W W_D$. Optionally the span can be calculated from a specified ratio to the span of another wing; or the span can be calculated from a specified ratio to the radius of a designated rotor, $b = 2fR$. Optionally the wing span can be calculated from an appropriate specification of all wing panel widths.

Optionally for the tiltrotor configuration, the wing span can be calculated from the fuselage and rotor geometry: $b = 2(fR + d_{\text{fus}}) + w_{\text{fus}}$, where R is the rotor radius (cruise value for variable-diameter rotor), d_{fus} the rotor-fuselage clearance, and w_{fus} the fuselage width. Note that the corresponding option for the rotor hub position is $y_{\text{hub}} = \pm(fR + d_{\text{fus}} + {}^1\!/2 w_{\text{fus}})$. Optionally the wing span can be calculated from the rotor hub position: $b = 2|y_{\text{hub}}|$ (regardless of how the rotor position is determined). As implemented, symmetry is not assumed; rather the radius or hub position of the outermost designated rotors is used.

The wing is at position z^F, where the aerodynamic forces act. The component axes are the aircraft body axes, $C^{BF} = I$.

The wing planform is defined in terms of one or more wing panels (fig. 13-1). Symmetry of the wing is assumed. The number of panels is P, with the panel index $p = 1$ to P. The wing span station η is scaled with the semi-span: $y = \eta(b/2), \eta = 0$ to 1. Each panel is a trapezoid, with a straight aerodynamic center and linear taper. The aerodynamic center locus (in wing axes) is defined by sweep Λ_p; dihedral δ_p; and offsets (x_{Ip}, z_{Ip}) at the inboard edge relative to the aerodynamic center of the previous panel. The wing position z^F is the mean aerodynamic center. The offset $(\bar{x}_A, \bar{z}_A)$ of the mean aerodynamic center from the root chord aerodynamic center is calculated (so the wing planform can be drawn; typically the aerodynamic center is drawn as the quarter-chord). Outboard panel edges are at η_{Ep} (input or calculated). A panel is characterized by span b_p (each side), mean chord c_p, and area $S_p = 2b_p c_p$ (both sides). The taper is defined by inboard and outboard chord ratios, $\lambda = c/c_{\text{ref}}$ (where c_{ref} is a panel or wing reference chord, depending on the options for describing the geometry).

The span for each panel (if there are more than two panels) can be a fixed input; a fixed ratio of the wing span, $b_p = f_{bp}(b/2)$; or free. The panel outboard edge (except at the wing tip) can be at a fixed input position y_{Ep}; at a fixed station $\eta_{Ep}, y_p = \eta_{Ep}(b/2)$; calculated from the rotor radius, $y_p = fR$; calculated from the fuselage and rotor geometry, $y_p = fR + d_{\text{fus}} + {}^1\!/2 w_{\text{fus}}$ (for a designated rotor); calculated from
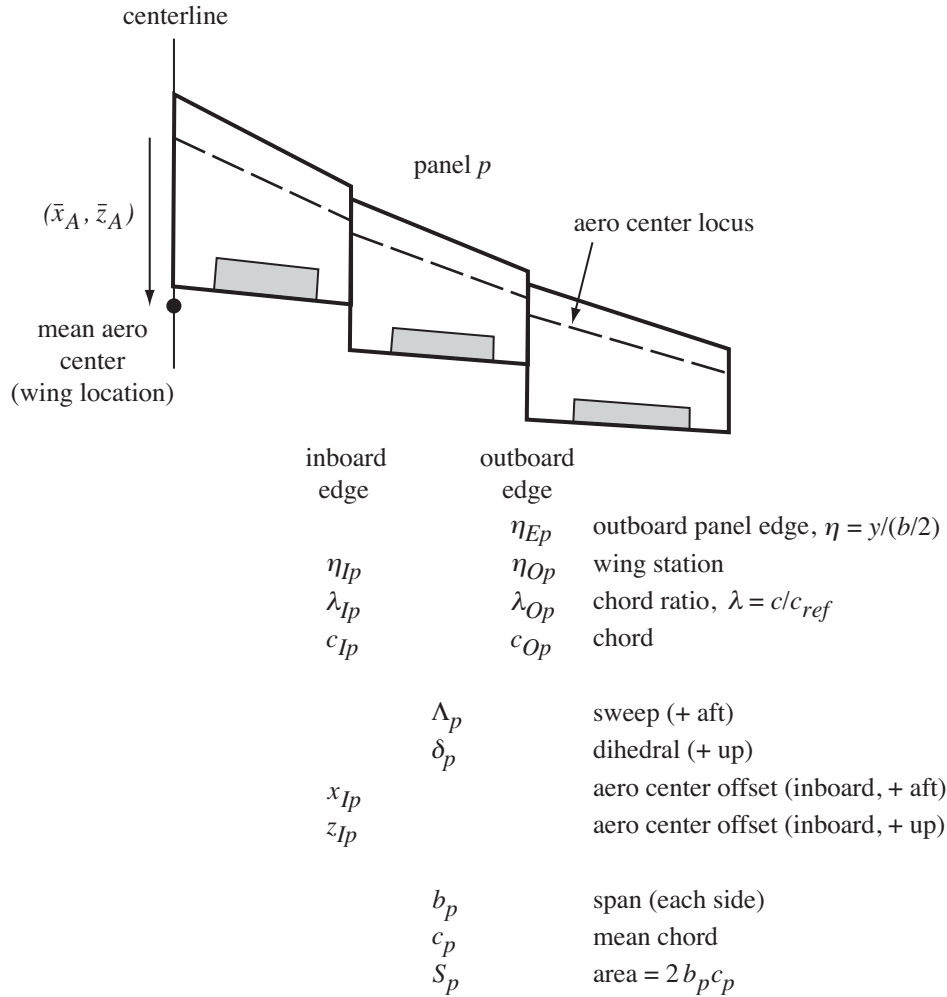
Figure 13-1. Wing geometry (symmetric, only right half-wing shown).

the hub position, $y_p = |y_{\text{hub}}|$ (for a designated rotor); or adjusted. An adjusted station is obtained from the last station and the span of this panel, $y_p = y_{p-1} + b_p$ or $y_p = y_{p-1} + f_{bp}(b/2)$; or from the next station and the span of the next panel, $y_p = y_{p+1} - b_{p+1}$ or $y_p = y_{p+1} - f_{b(p+1)}(b/2)$. The specification of panel spans and panel edges must be consistent, and sufficient to determine the wing geometry. Determining the panel edges requires the following steps:

a) Calculate the panel edges that are either at fixed values (input, or from width, or from hub position) or at fixed stations; root and tip edges are known.
b) Working from root to tip, calculate the adjusted panel edge y_p if panel span b_p or ratio f_{bp} is fixed, and if previous edge y_{p-1} is known.
c) Working from tip to root, calculate the adjusted panel edge y_p (if not yet known) if panel span b_{p+1} or ratio $f_{b(p+1)}$ is fixed, and if next edge y_{p+1} is known.

At the end of this process, all edges must be known and the positions y_p must be unique and sequential. If this geometry is being determined for a known span, then there must not be a fixed panel span or span

ratio that has not been used. Alternatively, if the wing span is being calculated from the specification of all panel widths, then the process must leave one and only one fixed panel span or span ratio that has not been used. Since the wing span is to be calculated, each panel edge is known in the form $y_p = c_0 + c_1 b/2$. Then the unused fixed panel span gives the equation $(c_0 + c_1 b/2)_O - (c_0 + c_1 b/2)_I = b_p$ (subscript O denotes outboard edge, subscript I denotes inboard edge), or the unused fixed panel span ratio gives the equation $(c_0 + c_1 b/2)_O - (c_0 + c_1 b/2)_I = f_p b/2$, which can be solved for the semispan $b/2$.

To complete the definition of the geometry, one of the following quantities is specified for each panel: panel area S_p; ratio of panel area to wing area, $f_s = S_p/S$; panel mean chord c_p; ratio of panel mean chord to wing mean chord, $f_c = c_p/c$; chord ratios $\lambda_I = c_I/c_{\text{ref}}$ and $\lambda_O = c_O/c_{\text{ref}}$ (taper); or free. The total wing area equals the sum of all panel areas:

$$S = \sum S_p + S \sum f_s + 2 \sum b_p c_p + 2c \sum b_p f_c + 2c_{\text{ref}} \sum b_p \frac{1}{2}(\lambda_I + \lambda_O)$$

If there is one or more taper specification (and no free), then c_{ref} is calculated from this equation for S, and the mean chord is $c_p = \frac{1}{2}(c_I + c_O) = c_{\text{ref}}\frac{1}{2}(\lambda_I + \lambda_O)$, $S_p = 2b_p c_p$. If there is one (and only one) free specification, then S_p is calculated from this equation for S, and the mean chord is $c_p = S_p/(2b_p)$, with $c_I = 2c_p/(1 + \lambda_O/\lambda_I)$, $c_O = 2c_p - c_I$.

Since the panels have linear taper ($c = c_{\text{ref}}\lambda$), the mean aerodynamic chord is

$$S\bar{c}_A = \int_{-b/2}^{b/2} c^2 dy = b \int_0^1 c_{\text{ref}}^2 \lambda^2 d\eta$$

$$= b \sum c_{\text{ref}}^2 \frac{1}{3}(\lambda_I^2 + \lambda_I \lambda_O + \lambda_O^2) \Delta\eta_p = \sum \frac{1}{3}(c_I^2 + c_I c_O + c_O^2) 2b_p$$

$$S = \int_{-b/2}^{b/2} c\, dy = b \int_0^1 c_{\text{ref}} \lambda\, d\eta$$

$$= b \sum c_{\text{ref}} \frac{1}{2}(\lambda_I + \lambda_O) \Delta\eta_p = \sum \frac{1}{2}(c_I + c_O) 2b_p$$

These expressions are evaluated from panel c_I and c_O, as calculated using λ_I and λ_O, or using the ratio λ_O/λ_I (c_{ref} may not be the same for all panels).

The mean aerodynamic center is the point where there is zero moment due to lift: $\bar{x}_A C_L S = \bar{x}_A \int c_\ell c\, dy = \int x c_\ell c\, dy$, with $c c_\ell = \ell(y)$ the spanwise lift distribution. Thus

$$\int_0^1 \ell(\eta)(\bar{x}_A - x_{AC}(\eta))\, d\eta = 0$$

The locus of section aerodynamic centers x_{AC} is described by the panel sweep Λ_p and the offset x_{Ip} at the inboard end of the panel. These offsets can be a fixed input, a fraction of the root chord, or a fraction of the panel inboard chord. Assuming elliptical loading ($\ell = \sqrt{1 - \eta^2}$) gives

$$\frac{\pi}{4} \bar{x}_A = \int_0^1 \ell(\eta) x_{AC} d\eta = \sum \int \sqrt{1 - \eta^2}\left(\widehat{x}_{Ip} + \frac{b}{2} \tan \Lambda_p\, \eta\right) d\eta$$

$$= \sum \left[\widehat{x}_{Ip} \frac{1}{2}\left(\eta\sqrt{1 - \eta^2} + \sin^{-1}\eta\right) - \frac{b}{2} \tan \Lambda_p \frac{1}{3}(1 - \eta^2)^{3/2}\right]_{\eta_I}^{\eta_O}$$

where $\widehat{x}_{Ip} = \sum_{q=2}^p \left(x_{Iq} + (b/2)\tan\Lambda_{q-1}(\eta_{O(q-1)} - \eta_{I(q-1)})\right) - (b/2)\tan\Lambda_p\eta_{Ip}$. The vertical position of the mean aerodynamic center is obtained in a similar fashion, from panel dihedral δ_p and offset z_{Ip} at the inboard edge of the panel. Assuming uniform loading ($\ell = 1$) gives

$$\bar{z}_A = \int_0^1 z_{AC} d\eta = \sum \int \left(\widehat{z}_{Ip} + \frac{b}{2} \tan \delta_p\, \eta\right) d\eta = \sum \left[\widehat{z}_{Ip}\eta + \frac{b}{2} \tan \delta_p \frac{1}{2}\eta^2\right]_{\eta_I}^{\eta_O}$$

Then $(\bar{x}_A, \bar{z}_A)$ is the offset of the mean aerodynamic center from the root chord aerodynamic center. Finally,

$$\Lambda = \tan^{-1}\left(\sum \frac{b_p}{b/2}\tan\Lambda_p\right)$$

$$\delta = \tan^{-1}\left(\sum \frac{b_p}{b/2}\tan\delta_p\right)$$

$$\lambda = \frac{2c}{c_{\text{root}}} - 1$$

are the wing overall sweep, dihedral, and taper.

The wing contribution to the aircraft operating length is $x_{\text{wing}} + (0.25c)\cos i$ (forward), $x_{\text{wing}} - (0.75c)\cos i$ (aft), and $y_{\text{wing}} \pm b/2$ (lateral).

13–2   Control and Loads

The control variables are flap δ_F, flaperon δ_f, aileron δ_a, and incidence i. The flaperon deflection can be specified as a fraction of flap deflection, or as an increment relative to the flap deflection, or the flaperon can be independent of the flap. The flaperon and aileron are the same surface, generating symmetric and antisymmetric loads, respectively, hence with different connections to pilot controls.

With more than one wing panel, each panel can have control variables: flap δ_{Fp}, flaperon δ_{fp}, aileron δ_{ap}, and incidence i_p. The outboard panel ($p \geq 2$) control or incidence can be specified independently, or in terms of the root panel ($p = 1$) control or incidence (either fraction or increment).

Each control is described by the ratio of the control surface chord to the wing panel chord, $\ell_f = c_f/c_p$; and by the ratio of the control surface span to wing panel span, $f_b = b_f/b_p$, such that the control surface area is obtained from the panel area by $S_f = \ell_f f_b S_p$.

Optionally the wing can have flow control. The momentum coefficient C_μ is a control, allowing it to be connected to an aircraft control and thus set (or trimmed) for each flight state.

The wing position z^F can be the pivot of a wing tilted at incidence angle i, with the aerodynamic forces acting at z^F_{ac} and the wing center-of-gravity at z^F_{cg}. The aerodynamic center offset from the pivot is specified as SL/BL/WL increments at zero incidence, so

$$z^F_{ac} = z^F + Y_{-i}\begin{pmatrix} -\text{dSL} \\ \text{dBL} \\ -\text{dWL} \end{pmatrix} = z^F + \begin{pmatrix} -\cos i(\text{dSL}) - \sin i(\text{dWL}) \\ \text{dBL} \\ \sin i(\text{dSL}) - \cos i(\text{dWL}) \end{pmatrix}$$

is where the wing velocity is evaluated and the wing forces act. The center of gravity offset from the pivot is specified as SL/WL increments at zero incidence, so the aircraft center of gravity shift due to wing tilt is

$$\Delta z^F_{cg} = \frac{W_{\text{wing}}}{W}(Y_{-i} - I)\begin{pmatrix} -\text{dSL} \\ 0 \\ -\text{dWL} \end{pmatrix} = \frac{W_{\text{wing}}}{W}\begin{pmatrix} -(\cos i - 1)(\text{dSL}) - \sin i(\text{dWL}) \\ 0 \\ \sin i(\text{dSL}) - (\cos i - 1)(\text{dWL}) \end{pmatrix}$$

where W is the gross weight and W_{wing} is the wing weight, and the aircraft center-of-gravity is specified for zero wing tilt. This model for aerodynamic center and center of gravity shifts should only be used when all wing panels have the same incidence.

13–3 Aerodynamics

The aerodynamic velocity of the wing relative to the air, including interference, is calculated in component axes, v^B. The angle of attack α_{wing} (hence C^{BA}) and dynamic pressure q are calculated from v^B. The reference area for the wing aerodynamic coefficients is the planform area, S. The wetted area contribution is twice the exposed area: $S_{\text{wet}} = 2(S - c w_{\text{fus}})$, where w_{fus} is the fuselage width.

The wing vertical drag can be fixed, specified as a drag area $(D/q)_V$; or the drag can be scaled, specified as a drag coefficient C_{DV} based on the wing area; or calculated from an airfoil section drag coefficient (for -90 deg angle of attack) and the wing area immersed in the rotor wake:

$$C_{DV} = \frac{1}{S} c_{d90} \left(S - S_{\text{center}} - f_{d90} b_F c_F (1 - \cos \delta_F) - f_{d90} b_f c_f (1 - \cos \delta_f) \right)$$

The term $S_{\text{center}} = c(w_{\text{fus}} + 2d_{\text{fus}})$ (where w_{fus} is the fuselage width and d_{fus} the rotor-fuselage clearance) is the area not immersed in the rotor wake, and is used only for tiltrotors. The last two terms account for the change in wing area due to flap and flaperon deflection, with an effectiveness factor f_{d90}.

From the control surface deflection and geometry, the lift coefficient, maximum lift angle, moment coefficient, and drag coefficient increments are evaluated: ΔC_{Lf}, $\Delta \alpha_{\max f}$, ΔC_{Mf}, and ΔC_{Df}. These increments are the sum of contributions from flap and flaperon deflection, hence weighted by the control surface area. The drag coefficient increment includes the contribution from aileron deflection.

The momentum coefficient $C_\mu = \dot{m}V_j/qS$ is a control of the wing, where $\dot{m}V_j$ is the momentum flux, q is the free stream dynamic pressure, and S the wing area. The momentum flux (gross force) required is supplied by the compressor or jet: $F_{Greq} = \dot{m}V_j = qSC_\mu$. The engine group or jet group performance includes the efficiency of the flow control ducts and nozzle. The compressor model also gives the engine group power required to produce F_{Greq} (summed over all components with flow control). From the momentum coefficient, the increments of lift coefficient, maximum lift angle, moment coefficient, and drag coefficient are evaluated: $\Delta C_{L\mu}$, $\Delta \alpha_{\max \mu}$, $\Delta C_{M\mu}$, $\Delta C_{D\mu}$.

13-3.1 Lift

The wing lift is defined in terms of lift-curve slope $C_{L\alpha}$ and maximum lift coefficient $C_{L\max}$ (based on wing planform area). The three-dimensional lift-curve slope is input directly or calculated from the two-dimensional lift-curve slope:

$$C_{L\alpha} = \frac{c_{\ell\alpha}}{1 + c_{\ell\alpha}(1 + \tau)/(\pi A\!R)}$$

where τ accounts for non-elliptical loading. Optionally the input incompressible lift-curve slope is multiplied by the compressibility factor F_c. The effective angle of attack is $\alpha_e = \alpha_{\text{wing}} + i - \alpha_{zl}$, where α_{zl} is the angle of zero lift; in reverse flow ($|\alpha_e| > 90$), $\alpha_e \leftarrow \alpha_e - 180 \operatorname{sign}\alpha_e$. Let $\alpha_{\max} = C_{L\max}/C_{L\alpha}$ be the angle-of-attack increment (above or below zero lift angle) for maximum lift. Including the change of maximum lift angle caused by control deflection and flow control, $A_{\max} = \alpha_{\max} + \Delta\alpha_{\max f} + \Delta\alpha_{\max\mu}$ and $A_{\min} = -\alpha_{\max} + \Delta\alpha_{\max f} + \Delta\alpha_{\max\mu}$. Then

$$C_L = \begin{cases} C_{L\alpha}\alpha_e + \Delta C_{Lf} + \Delta C_{L\mu} & A_{\min} \leq \alpha_e \leq A_{\max} \\[2ex] (C_{L\alpha}A_{\max} + \Delta C_{Lf} + \Delta C_{L\mu}) \max\left(0, \dfrac{f\pi/2 - |\alpha_e|}{f\pi/2 - |A_{\max}|}\right) & \alpha_e > A_{\max} \\[2ex] (C_{L\alpha}A_{\min} + \Delta C_{Lf} + \Delta C_{L\mu}) \max\left(0, \dfrac{f\pi/2 - |\alpha_e|}{f\pi/2 - |A_{\min}|}\right) & \alpha_e < A_{\min} \end{cases}$$

(for zero lift at $f90$ deg angle of attack, $f = .9$). Note that $C_{L\alpha}A_{\max} + \Delta C_{Lf} + \Delta C_{L\mu} = C_{L\alpha}\alpha_{\max} + \Delta C_{L\max f} + \Delta C_{L\max\mu}$. In sideward flight, $C_L = 0$. Finally, $L = qSC_L$ is the lift force.

13-3.2   Pitch Moment

The wing pitch moment coefficient is $C_M = C_{Mac} + \Delta C_{Mf} + \Delta C_{M\mu}$. Then $M = qScC_M$ is the pitch moment.

13-3.3   Roll Moment

The only wing roll moment considered is that produced by aileron control. Typically the flaperon and aileron are the same surface, but they are treated separately in this model. The aileron geometry is specified as for the flaperon and flap, hence includes both sides of the wing. The lift coefficient increment ΔC_{La} is evaluated as for the flaperon, so one-half of this lift acts up (on the right side) and one-half acts down. The roll moment is then $M_x = 2(\Delta L_a/2)y$, where y is the lateral position of the aileron aerodynamic center, measured from the wing centerline (defined as a fraction of the wing semi-span).

The roll moment coefficient is $C_\ell = -\frac{y}{b/2}\frac{1}{2}\Delta C_{La}$. Then $M_x = qSbC_\ell$ is the roll moment.

13-3.4   Drag

The drag area or drag coefficient is defined for forward flight and vertical flight. The effective angle of attack is $\alpha_e = \alpha_{\text{wing}} + i - \alpha_{D\min}$, where $\alpha_{D\min}$ is the angle of minimum drag; in reverse flow ($|\alpha_e| > 90$), $\alpha_e \leftarrow \alpha_e - 180\,\text{sign}\alpha_e$. For angles of attack less than a transition angle α_t, the drag coefficient equals the forward flight (minimum) drag C_{D0}, plus an angle of attack term and the control increment. If the angle of attack is greater than a separation angle $\alpha_s < \alpha_t$, there is an additional drag increase. Thus if $|\alpha_e| \leq \alpha_t$, the profile drag is

$$C_{Dp} = C_{D0}\left(1 + K_d|\alpha_e|^{X_d} + K_s(|\alpha_e| - \alpha_s)^{X_s}\right) + \Delta C_{Dc} + \Delta C_{Df} + \Delta C_{D\mu}$$

where the separation (K_s) term is present only for $|\alpha_e| > \alpha_s$; and otherwise

$$C_{Dt} = C_{D0}\left(1 + K_d|\alpha_t|^{X_d} + K_s(|\alpha_t| - \alpha_s)^{X_s}\right) + \Delta C_{Dc} + \Delta C_{Df} + \Delta C_{D\mu}$$

$$C_{Dp} = C_{Dt} + (C_{DV} - C_{Dt})\sin\left(\frac{\pi}{2}\frac{|\alpha_e| - \alpha_t}{\pi/2 - \alpha_t}\right)$$

ΔC_{Dc} is the compressibility drag increment. Optionally there might be no angle of attack variation at low angles ($K_d = 0$ and/or $K_s = 0$), or quadratic variation ($X_d = 2$), or cubic variation for the separation term ($X_s = 3$). For sideward flight ($v_x^B = 0$) the drag is obtained using $\phi_v = \tan^{-1}(-v_z^B/v_y^B)$ to interpolate the vertical coefficient: $C_D = C_{D0}\cos^2\phi_v + C_{DV}\sin^2\phi_v$. The induced drag is obtained from the lift coefficient, aspect ratio, and Oswald efficiency e:

$$C_{Di} = \frac{(C_L - C_{L0})^2}{\pi e A\!R}$$

Conventionally the Oswald efficiency e represents the wing parasite drag variation with lift, as well as the induced drag (hence the use of C_{L0}). If C_{Dp} varies with angle of attack, then e is just the span efficiency factor for the induced power (and C_{L0} should be zero). The wing-body interference is specified as a drag area, or a drag coefficient based on the wing area. Then

$$D = qSC_D = qS\left(C_{Dp} + C_{Di} + C_{Dwb}\right)$$

is the drag force. The other forces and moments are zero.

13-3.5   Wing Panels

The wing panels can have separate controls, different incidence angles, and different interference from the rotors. Thus the lift, drag, and moment coefficients are evaluated separately for each panel, based on the panel area S_p and mean chord c_p. The coefficient increments due to control surface deflection are calculated using the ratio of the control surface area to panel area, $S_f/S_p = \ell_f f_b$. The lateral position of the aileron aerodynamic center is $\eta_a b_p$ from the panel inboard edge, so $y/(b/2) = \eta_{E(p-1)} + \eta_a b_p/(b/2)$ from the wing centerline. Then the total wing loads are:

$$L = \sum q_p S_p C_{Lp}$$
$$M = \sum q_p S_p c_p C_{Mp}$$
$$M_x = \sum q_p S_p b C_{\ell p}$$
$$D = \sum q_p S_p C_{Dpp} + \langle qS \rangle (C_{Di} + C_{Dwb})$$

The sums are over all panels (left and right). The reference area is $S = \sum S_p$, accounting for possible absence of wing extensions. The total wing coefficients are based on $\langle qS \rangle = \sum q_p S_p$. The three-dimensional lift-curve slope $C_{L\alpha}$ is calculated for the entire wing and used for each panel. The induced drag is calculated for the entire wing, from the total C_L. Since $C^{BF} = I$,

$$F^F = C^{FB} C^{BA} F^A = \sum C_p^{BA} q_p S_p \begin{pmatrix} -C_{Dpp} - C_{Di} - C_{Dwb} \\ 0 \\ -C_L \end{pmatrix}$$

is the wing aerodynamic force.

13-3.6   Interference

With more than one wing, the interference velocity at other wings is proportional to the induced velocity of the wing producing the interference: $v_{\text{int}}^F = K_{\text{int}} v_{\text{ind}}^F$. The induced velocity is obtained from the induced drag, assumed to act in the k^B direction: $\alpha_{\text{ind}} = v_{\text{ind}}/|v^B| = C_{Di}/C_L = C_L/(\pi e A\!\!R)$, $v_{\text{ind}}^F = C^{FB} k^B |v^B| \alpha_{\text{ind}}$. For tandem wings, typically $K_{\text{int}} = 2$ for the interference of the front wing on the aft wing, and $K_{\text{int}} = 0$ for the interference of the aft wing on the front wing. For biplane wings, the mutual interference is typically $K_{\text{int}} = 0.7$ (upper on lower, and lower on upper). The induced drag is then

$$C_{Di} = \frac{(C_L - C_{L0})^2}{\pi e A\!\!R} + C_L \sum_{\text{other wings}} \alpha_{\text{int}}$$

$$\alpha_{\text{int}} = K_{\text{int}} \alpha_{\text{ind}} = K_{\text{int}} \left(\frac{C_L}{\pi e A\!\!R} \right)_{\text{other wing}}$$

The induced velocity from the rotors is included in the angle of attack of the wing. The rotor interference must also be accounted for in the wing induced power:

$$C_{Di} = \frac{(C_L - C_{L0})^2}{\pi e A\!\!R} + C_L \left[\sum_{\text{other wings}} K_{\text{int}} \alpha_{\text{ind}} + \sum_{\text{rotors}} C_{\text{int}} \alpha_{\text{ind}} \right]$$

The angle $\alpha_{\text{ind}} = v_{\text{ind}}/V$ is obtained from the rotor induced velocity λ_i. If the interference is wing-like, $v_{\text{ind}} = \Omega R \kappa \lambda_i$ (so $v_{\text{ind}} \propto L/\rho b^2 V \propto T/2\rho A V$). If the interference is propeller-like, $v_{\text{ind}} = V \kappa \lambda_i$ (so

$v_{\text{ind}} \propto \Gamma/b \propto T/2\rho A\Omega R)$. The interference factor can be evaluated from wing induced drag calculations: $C_{\text{int}} = \Delta C_{Di}/(C_L \kappa \lambda_i/\mu)$ or $C_{\text{int}} = \Delta C_{Di}/(C_L \kappa \lambda_i)$. Typically for tiltrotors the interference is wing-like, with $C_{\text{int}} \cong -0.06$.

The wing interference at the tail produces an angle-of-attack change $\epsilon = E(C_L/C_{L\alpha})$, where $E = d\epsilon/d\alpha$ is an input factor determined by the aircraft geometry. Then from the velocity v^B of the wing,

$$v_{\text{int}}^F = C^{FB} \begin{pmatrix} -\epsilon v_z^B \\ 0 \\ \epsilon v_x^B \end{pmatrix}$$

is the interference velocity at the tail.

The wing interference at the rotor can produce interference power. The induced velocity at the rotor disk is $v_{\text{int}}^F = K_{\text{int}} v_{\text{ind}}^F$, with $v_{\text{ind}}^F = C^{FB} k^B |v^B| \alpha_{\text{ind}}$ again. Separate interference factors K_{int} are used for the components of the interference velocity parallel to and perpendicular to the rotor force vector (roughly normal to and in the plane of the rotor disk).

13–4   Wing Extensions

The wing can have extensions, defined as wing portions of span b_X at each wing tip. For the tiltrotor configuration in particular, the wing weight depends on the distribution of wing area outboard (the extension) and inboard of the rotor and nacelle location. Wing extensions are defined as a set of wing panels at the tip. The extension span and area are the sum of the panel quantities, $b_X = \sum_{\text{ext}} b_p$ and $S_X = \sum_{\text{ext}} S_p$. The inboard span and area are then $b_I = b - 2b_X$, $S_I = S - S_X$. Optionally the wing extensions can be considered a kit, hence the extensions can be absent for designated flight conditions or missions. As a kit, the wing extension weight is considered fixed useful load. With wing extensions removed, the aerodynamic analysis considers only the remaining wing panels. The total wing coefficients are then based on the area without the extensions. For the induced drag and interference, the effective aspect ratio is then reduced by the factor $(b_I/b)^2$, since the lift and drag coefficients are still based on total wing area S.

13–5   Wing Kit

The wing can be a kit, the kit weight an input fraction of the total wing weight. The wing kit weight can be part of the wing group, or considered fixed useful load. With the kit removed, there are no aerodynamic loads or aerodynamic interference generated by the wing, and the wing kit weight is omitted.

13–6   Weights

The wing group consists of: basic structure (primary structure, consisting of torque box and spars, plus extensions); fairings (leading edge and trailing edge); fittings (non-structural); fold/tilt structure; and control surfaces (flaps, ailerons, flaperons, and spoilers). There are separate models for a tiltrotor or tiltwing configuration and for other configurations (including a compound helicopter).

The AFDD wing weight models are based on parameters for the basic wing plus the wing tip extensions (not the total wing and extensions). The tiltrotor wing model requires the weight on the wing tips (both sides), consisting of: rotor group, engine system, drive system (except drive shaft), engine

section or nacelle group, air induction group, rotary wing and conversion flight controls, hydraulic group, trapped fluids, and wing extensions. An adjustment (factor and increment) of this calculated weight can be used; a negative increment is required when the engine and transmission are not at the tip location with the rotor.

Empennage

The aircraft can have one or more tail surfaces, or no tail surface. Each tail is designated as horizontal or vertical, affecting some parameter definitions.

14–1 Geometry

The tail is described by planform area S, span b, chord $c = S/b$, and aspect ratio $A\!R = b^2/S$. The tail volume can be referenced to rotor radius and disk area, $V = S\ell/RA$; to wing area and chord for horizontal tails, $V = S\ell/S_w c_w$; or to wing area and span for vertical tails, $V = S\ell/S_w b_w$. Here the tail length is $\ell = |x_{ht} - x_{cg}|$ or $\ell = |x_{vt} - x_{cg}|$ for horizontal tail or vertical tail, respectively. The geometry is specified in terms of S or V; and b, or $A\!R$, or c. The elevator or rudder is described by the ratio of control surface chord to tail chord, c_f/c; and the ratio of control surface span to tail span, b_f/b.

The tail contribution to the aircraft operating length is $x_{\text{tail}} + 0.25c$ (forward), $x_{\text{tail}} - 0.75c$ (aft), and $y_{\text{tail}} \pm (b/2)C$ (lateral), where $C = \cos\phi$ for a horizontal tail and $C = \cos(\phi - 90)$ for a vertical tail.

14–2 Control and Loads

The tail is at position z^F, where the aerodynamic forces act. The scaled input for tail position can be referenced to the fuselage length, or to the rotor radius.

The horizontal tail can have a cant angle ϕ (positive tilt to left, becomes vertical tail for $\phi = 90$ deg). Thus the component axes are given by $C^{BF} = X_{-\phi}$. The control variables are elevator δ_e and incidence i.

The convention for nominal orientation of the vertical tail is positive lift to the left, so aircraft sideslip (positive to right) generates positive tail angle of attack and positive tail lift. The vertical tail can have a cant angle ϕ (positive tilt to right, becomes horizontal tail for $\phi = 90$), so the component axes are given by $C^{BF} = X_{-90+\phi}$. The control variables are rudder δ_r and incidence i.

14–3 Aerodynamics

The aerodynamic velocity of the tail relative to the air, including interference, is calculated in component axes, v^B. The angle of attack α_{tail} (hence C^{BA}) and dynamic pressure q are calculated from v^B. The reference area for the tail aerodynamic coefficients is the planform area, S. The wetted area contribution is $S_{\text{wet}} = 2S$. From the elevator or rudder deflection and geometry, the lift coefficient, maximum lift angle, and drag coefficient increments are evaluated: ΔC_{Lf}, $\Delta\alpha_{\text{max}f}$, and ΔC_{Df}.

14-3.1   Lift

The tail lift is defined in terms of lift-curve slope $C_{L\alpha}$ and maximum lift coefficient C_{Lmax} (based on tail planform area). The three-dimensional lift-curve slope is input directly or calculated from the two-dimensional lift-curve slope:

$$C_{L\alpha} = \frac{c_{\ell\alpha}}{1 + c_{\ell\alpha}(1 + \tau)/(\pi A\!\!R)}$$

where τ accounts for non-elliptical loading. Optionally the input incompressible lift-curve slope is multiplied by the compressibility factor F_c. The effective angle of attack is $\alpha_e = \alpha_{\text{tail}} + i - \alpha_{zl}$, where α_{zl} is the angle of zero lift; in reverse flow ($|\alpha_e| > 90$), $\alpha_e \leftarrow \alpha_e - 180\,\text{sign}\alpha_e$. Let $\alpha_{\max} = C_{Lmax}/C_{L\alpha}$ be the angle-of-attack increment (above or below zero lift angle) for maximum lift. Including the change of maximum lift angle caused by control deflection, $A_{\max} = \alpha_{\max} + \Delta\alpha_{\max f}$ and $A_{\min} = -\alpha_{\max} + \Delta\alpha_{\max f}$. Then

$$C_L = \begin{cases} C_{L\alpha}\alpha_e + \Delta C_{Lf} & A_{\min} \leq \alpha_e \leq A_{\max} \\[2mm] (C_{L\alpha}A_{\max} + \Delta C_{Lf})\,\max\left(0, \dfrac{f\pi/2 - |\alpha_e|}{f\pi/2 - |A_{\max}|}\right) & \alpha_e > A_{\max} \\[3mm] (C_{L\alpha}A_{\min} + \Delta C_{Lf})\,\max\left(0, \dfrac{f\pi/2 - |\alpha_e|}{f\pi/2 - |A_{\min}|}\right) & \alpha_e < A_{\min} \end{cases}$$

(for zero lift at $f90$ deg angle of attack, $f = .9$). Note that $C_{L\alpha}A_{\max} + \Delta C_{Lf} = C_{L\alpha}\alpha_{\max} + \Delta C_{Lmaxf}$. In sideward flight (defined by $(v_x^B)^2 + (v_z^B)^2 < (0.05|v^B|)^2)$, $C_L = 0$. Finally, $L = qSC_L$ is the lift force.

14-3.2   Drag

The drag area or drag coefficient is defined for forward flight and vertical flight. The effective angle of attack is $\alpha_e = \alpha_{\text{tail}} + i - \alpha_{Dmin}$, where α_{Dmin} is the angle of minimum drag; in reverse flow ($|\alpha_e| > 90$), $\alpha_e \leftarrow \alpha_e - 180\,\text{sign}\alpha_e$. For angles of attack less than a transition angle α_t, the drag coefficient equals the forward flight (minimum) drag C_{D0}, plus an angle-of-attack term and the control increment. Thus if $|\alpha_e| \leq \alpha_t$, the profile drag is

$$C_{Dp} = C_{D0}\left(1 + K_d|\alpha_e|^{X_d}\right) + \Delta C_{Dc} + \Delta C_{Df}$$

and otherwise

$$C_{Dt} = C_{D0}\left(1 + K_d|\alpha_t|^{X_d}\right) + \Delta C_{Dc} + \Delta C_{Df}$$
$$C_{Dp} = C_{Dt} + (C_{DV} - C_{Dt})\sin\left(\frac{\pi}{2}\frac{|\alpha_e| - \alpha_t}{\pi/2 - \alpha_t}\right)$$

ΔC_{Dc} is the compressibility drag increment. Optionally there might be no angle-of-attack variation at low angles ($K_d = 0$), or quadratic variation ($X_d = 2$). In sideward flight (defined by $(v_x^B)^2 + (v_z^B)^2 < (0.05|v^B|)^2)$, the drag is obtained using $\phi_v = \tan^{-1}(-v_z^B/v_y^B)$ to interpolate the vertical coefficient: $C_{Dp} = C_{D0}\cos^2\phi_v + C_{DV}\sin^2\phi_v$. The induced drag is obtained from the lift coefficient, aspect ratio, and Oswald efficiency e:

$$C_{Di} = \frac{(C_L - C_{L0})^2}{\pi e A\!\!R}$$

Conventionally the Oswald efficiency e can represent the tail parasite drag variation with lift, as well as the induced drag (hence the use of C_{L0}). Then

$$D = qSC_D = qS\left(C_{Dp} + C_{Di}\right)$$

is the drag force. The other forces and moments are zero.

14–4   V-Tail

A V-tail is modeled as a pair of horizontal and vertical tails, each sized by area or tail volume. This model follows references 1 and 2. The aerodynamic loads are calculated separately for each tail. The weight is calculated only for the second tail, using the V-tail area and aspect ratio. The equivalence of the V-tail and conventional tail gives $S_{ht} = S_V \cos^2 \delta$ and $S_{vt} = S_V \sin^2 \delta$. Hence the V-tail dihedral angle is $\delta = \tan^{-1} \sqrt{S_{vt}/S_{ht}}$, and the cant angle $\phi = 0$ for both. Assuming the same aspect ratio for the V-tail and horizontal tail, it follows:

$$S_V = S_{ht} + S_{vt}$$

$$A\!R_V = A\!R_{ht}$$

and then the V-tail span is $b_V = \sqrt{A\!R_V S_V} = b_{ht}/\cos \delta$. The location (which should be the same for the two tails) is the midpoint of the V-tail, vertically and laterally. An upward V-tail and a downward V-tail are identical in this model.

14–5   Weights

The empennage group consists of the horizontal tail, vertical tail, and tail-rotor. The tail plane weight consists of the basic structure and fold structure. The tail weight (empennage group) model depends on the configuration: helicopters and compounds, or tiltrotors and tiltwings. Separate weight models are available for horizontal and vertical tails.

The AFDD tail weight model depends on the design dive speed at sea level (input or calculated). The calculated dive speed is $V_{\text{dive}} = 1.25 V_{\text{max}}$, from the maximum speed at the design gross weight and sea level standard conditions.

14–6   References

1) Raymer, D.P. *Aircraft Design: A Conceptual Approach*. Fourth Edition. Reston, Virginia: American Institute of Aeronautics and Astronautics, 2006.

2) Purser, P.E., and Campbell, J.P. "Experimental Verification of a Simplified Vee-Tail Theory and Analysis of Available Data on Complete Models with Vee Tails." NACA Report 823, 1945.

Propulsion System

The aircraft propulsion system can be constructed from a number of components: propulsion groups, engine groups, jet groups, charge groups, and fuel tank systems. Figure 15-1 illustrates the configuration possibilities.

The aircraft can have one or more propulsion groups, or none. Each propulsion group is a set of components (rotors) and engine groups, connected by a drive system. The components define the power required, and the engine groups define the power available. There are one or more drive states, with a set of gear ratios for each state. The power required equals the sum of component power, transmission losses, and accessory losses.

An engine group consists of one or more engines of a specific type. An engine group transfers power by shaft torque, so it is associated with a propulsion group. For each engine type an engine model is defined. The engine model describes a particular engine, used in one or more engine groups. The models include turboshaft engines (perhaps convertible, for turbojet operation or reaction drive), reciprocating engines, compressors, electric motors, electric generators, and generator-motors.

The aircraft can have one or more jet groups, or none. A jet group produces a force on the aircraft. A jet model describes a particular jet, used in one or more jet groups. The models include turbojet and turbofan engines (perhaps convertible, for reaction drive), reaction drive, and a simple force. A reaction drive supplies a blade force that provides the rotor power required.

The aircraft can have one or more charge groups, or none. A charge group generates energy for the aircraft. A charge model describes a particular charger, used in one or more charge groups. The models include fuel cells, solar cells, and a simple charger.

There are one or more fuel tank systems for the aircraft. Fuel tank systems are associated with the engine groups, jet groups, and charge groups. Fuel quantity is measured as either weight or energy. Fuels considered include jet fuel, gasoline, diesel, and hydrogen. Storage systems considered include batteries, capacitors, and flywheels.

15–1 Referred Performance

Referred performance parameters are used for propulsion system components that operate with air. The operating condition and atmosphere give the standard conditions (temperature T_{std} and pressure p_{std}) for a specified pressure altitude; the sea-level standard conditions (temperature T_0 and pressure p_0); and the operating temperature T and pressure p. Here the temperatures are °R or °K. The performance characteristics depend on the temperature ratio $\theta = T/T_0$ and pressure ratio $\delta = p/p_0$. The flight Mach number $M = V/c_s = V/c_{s0}\sqrt{\theta}$ is obtained from the aircraft speed V.
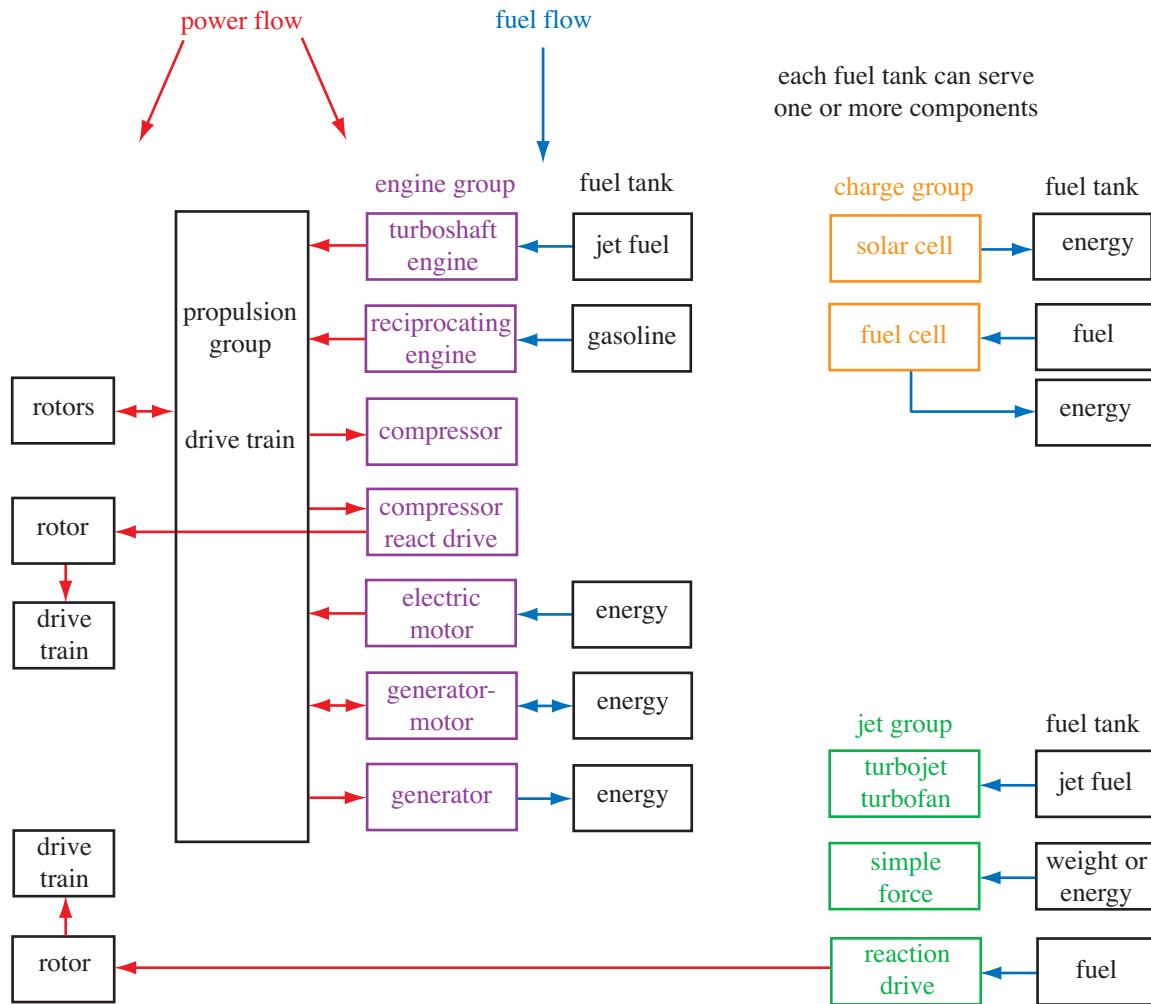
Figure 15-1. Propulsion system components.

The referred or corrected performance parameters are:

power	$\dfrac{P}{\delta\sqrt{\theta}}$	fuel flow	$\dfrac{\dot{w}}{\delta\sqrt{\theta}}$
mass flow	$\dfrac{\dot{m}}{\delta/\sqrt{\theta}}$	force	$\dfrac{F}{\delta}$
specific power	$\dfrac{P/\dot{m}}{\theta}$	rotational speed	$\dfrac{N}{\sqrt{\theta}}$

where P is power, $\dot{m}$ is mass flow, $\dot{w}$ is fuel flow, F is a force, and N is a rotational speed. The performance at sea-level-standard static conditions is indicated by subscript 0.

15–2 Engine Ratings

Engine performance depends on the engine rating. Each engine rating has specific operating limitations, most importantly an operating time limit intended to avoid damage to the engine.

The power available from a turboshaft engine depends on the engine rating. Typical engine ratings are given in table 15-1. Engine power is generally specified in terms of sea level standard (SLS) static MCP. Takeoff typically uses MRP. CRP or ERP is restricted to use in one-engine inoperative (OEI) emergencies.

Table 15-1. Typical turboshaft engine ratings.

rating	description	time limit
MCP	maximum continuous power	∞
IRP	intermediate rated power	30 min
MRP	maximum rated power	10 min
CRP	contingency rated power	2.5 min
ERP	emergency rated power	1.0 min

The thrust available from a turbojet or turbofan engine depends on the engine rating. Typical engine ratings are given in table 15-2.

Table 15-2. Typical jet engine ratings.

rating	description	time limit
MCT	maximum continuous thrust	∞
MTO	maximum takeoff thrust	5 min

15–3 Efficiency from Equivalent Circuit

The efficiency of an electrical device can be expressed in terms of its power by considering an equivalent circuit, defined by internal resistance R and current I_0. The total voltage is $V_x = V + IR$ and the total current is $I_x = I + I_0$. Then the useful power is

$$P = VI = (V_x - IR)I = V_x I_x - I^2 R - V_x I_0 = P_{\text{total}} - P_{\text{loss}}$$

and the efficiency is

$$\eta = \frac{P}{P_{\text{total}}} = \frac{P}{P + P_{\text{loss}}} = \frac{P}{P + P^2 R/V^2 + P_0}$$

In terms of a reference power, let

$$R/V^2 = \frac{1}{P_{\text{ref}}} \left(\frac{1}{\eta_{\text{ref}}} - 1 \right)$$

$$P_0 = c P_{\text{ref}}$$

Then

$$\frac{1}{\eta} = 1 + P(R/V^2) + P_0/P = 1 + \frac{P}{P_{\text{ref}}} \left(\frac{1}{\eta_{\text{ref}}} - 1 \right) + c \frac{P_{\text{ref}}}{P}$$

So $\eta = (1/\eta_{\text{ref}} + c)^{-1}$ at $P = P_{\text{ref}}$. The efficiency decreases with P because of the internal resistance, but is zero at $P = 0$ because of the internal current term.

15–4   Control and Loads

Geometry and control are defined for engine groups, jet groups, and charge groups. The group amplitude A and mode B are control variables:

$$A = A_0 + T_A c_{AC}$$
$$B = B_0 + T_B c_{AC}$$

with A_0 and B_0 zero, constant, or a function of flight speed (piecewise linear input). The amplitude can be power (engine group), thrust (jet group), or power (charge group). The mode can be mass flow (for convertible engines), or power flow (for generator-motor).

The group orientation is specified by selecting a nominal direction e_{f0} in body axes (positive or negative x-, y-, or z-axis; usually thrust forward, hence positive x-axis); then applying a yaw angle ψ; and then an incidence or tilt angle i (table 15-3). The yaw and incidence angles can be connected to the aircraft controls c_{AC}:

$$\psi = \psi_0 + T_\psi c_{AC}$$
$$i = i_0 + T_i c_{AC}$$

with ψ_0 and i_0 zero, constant, or a function of flight speed (piecewise linear input). Hence the incidence and yaw angles can be fixed orientation or can be control variables. Optionally the lateral position of the group can be set equal to that of a designated rotor (useful for tiltrotors when the rotor hub lateral position is calculated from the clearance or wing geometry).

The group produces a force T, acting in the direction of the group; and a drag D, acting in the wind direction. The group is at location z^F. The force and moment acting on the aircraft in body axes are thus:

$$F^F = e_f T + e_d D$$
$$M^F = \widetilde{\Delta z^F} F^F$$

where $\Delta z^F = z^F - z^F_{cg}$, e_f is the force direction, and e_d is the drag direction. The velocity relative to the air gives $e_d = -v^F / |v^F|$ (no interference). The group axes are $C^{BF} = U_i V_\psi$, where U and V depend on the nominal direction, as described in table 15-3. The force direction is $e_f = C^{FB} e_{f0}$.

For a tiltrotor aircraft, one of the aircraft controls is the nacelle angle, with the convention $\alpha_{\text{tilt}} = 0$ for cruise and $\alpha_{\text{tilt}} = 90$ deg for helicopter mode. The incidence angle is then connected to α_{tilt} by defining the matrix T_i appropriately. If the nominal direction is defined for airplane mode ($+x$), then $i = \alpha_{\text{tilt}}$ should be used; if the nominal direction is defined for helicopter mode ($-z$), then $i = \alpha_{\text{tilt}} - 90$ should be used.

Table 15-3. Group orientation.

nominal (F axes)		e_{f0}	incidence, + for force	yaw, + for force	$C^{BF} = U_i V_\psi$
x	forward	i	up	right	$Y_i Z_\psi$
$-x$	aft	$-i$	up	right	$Y_{-i} Z_{-\psi}$
y	right	j	aft	up	$Z_i X_{-\psi}$
$-y$	left	$-j$	aft	up	$Z_{-i} X_\psi$
z	down	k	aft	right	$Y_{-i} X_{-\psi}$
$-z$	up	$-k$	aft	right	$Y_i X_\psi$

15–5   Nacelle Drag

An aerodynamic model is defined for engine groups, jet groups, and charge groups. The group includes a nacelle, which contributes to the aircraft drag. The component drag contributions must be consistent. The pylon is the rotor support and the nacelle is the engine support. The drag model for a tiltrotor aircraft with tilting engines would use the pylon drag (and no nacelle drag), since the pylon is connected to the rotor shaft axes; with non-tilting engines it would use the nacelle drag as well.

The nacelle drag acts at the group location z^F. The nacelle axes are the group axes, hence C^{BF} is calculated as described previously (see table 15-1). For the nominal direction forward ($+x$-axis), the nacelle z-axis is downward and the x-axis is forward; zero incidence angle corresponds to zero angle of attack; and 90 deg incidence angle corresponds to 90 deg angle of attack (vertical drag). The velocity, angle of attack, and dynamic pressure are calculated at the nacelle (without interference). The reference area for the nacelle drag coefficient is the nacelle wetted area. The wetted area is input, or calculated from the weight w:

$$S_{\text{wet}} = k\left(w/N\right)^{2/3}$$

where N is the number of engines, jets, or chargers; and the units of k are ft^2/lb$^{2/3}$ or m^2/kg$^{2/3}$. The reference area is then $S_{\text{nac}} = N S_{\text{wet}}$. The nacelle area is included in the aircraft wetted area if the drag coefficient is nonzero. The drag area or drag coefficient is defined for forward flight and for vertical flight. The drag coefficient at angle of attack α is

$$C_D = C_{D0} + (C_{DV} - C_{D0})|\sin\alpha|^{X_d}$$

typically using $X_d = 2$. In sideward flight, $C_D = C_{D0}$ is used. The nacelle drag is $D_{\text{nac}} = q S_{\text{nac}} C_D$.

Fuel Tank

The fuel quantity stored and burned can be measured in weight or energy. Each component (engine group, jet group, charge group, or equipment) that uses or generates fuel is associated with a fuel tank system of the appropriate type. The unit of fuel energy is Mega-Joules (MJ). For reference, 1 British Thermal Unit (BTU) = 1055.056 Joule and 1 kW-hr = 3.6 MJ.

16–1   Fuel Tank System Store and Burn Weight

For fuel use measured by weight, the fuel properties are density ρ_{fuel} (weight per volume, lb/gal or kg/liter) and specific energy e_{fuel} (MJ/kg). Table 16-1 gives the properties of a number of aviation fuels, based on military and industry specifications (refs. 1–3). Fuels considered include jet fuel, gasoline, diesel, and hydrogen. From the fuel weight W_{fuel}, the energy is $E_{\text{fuel}} = e_{\text{fuel}}W_{\text{fuel}}$ (MJ) and the volume is $V_{\text{fuel}} = W_{\text{fuel}}/\rho_{\text{fuel}}$ (gallons or liters). A motive device has a fuel flow $\dot{w}$ (lb/hour or kg/hour), and its specific fuel consumption is $\text{sfc} = \dot{w}/P$ or $\text{sfc} = \dot{w}/T$.

16-1.1   Fuel Capacity

The fuel tank capacity $W_{\text{fuel}-\text{cap}}$ (maximum usable fuel weight) is determined from designated sizing missions. The maximum mission fuel required, $W_{\text{fuel}-\text{miss}}$ (excluding reserves and any fuel in auxiliary tanks, and ignoring any refueling), gives

$$W_{\text{fuel}-\text{cap}} = \max(f_{\text{fuel}-\text{cap}}W_{\text{fuel}-\text{miss}}, W_{\text{fuel}-\text{miss}} + W_{\text{reserve}})$$

or from mission fuel plus reserves

$$W_{\text{fuel}-\text{cap}} = d_{\text{fuel}-\text{cap}} + f_{\text{fuel}-\text{cap}}(W_{\text{fuel}-\text{miss}} + W_{\text{reserve}})$$

or from maximum fuel quantity in the tank during the mission

$$W_{\text{fuel}-\text{cap}} = d_{\text{fuel}-\text{cap}} + f_{\text{fuel}-\text{cap}}W_{\text{fuel}-\text{max}}$$

where $f_{\text{fuel}-\text{cap}} \geq 0$ is an input factor and $d_{\text{fuel}-\text{cap}} \geq 0$ is an input increment. Alternatively, the fuel tank capacity $W_{\text{fuel}-\text{cap}}$ can be input. The corresponding volumetric fuel tank capacity is $V_{\text{fuel}-\text{cap}} = W_{\text{fuel}-\text{cap}}/\rho_{\text{fuel}}$.

For missions that are not used to size the fuel tank, the fuel weight may be fallout, or the fuel weight may be specified (with or without auxiliary tanks). The fuel weight for a flight condition or the start of a mission can be specified as an increment d, plus a fraction f of the fuel tank capacity, plus auxiliary tanks:

$$W_{\text{fuel}} = \min(d_{\text{fuel}} + f_{\text{fuel}}W_{\text{fuel}-\text{cap}}, W_{\text{fuel}-\text{cap}}) + \sum N_{\text{auxtank}}W_{\text{aux}-\text{cap}}$$

where $W_{\text{aux}-\text{cap}}$ is the capacity of each auxiliary fuel tank. The fuel capacity of the wing can be estimated from

$$W_{\text{fuel}-\text{wing}} = \rho_{\text{fuel}} \sum f c_{tb} t_w b_w$$

where c_{tb} is the torque box chord, t_w the wing thickness, and b_w the wing span; and f is the input fraction of the wing torque box that is filled by primary fuel tanks, for each wing. This calculation is performed in order to judge whether fuel tanks outside the wing are needed.

16-1.2   Fuel Reserves

Mission fuel reserves can be specified in several ways for each mission. Fuel reserves can be defined in terms of specific mission segments, for example 200 miles plus 20 minutes at V_{be}. Fuel reserves can be an input fraction of the fuel burned by all (except reserve) mission segments, so $W_{\text{fuel}} = (1 + f_{\text{res}})W_{\text{fuel}-\text{miss}}$. Fuel reserves can be an input fraction of the fuel capacity, so $W_{\text{fuel}} = W_{\text{miss}-\text{seg}} + f_{\text{res}}W_{\text{fuel}-\text{cap}}$. If more than one criterion for reserve fuel is specified, the maximum reserve is used.

16-1.3   Auxiliary Fuel Tank

Auxiliary fuel tanks are defined in one or more sizes. The capacity of each auxiliary fuel tank, $W_{\text{aux}-\text{cap}}$, is an input parameter. The number of auxiliary fuel tanks on the aircraft, N_{auxtank} for each size, can be specified for the flight condition or mission segment. Alternatively (if the mission is not used to size the fuel tank), the number of auxiliary fuel tanks at the start of the mission can be determined from the mission fuel.

Figure 16-1 describes the process for determining N_{auxtank} from the required fuel weight W_{fuel} and the aircraft maximum fuel capacity $W_{\text{fuel}-\text{max}} = W_{\text{fuel}-\text{cap}} + \sum N_{\text{auxtank}}W_{\text{aux}-\text{cap}}$. The fuel weight adjustment ΔW_{fuel} is made if fuel weight is fallout from fixed gross weight and payload, accounting for the operating weight update when N_{auxtank} changes. If the auxiliary tank weight is greater than the increment in fuel weight needed, then the fallout fuel weight $W_{\text{fuel}} = W_G - W_O - W_{\text{pay}}$ can not be achieved; in such a case, the fuel weight is capped at the maximum fuel capacity and the payload weight adjusted instead. The tanks changed can be the first size, the first size already used, or a designated size. The tanks can be added or dropped in groups of n ($n = 2$ for pairs).

The weight and drag of N_{auxtank} tanks are included in the performance calculation. Optionally the number of auxiliary tanks required can be calculated at the beginning of designated mission segments (based on the aircraft fuel weight at that point), and tanks dropped if no longer needed. The weight of the auxiliary fuel tanks is an input fraction of the tank capacity: $W_{\text{auxtank}} = \sum f_{\text{auxtank}}N_{\text{auxtank}}W_{\text{aux}-\text{cap}}$.

16-1.4   Auxiliary Fuel Tank Drag

The auxiliary fuel tanks are located at position z^F. The drag area for one auxiliary tank is specified, $(D/q)_{\text{auxtank}}$. The velocity relative to the air gives the drag direction $e_d = -v^F/|v^F|$ and dynamic pressure $q = \frac{1}{2}\rho|v^F|^2$ (no interference). Then

$$F^F = e_d \, q \, N_{\text{auxtank}}(D/q)_{\text{auxtank}}$$

is the total drag force, calculated for each auxiliary tank size.

16-1.5   Weights

The fuel system consists of the tanks (including support) and the plumbing. For the fractional model, the fuel tank weight is $W_{\text{tank}} = \chi_{\text{tank}}f_{\text{tank}}W_{\text{fuel}-\text{cap}}$.

```
if  W_fuel > W_fuel−max
    for designated auxiliary tank
        N_auxtank = N_auxtank + n
        if fixed total weight:  ΔW_fuel = −nf_auxtank W_aux−cap
        ΔW_fuel−max = nW_aux−cap
        repeat if  W_fuel > W_fuel−max
    if fixed total weight and  W_fuel ≤ W_fuel−max − nW_aux−cap
        N_auxtank = N_auxtank − n
        ΔW_fuel−max = −nW_aux−cap
        W_fuel = W_fuel−max  (capped)
else if  W_fuel < W_fuel−max
    for designated auxiliary tank (then for last nonzero N_auxtank)
        N_auxtank = N_auxtank − n
        if fixed total weight:  ΔW_fuel = nf_auxtank W_aux−cap
        ΔW_fuel−max = −nW_aux−cap
        repeat if  W_fuel < W_fuel−max
    undo last increment
        N_auxtank = N_auxtank + n
        if fixed total weight:  ΔW_fuel = −nf_auxtank W_aux−cap
        ΔW_fuel−max = nW_aux−cap
```

Figure 16-1. Outline of N_{auxtank} calculation.

The weight of the auxiliary fuel tanks is part of the fixed useful load; it is an input fraction of the tank capacity: $W_{\mathrm{auxtank}} = \sum f_{\mathrm{auxtank}} N_{\mathrm{auxtank}} W_{\mathrm{aux-cap}}$.

The AFDD weight model for the plumbing requires the fuel flow rate (for all engines), calculated for the takeoff rating and conditions.

16–2 Fuel Tank System Store and Burn Energy

For fuel use and storage measured by energy, there is no weight change as energy is used. The energy storage (tank) is characterized by specific energy e_{tank} (MJ/kg or kWh/kg) and energy density ρ_{tank} (MJ/liter or kWh/liter). Table 16-2 gives the properties of a number of systems. The tank weight and volume are obtained from the fuel energy $E_{\mathrm{fuel-cap}}$ (MJ). The fuel weight W_{fuel} is zero. A motive device has an energy flow $\dot{E}$ (MJ/hr or kW), and its specific fuel consumption is $\mathrm{sfc} = \dot{E}/P$ (inverse of efficiency). An equivalent fuel flow is $\dot{w}_{eq} = \dot{E}/e_{\mathrm{ref}}$, based on the specific energy e_{ref} (MJ/kg) of the first fuel tank that burns weight (or GP-4, $e_{\mathrm{ref}} = 42.8$ MJ/kg). The corresponding equivalent specific fuel consumption is $\mathrm{sfc}_{eq} = \dot{w}_{eq}/P$. The specific power is π_{tank} (kW/kg).

Storage systems considered include batteries, capacitors, and flywheels. A battery (or capacitor) stores charge (A-hr), so the capacity is expressed as energy for a nominal voltage. Variation of the voltage with operation affects the efficiency of the relation between useful power and the rate of change of the energy stored. Each fuel tank system that stores and burns energy has a battery model for computation of the charge/discharge efficiency.

The components associated with a fuel tank system (engine groups, jet groups, charge groups) define the total energy flow $\dot{E}_{\mathrm{comp}}$ (charge or discharge). The fuel tank system energy flow (due to

equipment power and distribution losses) is also included in $\dot{E}_{\text{comp}}$. Accounting for efficiency and losses gives the battery energy flow $\dot{E}_{\text{batt}} = \dot{E}_{\text{comp}} + P_{\text{loss}} + P_{\text{TMS}}$. The battery model gives P_{loss} and efficiency η_{batt} ($\eta_{\text{batt}} = \dot{E}_{\text{comp}}/(\dot{E}_{\text{comp}} + P_{\text{loss}})$ for discharge, $\eta_{\text{batt}} = (\dot{E}_{\text{comp}} + P_{\text{loss}})/\dot{E}_{\text{comp}}$ for charge). The system efficiency is $\eta_{\text{sys}} = \dot{E}_{\text{comp}}/\dot{E}_{\text{batt}}$ or $\eta_{\text{sys}} = \dot{E}_{\text{batt}}/\dot{E}_{\text{comp}}$. The battery power margin is $P_{\text{max}} - |\dot{E}_{\text{batt}}|$. P_{TMS} is the power required for the thermal management system, calculated from the rejected power $P_{\text{rej}} = P_{\text{loss}}$ (the dependence of the battery efficiency, hence P_{loss}, on P_{TMS} is neglected in order to avoid an iterative solution). Accounting for battery capacity (reduced by current or fade) then gives the effective energy flow $\dot{E}_{\text{eff}}$ (the fuel tank capacity is not adjusted). The change in stored energy is calculated from $\dot{E}_{\text{eff}}$ and time. The convention is that energy flow $\dot{E} > 0$ for discharge, and $\dot{E} < 0$ for charge. Power and current are positive for both discharge and charge.

16-2.1 Fuel Capacity

The fuel tank capacity $E_{\text{fuel-cap}}$ (maximum usable fuel energy) is determined from designated sizing missions. The maximum mission fuel required, $E_{\text{fuel-miss}}$ (excluding reserves and any fuel in auxiliary tanks, and ignoring any refueling), gives

$$E_{\text{fuel-cap}} = \max(f_{\text{fuel-cap}} E_{\text{fuel-miss}}, E_{\text{fuel-miss}} + E_{\text{reserve}})$$

or from mission fuel plus reserves

$$E_{\text{fuel-cap}} = d_{\text{fuel-cap}} + f_{\text{fuel-cap}}(E_{\text{fuel-miss}} + E_{\text{reserve}})$$

or from maximum fuel quantity in the tank during the mission

$$E_{\text{fuel-cap}} = d_{\text{fuel-cap}} + f_{\text{fuel-cap}} E_{\text{fuel-max}}$$

where $f_{\text{fuel-cap}} \geq 0$ is an input factor and $d_{\text{fuel-cap}} \geq 0$ is an input increment. Alternatively, the fuel tank capacity $E_{\text{fuel-cap}}$ can be input. The corresponding fuel tank weight is $W_{\text{tank}} = \chi_{\text{tank}} E_{\text{fuel-cap}}/e_{\text{tank}}$ (lb or kg) and the fuel tank volume is $V_{\text{tank}} = E_{\text{fuel-cap}}/\rho_{\text{tank}}$ (gallons or liters). The corresponding power capacity is $P_{\text{cap}} = (\pi_{\text{tank}}/e_{\text{tank}}) E_{\text{fuel-cap}}$ (kW and MJ). Optionally the maximum mission battery discharge power gives P_{cap}, from which $E_{\text{fuel-cap}} = \max(E_{\text{fuel-cap}}, (e_{\text{tank}}/\pi_{\text{tank}}) P_{\text{cap}})$ (MJ from kW).

For missions that are not used to size the fuel tank, the fuel energy may be fallout, or the fuel energy may be specified (with or without auxiliary tanks). The fuel energy for a flight condition or the start of a mission can be specified as an increment d, plus a fraction f of the fuel tank capacity, plus auxiliary tanks:

$$E_{\text{fuel}} = \min(d_{\text{fuel}} + f_{\text{fuel}} E_{\text{fuel-cap}}, E_{\text{fuel-cap}}) + \sum N_{\text{auxtank}} E_{\text{aux-cap}}$$

where $E_{\text{aux-cap}}$ is the capacity of each auxiliary fuel tank.

16-2.2 Fuel Reserves

Mission fuel reserves can be specified in several ways for each mission. Fuel reserves can be defined in terms of specific mission segments, for example 200 miles plus 20 minutes at V_{be}. Fuel reserves can be an input fraction of the fuel burned by all (except reserve) mission segments, so $E_{\text{fuel}} = (1 + f_{\text{res}}) E_{\text{fuel-miss}}$. Fuel reserves can be an input fraction of the fuel capacity, so $E_{\text{fuel}} = E_{\text{miss-seg}} + f_{\text{res}} E_{\text{fuel-cap}}$. If more than one criterion for reserve fuel is specified, the maximum reserve is used.

$$\texttt{if } E_{\text{fuel}} > E_{\text{fuel}-\text{max}}$$

$$\texttt{for designated auxiliary tank}$$

$$N_{\text{auxtank}} = N_{\text{auxtank}} + n$$

$$\Delta E_{\text{fuel}-\text{max}} = nE_{\text{aux}-\text{cap}}$$

$$\texttt{repeat if } E_{\text{fuel}} > E_{\text{fuel}-\text{max}}$$

$$\texttt{else if } E_{\text{fuel}} < E_{\text{fuel}-\text{max}}$$

$$\texttt{for designated auxiliary tank}$$

$$\texttt{(then for last nonzero } N_{\text{auxtank}})$$

$$N_{\text{auxtank}} = N_{\text{auxtank}} - n$$

$$\Delta E_{\text{fuel}-\text{max}} = -nE_{\text{aux}-\text{cap}}$$

$$\texttt{repeat if } E_{\text{fuel}} < E_{\text{fuel}-\text{max}}$$

$$\texttt{undo last increment}$$

$$N_{\text{auxtank}} = N_{\text{auxtank}} + n$$

$$\Delta E_{\text{fuel}-\text{max}} = nE_{\text{aux}-\text{cap}}$$

Figure 16-2. Outline of N_{auxtank} calculation.

16-2.3   Auxiliary Fuel Tank

Auxiliary fuel tanks are defined in one or more sizes. The capacity of each auxiliary fuel tank, $E_{\text{aux}-\text{cap}}$, is an input parameter. The number of auxiliary fuel tanks on the aircraft, N_{auxtank} for each size, can be specified for the flight condition or mission segment. Alternatively (if the mission is not used to size the fuel tank), the number of auxiliary fuel tanks at the start of the mission can be determined from the mission fuel.

Figure 16-2 describes the process for determining N_{auxtank} from the required fuel energy E_{fuel} and the aircraft maximum fuel capacity $E_{\text{fuel}-\text{max}} = E_{\text{fuel}-\text{cap}} + \sum N_{\text{auxtank}}E_{\text{aux}-\text{cap}}$. The tanks changed can be the first size, the first size already used, or a designated size. The tanks can be added or dropped in groups of n ($n = 2$ for pairs).

The drag of N_{auxtank} tanks is included in the performance calculation. Optionally the number of auxiliary tanks required can be calculated at the beginning of designated mission segments (based on the aircraft fuel energy at that point), and tanks dropped if no longer needed. The weight of the auxiliary fuel tanks is obtained from e_{auxtank} (MJ/kg or kWh/kg) and the tank capacity: $W_{\text{auxtank}} = \sum N_{\text{auxtank}}E_{\text{aux}-\text{cap}}/e_{\text{auxtank}}$.

16-2.4   Thermal Management System

The thermal management system deals with the battery rejected heat $P_{\text{rej}} = P_{\text{loss}}$. The system mass flow $\dot{m}_{\text{TMS}}$ depends on P_{rej}; the power P_{TMS} and gross thrust F_G depend on the mass flow. The net jet thrust is $F_N = F_G - \dot{m}_{\text{TMS}}V$. The size of the thermal management system (including weight W_{TMS}) is determined by a design rejected heat $P_{\text{rej}-\text{design}}$, perhaps specified as a fraction of the battery power capacity P_{cap}.

16-2.5   Distribution Losses

Losses from power distribution (including wiring resistance) are assumed to scale with current-squared: $P_{\text{dist}} = I^2R = x^2C^2R$. As a simple model, these losses are specified in terms of an efficiency

at maximum power, so $x_{mbd}^2 C^2 R = (1 - \eta_{\text{dist}})P_{\text{cap}}$. Then

$$P_{\text{dist}} = \left(\frac{x}{x_{mbd}}\right)^2 (1 - \eta_{\text{dist}})P_{\text{cap}} = \left(\frac{\dot{E}_{\text{comp}}}{P_{\text{cap}}}\right)^2 (1 - \eta_{\text{dist}})P_{\text{cap}}$$

gives the distribution loss from the component energy flow.

16-2.6   Weights

The fuel system consists of the tanks (including support) and the plumbing. The plumbing weight here means the power distribution (wiring). The fuel tank weight is

$$W_{\text{tank}} = E_{\text{fuel-cap}}/e_{\text{tank}} + W_{\text{BMS}} + W_{\text{TMS}}$$

(lb or kg) and the fuel tank volume is $V_{\text{fuel-cap}} = E_{\text{fuel-cap}}/\rho_{\text{tank}}$ (gallons or liters). The battery management system (BMS) weight is a fraction of the basic tank weight: $W_{\text{BMS}} = f_{\text{BMS}}(E_{\text{fuel-cap}}/e_{\text{tank}})$. W_{TMS} is the weight of the thermal management system. The wiring weight is proportional to the wiring length, plus a fraction of the basic tank weight:

$$W_{\text{wire}} = U_{\text{wire}}\ell_{\text{wire}} + f_{\text{wire}}(E_{\text{fuel-cap}}/e_{\text{tank}})$$

Alternatively, the wiring weight can be input as part of the electrical group weight.

The length of the wiring ℓ_{wire} can be input or calculated. The calculated length is the sum of the longitudinal, lateral, and vertical distances from the fuel tank (at position z^F) to all components that use the energy from this tank: $\ell_{\text{wire}} = f \sum(|\Delta x| + |\Delta y| + |\Delta z|)$, where f is an input factor.

The weight of the auxiliary fuel tanks is part of the fixed useful load; it is obtained from e_{auxtank} (MJ/kg or kWh/kg) and the tank capacity: $W_{\text{auxtank}} = \sum N_{\text{auxtank}}E_{\text{aux-cap}}/e_{\text{auxtank}}$.

16–3   Equipment Power

Systems that supply equipment power can directly use fuel. Equipment power (similar to drive train accessory losses) is calculated as the sum of an input constant; terms that scale with air density and temperature; a term that scales aircraft weight; deice power loss (if deice system is on); and an increment specified for each flight state:

$$P_{\text{eq}} = P_{\text{eq0}} + P_{\text{eqd}}\sigma + P_{\text{eqt}}\theta + P_{\text{eqw}} + P_{\text{eqi}} + dP_{\text{eq}}$$

where $\sigma = \rho/\rho_0$ is the density ratio and $\theta = T/T_0$ is the temperature ratio. The power scaling with aircraft weight is $P_{\text{eqw}} = K_{\text{eq}}W_{MTO}^{X_{\text{eq}}}$ (where W_{MTO} is the maximum takeoff gross weight). The fuel flow or energy flow is then obtained from an input specific fuel consumption: $\dot{w}_{\text{eq}} = (\text{sfc})P_{\text{eq}}$ or $\dot{E}_{\text{eq}} = (\text{sfc})P_{\text{eq}}$. The energy sfc is the inverse of an efficiency: sfc=$3.6/\eta$ for energy flow in MJ/hr and power in kW.

16–4   Jet Force and Cooling Drag

The thermal management system produces a jet thrust $F_N = F_G - \dot{m}V$, acting in the wind direction. The drag associated with battery cooling is $D_{\text{cool}} = (1 - \eta)\dot{m}V = (1 - \eta)(\rho AV)V$, where A is the inlet area, η the ram recovery efficiency, and V the flight speed. So the drag area is specified, estimated from $(D/q)_{\text{cool}} = 2(1 - \eta)AV$.

The jet force and cooling drag act in the wind direction. The fuel tank is at location z^F. The force and moment acting on the aircraft in body axes are thus:

$$F^F = e_d F_N + e_d D_{\text{cool}}$$
$$M^F = \widetilde{\Delta z^F} F^F$$

where $\Delta z^F = z^F - z^F_{cg}$, and $e_d = -v^F/|v^F|$ is the drag direction.

16–5 References

1) Department of Defense Military Specification. "Glossary of Definitions, Ground Rules, and Mission Profiles to Define Air Vehicle Performance Capability." MIL-STD-3013A, September 2008.

2) Department of Defense Detail Specification. "Turbine Fuel, Aviation, Grades JP-4 and JP-5." MIL-DTL-5624U, September 1998.

3) Department of Defense Detail Specification. "Turbine Fuel, Aviation, Kerosene Type, JP-8 (NATO F-34), NATO F-35, and JP-8+100 (NATO F-37)." MIL-DTL-83133H, October 2011.

Table 16-1. Fuel properties.

fuel	specification	density		specific energy			energy dens.
		lb/gal	kg/L	MJ/kg	BTU/lb	lb/hp-hr	MJ/L
gasoline	MIL-STD-3013A	6.0*	0.719	43.50	18700*	0.136	31.3
diesel		7.0	0.839	43.03	18500	0.138	36.1
		6.84–7.05	0.820–0.845	43.0	18487	0.138	35.8
Jet A/A-1	MIL-STD-3013A	6.7*	0.803	42.80	18400*	0.138	34.4
		6.84/6.71	0.820/0.804	42.8	18401	0.138	34.8
JP-4		6.5	0.779	42.80	18400	0.138	33.3
	MIL-DTL-5624U	6.23–6.69	0.751*–0.802*	42.8*	18401	0.138	32.2
JP-5	MIL-STD-3013A	6.6*	0.791	42.57	18300*	0.139	33.7
	alternate design	6.8*	0.815	42.91	18450*	0.138	35.0
	MIL-DTL-5624U	6.58–7.05	0.788*–0.845*	42.6*	18315	0.139	34.8
JP-8	MIL-STD-3013A	6.5*	0.779	42.80	18400*	0.138	33.3
	alternate design	6.8*	0.815	43.19	18570*	0.137	35.2
	MIL-DTL-83133H	6.45–7.01	0.775*–0.840*	42.8*	18401	0.138	34.6
LPG (methane)		3.75	0.45	50.	21496	0.118	111.
hydrogen (700 bar)		0.328	0.03930	120.	51591	0.0493	4.72
high heating value		0.328	0.03930	140.	60190	0.0423	5.50
hydrogen (liquid)		0.592	0.07099	120.	51591	0.0493	8.52

*specification value

Table 16-2. Energy storage properties.

		tank specific energy		tank energy density		efficiency	power
		MJ/kg	kW-hr/kg	MJ/L	kW-hr/m^3		kW/kg
lead-acid battery		0.11–0.14	0.03–0.04	0.22–0.27	60–75	70–90%	0.18
nickel-cadmium battery		0.14–0.20	0.04–0.06	0.18–0.54	50–150	70–90%	0.15
lithium-ion	state-of-art	0.54–0.90	0.15–0.25	0.90–1.30	250–360	~99%	1.80
	+5 years	1.26	0.35	1.80	500		
	+15 years	2.34	0.65	2.25	625		
ultracapacitor		0.01–0.11	0.004-0.03	0.02–0.16	6–45		1.00
flywheel	steel	0.11	0.03			~90%	
	graphite	0.90	0.25				

Propulsion Group

The propulsion group is a set of components and engine groups, connected by a drive system. The engine model describes a particular engine, used in one or more engine group. The components (rotors) define the power required. The engine groups define the power available. Figure 17-1 illustrates the power flow.

17–1 Drive System

The drive system defines gear ratios for all the components it connects. The gear ratio is the ratio of the component rotational speed to that of the primary rotor. There is one primary rotor per propulsion group (for which the reference tip speed is specified); other components are dependent (for which a gear ratio is specified). There can be more than one drive system state, in order to model a multiple-speed or variable-speed transmission. Each drive system state corresponds to a set of gear ratios.

For the primary rotor, a reference tip speed $V_{\text{tip}-\text{ref}}$ is defined for each drive system state. By convention, the "hover tip speed" refers to the reference tip speed for drive state #1. If the sizing task changes the hover tip speed, then the ratios of the reference tip speeds at different engine states are kept constant. By convention, the gear ratio of the primary rotor is $r = 1$. For dependent rotors, either the gear ratio is specified (for each drive system state) or a tip speed is specified and the gear ratio is calculated ($r = \Omega_{\text{dep}}/\Omega_{\text{prim}}$, $\Omega = V_{\text{tip}-\text{ref}}/R$). For the engine group, either the gear ratio is specified (for each drive system state) or the gear ratio is calculated from the specification engine turbine speed $\Omega_{\text{spec}} = (2\pi/60)N_{\text{spec}}$ and the reference tip speed of the primary rotor ($r = \Omega_{\text{spec}}/\Omega_{\text{prim}}$, $\Omega_{\text{prim}} = V_{\text{tip}-\text{ref}}/R$). The latter option means the specification engine turbine speed N_{spec} corresponds to $V_{\text{tip}-\text{ref}}$ for all drive system states. To determine the gear ratios, the reference tip speed and radius are used, corresponding to hover.

The flight state specifies the tip speed of the primary rotor and the drive system state, for each propulsion group. The drive system state defines the gear ratio for dependent rotors and the engine groups. From the rotor radius, the rotational speed of the primary rotor is obtained ($\Omega_{\text{prim}} = V_{\text{tip}}/R$); from the gear ratios, the rotational speed of dependent rotors ($\Omega_{\text{dep}} = r\Omega_{\text{prim}}$) and the engine groups ($N = (60/2\pi)r_{\text{eng}}\Omega_{\text{prim}}$) are obtained; and from the rotor radius, the tip speed of the dependent rotor ($V_{\text{tip}} = \Omega_{\text{dep}}R$) is obtained. The flight state specification of the tip speed can be an input value, the reference tip speed, a function of flight speed or a conversion schedule, or one of several default values. These relationships between tip speed and rotational speed use the actual radius of the rotors in the flight state, which for a variable-diameter rotor may not be the same as the reference, hover radius.

A designated drive system state can have a variable speed (variable gear ratio) transmission, by introducing a factor f_{gear} on the gear ratio when the speeds of the dependent rotors and engines are
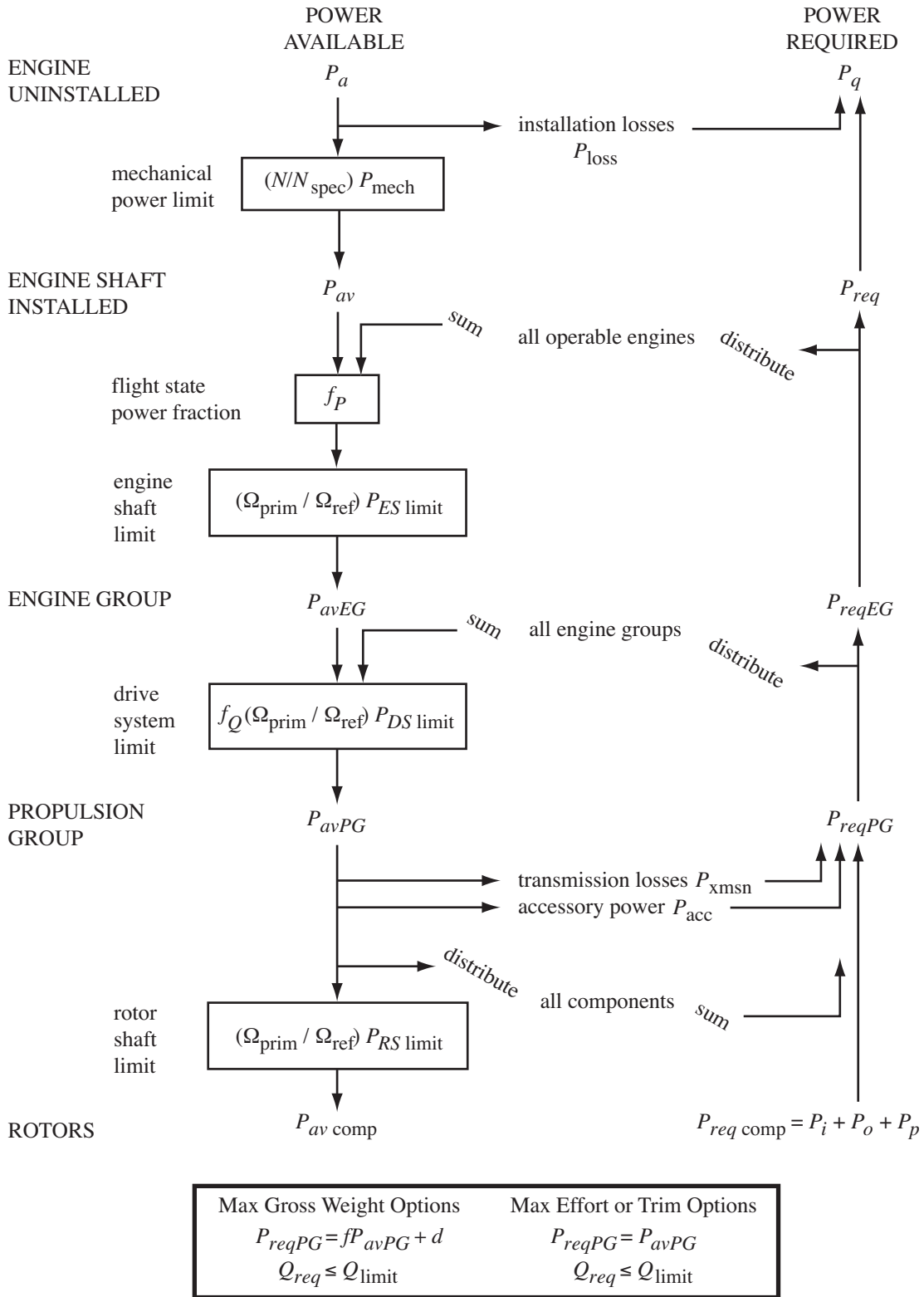
POWER
AVAILABLE

POWER
REQUIRED

ENGINE
UNINSTALLED

P_a

P_q

installation losses
P_{loss}

mechanical
power limit

$(N/N_{\text{spec}})\, P_{\text{mech}}$

ENGINE SHAFT
INSTALLED

P_{av}

P_{req}

sum   all operable engines   distribute

flight state
power fraction

f_P

engine
shaft
limit

$(\Omega_{\text{prim}} / \Omega_{\text{ref}})\, P_{ES\,\text{limit}}$

ENGINE GROUP

P_{avEG}

P_{reqEG}

sum   all engine groups   distribute

drive
system
limit

$f_Q(\Omega_{\text{prim}} / \Omega_{\text{ref}})\, P_{DS\,\text{limit}}$

PROPULSION
GROUP

P_{avPG}

P_{reqPG}

transmission losses P_{xmsn}
accessory power P_{acc}

distribute   all components   sum

rotor
shaft
limit

$(\Omega_{\text{prim}} / \Omega_{\text{ref}})\, P_{RS\,\text{limit}}$

ROTORS

$P_{av\,\text{comp}}$

$P_{req\,\text{comp}} = P_i + P_o + P_p$

Max Gross Weight Options	Max Effort or Trim Options
$P_{reqPG} = fP_{avPG} + d$	$P_{reqPG} = P_{avPG}$
$Q_{req} \le Q_{\text{limit}}$	$Q_{req} \le Q_{\text{limit}}$

Figure 17-1. Power flow.

evaluated. The factor f_{gear} is a component control, which can be connected to an aircraft control and thus set for each flight state.

An optional conversion schedule is defined in terms of two speeds: hover and helicopter mode for speeds below $V_{C\text{hover}}$, cruise mode for speeds above $V_{C\text{cruise}}$, and conversion mode for speeds between $V_{C\text{hover}}$ and $V_{C\text{cruise}}$. The tip speed is $V_{\text{tip}-\text{hover}}$ in helicopter and conversion mode, and $V_{\text{tip}-\text{cruise}}$ in airplane mode. Drive system states are defined for helicopter, cruise, and conversion mode flight. The flight state specifies the nacelle tilt angle, tip speeds, control state, and drive system state, including the option to obtain any or all of these quantities from the conversion schedule.

Several default values of the tip speed are defined for use by the flight state, including cruise, maneuver, one-engine inoperative, drive system limit conditions, and a function of flight speed (piecewise linear input). Optionally these default values can be input as a fraction of the hover tip speed. Optionally the tip speed can be calculated from $\mu = V/V_{\text{tip}}$, so $V_{\text{tip}} = V/\mu$; or from $M_{at} = M_{\text{tip}}\sqrt{(1+\mu)^2 + \mu_z^2}$, so $V_{\text{tip}} = \sqrt{(c_s M_{at})^2 - V_z^2} - V$. Optionally the tip speed can be the minimum of the input value or that for M_{at}.

The sizing task might change the hover tip speed (reference tip speed for drive system state #1), the reference tip speed of a dependent rotor, a rotor radius, or the specification engine turbine speed N_{spec}. In such cases the gear ratios and other parameters are recalculated. Note that it is not consistent to change the reference tip speed of a dependent rotor if the gear ratio is a fixed input.

An increment on the primary rotor rotational speed (or primary engine group, if there are no rotors) is a control variable of the propulsion group.

17–2   Power Required

The component power required P_{comp} is evaluated for a specified flight condition, as the sum of the power required by all the components of the propulsion group. The total power required for the propulsion group is obtained by adding the transmission losses and accessory power:

$$P_{reqPG} = P_{\text{comp}} + P_{\text{xmsn}} + P_{\text{acc}}$$

The transmission losses are calculated as an input fraction ℓ_{xmsn} of the component power, plus windage loss:

$$P_{\text{xmsn}} = \ell_{\text{xmsn}} f_{\text{xmsn}} |P_{\text{comp}}| + P_{\text{windage}} (\Omega_{\text{prim}}/\Omega_{\text{ref}})$$

The factor f_{xmsn} can equal 1, or can include a function of the drive shaft limit (increasing the losses at low power):

$$f_{\text{xmsn}} |P_{\text{comp}}| = \begin{cases} \frac{1}{2} P_{X\text{limit}} & Q < \frac{1}{4} \\ \left(\frac{7}{3} - \frac{4}{3}Q\right)|P_{\text{comp}}| & \frac{1}{4} < Q < 1 \\ |P_{\text{comp}}| & 1 < Q \end{cases}$$

where $Q = |P_{\text{comp}}|/P_{X\text{limit}}$, $P_{X\text{limit}} = r P_{DS\text{limit}}$, and $r = N/N_{\text{spec}} = \Omega_{\text{prim}}/\Omega_{\text{ref}}$. Accessory losses are calculated as the sum of an input constant; terms that scale with air density and rotor speed; a fraction of power required (such as environmental control unit (ECU) losses); infared suppressor fan loss (if IRS system is on); deice power loss (if deice system is on); and an increment specified for each flight state:

$$P_{\text{acc}} = P_{\text{acc}0} + P_{\text{acc}d}\sigma + P_{\text{acc}n}\sigma(\Omega_{\text{prim}}/\Omega_{\text{ref}}) + \ell_{\text{acc}}(|P_{\text{comp}}| + P_{\text{xmsn}}) + \ell_{\text{IRfan}}\sigma N_{\text{eng}} P_{\text{eng}} + P_{\text{acc}i} + dP_{\text{acc}}$$

where $\sigma = \rho/\rho_0$ is the density ratio.

The power required for the propulsion group must be distributed to the engine groups. With only one engine group, $P_{reqEG} = P_{reqPG}$. An engine group power can be fixed at $P_{reqEG} = (N_{\mathrm{eng}} - N_{\mathrm{inop}})A$, where A is the input power amplitude; or fraction A of engine power available, $P_{reqEG} = (N_{\mathrm{eng}} - N_{\mathrm{inop}})AP_{av}$; or fraction A of engine rated power $P_{reqEG} = (N_{\mathrm{eng}} - N_{\mathrm{inop}})AP_{\mathrm{eng}}$. The power required for the remaining (perhaps all) engine groups is distributed proportional to the engine rated power:

$$P_{reqEG} = \left(P_{reqPG} - \sum_{\mathrm{fixed}} P_{reqEG}\right) \frac{(N_{\mathrm{eng}} - N_{\mathrm{inop}})P_{\mathrm{eng}}}{\sum_{\mathrm{notfixed}}(N_{\mathrm{eng}} - N_{\mathrm{inop}})P_{\mathrm{eng}}}$$

omitting engine groups that do not supply shaft power. If the sum of the fixed P_{reqEG} exceeds the propulsion group power required, or if the power is fixed for all engine groups, then each is scaled by the ratio $P_{reqPG}/\sum_{\mathrm{fixed}} P_{reqEG}$. The fuel flow of the propulsion group is obtained from the sum over the engine groups: $\dot{w}_{reqPG} = \sum \dot{w}_{reqEG}$.

17–3   Geometry

The length of the drive system ℓ_{DS} can be input or calculated. The calculated length is the sum of the longitudinal, lateral, and vertical distances from the primary rotor hub to the other hub locations, for all rotors in the propulsion group: $\ell_{DS} = f \sum(|\Delta x| + |\Delta y| + |\Delta z|)$, where f is an input factor. Alternatively, the drive system length can be scaled with the radius of the primary rotor: $\ell_{DS} = fR$.

17–4   Drive System limit

The drive system limit is defined as a power limit, $P_{DS\mathrm{limit}}$. The limit is properly a torque limit, $Q_{DS\mathrm{limit}} = P_{DS\mathrm{limit}}/\Omega_{\mathrm{ref}}$, but is expressed as a power limit for clarity. The drive system limit can be specified as follows (with f_{limit} an input factor):

a) Input $P_{DS\mathrm{limit}}$.
b) From the engine takeoff power limit, $P_{DS\mathrm{limit}} = f_{\mathrm{limit}} \sum N_{\mathrm{eng}}P_{\mathrm{eng}}$ (summed over all engine groups).
c) From the power available at the transmission sizing conditions and missions, $P_{DS\mathrm{limit}} = f_{\mathrm{limit}}(\Omega_{\mathrm{ref}}/\Omega_{\mathrm{prim}}) \sum N_{\mathrm{eng}}P_{av}$ (largest of all conditions and segments).
d) From the power required at the transmission sizing conditions and missions, $P_{DS\mathrm{limit}} = f_{\mathrm{limit}}(\Omega_{\mathrm{ref}}/\Omega_{\mathrm{prim}}) \sum N_{\mathrm{eng}}P_{req}$ (largest of all conditions and segments).

The drive system limit is a limit on the entire propulsion system. To account for differences in the distribution of power through the drive system, limits are also used for the torque of each rotor shaft ($P_{RS\mathrm{limit}}$) and of each engine group ($P_{ES\mathrm{limit}}$). The engine shaft limit is calculated as for the drive system limit, without the sum over engine groups. The rotor shaft limit is either input or calculated from the rotor power required at the transmission sizing flight conditions. The power limit is associated with a reference rotational speed, and when applied the limit is scaled with the rotational speed of the flight state. The rotation speed for the drive system limit $P_{DS\mathrm{limit}}$ is the hover speed of the primary rotor of the propulsion group (for the first drive state). The rotation speed for the engine shaft limit $P_{ES\mathrm{limit}}$ is the corresponding engine turbine speed. The rotation speed for the rotor shaft limit $P_{RS\mathrm{limit}}$ is the corresponding speed of that rotor.

The drive system limits can be specified for several levels, analogous to engine ratings. Typically higher torque limits are associated with short duration operation. For each drive system rating, a torque factor x is specified. Then from the rating for a flight condition or mission segment, the torque limit is

$x f_Q Q_{\text{limit}}$, with Q_{limit} obtained from $P_{DS\text{limit}}$, and including a specified torque fraction f_Q. The torque factor x is used for engine shaft limits and rotor shaft limits as well. Optionally the drive system rating can be scheduled with flight speed.

17–5 Weights

The drive system consists of gear boxes and rotor shafts, drive shafts, rotor brakes, clutches, and gas drive. The drive system weight depends on the rotor and engine rotational speeds, evaluated for the propulsion group primary rotor and a specified engine group, at a specified drive system state (gear ratio).

The AFDD drive system weight model depends on f_Q, the second (main or tail) rotor rated torque as a fraction of the total drive system rated torque; and on f_P, the second (main or tail) rotor rated power as a fraction of the total drive system rated power. These parameters are related by the rotational speeds of the two rotors: $f_P = f_Q \Omega_{\text{other}}/\Omega_{\text{main}}$. Typically $f_P = f_Q = 0.6$ for twin rotors (tandem, coaxial, and tiltrotor configurations). For the single-main-rotor and tail-rotor configuration, typically $f_Q = 0.03$ and $f_P = 0.15$ (0.18 with a two-bladed teetering main-rotor).

Engine Group

The engine group consists of one or more engines of a specific type. An engine group transfers power by shaft torque, so it is associated with a propulsion group. For each engine type an engine model is defined. The engine model describes a particular engine, used in one or more engine groups.

The models include turboshaft engines (perhaps convertible, for turbojet operation or reaction drive), reciprocating engines, compressors, electric motors, electric generators, and generator-motors.

18–1   Engine Group Performance

The engine size is described by the power P_{eng}, which is the sea-level static power available per engine at a specified takeoff rating. The number of engines N_{eng} is specified for each engine group.

If the sizing task determines the engine power for a propulsion group, the power P_{eng} of at least one engine group is found (including the first engine group). The total power required is $P_{PG} = r \sum N_{\text{eng}} P_{\text{eng}}$, where $r = \max(P_{reqPG}/P_{avPG})$. The sized power is $P_{\text{sized}} = P_{PG} - \sum_{\text{fixed}} N_{\text{eng}} P_{\text{eng}}$. Then the sized engine power is $P_{\text{eng}} = f_n P_{\text{sized}}/N_{\text{eng}}$ for the n-th engine group (with $f_1 = \sum_{n \neq 1, \text{sized}} f_n$ for the first group). If an engine group does not consume power (compressor or generator) or does not contribute to shaft power (converted), the size is scaled with $r = \max(P_{reqEG}/P_{avEG})$. Optionally the sized power can be related to an engine group of other propulsion groups: $P_{\text{eng}} = \max(P_{\text{eng}}, f_E P_{\text{eng}-\text{other}})$.

The propulsion group power available is obtained from the sum over the engine groups: $P_{avPG} = \sum P_{avEG}$.

The propulsion group component power P_{comp} includes compressor power, generator power required, and generator-motor power when it is producing energy.

The flight condition information includes the altitude, temperature, flight speed, and primary rotor speed; a power fraction f_P; and the states of the engine, drive system, and infrared suppressor (IRS). The engine turbine speed is $N = (60/2\pi)r_{\text{eng}}\Omega_{\text{prim}}$, where Ω_{prim} is the current rotor speed and r_{eng} is the gear ratio (depending on the drive system state, including a factor f_{gear} for a variable speed transmisson). If the reference primary rotor speed Ω_{prim} corresponds to the specification turbine speed N_{spec}, then $r_{\text{eng}} = \Omega_{\text{spec}}/\Omega_{\text{prim}}$; alternatively, the engine gear ratio can be a fixed input.

The drive system limit at the flight condition is $rxf_Q P_{DS\text{limit}}$, where $r = \Omega_{\text{prim}}/\Omega_{\text{ref}}$, x is the rating factor, and f_Q is a specified torque fraction. Optionally this limit is applied to the propulsion group power: $P_{avPG} = \min(P_{avPG}, rxf_Q P_{DS\text{limit}})$. Similarly the engine shaft limit at the flight condition is optionally applied to the engine group power: $P_{avEG} = \min(P_{avEG}, rxP_{ES\text{limit}})$.

18–2   Turboshaft Engine

Turboshaft engine performance is obtained from the Referred Parameter Turboshaft Engine Model (RPTEM).

18-2.1   Power Available

Given the flight condition and engine rating, the power available P_a is calculated from the specific power $SP_a = P_a/\dot{m}_a$ and mass flow $\dot{m}_a$:

$$\frac{SP_a}{\theta} = SP_0 g_{sp}(\theta, M, n)$$

$$\frac{\dot{m}_a}{\delta/\sqrt{\theta}} = \dot{m}_0 g_m(\theta, M, n)$$

$$\frac{P_a}{\delta\sqrt{\theta}} = P_0 g_p(\theta, M, n)$$

as functions of temperature ratio $\theta = T/T_0$, Mach number M, and referred engine turbine speed $n = N/\sqrt{\theta}$.

In the engine model, installation losses P_{loss} are subtracted from P_a ($P_{av} = P_a - P_{\text{loss}}$), and then the mechanical limit is applied: $P_{av} = \min(P_{av}, rP_{\text{mech}R})$, $r = N/N_{\text{spec}}$. The mechanical limit is properly a torque limit, $Q_{\text{mech}} = P_{\text{mech}}/N_{\text{spec}}$, but is expressed as a power limit for clarity.

The engine model gives the performance of a single engine. The power available of the engine group is obtained by multiplying the single engine power by the number of engines operational (total number of engines less inoperable engines):

$$P_{avEG} = f_P(N_{\text{eng}} - N_{\text{inop}})P_{av}$$

including a specified power fraction f_P.

18-2.2   Performance at Power Required

The engine performance (mass flow, fuel flow, and gross jet thrust) is calculated for a specified power required P_q and flight condition:

$$\frac{\dot{m}_{req}}{\delta/\sqrt{\theta}} = \dot{m}_{0C} g_m(q, \theta, M, n)$$

$$\frac{\dot{w}_{req}}{\delta\sqrt{\theta}} = \dot{w}_{0C} g_w(q, \theta, M, n)$$

$$\frac{F_g}{\delta} = F_{g0C} g_f(q, \theta, M, n)$$

as functions of $q = P_q/(P_{0C}\delta\sqrt{\theta})$, temperature ratio $\theta = T/T_0$, Mach number M, and referred engine turbine speed $n = N/(N_{\text{spec}}\sqrt{\theta})$.

The engine model deals with a single engine. The power required of a single engine is obtained by dividing the engine group power by the number of engines operational (total number of engines less inoperable engines):

$$P_{req} = P_{reqEG}/(N_{\text{eng}} - N_{\text{inop}})$$

In the engine model, installation losses P_{loss} are added to P_{req}: $P_q = P_{req} + P_{\text{loss}}$.

The engine model gives the performance of a single engine. The performance of the engine group is obtained by multiplying the single engine characteristics by the number of engines operational (total number of engines less inoperable engines):

$$\dot{m}_{reqEG} = (N_{\text{eng}} - N_{\text{inop}})\dot{m}_{req}$$
$$\dot{w}_{reqEG} = (N_{\text{eng}} - N_{\text{inop}})\dot{w}_{req}K_{ffd}$$
$$F_{NEG} = (N_{\text{eng}} - N_{\text{inop}})F_N$$
$$D_{\text{aux}EG} = (N_{\text{eng}} - N_{\text{inop}})D_{\text{aux}}$$

The fuel flow has also been multiplied by a factor K_{ffd} accounting for deterioration of the engine efficiency.

18-2.3   Installation

The difference between installed and uninstalled power is the inlet and exhaust losses P_{loss}: $P_{av} = P_a - P_{\text{loss}}$ and $P_{req} = P_q - P_{\text{loss}}$. The inlet ram recovery efficiency η_d is included in the engine model calculations. The inlet and exhaust losses are modeled as fractions of power available or power required: $P_{\text{loss}} = (\ell_{in} + \ell_{ex})P_a$ or $P_{\text{loss}} = (\ell_{in} + \ell_{ex})P_q$. The installed gross jet thrust is $F_G = K_{fgr}F_g$, where K_{fgr} accounts for exhaust effects. The net jet thrust is $F_N = F_G - \dot{m}_{req}V$. The momentum drag of the auxiliary air flow is a function of the mass flow $\dot{m}_{\text{aux}} = f_{\text{aux}}\dot{m}_{req}$:

$$D_{\text{aux}} = (1 - \eta_{\text{aux}})\dot{m}_{\text{aux}}V = (1 - \eta_{\text{aux}})f_{\text{aux}}\dot{m}_{req}V$$

where η_{aux} is the ram recovery efficiency. Exhaust losses (ℓ_{ex}) and auxiliary air flow parameters (η_{aux}, f_{aux}) are defined for IR suppressor on and off. Inlet particle separator loss is added to the inlet losses (ℓ_{in}).

18-2.4   Convertible Engine: Turbojet/Turbofan

The engine mode B is the mass flow fraction diverted for a convertible engine: $B = 0$ for all mass flow to the power turbine (turboshaft operation), and $B = 1$ for all mass flow to the jet exhaust or a fan (turbojet/turbofan operation).

A separate engine model defines the performance for turbojet/turbofan operation ($B = 1$). The engine group power P_{reqEG} is prescribed, as a measure of the jet thrust, and this engine does not contribute to the propulsion group shaft power available. The turbojet/turbofan thrust is the engine group net jet thrust, $F_N = F_G - \dot{m}_{req}V$.

18-2.5   Convertible Engine: Reaction Drive

The engine mode B is the mass flow fraction diverted for a convertible engine: $B = 0$ for all mass flow to the power turbine (turboshaft operation), and $B = 1$ for all mass flow to the rotor (reaction jet operation).

A separate engine model defines the engine performance for reaction jet operation ($B = 1$). The engine group power is fixed by amplitude input, or adjusted to produce the required rotor reaction force (momentum flux) F_{Greq}, and this engine does not contribute to the propulsion group shaft power available. The gross jet thrust is directed to the reaction drive, so the net jet thrust of the engine group is $F_N = (F_G - F_{Gq}) - \dot{m}V$.

18–3   Turboshaft Engine Tabular Model

Tabular models of the engine performance are implemented, suitable for use with data from an exercise of an engine deck. Two table formats are defined (E and H). Input factors K_p, K_w and K_f can account for technology level, respectively for power available, fuel flow, and jet thrust. The tables are used with linear interpolation.

The tables are usually for an installed engine, including losses. Hence $P_{\text{loss}} = 0$ ($\ell_{in} = \ell_{ex} = 0$) with this model, and $P_{av} = P_a$, $P_{req} = P_q$. Mechanical limits are included in the power available data. Engine mass flow is not considered, so $D_{\text{aux}} = 0$, and the table is for net jet thrust. The fuel flow is multiplied by the factor K_{ffd}, accounting for deterioration of the engine efficiency. The engine is not scaled. The engine weight $W_{\text{one eng}}$ is fixed. The turbine speed N_{spec} is fixed.

18-3.1   Table Format E

The tables are for power available P_a, fuel flow $\dot{w}$, and net jet thrust F_N as a function of altitude h, flight speed V, and rating R. The altitude influence is for a selected atmosphere, hence for a temperature variation with altitude. The ratings reflect engine throttle setting, including partial power. Power turbine speed variation is not considered.

Given the altitude, speed, and rating, the power table $T_p(h, V, R)$ is interpolated. Then the power available is $P_a = K_p T_p$. For fixed altitude and speed, the table $T_p(h, V, R)$ implies a variation of power with rating; and the tables for fuel flow and jet thrust can be interpreted as $T_w(h, V, P_q)$ and $T_f(h, V, P_q)$. Given altitude, speed, and power required, the tables are interpolated; then the fuel flow is $\dot{w} = K_w T_w$ and the net jet thrust is $F_N = K_f T_f$.

18-3.2   Table Format H

The tables are for power available P_a and fuel flow $\dot{w}$ as a function of altitude h, temperature τ, flight speed V, and rating R. The power available is the product of static power P_0 and a ram factor f_{RAM}: $P_a = K_p P_0(h, \tau, R) f_{\text{RAM}}(V, \tau, h)$. Optionally, $f_{\text{RAM}} = (\delta_M \sqrt{\theta_M})^{X_{pa}}$, where $\delta_M = (1 + 0.2M^2)$ and $\delta_M = (1 + 0.2\eta_d M^2)^{3.5}$.

The reference fuel flow $\dot{w}_{\text{ref}} = \dot{w}/\delta^{X_{df}} \theta^{X_{rf}}$ is a function of reference power required $P_{q\text{ref}} = P_q/\delta^{X_{dp}} \theta^{X_{rp}}$: $\dot{w} = K_w \delta^{X_{df}} \theta^{X_{rf}} \dot{w}_{\text{ref}}(P_{q\text{ref}})$. Note that referred power and referred fuel flow are based on $\delta\sqrt{\theta}$. Alternatively, the fuel flow is a function of power, altitude, and temperature: $\dot{w}(P_q, h, \tau)$. The net jet thrust is zero.

18–4   Reciprocating Engine

Reciprocating engine performance is obtained from the Reciprocating Engine Model.

18-4.1   Power Available

Given the flight condition and engine rating, the power available P_a is calculated from:

$$P_a = P_0 g_p(\sigma, \theta, M, r)$$

as a function of density ratio $\sigma = \rho/\rho_0$, temperature ratio $\theta = T/T_0$, Mach number M, and engine speed ratio $r = N/N_{\text{spec}}$.

Installation losses P_{loss} are subtracted from P_a ($P_{av} = P_a - P_{\mathrm{loss}}$), and then the mechanical limit is applied: $P_{av} = \min(P_{av}, rP_{\mathrm{mech}R})$. The mechanical limit is properly a torque limit, $Q_{\mathrm{mech}} = P_{\mathrm{mech}}/N_{\mathrm{spec}}$, but is expressed as a power limit for clarity. The power available of the engine group is obtained by multiplying the single engine power by the number of engines operational (total number of engines less inoperable engines): $P_{avEG} = f_P(N_{\mathrm{eng}} - N_{\mathrm{inop}})P_{av}$, including a specified power fraction f_P.

18-4.2   Performance at Power Required

The engine performance for a specified power required P_q is:

$$\dot{w}_{req} = \dot{w}_0 g_w(q, r)$$
$$\dot{m}_{req} = \dot{m}_0 g_m(q, r)$$
$$F_g = F_{g0} g_f(q, r)$$

where $q = P_q/P_0$ and $r = N/N_{\mathrm{spec}}$. Installation losses P_{loss} are added to P_{req}: $P_q = P_{req} + P_{\mathrm{loss}}$, $P_{req} = P_{reqEG}/(N_{\mathrm{eng}} - N_{\mathrm{inop}})$.

The performance of the engine group is obtained by multiplying the single engine characteristics by the number of engines operational (total number of engines less inoperable engines):

$$\dot{m}_{reqEG} = (N_{\mathrm{eng}} - N_{\mathrm{inop}})\dot{m}_{req}$$
$$\dot{w}_{reqEG} = (N_{\mathrm{eng}} - N_{\mathrm{inop}})\dot{w}_{req}K_{ffd}$$
$$F_{NEG} = (N_{\mathrm{eng}} - N_{\mathrm{inop}})F_N$$
$$D_{\mathrm{aux}EG} = (N_{\mathrm{eng}} - N_{\mathrm{inop}})D_{\mathrm{aux}}$$

The fuel flow has also been multiplied by a factor K_{ffd} accounting for deterioration of the engine efficiency.

18-4.3   Installation

The difference between installed and uninstalled power is the installation loss: $P_{av} = P_a - P_{\mathrm{loss}}$ and $P_{req} = P_q - P_{\mathrm{loss}}$. This loss is modeled by an efficiency factor: $P_{\mathrm{loss}} = (1 - \eta_{\mathrm{loss}})P_a$ or $P_{\mathrm{loss}} = (1 - \eta_{\mathrm{loss}})P_q$, where $\eta_{\mathrm{loss}} = \ell_{in} + \ell_{ex}$. The installed gross jet thrust is $F_G = K_{fgr}F_g$, where K_{fgr} accounts for exhaust effects. The net jet thrust is $F_N = F_G - \dot{m}_{req}V$. The momentum drag of the auxiliary air flow is a function of the mass flow $\dot{m}_{\mathrm{aux}} = f_{\mathrm{aux}}\dot{m}_{req}$:

$$D_{\mathrm{aux}} = (1 - \eta_{\mathrm{aux}})\dot{m}_{\mathrm{aux}}V = (1 - \eta_{\mathrm{aux}})f_{\mathrm{aux}}\dot{m}_{req}V$$

where η_{aux} is the ram recovery efficiency.

18–5   Compressor

A compressor converts input shaft power to a jet velocity and thrust. The shaft power contributes to the propulsion group power required. The compressor does not use fuel.

18-5.1   Power Available

Given the flight condition and engine rating, the power available P_a is calculated from the specific power $SP_a = P_a/\dot{m}_a$ and mass flow $\dot{m}_a$:

$$\frac{SP_a}{\theta} = SP_0 g_{sp}(\theta, M, n)$$

$$\frac{\dot{m}_a}{\delta/\sqrt{\theta}} = \dot{m}_0 g_m(\theta, M, n)$$

$$\frac{P_a}{\delta\sqrt{\theta}} = P_0 g_p(\theta, M, n)$$

as functions of temperature ratio $\theta = T/T_0$, Mach number M, and referred compressor speed $n = N/\sqrt{\theta}$.

Installation losses P_{loss} are subtracted from P_a ($P_{av} = P_a - P_{\text{loss}}$), and then the mechanical limit is applied: $P_{av} = \min(P_{av}, rP_{\text{mech}R})$, $r = N/N_{\text{spec}}$. The mechanical limit is properly a torque limit, $Q_{\text{mech}} = P_{\text{mech}}/N_{\text{spec}}$, but is expressed as a power limit for clarity.

The compressor model gives the performance of a single compressor. The power available of the engine group is obtained by multiplying the single compressor power by the number of engines operational (total number of engines less inoperable engines):

$$P_{avEG} = f_P(N_{\text{eng}} - N_{\text{inop}})P_{av}$$

including a specified power fraction f_P.

18-5.2   Performance at Power Required

The compressor performance (mass flow and gross jet thrust) is calculated for a specified power required P_q and flight condition:

$$\frac{\dot{m}_{req}}{\delta/\sqrt{\theta}} = \dot{m}_{0C} g_m(q, \theta, M, n)$$

$$\frac{F_g}{\delta} = F_{g0C} g_f(q, \theta, M, n)$$

as functions of $q = P_q/(P_{0C}\delta\sqrt{\theta})$, temperature ratio $\theta = T/T_0$, Mach number M, and referred compressor speed $n = N/\sqrt{\theta}$).

The power required of a single compressor is obtained by dividing the engine group power by the number of engines operational (total number of engines less inoperable engines):

$$P_{req} = P_{reqEG}/(N_{\text{eng}} - N_{\text{inop}})$$

Accounting for installation losses gives the uninstalled power required $P_q = P_{req} + P_{\text{loss}}$.

The compressor model gives the performance of a single compressor. The performance of the engine group is obtained by multiplying the single compressor characteristics by the number of engines operational:

$$\dot{m}_{reqEG} = (N_{\text{eng}} - N_{\text{inop}})\dot{m}_{req}$$

$$F_{NEG} = (N_{\text{eng}} - N_{\text{inop}})F_N$$

$$D_{\text{aux}EG} = (N_{\text{eng}} - N_{\text{inop}})D_{\text{aux}}$$

$$P_{\text{comp}} = (N_{\text{eng}} - N_{\text{inop}})P_q K_{ffd}$$

The component power is the product of the uninstalled power required and the number of operational engines, and a factor K_{ffd} accounting for deterioration of the engine efficiency.

18-5.3   Installation

The difference between installed and uninstalled power is the inlet and exhaust losses P_{loss}: $P_{av} = P_a - P_{\text{loss}}$ and $P_{req} = P_q - P_{\text{loss}}$. The inlet and exhaust losses are modeled as fractions of power available or power required: $P_{\text{loss}} = (\ell_{in} + \ell_{ex})P_a$ or $P_{\text{loss}} = (\ell_{in} + \ell_{ex})P_q$. The installed gross jet thrust is $F_G = K_{fgr}F_g$, where K_{fgr} accounts for exhaust effects. The net jet thrust is $F_N = F_G - \dot{m}_{req}V$. The momentum drag of the auxiliary air flow is a function of the mass flow $\dot{m}_{\text{aux}} = f_{\text{aux}}\dot{m}_{req}$:

$$D_{\text{aux}} = (1 - \eta_{\text{aux}})\dot{m}_{\text{aux}}V = (1 - \eta_{\text{aux}})f_{\text{aux}}\dot{m}_{req}V$$

where η_{aux} is the ram recovery efficiency.

18-5.4   Compressor for Reaction Drive or Flow Control

If the compressor supplies the jet force for rotor reaction drive, then the engine group power required P_{EGreq} is fixed by ampitude input, or adjusted to produce the required rotor reaction force (momentum flux)

$$F_{Greq} = \dot{m}_{\text{react}}V_{\text{react}} = F_{\text{react}} + \dot{m}_{\text{react}}\Omega r_{\text{react}}$$

If the compressor supplies the momentum flux for flow control, then the engine group power required P_{EGreq} is fixed by ampitude input, or adjusted to produce the required momentum flux $F_{Greq} = \sum F_{Greq\ \text{comp}}$, summed over all components using the flow control. The gross jet thrust is directed to the reaction drive or flow control, so the net jet thrust of the engine group is $F_N = (F_G - F_{Gq}) - \dot{m}V$, from the gross thrust, mass flux used by the rotor or flow control $F_{Gq} = F_{Greq}/(N_{\text{eng}} - N_{\text{inop}})$, and momentum drag.

18–6   Electric Motor or Generator

A motor converts electrical energy (fuel) to shaft power. A generator converts input shaft power to electrical energy, and the shaft power contributes to the propulsion group power required.

The power available is related to the size P_{eng}. Given the flight condition and motor rating, the uninstalled power available P_a is calculated, including a torque limit. In the motor model, installation losses P_{TMS} are subtracted from P_a: $P_{av} = P_a - P_{\text{TMS}}$ (added for generator). The motor model gives the performance of a single engine. The power available of the engine group is obtained by multiplying the single engine power by the number of engines operational (total number of engines less inoperable engines):

$$P_{avEG} = f_P(N_{\text{eng}} - N_{\text{inop}})P_{av}$$

including a specified power fraction f_P.

From the engine group power required P_{reqEG}, the power required of a single engine is

$$P_{req} = P_{reqEG}/(N_{\text{eng}} - N_{\text{inop}})$$

In the motor model, installation losses P_{TMS} are added to P_{req}: $P_q = P_{req} + P_{\text{TMS}}$ (subtracted for generator).

18-6.1   Motor

The motor power required determines the energy flow from the fuel tank. The energy flow is calculated for P_q and a specified flight condition:

$$\dot{E}_{req} = \dot{E}_{0C}g_e(q, n) = P_q/\eta_{\text{motor}}$$

as a function of $q = P_q/P_{\mathrm{eng}}$ and engine speed $n = N/N_{\mathrm{spec}}$. The motor shaft power required is P_{req}, and the gross (uninstalled) power is $P_q = P_{req} + P_{\mathrm{TMS}}$. The rejected heat is $P_{\mathrm{rej}} = \dot{E}_{req} - P_q = P_q(1 - \eta_{\mathrm{motor}})/\eta_{\mathrm{motor}}$. From η_{motor} evaluated for P_{req}, P_{rej} gives the mass flow $\dot{m}_{\mathrm{TMS}}$, and then the power P_{TMS} and gross thrust F_G. Then η_{motor} is evaluated for $P_q = P_{req} + P_{\mathrm{TMS}}$, and the electrical power required is $\dot{E}_{req} = P_q/\eta_{\mathrm{motor}}$.

The motor model gives the performance of a single engine. The performance of the engine group is obtained by multiplying the single engine characteristics by the number of engines operational:

$$\dot{E}_{reqEG} = (N_{\mathrm{eng}} - N_{\mathrm{inop}})\dot{E}_{req}K_{ffd}$$

The energy flow has also been multiplied by a factor K_{ffd} accounting for deterioration of the engine efficiency.

18-6.2 Generator

The generator energy flow to the fuel tank defines the power required. The energy flow is calculated for P_q and a specified flight condition:

$$\dot{E}_{req} = \dot{E}_{0C}g_e(q, n) = P_q\eta_{\mathrm{motor}}$$

as a function of $q = P_q/P_{\mathrm{eng}}$ and engine speed $n = N/N_{\mathrm{spec}}$. The generator shaft power required is P_{req}, and the net (uninstalled) power is $P_q = P_{req} - P_{\mathrm{TMS}}$. The rejected heat is $P_{\mathrm{rej}} = P_q - \dot{E}_{req} = P_q(1 - \eta_{\mathrm{motor}})$. From η_{motor} evaluated for P_{req}, P_{rej} gives the mass flow $\dot{m}_{\mathrm{TMS}}$, and then the power P_{TMS} and gross thrust F_G. Then η_{motor} is evaluated for $P_q = P_{req} - P_{\mathrm{TMS}}$, and the electrical power produced is $\dot{E}_{req} = P_q\eta_{\mathrm{motor}}$.

The motor model gives the performance of a single engine. The performance of the engine group is obtained by multiplying the single engine characteristics by the number of engines operational:

$$\dot{E}_{reqEG} = (N_{\mathrm{eng}} - N_{\mathrm{inop}})\dot{E}_{req}$$
$$P_{\mathrm{comp}} = (N_{\mathrm{eng}} - N_{\mathrm{inop}})P_{req}K_{ffd}$$

The component power is the product of the power required and the number of operational engines, and a factor K_{ffd} accounting for deterioration of the engine efficiency.

18-6.3 Generator-Motor

The engine mode B is the direction of power flow for a generator-motor: B positive for motor operation, and B negative for generator operation. Separate motor models are used for the two modes.

18-6.4 Installation (Thermal Management System)

The difference between installed and uninstalled power is the thermal management system losses P_{TMS}: $P_{av} = P_a - P_{\mathrm{TMS}}$ and $P_{req} = P_q - P_{\mathrm{TMS}}$ (added for generator). The system mass flow $\dot{m}_{\mathrm{TMS}}$ depends on the motor rejected heat P_{rej}; the power P_{TMS} and gross thrust F_G depend on the mass flow. The net jet thrust is $F_N = F_G - \dot{m}_{\mathrm{TMS}}V$. The size of the thermal management system (including weight) is determined by a design rejected heat $P_{\mathrm{rej-design}}$, perhaps specified as a fraction of the motor size P_{eng}.

The motor model gives the performance of the thermal management system for a single engine. The performance of the engine group is obtained by multiplying the single engine characteristics by the

number of engines operational:

$$P_{\text{TMS}EG} = (N_{\text{eng}} - N_{\text{inop}})P_{\text{TMS}}$$
$$\dot{m}_{reqEG} = (N_{\text{eng}} - N_{\text{inop}})\dot{m}_{\text{TMS}}$$
$$F_{NEG} = (N_{\text{eng}} - N_{\text{inop}})F_N$$

18–7   Control and Loads

The engine power amplitude A and mode B are control variables. The mode can be mass flow (for convertible engines), or power flow (for generator-motor). In distributing the propulsion group power required to the engine groups, an engine group power can be fixed at $P_{reqEG} = (N_{\text{eng}} - N_{\text{inop}})A$; or fraction A of engine power available, $P_{reqEG} = (N_{\text{eng}} - N_{\text{inop}})AP_{av}$; or fraction A of engine rated power $P_{reqEG} = (N_{\text{eng}} - N_{\text{inop}})AP_{\text{eng}}$.

The engine orientation is specified by selecting a nominal direction e_{f0} in body axes (positive or negative x-, y-, or z-axis; usually thrust forward, hence positive x-axis); then applying a yaw angle ψ; and then an incidence or tilt angle i. The yaw and incidence angles can be control variables.

The engine group produces a jet thrust F_N, acting in the direction of the engine (actually, $F_N = F_G - \dot{m}V$, with the gross thrust in the direction of the engine and the momentum thrust in the wind direction; $F_N = (F_G - F_{Gq}) - \dot{m}V$ for reaction drive or flow control); a momentum drag D_{aux} (if the component has mass flow), a nacelle drag D_{nac}, and cooling drag D_{cool} (all acting in the wind direction). The engine group is at location z^F. The force and moment acting on the aircraft in body axes are thus:

$$F^F = (e_f F_G + e_d \dot{m}V) + e_d(D_{\text{aux}} + D_{\text{nac}} + D_{\text{cool}})$$
$$M^F = \widetilde{\Delta z^F} F^F$$

where $\Delta z^F = z^F - z^F_{cg}$, e_f is the engine thrust direction and e_d is the drag direction.

18–8   Nacelle and Cooling Drag

The engine group includes a nacelle, which contributes to the aircraft drag. The component drag contributions must be consistent. The pylon is the rotor support and the nacelle is the engine support. The drag model for a tiltrotor aircraft with tilting engines would use the pylon drag (and no nacelle drag), since the pylon is connected to the rotor shaft axes; with non-tilting engines it would use the nacelle drag as well.

The reference area for the nacelle drag coefficient is the nacelle wetted area. The wetted area per engine is input, or calculated either from the engine system (engine, exhaust, and accessories) weight or from the engine system plus drive system weight:

$$S_{\text{wet}} = k\left(w/N_{\text{eng}}\right)^{2/3}$$

where $w = W_{ES}$ or $w = W_{ES} + W_{gbrs}/N_{EG}$, and the units of k are ft^2/lb$^{2/3}$ or m^2/kg$^{2/3}$. The reference area is then $S_{\text{nac}} = N_{\text{eng}}S_{\text{wet}}$.

The drag associated with engine group cooling is $D_{\text{cool}} = (1 - \eta)\dot{m}V = (1 - \eta)(\rho AV)V$, where A is the inlet area, η the ram recovery efficiency, and V the flight speed. So the drag area is specified, estimated from $(D/q)_{\text{cool}} = 2(1 - \eta)AV$.

18–9 Weights

The component weight consists of engine system, engine section or nacelle group, and air induction group. The engine system consists of engine, exhaust system, and accessories. The engine section or nacelle group consists of engine support, engine cowling, and pylon support. These weights are for the engine group, consisting of N_{eng} engines. The engine system weight $W_{ES} = W_{\text{eng}} + W_{\text{exh}} + W_{\text{acc}}$ is used for the engine nacelle wetted area, the rotor pylon wetted area, and the rotor moving weight. The rotor group includes the rotor support structural weight; this must be consistent with the use of the engine support weight and pylon support weight.

Jet Group

A jet group produces a force on the aircraft, possibly used for lift, propulsion, or control. A jet model describes a particular jet, used in one or more jet groups. The models include turbojet and turbofan engines (perhaps convertible, for reaction drive), reaction drive, and a simple force. A reaction drive supplies a blade force that provides the rotor power required.

19–1   Jet Group Performance

The jet size is described by the thrust T_{jet}, which is the sea-level static thrust available per jet at a specified takeoff rating. The number of jets N_{jet} is specified for each jet group.

If the sizing task determines the jet thrust, the total thrust required is $T_{JG} = rN_{\text{jet}}T_{\text{jet}}$, where $r = \max(T_{reqJG}/T_{avJG})$.

The flight condition information includes the altitude, temperature, and flight speed; and a thrust fraction f_T.

19–2   Turbojet or Turbofan

The thrust of a turbojet is

$$T = \dot{m}\big((1+f)V_e - V\big) + (p_e - p_{\text{atm}})A_e$$

where $\dot{m}$ is the mass flow; $f = \dot{w}/\dot{m}$ is the fuel-air ratio; and V_e, p_e, and A_e are the velocity, pressure, and area at the exit. The pressure term is zero or small, and the fuel-air ratio is small, so the net thrust is approximately

$$T = \dot{m}(V_e - V) = T_G - T_{\text{mom}}$$

from the gross thrust $T_G = \dot{m}V_e$, and the inlet-momentum or ram drag $T_{\text{mom}} = \dot{m}V$.

The thrust of a turbofan is

$$T = \dot{m}\big((1+f)V_e - V\big) + \dot{m}_{\text{fan}}\big(V_{\text{efan}} - V\big) = \dot{m}\big((1+f)V_e + \beta V_{\text{efan}}\big) - \dot{m}(1+\beta)V = T_G - T_{\text{mom}}$$

with bypass ratio $\beta = \dot{m}_{\text{fan}}/\dot{m}$.

Turbojet or turbofan performance is obtained from the Referred Parameter Jet Engine Model (RP-JEM), based on references 1 and 2. The referred thrust, specific thrust $ST = T/\dot{m}$, and specific fuel con-

sumption $\text{sfc} = \dot{w}/T$ are functions of the flight Mach number M and compressor speed $n = N/(N_0\sqrt{\theta})$:

$$\frac{T}{\delta} = G_t(M, n)$$

$$\frac{T/\dot{m}}{\sqrt{\theta}} = G_{st}(M, n)$$

$$\frac{\dot{w}/T}{\sqrt{\theta}} = \frac{1}{\eta_b}G_{\text{sfc}}(M, n)$$

The independent variable can be the compressor speed, or the turbine inlet temperature, or the fuel flow. The combustion efficiency η_b depends on the atmosphere (altitude and temperature), hence the specific fuel consumption is not a function of just M and n.

19-2.1   Thrust Available

Given the flight condition and engine rating, the thrust available T_a is calculated from the specific thrust $ST_a = T_{Ga}/\dot{m}_a$ and mass flow $\dot{m}_a$:

$$\frac{ST_a}{\sqrt{\theta}} = ST_0 g_{st}(\theta, M)$$

$$\frac{\dot{m}_a}{\delta/\sqrt{\theta}} = \dot{m}_0 g_m(\theta, M)$$

$$\frac{T_a}{\delta} = T_0 g_t(\theta, M) - \frac{\dot{m}_a}{\delta/\sqrt{\theta}}(1 + \beta)\frac{V}{\sqrt{\theta}}$$

as functions of temperature ratio $\theta = T/T_0$ and Mach number M.

In the jet model, installation losses T_{loss} are subtracted from T_a ($T_{av} = T_a - T_{\text{loss}}$), and then the mechanical limit is applied: $T_{av} = \min(T_{av}, T_{\text{mech}R})$.

The jet model gives the performance of a single jet. The thrust available of the jet group is obtained by multiplying the single jet thrust by the number of jets operational (total number of jets less inoperable jets):

$$T_{avJG} = f_T(N_{\text{jet}} - N_{\text{inop}})T_{av}$$

including a specified thrust fraction f_T.

19-2.2   Performance at Thrust Required

The thrust required of a single jet is obtained by dividing the jet group thrust by the number of jets operational (total number of jets less inoperable jets):

$$T_{req} = T_{reqJG}/(N_{\text{jet}} - N_{\text{inop}})$$

In the jet model, installation losses T_{loss} are added to T_{req} ($T_q = T_{req} + T_{\text{loss}}$).

The jet performance (mass flow and fuel flow) is calculated for a specified thrust required T_q and flight condition:

$$\frac{\dot{m}_{req}}{\delta/\sqrt{\theta}} = \dot{m}_{0C} g_m(t, \theta, M)$$

$$\frac{\dot{w}_{req}}{\delta\sqrt{\theta}} = \dot{w}_{0C} g_w(t, \theta, M)$$

as functions of the referred gross thrust $t = T_{Gq}/(T_{0C}\delta)$ (where $T_{Gq} = T_q + \dot{m}(1 + \beta)V$), temperature ratio $\theta = T/T_0$, and Mach number M. The specific thrust is $ST = T_{Gq}/\dot{m}$. The net force is $F_N = T_{req} = F_G - \dot{m}_{req}(1 + \beta)V = F_G - F_{\text{mom}}$.

The jet model gives the performance of a single jet. The performance of the jet group is obtained by multiplying the single jet characteristics by the number of jets operational:

$$\dot{m}_{reqJG} = (N_{\text{jet}} - N_{\text{inop}})\dot{m}_{req}$$

$$\dot{w}_{reqJG} = (N_{\text{jet}} - N_{\text{inop}})\dot{w}_{req}K_{ffd}$$

$$D_{\text{aux}JG} = (N_{\text{jet}} - N_{\text{inop}})D_{\text{aux}}$$

The fuel flow has also been multiplied by a factor K_{ffd} accounting for deterioration of the jet efficiency.

19-2.3   Installation

The difference between installed and uninstalled thrust is the inlet and exhaust losses T_{loss}: $T_{av} = T_a - T_{\text{loss}}$ and $T_{req} = T_q - T_{\text{loss}}$. The inlet and exhaust losses are modeled as fractions of thrust available or thrust required: $T_{\text{loss}} = (\ell_{in} + \ell_{ex})T_a$ or $T_{\text{loss}} = (\ell_{in} + \ell_{ex})T_q$. The momentum drag of the auxiliary air flow is a function of the mass flow $\dot{m}_{\text{aux}} = f_{\text{aux}}\dot{m}_{req}$:

$$D_{\text{aux}} = (1 - \eta_{\text{aux}})\dot{m}_{\text{aux}}V = (1 - \eta_{\text{aux}})f_{\text{aux}}\dot{m}_{req}V$$

where η_{aux} is the ram recovery efficiency. Exhaust losses (ℓ_{ex}) and auxiliary air flow parameters (η_{aux}, f_{aux}) are defined for IR suppressor on and off.

19-2.4   Convertible Engine: Reaction Drive

The jet mode B is the mass flow fraction diverted for a convertible engine: $B = 0$ for all mass flow to the exhaust (turbojet operation), and $B = 1$ for all mass flow to the rotor (reaction jet operation).

A separate jet model defines the engine performance for $B = 1$. The jet thrust required is fixed by amplitude input, or adjusted to produce the required rotor reaction force (momentum flux) $T_{reqJG} = F_{Greq}$. The mass flow and fuel flow follow. The net force of the jet is $F_N = (T_{req} - F_{Gq}) - \dot{m}_{req}(1 + \beta)V$.

Thrust, specific thrust, and mass flow are defined by the jet model for turbojet operation. The specific fuel consumption of the jet model for reaction jet operation is scaled proportional to the sfc scaling for turbojet operation.

19–3   Reaction Drive or Flow Control

Rotor power can be supplied by a reaction drive, using cold or hot air ejected out of the blade tips or trailing edges. Helicopters have also been designed with ram jets on the blade tips, or with jet flaps on the blade trailing edges that use compressed air generated in the fuselage. The jet group performance includes the efficiency of the blade duct and nozzle, perhaps even with tip burning. The fuselage and wing can have flow control, produced by momentum flux through slots on the component. The jet group performance includes the efficiency of the ducting and slots.

For reaction drive and flow control, the thrust available and thrust required exclude the momentum drag, which is only accounted for in the total jet group load. For reaction drive, the net jet thrust required is fixed by amplitude input, or adjusted to produce the required rotor reaction force (momentum flux)

$$T_{reqJG} = F_{Greq} = \dot{m}_{\text{react}}V_{\text{react}} = F_{\text{react}} + \dot{m}_{\text{react}}\Omega r_{\text{react}}$$

For flow control, the net jet thrust required is fixed by amplitude input, or adjusted to produce the required momentum flux $F_{Greq} = \sum F_{Greq\,\mathrm{comp}}$, summed over all components using the flow control. The net force of the jet is $F_N = (T_{req} - F_{Gq}) - \dot{m}_{req}(1+\beta)V = F_G - F_{\mathrm{mom}}$, from $F_{Gq} = F_{Greq}/(N_{\mathrm{jet}} - N_{\mathrm{inop}})$. From the required gross thrust, the jet performance (mass flow and fuel flow) is calculated.

19–4   Simple Force

For the simple force model, the design maximum thrust is $T_{\max}$ (per jet). The thrust available is thus $T_a = T_{\max}$. The force generation can use fuel as weight or as energy.

If the component burns fuel weight, the fuel flow is calculated from an input thrust-specific fuel consumption: $\dot{w}_{req} = T_q(\mathrm{sfc}) = \dot{w}_{0C}q$, where $q = T_q/T_{\max}$. Units of sfc are pound/hour/pound or kilogram/hour/Newton. Then

$$\dot{w}_{reqJG} = (N_{\mathrm{jet}} - N_{\mathrm{inop}})\dot{w}_{req}K_{ffd}$$

The fuel flow has also been multiplied by a factor K_{ffd} accounting for deterioration of the jet efficiency.

If the component uses fuel energy, the energy flow is calculated from an input thrust-specific fuel consumption: $\dot{E}_{req} = T_q(\mathrm{sfc}) = \dot{E}_{0C}q$, where $q = T_q/T_{\max}$. Units of sfc are MJ/hour/pound or MJ/hour/Newton. Then

$$\dot{E}_{reqJG} = (N_{\mathrm{jet}} - N_{\mathrm{inop}})\dot{E}_{req}K_{ffd}$$

The energy flow has also been multiplied by a factor K_{ffd} accounting for deterioration of the jet efficiency.

The simple force weight is calculated from specific weight S plus a fixed increment: $W = ST_{\max} + \Delta W$. This weight is identified as either engine system or propeller/fan installation weight, both of the propulsion group; or tail-rotor, of empennage group.

19–5   Control and Loads

The jet thrust amplitude A and mode B are control variables. The mode can be mass flow (for convertible engines). The jet group thrust can be fixed at $T_{reqJG} = (N_{\mathrm{jet}} - N_{\mathrm{inop}})A$; or fraction A of jet thrust available, $T_{reqJG} = (N_{\mathrm{jet}} - N_{\mathrm{inop}})AT_{av}$; or fraction A of jet rated thrust $T_{reqJG} = (N_{\mathrm{jet}} - N_{\mathrm{inop}})AT_{\mathrm{jet}}$.

The jet orientation is specified by selecting a nominal direction e_{f0} in body axes (positive or negative x-, y-, or z-axis; usually thrust forward, hence positive x-axis); then applying a yaw angle ψ; and then an incidence or tilt angle i. The yaw and incidence angles can be control variables.

The jet group produces a jet thrust T, acting in the direction of the engine (actually, the gross thrust in the direction of the engine and the momentum thrust in the wind direction); a momentum drag D_{aux}, a nacelle drag D_{nac}, and cooling drag D_{cool} (all acting in the wind direction). The jet group is at location z^F. The force and moment acting on the aircraft in body axes are thus:

$$F^F = (e_f F_G + e_d F_{\mathrm{mom}}) + e_d(D_{\mathrm{aux}} + D_{\mathrm{nac}} + D_{\mathrm{cool}})$$
$$M^F = \widetilde{\Delta z^F} F^F$$

where $\Delta z^F = z^F - z^F_{cg}$, e_f is the jet thrust direction and e_d is the drag direction. The gross thrust is $F_G = F_N + F_{\mathrm{mom}}$, $F_{\mathrm{mom}} = \dot{m}_{reqJG}(1+\beta)V$.

19–6   Nacelle and Cooling Drag

The jet group includes a nacelle, which contributes to the aircraft drag. The reference area for the nacelle drag coefficient is the nacelle wetted area. The wetted area per jet is input, or calculated from the engine system (engine, exhaust, and accessories) weight:

$$S_{\text{wet}} = k\left(w/N_{\text{jet}}\right)^{2/3}$$

where $w = W_{ES}$, and the units of k are ft^2/lb$^{2/3}$ or m^2/kg$^{2/3}$. The reference area is then $S_{\text{nac}} = N_{\text{jet}} S_{\text{wet}}$.

The drag associated with jet group cooling is $D_{\text{cool}} = (1 - \eta)\dot{m}V = (1 - \eta)(\rho AV)V$, where A is the inlet area, η the ram recovery efficiency, and V the flight speed. So the drag area is specified, estimated from $(D/q)_{\text{cool}} = 2(1 - \eta)AV$.

19–7   Weights

The component weight consists of engine system, engine section or nacelle group, and air induction group. The engine system consists of engine, exhaust system, and accessories. The engine section or nacelle group consists of engine support, engine cowling, and pylon support. These weights are for the jet group, consisting of N_{jet} engines. The engine system weight $W_{ES} = W_{\text{eng}} + W_{\text{exh}} + W_{\text{acc}}$ is used for the engine nacelle wetted area.

19–8   References

1) Sanders, N.D. "Performance Parameters for Jet-Propulsion Engines." NACA TN 1106, July 1946.

2) Hill, P.G., and Peterson, C.R. *Mechanics and Thermodynamics of Propulsion*. Reading, MA: Addison-Wesley Publishing Company, Inc., 1965.

Charge Group

A charge group generates energy for the aircraft. A charger model describes a particular charger, used in one or more charge groups. The models include fuel cells, solar cells, and a simple charger.

20–1 Charge Group Performance

The charger size is described by the power P_{chrg}, which is the sea-level static power available per charger at a specified takeoff rating. The number of chargers N_{chrg} is specified for each charge group.

If the sizing task determines the charger power, the charger power required is rP_{chrg}, where $r = \max(P_{reqCG}/P_{avCG})$.

The flight condition information includes the altitude, temperature, and flight speed; and a power fraction f_P.

Given the flight condition and the charger rating, the charger power available is

$$P_{av} = \dot{E}_{a\mathrm{cell}} = P_0 g_p(\delta, M)$$

as a function of pressure ratio $\delta = p/p_0$ and Mach number M. The power available is obtained by multiplying the single charger power by the number of chargers operational (total number of chargers less inoperable chargers):

$$P_{avCG} = f_P(N_{\mathrm{chrg}} - N_{\mathrm{inop}})P_{av}$$

including a specified power fraction f_P.

The energy flow to the fuel tank gives the charge group power required: $P_{reqCG} = \dot{E}_{reqCG}$. The power required of a single charger is

$$P_{req} = P_{reqCG}/(N_{\mathrm{chrg}} - N_{\mathrm{inop}})$$

The cell energy flow required $\dot{E}_{q\mathrm{cell}}$ is $\dot{E}_{q\mathrm{cell}} = P_{0C} g_e(q)$, where $q = P_{req}/P_{0C}$. Then

$$P_{req\mathrm{total}} = (N_{\mathrm{chrg}} - N_{\mathrm{inop}})\dot{E}_{q\mathrm{cell}}$$

is the total cell power required.

20–2 Fuel Cell

A fuel cell burns a fuel (typically hydrogen) and generates electrical energy. For a specified flight condition, the cell power available and power required are

$$P_{av} = \dot{E}_{a\mathrm{cell}} = P_0 g_p(\delta, M)$$
$$\dot{E}_{q\mathrm{cell}} = P_{0C} g_e(q)$$

where $q = P_{req}/P_{0C}$. The fuel cell performance (mass flow and fuel flow) is

$$\dot{m}_{req} = \dot{m}_{0C} g_m(q, \delta, M)$$
$$\dot{w}_{req} = \dot{w}_{0C} g_w(q, \delta, M)$$

Installation losses are included in the specific fuel consumption. The net thrust is the inlet momentum drag, $F_N = -\dot{m}_{req}V$. The charger model gives the performance of a single charger. The performance of the charge group is obtained by multiplying the single charger characteristic by the number of chargers operational:

$$\dot{m}_{reqCG} = (N_{chrg} - N_{inop})\dot{m}_{req}$$
$$\dot{w}_{reqCG} = (N_{chrg} - N_{inop})\dot{w}_{req}K_{ffd}$$
$$F_{NCG} = (N_{chrg} - N_{inop})F_N$$
$$D_{auxCG} = (N_{chrg} - N_{inop})D_{aux}$$

The fuel flow has also been multiplied by a factor K_{ffd} accounting for deterioration of the engine efficiency. The momentum drag of the auxiliary air flow is a function of the mass flow $\dot{m}_{aux} = f_{aux}\dot{m}_{req}$:

$$D_{aux} = (1 - \eta_{aux})\dot{m}_{aux}V = (1 - \eta_{aux})f_{aux}\dot{m}_{req}V$$

where η_{aux} is the ram recovery efficiency.

20–3   Solar Cell

The power available from solar radiation is approximately 1.36 kW/m^2, reduced by atmospheric effects (absorption, reflection, and scattering) to about 1.00 kW/m^2. The average solar radiation in the continental United States is 3.5–7.0 (kW/m^2)(hour/day), hence approximately 15 to 25% of the available power. Typical efficiencies of solar cells are 10–35%, with some sensitivity to temperature. The solar cell is characterized by power density (W/m^2) and weight density (kg/m^2).

For a specified flight condition, the cell energy flow is

$$P_{av} = \dot{E}_{acell} = P_0 g_p(\delta, M)$$
$$\dot{E}_{qcell} = P_{0C} g_e(q)$$

where $q = P_{req}/P_{0C}$.

20–4   Simple Charger

For the simple charger model, the design maximum power is P_{max} (per charger). The power available is $P_{av} = P_{max}$. The energy flow is $\dot{E}_{qcell} = P_{0C} g_e(q) = P_{req}/\eta_{chrg}$, where η_{chrg} is the charger efficiency. The weight is calculated from specific weight S plus a fixed increment: $W = SP_{max} + \Delta W$.

20–5   Control and Loads

The charger power amplitude A and mode B are control variables. The charge group power can be fixed at $\dot{E} = P_{reqCG} = (N_{chrg} - N_{inop})A$; or fraction A of charger rated power $\dot{E} = P_{reqCG} = (N_{chrg} - N_{inop})AP_{chrg}$.

The charger orientation is specified by selecting a nominal direction e_{f0} in body axes (positive or negative x-, y-, or z-axis); then applying a yaw angle ψ; and then an incidence or tilt angle i. The yaw and incidence angles can be control variables.

The charge group produces a jet thrust F_N (actually, $F_N = F_G - \dot{m}V$, with the gross thrust in the direction of the engine and the momentum thrust in the wind direction), acting in the direction of the charger; a momentum drag D_{aux} (if the component has mass flow), a nacelle drag D_{nac}, and cooling drag D_{cool} (all acting in the wind direction). The charge group is at location z^F. The force and moment acting on the aircraft in body axes are thus:

$$F^F = (e_f F_G + e_d \dot{m}V) + e_d(D_{\text{aux}} + D_{\text{nac}} + D_{\text{cool}})$$
$$M^F = \widetilde{\Delta z^F} F^F$$

where $\Delta z^F = z^F - z^F_{cg}$, e_f is the charger thrust direction and e_d is the drag direction.

20–6   Nacelle and Cooling Drag

The charge group includes a nacelle, which contributes to the aircraft drag. The reference area for the nacelle drag coefficient is the nacelle wetted area. The wetted area per charger is input, or calculated from the engine system (engine, exhaust, and accessories) weight:

$$S_{\text{wet}} = k\left(w/N_{\text{chrg}}\right)^{2/3}$$

where $w = W_{ES}$, and the units of k are ft^2/lb$^{2/3}$ or m^2/kg$^{2/3}$. The reference area is then $S_{\text{nac}} = N_{\text{chrg}} S_{\text{wet}}$.

The drag associated with charge group cooling is $D_{\text{cool}} = (1 - \eta)\dot{m}V = (1 - \eta)(\rho AV)V$, where A is the inlet area, η the ram recovery efficiency, and V the flight speed. So the drag area is specified, estimated from $(D/q)_{\text{cool}} = 2(1 - \eta)AV$.

20–7   Weights

The component weight consists of the engine system. The engine system consists of the charger weight. These weights are for the charge group, consisting of N_{chrg} chargers. The engine system weight $W_{ES} = N_{\text{chrg}} W_{\text{one chrg}}$ is used for the charger nacelle wetted area.

Referred Parameter Turboshaft Engine Model

Aircraft gas turbine engine performance capabilities are formally specified by computer programs known as engine decks, which are created by engine manufacturers in an industry-standard format. Engine decks are typically based on thermodynamic cycle analysis using real engine component performance maps. The most important performance maps for turboshaft engines are compressor, gas generator turbine, and power turbine. These component performance maps are critical to obtaining realistic off-design engine performance. Design and analysis codes calculate aircraft performance for a very wide range of operating conditions. Thus engine performance must be realistic even far from the engine design point. A simple thermodynamic cycle analysis that assumes design point component efficiencies everywhere is not realistic for such an application. Rather than developing models for component performance, the approach taken is to use a model for the total engine performance. The engine is not being designed.

The Referred Parameter Turboshaft Engine Model (RPTEM) is based on curve-fits of performance data for existing or projected engines over a range of operating conditions. The curve-fits are typically obtained by exercising an engine deck. The use of referred parameters tends to collapse the data, and provides a basis for scaling the engine. The operating condition is described by pressure altitude, ambient air temperature, flight Mach number, power turbine speed, exhaust nozzle area, and either engine rating or engine power required. These curve-fits, typically based on real engines, are scaled to the required size and adjusted to the appropriate technology level to represent a notional engine. Engine size is represented by mass flow. Engine technology is represented by specific power available and specific fuel consumption at maximum continuous power (MCP), sea level/standard day (SLS), static (zero airspeed) conditions. Engine installation effects (inlet and exhaust losses) are also modeled.

The use of referred parameters to curve-fit engine performance data was suggested by David Woodley from Boeing during the JVX program (1983). The RPTEM was developed and documented by Michael P. Scully and Henry Lee of ASRAO, U.S. Army Aeroflightdynamics Directorate (AFDD), with a subsequent implementation written by Sam Ferguson (1995).

21–1   Operating Environment

The operating condition and atmosphere give the standard conditions (temperature T_{std} and pressure p_{std}) for a specified pressure altitude; the sea-level standard conditions (temperature T_0 and pressure p_0); and the operating temperature T and pressure p. Here the temperatures are °R or °K. The engine characteristics depend on the temperature ratio $\theta = T/T_0$ and pressure ratio $\delta = p/p_0$.

The flight Mach number $M = V/c_s = V/c_{s0}\sqrt{\theta}$ is obtained from the aircraft speed V.

The inlet ram air temperature ratio and pressure ratio are obtained then from M and the inlet ram recovery efficiency η_d:

$$\theta_M = \left(1 + \frac{\gamma - 1}{2} M^2\right) = \left(1 + 0.2M^2\right)$$

$$\delta_M = \left(1 + \frac{\gamma - 1}{2} \eta_d M^2\right)^{\frac{\gamma}{\gamma-1}} = \left(1 + 0.2\eta_d M^2\right)^{3.5}$$

where the ratio of specific heats $\gamma = 1.4$.

21–2 Performance Characteristics

The engine performance is described by: the power available P_a, at each engine rating and the specification engine turbine speed N_{spec}; the mass flow $\dot{m}$ and fuel flow $\dot{w}$ required to produce power required P_q at engine turbine speed N; and the gross jet thrust F at a given power required P_q. Then the specific power is $SP = P/\dot{m}$, and the specific fuel consumption is $\text{sfc} = \dot{w}/P$.

The reference performance is at sea-level-standard static conditions (subscript 0), and MCP (subscript C). For each rating R, the performance is characterized by the following quantities for sea-level-standard static conditions: power P_{0R}, specific power SP_{0R}, and mechanical power limit $P_{\text{mech}R}$. The mass flow is then $\dot{m}_{0R} = P_{0R}/SP_{0R}$. The gross jet thrust F_{g0C} is given at MCP. These characteristics are at the specification turbine speed N_{spec}.

The installed power required P_{req} and power available $P_{av} > P_{req}$ are measured at the engine output shaft. In addition to shaft power, the engine exhaust produces a net jet thrust F_N, from mass flow that goes through the engine core. The fuel flow and mass flow are the total required to produce the shaft power and jet thrust. The forces produced by mass flow that does not go through the engine core (such as infrared suppressor or cooling air) are treated as momentum drag D_{aux}.

The difference between net and gross jet thrust is the momentum drag: $F_n = F_g - \dot{m}_{req}V = \dot{m}_{req}(V_j - V)$, where V_j is the engine jet exhaust velocity. Note that traditional units for mass flow are pound/sec (pps), while this equation requires slug/sec ($\dot{m}_{req}/g$ replaces $\dot{m}_{req}$).

The uninstalled power required is P_q, the power available P_a, the gross jet thrust F_g, and net jet thrust F_n. The engine model calculates P_a as a function of flight condition and engine rating; or calculates engine mass flow, fuel flow, and jet thrust at P_q.

21–3 Installation

The difference between installed and uninstalled power is the inlet and exhaust losses P_{loss}:

$$P_{av} = P_a - P_{\text{loss}}$$

$$P_{req} = P_q - P_{\text{loss}}$$

The inlet ram recovery efficiency η_d (through δ_M) is included in the engine model calculations. The inlet and exhaust losses are modeled as fractions of power available or power required: $P_{\text{loss}} = (\ell_{in} + \ell_{ex})P_a$ or $P_{\text{loss}} = (\ell_{in} + \ell_{ex})P_q$. So

$$P_{av} = P_a(1 - \ell_{in} - \ell_{ex})$$

$$P_q = P_{req}/(1 - \ell_{in} - \ell_{ex})$$

The engine model gives uninstalled power and the gross thrust F_g for a nominal exhaust nozzle area. The gross jet thrust F_G and exhaust power loss (ℓ_{ex}) are both functions of the exhaust nozzle area. Smaller exhaust nozzle areas increase exhaust losses and increase gross thrust. Thus the ratio of installed to uninstalled thrust is approximated by a function of the exhaust power loss:

$$F_G/F_g = K_{fgr} = K_{fgr0} + K_{fgr1}\ell_{ex} + K_{fgr2}\ell_{ex}^2 + K_{fgr3}\ell_{ex}^3$$

so the net installed jet thrust is

$$F_N = K_{fgr}F_g - \dot{m}_{req}V$$

The momentum drag of the auxiliary air flow is a function of the mass flow $\dot{m}_{\text{aux}} = f_{\text{aux}}\dot{m}_{req}$:

$$D_{\text{aux}} = (1 - \eta_{\text{aux}})\dot{m}_{\text{aux}}V = (1 - \eta_{\text{aux}})f_{\text{aux}}\dot{m}_{req}V$$

where η_{aux} is the ram recovery efficiency. Exhaust losses (ℓ_{ex}) and auxiliary air flow parameters (η_{aux}, f_{aux}) are defined for IR suppressor on and off. Inlet particle separator loss is added to the inlet losses (ℓ_{in}).

21–4 Power Turbine Speed

The shaft power available is a function of the gas power available P_G and the power turbine efficiency η_t: $P_a = \eta_t P_G$. Generally the power turbine speed N has a significant effect on η_t, but almost no effect on P_G. The model used for the efficiency variation is $\eta_t \cong 1 - |(N/N_{\text{opt}}) - 1|^{X_{N\eta}}$, where N_{opt} is the speed for peak efficiency, hence

$$\frac{P(N)}{P(N_{\text{spec}})} = \frac{\eta_t(N)}{\eta_t(N_{\text{spec}})} = \frac{1 - |(N/N_{\text{opt}}) - 1|^{X_{N\eta}}}{1 - |(N_{\text{spec}}/N_{\text{opt}}) - 1|^{X_{N\eta}}}$$

Two approximations for the optimum turbine speed are used. The first is a linear expression in $p = P/(P_{0R}\delta\sqrt{\theta})$:

$$N_{\text{opt}} = N_{\text{spec}}\sqrt{\theta}\left(K_{NoptA}\sqrt{\theta_M} + \frac{K_{NoptB}}{\delta_M}p\right)$$

and the second is a cubic function of $p = P/(P_{0C}\delta\sqrt{\theta})$:

$$N_{\text{opt}} = N_{\text{opt}0C}\sqrt{\theta}\left(K_{Nopt0} + K_{Nopt1}p + K_{Nopt2}p^2 + K_{Nopt3}p^3\right)[\theta_M]^{X_{Nopt}}$$

The second expression is based on a larger data sample. For power available calculations, $P = P_a(N_{\text{spec}})$; for power required calculations, $P = P_q(N)$.

21–5 Power Available

Given the flight condition and engine rating, the power available P_a is calculated as follows. The specific power and referred mass flow (at N_{spec}, relative to SP_0 and $\dot{m}_0$ for this rating) are approximated by functions of the ambient temperature ratio θ and inlet ram air ratios:

$$SP_a(N_{\text{spec}}) = SP_0\,\theta\,K_{spa}\left[\delta_M\sqrt{\theta_M}\right]^{X_{spa}}$$

$$\dot{m}_a(N_{\text{spec}}) = \dot{m}_0\left(\delta/\sqrt{\theta}\right)e^{K_{mfa}}\left[\delta_M\sqrt{\theta_M}\right]^{X_{mfa}}$$

where the static lapse rate (K_{spa}, K_{mfa}) and ram air exponents (X_{spa}, X_{mfa}) are piecewise linear functions of θ. The power available is then

$$P_a(N_{\text{spec}}) = P_0 \frac{SP_a(N_{\text{spec}})}{SP_0} \frac{\dot{m}_a(N_{\text{spec}})}{\dot{m}_0} = P_0 \left(\delta\sqrt{\theta}\right) K_{spa} e^{K_{mfa}} \left[\delta_M \sqrt{\theta_M}\right]^{X_{spa}+X_{mfa}}$$

This expression for $\dot{m}_a$ is used only to calculate P_a; elsewhere the $\dot{m}_q$ expression below (for performance at power required) is used to obtain the mass flow at a power P_q. Finally

$$P_a(N) = P_a(N_{\text{spec}}) \frac{1 - |(N/N_{\text{opt}}) - 1|^{X_{N\eta}}}{1 - |(N_{\text{spec}}/N_{\text{opt}}) - 1|^{X_{N\eta}}}$$

is the power available at turbine speed N. Installation losses P_{loss} are subtracted from P_a ($P_{av} = P_a - P_{\text{loss}}$), and then the mechanical limit is applied: $P_{av} = \min(P_{av}, r P_{\text{mechR}})$, $r = N/N_{\text{spec}}$. The mechanical limit is properly a torque limit, $Q_{\text{mech}} = P_{\text{mech}}/N_{\text{spec}}$, but is expressed as a power limit for clarity.

For a simple model, the power available can be constant, $P_a = P_0$; or the referred power can be constant, $P_a = P_0(\delta\sqrt{\theta})$; without any effect of power turbine speed.

21-5.1   Piecewise Linear Parameters

Several parameters K are input as piecewise linear functions of the temperature ratio θ. One format of the input is a set of I regions, with the function in the i-th region given by $K = K_{0i} + K_{1i}\theta$. The break point between the i and $i-1$ regions is

$$\theta_{bi} = -\frac{K_{0i} - K_{0(i-1)}}{K_{1i} - K_{1(i-1)}}$$

$$K_{bi} = K_{0i} + K_{1i}\theta_b$$

for $i = 2$ to I. Another format is a set of $I + 1$ break points, with the values at the i-th point (θ_{bi}, K_{bi}). Then the coefficients between i and $i + 1$ are

$$K_{0i} = \frac{K_{bi}\theta_{b(i+1)} - K_{b(i+1)}\theta_{bi}}{\theta_{b(i+1)} - \theta_{bi}}$$

$$K_{1i} = \frac{-K_{bi} + K_{b(i+1)}}{\theta_{b(i+1)} - \theta_{bi}}$$

for $i = 1$ to I. The interpolation is performed using $K = K_{0i} + K_{1i}\theta$ for θ in the range θ_{bi} to $\theta_{b(i+1)}$. Outside the defined regions, the linear expression is continued; hence θ_{bi} is used only for $i = 2$ to I.

21–6   Performance at Power Required

The engine performance (mass flow, fuel flow, and gross jet thrust) is calculated for a specified power required P_q (which might equal the power available), flight condition, and engine rating. Installation losses P_{loss} are added to P_{req} ($P_q = P_{req} + P_{\text{loss}}$). The referred quantities (relative to SLS static MCP quantities) are approximated by cubic functions of $q = P_q(N_{\text{spec}})/(P_{0C}\delta\sqrt{\theta})$:

$$\dot{w}_{req} = \dot{w}_{0C} \left(\delta\sqrt{\theta}\right) \left(K_{ffq0} + K_{ffq1}q + K_{ffq2}q^2 + K_{ffq3}q^3\right) [\theta_M]^{-X_{ffq}}$$

$$\dot{m}_{req} = \dot{m}_{0C} \left(\delta/\sqrt{\theta}\right) \left(K_{mfq0} + K_{mfq1}q + K_{mfq2}q^2 + K_{mfq3}q^3\right) [\theta_M]^{X_{mfq}}$$

$$F_g = F_{g0C} \left(\delta\right) \left(K_{fgq0} + K_{fgq1}q + K_{fgq2}q^2 + K_{fgq3}q^3\right) [\theta_M]^{X_{fgq}}$$

at N_{spec}, with $\dot{w}_{0C} = \mathrm{sfc}_{0C} P_{0C}$. The mass flow and fuel flow are primarily functions of the gas power P_G, and are assumed to be independent of η_t, hence independent of turbine speed. However, these equations are functions of $P_q(N_{\mathrm{spec}})$, obtained from $P_q(N)$ using

$$P_q(N_{\mathrm{spec}}) = P_q(N)\, \frac{1 - |(N_{\mathrm{spec}}/N_{\mathrm{opt}}) - 1|^{X_{N\eta}}}{1 - |(N/N_{\mathrm{opt}}) - 1|^{X_{N\eta}}}$$

Then the installed net jet thrust F_N and momentum drag D_{aux} are calculated.

For a simple model, the referred performance can be constant:

$$\dot{w}_{req} = \dot{w}_{0C}\,(\delta\sqrt{\theta})q = \mathrm{sfc}_{0C} P_q$$
$$\dot{m}_{req} = \dot{m}_{0C}\,(\delta/\sqrt{\theta})$$
$$F_g = F_{g0C}\,(\delta)$$

without any effect of power turbine speed.

21–7   Scaling

The parameters of the engine model can be defined for a specific engine, but it is also necessary to scale the parameters as part of the aircraft sizing task, in order to define an engine for a specified power. In addition, advanced technology must be represented in the model. Scaling and advanced technology are handled in terms of specific power and specific fuel consumption (at SLS static conditions, MCP, and N_{spec}). Figures 21-1 through 21-3 present historical data for engine specific fuel consumption, weight, and specific power; the variation at a given power reflects technology insertion with time.

The engine model includes reference values of the engine performance parameters: P_{0R}, SP_{0R}, $P_{\mathrm{mech}R}$, sfc_{0C}, SF_{0C}, N_{spec}, and $N_{\mathrm{opt}0C}$. Mass flow and fuel flow are obtained from $\dot{m}_{0R} = P_{0R}/SP_{0R}$ and $\dot{w}_{0C} = \mathrm{sfc}_{0C} P_{0C}$. The reference power at each engine rating R defines a ratio to MCP: $r_{p0R} = P_{0R}/P_{0C}$. Similarly for specific power and mechanical limits: $r_{s0R} = SP_{0R}/SP_{0C}$ and $r_{m0R} = P_{\mathrm{mech}R}/P_{0C}$. These ratios are kept fixed when the engine is scaled.

The engine size is specified as takeoff power $P_{to} = P_{\mathrm{eng}}$, which is the power at rating R, for SLS static conditions and specification turbine speed N_{spec}. Hence the MCP is $P_{0C} = P_{to}/r_{p0R}$, and the power at all other ratings follows. If P_{0C} is not equal to the reference value of the engine model, then the engine is scaled. To reflect advanced technology, the specific power, specific fuel consumption, and specification turbine speed can be specified: $SP_{0C} = SP_{\mathrm{tech}}$, $\mathrm{sfc}_{0C} = \mathrm{sfc}_{\mathrm{tech}}$, and $N_{\mathrm{spec}} = N_{\mathrm{tech}}$ (replacing the engine model reference values). The default values are the reference values of the engine model. In the following paragraph, the subscript "tech" refers to these quantities; the subscript "ref" means the engine model reference values.

The engine technology parameters SP_{0C} and sfc_{0C} are assumed to vary linearly with mass flow $\dot{m}_{0C}$ up to a limit $\dot{m}_{\mathrm{lim}}$, and constant thereafter at SP_{lim} and $\mathrm{sfc}_{\mathrm{lim}}$. The mass flow at the technology condition is $\dot{m}_{\mathrm{tech}} = P_{\mathrm{ref}}/SP_{\mathrm{tech}}$, with the technology values SP_{tech} and $\mathrm{sfc}_{\mathrm{tech}}$. The intercept values are projected from the technology values: $K_{sp0} = SP_{\mathrm{tech}} - K_{sp1}\dot{m}_{\mathrm{tech}}$, $K_{sp1} = (SP_{\mathrm{lim}} - SP_{\mathrm{tech}})/(\dot{m}_{\mathrm{lim}} - \dot{m}_{\mathrm{tech}})$; and similarly for sfc. Then for $\dot{m}_{0C} < \dot{m}_{\mathrm{lim}}$

$$SP_{0C} = K_{sp0} + K_{sp1}\dot{m}_{0C}$$
$$\mathrm{sfc}_{0C} = K_{sfc0} + K_{sfc1}\dot{m}_{0C}$$

and for $\dot{m}_{0C} \geq \dot{m}_{\text{lim}}$

$$SP_{0C} = SP_{\text{lim}}$$
$$\text{sfc}_{0C} = \text{sfc}_{\text{lim}}$$

These equations are used if $\dot{m}_{\text{lim}} > \dot{m}_{\text{tech}}$. Otherwise the model sets $K_{sp1} = K_{sfc1} = 0$, so there is no variation with scale: $SP_{0C} = SP_{\text{tech}}$ and $\text{sfc}_{0C} = \text{sfc}_{\text{tech}}$. Usually the effect of size gives $K_{sp2} \geq 0$ and $K_{sfc2} \leq 0$. The power at the limit is $P_{\text{lim}} = SP_{\text{lim}}\dot{m}_{\text{lim}}$. Figure 21-4 illustrates the scaling of SP, sfc, and SW.

Using $\dot{m}_{0C} = P_{0C}/SP_{0C}$, the specific power equation can be solved for the mass flow given the power:

$$\dot{m}_{0C} = \begin{cases} P_{0C}/K_{sp0} & K_{sp1} = 0 \\ P_{0C}/SP_{\text{lim}} & P_{0C} \geq P_{\text{lim}} \\ \sqrt{(\frac{K_0}{2K_1})^2 + \frac{P_{0C}}{K_1}} - \frac{K_0}{2K_1} & \text{otherwise} \end{cases}$$

From this mass flow, SP_{0C} and sfc_{0C} are calculated, hence the fuel flow $\dot{w}_{0C} = \text{sfc}_{0C}P_{0C}$. The specific thrust available at MCP is assumed to be constant, and the specification power turbine speed decreases with the mass flow:

$$F_{g0C} = SF_{0C}\,\dot{m}_{0C}$$
$$N_{\text{spec}} = \left(N_{\text{spec}} - K_{Ns2}/\sqrt{\dot{m}_{0C}}\right)_{\text{tech}} + K_{Ns2}/\sqrt{\dot{m}_{0C}} = K_{Ns1} + K_{Ns2}/\sqrt{\dot{m}_{0C}}$$
$$N_{\text{opt}0C} = \frac{(N_{\text{opt}0C})_{\text{ref}}}{N_{\text{tech}}}\,N_{\text{spec}} = K_{No}N_{\text{spec}}$$

Then the power and specific power at all ratings R are obtained from the ratios: $P_{0R} = r_{p0R}P_{0C}$, $SP_{0R} = r_{s0R}SP_{0C}$, and $P_{\text{mech}R} = r_{m0R}P_{0C}$.

The actual (perhaps scaled) values of the performance parameters are available for the engine group: $P_{0R}, SP_{0R}, P_{\text{mech}R}, \text{sfc}_{0C}, F_{g0C}, N_{\text{spec}}$, and $N_{\text{opt}0C}$.

21–8   Engine Speed

The model as described in the previous sections may not adequately account for variation of engine performance with engine speed, so it is also possible to define the parameters corresponding to a set of engine speed ratios $r = N/N_{\text{spec}}$. Then the engine performance and power available quantities are linearly interpolated to obtain the values at the required engine speed N. If this option is used, then the correction based on $P(N)/P(N_{\text{spec}}) = \eta_t(N)/\eta_t(N_{\text{spec}})$ is not applied.

21–9   Weight

The engine weight can be a fixed input value, calculated as a function of power, or scaled with engine mass flow. As a function of power, the weight of one engine is:

$$W_{\text{one eng}} = K_{0\text{eng}} + K_{1\text{eng}}P + K_{2\text{eng}}P^{X_{\text{eng}}}$$

where P is the installed takeoff power (SLS static, specified rating) per engine. A constant weight per power W/P is given by using only $K_{1\text{eng}}$. Alternatively, the specific weight $SW = P/W$ can be scaled
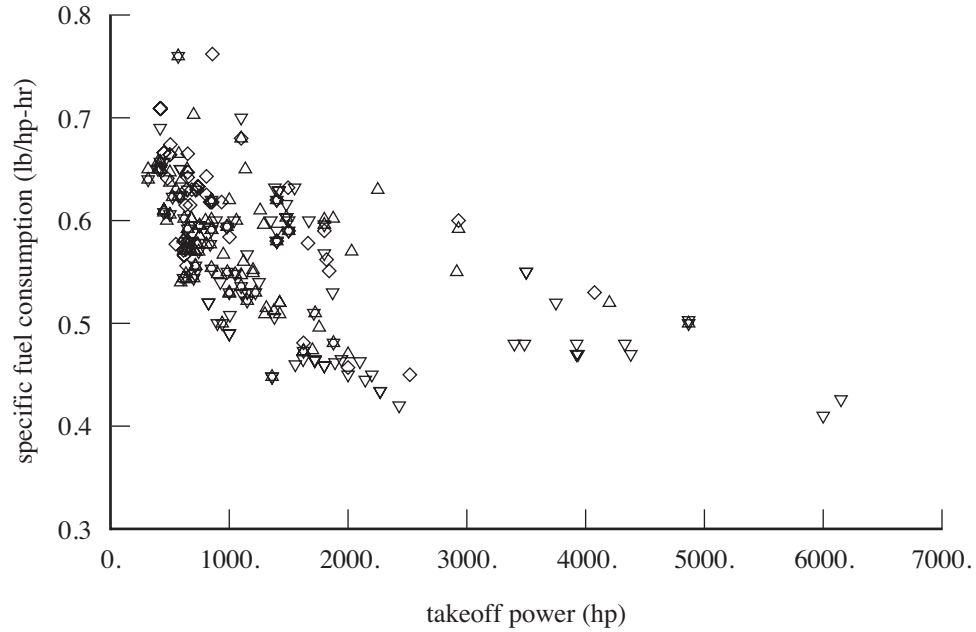
Figure 21-1. Historical data for turboshaft engine specific fuel consumption.

with the mass flow $\dot{m}_{0C}$. The scaling is determined from the specific weight SW_{ref} at the mass flow $\dot{m}_{\text{tech}}$, and the limit SW_{lim} at $\dot{m}_{\text{lim}}$. Then

$$SW = \begin{cases} K_{sw0} + K_{sw1}\dot{m}_{0C} & \dot{m}_{0C} < \dot{m}_{\text{lim}} \\ SW_{\text{lim}} & \dot{m}_{0C} \geq \dot{m}_{\text{lim}} \end{cases}$$

(if $\dot{m}_{\text{lim}} < \dot{m}_{\text{tech}}$, then $K_{sw1} = 0$, so $SW = SW_{\text{ref}}$) and $W_{\text{one eng}} = P/SW$.

21–10   Units

In this engine model, only the reference values and scaling constants are dimensional. Conventional English units and SI units are shown in table 21-1. Units of specific power and specific fuel consumption follow from these conventions.

Table 21-1. Conventional units.

	power P	mass flow $\dot{m}$	fuel flow $\dot{w}$	force F	turbine speed N
English:	horsepower	pound/sec	pound/hour	pound	rpm
SI:	kiloWatt	kilogram/sec	kilogram/hour	Newton	rpm

21–11   Typical Parameters

Typical values of the principal parameters describing the engine and its performance are given in table 21-2 for several generic engine sizes. These values represent good current technology. Advanced

technology can be introduced by reducing the specific fuel consumption and weight, and increasing the specific power. Typical ratios of the power, specific power, and mass flow to the values at MCP are given in table 21-3 for several ratings. Figure 21-5 shows typical performance characteristics: fuel flow $\dot{w}$, mass flow $\dot{m}$, and net jet thrust F_g variation with power P and speed (referred quantities, normalized, at N_{spec}). Figure 21-6 shows typical variation of the power available with engine turbine speed. Figures 21-7 to 21-12 show typical power available characteristics: specific power SP, mass flow $\dot{m}$, and power P variation with temperature ratio θ, for static and 200 knots conditions and several engine ratings (referred quantities, normalized, at N_{spec}).

Table 21-2. Typical engine performance parameters.

power (MCP)	P_{0C}	500	1000	2000	4000	8000	16000	hp
specific power	SP_{0C}	116	125	134	143	153	164	hp/lb/sec
mechanical limit	P_{mech}	750	1500	3000	6000	12000	24000	hp
specific fuel cons.	sfc_{0C}	0.54	0.48	0.44	0.40	0.38	0.35	lb/hp-hr
specific jet thrust	SF_{0C}	5.9	7.3	8.9	11.0	13.6	16.7	lb/lb/sec
gross jet thrust	F_{g0C}	25	58	134	308	707	1625	lb
mass flow	$\dot{m}_{0C}$	4.3	8.0	15.0	27.9	52.1	97.3	lb/sec
turbine speed	N_{spec}	35600	26100	19100	14000	10200	7500	rpm
weight	W/P_{to}	0.34	0.23	0.18	0.16	0.16	0.15	lb/hp

Table 21-3. Typical parameter ratios for various ratings (percent).

		IRP	MRP	CRP	P_{mech}
power	P_0/P_{0C}	120	127	133	150
specific power	SP_0/SP_{0C}	117	123	128	
mass flow	$\dot{m}_0/\dot{m}_{0C}$	102.6	103.2	103.9	
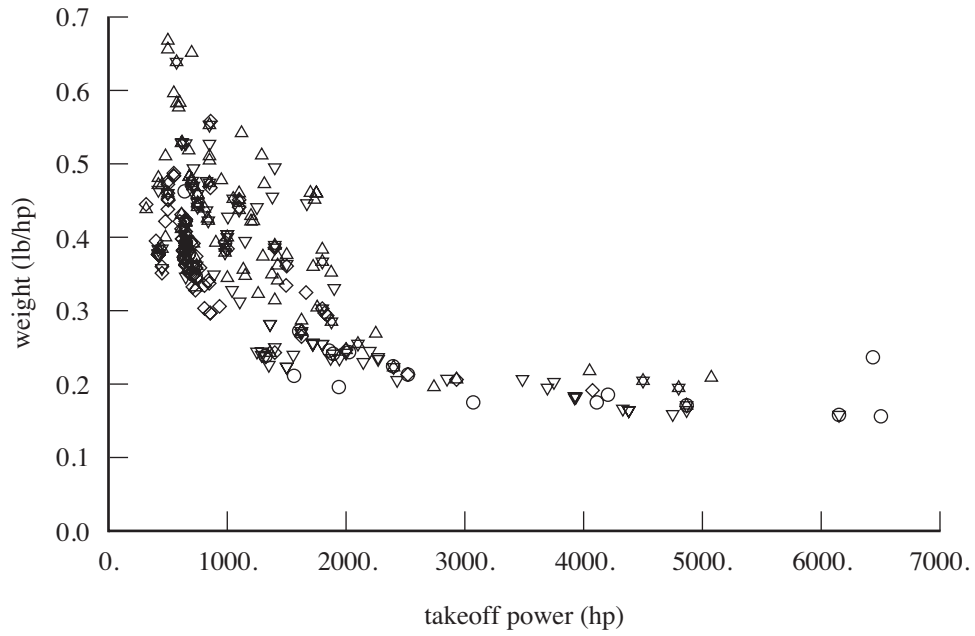
Figure 21-2. Historical data for turboshaft engine weight.
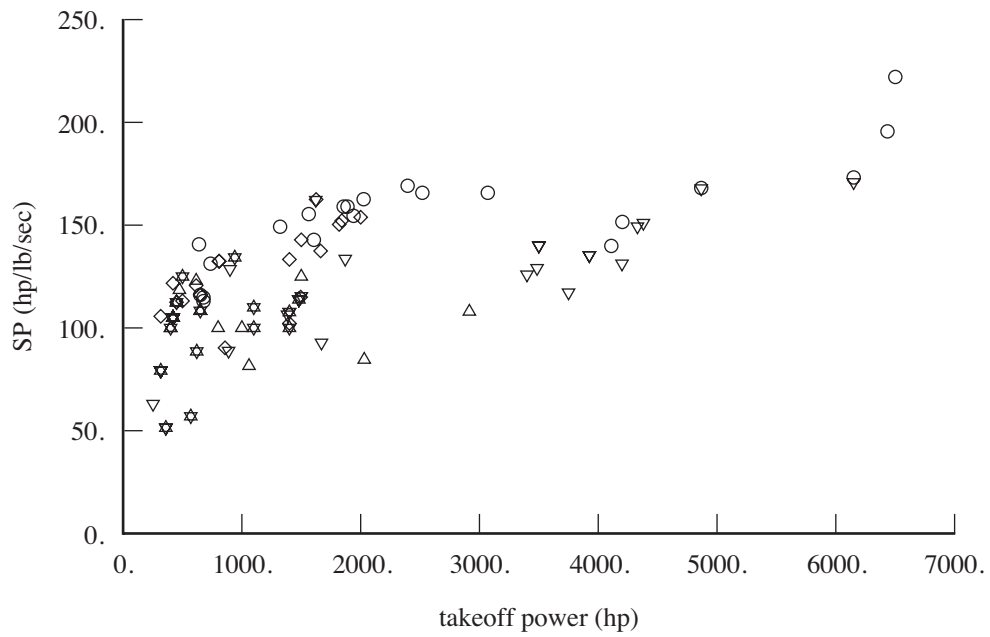
Figure 21-3. Historical data for turboshaft engine specific power.
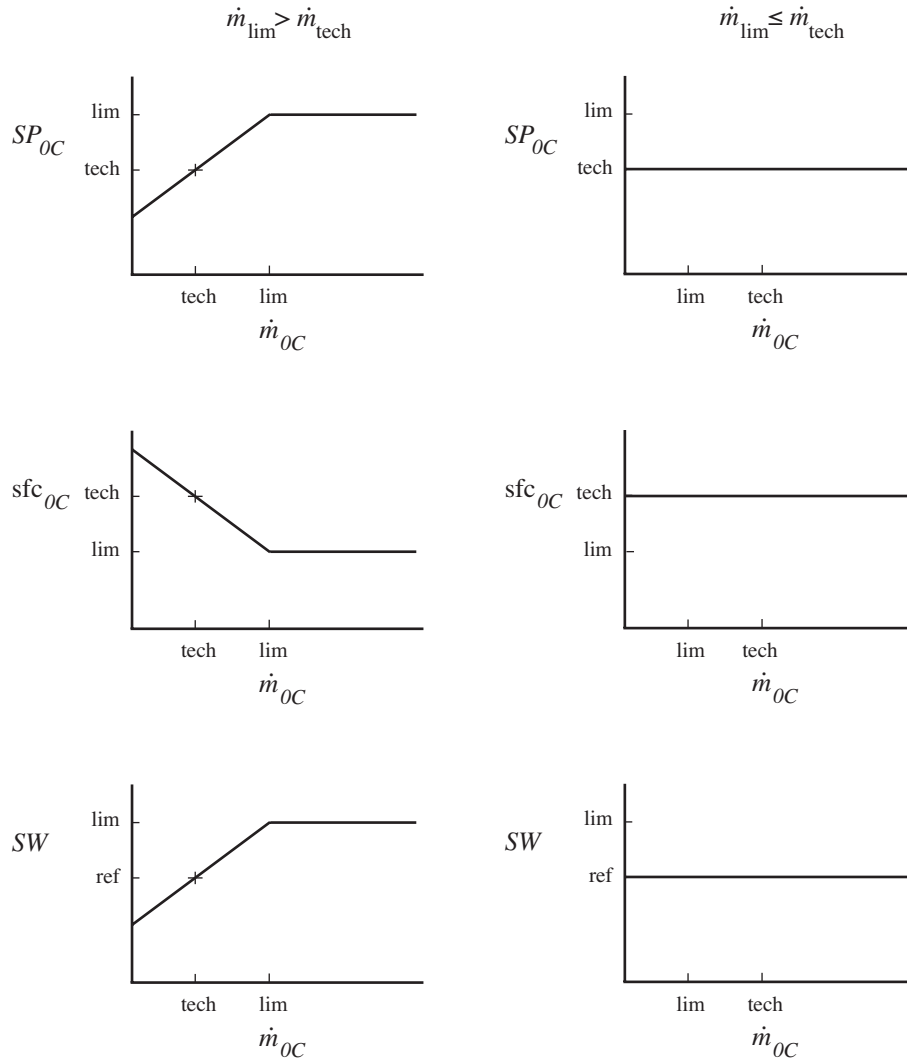
Figure 21-4. Turboshaft engine scaling; $SP = P/\dot{m}$, sfc $= \dot{w}/P$, and $SW = P/W$.
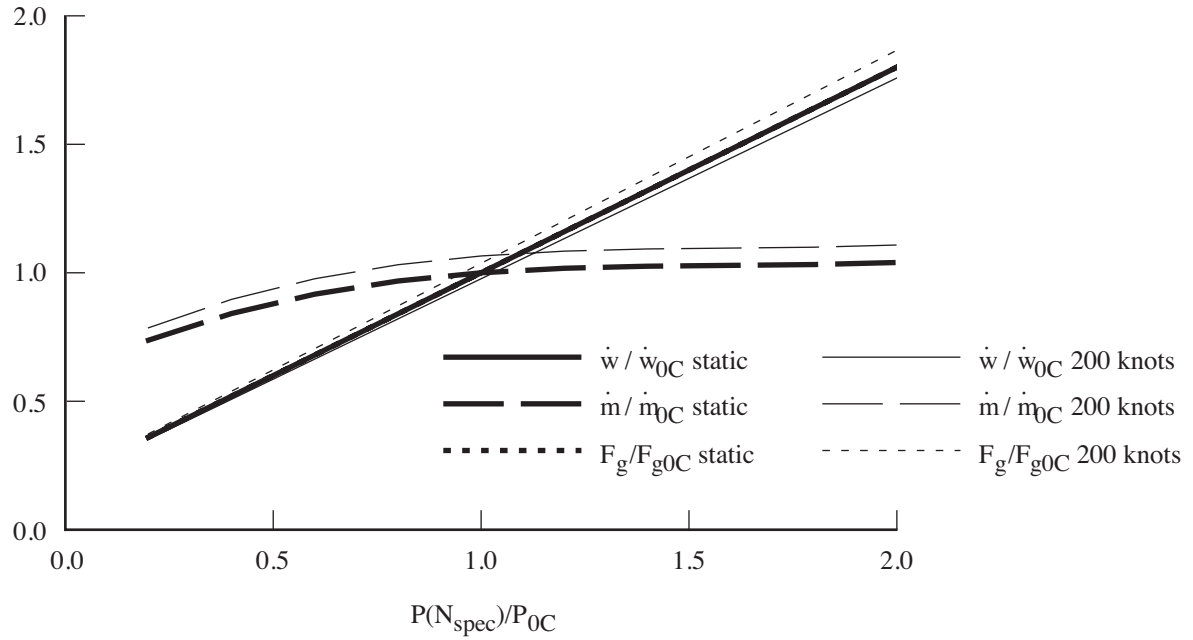
Figure 21-5. Fuel flow, mass flow, and net jet thrust variation with power.
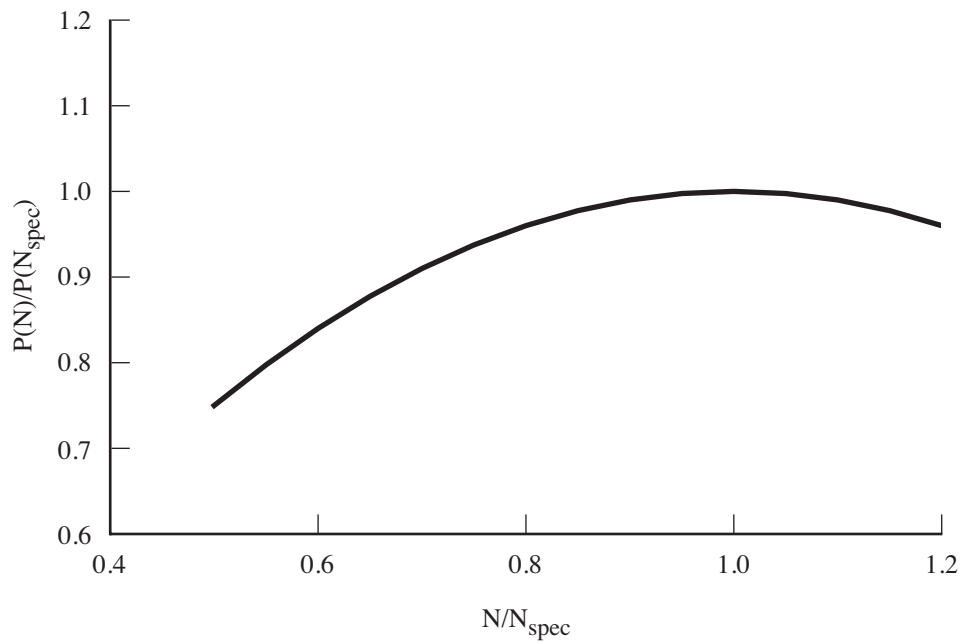
Figure 21-6. Power variation with turbine speed.
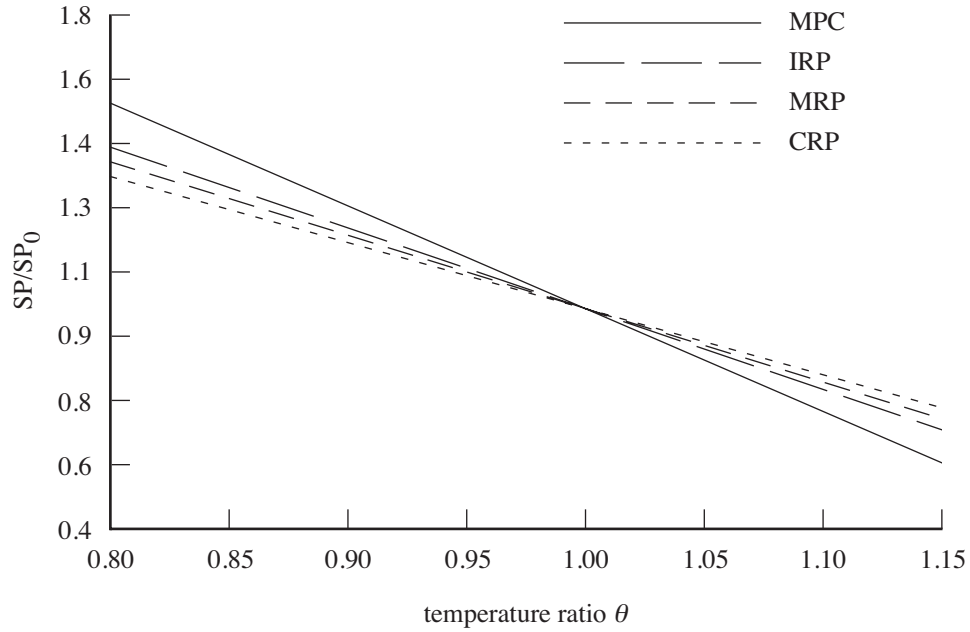
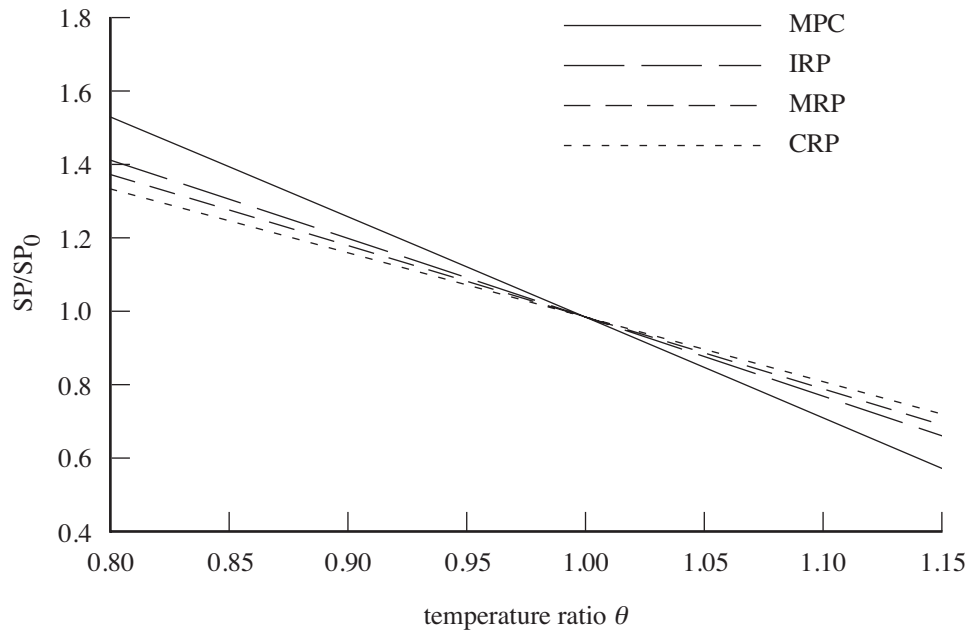Figure 21-7. Specific power variation with temperature ratio, static.

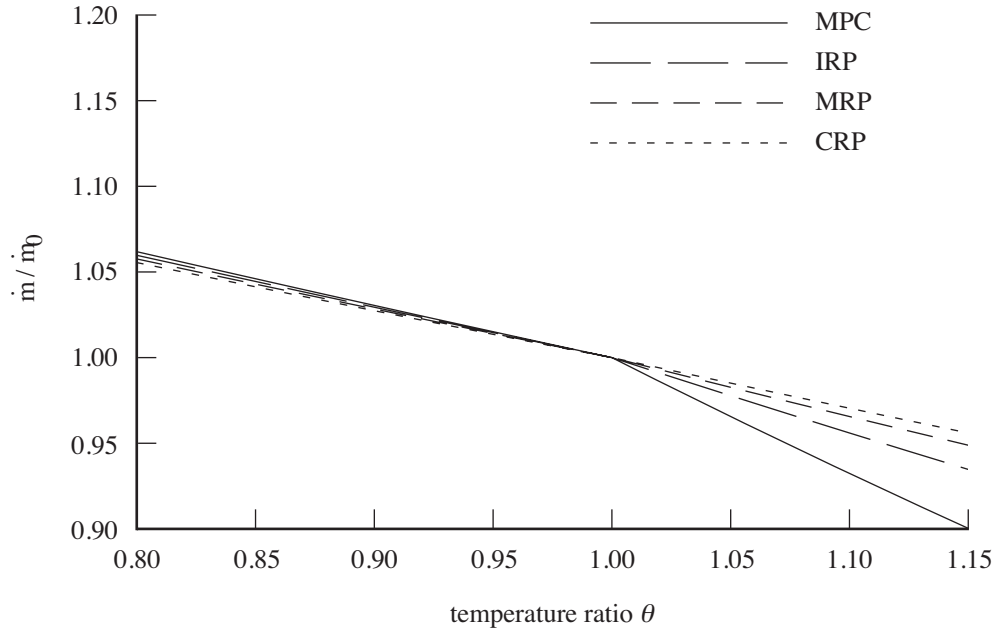Figure 21-8. Specific power variation with temperature ratio, 200 knots.

Figure 21-9. Mass flow variation with temperature ratio, static.
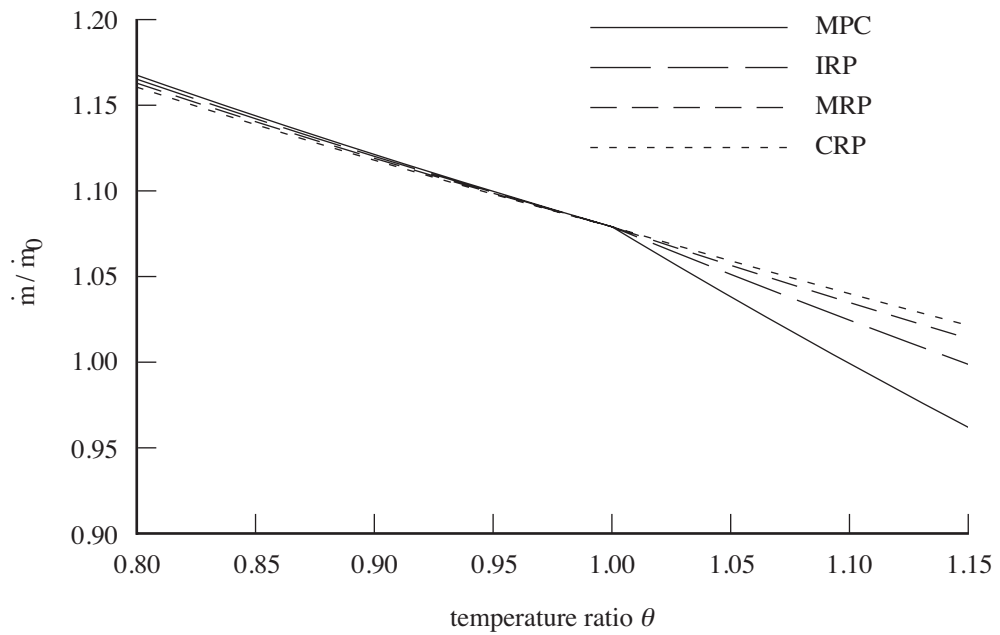
Figure 21-10. Mass flow variation with temperature ratio, 200 knots.
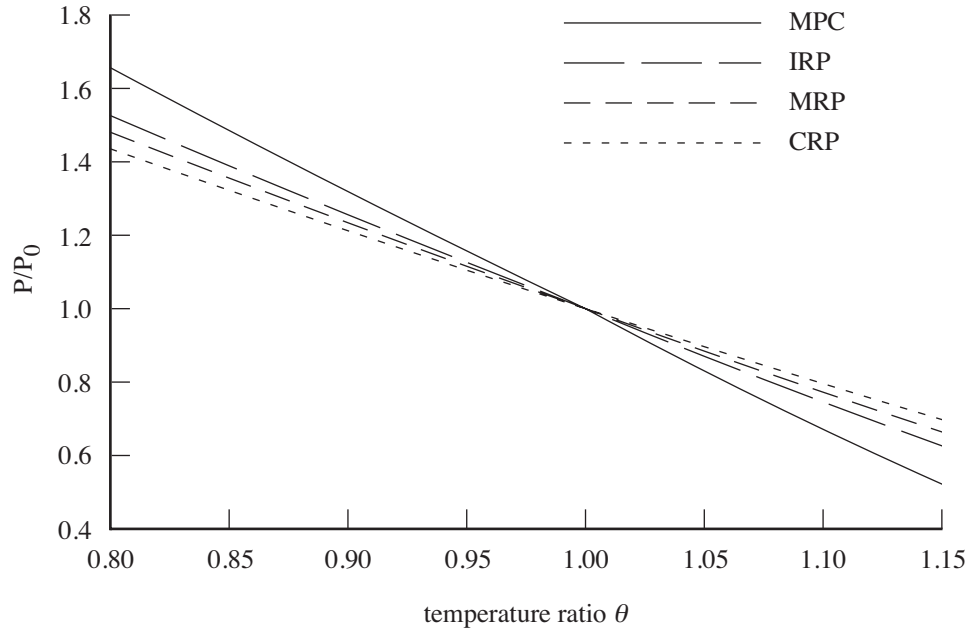
Figure 21-11. Power variation with temperature ratio, static.
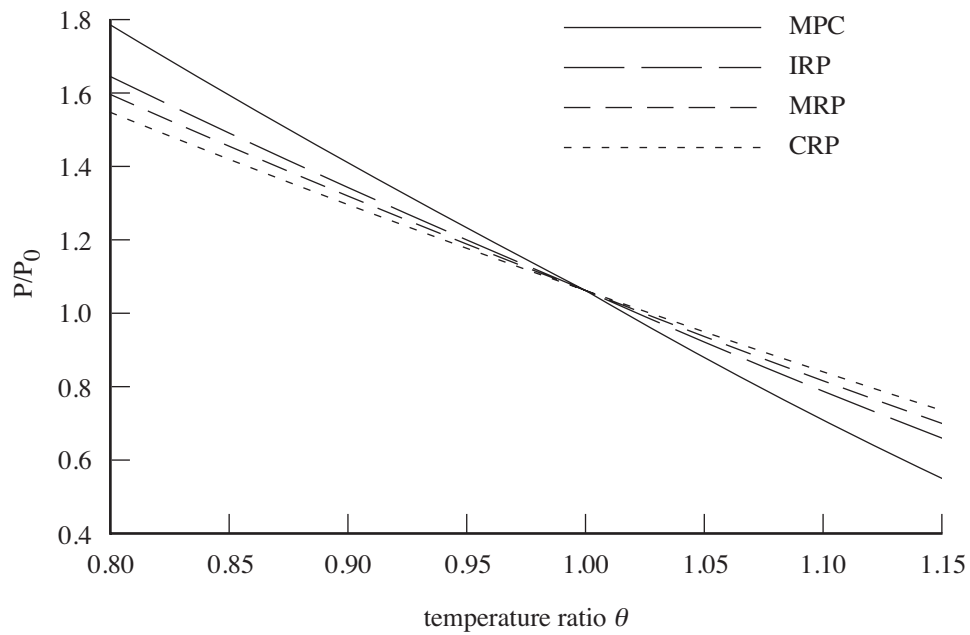
Figure 21-12. Power variation with temperature ratio, 200 knots.

Reciprocating Engine Model

A reciprocating engine generates power by means of reciprocating pistons that convert pressure into a rotating motion. Each piston is inside a cylinder, into which a gas is introduced, either already under pressure (external combustion, as in a steam engine) or heated inside the cylinder by ignition of a fuel-air mixture (internal combustion). The hot gases expand, pushing the piston to the bottom of the cylinder. Here only internal combustion engines are considered, perhaps more appropriately identified as intermittent combustion engines, to distinguish them from the continuous combustion of gas turbines and jet engines. An internal combustion engine is either a spark-ignition engine, where a spark plug initiates the combustion; or a compression-ignition engine, where the air within the cylinder is compressed, heating it and thereby igniting fuel. Internal combustion engine performance is described in reference 1.

Design and analysis codes calculate aircraft performance for a very wide range of operating conditions. Thus engine performance must be realistic even far from the engine design point. A simple thermodynamic cycle analysis is not realistic for such an application. Rather than developing high fidelity analyses, the approach taken is to use a model for the total engine performance. The engine is not being designed.

The Reciprocating Engine Model is based on curve-fits of performance data for existing or projected engines over a range of operating conditions. The operating condition is described by pressure altitude, ambient air temperature, flight Mach number, engine speed, and either engine rating or engine power required. These curve-fits, typically based on real engines, are scaled to the required size and adjusted to the appropriate technology level to represent a notional engine. Engine size is represented by power. Engine technology is represented by specific fuel consumption and power-to-weight ratio. Engine installation losses are also modeled.

22–1 Operating Environment

The operating condition and atmosphere give the standard conditions (temperature T_{std} and pressure p_{std}) for a specified pressure altitude; the sea-level standard conditions (temperature T_0 and pressure p_0); and the operating temperature T and pressure p. Here the temperatures are °R or °K. The engine characteristics depend on the density ratio $\sigma = \rho/\rho_0$ and the temperature ratio $\theta = T/T_0$.

The flight Mach number $M = V/c_s = V/c_{s0}\sqrt{\theta}$ is obtained from the aircraft speed V.

The inlet ram air temperature ratio and pressure ratio are obtained then from M and the inlet ram

recovery efficiency η_d:

$$\theta_M = \left(1 + \frac{\gamma - 1}{2}M^2\right) = \left(1 + 0.2M^2\right)$$

$$\delta_M = \left(1 + \frac{\gamma - 1}{2}\eta_d M^2\right)^{\frac{\gamma}{\gamma - 1}} = \left(1 + 0.2\eta_d M^2\right)^{3.5}$$

where the ratio of specific heats $\gamma = 1.4$. The ram density ratio is $\sigma_M = \delta_M/\theta_M$. The engine mass flow and power available depend on $(\sigma + \Delta\sigma_M)$, where $\Delta\sigma_M = \sigma_M - 1$

22–2 Reciprocating Engine Performance

The engine work per cycle for a reciprocating engine is $W = Pn_c/R$; where P is the power, $R = N/2\pi$ the rotational speed (rev/sec, N in rad/sec), n_c the revolutions per cycle ($n_c = 2$ for a 4-stroke engine, $n_c = 1$ for a 2-stroke engine), and R/n_c is the cycles per second. The piston area is A_p, bore b, stroke k; the engine displacement $V_d = A_p k$ (swept volume). The mean piston speed is defined as $s = 2Rk$ ($1/2R$ is the time for one stroke). Thus $A_p s = 2RV_d$. The mean effective pressure (mep) is defined as

$$\text{mep} = \frac{W}{V_d} = \frac{Pn_c}{V_d R} = \frac{P}{A_p s/2n_c} = \frac{Qn_c 2\pi}{V_d}$$

from the power $P = NQ = 2\pi RQ$; so mep is the specific torque. The power and torque can be written in terms of mep:

$$P = (\text{mep})V_d R/n_c = (\text{mep})A_p s/2n_c$$

$$Q = (\text{mep})V_d/2\pi n_c = (\text{mep})A_p k/2\pi n_c$$

The specific output is $P/A_p = (\text{mep})s/2n_c$.

The engine output is the brake horsepower (BHP), which equals indicated power less friction power:

$$\text{BHP} = \text{IHP} - \text{FHP} = \text{IHP} - (\text{MHP} + \text{PHP} + \text{CHP} + \text{AHP} - \text{THP})$$

The friction power is composed of losses due to mechanical friction (MHP), pumping (PHP, the work of the piston during inlet and exhaust strokes), compressor or supercharger (CHP), auxiliary or accessories (AHP, such as oil pump, water pump, cooling fan, generator), and exhaust turbine (THP, treated as negative friction). The mechanical efficiency is $\eta = \text{BHP}/\text{IHP}$.

The sum of airflow ($\dot{m} = \dot{w}_a$) and fuel flow ($\dot{w} = \dot{w}_f$) is the charge flow: $\dot{w}_c = \dot{w}_f + \dot{w}_a$. The fuel-air ratio is $F = \dot{w}_f/\dot{w}_a$. The stoichiometric (chemically correct) fuel-air ratio is about $F_{\text{stoich}} = 0.068$ for both gasoline and diesel. Rich mixture (fuel-air ratio) means extra fuel and lean mixture means extra air. The fuel-air equivalence ratio is $\phi = F/F_{\text{stoich}}$. The mass flow is

$$\dot{m} = e_v \dot{m}_{\text{ideal}} = e_v \rho_i V_d(R/n_c) = e_v \rho_i A_p s/2n_c = \sigma_i N(e_v \rho_0 V_d/(2\pi n_c))$$

where ρ_i is the density at the intake and $e_v = \dot{m}/\dot{m}_{\text{ideal}}$ is the volumetric efficiency. Including the influence of forward speed, $\rho_i = \rho_0(\sigma + \Delta\sigma_M)$. Typically $e_v = 0.8$ to 0.9. The fuel flow is then $\dot{w} = F\dot{m}$. The indicated power P_i is the product of the fuel flow, fuel specific energy ($e_{\text{fuel}} = JQ_c$, from the heat of combustion Q_c and Joule's constant J relating work and heat), and thermal efficiency:

$$P_i = e_{\text{fuel}}\eta_{th}\dot{w} = (e_{\text{fuel}}\eta_{th}F)\dot{m} = \sigma_i N(e_{\text{fuel}}\eta_{th}F)(e_v \rho_0 V_d/(2\pi n_c))$$

The specific fuel consumption is

$$\text{sfc} = \dot{w}/P = 1/(e_{\text{fuel}}\eta_{th})$$

and the power to mass flow ratio is

$$P/\dot{m} = e_{\text{fuel}}\eta_{th}F = F/\text{sfc} = (\text{mep})/(e_v\rho_i)$$

These equations all have constant factors when conventional units are used.

From $\dot{m} \sim e_v\rho_i$ and $P \sim e_v\rho_i\eta_{th}F$, it follows that mass flow and power are proportional to the density ratio $\sigma = \delta/\theta$. For unsupercharged engines at constant fuel-air fraction and constant throttle, typically $e_v\rho_i \sim p/\sqrt{T}$ (ref. 1). Similarly, operating curves give $\pm1\%$ power change for $\mp6°C$ at constant pressure altitude and manifold pressure, implying again $P \sim 1/\sqrt{T}$. The implication is that mass flow, indicated power, and imep are proportional to $\delta/\sqrt{\theta} = \sigma\sqrt{\theta}$.

The engine has a limit on bmep (hence torque) corresponding to detonation limits (spark-ignition) or thermal and loads limits (compression-ignition) at maximum power. This limit is defined in terms of power at a specified engine speed. Load limits associated with mean piston speed give an engine speed limit. Maximum throttle, encountered at a critical altitude or critical density, can be expressed as a power limit that varies with engine speed. These limits depend on the rating, fuel-air ratio (lean or rich), and supercharger speed.

22–3   Performance Characteristics

The engine performance is described by the uninstalled power available P_a, as a function of flight condition and engine rating; the mass flow $\dot{m}$ and fuel flow $\dot{w}$ required to produce uninstalled power required P_q; and the gross jet thrust. The installed power required P_{req} and power available $P_{av} > P_{req}$ are measured at the engine output shaft. In addition to shaft power, the engine exhaust can produce a net jet thrust F_N, from mass flow that goes through the engine. The difference between net and gross jet thrust is the momentum drag: $F_n = F_g - \dot{m}_{req}V = \dot{m}_{req}(V_j - V)$, where V_j is the engine jet exhaust velocity. The forces produced by mass flow that does not go through the engine (such as cooling air) are treated as momentum drag D_{aux}.

The engine performance depends on the rating. Typical ratings include maximum continuous power (MCP) and takeoff power. The rating structure of the model includes dependence on fuel-air ratio (lean or rich) and supercharger speed.

The reference performance is at sea-level-standard static conditions (subscript 0). For each rating R, the performance is characterized by the following quantities: power P_{0R} and specific fuel consumption sfc_{0R}; fuel-air ratio F_{0R}; specific thrust SF_{0R}; mean effective pressure (mechanical) power limit $P_{\text{mep}R}$ and critical power $P_{\text{crit}R}$; and a reference engine speed N_{0R}. Then the mass flow is $\dot{m}_{0R} = \dot{w}_{0R}/F_{0R}$, and the fuel flow $\dot{w}_{0R} = \text{sfc}_{0R}P_{0R}$. The gross jet thrust is $F_{g0R} = SF_{0R}\dot{m}_{0R}$ ($SF_{0R} = V_{j0R}$). The specification engine speed is N_{spec}.

22–4   Installation

The difference between installed and uninstalled power is the installation loss: $P_{av} = P_a - P_{\text{loss}}$ and $P_{req} = P_q - P_{\text{loss}}$. This loss is modeled by an efficiency factor: $P_{\text{loss}} = (1-\eta_{\text{loss}})P_a$ or $P_{\text{loss}} = (1-\eta_{\text{loss}})P_q$, where $\eta_{\text{loss}} = \ell_{in} + \ell_{ex}$. The engine model gives the gross thrust F_g. The installed gross jet thrust is

$F_G = K_{fgr}F_g$, where K_{fgr} accounts for exhaust effects. The net jet thrust is $F_N = F_G - \dot{m}_{req}V$. The momentum drag of the auxiliary air flow is a function of the mass flow $\dot{m}_{aux} = f_{aux}\dot{m}_{req}$:

$$D_{aux} = (1 - \eta_{aux})\dot{m}_{aux}V = (1 - \eta_{aux})f_{aux}\dot{m}_{req}V$$

where η_{aux} is the ram recovery efficiency.

22–5   Power Available

Given the flight condition and engine rating, the power available P_a is calculated from:

$$P_a = P_0 K_p(\sigma + K_{ram}\Delta\sigma_M)r^{X_{pN}}\theta^{X_{p\theta}}$$

where the engine speed ratio is $r = N/N_0$. The factor K_p is included so P_0 is a relevant reference power, instead of the power extrapolated to sea-level conditions. K_{ram} permits adjustment (or suppression) of the effect of flight speed. The exponents X_{pN} and $X_{p\theta}$ are obtained by fitting the engine operating curves. The reference engine speed N_0 corresponds to P_0.

Maximum throttle, encountered at a critical altitude or critical density, is expressed as a power limit that varies with engine speed: $P_a = \min(P_a, P_{critR}r^{X_{crit}})$, where $r = N/N_0$. The exponent X_{crit} is obtained by fitting the engine operating curves.

Installation losses P_{loss} are subtracted from P_a ($P_{av} = P_a - P_{loss}$), and then the mechanical limit (mean effective pressure) is applied: $P_{av} = \min(P_{av}, rP_{mechR})$, $r = N/N_{spec}$. The mechanical limit is properly a torque limit, $Q_{mech} = P_{mech}/N_{spec}$, but is expressed as a power limit for clarity. The mep limit P_{mepR} is specified at the reference engine speed N_{0R}, hence $P_{mechR} = P_{mepR}(N_{spec}/N_{0R})$.

22–6   Performance at Power Required

The engine performance (fuel flow, mass flow, and gross jet thrust) is calculated for a specified power required P_q (which might equal the power available), flight condition, and engine rating. Installation losses P_{loss} are added to P_{req} ($P_q = P_{req} + P_{loss}$). The fuel flow and fuel-air ratio are approximated by cubic functions of $q = P_q/P_0$:

$$\dot{w}_{req} = \dot{w}_0 \left(K_{ffq0} + K_{ffq1}q + K_{ffq2}q^2 + K_{ffq3}q^3\right)r^{X_{ffq}}\sigma^{X_{ff\sigma}} = \dot{w}_0 K_{ffq}(q)r^{X_{ffq}}\sigma^{X_{ff\sigma}}$$

$$F_{req} = F_0 \left(K_{Fq0} + K_{Fq1}q + K_{Fq2}q^2 + K_{Fq3}q^3\right)r^{X_{Fq}} = F_0 K_{Fq}(q)r^{X_{Fq}}$$

where the engine speed ratio is $r = N/N_0$. The reference engine speed N_0 corresponds to P_0. Alternatively, $K_{ffq}(q)$ and $K_{Fq}(q)$ can be specified as piecewise-linear functions of q. The constants (K_{ffqn}, K_{Fqn}) and exponents (X_{ffq}, X_{Fq}) are obtained by fitting the engine operating curves for sfc and F. For constant specific fuel consumption, $K_{ffq}(q) = q$. For constant fuel-air ratio, $K_{Fq}(q) = 1$; for fuel-air ratio proportional to power, $K_{Fq}(q) = q$. If F_{stoich} is used for F_0, the fuel-air equivalence ratio is $\phi = F/F_{stoich} = K_{Fq}(q)r^{X_{Fq}}$.

The mass flow is $\dot{m} = \dot{w}/F$. The gross jet thrust is $F_g = SF_{0R}\dot{m}$. The installed net jet thrust F_N and momentum drag D_{aux} are calculated from the mass flow.

22–7   Scaling

The parameters of the engine model can be defined for a specific engine, but it is also necessary to scale the parameters as part of the aircraft sizing task, in order to define an engine for a specified power.

The engine model includes reference values of the engine performance parameters: P_{0R}, sfc_{0R}, F_{0R}, SF_{0R}, $P_{\text{mep}R}$, $P_{\text{crit}R}$, N_{0R}, N_{spec}. The actual (perhaps scaled) values are available to the engine group.

The engine size is specified as takeoff power $P_{to} = P_{\text{eng}}$, which is the power P_{0X} at rating $R = X$, for SLS static conditions. If P_{0X} is not equal to the reference value of the engine model, then the engine is scaled. The reference power at each engine rating R defines a ratio to MCP: $r_{p0R} = P_{0R}/P_{0C}$. The reference engine speed gives the ratio $r_{N0R} = N_{0R}/N_{\text{spec}}$. These ratios are kept fixed when the engine is scaled. Hence the MCP is $P_{0C} = P_{to}/r_{p0R}$, and the power at all other ratings follows.

The size of geometrically scaled engines can be represented by the cylinder bore b. The specific output, mean piston speed, and specific fuel consumption are relatively weak functions of size (ref. 1). It is assumed that $P/A_p \sim b^{-X_o}$, $s \sim b^{-X_s}$, $\text{sfc} \sim b^{-X_f}$. Hence $P \sim b^{2-X_o}$ and $N_{\text{spec}} \sim s/k \sim b^{-1-X_s}$. Including scaling terms linear with the power gives

$$\text{sfc}_{0R} = \text{sfc}_{0R\text{ref}}\left(1 + K_{\text{sfc}1}(p^{-X_f/(2-X_o)} - 1) + K_{\text{sfc}2}(p-1)\right)$$

$$N_{\text{spec}} = N_{\text{spec}-\text{ref}}\left(1 + K_{N\text{spec}1}(p^{-(1+X_s)/(2-X_o)} - 1) + K_{N\text{spec}2}(p-1)\right)$$

where $p = P_{\text{eng}}/P_{0X\text{ref}}$. For example, $X_o \cong 0.2$, $X_s \cong 0.3$, $X_f \cong 0.1$ for diesel engines (ref. 1).

P_{mep} and P_{crit} scale with power:

$$P_{0R} = P_{0C}r_{p0R}$$

$$P_{\text{crit}R} = P_{0R}r_{\text{crit}R}$$

$$P_{\text{mech}R} = P_{\text{mep}R}(N_{\text{spec}}/N_{0R}) = P_{0R}r_{\text{mep}R}$$

where $r_{\text{crit}R} = P_{\text{crit}-\text{ref}R}/P_{0R\text{ref}}$ and $r_{\text{mep}R} = (P_{\text{mep}-\text{ref}}/P_{0R\text{ref}})(N_{\text{spec}-\text{ref}}/N_{0R\text{ref}})$. The engine speed scales with N_{spec}: $N_{0R} = N_{\text{spec}}r_{N0R}$.

22–8   Weight

The engine weight can be a fixed input value, or calculated as a function of power. As a function of power, the weight of one engine is:

$$W_{\text{one eng}} = K_{0\text{eng}} + K_{1\text{eng}}P + K_{2\text{eng}}P^{X_{\text{eng}}}$$

where P is the installed takeoff power (SLS static, specified rating) per engine. A constant weight per power W/P is given by using only $K_{1\text{eng}}$.

22–9   Units

In this engine model, only the reference values and scaling constants are dimensional. Conventional English units and SI units are shown in table 22-1. Units of specific power and specific fuel consumption follow from these conventions.

Table 22-1. Conventional units.

	power P	mass flow $\dot{m}$	fuel flow $\dot{w}$	force	engine speed N	mep
English:	horsepower	pound/sec	pound/hour	pount	rpm	pound/in^2
SI:	kiloWatt	kilogram/sec	kilogram/hour	Newton	rpm	kN/m^2

22–10   Typical Parameters

The maximum brake mean effective pressure is 125–300 lb/in^2 (850–2050 kN/m^2). Reciprocating aircraft engine mean piston speed at maximum power is typically 10-16 m/sec for geared engines. Direct drive aircraft engine speeds are typically limited by propeller tip speed. Stoichiometric (exact chemical) fuel-air ratios are given in table 22-2 for a number of fuels. The fuel-air equivalence ratio is $\phi = F/F_{\text{stoich}}$.

Spark-ignition engines can run at stoichiometric fuel-air ratio ($\phi = 1$). Typical operating fuel-air ratios are $\phi = 0.8$ to 1.5, with rich values up to $\phi = 1.8$ for takeoff power. Modern "lean-burn" spark-ignition engines use Gasoline Direct Injection (GDI) to get ϕ below 0.3 at low power. Highly supercharged engines at maximum power run rich. From reference 1, the highest power for each throttle setting is at $\phi \cong 1.1$, with best brake economy at $\phi = 0.9$ (full load) to $\phi = 1.1$ (zero load).

Compression-ignition engines run lean, limited by incomplete combustion. Typical operating fuel-air ratios are $\phi = 0.2$ to 0.8, with bmep increasing with ϕ. Best break economy is near $\phi = 0.5$ (ref. 1).

Table 22-2. Stoichiometric fuel-air ratios.

fuel	F_{stoich}	fuel	F_{stoich}
gasoline	0.0678–0.0685	propane	0.0638
diesel	0.0690–0.0697	ethanol	0.111
hydrogen	0.0292	methanol	0.155
natural gas	0.058	methane	0.0581

22–11   References

1) Taylor, C.F. *The Internal-Combustion Engine in Theory and Practice*. *Volume 1: Thermodynamics, Fluid Flow, Performance*. Second edition. Cambridge, MA: MIT Press, 1966.

Compressor Model

A compressor converts input shaft power to a jet velocity and thrust. The shaft power contributes to the propulsion group power required. The compressor does not use fuel.

The operating condition is described by pressure altitude, ambient air temperature, flight Mach number, and either compressor rating or power required. The parametric model is scaled to the required size and adjusted to the appropriate technology level to represent a notional compressor. Compressor size is represented by mass flow. Technology is represented by specific power available at maximum continuous power (MCP), sea level/standard day (SLS), static (zero airspeed) conditions. Installation effects are also modeled.

23–1 Operating Environment

The operating condition and atmosphere give the standard conditions (temperature T_{std} and pressure p_{std}) for a specified pressure altitude; the sea-level standard conditions (temperature T_0 and pressure p_0); and the operating temperature T and pressure p. Here the temperatures are °R or °K. The characteristics depend on the temperature ratio $\theta = T/T_0$ and pressure ratio $\delta = p/p_0$.

The flight Mach number $M = V/c_s = V/c_{s0}\sqrt{\theta}$ is obtained from the aircraft speed V.

The inlet ram air temperature ratio and pressure ratio are obtained then from M and the inlet ram recovery efficiency η_d:

$$\theta_M = \left(1 + \frac{\gamma - 1}{2}M^2\right) = \left(1 + 0.2M^2\right)$$

$$\delta_M = \left(1 + \frac{\gamma - 1}{2}\eta_d M^2\right)^{\frac{\gamma}{\gamma-1}} = \left(1 + 0.2\eta_d M^2\right)^{3.5}$$

where the ratio of specific heats $\gamma = 1.4$.

23–2 Performance Characteristics

The uninstalled power required is P_q, the power available P_a, the gross jet thrust F_G, and net jet thrust F_N. The compressor model calculates P_a as a function of flight condition and rating; or calculates jet thrust and mass flow at P_q. The specific power is $SP = P/\dot{m}$, the specific thrust is $ST = F_G/\dot{m}$. The forces produced by mass flow that does not go through the core are treated as momentum drag D_{aux}.

The reference performance is at sea-level-standard static conditions (subscript 0), and MCP (subscript C). For each rating R, the performance is characterized by the following quantities for sea-level-standard static conditions: Power P_{0R}, specific power SP_{0R}, and mechanical power limit $P_{\mathrm{mech}R}$. The mass flow is then $\dot{m}_{0R} = P_{0R}/SP_{0R}$.

The difference between net and gross jet thrust is the momentum drag: $F_N = F_G - \dot{m}_{req}V$. Note that traditional units for mass flow are pound/sec (pps), while this equation requires slug/sec ($\dot{m}_{req}/g$ replaces $\dot{m}_{req}$).

23–3   Installation

The difference between installed and uninstalled power is the inlet and exhaust losses P_{loss}:

$$P_{av} = P_a - P_{\text{loss}}$$
$$P_{req} = P_q - P_{\text{loss}}$$

The inlet and exhaust losses are modeled as fractions of power available or power required: $P_{\text{loss}} = (\ell_{in} + \ell_{ex})P_a$ or $P_{\text{loss}} = (\ell_{in} + \ell_{ex})P_q$. So

$$P_{av} = P_a(1 - \ell_{in} - \ell_{ex})$$
$$P_q = P_{req}/(1 - \ell_{in} - \ell_{ex})$$

The momentum drag of the auxiliary air flow is a function of the mass flow $\dot{m}_{\text{aux}} = f_{\text{aux}}\dot{m}_{req}$:

$$D_{\text{aux}} = (1 - \eta_{\text{aux}})\dot{m}_{\text{aux}}V = (1 - \eta_{\text{aux}})f_{\text{aux}}\dot{m}_{req}V$$

where η_{aux} is the ram recovery efficiency.

23–4   Power Available

Given the flight condition and engine rating, the power available P_a is calculated as follows. The specific power and referred mass flow (relative to SP_0 and $\dot{m}_0$ for this rating) are approximated by functions of the ambient temperature ratio θ, here just:

$$SP_a = SP_0\,\theta\left[\delta_M\sqrt{\theta_M}\right]^{X_{spa}}$$
$$\dot{m}_a = \dot{m}_0\left(\delta/\sqrt{\theta}\right)\left[\delta_M\sqrt{\theta_M}\right]^{X_{mfa}}$$

The power available is then

$$P_a = SP_a\dot{m}_a = P_0\left(\delta\sqrt{\theta}\right)\left[\delta_M\sqrt{\theta_M}\right]^{X_{spa}+X_{mfa}}$$

This expression for $\dot{m}_a$ is used only to calculate P_a; elsewhere the $\dot{m}_q$ expression below (for performance at power required) is used to obtain the mass flow at a power P_q. The influence of compressor rotational speed is not considered.

23–5   Performance at Power Required

The compressor performance (mass flow and gross jet thrust) is calculated for a specified power required P_q, flight condition, and rating. Installation losses P_{loss} are added to P_{req} ($P_q = P_{req} + P_{\text{loss}}$). The referred quantities (relative to SLS static MCP quantities) are approximated by functions of $q = P_q/(P_{0C}\delta\sqrt{\theta})$:

$$\dot{m}_{req} = \dot{m}_{0C}\left(\delta/\sqrt{\theta}\right)\left(K_{mfq0} + K_{mfq1}q + K_{mfq2}q^2 + K_{mfq3}q^3\right)[\theta_M]^{X_{mfq}}$$
$$F_g = F_{g0C}(\delta)\left(K_{fgq0} + K_{fgq1}q + K_{fgq2}q^2 + K_{fgq3}q^3\right)[\theta_M]^{X_{fgq}}$$

Then the installed net jet thrust F_N and momentum drag D_{aux} are calculated. The influence of compressor rotational speed is not considered.

For reaction drive, the power and mass flow are obtained to produce the required rotor reaction force

$$F_{Greq} = \dot{m}_{\text{react}} V_{\text{react}} = F_{\text{react}} + \dot{m}_{\text{react}} \Omega r_{\text{react}}$$

Arrays of

$$q = P_q/(P_{0C}(\delta\sqrt{\theta})$$
$$K_{fgq} = F_g/(F_{g0C}(\delta)\,[\theta_M]^{X_{fgq}}) = K_{fgq0} + K_{fgq1}q + K_{fgq2}q^2 + K_{fgq3}q^3$$
$$K_{mfq} = \dot{m}/(\dot{m}_{0C}(\delta/\sqrt{\theta})\,[\theta_M]^{X_{mfq}}) = K_{mfq0} + K_{mfq1}q + K_{mfq2}q^2 + K_{mfq3}q^3$$

hence net force

$$f_n = F_{\text{react}}/(F_{g0C}(\delta)\,[\theta_M]^{X_{fgq}})$$

are interpolated given F_{react} and Ωr_{react} to find q.

For flow control drive, the power and mass flow are obtained to produce the required rotor momentum flux $F_{Greq} = \sum F_{Greq\,\text{comp}}$, summed over all components using the flow control. Arrays of q, K_{fgq}, K_{mfq} and hence net force f_n are interpolated given F_{Greq} to find q.

23–6   Scaling

The parameters of the compressor model can be defined for a specific compressor, but it is also necessary to scale the parameters as part of the aircraft sizing task, in order to define a compressor for a specified power. In addition, advanced technology must be represented in the model. Scaling and advanced technology are handled in terms of specific power and specific thrust (at SLS static conditions and MCP).

The compressor model includes reference values of the performance parameters: P_{0R}, SP_{0R}, and $P_{\text{mech}R}$. Mass flow is obtained from $\dot{m}_{0R} = P_{0R}/SP_{0R}$. The reference power at each rating R defines a ratio to MCP: $r_{p0R} = P_{0R}/P_{0C}$. Similarly for specific power and mechanical limits: $r_{s0R} = SP_{0R}/SP_{0C}$ and $r_{m0R} = P_{\text{mech}R}/P_{0C}$. These ratios are kept fixed when the compressor is scaled.

The compressor size is specified as power P_{eng}, which is the power at takeoff rating R, for SLS static conditions. Hence the MCP power is $P_{0C} = P_{to}/r_{p0R}$, and the power at all other ratings follows. If P_{0C} is not equal to the reference value of the compressor model, then the compressor is scaled.

23–7   Weight

The compressor weight can be a fixed input value, or calculated as a function of power. As a function of power, the weight of one compressor is:

$$W_{\text{one eng}} = K_{0\text{comp}} + K_{1\text{comp}}P + K_{2\text{comp}}P^{X_{\text{comp}}}$$

where P is the installed power (SLS static, specified rating) per compressor. A constant weight per power W/P is given by using only $K_{1\text{comp}}$.

Motor Model

A motor converts electrical energy (fuel) to shaft power. A generator converts input shaft power to electrical energy, and the shaft power contributes to the propulsion group power required. The model follows references 1 to 4.

24–1   Motor Characteristics

The motor or generator size is defined by the maximum power available, $P_{\max} = P_{\mathrm{eng}}$, and a peak torque Q_{peak}. For clarity, the torque limit is expressed as a power limit P_{peak}, such that $Q_{\mathrm{peak}} = P_{\mathrm{peak}}/N_{\mathrm{spec}}$. So P_{peak} is the torque limit at N_{spec}, and $P_{\mathrm{peak}}(N/N_{\mathrm{spec}})$ is the torque limit at engine rotation speed N. The ratio of maximum power and peak torque is the base rotational speed: $P_{\max}/Q_{\mathrm{peak}} = N_{\mathrm{base}}$; at N_{base}, the power and torque limits coincide. It follows that $N_{\mathrm{base}} = (P_{\max}/P_{\mathrm{peak}})N_{\mathrm{spec}}$. Figure 24-1 shows the maximum power and peak torque for a number of motors. The base rotational speed is typically $N_{\mathrm{base}} = 700$–7000 rpm, lower for high-torque motors. The ratio of maximum power to continuous power is typically MRP/MCP = 1.25–3 (fig. 24-2).

Motor weight depends primarily on the peak torque. Figure 24-3 shows the weight for a number of motors. For a given size, there is a wide range of Q/W values. Here high torque-to-weight is defined as $Q/W > 3.5$ ft-lb/lb. Motor electrical controller weight is typically 10–30% of the basic motor weight, up to 70% for small motors. The density is typically 100–250 lb/ft^3 (fig. 24-4).

The engine performance is described by the power available P_a at each engine rating, and the energy flow $\dot{E}$ required to produce the power required P_q. The specific fuel consumption is $\mathrm{sfc} = \dot{E}/P$ (inverse of efficiency). For each rating R, the maximum power is P_{0R}, and the torque limit is $P_{\mathrm{peak}R}$.

24–2   Installation (Thermal Management System)

The difference between installed and uninstalled power is the thermal management system losses P_{TMS}: $P_{av} = P_a - P_{\mathrm{TMS}}$ and $P_{req} = P_q - P_{\mathrm{TMS}}$ (added for generator). The system mass flow $\dot{m}_{\mathrm{TMS}}$ depends on the motor rejected heat P_{rej}; the power P_{TMS} and gross thrust F_G depend on the mass flow. The net jet thrust is $F_N = F_G - \dot{m}_{\mathrm{TMS}}V$. The size of the thermal management system (including weight) is determined by a design rejected heat $P_{\mathrm{rej-design}}$, perhaps specified as a fraction of the motor size P_{eng}.

The motor shaft power required is P_{req}, and the gross (uninstalled) power is $P_q = P_{req} + P_{\mathrm{TMS}}$. The rejected heat is $P_{\mathrm{rej}} = \dot{E}_{req} - P_q = P_q(1 - \eta_{\mathrm{motor}})/\eta_{\mathrm{motor}}$. From η_{motor} evaluated for P_{req}, P_{rej} gives the mass flow $\dot{m}_{\mathrm{TMS}}$, and then the power P_{TMS} and gross thrust F_G. Then η_{motor} is evaluated for $P_q = P_{req} + P_{\mathrm{TMS}}$, and the electrical power required is $\dot{E}_{req} = P_q/\eta_{\mathrm{motor}}$.

The generator shaft power required is P_{req}, and the net (uninstalled) power is $P_q = P_{req} - P_{\mathrm{TMS}}$. The rejected heat is $P_{\mathrm{rej}} = P_q - \dot{E}_{req} = P_q(1 - \eta_{\mathrm{motor}})$. From η_{motor} evaluated for P_{req}, P_{rej} gives the mass

flow $\dot{m}_{\text{TMS}}$, and then the power P_{TMS} and gross thrust F_G. Then η_{motor} is evaluated for $P_q = P_{req} - P_{\text{TMS}}$, and the electrical power produced is $\dot{E}_{req} = P_q\eta_{\text{motor}}$.

Using referred variables, the mass flow, power, and gross thrust are

$$\left(\frac{\dot{m}_{\text{TMS}}}{\delta/\sqrt{\theta}}\right) = K_{\text{TMS}m0} + K_{\text{TMS}m1}\left(\frac{P_{\text{rej}}}{\delta\sqrt{\theta}}\right)^{X_{\text{TMS}m}}$$

$$\left(\frac{P_{\text{TMS}}}{\delta\sqrt{\theta}}\right) = K_{\text{TMS}p0} + K_{\text{TMS}p1}\left(\frac{\dot{m}_{\text{TMS}}}{\delta/\sqrt{\theta}}\right)^{X_{\text{TMS}p}}$$

$$\left(\frac{F_G}{\delta}\right) = K_{\text{TMS}f0} + K_{\text{TMS}f1}\left(\frac{\dot{m}_{\text{TMS}}}{\delta/\sqrt{\theta}}\right)^{X_{\text{TMS}f}}$$

The weight of the thermal management system is obtained from the design rejected heat $P_{\text{rej}-\text{design}}$:

$$W_{\text{TMS}} = K_{\text{TMS}w0} + K_{\text{TMS}w1}P_{\text{rej}-\text{design}}^{X_{\text{TMS}wp}}\dot{m}_{\text{rej}-\text{design}}^{X_{\text{TMS}wm}}$$

where $\dot{m}_{\text{rej}-\text{design}}$ is calculated from $P_{\text{rej}-\text{design}}$.

24–3   Power Available and Performance at Power Required

Given the flight condition and engine rating R, the power available is

$$P_a = \min\left(P_{0R}, rP_{\text{peak}R}\right)$$

where $r = N/N_{\text{spec}}$, and N is the motor rotational speed.

The performance is calculated for a specified power required (which might equal the power available), flight condition, and engine rating. The motor power required determines the energy flow:

$$\dot{E}_{req} = \dot{E}_{0C}g_e(q, n) = P_q/\eta_{\text{motor}}$$

where $q = P_q/P_{\text{eng}}$, $n = N/N_{\text{spec}}$, and η_{motor} is the motor efficiency. The generator energy flow to the fuel tank is related to the power required:

$$\dot{E}_{req} = \dot{E}_{0C}g_e(q, n) = P_q\eta_{\text{motor}}$$

where η_{motor} is the generator efficiency.

24–4   Efficiency

Motor loss sources include copper (internal resistance, proportional to current-squared hence torque-squared), iron core (eddy current and hysteresis, proportional to rotational speed), and mechanical (friction, proportional to speed, and windage, proportional to speed-cubed). Equivalent circuit and efficiency map models are implemented. The efficiency can also be a fixed value. The efficiency is evaluated at the uninstalled power P_q.

24-4.1   Equivalent Circuit

The motor or generator efficiency as a function of power is estimated considering an equivalent circuit, defined by internal resistance R and current I_0. The voltage is $V = V_o - IR$. The efficiency is

$\eta_{\text{motor}} = P/(P + P_{\text{loss}})$, from the power loss $P_{\text{loss}} = I^2 R + P_0$. For small loss, $I \cong P/V_o$. In terms of P_{eng}, let

$$R/V_o^2 = \frac{1}{P_{\text{eng}}} \left(\frac{1}{\eta_{\text{ref}}} - 1 \right)$$

$$P_0 = c P_{\text{eng}}$$

Then

$$\frac{1}{\eta_{\text{motor}}} = 1 + P(R/V_o^2) + P_0/P = 1 + \frac{P}{P_{\text{eng}}} \left(\frac{1}{\eta_{\text{ref}}} - 1 \right) + c \frac{P_{\text{eng}}}{P}$$

So $\eta_{\text{motor}} = (1/\eta_{\text{ref}} + c)^{-1}$ at $P = P_{\text{eng}}$. The efficiency decreases with P because of the internal resistance, but is zero at $P = 0$ because of the internal current term.

24-4.2   Efficiency Map

The motor or generator power loss is described as a polynomial in the motor torque and rotational speed:

$$P_{\text{loss}} = P_{\text{eng}} f_{\text{loss}} \sum_{i=0}^{3} \sum_{j=0}^{3} C_{ij} t^i n^j$$

where $q = P_q/P_{\text{eng}}$, $n = N/N_{\text{spec}}$, and $t = q/n$. The factor f_{loss} allows adjustment of the peak efficiency. Controller losses, including power conversion and conditioning, are represented by an efficiency η_{cont}. Then

$$\eta_{\text{motor}} = \eta_{\text{cont}} \frac{P_q}{P_q + P_{\text{loss}}} = \eta_{\text{cont}} \frac{q}{q + P_{\text{loss}}/P_{\text{eng}}}$$

is the motor or generator efficiency. Constant efficiency implies just $C_{11} = 1/\eta - 1$. The copper, iron, and windage losses imply $P_{\text{loss}} = K_c Q^2 + K_i N + K_w N^3 + K_0 = P_{\text{eng}}(C_{20} t^2 + C_{01} n + C_{03} n^3 + C_{00})$.

Consider an efficiency map with a peak η_0 at $t_0 = Q_0/Q_{\text{eng}}$ and $n_0 = N_0/N_{\text{spec}}$. Taking the derivatives of $(C_{20} t^2 + C_{01} n + C_{03} n^3 + C_{00}) = nt(1 - \eta)/\eta$ with respect to t and n, and evaluating at t_0 and n_0 so $\partial \eta/\partial t = \partial \eta/\partial n = 0$, gives

$$C_{01} = \frac{t_0}{4} \frac{1 - \eta_0}{\eta_0} - \frac{3 C_{00}}{2 n_0}$$

$$C_{03} = \frac{t_0}{4 n_0^2} \frac{1 - \eta_0}{\eta_0} + \frac{C_{00}}{2 n_0^3}$$

$$C_{20} = \frac{n_0}{2 t_0} \frac{1 - \eta_0}{\eta_0}$$

(ref. 2). Note that $C_{01} > 0$ requires $C_{00} < (n_0 t_0/6)(1 - \eta_0)/\eta_0$.

In general, expanding the efficiency in t and n about the peak gives

$$\frac{1}{\eta} = \frac{P_{\text{loss}}}{QN} + 1 = \frac{P_{\text{loss}}}{P_{\text{eng}}} \frac{1}{tn} + 1 \cong \frac{1}{\eta_0} \left(1 + a(\widetilde{t} - 1)^2 + b(\widetilde{t} - 1)(\widetilde{n} - 1) + c(\widetilde{n} - 1)^2 \right)$$

where $\widetilde{t} = t/t_0$ and $\widetilde{n} = n/n_0$. Hence

$$\frac{P_{\text{loss}}}{P_{\text{eng}}} = tn \left(\frac{1}{\eta} - 1 \right) = tn \left[\frac{1}{\eta_0} \left(1 + a(\widetilde{t} - 1)^2 + b(\widetilde{t} - 1)(\widetilde{n} - 1) + c(\widetilde{n} - 1)^2 \right) - 1 \right] + d(\widetilde{t} - 1)^2 (\widetilde{n} - 1)^2$$

The last term is introduced so the efficiency is zero at zero torque or zero speed.

Alternatively, consider an equivalent circuit defined by internal resistance R, no load current I_0, motor constant K_v, and friction K_f. So $V = V_m + IR$ (back emf $V_m = N/K_v$), $I = I_m + I_0$ ($I_m = Q_m N/V_m = K_v Q_m$), $Q = Q_m - Q_f$ (output torque, $Q_f = K_f N$). The input power is $VI = (N/K_v + IR)I$, with $I = K_v Q_m + I_0 = K_v(Q + K_f N) + I_0$. The shaft power is QN. Then the power loss is

$$\begin{aligned} P_{\text{loss}} = VI - QN &= \Big(N + (I/K_v)K_v^2 R \Big)(I/K_v) - QN \\ &= N\Big(Q + K_f N + I_0/K_v \Big) + \Big(Q + K_f N + I_0/K_v \Big)^2 K_v^2 R - QN \\ &= N\Big(K_f N + I_0/K_v \Big) + \Big(Q + K_f N + I_0/K_v \Big)^2 K_v^2 R \end{aligned}$$

This expression does not show a peak efficiency as a function of torque and speed.

24–5   Weight

The motor or generator weight can be a fixed input value, or calculated as a function of power or torque. As a function of power, the weight of one engine is:

$$W_{\text{one eng}} = K_{0\text{motor}} + K_{1\text{motor}}P + K_{2\text{motor}}P^{X_{\text{motor}}}Q^{X_{q\text{motor}}}S_{\text{nac}}^{X_{s\text{motor}}} + K_{\text{ESC}}P^{X_{\text{ESC}}} + W_{\text{TMS}}$$

where $P = P_{\text{eng}}$, and $Q = P_{\text{peak}R}/N_{\text{spec}}$ (ft-lb or m-N) for takeoff rating R. Note then $P^X Q^{X_q} = P^{X+X_q}N^{-X_q} = N^X Q^{X+X_q}$. The engine group nacelle area S_{nac} is included as a surrogate for motor size. The last terms account for the weight of the electronic speed control system and the thermal management system.

Motor weight depends primarily on the peak torque. For the NASA15 model, the weight is:

$$W_{\text{one eng}} = 0.5382 f_{\text{design}}Q^{0.8129} + K_{\text{ESC}}P^{X_{\text{ESC}}} + W_{\text{TMS}}$$

where $Q = P_{\text{peak}R}/N_{\text{spec}}$ (ft-lb) for takeoff rating R. The structural design factor $f_{\text{design}} = 1.0$ for high torque-to-weight motors ($Q/W > 3.5$ ft-lb/lb), and $f_{\text{design}} = 2.5606$ for others. Based on 64 motors, the average error is 27.5% (fig. 24-5); 25% for high torque-to-weight and 29% for others. Including either maximum rotational speed or maximum power does not reduce the weight estimation error significantly. Considering only the high torque-to-weight motors, the weight is

$$W_{\text{one eng}} = 0.3928 Q^{0.8587} + K_{\text{ESC}}P^{X_{\text{ESC}}} + W_{\text{TMS}}$$

Based on 25 motors, the average error is 21.8% (fig. 24-5). This sensitivity of motor weight to output torque capability is somewhat greater than the drive system trend.

24–6   Scaling

The parameters of the motor model can be defined for a specific motor, but it is also necessary to scale the parameters as part of the aircraft sizing task, in order to define a motor for a specified power. In addition, advanced technology must be represented in the model.

The motor model includes reference values of the performance parameters: P_{0R}, $P_{\text{peak}R}$. The reference power at each rating R defines a ratio to MCP: $r_{p0R} = P_{0R}/P_{0C}$. Similarly for torque limits:

$r_{m0R} = P_{\text{peak}R}/P_{0C}$. These ratios are kept fixed when the motor is scaled. The specification motor speed and base rotational speed are scaled with the power:

$$N_{\text{spec}} = N_{\text{spec−ref}} \left(\frac{P_{\text{ref}}}{P_{\text{eng}}} \right)^{K_{Ns}}$$

$$N_{\text{base}} = N_{\text{base−ref}} \left(\frac{P_{\text{ref}}}{P_{\text{eng}}} \right)^{K_{Nb}}$$

The speed ratio N/N_{spec} influences the efficiency map and the torque limit. Then $P_{\text{peak}R}$ scales as

$$P_{\text{peak}R} = P_{\text{peak}R−\text{ref}} \frac{P_{\text{eng}}}{P_{\text{ref}}} \frac{N_{\text{spec}}/N_{\text{spec−ref}}}{N_{\text{base}}/N_{\text{base−ref}}} = r_{m0R} P_{\text{eng}} \left(\frac{P_{\text{ref}}}{P_{\text{eng}}} \right)^{K_{Ns}−K_{Nb}}$$

So $P_{\text{peak}R}$ just scales with P_{eng} if $K_{Ns} = K_{Nb}$. Depending on the design approach, K_{Nb} ranges from about 0.8 to zero (constant N_{base}) to about $−1.2$ (N_{base} increase with power).

The motor size is specified as power P_{eng}, which is the power at takeoff rating R, for sea level standard (SLS) static conditions. Hence the MCP power is $P_{0C} = P_{to}/r_{p0R}$, and the power at all other ratings follows. If P_{0C} is not equal to the reference value of the motor model, then the motor is scaled.

24–7 References

1) McDonald, R.A. "Electric Motor Modeling for Conceptual Aircraft Design." AIAA Paper No. 2013-0941, January 2013.

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3) Sinsay, J.D.; Alonso, J.J.; Kontinos, D.A.; Melton, J.E.; and Grabbe, S. "Air Vehicle Design and Technology Considerations for an Electric VTOL Metro-Regiional Public Transportation System." AIAA Paper No. 2012-5404, September 2012.

4) Datta, A., and Johnson, W. "Powerplant Design and Performance Analysis of a Manned All-Electric Manned Helicopter." Journal of Propulsion and Power, Vol. 30, No. 2 (March-April 2014).
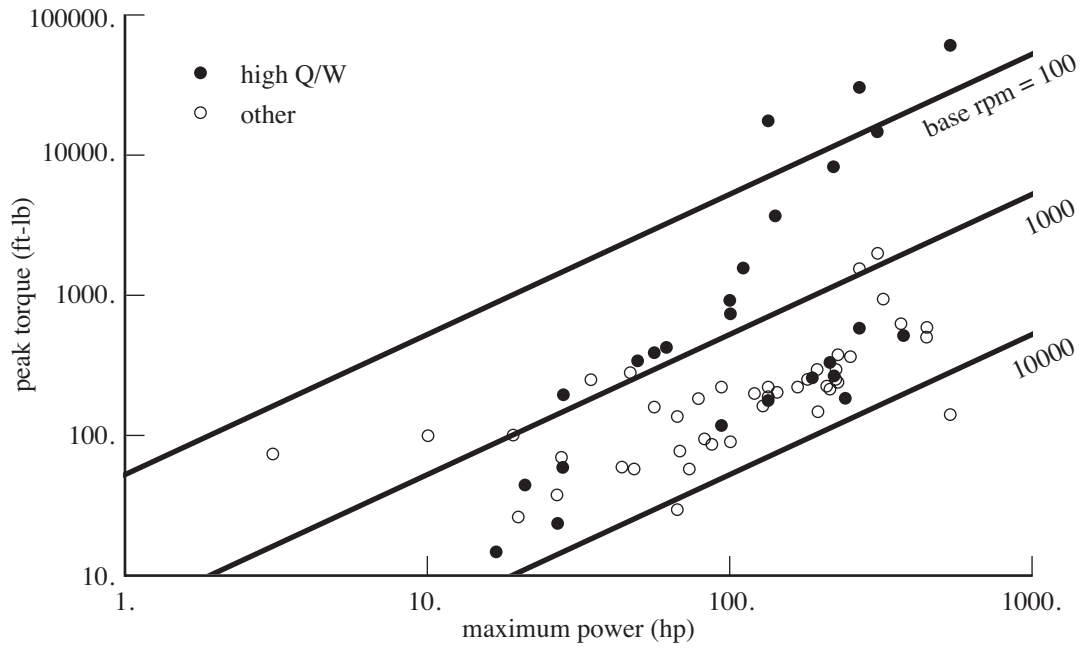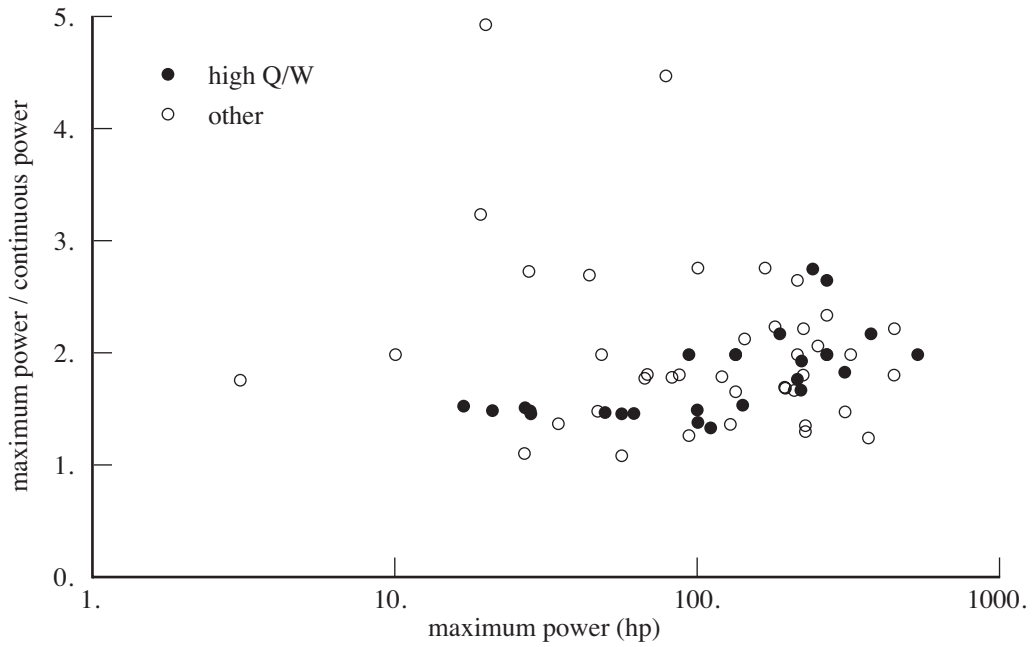
Figure 24-1. Motor maximum power and peak torque.

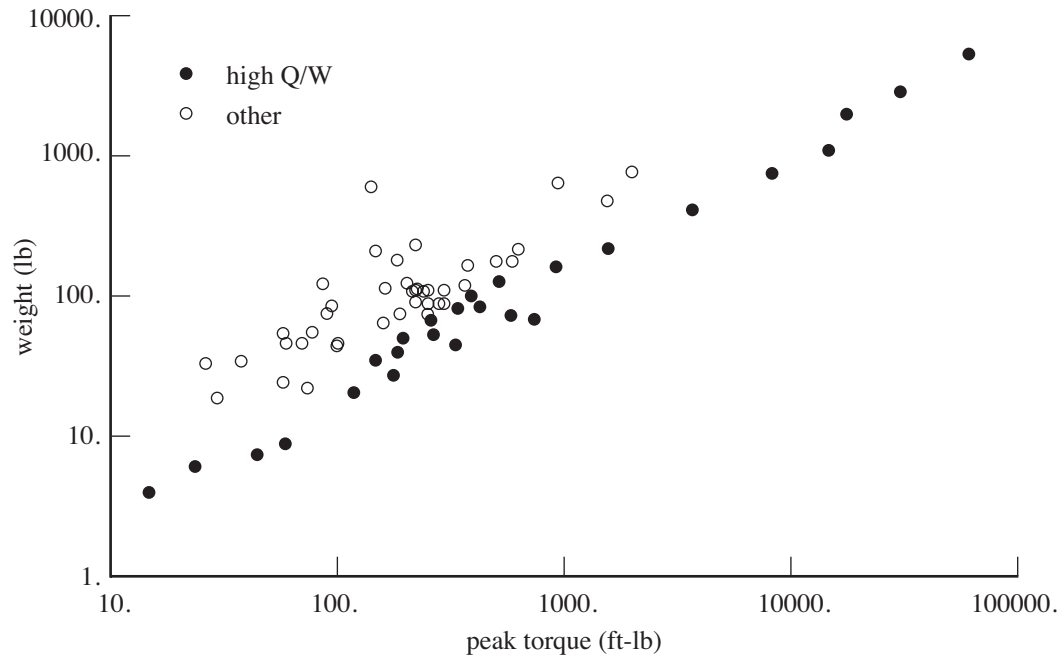Figure 24-2. Motor maximum power and continuous power (MRP/MCP).

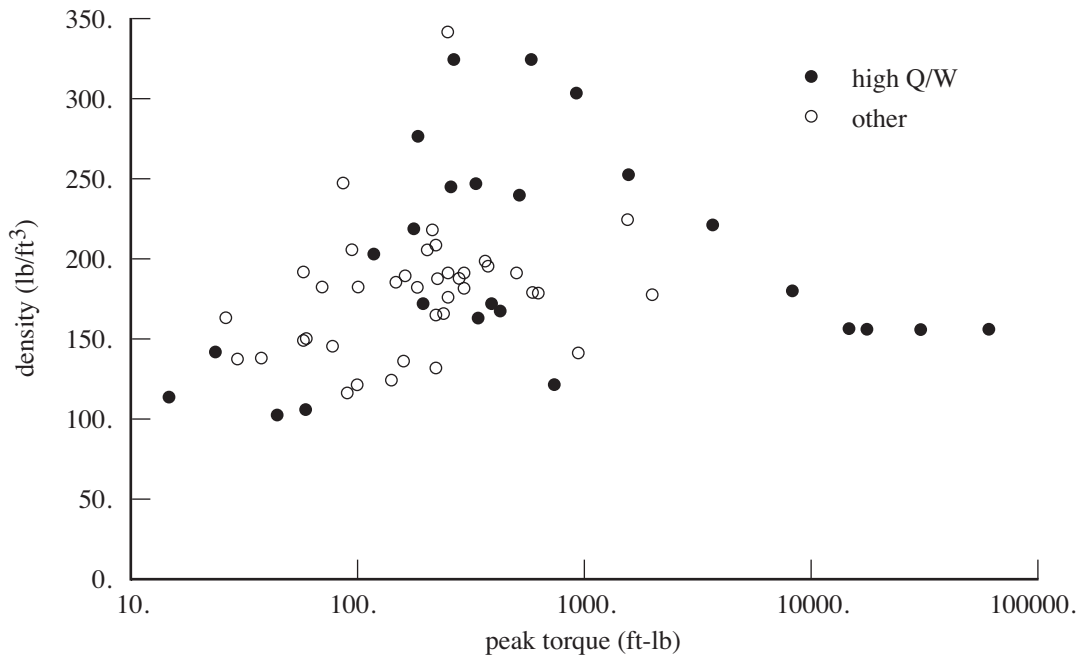Figure 24-3. Motor weight.

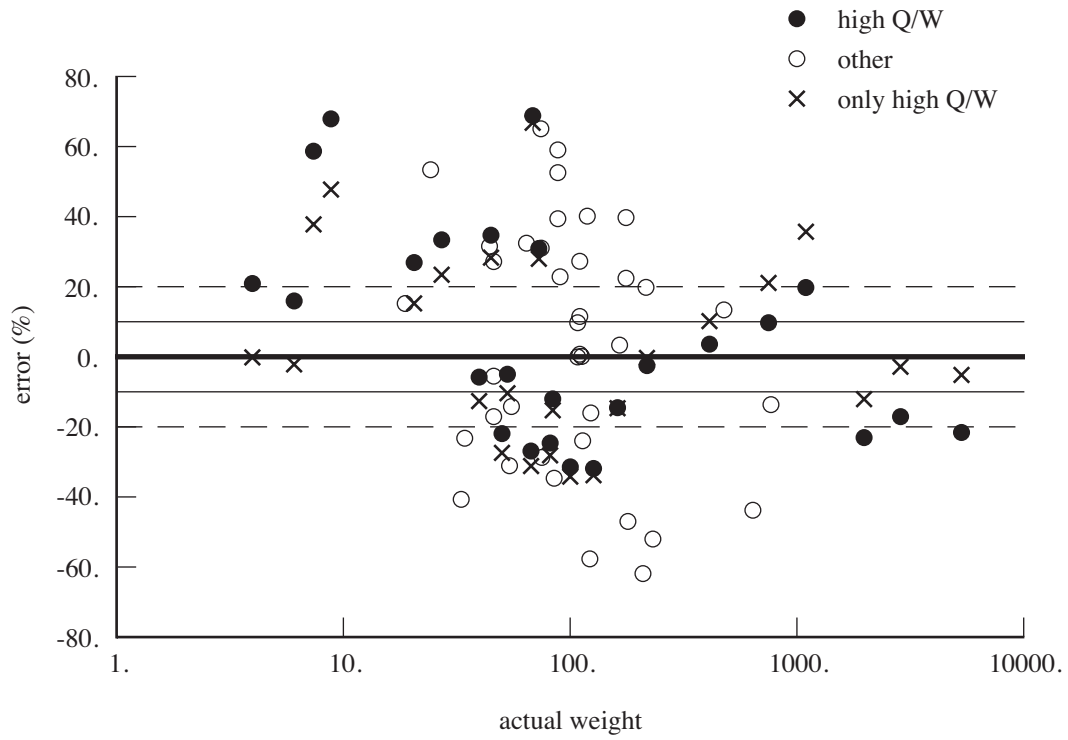Figure 24-4. Motor density.

Figure 24-5. Motor weight (NASA15).

Referred Parameter Jet Engine Model

Design and analysis codes calculate aircraft performance for a very wide range of operating conditions. Thus the jet performance model must be realistic even far from the design point. A simple thermodynamic cycle analysis that assumes design point component efficiencies everywhere is not realistic for such an application. Rather than developing models for component performance, the approach taken is to use a model for the total turbojet or turbofan performance. The jet is not being designed.

The Referred Parameter Jet Engine Model (RPJEM) is based on curve-fits of performance data for existing or projected jets over a range of operating conditions. The use of referred parameters tends to collapse the data and provides a basis for scaling the jet. The operating condition is described by pressure altitude, ambient air temperature, flight Mach number, and either jet rating or jet thrust required. The parametric model is scaled to the required size and adjusted to the appropriate technology level to represent a notional jet. Jet size is represented by mass flow. Jet technology is represented by specific thrust available and specific fuel consumption at maximum continuous thrust (MCT), sea level/standard day (SLS), static (zero airspeed) conditions. Jet installation effects are also modeled.

25–1   Operating Environment

The operating condition and atmosphere give the standard conditions (temperature T_{std} and pressure p_{std}) for a specified pressure altitude; the sea-level standard conditions (temperature T_0 and pressure p_0); and the operating temperature T and pressure p. Here the temperatures are °R or °K. The characteristics depend on the temperature ratio $\theta = T/T_0$ and pressure ratio $\delta = p/p_0$.

The flight Mach number $M = V/c_s = V/c_{s0}\sqrt{\theta}$ is obtained from the aircraft speed V.

The inlet ram air temperature ratio and pressure ratio are obtained then from M and the inlet ram recovery efficiency η_d:

$$\theta_M = \left(1 + \frac{\gamma - 1}{2}M^2\right) = \left(1 + 0.2M^2\right)$$

$$\delta_M = \left(1 + \frac{\gamma - 1}{2}\eta_d M^2\right)^{\frac{\gamma}{\gamma-1}} = \left(1 + 0.2\eta_d M^2\right)^{3.5}$$

where the ratio of specific heats $\gamma = 1.4$.

25–2   Performance Characteristics

The uninstalled thrust required is T_q, the thrust available T_a. The jet model calculates T_a as a function of flight condition and engine rating; or calculates mass flow and fuel flow at T_q. The specific thrust is $ST = T/\dot{m}$, and the specific fuel consumption is $\text{sfc} = \dot{w}/T$. A turbofan engine has a bypass

ratio $\beta = \dot{m}_{\text{fan}}/\dot{m}$. The forces produced by mass flow that does not go through the core or fan are treated as momentum drag D_{aux}.

The reference performance is at sea-level-standard static conditions (subscript 0), and MCT (subscript C). For each rating R, the performance is characterized by the following quantities for sea-level-standard static conditions: thrust T_{0R}, specific thrust ST_{0R}, and mechanical thrust limit $T_{\text{mech}R}$. The mass flow is then $\dot{m}_{0R} = T_{0R}/ST_{0R}$.

The difference between net and gross thrust is the momentum drag: $T_N = T_G - \dot{m}(1 + \beta)V$. Note that traditional units for mass flow are pound/sec (pps), while this equation requires slug/sec ($\dot{m}_{req}/g$ replaces $\dot{m}_{req}$).

25–3   Installation

The difference between installed and uninstalled thrust is the inlet and exhaust losses T_{loss}: $T_{av} = T_a - T_{\text{loss}}$ and $T_{req} = T_q - T_{\text{loss}}$. The inlet and exhaust losses are modeled as fractions of thrust available or thrust required: $T_{\text{loss}} = (\ell_{in} + \ell_{ex})T_a$ or $T_{\text{loss}} = (\ell_{in} + \ell_{ex})T_q$. The momentum drag of the auxiliary air flow is a function of the mass flow $\dot{m}_{\text{aux}} = f_{\text{aux}}\dot{m}_{req}$:

$$D_{\text{aux}} = (1 - \eta_{\text{aux}})\dot{m}_{\text{aux}}V = (1 - \eta_{\text{aux}})f_{\text{aux}}\dot{m}_{req}V$$

where η_{aux} is the ram recovery efficiency. Exhaust losses (ℓ_{ex}) and auxiliary air flow parameters (η_{aux}, f_{aux}) are defined for infrared suppressor on and off.

25–4   Thrust Available

Given the flight condition and jet rating, the thrust available T_a is calculated as follows. The gross specific thrust and referred mass flow (relative to ST_0 and $\dot{m}_0$ for this rating) are approximated by functions of the ambient temperature ratio θ, here just:

$$ST_a = ST_0\,\sqrt{\theta}\left[\delta_M\,\sqrt{\theta_M}\right]^{X_{sta}}$$

$$\dot{m}_a = \dot{m}_0\left(\delta/\sqrt{\theta}\right)\left[\delta_M\,\sqrt{\theta_M}\right]^{X_{mfa}}$$

The thrust available is then

$$T_a = ST_a\dot{m}_a - \dot{m}_a(ST)_{\text{mom}} = T_0\delta\left[\delta_M\,\sqrt{\theta_M}\right]^{X_{sta}+X_{mfa}} - \dot{m}_a(1 + \beta)V$$

This expression for $\dot{m}_a$ is used only to calculate T_a; elsewhere the $\dot{m}_q$ expression below (for performance at thrust required) is used to obtain the mass flow at a thrust T_q.

25–5   Performance at Thrust Required

The jet performance (mass flow and fuel flow) is calculated for a specified thrust required T_q, flight condition, and jet rating. The referred quantities (relative to SLS static MCT quantities) are approximated by functions of referred gross thrust $t = T_{Gq}/(T_{0C}\delta)$:

$$\dot{w}_{req} = \dot{w}_{0C}\left(\delta\sqrt{\theta}\right)\left(K_{ffq0} + K_{ffq1}t + K_{ffq2}t^2 + K_{ffq3}t^3\right)[\theta_M]^{-X_{ffq}}$$

$$\dot{m}_{req} = \dot{m}_{0C}\left(\delta/\sqrt{\theta}\right)\left(K_{mfq0} + K_{mfq1}t + K_{mfq2}t^2 + K_{mfq3}t^3\right)[\theta_M]^{X_{mfq}}$$

where $T_{Gq} = T_q + \dot{m}(1 + \beta)V$. Arrays of

$$t = t_{Gq}/(T_{0C}\delta)$$

$$K_{mfq} = \dot{m}/(\dot{m}_{0C}(\delta/\sqrt{\theta})\,[\theta_M]^{X_{mfq}}) = K_{mfq0} + K_{mfq1}t + K_{mfq2}t^2 + K_{mfq3}t^3$$

hence net thrust

$$t_n = t_q/(T_{0C}\delta)$$

are interpolated given T_q to find mass flow $\dot{m}$, and then fuel flow $\dot{w}$.

25–6   Reaction Drive or Flow Control

For reaction drive or flow control, the thrust available and thrust required exclude the momentum drag, which is only accounted for in the total jet group load. The thrust available is then

$$T_a = T_0\delta\left[\delta_M\sqrt{\theta_M}\right]^{X_{sta}+X_{mfa}}$$

The gross jet thrust required is

$$T_{reqJG} = F_{Greq} = \dot{m}_{\text{react}}V_{\text{react}} = F_{\text{react}} + \dot{m}_{\text{react}}\Omega r_{\text{react}}$$

for reaction drive; or $T_{reqJG} = F_{Greq} = \sum F_{Greq\ \text{comp}}$ for flow control (summed over all components using the flow control). So

$$T_{Gq} = T_{reqJG}/(N_{\text{jet}} - N_{\text{inop}}) + T_{\text{loss}} = T_{reqJG}/((N_{\text{jet}} - N_{\text{inop}})(1 - \ell_{in} - \ell_{ex}))$$

Given the required net reaction force F_{react} (and Ωr_{react}), arrays of t, K_{mfq}, and t_n are interpolated to find mass flow $\dot{m}$, and then fuel flow $\dot{w}$.

25–7   Scaling

The parameters of the jet model can be defined for a specific turbojet or turbofan, but it is also necessary to scale the parameters as part of the aircraft sizing task, in order to define a jet for a specified thrust. In addition, advanced technology must be represented in the model. Scaling and advanced technology are handled in terms of specific thrust and specific fuel consumption (at SLS static conditions and MCT). Figures 25-1 through 25-3 present historical data for jet specific fuel consumption, weight, and specific thrust.

The jet model includes reference values of the performance parameters: T_{0R}, ST_{0R}, $T_{\text{mech}R}$, and sfc$_{0C}$. Mass flow and fuel flow are obtained from $\dot{m}_{0R} = T_{0R}/ST_{0R}$ and $\dot{w}_{0C} = \text{sfc}_{0C}T_{0C}$. The reference thrust at each rating R defines a ratio to MCT: $r_{t0R} = T_{0R}/T_{0C}$. Similarly for specific thrust and mechanical limits: $r_{s0R} = ST_{0R}/ST_{0C}$ and $r_{m0R} = T_{\text{mech}R}/T_{0C}$. These ratios are kept fixed when the jet is scaled.

The jet size is specified as takeoff thrust $T_{to} = T_{\text{eng}}$, which is the thrust at rating R, for SLS static conditions. Hence the MCT is $T_{0C} = T_{to}/r_{t0R}$, and the thrust at all other ratings follows. If T_{0C} is not equal to the reference value of the jet model, then the jet is scaled. To reflect advanced technology, the specific thrust and specific fuel consumption can be specified: $ST_{0C} = ST_{\text{tech}}$ and sfc$_{0C} = $ sfc$_{\text{tech}}$ (replacing the jet model reference values). The default values are the reference values of the jet model.
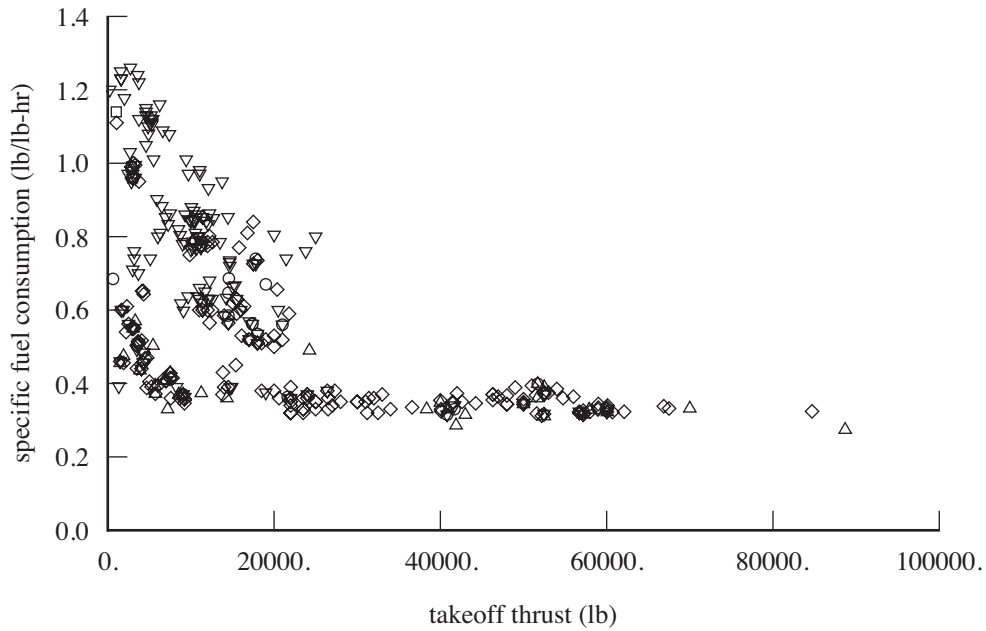
Figure 25-1. Historical data for turbojet and turbofan specific fuel consumption.

In the following paragraph, the subscript "tech" refers to these quantities; the subscript "ref" means the jet model reference values.

The jet technology parameters ST_{0C} and sfc_{0C} are assumed to vary linearly with mass flow $\dot{m}_{0C}$ up to a limit $\dot{m}_{\lim}$, and constant thereafter at $ST_{\lim}$ and $\mathrm{sfc}_{\lim}$. The mass flow at the technology condition is $\dot{m}_{\mathrm{tech}} = T_{\mathrm{ref}}/ST_{\mathrm{tech}}$, with the technology values ST_{tech} and $\mathrm{sfc}_{\mathrm{tech}}$. The intercept values are projected from the technology values: $K_{st0} = ST_{\mathrm{tech}} - K_{st1}\dot{m}_{\mathrm{tech}}$, $K_{st1} = (ST_{\lim} - ST_{\mathrm{tech}})/(\dot{m}_{\lim} - \dot{m}_{\mathrm{tech}})$; and similarly for sfc. Then for $\dot{m}_{0C} < \dot{m}_{\lim}$

$$ST_{0C} = K_{st0} + K_{st1}\dot{m}_{0C}$$

$$\mathrm{sfc}_{0C} = K_{sfc0} + K_{sfc1}\dot{m}_{0C}$$

and for $\dot{m}_{0C} \geq \dot{m}_{\lim}$

$$ST_{0C} = ST_{\lim}$$

$$\mathrm{sfc}_{0C} = \mathrm{sfc}_{\lim}$$

These equations are used if $\dot{m}_{\lim} > \dot{m}_{\mathrm{tech}}$. Otherwise the model sets $K_{st1} = K_{sfc1} = 0$, so there is no variation with scale: $ST_{0C} = ST_{\mathrm{tech}}$ and $\mathrm{sfc}_{0C} = \mathrm{sfc}_{\mathrm{tech}}$. Usually the effect of size gives $K_{st2} \geq 0$ and $K_{sfc2} \leq 0$. The thrust at the limit is $T_{\lim} = ST_{\lim}\dot{m}_{\lim}$. Figure 25-4 illustrates the scaling of SP, sfc, and SW.

Using $\dot{m}_{0C} = T_{0C}/ST_{0C}$, the specific thrust equation can be solved for the mass flow given the thrust:

$$\dot{m}_{0C} = \begin{cases} T_{0C}/K_{st0} & K_{st1} = 0 \\ T_{0C}/ST_{\lim} & T_{0C} \geq T_{\lim} \\ \sqrt{(\frac{K_0}{2K_1})^2 + \frac{T_{0C}}{K_1}} - \frac{K_0}{2K_1} & \text{otherwise} \end{cases}$$
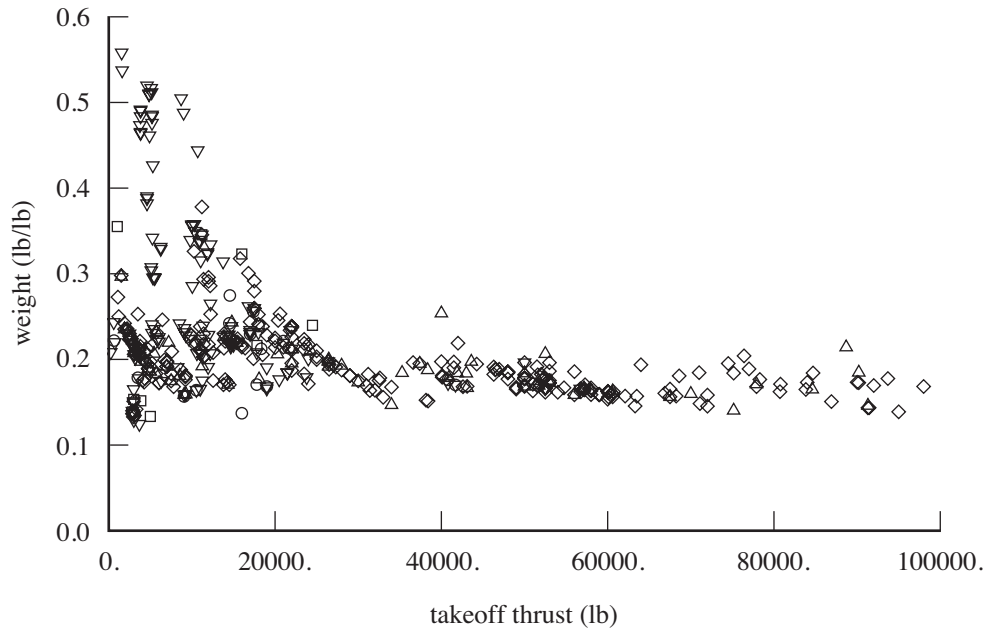
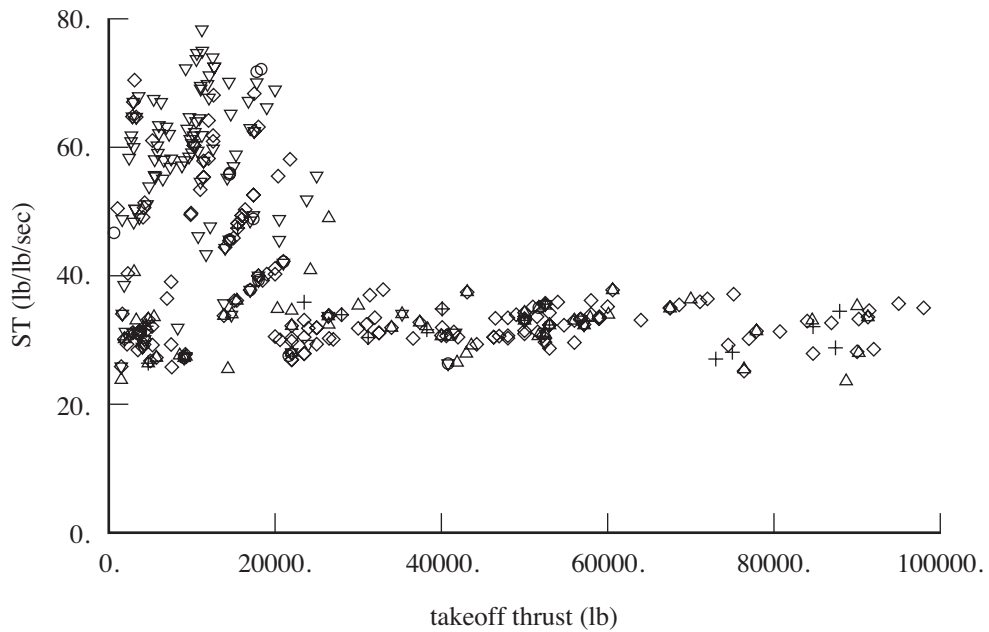Figure 25-2. Historical data for turbojet and turbofan weight.

Figure 25-3. Historical data for turbojet and turbofan specific thrust.

From this mass flow, ST_{0C} and sfc_{0C} are calculated, hence the fuel flow $\dot{w}_{0C} = \mathrm{sfc}_{0C}T_{0C}$. Then the thrust and specific thrust at all ratings R are obtained from the ratios: $T_{0R} = r_{t0R}T_{0C}$, $ST_{0R} = r_{s0R}ST_{0C}$, and $T_{\mathrm{mech}R} = r_{m0R}T_{0C}$.

The actual (perhaps scaled) values of the performance parameters are available for the jet group: T_{0R}, ST_{0R}, $T_{\mathrm{mech}R}$, and sfc_{0C}.

25–8 Weight

The jet weight can be a fixed input value, or calculated as a function of thrust. As a function of thrust, the weight of one jet is:

$$W_{\mathrm{one\ jet}} = K_{0\mathrm{jet}} + K_{1\mathrm{jet}}T + K_{2\mathrm{jet}}T^{X_{\mathrm{jet}}}$$

where T is the installed takeoff thrust (SLS static, specified rating) per jet. A constant weight per thrust W/T is given by using only $K_{1\mathrm{jet}}$.

25–9 Units

In this jet model, only the reference values and scaling constants are dimensional. Conventional English units and SI units are shown in table 25-1. Units of specific thrust and specific fuel consumption follow from these conventions.

Table 25-1. Conventional units.

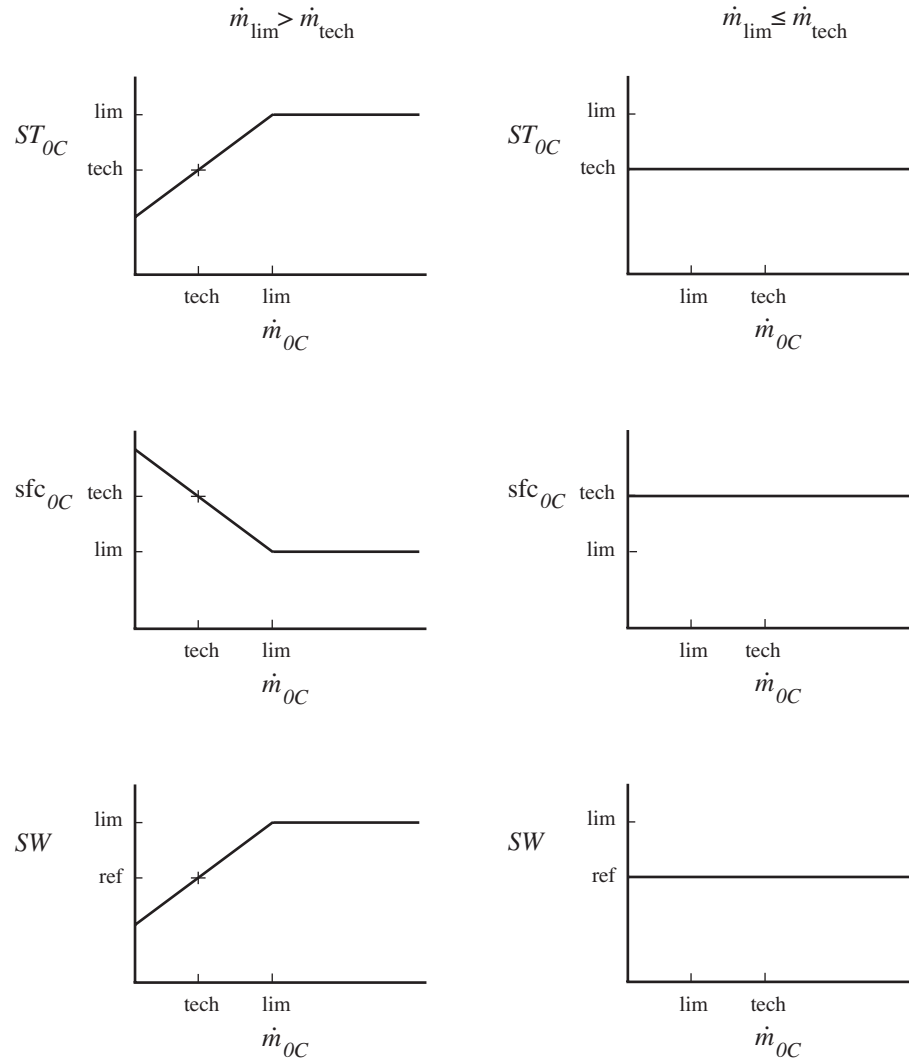|	thrust T	mass flow $\dot{m}$	fuel flow $\dot{w}$
English:	pound	pound/sec	pound/hour
SI:	Newton	kilogram/sec	kilogram/hour

Figure 25-4. Turbojet or turbofan scaling; $ST = T/\dot{m}$, sfc $= \dot{w}/T$, and $SW = T/W$.

Fuel Cell Model

A fuel cell burns a fuel (typically hydrogen) and generates electrical energy, which is stored in a fuel tank system or used directly by a motor. The energy flow defines the power required. The power available is related to the size P_{chrg}. The model follows reference 1.

26–1   Operating Environment

The operating condition and atmosphere give the standard conditions (temperature T_{std} and pressure p_{std}) for a specified pressure altitude; the sea-level standard conditions (temperature T_0 and pressure p_0); and the operating temperature T and pressure p. Here the temperatures are °R or °K. The engine characteristics depend on the temperature ratio $\theta = T/T_0$ and pressure ratio $\delta = p/p_0$.

The flight Mach number $M = V/c_s = V/c_{s0}\sqrt{\theta}$ is obtained from the aircraft speed V.

The inlet ram air temperature ratio and pressure ratio are obtained then from M and the inlet ram recovery efficiency η_d:

$$\theta_M = \left(1 + \frac{\gamma - 1}{2}M^2\right) = \left(1 + 0.2M^2\right)$$

$$\delta_M = \left(1 + \frac{\gamma - 1}{2}\eta_d M^2\right)^{\frac{\gamma}{\gamma-1}} = \left(1 + 0.2\eta_d M^2\right)^{3.5}$$

where the ratio of specific heats $\gamma = 1.4$.

26–2   Fuel Cell Performance

The performance of a fuel cell is characterized by the single cell behavior: voltage v_c as a function of current density i_c. The corresponding power density is $p_c = v_c i_c$. The maximum possible voltage is the cell ideal reversible voltage E_h, obtained when all the energy available is converted to electrical energy. The efficiency of the cell is $\eta = v_c/E_h$. From the fuel specific energy e_{fuel} (MJ/kg or kWh/kg), the specific fuel consumption is then $\mathrm{sfc} = \dot{w}/P = e_{\mathrm{fuel}}/\eta = e_{\mathrm{fuel}}E_h/v_c$.

Figure 26-1 shows the typical performance of a proton exchange membrane fuel cell. For hydrogen, $E_h = 1.472$ volts. The voltage is reduced by activation losses (low current), Ohmic losses, and mass transport losses (high current). The performance depends on the cell pressure δ_c, increasing as the operating pressure increases. A voltage reduction corresponds to a reduction in efficiency. There is a maximum power density $p_{\max}$ at $v_{\max}$, where $dv_c/di_c = -v_c/i_c$. The system is designed to operate at a current below the maximum power.

The fuel cell system consists of n_c individual cells in series, with total area A. Then $V = n_c v_c$, $I = A i_c$, and the total power is $P = VI = n_c A p_c$. The total fuel flow is proportional to the current, $\dot{w} = \mathrm{sfc} P = e_{\mathrm{fuel}} E_h n_c A i_c$.

Consider a fuel cell with maximum continuous power P_D (MCP), designed for current density i_d, from which v_d and p_d are determined. The design point is sea level static conditions, nominal temperature, and design cell pressure. The controllers, wiring, compressor, and other subsystems are designed for this current, so the total system weight will depend on P_D. As i_d decreases, less fuel is used (larger efficiency) but the weight is more (larger $n_c A$). The fuel cell can operate for a short time at a higher power (MRP). The power density $p_c = (\mathrm{MRP}/\mathrm{MCP}) p_d$ gives the corresponding current density i_{MRP}. There is also an absolute maximum on the power. The cell efficiency at the design point is $\eta_D = v_d / E_h$, hence $\mathrm{sfc}_D = (e_{\mathrm{fuel}} E_h / v_d) / \eta_{\mathrm{chrg}}$ and the fuel flow is $\dot{w}_D = \mathrm{sfc}_D P_D$. The system specific fuel consumption can include additional losses in terms of an efficiency η_{chrg}.

The cell pressure depends on the atmospheric pressure ratio δ, inlet ram pressure ratio δ_M, and compressor pressure ratio π_C: $\delta_c = \delta \delta_M \pi_C$. It is assumed that the cell temperature is maintained at the design value. The cell performance $v_c(i_c)$ is required at the design conditions ($\delta_c = \pi_C$) and at the operating conditions of the mission. The performance dependence on cell pressure can be captured by scaling the current with $\delta_c^{X_{fc}}$: given the reference characteristics $v_{\mathrm{ref}}(i_{\mathrm{ref}})$ for $\delta_c = 1$,

$$v_c = v_{\mathrm{ref}}(i_{\mathrm{ref}} = i_c / \delta_c^{X_{fc}})$$

Typically $X_{fc} = 0.38$; increasing δ_c then reduces i_{ref}, increasing v_c and the efficiency. This scaling leaves the voltage at maximum power unchanged. The reference characteristics are only needed to maximum power. From nominal performance at the design pressure ratio, $\delta_{\mathrm{nom}} = \pi_C$, the reference characteristics are $v_{\mathrm{ref}} = v_{\mathrm{nom}}$ at $i_{\mathrm{ref}} = i_{\mathrm{nom}} / \delta_{\mathrm{nom}}^{X_{fc}}$.

The ratio of mass flow and fuel flow follows from the chemistry of the reaction. For hydrogen and air

$$K_{mf} = \frac{\lambda_A}{\lambda_H} \frac{m_A}{x_O m_H} = \frac{\lambda_A}{\lambda_H} 68.59$$

The molar masses of hydrogen and air are $m_H = 2.016$ and $m_A = 28.97$ g/mole; $x_O = 0.2095$ is the molar fraction of oxygen in air. The supply ratio $\lambda_A / \lambda_H = 1/2$ from stoichiometry, and typically $\lambda_A / \lambda_H \cong 1.25$ in practice.

The fuel cell is described by the static power P_{0R} at rating R, the MCP specific fuel consumption sfc_{0C}, the design current i_d and compressor pressure ratio π_C, the mass flow ratio K_{mf}, and the cell characteristics $v_{\mathrm{ref}}(i_{\mathrm{ref}})$. The cell characteristics are only used as ratios to the design values.

26–3   Power Available

Given the flight condition and charger rating, the power available $P_{av} = \dot{E}_{a\mathrm{cell}}$ is

$$P_{av} = P_{0C} \frac{p_a}{p_d}$$

where p_a is obtained from the rated current i_R. From $v_R = v_{\mathrm{ref}}(i_{\mathrm{ref}} = i_R / \delta_c^{X_{fc}})$, $p_R = v_R i_R$. The maximum power density $p_{\max} = v_{\max} i_{\max}$ is obtained from $i_{\max} = \delta_c^{X_{fc}} i_{\mathrm{ref}}(v_{\mathrm{ref}} = v_{\max})$. Then $p_a = \min(p_R, p_{\max})$.

26–4   Performance at Power Required

The fuel cell performance (fuel flow and mass flow) is calculated for a specified power required $P_{req} = \dot{E}_{q\text{cell}}$ and flight condition. The current i_q required at $p_q = p_d(P_{req}/P_{0C})$ is obtained from $p_c = i_c v_{\text{ref}}(i_{\text{ref}} = i_c/\delta_c^{X_{fc}})$ as a function of i_c. Then

$$\dot{w}_{req} = \dot{w}_{0C}(i_q/i_d)$$
$$\dot{m}_{req} = K_{mf}\dot{w}_{req}$$

where $\dot{w}_{0C} = \text{sfc}_{0C}P_{0C}$.

26–5   Simple Model

Constant v_{ref} implies constant efficiency and specific fuel consumption, and so p_c proportional to i_c with no maximum power density. Then the rated power available and specific fuel consumption are constant: $P_{av} = P_{0R}$, $\dot{w}_{req} = \dot{w}_{0C}(P_{req}/P_{0C}) = \text{sfc}_{0C}P_{req}$, $\dot{m}_{req} = K_{mf}\dot{w}_{req}$.

26–6   Weight

The fuel cell weight can be a fixed input value, or calculated as a function of power. As a function of power, the weight of one charger is:

$$W_{\text{one chrg}} = K_{0\text{cell}} + K_{1\text{cell}}P + K_{2\text{cell}}P^{X_{\text{cell}}}$$

where P is the installed power (SLS static, specified rating) per charger. A constant weight per power W/P is given by using only $K_{1\text{cell}}$.

26–7   References

1) Datta, A., and Johnson, W. "Powerplant Design and Performance Analysis of a Manned All-Electric Manned Helicopter." Journal of Propulsion and Power, Vol. 30, No. 2 (March-April 2014).
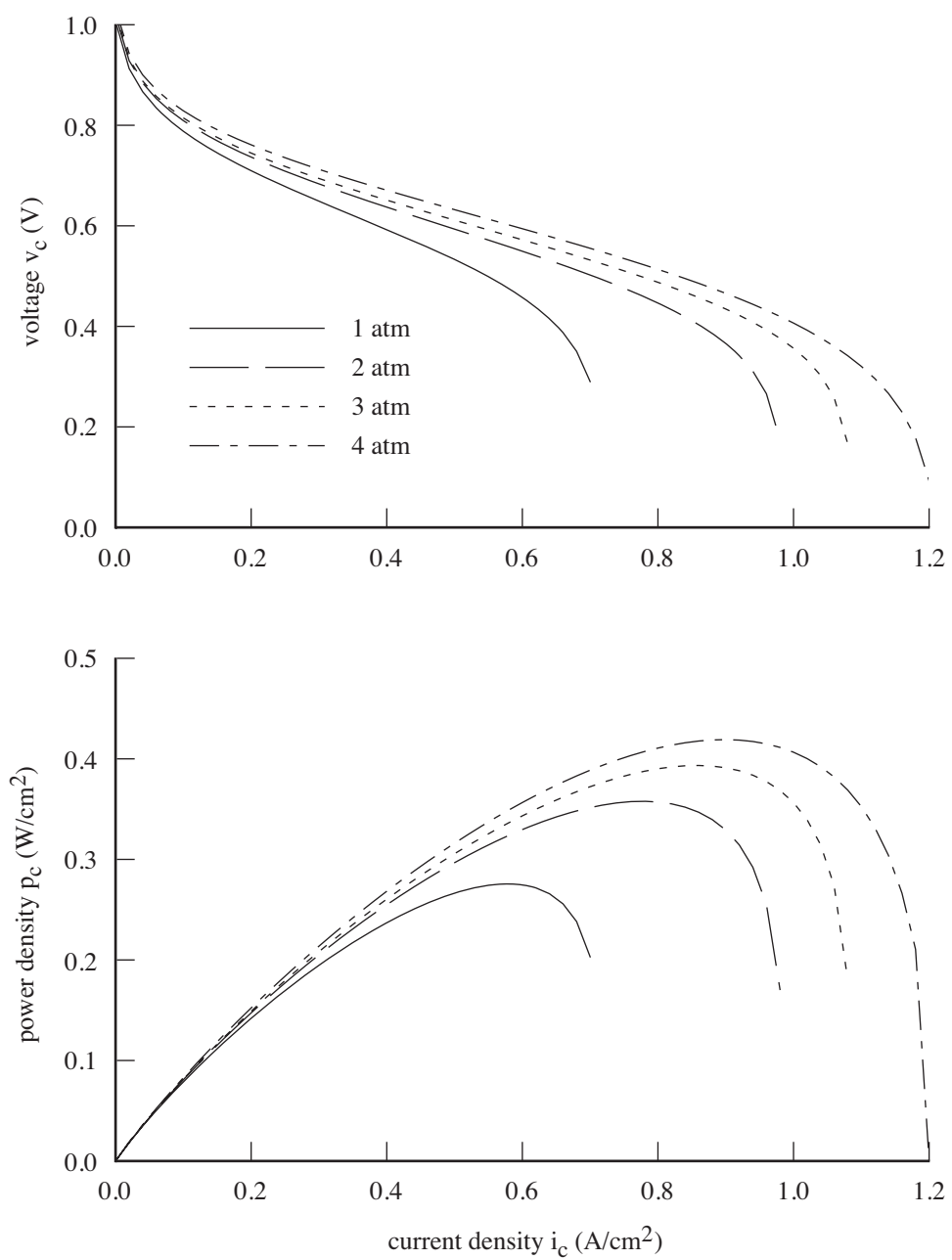
Figure 26-1. Characteristic performance of a proton exchange membrane fuel cell.

Solar Cell Model

A solar cell generates electrical energy, which is stored in a fuel tank system. The energy flow defines the power required. The power available is related to the size P_{chrg}. The solar cell is characterized by power density e_{solar} (W/m^2) and weight density σ_{solar} (kg/m^2). From the size P_{chrg}, the area is $A_{\text{solar}} = P_{\text{chrg}}/e_{\text{solar}}$; and then the weight is $W_{\text{one chrg}} = W_{\text{solar}} = A_{\text{solar}}\sigma_{\text{solar}}$.

Given the flight condition and the charger rating, the cell power available is $P_{av} = \dot{E}_{a\text{cell}} = P_0$. The power required P_{req} is defined by the charge group energy flow. The cell power required is

$$\dot{E}_{q\text{cell}} = P_{0C}g_e(q) = P_{req}/\eta_{\text{chrg}}$$

where $q = P_{req}/P_{0C}$, and η_{chrg} is the solar cell efficiency.

The solar cell efficiency as a function of power is estimated considering an equivalent circuit, defined by internal resistance R and current I_0. The voltage is $V = V_o - IR$. The efficiency is $\eta_{\text{chrg}} = P/(P + P_{\text{loss}})$, from the power loss $P_{\text{loss}} = I^2R + P_0$. For small loss, $I \cong P/V_o$. In terms of P_{chrg}, let

$$R/V_o^2 = \frac{1}{P_{\text{chrg}}}\left(\frac{1}{\eta_{\text{ref}}} - 1\right)$$

$$P_0 = cP_{\text{chrg}}$$

Then

$$\frac{1}{\eta_{\text{chrg}}} = 1 + P(R/V_o^2) + P_0/P = 1 + \frac{P}{P_{\text{chrg}}}\left(\frac{1}{\eta_{\text{ref}}} - 1\right) + c\frac{P_{\text{chrg}}}{P}$$

So $\eta_{\text{chrg}} = (1/\eta_{\text{ref}} + c)^{-1}$ at $P = P_{\text{chrg}}$. The efficiency decreases with P because of the internal resistance, but is zero at $P = 0$ because of the internal current term. Alternatively, the efficiency can be a fixed value.

A simple model requires the power density e_{solar} and weight density σ_{solar}, with efficiency included in e_{solar} ($\eta_{\text{chrg}} = 1$).

Battery Model

A battery is a fuel tank system for which the fuel quantity stored and burned is measured in energy. The unit of fuel energy is Mega-Joules (MJ). For reference, 1 kW-hr = 3.6 MJ. The operating state affects the efficiency of the relation between useful power and the rate of change of the energy stored. The battery model produces the charge/discharge efficiency. The battery model can be used for capacitors and flywheels as well. References 1–6 provide background for the model.

The components associated with a fuel tank system (engine groups, jet groups, charge groups) define the total energy flow $\dot{E}_{\text{comp}}$ (charge or discharge). The fuel tank system energy flow (due to equipment power and distribution losses) is also included in $\dot{E}_{\text{comp}}$. Accounting for efficiency and losses gives the battery energy flow $\dot{E}_{\text{batt}} = \dot{E}_{\text{comp}} + P_{\text{loss}} + P_{\text{TMS}}$. The battery model gives P_{loss} and efficiency η_{batt} ($\eta_{\text{batt}} = \dot{E}_{\text{comp}}/(\dot{E}_{\text{comp}} + P_{\text{loss}})$ for discharge, $\eta_{\text{batt}} = (\dot{E}_{\text{comp}} + P_{\text{loss}})/\dot{E}_{\text{comp}}$ for charge). The system efficiency is $\eta_{\text{sys}} = \dot{E}_{\text{comp}}/\dot{E}_{\text{batt}}$ or $\eta_{\text{sys}} = \dot{E}_{\text{batt}}/\dot{E}_{\text{comp}}$. The battery power margin is $P_{\max} - |\dot{E}_{\text{batt}}|$. P_{TMS} is the power required for the thermal management system, calculated from the rejected power $P_{\text{rej}} = P_{\text{loss}}$ (the dependence of the battery efficiency, hence P_{loss}, on P_{TMS} is neglected in order to avoid an iterative solution). Accounting for battery capacity (reduced by current or fade) then gives the effective energy flow $\dot{E}_{\text{eff}}$ (the fuel tank capacity is not adjusted). The change in stored energy is calculated from $\dot{E}_{\text{eff}}$ and time. The convention is that energy flow $\dot{E} > 0$ for discharge, and $\dot{E} < 0$ for charge. Power and current are positive for both discharge and charge.

The battery capacity is $E_{\text{fuel}-\text{cap}}$ (maximum usable fuel energy). The battery is characterized by specific energy e_{tank} (MJ/kg or kWh/kg) and energy density ρ_{tank} (MJ/liter or kWh/liter), so the tank weight and volume are obtained from the capacity. The current amount of energy stored is E_{fuel}. The state-of-charge is $s = E_{\text{fuel}}/E_{\text{fuel}-\text{cap}}$; the depth-of-discharge is $d = 1 - s$. Typically 20–30% of the stored charge is not usable at nominal conditions, which is accounted for in the capacity through the specific energy and energy density values. State-of-charge and depth-of-discharge are here based on the usable energy capacity $E_{\text{fuel}-\text{cap}}$. Relative the actual cell capacity, the depth-of-discharge is $d_{\text{act}} = d_{\min} + (d_{\max} - d_{\min})d_{\text{use}}$, where $d_{\min}$ and $d_{\max}$ correspond to the usable charge range. The actual battery capacity is then $E_{\text{fuel}-\text{act}} = E_{\text{fuel}-\text{cap}}/(d_{\max} - d_{\min})$.

The charge capacity is C (A-hr). C is the actual capacity at low current and a reference temperature. The corresponding actual energy capacity is obtained for a reference voltage, $E = CV_{\text{ref}}$ (W-hr, expressed in MJ). Then the specific energy (conventional units W-hr/kg) gives the weight, and the energy density (conventional units W-hr/liter) gives the volume. The current is measured in terms of the charge capacity: $I = xC$ (A), corresponding to a discharge time of $1/x$ hours. The units of x are 1/hr. The maximum burst discharge current (mbd) is x_{mbd}.

The usable capacity decreases with time and duty cycles, depending on the temperature and current. This factor is assumed to be constant during a mission or flight condition. Capacity fade is accounted

for using a factor $f_{\text{fade}} < 1$ on the available capacity or on the energy flow. Thus the effective energy flow is obtained by dividing the actual energy flow by f_{fade}.

The capacity depends on the discharge current. The Peukert model assumes $I^k T = \text{constant}$, where T is the discharge time for current $I = xC_{\text{ref}}$. The Peukert coefficient $k = 1.2$ to 1.3 for lead-acid batteries, and $k = 1.01$ to 1.05 for lithium-ion batteries (weak dependence). Thus the capacity $C = IT \sim 1/x^{k-1}$. An increase in current by a factor of 10 means a 2–11% reduction of capacity for lithium-ion batteries. For a larger current, the battery reaches a specified discharge voltage sooner, hence effectively has a reduced capacity.

The specific power is π_{batt} (kW/kg). The power capacity is obtained at the maximum burst discharge current:
$$P_{\text{cap}} = I_{mbd}V_{\text{ref}} = x_{mbd}CV_{\text{ref}} = x_{mbd}E_{\text{fuel}-\text{act}} = x_{mbd}E_{\text{fuel}-\text{cap}}/(d_{\max} - d_{\min})$$
(for P_{cap} in W and $E_{\text{fuel}-\text{act}}$ in W-hr). Hence given x_{mbd}, $\pi_{\text{batt}} = x_{mbd}e_{\text{tank}}/(3.6(d_{\max} - d_{\min}))$ (kW/kg from MJ/kg). Typically x_{mbd} is smaller for large capacity, implying a trade between high specific power and high specific energy.

28–1 Thermal Management System

The thermal management system deals with the battery rejected heat $P_{\text{rej}} = P_{\text{loss}}$. The system mass flow $\dot{m}_{\text{TMS}}$ depends on P_{rej}; the power P_{TMS} and gross thrust F_G depend on the mass flow. The net jet thrust is $F_N = F_G - \dot{m}_{\text{TMS}}V$. The size of the thermal management system (including weight) is determined by a design rejected heat $P_{\text{rej}-\text{design}}$, perhaps specified as a fraction of the battery power capacity P_{cap}.

Using referred variables, the mass flow, power, and gross thrust are

$$\left(\frac{\dot{m}_{\text{TMS}}}{\delta/\sqrt{\theta}}\right) = K_{\text{TMS}m0} + K_{\text{TMS}m1}\left(\frac{P_{\text{rej}}}{\delta\sqrt{\theta}}\right)^{X_{\text{TMS}m}}$$

$$\left(\frac{P_{\text{TMS}}}{\delta\sqrt{\theta}}\right) = K_{\text{TMS}p0} + K_{\text{TMS}p1}\left(\frac{\dot{m}_{\text{TMS}}}{\delta/\sqrt{\theta}}\right)^{X_{\text{TMS}p}}$$

$$\left(\frac{F_G}{\delta}\right) = K_{\text{TMS}f0} + K_{\text{TMS}f1}\left(\frac{\dot{m}_{\text{TMS}}}{\delta/\sqrt{\theta}}\right)^{X_{\text{TMS}f}}$$

The weight of the thermal management system is obtained from the design rejected heat $P_{\text{rej}-\text{design}}$:

$$W_{\text{TMS}} = K_{\text{TMS}w0} + K_{\text{TMS}w1}P_{\text{rej}-\text{design}}^{X_{\text{TMS}wp}}\dot{m}_{\text{rej}-\text{design}}^{X_{\text{TMS}wm}}$$

where $\dot{m}_{\text{rej}-\text{design}}$ is calculated from $P_{\text{rej}-\text{design}}$.

28–2 Equivalent Circuit Model

The discharge or charge efficiency as a function of power is estimated considering an equivalent circuit, defined by internal resistance R and current I_0. The voltage is $V = V_o - IR$. The efficiency is $\eta_{\text{batt}} = P/(P + P_{\text{loss}})$, from the power loss $P_{\text{loss}} = I^2R + P_0$. For small loss, $I \cong P/V_o$. In terms of a reference power $P_{\text{ref}} = P_{\text{cap}}$, and battery power $P = |\dot{E}_{\text{comp}}|$, let

$$R/V_o^2 = \frac{1}{P_{\text{ref}}}\left(\frac{1}{\eta_{\text{ref}}} - 1\right)$$

$$P_0 = cP_{\text{ref}}$$

Then

$$\frac{1}{\eta_{\mathrm{batt}}} = 1 + P(R/V_o^2) + P_0/P = 1 + \frac{P}{P_{\mathrm{ref}}}\left(\frac{1}{\eta_{\mathrm{ref}}} - 1\right) + c\frac{P_{\mathrm{ref}}}{P}$$

So $\eta_{\mathrm{batt}} = (1/\eta_{\mathrm{ref}} + c)^{-1}$ at $P = P_{\mathrm{ref}}$. The efficiency decreases with P because of the internal resistance, but is zero at $P = 0$ because of the internal current term. Alternatively, the efficiency can be a fixed value. The current is $x = I/C \cong P/V_0C \cong x_{mbd}P/P_{\mathrm{ref}} = x_{mbd}|\dot{E}_{\mathrm{comp}}|/P_{\mathrm{cap}}$. Given the maximum current $x_{\max}$, the maximum battery power is $P_{\max} = (x_{\max}/x_{mbd})P_{\mathrm{cap}}$.

For discharge, the battery energy flow is $\dot{E}_{\mathrm{batt}} = \dot{E}_{\mathrm{comp}}/\eta_{\mathrm{batt}} = \dot{E}_{\mathrm{comp}} + P_{\mathrm{loss}}$. Then the effective energy flow is $\dot{E}_{\mathrm{eff}} = \dot{E}_{\mathrm{batt}}/f_{\mathrm{fade}}$. For charge (negative energy flow), the battery energy flow is $\dot{E}_{\mathrm{batt}} = \dot{E}_{\mathrm{comp}}\eta_{\mathrm{batt}} = \dot{E}_{\mathrm{comp}} + P_{\mathrm{loss}}$. Then the effective energy flow is $\dot{E}_{\mathrm{eff}} = \dot{E}_{\mathrm{batt}}/f_{\mathrm{fade}}$. The battery power margin is $P_{\max} - |\dot{E}_{\mathrm{batt}}|$.

28–3 Lithium-Ion Battery Model

The capacity and power of currently available lithium-ion batteries are shown in figure 28-1. The power shown corresponds to the maximum burst discharge current, $P_{\mathrm{cap}} = x_{mbd}E_{\mathrm{fuel-act}}$. The cell reference voltage is typically $V_{\mathrm{ref}} = 4.2$ V. The maximum continous discharge current is in the range x_{mcd} = 1–30 (fig. 28-2). The maximum burst discharge current (which equals the power divided by capacity) is in the range x_{mbd} = 10–50 (fig. 28-3), with $x_{mbd} \cong 10$ for large capacity. Values of x_{mbd} above 100 are achievable, but such high-power/short-duration capability will not have much application to aircraft. The maximum charge current is typically $x_{CC\max}$ = 1–5 (fig. 28-4). Figure 28-5 shows the corresponding specific energy e_{tank} and specific power $\pi_{\mathrm{batt}} = x_{mbd}e_{\mathrm{tank}}$. The specific energy is typically e_{tank} = 50–200 W-hr/kg = 0.2–0.7 MJ/kg. Figure 28-6 shows the specific energy e_{tank} (W-hr/kg) and the energy density ρ_{tank} (W-hr/liter) (based on the battery actual capacity). The battery density is 2–3 kg/liter for all configurations (cylindrical, pouch, prismatic).

The battery temperature is likely controlled, for safety and efficiency. The operating temperature depends on the environment ambient temperature, internal heat production, and active heating or cooling. Internal heat production includes that due to internal resistance (I^2R), and radiation and convection (roughly proportional to the difference between cell and ambient temperatures). The cell temperature T_c is defined for a flight condition, or at the start of a mission. Without active temperature control, during a mission the cell temperature changes: dT_c/dt is proportional to the internal heat production. With temperature control, there is a power loss associated with the control system, depending on the cell temperature relative to ambient, hence on the internal heat production. As a simple model, it is assumed that this loss is a fraction f_{TC} of the total power.

28-3.1 Discharge Characteristics

The lithium-ion battery voltage during discharge depends on the discharge capacity (depth-of-discharge d), current ($I = xC$), and cell temperature (T_c). Figure 28-7 shows typical variation of the voltage V at constant current and constant temperature. The nominal energy capacity $E = CV_{\mathrm{ref}}$ is obtained from the nominal charge capacity C and a reference voltage V_{ref}. The open-circuit voltage V_d is the voltage at zero current, a reference temperature T_{ref}, and 100% charge. Typically the cell voltage is V_d = 4.2 V, and V_d is used as the reference V_{ref}. The actual battery capacity is determined by the voltage reaching a critical value V_{crit} (typically 2–3 V). Usually V decreases gradually with d, so the voltage provides a measure of depth-of-discharge. The decrease of V with current is primarily due to internal resistance. The voltage also decreases as temperature decreases (fig. 28-7). There is a maximum

discharge current (continuous or burst), which gives the rated power: $P_{\text{cap}} = I_{mbd}V_{\text{ref}} = x_{mbd}CV_{\text{ref}}$. The actual battery power supplied is $P = IV$.

The voltage variation with current is primarily due to the internal resistance: $V = V_o - IR$. The open circuit voltage decreases with depth-of-discharge: $V_o = V_d F_V(d)$, such that $F_V = 1$ at $d = 0$ and $F_V = f_{\text{crit}} = V_{\text{crit}}/V_d$ at $d = 1$; and $V_d = f_d V_{\text{ref}}$. For large discharge (d approaching 1), there is a secondary influence of current, which can be modelled by adjusting the discharge by a factor k. Then the capacity is given by $F_V(kd) = f_{\text{crit}}$. Variations of the voltage with current are scaled using $I = xC = (x_{mbd}C)(x/x_{mbd})$. The influence of temperature on the open circuit voltage V_o is reasonably accounted for by an increment proportional to the cube of the temperature difference $\Delta T = T_c - T_{\text{ref}}$.

The model for the discharge voltage is

$$V = V_d F_V(kd) + k_{VT}\Delta T^3 - IR = V_d F_V(kd) + k_{VT}\Delta T^3 - V_{\text{ref}}\frac{x_{mbd}CR}{V_{\text{ref}}}(x/x_{mbd})$$

with

$$k = 1 + k_{dI}I - k_{dT}\Delta T = 1 + (k_{dI}x_{mbd}C)(x/x_{mbd}) - k_{dT}\Delta T$$

The open circuit voltage function $F_V(d)$ is derived from $V(d)$ as a function of I and T_c: $F_V = (V - k_{VT}\Delta T^3 + IR)/V_d$ at kd. The model parameters R, k_{VT}, k_{dI}, and k_{dT} are adjusted for a good fit to the measured discharge characateristics. Figure 28-8 shows $F_V(d)$ obtained for a number of lithium-ion batteries. Typically $f_{\text{crit}} \cong 0.6$. That $d \neq 1$ at $F_V = f_{\text{crit}}$ reflects the definition of nominal capacity for the battery characteristics. Table 28-1 gives a typical $F_V(d)$ function. The function extends below f_{crit} since high temperature can increase the voltage. Figure 28-9 shows the current parameters $x_{mbd}CR/V_{\text{ref}}$ and $k_{dI}x_{mbd}C$ for a number of batteries. Generally $x_{mbd}CR/V_{\text{ref}} = 0.05$–0.20 and $k_{dI}x_{mbd}C = 0$–0.25. Figure 28-10 shows the temperature parameters k_{VT} and k_{dT} for a number of batteries.

Table 28-1. Lithium-ion battery open circuit voltage.

depth-of-charge d	F_V	depth-of-charge d	F_V
0.00	1.000	0.92	0.842
0.10	0.970	0.93	0.835
0.20	0.950	0.94	0.826
0.30	0.930	0.95	0.815
0.40	0.915	0.96	0.800
0.50	0.900	0.97	0.780
0.60	0.890	0.98	0.750
0.70	0.880	0.99	0.700
0.80	0.870	1.00	0.600
0.90	0.850	1.01	0.400
0.91	0.847	1.02	0.000

The battery power available to the components is

$$\dot{E}_{\text{comp}} = P = IV = I\big(V_d F_V(kd) + k_{VT}\Delta T^3\big) - I^2R - P_{TC} = \dot{E}_{\text{batt}} - P_{\text{loss}}$$

including power loss for temperature control. Hence

$$\dot{E}_{\text{batt}} = P_{\text{cap}}\xi\big(f_d F_V(kd) + (k_{VT}/V_{\text{ref}})\Delta T^3\big)$$
$$P_{\text{loss}} = P_{\text{cap}}\widehat{R}\xi^2 + f_{TC}|\dot{E}_{\text{comp}}|$$
$$k = 1 + \widehat{k}_{dI}\xi - k_{dT}\Delta T$$

where $\xi = x/x_{mbd}$, $\widehat{R} = x_{mbd}CR/V_{\mathrm{ref}}$, and $\widehat{k}_{dI} = k_{dI}x_{mbd}C$. The battery capacity for a given current and temperature is obtained by finding kd_{crit} such that

$$V = V_d F_V(kd_{\mathrm{crit}}) + k_{VT}\Delta T^3 - IR = V_d f_{\mathrm{crit}}$$

or

$$F_V(kd_{\mathrm{crit}}) = f_{\mathrm{crit}} + (\widehat{R}\xi - (k_{VT}/V_{\mathrm{ref}})\Delta T^3)/f_d$$

Then $d_{\mathrm{crit}} = (kd_{\mathrm{crit}})/k$, and the effective capacity is $d_{\mathrm{crit}}f_{\mathrm{fade}}E_{\mathrm{fuel-act}}$. Rather than change the fuel tank capacity, an effective fuel flow is used: $\dot{E}_{\mathrm{eff}} = \dot{E}_{\mathrm{batt}}/(d_{\mathrm{crit}}f_{\mathrm{fade}})$.

For a flight condition or mission segment, the total energy flow required is $\dot{E}_{\mathrm{comp}}$. The temperature T_c is specified. The depth-of-discharge d_{act} is obtained from $d = 1 - s = 1 - E_{\mathrm{fuel}}/E_{\mathrm{fuel-cap}}$. The discharge current $x = x_{mbd}\xi$ is calculated from the required energy flow:

$$\xi = \frac{(\dot{E}_{\mathrm{comp}} + P_{\mathrm{loss}})/P_{\mathrm{cap}}}{f_d F_V(kd) + (k_{VT}/V_{\mathrm{ref}})\Delta T^3}$$

Successive-substitution solution for ξ, with f_{crit} as the minimum for F_V, converges in about 10 iterations. Since F_V is a monotonically decreasing function of d, d_{crit} is obtained from x by interpolating $d(F_V)$. Then $\dot{E}_{\mathrm{batt}} = \dot{E}_{\mathrm{comp}} + P_{\mathrm{loss}}$, and the battery efficiency is $\eta_{\mathrm{batt}} = \dot{E}_{\mathrm{comp}}/\dot{E}_{\mathrm{batt}}$. The effective energy flow is $\dot{E}_{\mathrm{eff}} = \dot{E}_{\mathrm{batt}}/(d_{\mathrm{crit}}f_{\mathrm{fade}})$. The maximum battery discharge power is $P_{\mathrm{max}} = \dot{E}_{\mathrm{batt}}$ evaluated at x_{mbd}, or at a specified discharge current. The battery power margin is $P_{\mathrm{max}} - |\dot{E}_{\mathrm{batt}}|$.

28-3.2   Charge Characteristics

Lithium-ion battery charging is controlled for safety, with limits on minimum and maximum voltage, maximum current, and minimum and maximum temperature. Standard charging procedure consists of constant current (CC) followed by constant voltage (CV). Figure 28-11 shows the voltage, current, and capacity variation with charging time. The charging current is $I_{CC} = x_{CC}C$; the maximum charging current is typically $x_{CC\mathrm{max}} = 1$–5 (fig. 28-4). The charging voltage is $V_c = f_c V_{\mathrm{ref}}$; typically $V_c = V_{\mathrm{ref}} = 4.2$ V. The state-of-charge is s.

During the CC phase, the voltage starts at a fraction k_{cV} (typically 0.05–0.20) below V_c and increases with time, reaching V_c at time $t_c = \sigma/x_{CC}$. The charging model assumes linear variation of the voltage. During the CV phase, the voltage is fixed at V_c, while the current decreases rapidly. The charging model assumes the current varies inversely with the square of the time increment since t_c. The charge is then the integral of the current, $Q = \int_0^t I\, dt$; and the state-of-charge $s = Q/C$. The charging power is $P = IV$. Table 28-2 gives the charging model as a function of normalized time $\tau = t/t_c - 1 = tx_{CC}/\sigma - 1$.

At $t = t_c$ ($\tau = 0$), the current and voltage have the CC/CV values ($I = x_{CC}C$, $V = V_c$), the state-of-charge $s = \sigma$, and the charging power is at the peak ($P = x_{CC}CV_c = P_{\mathrm{cap}}(x_{CC}/x_{mbd})f_c$). For $s = 1$ at large time, k_{cI} must be related to σ: $1/\sigma = 1 + 1/k_{cI}$; the measured CV capacity is matched better with smaller values for k_{cI}. Figure 28-12 shows the charging parameters k_{cV} and k_{cI} for a number of batteries.

Table 28-2. Lithium-ion battery charging model.

	CC phase, $t < t_c, \tau = -1$ to $0, s < \sigma$	CV phase, $t > t_c, \tau > 0, s > \sigma$
current	$I = xC = x_{CC}C$	$I = xC = \dfrac{x_{CC}C}{(1 + k_{cI}\tau)^2}$
voltage	$V = V_c(1 + k_{cV}\tau)$	$V = V_c$
charge	$Q = x_{CC}Ct = \sigma C(1 + \tau)$	$Q = \sigma C \dfrac{1 + (k_{cI} + 1)\tau}{1 + k_{cI}\tau}$
state-of-charge	$s = x_{CC}t = \sigma(1 + \tau)$	$s = \sigma \dfrac{1 + (k_{cI} + 1)\tau}{1 + k_{cI}\tau}$
power	$P = xCV = x_{CC}CV_c(1 + k_{cV}\tau)$	$P = xCV_c = x_{CC}CV_c \dfrac{1}{(1 + k_{cI}\tau)^2}$
time	$\tau = \frac{s}{\sigma} - 1$	$\tau = \dfrac{\frac{s}{\sigma} - 1}{1 - k_{cI}(\frac{s}{\sigma} - 1)}$

For small charging current x_{CC}, the voltage V_c is reached at the nominal charging time $t_c = 1/x_{CC}$. So the CC phase is the entire process, and $s = x_{CC}t$. For higher current, the voltage V_c is reached sooner, at $t_c = \sigma/x_{CC}$ with $\sigma < 1$. Then the CV phase is required to complete the charging. For a given battery, $\sigma = 1$ at low charge current, and σ decreases with increasing x_{CC}. Figure 28-13 shows $1/\sigma$ and $1 + 1/k_{cI}$ for a number of batteries. An approximation for the variation of σ is

$$\frac{1}{\sigma} = 1 + \frac{1}{k_{cI}} = 1 + k_\sigma(x_{CC} - 0.2)$$

above $x_{CC} = 0.2$, with k_σ depending on the battery.

For a flight condition or mission segment, the total energy flow delivered to the battery is $\dot{E}_{\text{comp}}$. The temperature T_c is specified. The state-of-charge s_{act} is obtained from $s = E_{\text{fuel}}/E_{\text{fuel-cap}}$. The charge current $x = x_{mbd}\xi$ is calculated from the required energy flow, $|\dot{E}_{\text{batt}}| = P = IV = \xi x_{mbd}CV = P_{\text{cap}}\xi V/V_{\text{ref}}$. In the CC phase ($\tau < 0, s < \sigma$):

$$\xi = \left(\left(|\dot{E}_{\text{comp}}| - P_{\text{loss}} \right)/P_{\text{cap}} \right)/f_c(1 + k_{cV}\tau)$$

and $x_{CC} = x$. In the CV phase ($\tau > 0, s > \sigma$):

$$\xi = \left(\left(|\dot{E}_{\text{comp}}| - P_{\text{loss}} \right)/P_{\text{cap}} \right)/f_c$$

and $x_{CC} = x(1 + k_{cI}\tau)^2$. Successive-substitution solution for ξ converges in about 10 iterations. First the CC solution is found, then if $\tau > 0$ the CV solution is found (with an upper limit on x_{CC}). Then $\dot{E}_{\text{batt}} = \dot{E}_{\text{comp}} + P_{\text{loss}}$, and the battery efficiency is $\eta_{\text{batt}} = \dot{E}_{\text{batt}}/\dot{E}_{\text{comp}}$. The effective energy flow is $\dot{E}_{\text{eff}} = \dot{E}_{\text{batt}}/f_{\text{fade}}$. The maximum battery charging power is $P_{\text{max}} = P$ evaluated at current $x_{CC\text{max}}$, or at a specified charge current x_{CC}. The battery power margin is $P_{\text{max}} - |\dot{E}_{\text{batt}}|$.

28–4   References

1) Gao, L.; Liu, S.; and Dougal, R.A. "Dynamic Lithium-Ion Battery Model for System Simulation." IEEE Transactions on Components and Packaging Technologies, 25:3 (September 2002).
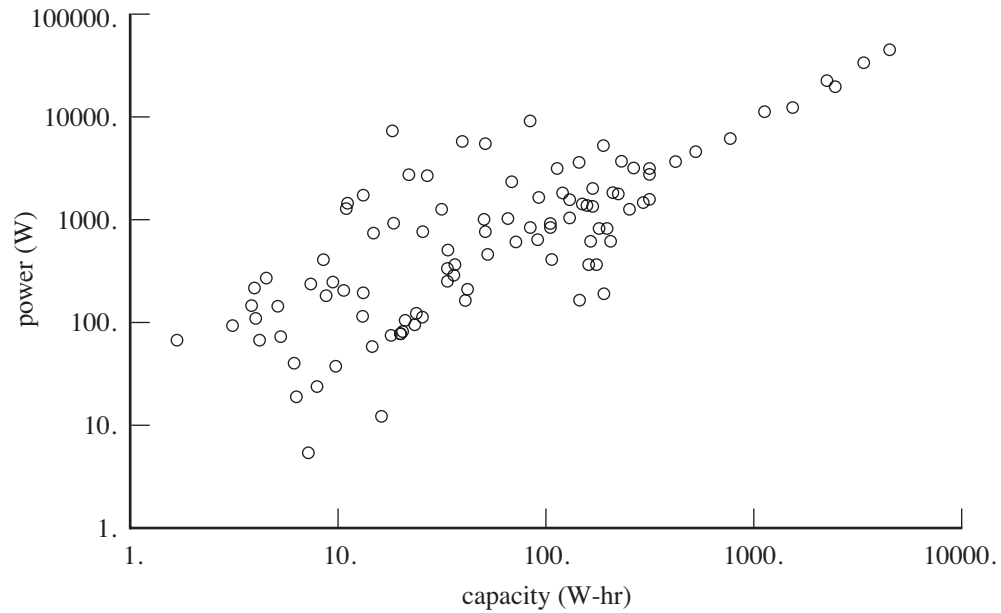
Figure 28-1. Lithium-ion battery capacity (W-hr) and power (W).

2) Ramadass, P.; Haran, B.; White, R.; and Popov, B.N. "Mathematical Modeling of the Capacity Fade of Li-Ion Cells." Journal of Power Sources, *123*:2 (September 2003).

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5) Datta, A., and Johnson, W. "Powerplant Design and Performance Analysis of a Manned All-Electric Manned Helicopter." Journal of Propulsion and Power, Vol. 30, No. 2 (March-April 2014).

6) Barth, A.; Feil, R.; Konkak, K.; and Hajek, M. "Conceptual Study for an Autonomous Rotorcraft for Extreme Altitudes." Fortieth European Rotorcraft Forum, Southampton, UK, September 2014.
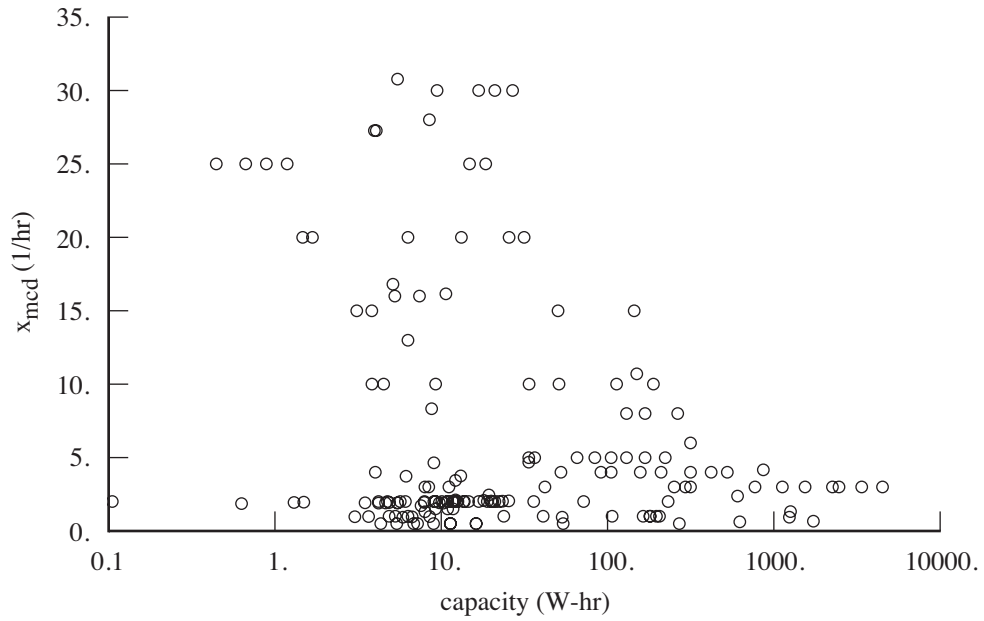
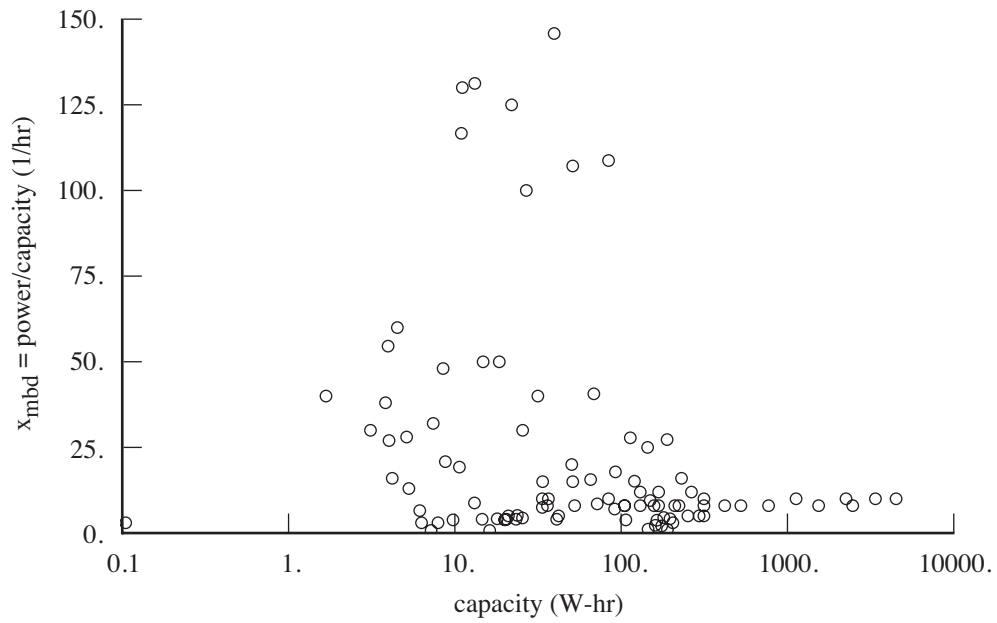Figure 28-2. Lithium-ion battery maximum continuous discharge current.

Figure 28-3. Lithium-ion battery maximum burst discharge current.
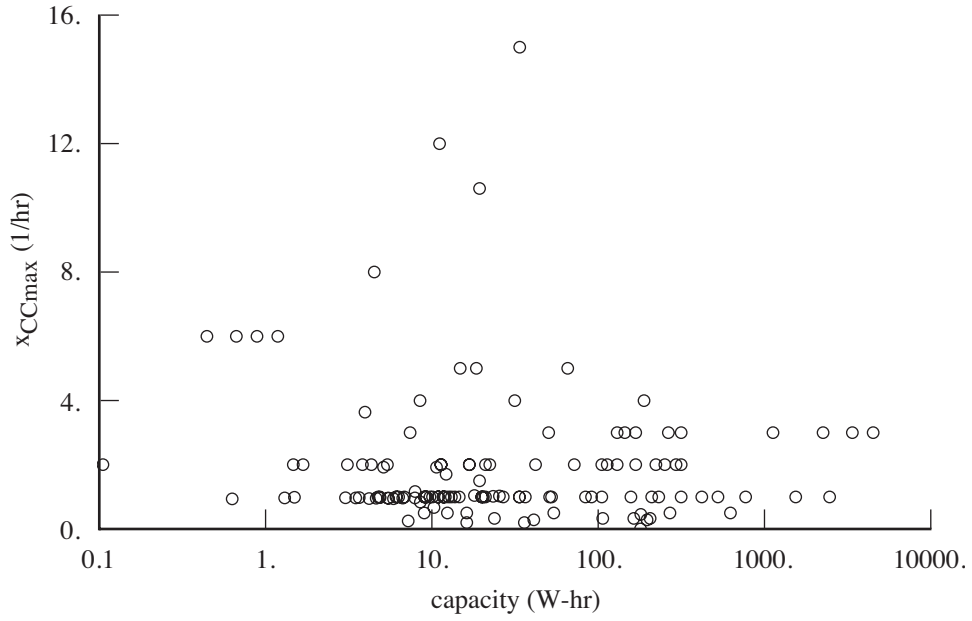
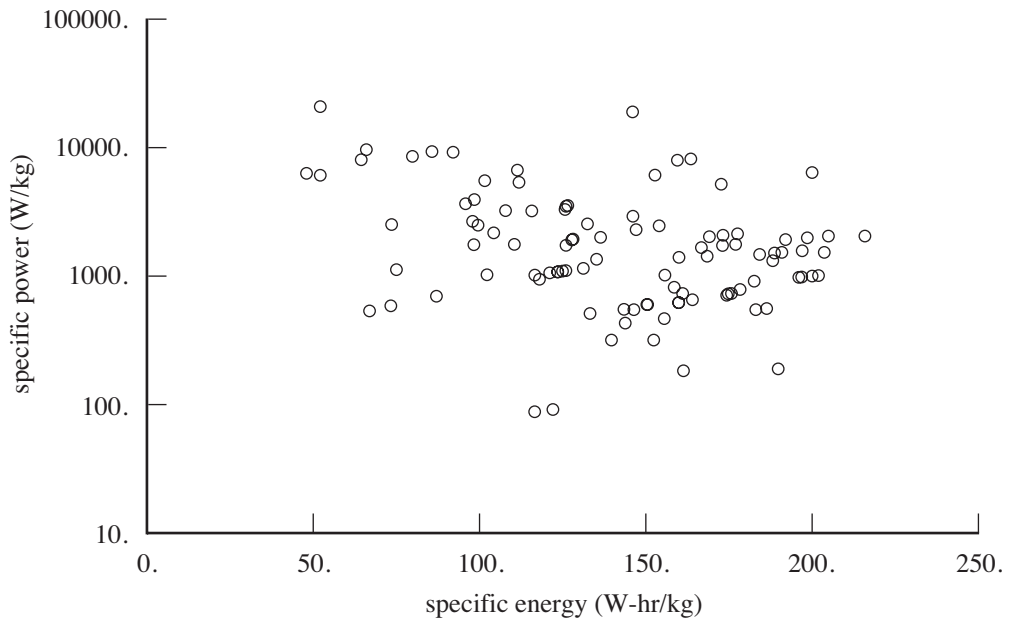Figure 28-4. Lithium-ion battery maximum charge current.

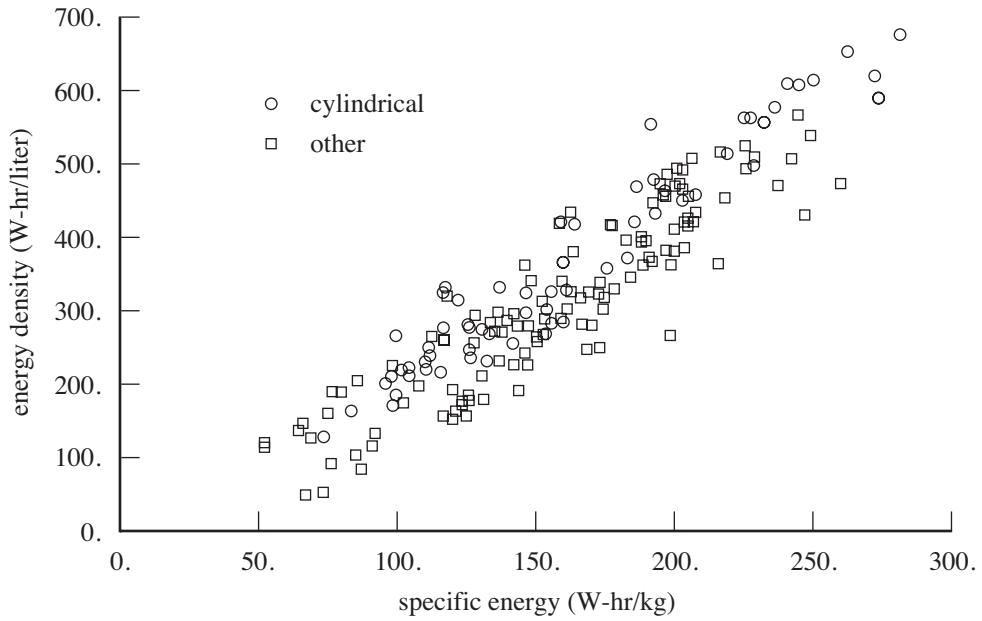Figure 28-5. Lithium-ion battery specific energy (W-hr/kg) and specific power (W/kg).

Figure 28-6. Lithium-ion battery specific energy (W-hr/kg) and energy density (W-hr/liter).
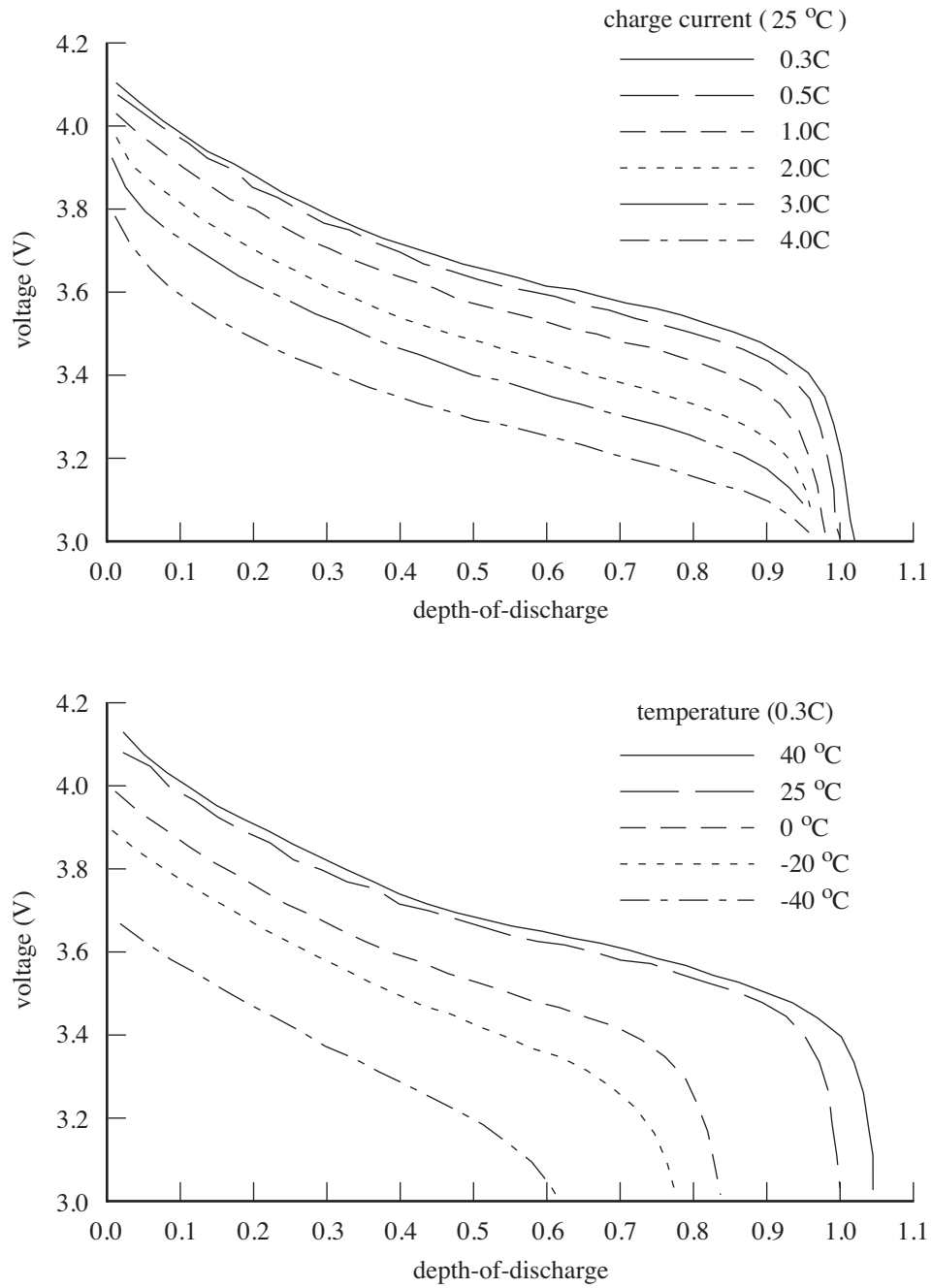
Figure 28-7. Lithium-ion battery discharge characteristics.

Figure 28-8. Lithium-ion battery open circuit voltage function.

Figure 28-9. Lithium-ion battery discharge model, current parameters.

Figure 28-10. Lithium-ion battery discharge model, temperature parameters.
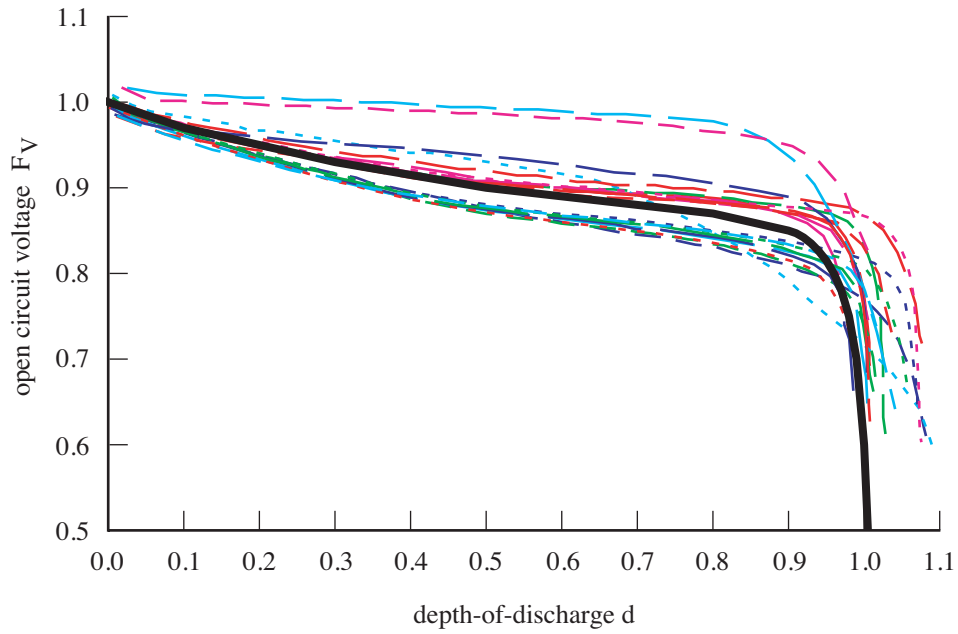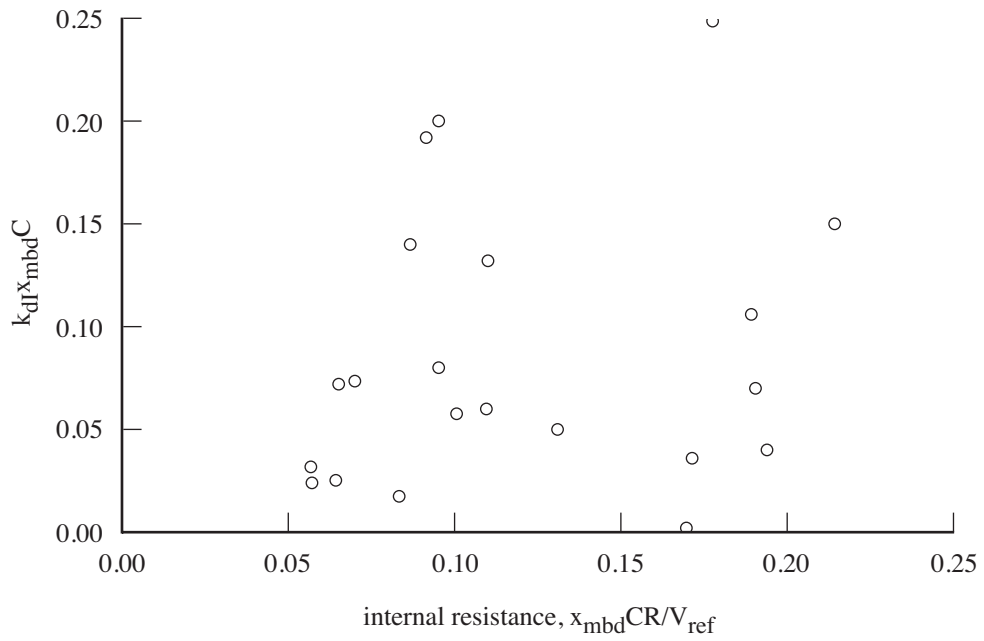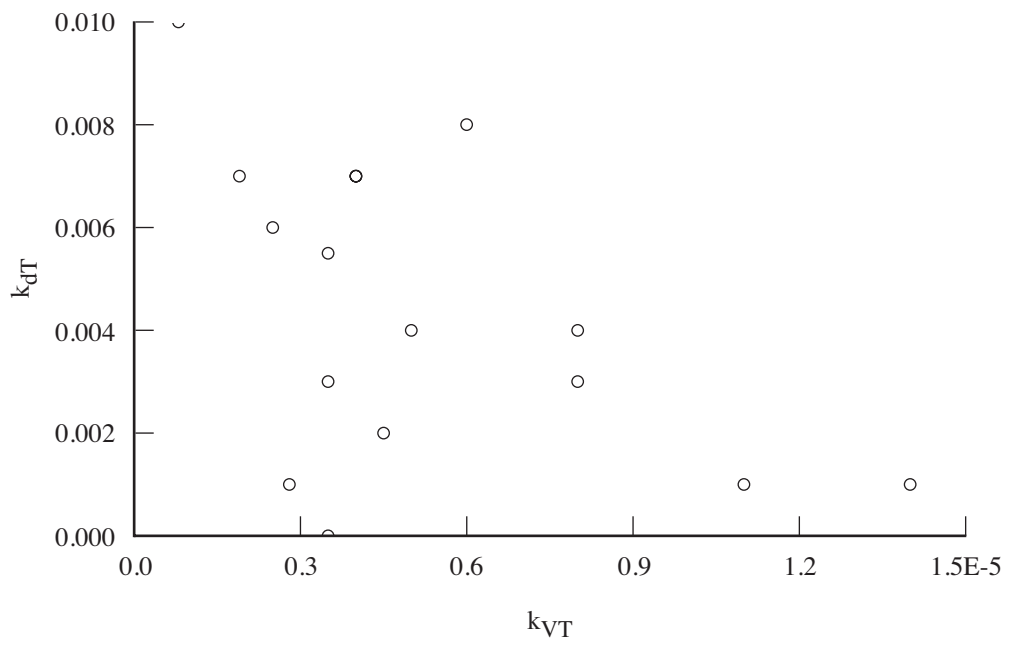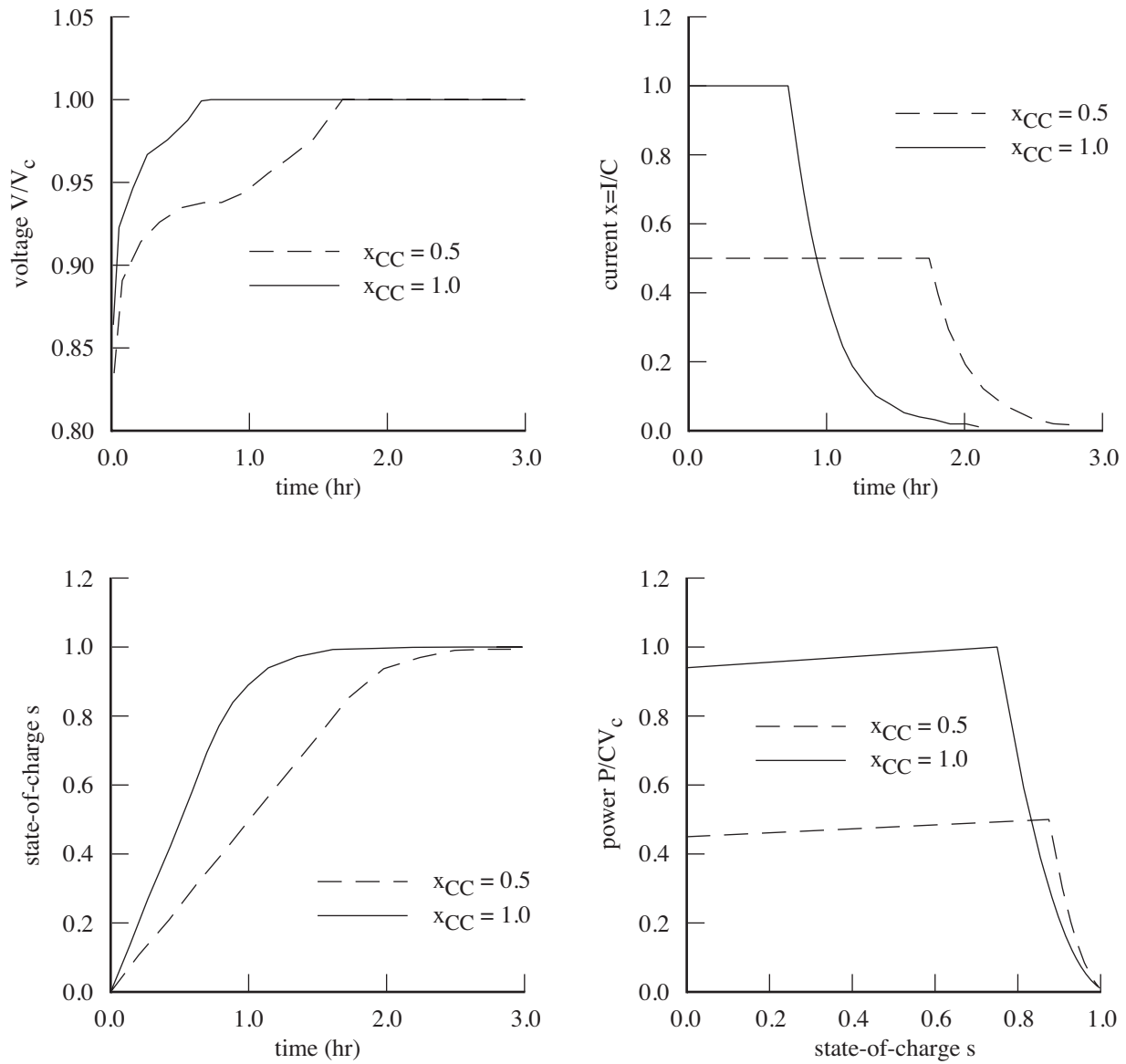
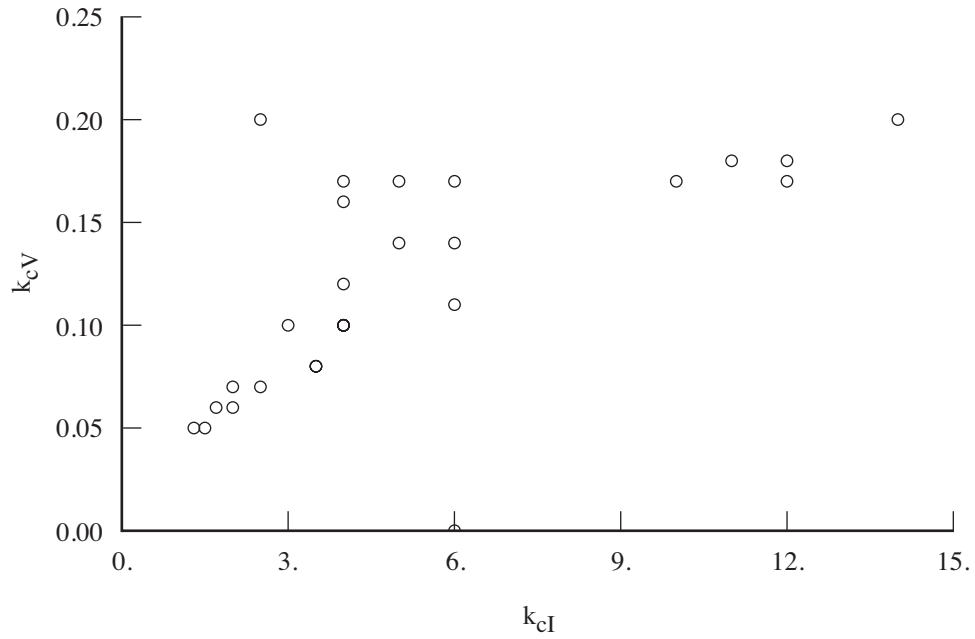Figure 28-11. Lithium-ion battery charge characteristics.

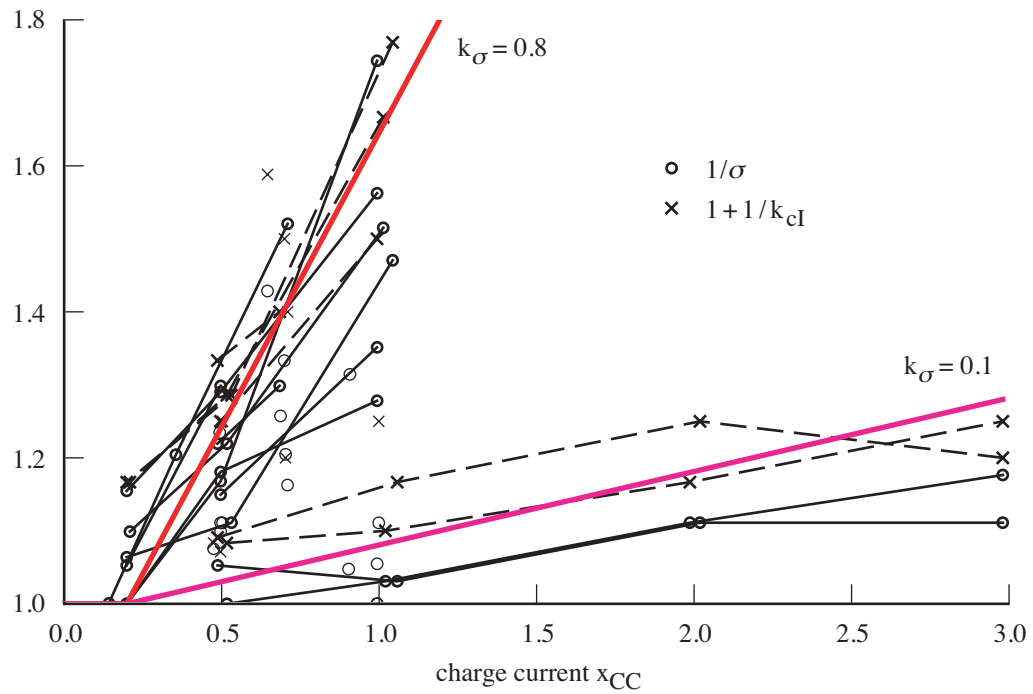Figure 28-12. Lithium-ion battery charge model parameters.

Figure 28-13. Lithium-ion battery charge model, CV parameters.

AFDD Weight Models

This chapter presents the rotorcraft weight models developed by the U.S. Army Aeroflightdynamics Directorate (AFDD). For some weight groups several models are available, designated AFDDnn. The weights are estimated from parametric equations, based on the weights of existing turbine-powered helicopters and tiltrotors. The figures of this chapter compare the weights calculated from these equations with the actual weights. The results of these equations are the weight in pounds, and the units of the parameters are noted in the tables. Technology factors χ are included in the weight equations. The input usually includes a weight increment dW that can be added to the results of the weight model. Thus typically a component or element weight is obtained from $W = \chi w_{\mathrm{model}} + dW$. Weight of individual elements in a group can be fixed by using dW and setting the corresponding technology factor $\chi = 0$. With $\chi \neq 0$, the increment dW can account for something not included in the parametric model.

The weight models are implemented as part of the aircraft components. The weights are entered into the weight statement data structure (extended RP8A format) for each component, reflected in the organization of this chapter.

29–1 Wing Group

The wing group consists of: basic structure (primary structure, consisting of torque box and spars, plus extensions); fairings (leading edge and trailing edge); fittings (non-structural); fold/tilt structure; and control surfaces (flaps, ailerons, flaperons, and spoilers). There are separate models for a tiltrotor or tiltwing configuration and for other configurations (including compound helicopter).

29-1.1 Tiltrotor or Tiltwing Wing

Wing weight equations for a tiltrotor or tiltwing aircraft are based on methodology developed by Chappell and Peyran (refs. 1 and 2). The wing is sized primarily to meet torsional stiffness requirements. The primary structure weight is calculated from torque box and spar weights:

$$W_{\mathrm{box}} = A_{tb}\rho_{tb}b_w/e_{tb}$$
$$W_{\mathrm{spar}} = C_t A_{sp}\rho_{sp}b_w/e_{sp}$$
$$w_{\mathrm{prim}} = (W_{\mathrm{box}} + W_{\mathrm{spar}})f_{\mathrm{units}}$$
$$W_{\mathrm{prim}} = \chi_{\mathrm{prim}}w_{\mathrm{prim}}$$

A consistent mass-length-time system is used in the equations for W_{box} and W_{spar}, which therefore have units of slug or kg. The primary structure weight W_{prim} however has units of lb or kg, hence a conversion factor $f_{\mathrm{units}} = g$ is required for English units. The wing fairing (leading edge and trailing edge), control

surface (flaps, ailerons, flaperons, and spoilers), fittings (non-structural), and fold/tilt weights are:

$$w_{\text{fair}} = S_{\text{fair}} U_{\text{fair}} \qquad\qquad W_{\text{fair}} = \chi_{\text{fair}} w_{\text{fair}}$$

$$w_{\text{flap}} = S_{\text{flap}} U_{\text{flap}} \qquad\qquad W_{\text{flap}} = \chi_{\text{flap}} w_{\text{flap}}$$

$$w_{\text{fit}} = f_{\text{fit}} T_{\text{cap}} \qquad\qquad W_{\text{fit}} = \chi_{\text{fit}} w_{\text{fit}}$$

$$w_{\text{fold}} = f_{\text{fold}} \left(W_{\text{prim}} + W_{\text{fair}} + W_{\text{flap}} + W_{\text{fit}} + W_{\text{tip}} \right) \qquad W_{\text{fold}} = \chi_{\text{fold}} w_{\text{fold}}$$

The control surface area S_{flap} for a tiltrotor wing is the sum of the flap and flaperon areas. The fairing area is

$$S_{\text{fair}} = (b_w - w_{\text{attach}}) \, c_w \, (1 - w_{tb}) - S_{\text{flap}}$$

The wing extension weight is:

$$w_{\text{ext}} = S_{\text{ext}} U_{\text{ext}} \qquad W_{\text{ext}} = \chi_{\text{ext}} w_{\text{ext}}$$

$$w_{\text{efold}} = f_{\text{efold}} W_{\text{ext}} \qquad W_{\text{efold}} = \chi_{\text{efold}} w_{\text{efold}}$$

and these terms are added to W_{prim} and W_{fold}. The tiltrotor wing weight (and wing folding weight in fuselage group) depends on W_{tip}, the weight on the wing tips; which is the sum of rotor group, engine section or nacelle group, air induction group, engine system, drive system (except drive shaft), rotary wing and conversion flight controls, hydraulic group, trapped fluids, and wing extensions. An adjustment of this calculated weight can be used; a negative increment is required when the engine and transmission are not at the tip location with the rotor. The weight on wing tip is used as the fraction $f_{\text{tip}} = W_{\text{tip}}/W_{SD}$; the mass on the wing tip is M_{tip} (slug or kg).

To estimate the wing weights, the required stiffness is scaled with input frequencies (per rev) of the wing primary bending and torsion modes. First the torque box is sized to meet the torsional stiffness (frequency) requirement. Next spar cap area is added as required to meet the chord and beam bending frequency requirements. Finally spar cap area is added if necessary for a jump takeoff condition. Wing section form factors, relating typical airfoil and torque box geometry to ideal shapes, are input or calculated from the thickness-to-chord ratio and the torque box chord to wing chord ratio:

$$F_B = 0.073 \sin(2\pi(\tau_w - 0.151)/0.1365) + 0.14598\tau_w$$
$$+ 0.610 \sin(2\pi(w_{tb} + 0.080)/2.1560) - (0.4126 - 1.6309\tau_w)(w_{tb} - 0.131) + 0.0081$$

$$F_C = 0.640424 w_{tb}^2 - 0.89717 w_{tb} + 0.4615\tau_w + 0.655317$$

$$F_T = ((0.27 - \tau_w)/0.12)0.12739 \left(-0.96 + \sqrt{3.32 + 94.6788 w_{tb} - (w_{tb}/0.08344)^2} \right)$$
$$- 2.7545 w_{tb}^2 + 5.1799 w_{tb} - 0.2683$$

$$F_{VH} = 0.25 \sin(5.236 w_{tb}) + 0.325$$

for beam bending, chord bending, torsion, and spar cap vertical/horizontal bending. The ideal shape for torsional stiffness is a tube of radius t_w, so the torsional stiffness $J = F_T A_{tb} t_w^2/4$. The ideal shape for chord bending is two caps c_{tb} apart, so $I_{Ctb} = F_C A_{tb} c_{tb}^2/4$. The ideal shape for beam bending is two caps t_w apart, so $I_{Bsp} = F_{VH} A_{sp} t_w^2/4$ and $I_{Btb} = F_B A_{tb} t_w^2/4$. The torque box cross-sectional area is obtained from the wing torsion frequency;

$$GJ = (\omega_T \Omega)^2 \frac{1}{2}(b_w - w_{\text{attach}}) \frac{1}{2} M_{\text{tip}} r_{\text{pylon}}^2$$
$$A_{tb} = 4GJ/(G_{tb} F_T t_w^2)$$

It is assumed that between the points of attachment to the fuselage, the torque box is very stiff in torsion, but does deflect in bending. So the effective length is $\frac{1}{2}(b_w - w_{\text{attach}})$ for torsion, and $\frac{1}{2}b_w$ for bending. The spar cap cross-sectional area (in addition to torque box material) is obtained from beam and chord bending frequencies:

$$EI_C = (\omega_C \Omega)^2 \frac{1}{24} b_w^3 \frac{1}{2} M_{\text{tip}} f_{\text{mode}}$$

$$EI_B = (\omega_B \Omega)^2 \frac{1}{24} b_w^3 \frac{1}{2} M_{\text{tip}} f_{\text{mode}}$$

$$EI_{Ctb} = E_{tb} F_C A_{tb} c_{tb}^2 / 4$$
$$EI_{Csp} = EI_C - EI_{Ctb}$$
$$A_{Csp} = EI_{Csp} / (E_{sp} c_{tb}^2 / 4)$$

$$EI_{Btb} = E_{tb} F_B A_{tb} t_w^2 / 4$$
$$EI_{VH} = E_{sp} F_{VH} A_{Csp} t_w^2 / 4$$
$$EI_{Bsp} = EI_B - EI_{Btb} - EI_{VH}$$
$$A_{Bsp} = EI_{Bsp} / (E_{sp} t_w^2 / 4)$$
$$EI_{sp} = EI_{VH} + EI_{Bsp}$$

$$A_{sp} = A_{Csp} + A_{Bsp}$$

where EI_{Csp} and EI_{Bsp} are replaced by zero if negative (no additional spar material required). The factor $f_{\text{mode}} = 1 - f_{\text{tip}}$ is a mode shape correction for fuselage motion. Next the primary structure, fairing, flap, and fitting weights are calculated as before; and the sum $W_{\text{wing}} = W_{\text{prim}} + W_{\text{fair}} + W_{\text{flap}} + W_{\text{fit}}$. Additional spar cap material for a jump takeoff condition is obtained from the ultimate applied bending moment at the wing root:

$$M_U = T_{\text{cap}} \ell_w \left(0.75(1 - f_{\text{tip}}) - 0.375(\ell_w/b_w)(W_{\text{wing}}/W_{SD}) \right)$$

where T_{cap} is the maximum thrust capability of one rotor, equal to the greater of $n_{\text{jump}} W_{SD}/N_{\text{rotor}}$ or $(C_T/\sigma)\rho A_b V_{\text{tip}}^2$ (from an input C_T/σ at the jump takeoff condition, sea level standard, and hover rotor speed). The bending moment capacity of the wing is

$$M_{tb} = 2 EI_{Btb} \epsilon_U / t_w$$
$$M_{sp} = 2 C_m EI_{sp} \epsilon_U / t_w$$

Then the additional cross-section area is obtained from the moment deficit:

$$\Delta M = M_U - (M_{tb} + M_{sp})$$
$$\Delta A_{sp} = 2\Delta M / (\epsilon_U E_{sp} t_w)$$
$$\Delta W_{\text{spar}} = C_j \Delta A_{sp} \rho_{sp} b_w / e_{sp}$$

where ΔM is replaced by zero if negative (no additional spar material required). If ΔW_{spar} is positive, it is added to W_{spar}, and the primary structure, fairing, flap, and fitting weights are recalculated. Parameters are defined in table 29-1, including units as used in these equations. Here a consistent mass-length-time system is used, producing W_{box} and W_{spar} in slug or kg. Typically the input uses conventional English units for density (lb/in^3) and modulus (lb/in^2).

Table 29-1. Parameters for tiltrotor wing weight.

parameter	definition	units
$W_{\mathrm{box}}, W_{\mathrm{spar}}$	wing torque box and spar weights	slug or kg
W_{SD}	structural design gross weight	lb
$(C_T/\sigma)_{\mathrm{jump}}$	rotor maximum thrust capability (jump takeoff)	
n_{jump}	load factor at W_{SD} (jump takeoff)	
N_{rotor}	number of rotors	
b_w	wing span (length of torque box)	ft or m
$\ell_w = b_w - w_{fus}$	wing length (span less fuselage width)	ft or m
c_w	wing chord	ft or m
w_{tb}	torque box chord to wing chord ratio	
$c_{tb} = w_{tb}c_w$	torque box chord	ft or m
τ_w	wing airfoil thickness-to-chord ratio	
$t_w = \tau_w c_w$	wing thickness	ft or m
r_{pylon}	pylon radius of gyration (pitch inertia $= r_{\mathrm{pylon}}^2 M_{\mathrm{tip}}$)	ft or m
Ω	rotor speed for wing weight design condition	rad/sec
ω_T	wing torsion mode frequency (fraction rotor speed)	per rev
ω_B	wing beam bending mode frequency (fraction rotor speed)	per rev
ω_C	wing chord bending mode frequency (fraction rotor speed)	per rev
ρ_{tb}	density of torque box material	slug/ft^3 or kg/m^3
ρ_{sp}	density of spar cap material	slug/ft^3 or kg/m^3
G_{tb}	torque box shear modulus	lb/ft^2 or N/m^2
E_{tb}	torque box modulus	lb/ft^2 or N/m^2
E_{sp}	spar modulus	lb/ft^2 or N/m^2
ϵ_U	ultimate strain allowable (minimum of spar and torque box)	
C_t	weight correction for spar taper (equivalent stiffness)	
C_j	weight correction for spar taper (equivalent strength)	
C_m	strength correction for spar taper (equivalent stiffness)	
e_{tb}	structural efficiency factor, torque box	
e_{sp}	structural efficiency factor, spars	
U_{fair}	unit weight of leading and trailing-edge fairings	lb/ft^2 or kg/m^2
U_{flap}	unit weight of control surfaces	lb/ft^2 or kg/m^2
S_{fair}	area of leading and trailing-edge fairings	ft^2 or m^2
S_{flap}	area of control surfaces	ft^2 or m^2
f_{fit}	wing fittings and brackets (fraction maximum thrust of one rotor)	
f_{fold}	wing fold/tilt (fraction total weight excluding fold, including weight on tips)	
w_{attach}	width of wing structural attachments to body	ft or m
U_{ext}	unit weight of wing extension	lb/ft^2 or kg/m^2
S_{ext}	area of wing extensions (span b_{ext} times mean chord c_{ext})	ft^2 or m^2
f_{efold}	wing extension fold/tilt (fraction extension weight)	

29-1.2   Aircraft Wing

There are two models intended for the wing of a compound helicopter: area method and parametric method. For the area method (based on weight per unit area), the total wing weight excluding folding is:

$$w_{\text{wing}} = S_w U_w$$

and $f_{\text{prim}} = 1 - f_{\text{fair}} - f_{\text{flap}} - f_{\text{fit}}$. Typically $U_w = 5$ to 9 lb/ft^2. For the parametric method (AFDD93), the total wing weight including folding is:

$$w_{\text{wing}} = 5.66411 f_{LG\text{loc}} \left(\frac{W_{SD} f_L}{1000 \cos \Lambda_w} \right)^{0.847} n_z^{0.39579} S_w^{0.21754} A_w^{0.50016}$$

$$((1 + \lambda_w)/\tau_w)^{0.09359} (1 - b_{\text{fold}})^{-0.14356}$$

with $f_{\text{prim}} = 1 - f_{\text{fair}} - f_{\text{flap}} - f_{\text{fit}} - f_{\text{fold}}$; here $f_{LG\text{loc}} = 1.7247$ if the landing gear is on the wing, and 1.0 otherwise. The lift factor f_L accounts for wing relief factor and wing/rotor lift share. Based on 25 fixed-wing aircraft, the average error of the aircraft wing equation is 3.4% (fig. 29-1). Then the primary structure, secondary structure, and control surface weights are:

$$W_{\text{prim}} = \chi_{\text{prim}} f_{\text{prim}} w_{\text{wing}}$$

$$W_{\text{fair}} = \chi_{\text{fair}} f_{\text{fair}} w_{\text{wing}}$$

$$W_{\text{flap}} = \chi_{\text{flap}} f_{\text{flap}} w_{\text{wing}}$$

$$W_{\text{fit}} = \chi_{\text{fit}} f_{\text{fit}} w_{\text{wing}}$$

$$W_{\text{fold}} = \chi_{\text{fold}} f_{\text{fold}} w_{\text{wing}}$$

The wing extension weight is:

$$w_{\text{ext}} = S_{\text{ext}} U_{\text{ext}} \qquad\qquad W_{\text{ext}} = \chi_{\text{ext}} w_{\text{ext}}$$

$$w_{\text{efold}} = f_{\text{efold}} W_{\text{ext}} \qquad\qquad W_{\text{efold}} = \chi_{\text{efold}} w_{\text{efold}}$$

and these terms are added to W_{prim} and W_{fold}. Parameters are defined in table 29-2, including units as used in these equations.

Table 29-2. Parameters for aircraft wing weight.

parameter	definition	units
S_w	wing planform area (theoretical)	ft^2
U_w	unit weight of wing planform	lb/ft^2 or kg/m^2
W_{SD}	structural design gross weight	lb
f_L	lift factor	
n_z	design ultimate flight load factor at W_{SD}	g
Λ_w	wing sweep angle	deg
A_w	wing aspect ratio	
λ_w	wing taper ratio (tip chord/root chord)	
τ_w	wing airfoil thickness-to-chord ratio	
b_{fold}	fraction wing span that folds (0 to 1)	
f_{fair}	fairings (fraction total wing weight)	
f_{flap}	control surfaces (fraction total wing weight)	
f_{fit}	fittings (fraction total wing weight)	
f_{fold}	fold/tilt (fraction total wing weight)	
U_{ext}	unit weight of wing extension	lb/ft^2 or kg/m^2
S_{ext}	area of wing extensions (span b_{ext} times mean chord c_{ext})	ft^2
f_{efold}	wing extension fold/tilt (fraction extension weight)	

29–2   Rotor Group

The rotor group consists of: blades, hub and hinge, spinner, and blade fold structure. The blade and hub-hinge weights for the AFDD82 model are:

$$w_{\text{blade}} = 0.02606 N_{\text{rotor}} N_{\text{blade}}^{0.6592} R^{1.3371} c^{0.9959} V_{\text{tip}}^{0.6682} \nu_{\text{blade}}^{2.5279} \qquad\qquad W_{\text{blade}} = \chi_{\text{blade}} w_{\text{blade}}$$

$$w_{\text{hub}} = 0.003722 N_{\text{rotor}} N_{\text{blade}}^{0.2807} R^{1.5377} V_{\text{tip}}^{0.4290} \nu_{\text{hub}}^{2.1414} (W_{\text{blade}}/N_{\text{rotor}})^{0.5505} \qquad W_{\text{hub}} = \chi_{\text{hub}} w_{\text{hub}}$$

Based on 37 aircraft, the average error of the blade equation is 7.7% (fig. 29-2). Based on 35 aircraft, the average error of the hub equation is 10.2% (fig. 29-3). The blade and hub-hinge weights for the AFDD00 model are:

$$w_{\text{blade}} = 0.0024419 f_{\text{tilt}} N_{\text{rotor}} N_{\text{blade}}^{0.53479} R^{1.74231} c^{0.77291} V_{\text{tip}}^{0.87562} \nu_{\text{blade}}^{2.51048} \qquad\qquad W_{\text{blade}} = \chi_{\text{blade}} w_{\text{blade}}$$

$$w_{\text{hub}} = 0.0061182 N_{\text{rotor}} N_{\text{blade}}^{0.20373} R^{0.60406} V_{\text{tip}}^{0.52803} \nu_{\text{hub}}^{1.00218} (W_{\text{blade}}/N_{\text{rotor}})^{0.87127} \qquad W_{\text{hub}} = \chi_{\text{hub}} w_{\text{hub}}$$

where $f_{\text{tilt}} = 1.17940$ for tilting rotors; 1.0 otherwise. Based on 51 aircraft, the average error of the blade equation is 7.9% (fig. 29-4) and the average error of the hub equation is 12.2% (fig. 29-5). This equation for hub weight was developed using the actual blade weight. Using the blade weight equation instead gives

$$w_{\text{hub}} = 0.18370 N_{\text{rotor}} N_{\text{blade}}^{0.16383} R^{0.19937} V_{\text{tip}}^{0.06171} \nu_{\text{hub}}^{0.46203} (w_{\text{blade}}/N_{\text{rotor}})^{1.02958}$$

$$= 0.00037547 f_{\text{tilt}}^{1.02958} N_{\text{rotor}} N_{\text{blade}}^{0.71443} R^{1.99321} c^{0.79577} V_{\text{tip}}^{0.96323} \nu_{\text{hub}}^{0.46203} \nu_{\text{blade}}^{2.58473}$$

which really just adds chord and f_{tilt} to the regression parameters. Based on 51 aircraft, the average error of this equation is 9.2% (fig. 29-5). Thus using the blade weight equation results in a lower average error, and best represents legacy rotor systems. The hub weight equation using the actual blade weight is best for advanced technology rotors with blades lighter than trend. For teetering and gimballed rotors, the flap frequency ν should be the coning frequency. If the weight is evaluated separately for each rotor, then $N_{\text{rotor}} = 1$ should be used in the equations.

The fairing/spinner and blade fold weights are:

$$W_{\text{spin}} = \chi_{\text{spin}} \, 7.386 N_{\text{rotor}} D_{\text{spin}}^2$$

$$W_{\text{fold}} = \chi_{\text{fold}} \, f_{\text{fold}} W_{\text{blade}}$$

The blade weight is for all blades of the rotors. Typically $f_{\text{fold}} = 0.04$ for manual fold and $f_{\text{fold}} = 0.28$ for automatic fold. The rotor shaft, rotor support, and duct weights are:

$$W_{\text{shaft}} = \chi_{\text{shaft}} N_{\text{rotor}} \ell_{\text{shaft}} U_{\text{shaft}}$$

$$W_{\text{supt}} = \chi_{\text{supt}} \left(f_{\text{supt}} W_{MTO} + U_{\text{supt}} R \right)$$

$$W_{\text{duct}} = \chi_{\text{duct}} N_{\text{rotor}} S_{\text{duct}} U_{\text{duct}}$$

The rotor shaft length is $\ell_{\text{shaft}} = f_{\text{shaft}} R$. Parameters are defined in table 29-3, including units as used in these equations.

Table 29-3. Parameters for rotor weight.

parameter	definition	units
N_{rotor}	number of rotors	
N_{blade}	number of blades per rotor	
R	rotor radius in hover	ft
c	rotor mean geometric blade chord	ft
V_{tip}	rotor hover tip velocity	ft/sec
$\nu_{\text{blade}}, \nu_{\text{hub}}$	flap natural frequency (for weight estimate)	per rev
D_{spin}	spinner diameter	ft
f_{fold}	blade fold weight (fraction total blade weight)	
W_{MTO}	maximum takeoff weight	lb
f_{shaft}	rotor shaft length (fraction rotor radius)	
U_{shaft}	unit weight of rotor shaft	lb/ft or kg/m
f_{supt}	rotor support structure weight (fraction maximum takeoff weight)	
U_{supt}	unit weight of rotor support structure	lb/ft or kg/m
S_{duct}	area of duct	ft^2 or m^2
U_{duct}	unit weight of duct	lb/ft^2 or kg/m^2

29-2.1   Lift-Offset Rotor

For lift-offset rotors, the blade and hub weights can be calculated based on the methodology of reference 3. The blade weight is estimated based on the beam stiffness required to maintain the clearance s when the blade is loaded by the lift offset. The blade tip deflection is proportional to $\delta \propto PR^3/EI$, where EI is the bending stiffness. The beam loading is $P \propto n_z W_{SD} L / N_{\text{blade}}$. With A_{xs} the blade cross-section area, the moment of inertia $I \propto A_{xs} t^2$. The criterion is $\delta = h - s$. Hence the blade weight is $W_{\text{blade}} \propto \rho N_{\text{blade}} R A_{xs} \propto (\rho/E) n_z W_{SD} L R^4 / (t^2 (h-s))$ with E the elastic modulus, and ρ here the material density. The hub weight is estimated based on the structure in upper and lower hub plates required to react a tensile force $F = C_{\text{cent}} + M_{\text{bend}}/(x/2)$ due to combined centrifugal force and bending moment at the root; where the hub plate separation x scales with the blade thickness t. The centrifugal force $C_{\text{cent}} \propto (W_{\text{blade}}/N_{\text{blade}}) V_{\text{tip}}^2 / R$. The bending moment $M_{\text{bend}} \propto n_z W_{SD} R$. The limit tensile stress $\sigma \propto F/A_{xs}$ gives a criterion for the total hub arm area A_{xs}. The radius of the hub ℓ scales with the blade thickness t. Hence the hub weight is $W_{\text{hub}} \propto \rho N_{\text{blade}} \ell A_{xs} \propto (\rho/\sigma)\left(W_{\text{blade}} V_{\text{tip}}^2 t/R + K n_z W_{SD} R N_{\text{blade}}\right)$. The distribution factor K is determined from the calibration weights. The inter-rotor shaft weight is estimated based on the structure to react the hub moment caused by lift offset. The hub moment $M \propto n_z W_{SD} L R$. The shaft diameter d scales with the blade thickness t. The shaft length ℓ scales with the rotor separation h. The ultimate bending stress $\sigma = M/(I/c)$ gives a criterion for the area moment $I/c \propto d^2 w$, hence for the shaft wall thickness w. Then $W_{\text{shaft}} \propto \rho \ell d w \propto (\rho/\sigma) n_z W_{SD} L R h/t$ is the shaft weight.

The blade, hub-hinge, and inter-rotor shaft weights are:

$$w_{\text{blade}} = N_{\text{rotor}}\, 0.000083770\, wL\, R^3/(2(h-s)t_{.2R}^2) \qquad\qquad W_{\text{blade}} = \chi_{\text{blade}} w_{\text{blade}}$$

$$w_{\text{hub}} = N_{\text{rotor}} \left(0.17153\, w\, R N_{\text{blade}} + 0.000010534(W_{\text{blade}}/N_{\text{rotor}})V_{\text{tip}}^2 t_{.2R}/R \right) \qquad W_{\text{hub}} = \chi_{\text{hub}} w_{\text{hub}}$$

$$w_{\text{shaft}} = N_{\text{rotor}}\, 0.081304\, wL\, R^2 2h/t_{.2R} \qquad\qquad W_{\text{shaft}} = \chi_{\text{shaft}} w_{\text{shaft}}$$

where $w = n_z W_{SD}/1000$, and

$$t_{.2R} = \tau_{.2R} c \left(\frac{0.8 + 0.2\lambda}{0.5 + 0.5\lambda} \right)$$

is the blade thickness at $20\%R$. These equations were developed for the coaxial rotor configuration ($N_{\mathrm{rotor}} = 2$), and calibrated to the XH-59A weights. The material factors (ρ/E and ρ/σ) are included in the technology factors χ. These weight terms can be used separately, with other hub and blade and shaft weight models. Parameters are defined in table 29-4, including units as used in these equations.

Table 29-4. Parameters for lift-offset rotor weight.

parameter	definition	units
N_{rotor}	number of rotors	
N_{blade}	number of blades per rotor	
W_{SD}	structural design gross weight	lb
n_z	design ultimate flight load factor at W_{SD}	g
λ	blade taper ratio (tip chord/root chord)	
$\tau_{.2R}$	blade airfoil thickness-to-chord ratio (at $20\%R$)	
R	rotor radius	ft
c	blade mean chord	ft
h	coaxial rotor separation (fraction rotor diameter)	
s	coaxial rotor tip clearance (fraction rotor diameter)	
L	lift offset (M_{roll}/TR)	
V_{tip}	rotor hover tip velocity	ft/sec

29–3   Empennage Group

The empennage group consists of: horizontal tail, vertical tail, and tail-rotor. The tail plane weight consists of the basic structure and fold structure. There are two models for tail plane basic weight: area method and parametric method. For the area method (based on weight per unit area), the weight is $W_{\mathrm{tail}} = S_{\mathrm{tail}}U_{\mathrm{tail}}$. The parametric weight model depends on the aircraft configuration. The helicopter or compound model is AFDD82. The horizontal tail weight is:

$$\text{tiltrotor or tiltwing} \qquad W_{ht} = \chi_{ht}S_{ht}(0.00395S_{ht}^{0.2}V_{\mathrm{dive}} - 0.4885)$$

$$\text{helicopter or compound} \qquad W_{ht} = \chi_{ht}0.7176S_{ht}^{1.1881}A_{ht}^{0.3173}$$

Based on 13 aircraft, the average error of the helicopter horizontal tail equation is 22.4% (fig. 29-6). The vertical tail weight is:

$$\text{tiltrotor or tiltwing} \qquad W_{vt} = \chi_{vt}S_{vt}(0.00395S_{vt}^{0.2}V_{\mathrm{dive}} - 0.4885)$$

$$\text{helicopter or compound} \qquad W_{vt} = \chi_{vt}1.0460f_{\mathrm{tr}}S_{vt}^{0.9441}A_{vt}^{0.5332}$$

where $f_{\mathrm{tr}} = 1.6311$ if the tail-rotor is located on the vertical tail; 1.0 otherwise. Based on 12 aircraft, the average error of the helicopter vertical tail equation is 23.3% (fig. 29-7). V_{dive} is the design dive speed, calculated or input; $V_{\mathrm{dive}} = 1.25V_{\mathrm{max}}$, where V_{max} is the maximum speed at design gross weight and sea level standard conditions. The fold weight is a fraction of the basic weight: $W_{\mathrm{fold}} = \chi_{\mathrm{fold}}f_{\mathrm{fold}}W_{\mathrm{basic}}$, where $W_{\mathrm{basic}} = W_{ht}$ or W_{vt}.

The tail-rotor weight is:

$$W_{tr} = \chi_{tr}1.3778R_{tr}^{0.0897}(P_{DS\mathrm{limit}}R/V_{\mathrm{tip}})^{0.8951}$$

Based on 19 aircraft, the average error of the helicopter tail-rotor equation is 16.7% (fig. 29-8). The tail-rotor weight is calculated by the rotor component model, including rotor support and duct weights. Rotor weight models can be used for the tail-rotor. Parameters are defined in table 29-5, including units as used in these equations.

Table 29-5. Parameters for tail weight.

parameter	definition	units
S_{ht}	horizontal tail planform area	ft^2
S_{vt}	vertical tail planform area	ft^2
A_{ht}	horizontal tail aspect ratio	
A_{vt}	vertical tail aspect ratio	
V_{dive}	design dive speed at sea level	kts
R_{tr}	tail-rotor radius	ft
$P_{DS\text{limit}}$	drive system power limit (MCP)	hp
R	main-rotor radius in hover	ft
V_{tip}	main-rotor hover tip velocity	ft/sec
f_{fold}	fold (fraction basic weight excluding fold)	
S_{tail}	horizontal or vertical tail planform area	ft^2 or m^2
U_{tail}	unit weight of tail planform	lb/ft^2 or kg/m^2

29–4   Fuselage Group

The fuselage group consists of: basic structure; wing and rotor fold/retraction; tail fold/tilt; and marinization, pressurization, and crashworthiness structure. The AFDD84 model is a universal body weight equation, used for tiltrotor and tiltwing as well as for helicopter configurations. The AFDD82 model is a helicopter body weight equation, not used for tiltrotor or tiltwing configuration.

For the AFDD84 (UNIV) model, the basic structure weight is

$$w_{\text{basic}} = 25.41 f_{LG\text{loc}} f_{LG\text{ret}} f_{\text{ramp}} \left(\frac{W_{MTO}}{1000} \right)^{0.4879} \left(\frac{n_z W_{SD}}{1000} \right)^{0.2075} S_{\text{body}}^{0.1676} \ell^{0.1512}$$

$$W_{\text{basic}} = \chi_{\text{basic}} w_{\text{basic}}$$

where $f_{LG\text{loc}} = 1.1627$ if the landing gear is located on the fuselage, and 1.0 otherwise; $f_{LG\text{ret}} = 1.1437$ if the landing gear is on the fuselage and retractable, and 1.0 otherwise; $f_{\text{ramp}} = 1.2749$ if there is a cargo ramp, and 1.0 otherwise. Based on 35 aircraft, the average error of the body equation is 6.5% (fig. 29-9). The tail fold, wing and rotor fold, marinization, pressurization, and crashworthiness weights are:

$$w_{\text{tfold}} = f_{\text{tfold}} W_{\text{tail}} \qquad\qquad W_{\text{tfold}} = \chi_{\text{tfold}} w_{\text{tfold}}$$

$$w_{\text{wfold}} = f_{\text{wfold}} (W_{\text{wing}} + W_{\text{tip}}) \qquad\qquad W_{\text{wfold}} = \chi_{\text{wfold}} w_{\text{wfold}}$$

$$w_{\text{mar}} = f_{\text{mar}} W_{\text{basic}} \qquad\qquad W_{\text{mar}} = \chi_{\text{mar}} w_{\text{mar}}$$

$$w_{\text{press}} = f_{\text{press}} W_{\text{basic}} \qquad\qquad W_{\text{press}} = \chi_{\text{press}} w_{\text{press}}$$

$$w_{\text{cw}} = f_{\text{cw}} (W_{\text{basic}} + W_{\text{tfold}} + W_{\text{wfold}} + W_{\text{mar}} + W_{\text{press}}) \qquad\qquad W_{\text{cw}} = \chi_{\text{cw}} w_{\text{cw}}$$

Typically $f_{\text{tfold}} = 0.30$ for a folding tail, and $f_{\text{cw}} = 0.06$. For wing folding the weight on the wing tip (W_{tip}) is required (calculated as for the wing group). Parameters are defined in table 29-6, including units as used in these equations.

Table 29-6. Parameters for fuselage weight (AFDD84 model).

parameter	definition	units
W_{MTO}	maximum takeoff weight	lb
W_{SD}	structural design gross weight	lb
S_{body}	wetted area of body	ft^2
R	main-rotor radius	ft
n_z	design ultimate flight load factor at W_{SD}	g
ℓ	length of fuselage	ft
f_{tfold}	tail fold weight (fraction tail weight)	
f_{wfold}	wing and rotor fold weight (fraction wing/tip weight)	
W_{tail}	tail group weight	
$W_{\text{wing}} + W_{\text{tip}}$	wing group weight plus weight on wing tip	
f_{mar}	marinization weight (fraction basic body weight)	
f_{press}	pressurization (fraction basic body weight)	
f_{cw}	crashworthiness weight (fraction fuselage weight)	

For the AFDD82 (HELO) model, the basic structure weight is

$$w_{\text{basic}} = 5.896 f_{\text{ramp}} \left(\frac{W_{MTO}}{1000} \right)^{0.4908} n_z^{0.1323} S_{\text{body}}^{0.2544} \ell^{0.6100}$$

$$W_{\text{basic}} = \chi_{\text{basic}} w_{\text{basic}}$$

where $f_{\text{ramp}} = 1.3939$ if there is a cargo ramp, and 1.0 otherwise. Based on 30 aircraft, the average error of the body equation is 8.7% (fig. 29-10). The tail fold, wing and rotor fold, marinization, pressurization, and crashworthiness weights are:

$$w_{\text{tfold}} = f_{\text{tfold}} W_{\text{basic}} \qquad\qquad\qquad\qquad W_{\text{tfold}} = \chi_{\text{tfold}} w_{\text{tfold}}$$

$$w_{\text{wfold}} = f_{\text{wfold}}(W_{\text{basic}} + W_{\text{tfold}}) \qquad\qquad W_{\text{wfold}} = \chi_{\text{wfold}} w_{\text{wfold}}$$

$$w_{\text{mar}} = f_{\text{mar}} W_{\text{basic}} \qquad\qquad\qquad\qquad W_{\text{mar}} = \chi_{\text{mar}} w_{\text{mar}}$$

$$w_{\text{press}} = f_{\text{press}} W_{\text{basic}} \qquad\qquad\qquad\qquad W_{\text{press}} = \chi_{\text{press}} w_{\text{press}}$$

$$w_{\text{cw}} = f_{\text{cw}}(W_{\text{basic}} + W_{\text{tfold}} + W_{\text{wfold}} + W_{\text{mar}} + W_{\text{press}}) \qquad W_{\text{cw}} = \chi_{\text{cw}} w_{\text{cw}}$$

Typically $f_{\text{tfold}} = 0.05$ for a folding tail, and $f_{\text{cw}} = 0.06$. Parameters are defined in table 29-7, including units as used in these equations.

Table 29-7. Parameters for fuselage weight (AFDD82 model).

parameter	definition	units
W_{MTO}	maximum takeoff weight	lb
S_{body}	wetted area of body	ft^2
R	main-rotor radius	ft
n_z	design ultimate flight load factor at W_{SD}	g
ℓ	length of fuselage	ft
f_{tfold}	tail fold weight (fraction basic structure)	
f_{wfold}	wing and rotor fold weight (fraction basic structure and tail fold)	
f_{mar}	marinization weight (fraction basic body weight)	
f_{press}	pressurization (fraction basic body weight)	
f_{cw}	crashworthiness weight (fraction fuselage weight)	

29–5   Alighting Gear Group

The alighting gear group consists of: basic structure, retraction, and crashworthiness structure. There are two models, parametric (AFDD82) and fractional. The basic landing gear weight is:

$$\text{parametric} \qquad w_{LG} = 0.4013 W_{MTO}^{0.6662} N_{LG}^{0.5360} (W/S)^{0.1525}$$

$$\text{fractional} \qquad w_{LG} = f_{LG} W_{MTO}$$

and $W_{LG} = \chi_{LG} w_{LG}$. Typically $f_{LG} = 0.0325$ for wheeled gear, or $f_{LG} = 0.014$ for skid gear (fractional method). Based on 28 aircraft (wheeled gear), the average error of the parametric equation is 8.4% (fig. 29-11), 7.4% for rotorcraft only or 10.2% for fixed wing aircraft only. The retraction and crashworthiness weights are:

$$w_{LG\text{ret}} = f_{LG\text{ret}} W_{LG} \qquad\qquad W_{LG\text{ret}} = \chi_{LG\text{ret}} w_{LG\text{ret}}$$

$$w_{LG\text{cw}} = f_{LG\text{cw}} (W_{LG} + W_{LG\text{ret}}) \qquad W_{LG\text{cw}} = \chi_{LG\text{cw}} w_{LG\text{cw}}$$

Typically $f_{LG\text{ret}} = 0.08$, and $f_{LG\text{cw}} = 0.14$. Parameters are defined in table 29-8, including units as used in these equations.

Table 29-8. Parameters for landing gear weight.

parameter	definition	units
W_{MTO}	maximum takeoff weight	lb
f_{LG}	landing gear weight (fraction maximum takeoff weight)	
W/S	wing loading (1.0 for helicopter)	lb/ft^2
N_{LG}	number of landing gear assemblies	
$f_{LG\text{ret}}$	retraction weight (fraction basic weight)	
$f_{LG\text{cw}}$	crashworthiness weight (fraction basic and retraction weight)	

29–6   Engine Section or Nacelle Group and Air Induction Group

The engine section or nacelle group consists of: engine support structure, engine cowling, and pylon support structure. The weights (AFDD82 model) are:

$$W_{\text{supt}} = \chi_{\text{supt}} 0.0412 (1 - f_{\text{airind}}) (W_{\text{eng}}/N_{\text{eng}})^{1.1433} N_{\text{eng}}^{1.3762}$$

$$W_{\text{cowl}} = \chi_{\text{cowl}} 0.2315 S_{\text{nac}}^{1.3476}$$

$$W_{\text{pylon}} = \chi_{\text{pylon}} f_{\text{pylon}} W_{MTO}$$

Based on 12 aircraft, the average error of the engine support equation is 11.0% (fig. 29-12). Based on 12 aircraft, the average error of the engine cowling equation is 17.9% (fig. 29-13). The air induction group weight (AFDD82 model) is:

$$W_{\text{airind}} = \chi_{\text{airind}} 0.0412 f_{\text{airind}} (W_{\text{eng}}/N_{\text{eng}})^{1.1433} N_{\text{eng}}^{1.3762}$$

Typically $f_{\text{airind}} = 0.3$ (range 0.1 to 0.6). Based on 12 aircraft, the average error of the air induction equation is 11.0% (fig. 29-14).

Alternatively, the nacelle group weight (including support, cowling, and pylon) scales with power or thrust, and the air induction group weight scales with the nacelle area:

$$\text{engine group:} \qquad\qquad \text{jet group:}$$

$$W_{\text{cowl}} = \chi_{\text{cowl}} K_{\text{nac}} P^{X_{\text{nac}}} \qquad W_{\text{cowl}} = \chi_{\text{cowl}} K_{\text{nac}} T^{X_{\text{nac}}}$$

$$W_{\mathrm{airind}} = \chi_{\mathrm{airind}} S_{\mathrm{nac}} U_{\mathrm{airind}}$$

Parameters are defined in table 29-9, including units as used in these equations.

Table 29-9. Parameters for engine section, nacelle, and air induction weight.

parameter	definition	units
W_{MTO}	maximum takeoff weight	lb
W_{eng}	weight all main engines	lb
N_{eng}	number of main engines	
S_{nac}	wetted area of nacelles and pylon (less spinner)	ft^2
f_{airind}	air induction weight (fraction nacelle plus air induction)	
f_{pylon}	pylon support structure weight (fraction W_{MTO})	
P	engine group power (SLS static, specified rating)	hp
T	jet group thrust (SLS static, specified rating)	lb
U_{airind}	unit weight of air induction group	lb/ft^2 or kg/m^2

29–7   Propulsion Group

The propulsion group consists of the engine system, fuel system, and drive system.

29-7.1   Engine System

The engine system consists of the main engines, the engine exhaust system, and the engine accessories. The engine system weights are:

engine group: jet group:

$$W_{\mathrm{eng}} = \chi_{\mathrm{eng}} N_{\mathrm{eng}} W_{\mathrm{one\ eng}} \qquad\qquad W_{\mathrm{eng}} = \chi_{\mathrm{jet}} N_{\mathrm{jet}} W_{\mathrm{one\ jet}}$$

$$W_{\mathrm{exh}} = \chi_{\mathrm{exh}} N_{\mathrm{eng}} (K_{0\mathrm{exh}} + K_{1\mathrm{exh}} P) \qquad W_{\mathrm{exh}} = \chi_{\mathrm{exh}} N_{\mathrm{jet}} (K_{0\mathrm{exh}} + K_{1\mathrm{exh}} T)$$

$$W_{\mathrm{acc}} = \chi_{\mathrm{acc}} 2.0088\, f_{\mathrm{lub}} (W_{\mathrm{eng}}/N_{\mathrm{eng}})^{0.5919} N_{\mathrm{eng}}^{0.7858}$$

where $f_{\mathrm{lub}} = 1.4799$ if the accessory weight includes the lubrication system weight, 1.0 if the lubrication system weight is in the engine weight. The exhaust system weight is per engine, including any infrared suppressor. The accessory weight equation is the AFDD82 model. Based on 16 aircraft, the average error of the accessories equation is 11.5% (fig. 29-15). The engine system weight $W_{ES} = W_{\mathrm{eng}} + W_{\mathrm{exh}} + W_{\mathrm{acc}}$. Parameters are defined in table 29-10, including units as used in these equations.

Table 29-10. Parameters for engine system weight.

parameter	definition	units
N_{eng}	number of main engines	
N_{jet}	number of jets	
P	installed takeoff power (SLS static, specified rating) per engine	hp
T	installed takeoff thrust (SLS static, specified rating) per jet	lb
$K_{0\mathrm{exh}}, K_{1\mathrm{exh}}$	engine exhaust weight vs. power or thrust, constants	

29-7.2   Propeller/Fan Installation

The auxiliary propulsion or propeller weight is:

AFDD82 $\qquad W_{\text{at}} = \chi_{\text{at}} 0.0809484 N_{\text{at}} T_{\text{at}}^{1.04771} (T_{\text{at}}/A_{\text{at}})^{-0.07821}$

AFDD10 $\qquad W_{\text{at}} = \chi_{\text{at}} 9.90350 N_{\text{at}} P_{\text{at}}^{0.91996} N_{\text{blade}}^{-0.48578} \Omega_{\text{prop}}^{-0.45904} D_{\text{prop}}^{0.15690} f_m$

where T_{at} is at maximum speed, design gross weight, and SLS conditions, calculated or input. The material factor $f_m = 1$ for composite construction; 1.20 for wood; 1.31 for aluminum spar; and 1.44 for aluminum construction. Based on 16 aircraft, the average error of the AFDD10 equation is 10.5%. The propeller weight is calculated by the rotor component model, including rotor support and duct weights. Rotor weight models can be used for the propeller. Parameters are defined in table 29-11, including units as used in these equations.

Table 29-11. Parameters for propeller weight.

parameter	definition	units
N_{at}	number of auxiliary thrusters	
T_{at}	thrust per propeller	lb
A_{at}	auxiliary thruster disk area	ft^2
P_{at}	power per propeller	hp
N_{blade}	number of blades per propeller	
Ω_{prop}	propeller rotation speed at P_{at}	rpm
D_{prop}	propeller diameter	ft

29-7.3   Fuel System

The fuel system consists of tanks and support structure (including fuel tanks, bladders, supporting structure, filler caps, tank covers, and filler material for void and ullage), and fuel plumbing (including fuel system weight not covered by tank weight). There are two models for tank weight, parametric (AFDD82) and fractional. The fuel system weights (AFDD82 model) are:

$$W_{\text{tank}} = \chi_{\text{tank}} 0.4341 C_{\text{int}}^{0.7717} N_{\text{int}}^{0.5897} f_{\text{cw}} f_{\text{bt}}^{1.9491}$$

$$W_{\text{plumb}} = \chi_{\text{plumb}} \left[K_{0\text{plumb}} + K_{1\text{plumb}}(0.01 N_{\text{plumb}} + 0.06 N_{\text{eng}})(F/N_{\text{eng}})^{0.866} \right]$$

where $f_{\text{cw}} = 1.3131$ for ballistically survivable (UTTAS/AAH level) and 1.0 otherwise. The ballistic tolerance factor $f_{\text{bt}} = 1.0$ to 2.5. Based on 15 aircraft, the average error of the fuel tank equation is 4.6% (fig. 29-16). The fuel flow rate F is calculated for the takeoff power rating at static SLS conditions. $K_{1\text{plumb}}$ is a crashworthiness and survivability factor; typically $K_{1\text{plumb}} = 2$. $K_{0\text{plumb}}$ is the sum of weights for auxiliary fuel (typically up to 120 lb), in-flight refueling (up to 150 lb), pressure refueling (up to 150 lb), inerting system (up to 20 lb), etc.; typically $K_{0\text{plumb}} = 50$ to 250 lb. Alternatively, for the fractional model the fuel tank weight is $W_{\text{tank}} = \chi_{\text{tank}} f_{\text{tank}} W_{\text{fuel}-\text{cap}}$. The fractional model for the fuel plumbing weight is $W_{\text{plumb}} = \chi_{\text{plumb}}(W_{\text{tank}}/\chi_{\text{tank}}) f_{\text{plumb}}/(1 - f_{\text{plumb}})$. Parameters are defined in table 29-12, including units as used in these equations.

Table 29-12. Parameters for fuel system weight.

parameter	definition	units
N_{int}	number of internal fuel tanks	
C_{int}	internal fuel tank capacity	gallons
f_{bt}	ballistic tolerance factor	
N_{plumb}	total number of fuel tanks (internal and auxiliary) for plumbing	
N_{eng}	number of main engines	
$K_{0\text{plumb}}, K_{1\text{plumb}}$	plumbing weight, constants	lb
F	fuel flow rate	lb/hr
f_{tank}	fuel tank weight (fraction fuel capacity)	
f_{plumb}	fuel plumbing weight (fraction total fuel system weight)	

29-7.4   Drive System

The drive system consists of gear boxes and rotor shafts, drive shafts, and rotor brake. This distribution of drive system weights is based on the following functional definitions. Gearboxes are parts of the drive system that transmit power by gear trains, and the structure that encloses them. Rotor shafts are the structure (typically a shaft) that transmits power to the rotor. Drive shafts are the structure (typically a shaft) that transmits power in the propulsion system, but not directly to the rotor or by a gear train. The rotor brake weight encompasses components that can prevent the rotor from freely turning. The gear box and rotor shaft weights for the AFDD83 model are:

$$w_{gbrs} = 57.72 P_{DS\text{limit}}^{0.8195} f_Q^{0.0680} N_{\text{gb}}^{0.0663} (\Omega_{\text{eng}}/1000)^{0.0369} / \Omega_{\text{rotor}}^{0.6379}$$

$$W_{gb} = \chi_{gb}(1 - f_{rs})w_{gbrs}$$

$$W_{rs} = \chi_{rs} f_{rs} w_{gbrs}$$

Based on 30 aircraft, the average error of the gear box and rotor shaft equation is 7.7% (fig. 29-17). The gear box and rotor shaft weights for the AFDD00 model are:

$$w_{gbrs} = 95.7634 N_{\text{rotor}}^{0.38553} P_{DS\text{limit}}^{0.78137} \Omega_{\text{eng}}^{0.09899} / \Omega_{\text{rotor}}^{0.80686}$$

$$W_{gb} = \chi_{gb}(1 - f_{rs})w_{gbrs}$$

$$W_{rs} = \chi_{rs} f_{rs} w_{gbrs}$$

Based on 52 aircraft, the average error of the gear box and rotor shaft equation is 8.6% (fig. 29-18). Typically $f_{rs} = 0.13$ (range 0.06 to 0.20). Parameters are defined in table 29-13, including units as used in these equations.

The drive shaft (AFDD82 model) and rotor brake weights are:

$$W_{ds} = \chi_{ds} 1.166 Q_{DS\text{limit}}^{0.3828} x_{\text{hub}}^{1.0455} N_{\text{ds}}^{0.3909} (0.01 f_P)^{0.2693}$$

$$W_{rb} = \chi_{rb} 0.000871 W_{\text{blade}} (0.01 V_{\text{tip}})^2$$

where $f_P = f_Q \Omega_{\text{other}}/\Omega_{\text{main}}$. Based on 28 aircraft, the average error of the drive shaft equation is 16.0% (fig. 29-19). Based on 23 aircraft, the average error of the rotor brake equation is 25.1% (fig. 29-20). The clutch weight in the weight statement is associated with an auxiliary power unit, and is a fixed input

value. The conventional rotor drive system clutch and free wheeling device weights are included in the gear box and rotor shaft weight equations. Parameters are defined in table 29-14, including units as used in these equations.

Typically $f_P = f_Q = 60\%$ for twin main-rotors (tandem, coaxial, and tiltrotor); for a single main-rotor and tail-rotor, $f_Q = 3\%$ and $f_P = 15\%$ (18% for 2-bladed rotors).

Table 29-13. Parameters for drive system weight.

parameter	definition	units
$P_{DS\text{limit}}$	drive system power limit (MCP)	hp
N_{rotor}	number of main-rotors	
N_{gb}	number of gear boxes	
Ω_{rotor}	main-rotor rotation speed	rpm
Ω_{eng}	engine output speed	rpm
f_Q	second (main or tail) rotor torque limit*	%
f_{rs}	rotor shaft weight (fraction gear box and rotor shaft)	

*input as fraction of total drive system torque limit

Table 29-14. Parameters for drive shaft and rotor brake weight.

parameter	definition	units
$Q_{DS\text{limit}}$	$P_{DS\text{limit}}/\Omega_{\text{rotor}}$ (MCP)	hp/rpm
N_{ds}	number of intermediate drive shafts	
x_{hub}	length of drive shaft between rotors	ft
f_P	second (main or tail) rotor power limit*	%
V_{tip}	main-rotor tip speed	ft/sec

*input as fraction of total drive system power limit

29–8   Flight Controls Group

The flight controls group consists of cockpit controls, automatic flight control system, and system controls. System controls consist of fixed wing flight controls, rotary wing flight controls, and conversion (rotor tilt) flight controls. The weight equations model separately non-boosted controls (which do not see aerodynamic surface or rotor loads), boost mechanisms (actuators), and boosted controls (which are affected by aerodynamic surface or rotor loads). The load path goes from pilot, to cockpit controls, to non-boosted controls, to boost mechanisms, to boosted controls, and finally to the component.

The cockpit controls weight W_{cc} is fixed (input), or scaled with design gross weight: $W_{cc} = K_{cc}W_D^{X_{cc}}$. The automatic flight control system weight W_{afcs} is fixed (input).

Fixed wing flight controls consist of non-boosted flight controls and flight control boost mechanisms. The weights are:

$$\text{full controls} \qquad w = 0.91000 W_{MTO}^{0.6}$$
$$\text{only stabilizer controls} \qquad w = 0.01735 W_{MTO}^{0.64345} S_{ht}^{0.40952}$$

and then

$$W_{FWnb} = \chi_{FWnb} f_{FWnb}\, w$$
$$W_{FWmb} = \chi_{FWmb}(1 - f_{FWnb})\, w$$

For a helicopter, the stabilizer control equation is used. Parameters are defined in table 29-15, including units as used in these equations.

Table 29-15. Parameters for fixed wing flight control weight.

parameter	definition	units
S_{ht}	horizontal tail planform area	ft^2
W_{MTO}	maximum takeoff weight	lb
f_{FWnb}	fixed wing non-boosted weight	
	(fraction total fixed wing flight control weight)	

Rotary wing flight controls consist of non-boosted flight controls, flight control boost mechanisms, and boosted flight controls. The non-boosted flight control weight (AFDD82 model) is:

$$\text{fraction method} \qquad W_{RWnb} = \chi_{RWnb} f_{RWnb}(1 - f_{RW\text{hyd}})w_{fc}$$

$$\text{parametric method} \qquad W_{RWnb} = \chi_{RWnb} 2.1785 f_{nbsv} W_{MTO}^{0.3999} N_{\text{rotor}}^{1.3855}$$

where $f_{nbsv} = 1.8984$ for ballistically survivable (UTTAS/AAH level), 1.0 otherwise. The parametric method assumes the rotor flight controls are boosted and computes the weight of the non-boosted portion up to the control actuators. Based on 20 aircraft, the average error of the non-boosted flight controls equation is 10.4% (fig. 29-21). The flight control boost mechanism weight and boosted flight control weight (AFDD82 model) are:

$$w_{fc} = 0.2873 f_{mbsv}(N_{\text{rotor}}N_{\text{blade}})^{0.6257} c^{1.3286}(0.01V_{\text{tip}})^{2.1129} f_{RW\text{red}}^{0.8942}$$

$$W_{RWmb} = \chi_{RWmb}(1 - f_{RW\text{hyd}})w_{fc}$$

$$W_{RWb} = \chi_{RWb} 0.02324 f_{bsv}(N_{\text{rotor}}N_{\text{blade}})^{1.0042} N_{\text{rotor}}^{0.1155} c^{2.2296}(0.01V_{\text{tip}})^{3.1877}$$

where $f_{mbsv} = 1.3029$ and $f_{bsv} = 1.1171$ for ballistically survivable (UTTAS/AAH level); 1.0 otherwise; and $f_{RW\text{red}} = 1.0$ to 3.0. Typically $f_{RWnb} = 0.6$ (range 0.3 to 1.8); $f_{RW\text{hyd}} = 0.4$. Based on 21 aircraft, the average error of the boost mechanisms equation is 6.5% (fig. 29-22). Based on 20 aircraft, the average error of the boosted flight controls equation is 9.7% (fig. 29-23). Parameters are defined in table 29-16, including units as used in these equations.

Table 29-16. Parameters for rotary wing flight control weight.

parameter	definition	units
W_{MTO}	maximum takeoff weight	lb
N_{rotor}	number of main-rotors	
N_{blade}	number of blades per rotor	
c	rotor mean blade chord	ft
V_{tip}	rotor hover tip velocity	ft/sec
f_{RWnb}	rotary wing non-boosted weight (fraction boost mechanisms weight)	
$f_{RW\text{hyd}}$	rotary wing hydraulics weight	
	(fraction hydraulics plus boost mechanisms weight)	
$f_{RW\text{red}}$	flight control hydraulic system redundancy factor	

The conversion controls consist of non-boosted tilt controls, and tilt control boost mechanisms; they are used only for tilting rotor configurations. The weights are:

$$w_{CVmb} = f_{CVmb}W_{MTO} \qquad W_{CVmb} = \chi_{CVmb}w_{CVmb}$$
$$w_{CVnb} = f_{CVnb}W_{CVmb} \qquad W_{CVnb} = \chi_{CVnb}w_{CVnb}$$

Parameters are defined in table 29-17, including units as used in these equations.

Table 29-17. Parameters for conversion control weight.

parameter	definition	units
W_{MTO}	maximum takeoff weight	lb
f_{CVnb}	conversion non-boosted weight (fraction boost mechanisms weight)	
f_{CVmb}	conversion boost mechanisms weight (fraction maximum takeoff weight)	

29–9   Hydraulic Group

The hydraulic group consists of hydraulics for fixed wing flight controls, rotary wing flight controls, conversion (rotor tilt) flight controls, and equipment. The hydraulic weight for equipment, W_{EQhyd}, is fixed (input). The weights (AFDD82 model) are

$$W_{FW\mathrm{hyd}} = \chi_{FW\mathrm{hyd}}f_{FW\mathrm{hyd}}W_{FWmb}$$
$$W_{RW\mathrm{hyd}} = \chi_{RW\mathrm{hyd}}f_{RW\mathrm{hyd}}w_{fc}$$
$$W_{CV\mathrm{hyd}} = \chi_{CF\mathrm{hyd}}f_{CV\mathrm{hyd}}W_{CVmb}$$

Typically $f_{RW\mathrm{hyd}} = 0.4$. Parameters are defined in table 29-18, including units as used in these equations.

Table 29-18. Parameters for hydraulic group weight.

parameter	definition	units
$f_{FW\mathrm{hyd}}$	fixed wing hydraulics weight (fraction boost mechanisms weight)	
$f_{RW\mathrm{hyd}}$	rotary wing hydraulics weight (fraction hydraulics plus boost mechanisms weight)	
$f_{CV\mathrm{hyd}}$	conversion hydraulics weight (fraction boost mechanisms weight)	

29–10   Anti-Icing Group

The anti-icing group consists of the anti-ice system. The electrical system for anti-icing is part of the electrical group. The weights are obtained from the sum over all rotors, all wings, and all engines:

$$W_{DI\mathrm{elect}} = \chi_{DI\mathrm{elect}} \sum k_{\mathrm{elec}}A_{\mathrm{blade}}$$
$$W_{DI\mathrm{sys}} = \chi_{DI\mathrm{sys}} \left(\sum k_{\mathrm{rotor}}A_{\mathrm{blade}} + \sum k_{\mathrm{wing}}\ell_{\mathrm{wing}} + \sum k_{\mathrm{air}}W_{\mathrm{eng}} + \sum k_{\mathrm{jet}}W_{\mathrm{jet}} \right)$$

Parameters are defined in table 29-19, including units as used in these equations.

Table 29-19. Parameters for anti-icing group weight.

parameter	definition	units
A_{blade}	total blade area of rotor, from geometric solidity	ft^2 or m^2
ℓ_{wing}	wing length (wing span less fuselage width)	ft or m
k_{elect}	electrical system weight factor	
k_{rotor}	rotor deice system weight factor	
k_{wing}	wing deice system weight factor	
k_{air}	engine air intake deice system weight factor	
k_{jet}	jet air intake deice system weight factor	

29–11   Other Systems and Equipment

The following weights are fixed (input) in this model: auxiliary power group; instruments group; pneumatic group; electrical group (aircraft); avionics group (mission equipment); armament group (armament provisions and armor); furnishing and equipment group; environmental control group; and load and handling group. Typical fixed weights are given in table 29-20, based on medium to heavy helicopters and tiltrotors.

Table 29-20. Other systems and equipment weight.

group	typical weight (lb)
SYSTEMS AND EQUIPMENT	
flight controls group	
cockpit controls	100–125
automatic flight control system	35–200
flight control electronics, mechanical	35–100
flight control electronics, fly-by-wire	250
auxiliary power group	130–300
instruments group	150-250
hydraulic group	
equipment	50–300
electrical group	400–1000
avionics group (mission equipment)	400–1500
furnishings & equipment group	600–1000
crew only	100–200
environmental control group	50–250
anti-icing group	50–300
load & handling group	
internal	200–400
external	150–300
FIXED USEFUL LOAD	
crew	500–800
fluids (oil, unusable fuel)	50–150

29–12   Folding Weight

Folding weights are calculated in a number of groups: wing W_{fold} (including extensions), rotor W_{fold}, tail W_{fold}, fuselage W_{tfold} and W_{wfold}. These are the total weights for folding and the impact of folding on the group. A fraction f_{foldkit} of these weights can be in a kit, hence optionally removable. Thus of the total folding weight, the fraction f_{foldkit} is a kit weight in the fixed useful load of the weight statement, while the remainder is kept in the wing, rotor, or fuselage group weight.

29–13   Parametric Weight Correlation

Table 29-21 summarizes the statistics of the parametric weight estimation equations. Figure 29-24 shows the error of the calculated weight for the sum of all parametric weight, accounting on average for 42% of the empty weight. This sum is composed of the structural group (based on the AFDD00 equation for rotor blade and hub weights, and the AFDD84 equation for body weight), the propulsion group (based on the AFDD00 equation for drive system weight), and the flight controls group. Based on 42 aircraft, the average error of the sum of all parametric weight is 5.3%. The corresponding average error is 6.1% for the structural group (8.6% for the rotor group alone), 10.9% for the propulsion group, and 8.7% for the flight controls group.

Table 29-21. Statistics of parametric weight equations.

group		number of aircraft	average error (%)
wing		25	3.4
rotor blade	AFDD82	37	7.7
rotor hub	AFDD82	37	10.2
rotor blade	AFDD00	51	7.9
rotor hub	AFDD00	51	12.2
horizontal tail		13	22.4
vertical tail		12	23.3
tail-rotor		19	16.7
fuselage	AFDD82	30	8.7
fuselage	AFDD84	35	6.5
alighting gear		28	8.4
engine support		12	11.0
engine cowling		12	17.9
air induction		12	11.0
accessory		16	11.5
propeller	AFDD10	16	10.5
fuel tank		15	4.6
gear box + rotor shaft	AFDD83	30	7.7
gear box + rotor shaft	AFDD00	52	8.6
drive shaft		28	16.0
rotor brake		23	25.1
rotary wing flight controls	non-boosted	20	10.4
rotary wing flight controls	boost mechanisms	21	6.5
rotary wing flight controls	boosted	20	9.7

29–14   References

1) Chappell, D., and Peyran, R. "Methodology for Estimating Wing Weights for Conceptual Tilt-Rotor and Tilt-Wing Aircraft." SAWE Paper No. 2107, Category No. 23, May 1992.

2) Chappell, D.P. "Tilt-rotor Aircraft Wing Design." ASRO-PDT-83-1, 1983.

3) "Weight Trend Estimation for the Rotor Blade Group, Rotor Hub Group, and Upper Rotor Shaft of the ABC Aircraft." ASRO-PDT-83-2, 1983.
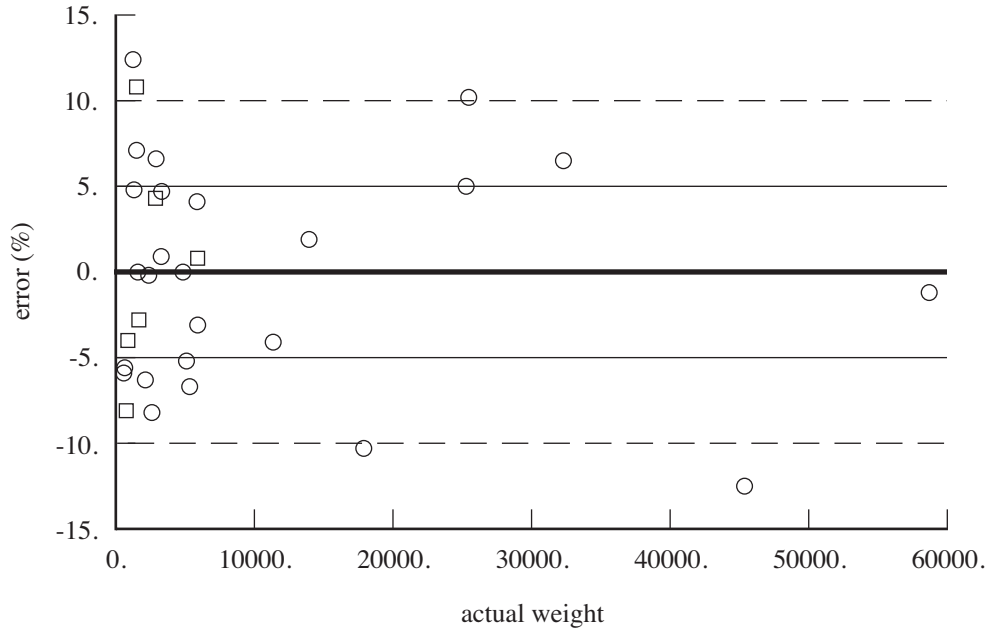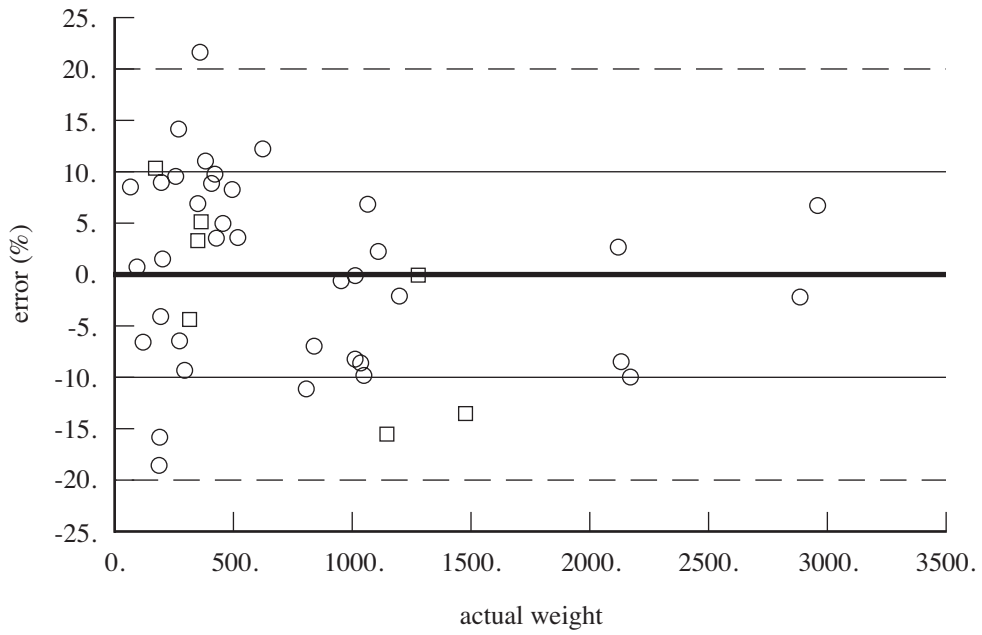
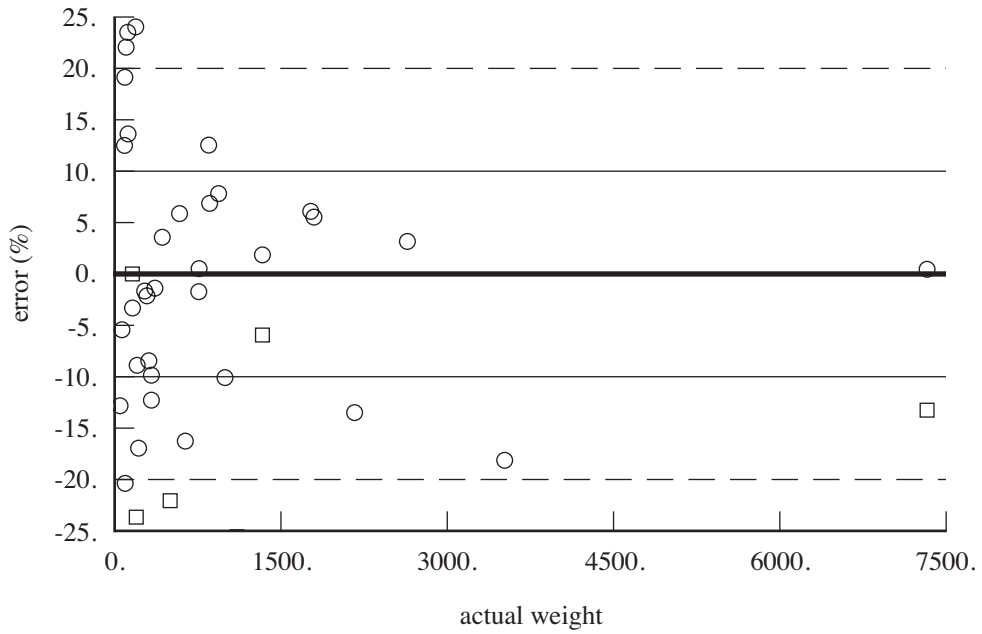Figure 29-1. Wing group (AFDD93).

Figure 29-2. Rotor group, blade weight (AFDD82).

Figure 29-3. Rotor group, hub weight (AFDD82).
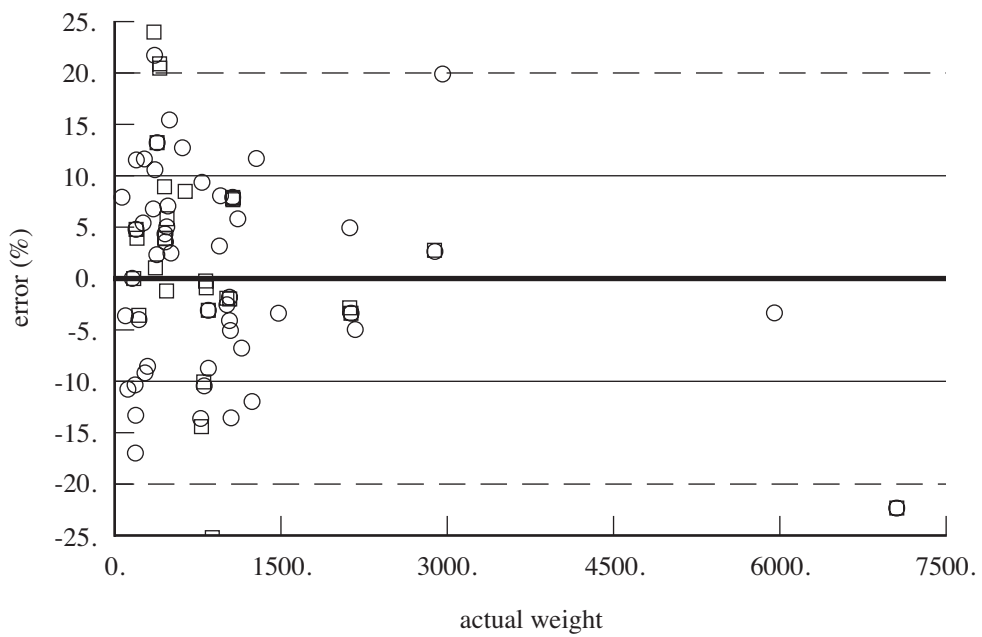
Figure 29-4. Rotor group, blade weight (AFDD00).
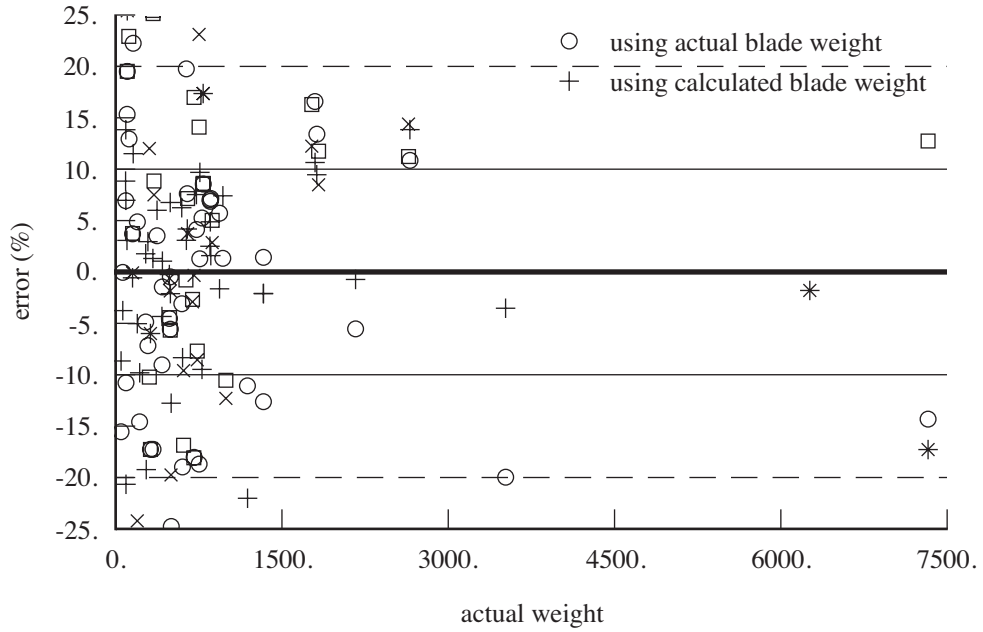
Figure 29-5. Rotor group, hub weight (AFDD00).
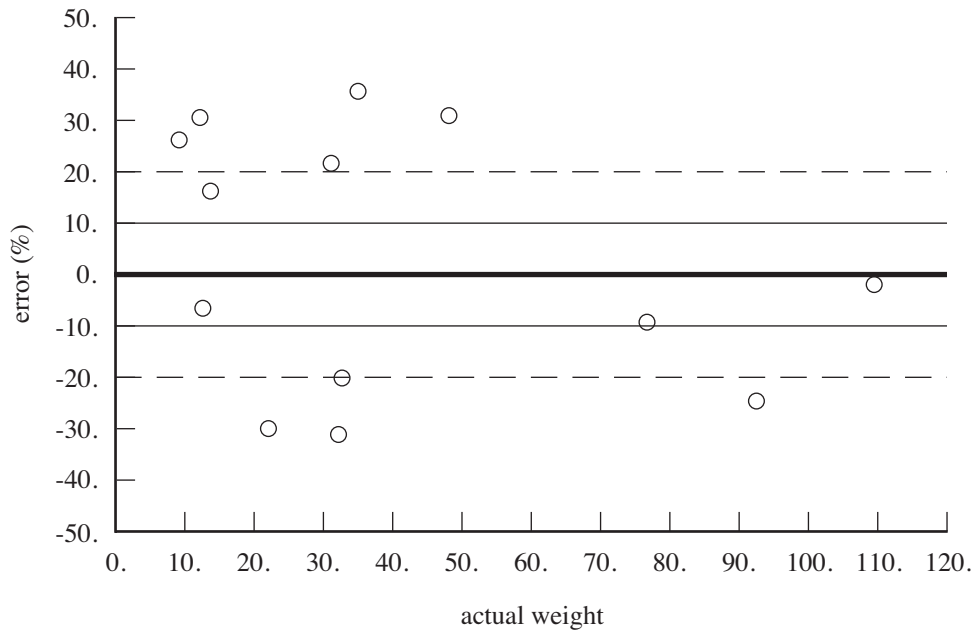
Figure 29-6. Empennage group, horizontal tail weight (AFDD82).
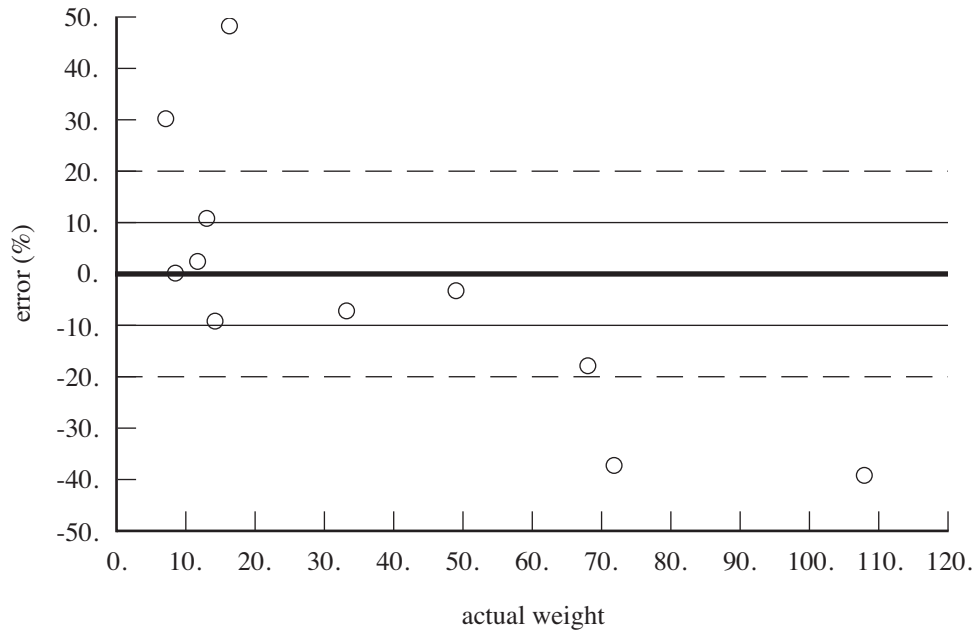
Figure 29-7. Empennage group, vertical tail weight (AFDD82).
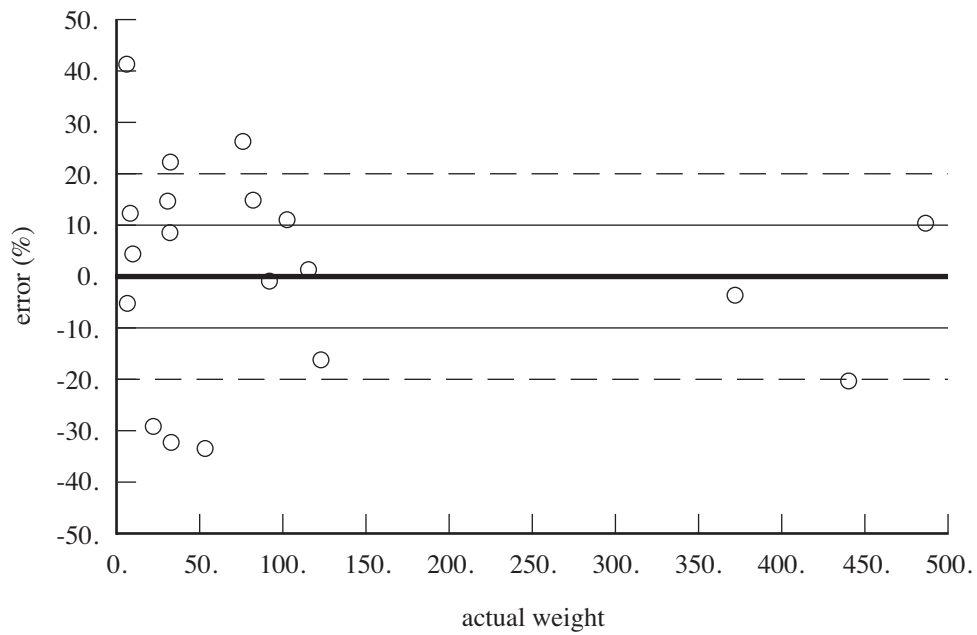
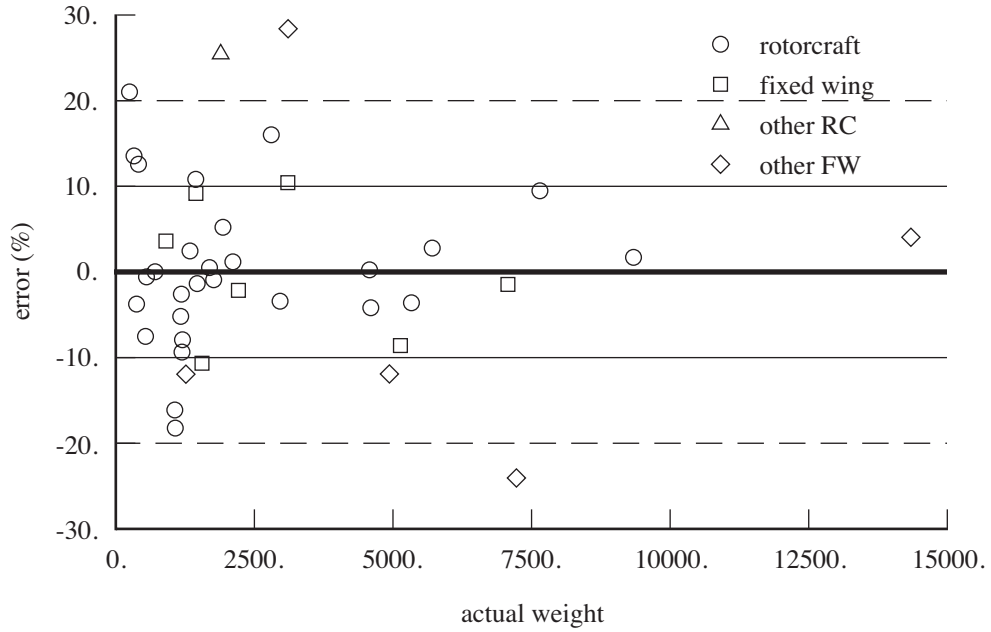Figure 29-8. Empennage group, tail-rotor weight (AFDD82).

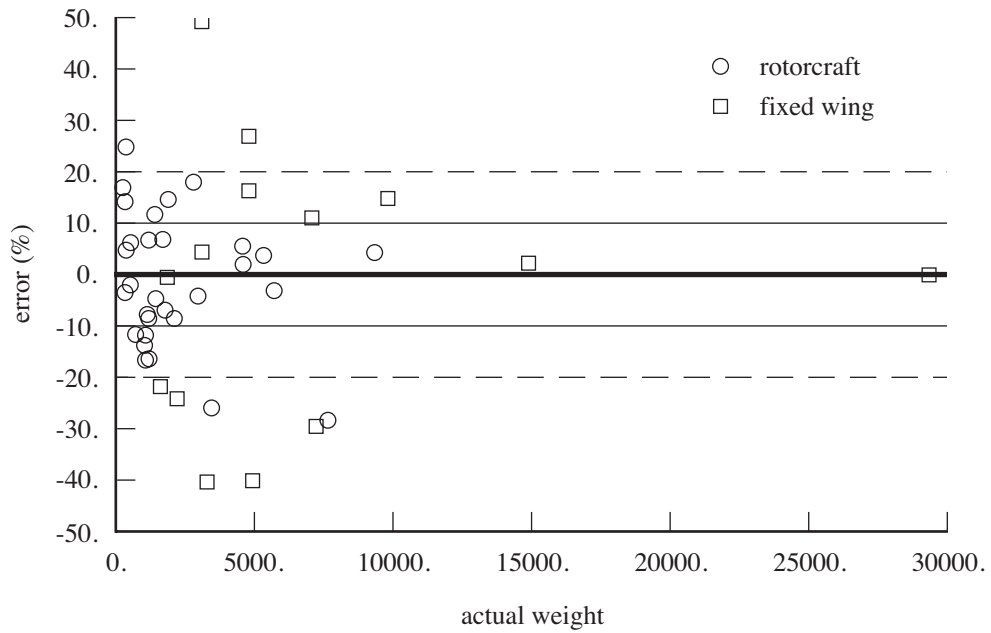Figure 29-9. Fuselage group, fuselage weight (AFDD84).

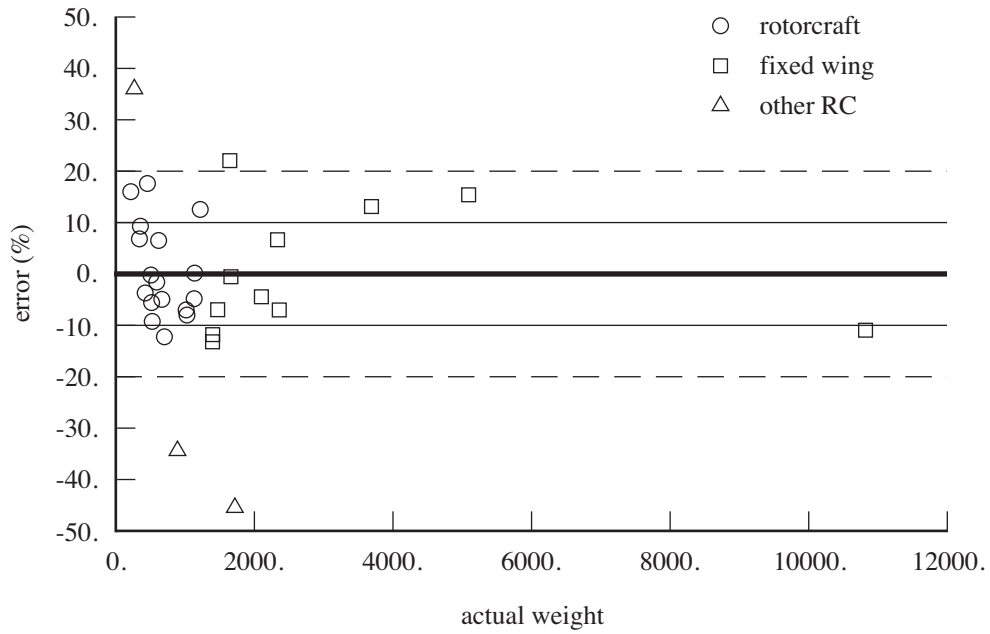Figure 29-10. Fuselage group, fuselage weight (AFDD82).

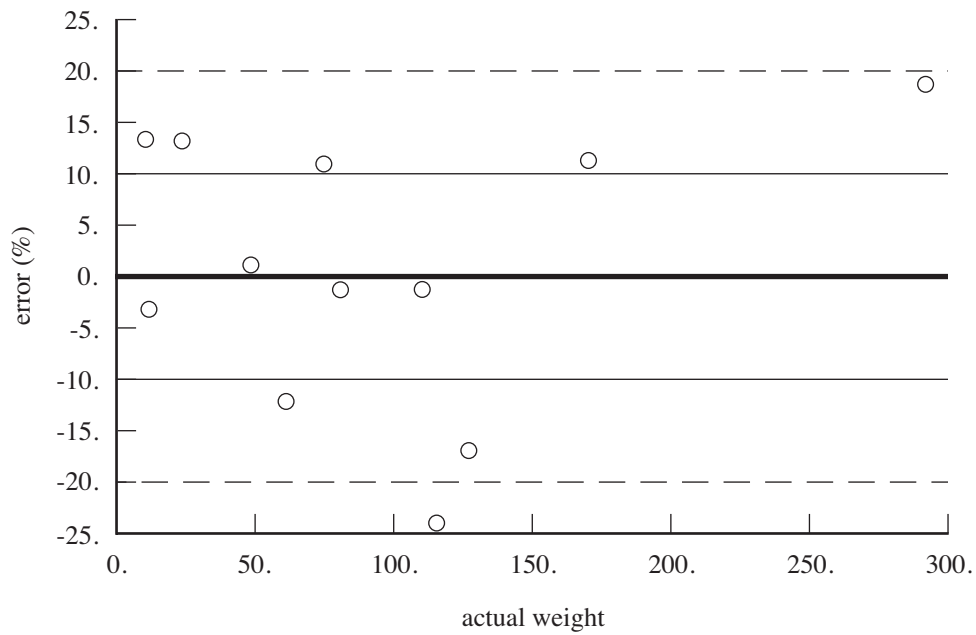Figure 29-11. Alighting gear group, landing gear weight (AFDD82).

Figure 29-12. Engine section or nacelle group, engine support weight (AFDD82).
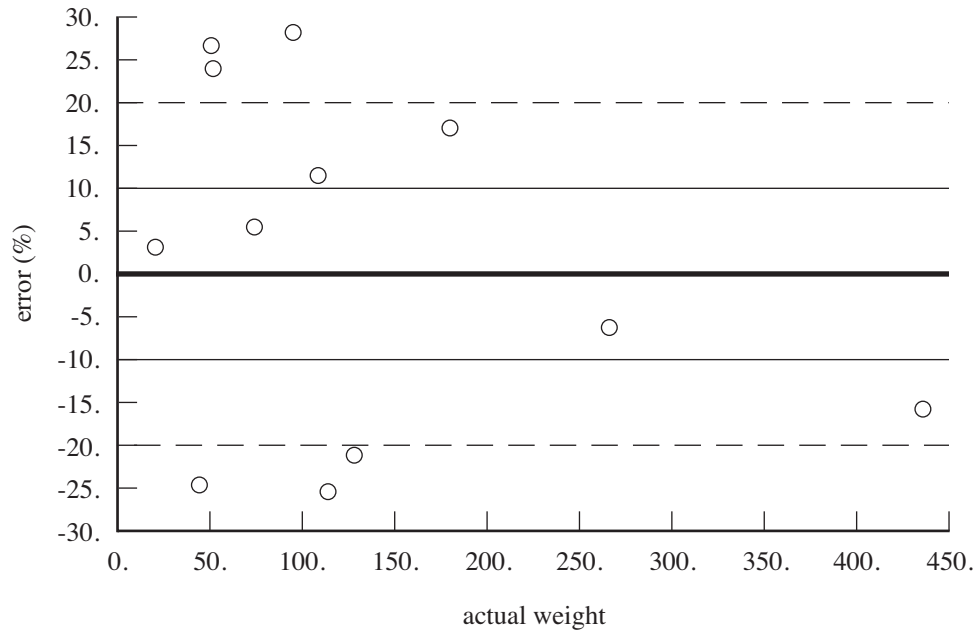
Figure 29-13. Engine section or nacelle group, cowling weight (AFDD82).
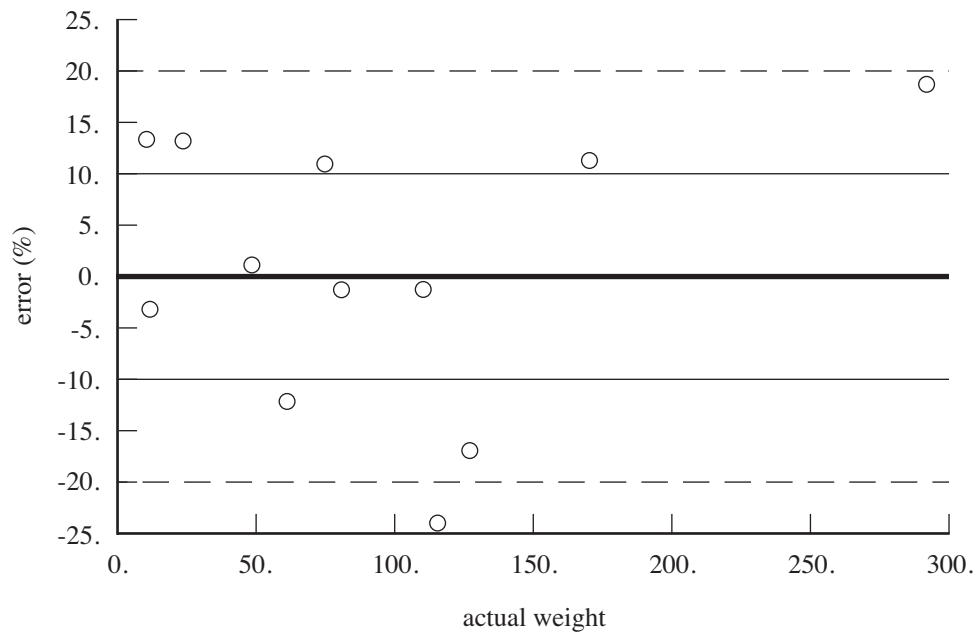

Figure 29-14. Air induction group, air induction weight (AFDD82).
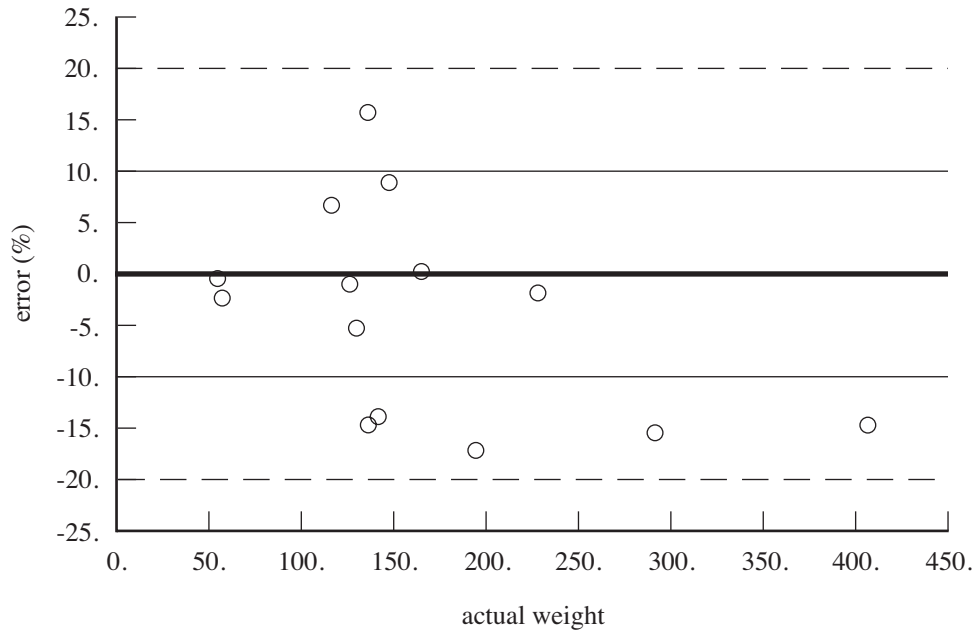
Figure 29-15. Propulsion group, accessories weight (AFDD82).
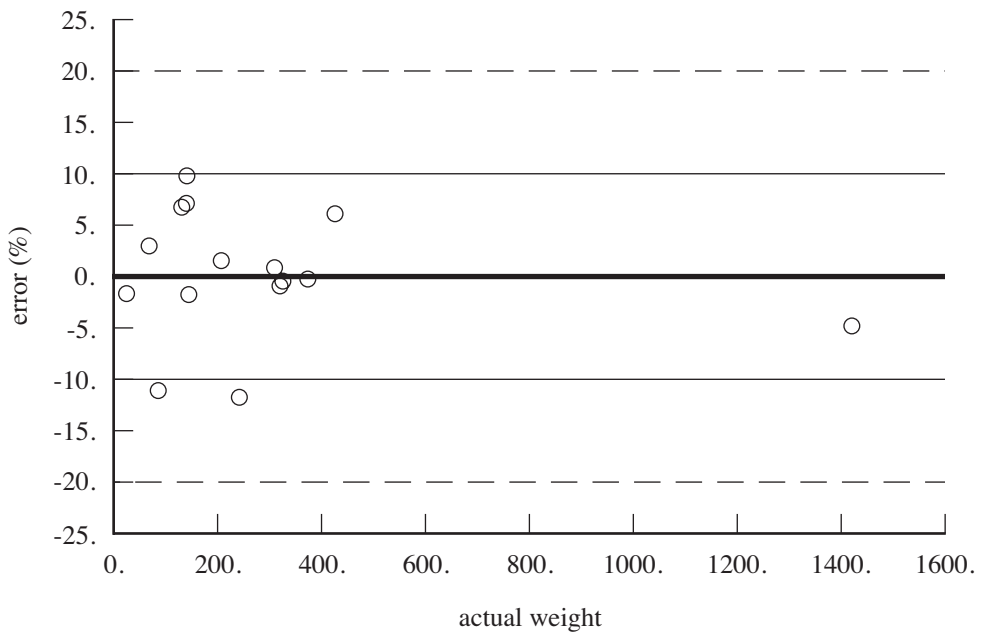
Figure 29-16. Propulsion group, fuel tank weight (AFDD82).
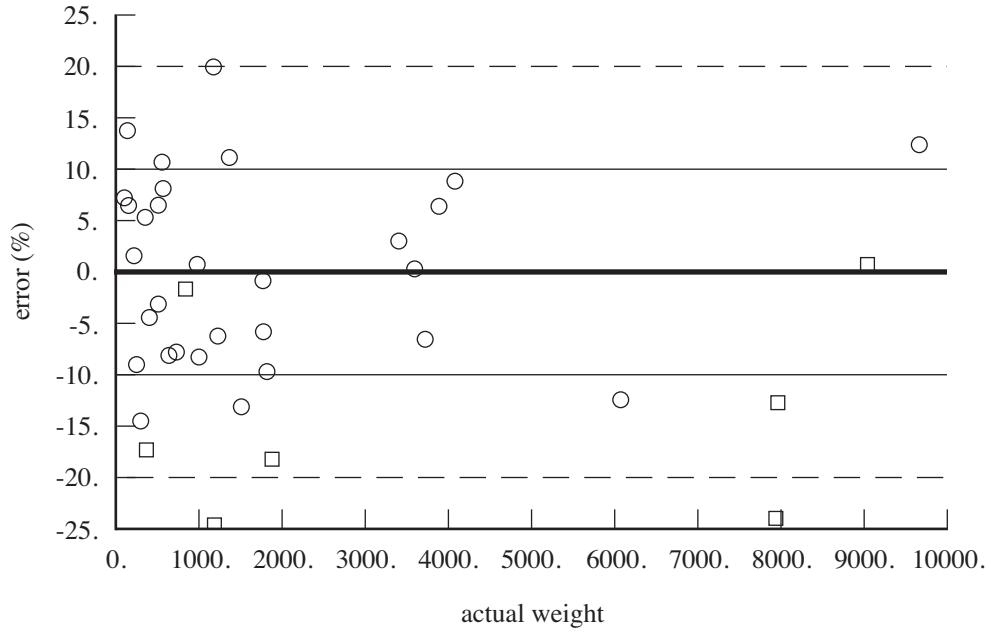
Figure 29-17. Propulsion group, gear box and rotor shaft weight (AFDD83).
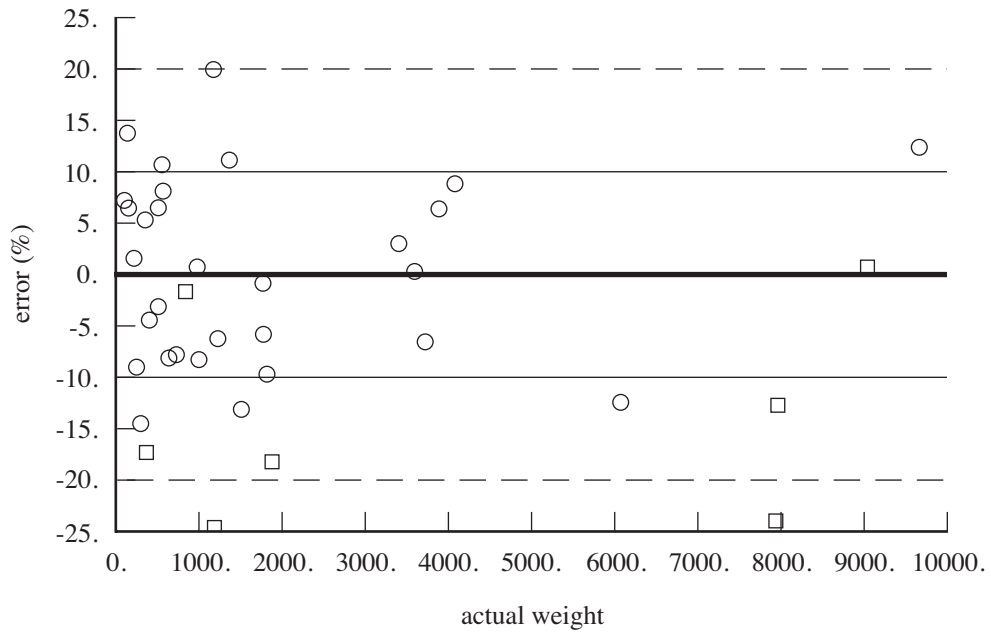
Figure 29-18. Propulsion group, gear box and rotor shaft weight (AFDD00).

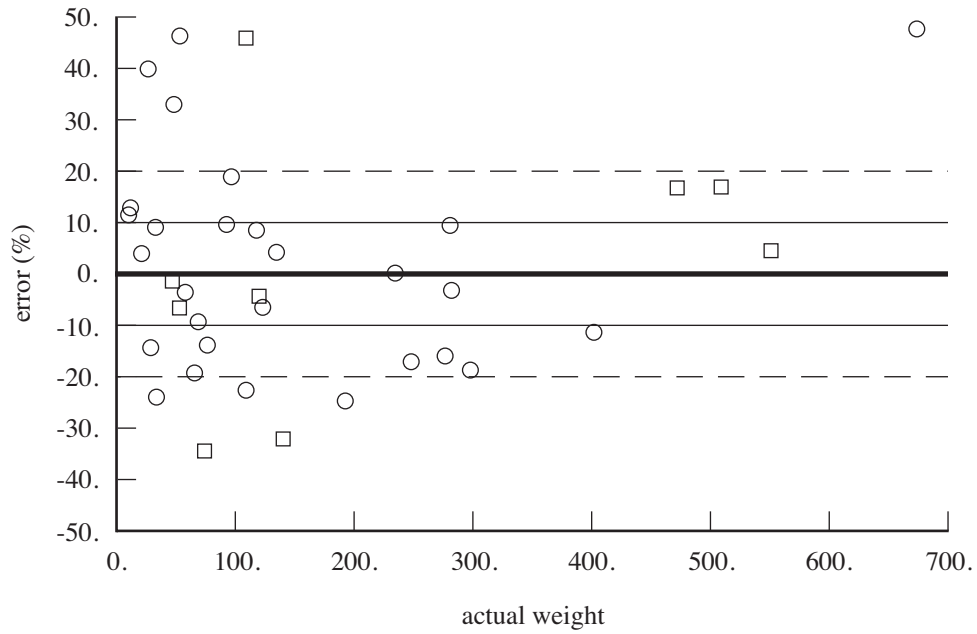Figure 29-19. Propulsion group, drive shaft weight (AFDD82).
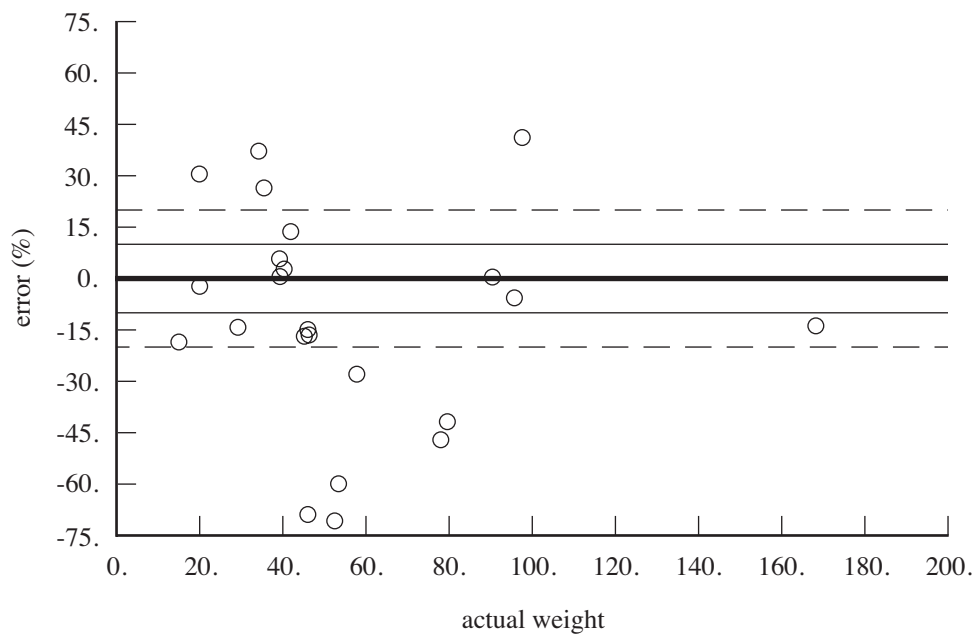

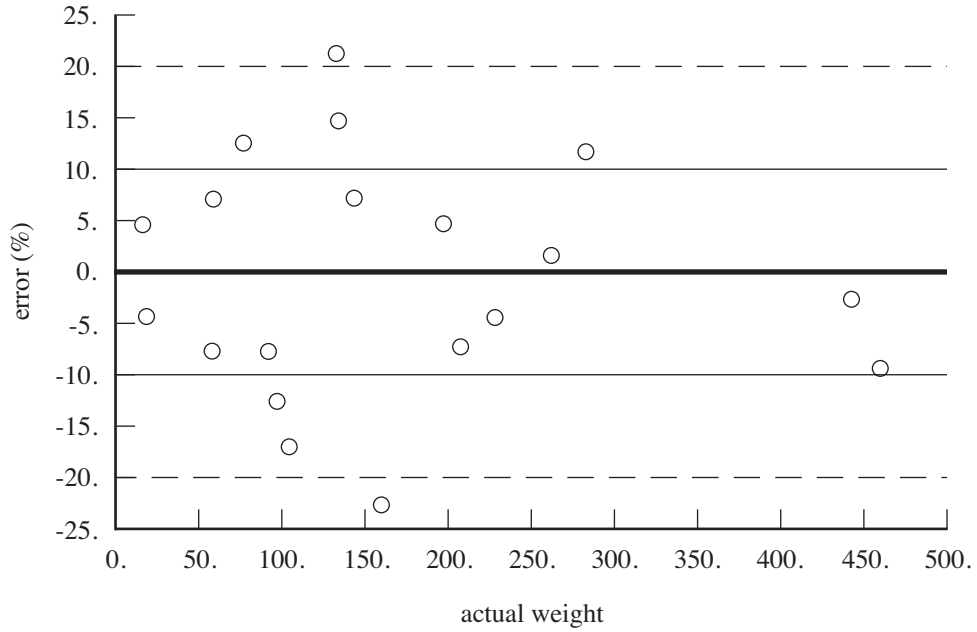
Figure 29-20. Propulsion group, rotor brake weight.

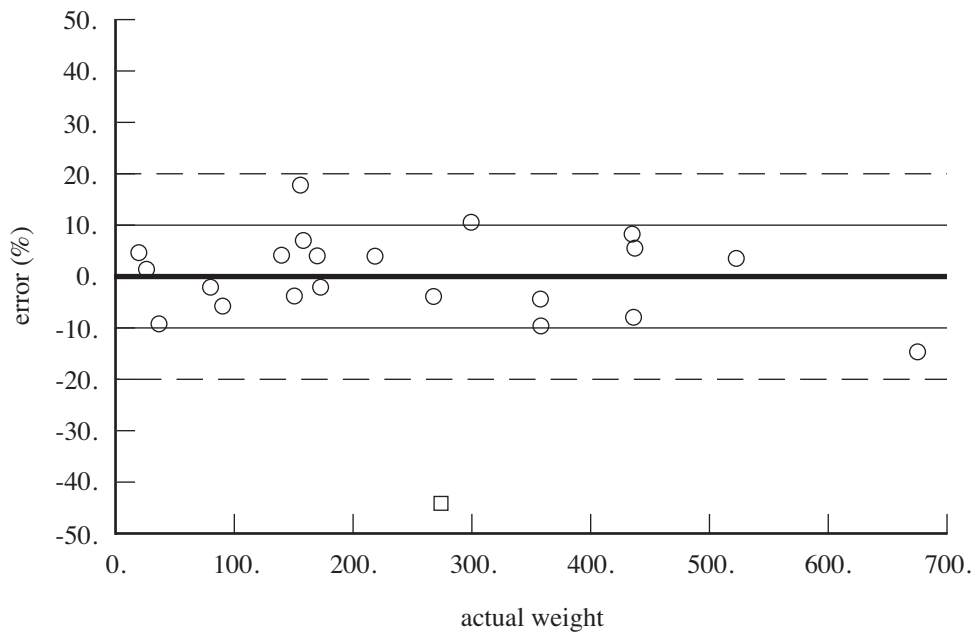Figure 29-21. Flight controls group, rotor non-boosted control weight (AFDD82).

Figure 29-22. Flight controls group, rotor boost mechanisms weight (AFDD82).
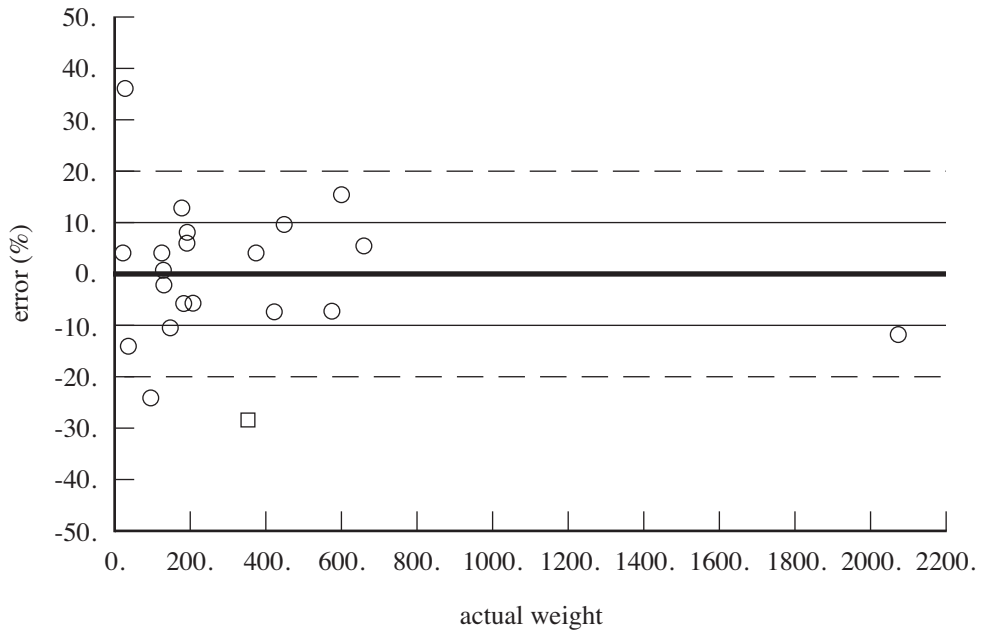
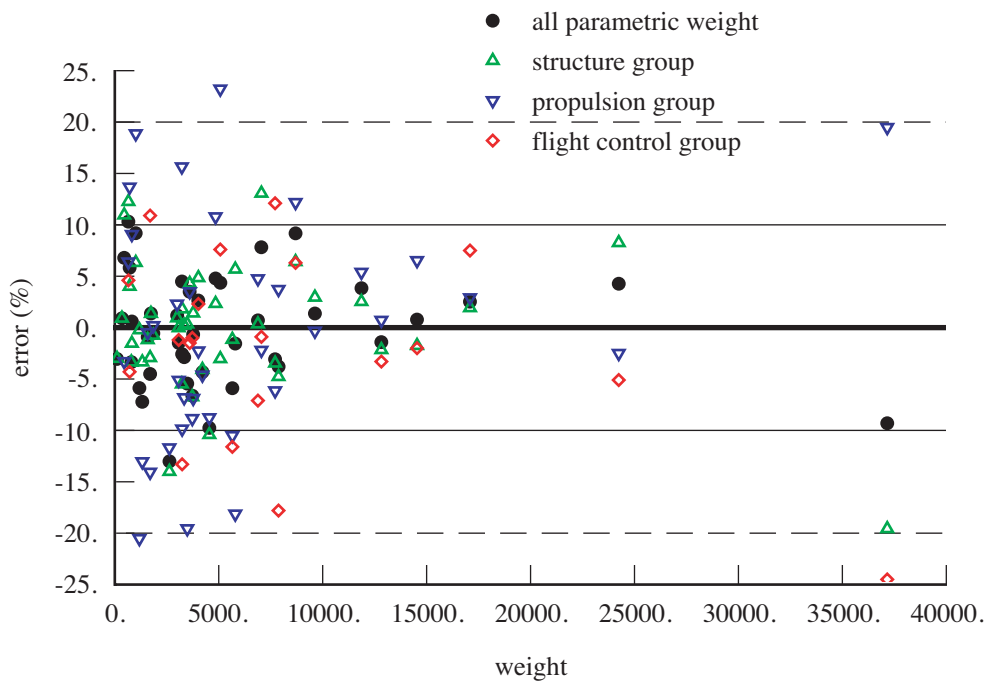Figure 29-23. Flight controls group, rotor boosted control weight (AFDD82).

Figure 29-24. Sum of all parametric weight.

Other Weight Models

This chapter presents aircraft weight models developed by a number of organizations: Boeing (refs. 1 to 4), GARTEUR (Aerospatiale and MBB, ref. 5), Tishchenko (refs. 6 and 3), Torenbeek (ref. 7), and Raymer (ref. 8). The weights are estimated from parametric equations, based on the weights of existing aircraft. In addition, generic weight estimation equations are implemented for some groups.

The results of these equations are the weight in pounds, and the units of the parameters are noted in the tables. Technology factors χ are included in the weight equations. The input usually includes a weight increment dW that can be added to the results of the weight model. Thus typically a component or element weight is obtained from $W = \chi w_{\mathrm{model}} + dW$. Weight of individual elements in a group can be fixed by using dW and setting the corresponding technology factor $\chi = 0$. With $\chi \neq 0$, the increment dW can account for something not included in the parametric model.

30–1   Wing Group

The wing group consists of: basic structure (primary structure, consisting of torque box and spars, plus extensions); fairings (leading edge and trailing edge); fittings (non-structural); fold/tilt structure; and control surfaces (flaps, ailerons, flaperons, and spoilers).

The total wing weight is:

Boeing

$$w_{\mathrm{wing}} = 0.0680 \left[\sqrt{n_z} W_D f_L \, S_w \log(b_w/w_f) \sqrt{\frac{1+\lambda_w}{2\tau_w}} \log V_{\mathrm{dive}} \log A_w \right]^{0.585}$$

GARTEUR

$$w_{\mathrm{wing}} = 0.00715 f_{TR} \left(1 + 0.12 f_{\mathrm{flap}}\right)^{0.4} \left(n_z W_D f_L\right)^{0.52} S_w^{0.7} A_w^{0.47} \left(\frac{2}{\tau_w}\right)^{0.4} (1.1 + \lambda_w/2)^{0.695}$$

The factor $f_{TR} = 1.116$ for tiltrotors, 1.0 otherwise.

Torenbeek (light aircraft, up to 12500 lb)

$$w_{\mathrm{wing}} = 0.00125(n_z^{0.55} W_{MTO} f_L) \left(\frac{b_w}{\cos \Lambda_w}\right)^{0.75} \left(1 + \sqrt{6.25 \cos \Lambda_w/b_w}\right) \left(\frac{b_w/\cos \Lambda_w t_w}{W_{MTO} f_L/S_w}\right)^{0.3}$$

Torenbeek (transport catagory aircraft)

$$w_{\mathrm{wing}} = 0.0017(n_z^{0.55} W_{MZF} f_L) \left(\frac{b_w}{\cos \Lambda_w}\right)^{0.75} \left(1 + \sqrt{6.25 \cos \Lambda_w/b_w}\right) \left(\frac{b_w/\cos \Lambda_w t_w}{W_{MZF} f_L/S_w}\right)^{0.3}$$

Raymer (cargo/transport)

$$w_{\text{wing}} = 0.0051(n_z W_D f_L)^{0.557} S_w^{0.649} A_w^{0.5} \frac{(1 + \lambda_w)^{0.1} S_c^{0.1}}{\tau_w^{0.4} \cos \Lambda_w}$$

Raymer (general aviation)

$$w_{\text{wing}} = 0.0087(n_z W_D f_L)^{0.49} S_w^{0.758} A_w^{0.6} \frac{\lambda_w^{0.04} V_{\text{dive}}^{0.012}}{\tau_w^{0.3} \cos \Lambda_w^{0.9}}$$

The lift factor f_L accounts for wing relief factor and wing/rotor lift share.

Then the primary structure, secondary structure, and control surface weights are calculated as for the AFDD aircraft wing model. The wing extension weights are calculated as for the AFDD model. Parameters are defined in table 30-1, including units as used in these equations.

Table 30-1. Parameters for wing weight.

parameter	definition	units
W_D	design gross weight	lb
W_{MTO}	maximum takeoff weight	lb
W_{MZF}	maximum zero fuel weight	lb
f_L	lift factor	
S_w	wing planform area (theoretical)	ft^2
b_w	wing span	ft
A_w	wing aspect ratio	
w_f	maximum fuselage width	ft
λ_w	wing taper ratio (tip chord/root chord)	
Λ_w	wing sweep angle	deg
τ_w	wing root airfoil thickness-to-chord ratio	
t_w	wing root airfoil thickness	ft
n_z	design ultimate flight load factor	g
V_{dive}	dive speed	kts
S_c	control surface area	ft^2
f_{flap}	ratio maximum lift coefficient with and without flaps	

30–2   Rotor Group

The rotor group consists of: blades, hub and hinge, spinner, and blade fold structure. The blade and hub-hinge weights are:

Boeing

$$w_{\text{blade}} = 0.0378 N_{\text{rotor}} \left[(n_z W_D f_{tb}/N_{\text{rotor}}) R^2 (R - r) N_{\text{blade}} c f_b f_{dr} \right]^{0.438}$$

$$w_{\text{hub}} = 0.00704 N_{\text{rotor}} \left[(W_{\text{blade}}/N_{\text{rotor}}) R \Omega^2 (P_{to} f_{th}/N_{\text{rotor}}) r^{1.82} N_{\text{blade}}^{1.5} f_{amd} \right]^{0.358}$$

with $f_b = 1.0$ for articulated rotor, 2.2 hingeless or teetering rotor; $f_{tb} = 0.247$ and $f_{th} = 0.577$ for tandem, 1.0 single main rotor; $f_{amd} = f_a f_m f_d$, design concept factor $f_a = 0.53$ for hingeless rotor and 1.0 otherwise, material factor $f_m = 1.0$ for steel and 0.56 titanium and 0.354 composite, development factor $f_d = 1.0$ early, 0.62 developed. The droop factor is $f_{dr} = \max(1, R^{1.6}/(1200 t_{0.25}))$. The blade attachment radius is typically $r/R = 0.08$ to 0.11.

GARTEUR

$$w_{\text{blade}} = 4.199 N_{\text{rotor}} f_{\text{root}} f_{cb} (R N_{\text{blade}} c)$$

$$w_{\text{hub}} = 0.004111 N_{\text{rotor}} \left[(W_{\text{blade}}/N_{\text{rotor}}) V_{\text{tip}}^2 (P_{to}/N_{\text{rotor}}) R^{0.82} N_{\text{blade}}^{1.5} \right]^{0.36}$$

with $f_{\text{root}} = 1.17$ for hingeless and teetering blades, 0.88 for composite articulated blades. The original model is extended to other configurations by introducing $f_{cb} = 0.56$ for tandem helicopter, 0.65 for tiltrotor.

Tishchenko

$$w_{\text{blade}} = 1.23 N_{\text{rotor}} f_{cb} (\sigma R^{2.7}/\bar{\lambda}^{0.7}) \left[1 + 0.011 \bar{R} (\lambda - \lambda_0) \right]$$

$$w_{\text{hub}} = 0.0000777 N_{\text{rotor}} f_{ch} N_{\text{blade}} (CF_{\text{blade}})^{1.35} \left[1 + 0.05 (N_{\text{blade}} - 4) \right]$$

where $\bar{\lambda} = \lambda/18$, blade aspect ratio $\lambda = R/c_{0.7}$, $\bar{R} = R/52.5$, $\bar{R}\lambda_0 = 12.4$, and the last factor in the blade weight equation has a minimum value of 1. For the hub weight, the centrifugal force per blade is $CF_{\text{blade}} = \int_0^R \Omega^2 r m \, dr \cong \Omega^2 \frac{1}{2} R (M_{\text{blade}}/(N_{\text{blade}} N_{\text{rotor}}))$. The original model is extended to other configurations by introducing $f_{cb} = 0.45$ for tandem helicopter, 0.80 for tiltrotor; and $f_{ch} = 1.11$ for tandem helicopter, 0.34 for tiltrotor, 0.53 for teetering rotor, 0.67 for hingeless rotor.

generic

$$w_{\text{blade}} = K_{\text{blade}} N_{\text{rotor}} N_{\text{blade}}^{X_{\text{bld}N}} R^{X_{\text{bld}R}} c^{X_{\text{bld}c}} V_{\text{tip}}^{X_{\text{bld}V}} \nu_{\text{blade}}^{X_{\text{bld}\nu}} (W_{MTO}/N_{\text{rotor}})^{X_{\text{bld}W}}$$

$$w_{\text{hub}} = K_{\text{hub}} N_{\text{rotor}} N_{\text{blade}}^{X_{\text{hub}N}} R^{X_{\text{hub}R}} c^{X_{\text{hub}c}} V_{\text{tip}}^{X_{\text{hub}V}} \nu_{\text{hub}}^{X_{\text{hub}\nu}} (W_{MTO}/N_{\text{rotor}})^{X_{\text{hub}W}}$$

using system units. If the the rotor is not a main rotor, T_{design} is used for W_{MTO}/N_{rotor}.

The blade weight is for all blades of the rotors. If the weight is evaluated separately for each rotor, then the leading $N_{\text{rotor}} = 1$ in these equations (with blade weight, design gross weight, and takeoff power per rotor). The fairing/spinner, blade fold, rotor support, and duct weights are calculated as for the AFDD model. Parameters are defined in table 30-2, including units as used in these equations.

Table 30-2. Parameters for rotor weight.

parameter	definition	units
N_{rotor}	number of rotors	
N_{blade}	number of blades per rotor	
R	rotor radius in hover	ft
r	center of rotation to blade attachment	ft
c	rotor mean geometric blade chord	ft
σ	rotor solidity	
$t_{0.25}$	blade thickness at 25%R	ft
W_D	design gross weight	lb
n_z	design limit flight load factor	g
Ω	rotor rotation speed	rpm
V_{tip}	hover tip speed	ft/sec
P_{to}	takeoff power	hp
$\nu_{\text{blade}}, \nu_{\text{hub}}$	flap natural frequency (for weight estimate)	per rev
W_{MTO}	maximum takeoff weight	
T_{design}	rotor design thrust	

30–3   Empennage Group

The empennage group consists of: horizontal tail, vertical tail, and tail-rotor. The tail plane weight consists of the basic structure and fold structure. The horizontal tail weight is:

GARTEUR

$$W_{ht} = \chi_{ht} 0.00000518 V_{\max}^{2.44} W_D^{0.35}$$

Torenbeek (V_{dive} up to 250 knots)

$$W_{\text{tail}} = \chi_{ht} 0.04 \big(n_z S_{\text{tail}}^2 \big)^{0.75}$$

Torenbeek (transport catagory aircraft)

$$W_{ht} = \chi_{ht} f_h S_{ht} 0.00346 \Big(S_{ht}^{0.2} V_{\text{dive}} / \sqrt{\cos \Lambda_{ht}} \Big)$$

where $f_h = 1.0$ for fixed stabilizer, 1.1 for variable-incidence tail.

Raymer (cargo/transport)

$$W_{ht} = \chi_{ht} 0.0379 f_h n_z^{0.10} W_D^{0.639} S_{ht}^{0.75} A_{ht}^{0.166} K_y^{0.704} \frac{(1 + S_e/S_{ht})^{0.1}}{(1 + w_f/b_{ht})^{0.25} L_t \cos \Lambda_{ht}}$$

Raymer (general aviation)

$$W_{ht} = \chi_{ht} 0.00354 (n_z W_D)^{0.414} S_{ht}^{0.896} A_{ht}^{0.043} \frac{V_{\text{dive}}^{0.336} \cos \Lambda_{ht}^{0.034}}{\lambda_{ht}^{0.02} \tau_{ht}^{0.12}}$$

where $f_h = 1.143$ for all-moving horizontal tail, 1.0 otherwise.

The vertical tail weight is:

GARTEUR

$$W_{vt} = \chi_{vt} 0.0253 S_{vt}^{0.508} W_D^{0.647}$$

Torenbeek (V_{dive} up to 250 knots)

$$W_{\text{tail}} = \chi_{vt} 0.04 \big(n_z S_{\text{tail}}^2 \big)^{0.75}$$

Torenbeek (transport catagory aircraft)

$$W_{vt} = \chi_{vt} f_v S_{vt} 0.00346 \Big(S_{vt}^{0.2} V_{\text{dive}} / \sqrt{\cos \Lambda_{vt}} \Big)$$

where $f_v = 1.0$ for fuselage-mounted horizontal tail, $f_v = 1 + 0.15 (S_{ht} h_t) / (S_{vt} b_{vt})$ for fin-mounted stabilizer (T-tail).

Raymer (cargo/transport)

$$W_{vt} = \chi_{vt} 0.0026 f_{v1} n_z^{0.536} W_D^{0.556} S_{vt}^{0.5} A_{vt}^{0.35} K_z^{0.875} \frac{1}{\tau_{\text{root}}^{0.5} L_t^{0.5} \cos \Lambda_{vt}}$$

Raymer (general aviation)

$$W_{vt} = \chi_{vt} 0.00382 f_{v2} (n_z W_D)^{0.376} S_{vt}^{0.873} A_{vt}^{0.357} \frac{V_{\text{dive}}^{0.244}}{\lambda_{vt}^{0.039} \tau_{vt}^{0.49} \cos \Lambda_{vt}^{0.224}}$$

where $f_{v1} = 1.0$ for conventional tail, 1.169 for T-tail; $f_{v2} = 1.0$ for conventional tail, 1.2 for T-tail.

The tail-rotor weight is:

Boeing

$$W_{tr} = \chi_{tr} 0.0297 \Big[r^{0.25} P_{tr}^{0.5} V_{\text{tip}} R N_{\text{blade}} c \Big]^{0.67}$$

GARTEUR

$$W_{tr} = \chi_{tr} 12.48 W_D^{0.556} \Big(R \sigma \tau_{0.7} \Big)^{1.372}$$

Fold weight is calculated as for the AFDD model. Parameters are defined in table 30-3, including units as used in these equations.

Table 30-3. Parameters for tail weight.

parameter	definition	units
$V_{\max}$	maximum speed	kts
V_{dive}	dive speed	kts
S	tail planform area	ft^2
b	tail span	
A	tail aspect ratio	
Λ	sweep angle	deg
λ	taper ratio (tip chord/root chord)	
h_t/b_{vt}	height horizontal tail above fin root (fraction span)	
P_{tr}	tail rotor transmission power limit	hp
V_{tip}	hover tip speed	ft/sec
R	radius	ft
N_{blade}	number of blades	
c	mean chord	ft
r	rotor blade attachment radius	ft
σ	solidity	
$\tau_{0.7}$	airfoil thickness-to-chord ratio at $0.7R$	
W_D	design gross weight	lb
n_z	design ultimate flight load factor	g
w_f/b_{ht}	fuselage width at horizontal tail (fraction span)	
L_t	tail length (wing to horizontal tail)	ft
S_c	control surface area	ft^2
K_y	aircraft pitch radius of gyration	ft
K_z	aircraft yaw radius of gyration	ft

30–4   Fuselage Group

The fuselage group consists of: basic structure; wing and rotor fold/retraction; tail fold/tilt; and marinization, pressurization, and crashworthiness structure. The basic structure weight is

Boeing

$$w_{\text{basic}} = 0.198 \left[(n_z W_{SD} S_{\text{body}}(\ell_c + \ell_r + \Delta\text{CG}))^{0.5} \log V_{\text{dive}} \right]^{0.8}$$

where $(\ell_c + \ell_r + \Delta\text{CG})/\ell_{\text{body}} \cong 1.0$ for tandem, and 0.3–0.6 for single main rotor (0.7–0.8 with ramp).

GARTEUR (airplane)

$$w_{\text{basic}} = 0.149 \left(n_z W_D S_{\text{body}} \ell_{\text{body}} \right)^{0.44}$$

GARTEUR (helicopter)

$$w_{\text{basic}} = 0.0280 R W_D^{0.67} n_z^{0.335}$$

Tishchenko

$$w_{\text{basic}} = 0.251 W_D^{0.25} S_{\text{body}}^{0.88} L_{rr}^{0.16(1+\alpha)}$$

where $\alpha = 0.0$ for single main rotor, 0.2 for tandem helicopter, 0.05 for side-by-side.

Torenbeek

$$w_{\text{basic}} = 0.021 S_{\text{body}}^{1.2} \left(V_{\text{dive}} \frac{L_t}{w_{\text{body}} + h_{\text{body}}} \right)^{0.5}$$

Raymer (cargo/transport)

$$w_{\text{basic}} = 0.3280 f_{\text{door}} f_{LG} (n_z W_D)^{0.5} S_{\text{body}}^{0.302} \ell_{\text{body}}^{0.25} x_w^{0.04} (\ell_{\text{body}}/h_{\text{body}})^{0.10}$$

Raymer (general aviation)

$$w_{\text{basic}} = 0.0103 (n_z W_D)^{0.177} S_{\text{body}}^{1.086} \frac{V_{\text{dive}}^{0.482}}{L_t^{0.051} (\ell_{\text{body}}/h_{\text{body}})^{0.072}}$$

$$w_{\text{press}} = 11.9 \big(V_{\text{press}} P_{\text{delta}} \big)^{0.271}$$

where $f_{\text{door}} = 1.0$ for no cargo door, 1.06 for one side cargo door, 1.12 for two side cargo doors or aft clamshell door, 1.25 for two side cargo doors and aft clamshell door ($f_{\text{door}} = 1 + .06 n_{\text{door}}$); $f_{LG} = 1.12$ for fuselage-mounted main landing gear, 1.0 otherwise; and $x_w = 1 + 0.75[(1+2\lambda_w)/(1+\lambda_w)] b_w \tan \Lambda_w / \ell_{\text{body}}$. For the weight penalty due to pressurization, $V_{\text{press}} \cong \ell_{\text{body}} w_{\text{body}} h_{\text{body}}$, and typically $P_{\text{delta}} = 8$ psi.

generic

$$w_{\text{basic}} = K_{\text{fus}} \left(\frac{W_{MTO}}{1000} \right)^{X_{\text{fus}W}} \left(\frac{n_z W_{SD}}{1000} \right)^{X_{\text{fus}n}} S_{\text{body}}^{X_{\text{fus}S}} \ell_{\text{body}}^{X_{\text{fus}\ell}}$$

using system units.

The tail fold, wing and rotor fold, marinization, pressurization, and crashworthiness weights are calculated as for the AFDD84 model. Parameters are defined in table 30-4, including units as used in these equations.

Table 30-4. Parameters for fuselage weight.

parameter	definition	units
W_{MTO}	maximum takeoff weight	
W_{SD}	structural design gross weight	lb
W_D	design gross weight	lb
n_z	design ultimate flight load factor at W_{SD}	g
S_{body}	wetted area of body	ft²
ℓ_c	length of cabin (nose to end of cabin floor)	ft
ℓ_r	length of rampwell	ft
ΔCG	center of gravity range at W_D	ft
V_{dive}	dive speed	kts
ℓ_{body}	fuselage length	ft
w_{body}	fuselage width	ft
h_{body}	fuselage height	ft
L_{rr}	rotor-rotor longitudinal separation	ft
L_t	tail length (wing to horizontal tail)	ft
n_{door}	number of cargo doors	
Λ_w	wing sweep angle	deg
λ_w	wing taper ratio (tip chord/root chord)	
b_w	wing span	
V_{press}	pressurized volume	ft³
P_{delta}	cabin pressure differential	psi

30–5 Engine Section or Nacelle Group and Air Induction Group

The engine section or nacelle group consists of: engine support structure, engine cowling, and pylon support structure. The weights (engine mounts and nacelle structure) are:

Boeing

$$W_{\text{supt}} = \chi_{\text{supt}} N_{\text{eng}} \left[(W_{\text{eng}}/N_{\text{eng}}) n_{clf} \right]^{0.41}$$
$$W_{\text{cowl}} = \chi_{\text{cowl}} 1.25 S_{\text{nac}}$$

The crash load factor is typically $n_{clf} = 8g$ for civil rotorcraft, $20g$ for military rotorcraft.

Raymer (cargo/transport)

$$W_{\text{cowl}} = \chi_{\text{cowl}} 0.6724 f_n \ell_{\text{nac}}^{0.10} w_{\text{nac}}^{0.294} n_z^{0.119} W_{\text{eng}}^{0.611} S_{\text{nac}}^{0.224}$$

where $f_n = 1.017$ for pylon-mounted nacelle, 1.0 otherwise. The Raymer model includes the air induction group weight. The nacelle length and width are calculated from $f = w_{\text{nac}}/\ell_{\text{nac}}$, assuming $S_{\text{nac}} = \pi w_{\text{nac}} \ell_{\text{nac}}$. Parameters are defined in table 30-5, including units as used in these equations.

Table 30-5. Parameters for engine section, nacelle, and air induction weight.

parameter	definition	units
W_{eng}	weight all main engines	lb
N_{eng}	number of main engines	
S_{nac}	wetted area of nacelles and pylon (less spinner)	ft^2
ℓ_{nac}	nacelle length	ft
w_{nac}	nacelle width	ft
n_z	design ultimate flight load factor	g

30–6 Propulsion Group

The propulsion group consists of the engine system, fuel system, and drive system.

30-6.1 Propeller/Fan Installation

The auxiliary propulsion or propeller weight is:

Boeing

$$W_{\text{at}} = \chi_{\text{at}} 0.0297 N_{\text{at}} \left[r^{0.25} P_{\text{at}}^{0.5} V_{\text{tip}} R N_{\text{blade}} c \right]^{0.67}$$

GARTEUR

$$W_{\text{at}} = \chi_{\text{at}} 0.5320 N_{\text{at}} R^{1.544} N_{\text{blade}}^{0.471} P_{\text{at}}^{0.376}$$

Torenbeek

$$W_{\text{at}} = \chi_{\text{at}} 0.186 N_{\text{at}} f_e \left(R P_{\text{at}} N_{\text{blade}}^{0.5} \right)^{0.78174}$$

generic

$$W_{\text{at}} = \chi_{\text{at}} K_{\text{at}} N_{\text{at}} N_{\text{blade}}^{X_{\text{at}N}} R^{X_{\text{at}R}} c^{X_{\text{at}c}} V_{\text{tip}}^{X_{\text{at}V}} P_{\text{at}}^{X_{\text{at}P}}$$

where $f_e = 1.0$ for turboprop, 1.333 for reciprocating engine. The propeller weight is calculated by the rotor component model, including rotor support and duct weights. Rotor weight models can be used for the propeller. Parameters are defined in table 30-6, including units as used in these equations.

Table 30-6. Parameters for propeller weight.

parameter	definition	units
N_{at}	number of auxiliary thrusters	
P_{at}	power per propeller	hp
V_{tip}	hover tip speed	ft/sec
R	radius	ft
N_{blade}	number of blades	
c	mean chord	ft
r	rotor blade attachment radius	ft

30-6.2   Fuel System

The fuel system consists of tanks and support structure (including fuel tanks, bladders, supporting structure, filler caps, tank covers, and filler material for void and ullage), and fuel plumbing (including fuel system weight not covered by tank weight). The fuel system weights are:

Torenbeek (integral tanks)

$$W_{\text{tank}} = \chi_{\text{tank}}\left[80(N_{\text{eng}} + N_{\text{int}} - 1) + 15C_{\text{int}}^{0.333}N_{\text{int}}^{0.5}\right]$$

Torenbeek (generic)

$$W_{\text{tank}} = \chi_{\text{tank}}K_{\text{tank}}C_{\text{int}}^{X_{\text{tank}}}$$

with $K_{\text{tank}} = 2$ and $X_{\text{tank}} = 0.667$ for single-engine reciprocating; $K_{\text{tank}} = 4.5$ and $X_{\text{tank}} = 0.60$ for multi-engine reciprocating; $K_{\text{tank}} = 3.2$ and $X_{\text{tank}} = 0.727$ for bladder tanks, turbofan or turboprop.

Raymer (cargo/transport)

$$W_{\text{tank}} = \chi_{\text{tank}}2.405C_{\text{int}}^{0.606}N_{\text{int}}^{0.5}\frac{1 + f_p}{1 + f_i}$$

Raymer (general aviation)

$$W_{\text{tank}} = \chi_{\text{tank}}2.49C_{\text{int}}^{0.726}N_{\text{int}}^{0.242}N_{\text{eng}}^{0.157}(1 + f_i)^{-0.363}$$

Parameters are defined in table 30-7, including units as used in these equations.

Table 30-7. Parameters for fuel system weight.

parameter	definition	units
N_{int}	number of internal fuel tanks	
C_{int}	internal fuel tank capacity	gallons
f_i	integral tank capacity (fraction total)	
f_p	protected (self-sealing) tank capacity (fraction total)	
N_{eng}	number of main engines	

30-6.3   Drive System

The drive system consists of gear boxes and rotor shafts, drive shafts, and rotor brake. This distribution of drive system weights is based on the following functional definitions. Gearboxes are parts of the drive system that transmit power by gear trains, and the structure that encloses them. Rotor shafts are the structure (typically a shaft) that transmits power to the rotor. Drive shafts are the structure (typically a shaft) that transmits power in the propulsion system, but not directly to the rotor or by a gear train. The rotor brake weight encompasses components that can prevent the rotor from freely turning. The gear box and rotor shaft weights are:

Boeing

$$w_{gbrs} = 250\left[(P_{DS\text{limit}}/\Omega_{mr})z_{mr}^{0.25}f_t\right]^{0.67}$$

Boeing (alternate)

$$w_{gbrs} = 300\left[1.1P/\Omega\right]^{0.8}$$

where $f_t = 1.0$ for single main rotor, $f_t = 1.3 \times 1.2 = 1.56$ for tandem main rotors. The number of drive stages is typically $z_{mr} \cong W_D/5000$. For the alternate model applied to the entire drive system, $P/\Omega = P_{DS\text{limit}}/\Omega_{mr}$. The alternate model is also applied to the tail rotor only, with $P/\Omega = P_{tr}/\Omega_{tr}$.

GARTEUR (helicopter)

$$w_{gbrs} = 252.3\left(P_{DS\text{limit}}/\Omega_{mr}\right)^{0.74}$$

GARTEUR (tiltrotor)

$$w_{gbrs} = 345.6\left(P_{DS\text{limit}}/\Omega_{mr}\right)^{0.8}$$

Tishchenko

$$w_{gbrs} = 199.3\left(P_{DS\text{limit}}/\Omega_{mr}\right)^{0.8} + 643\left(P_{tr}/\Omega_{tr}\right)^{0.8}$$

with $P_{tr} = f_{tr}P_{DS\text{limit}}$, typically $f_{tr} \cong 10\%$.

generic

$$w_{gbrs} = K_{gbrs}P_{DS\text{limit}}^{X_{gsP}}\Omega_{\text{eng}}^{X_{gse}}\Omega_{\text{rotor}}^{X_{gsr}}$$

using system units. These models are all implemented with an optional tail rotor drive weight increment $\Delta w_{gbrs} = K_{trgb}270(P_{tr}/\Omega_{tr})^{0.8}$. Then

$$W_{gb} = \chi_{gb}(1 - f_{rs})w_{gbrs}$$
$$W_{rs} = \chi_{rs}f_{rs}w_{gbrs}$$

Parameters are defined in table 30-8, including units as used in these equations.

Table 30-8. Parameters for drive system weight.

parameter	definition	units
$P_{DS\text{limit}}$	drive system power limit	hp
Ω_{mr}	main-rotor rotation speed	rpm
Ω_{rotor}	main-rotor rotation speed	rpm
Ω_{eng}	engine output speed	rpm
z_{mr}	number of stages in main-rotor drive	
P_{tr}	tail rotor drive system power limit	hp
Ω_{tr}	tail-rotor rotation speed	rpm
f_{tr}	tail-rotor power (fraction main-rotor power)	

30–7 Flight Controls Group and Hydraulic Group

The flight controls group consists of cockpit controls, automatic flight control system, and system controls. System controls consist of fixed wing flight controls, rotary wing flight controls, and conversion (rotor tilt) flight controls. The weight equations model separately non-boosted controls (which do not see aerodynamic surface or rotor loads), boost mechanisms (actuators), and boosted controls (which are affected by aerodynamic surface or rotor loads). The load path goes from pilot, to cockpit controls, to non-boosted controls, to boost mechanisms, to boosted controls, and finally to the component. The hydraulic group consists of hydraulics for fixed wing flight controls, rotary wing flight controls, and equipment.

Fixed wing flight controls consist of non-boosted flight controls and flight control boost mechanisms. The weights are:

GARTEUR

$$w = 0.0350 W_D \left(W_D / S_w \right)^{-0.36}$$

Raymer (cargo/transport)

$$w = 55.47 \frac{N_f^{0.554}}{1 + f_m} S_{cs}^{0.20} I_y^{0.07}$$

$$W_{FW\text{hyd}} = \chi_{FW\text{hyd}} 0.2673 N_f (\ell_{\text{body}} + b_w)^{0.937}$$

Raymer (general aviation)

$$w = 0.053 (n_z W_D / 10000)^{0.8} \ell_{\text{body}}^{1.536} b_w^{0.371}$$

generic

$$w = K_{FW} W_{MTO}^{X_{FW}}$$

with typically $N_f = 4$ to 7, $f_m N_f = 0$ to 2. Then

$$W_{FWnb} = \chi_{FWnb} f_{FWnb} \, w$$

$$W_{FWmb} = \chi_{FWmb} (1 - f_{FWnb}) \, w$$

$$W_{FW\text{hyd}} = \chi_{FW\text{hyd}} f_{FW\text{hyd}} W_{FWmb}$$

as for the AFDD model. Parameters are defined in table 30-9, including units as used in these equations.

Table 30-9. Parameters for fixed wing flight control weight.

parameter	definition	units
W_{MTO}	maximum takeoff weight	
W_D	design gross weight	lb
S_w	wing planform area (theoretical)	ft^2
N_f	number of functions performed by controls	
f_m	number of mechanical functions (fraction N_f)	
S_{cs}	total area of control surfaces	ft^2
I_y	yaw moment of inertia ($W_D K_y^2$)	lb-ft^2
ℓ_{body}	fuselage length	ft
b_w	wing span	ft

Rotary wing flight controls consist of non-boosted flight controls, flight control boost mechanisms, and boosted flight controls. The Boeing weight equations model the main rotor controls (from and including the power actuators, swashplate, up through the pitch links; identified as boost mechanisms plus boosted controls) and the rotor systems (between cockpit controls and rotor controls; identified as non-boosted controls) and hydraulics. The weights are:

Boeing

$$W_{cc} = \chi_{cc} 1.7 \Big[W_D \Big]^{0.41}$$

$$w_{mb+b} = 0.562 N_{\text{rotor}} \Big[c \left(RW_{\text{blade}} / N_{\text{rotor}} \right)^{0.5} \Big]^{1.11}$$

$$w_{nb+hyd} = 0.0906 N_{\text{rotor}} \Big[W_D / N_{\text{rotor}} \Big]^{0.84}$$

$$W_{RWmb} = \chi_{RWmb} (1 - f_{RWb}) w_{mb+b}$$

$$W_{RWb} = \chi_{RWb} f_{RWb} w_{mb+b}$$

$$W_{RWnb} = \chi_{RWnb} (1 - f_{RW\text{hyd}}) w_{nb+hyd}$$

$$W_{RW\text{hyd}} = \chi_{RW\text{hyd}} f_{RW\text{hyd}} w_{nb+hyd}$$

or

GARTEUR

$$w_{fc} = 0.0324 W_D^{1.0136}$$

Tishchenko

$$w_{fc} = 0.824 N_{\text{rotor}} N_{\text{blade}} c^2 R$$

generic

$$w_{fc} = K_{RW} (N_{\text{rotor}} N_{\text{blade}})^{X_{RWN}} R^{X_{RWR}} c^{X_{RWc}} (W_{MTO} / N_{\text{rotor}})^{X_{RWW}} (W_{\text{blade}} / N_{\text{rotor}})^{X_{RWb}}$$

$$W_{RWmb} = \chi_{RWmb} f_{RWmb} w_{fc}$$

$$W_{RWb} = \chi_{RWb} f_{RWb} w_{fc}$$

$$W_{RWnb} = \chi_{RWnb} (1 - f_{RWmb} - f_{RWb} - f_{RW\text{hyd}}) w_{fc}$$

$$W_{RW\text{hyd}} = \chi_{RW\text{hyd}} f_{RW\text{hyd}} w_{fc}$$

using system units for the generic model. Parameters are defined in table 30-10, including units as used in these equations.

Table 30-10. Parameters for rotary wing flight control weight.

parameter	definition	units
W_{MTO}	maximum takeoff weight	
W_D	design gross weight	lb
c	rotor mean blade chord	ft
R	rotor radius	ft
W_{blade}	weight rotor blades	lb
N_{rotor}	number of main rotors	
N_{blade}	number of blades	

30–8   Parametric Weight Correlation

Tables 30-11 and 30-12 summarize the statistics of the parametric weight estimation equations, for the AFDD data base and a data base from Boeing (ref. 4), respectively. The technology factor χ was adjusted for each equation to minimize the average error. Except for the wing group, the aircraft in the data base are all helicopters, in particular for the fuselage group and empennage group. The fuel system weight is for just the tanks; the technology factors are about 70% larger for both tanks and plumbing. The drive system weight includes the tail rotor.

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Table 30-11. Statistics of parametric weight equations, AFDD data base.

model	number of aircraft	average error (%)	technology factor
wing group			
Boeing	23	14.1	$\chi = 0.73$
GARTEUR	23	20.9	$\chi = 1.07$
Torenbeek	23	20.3	$\chi = 1.04$
Raymer (cargo/transport)	23	16.9	$\chi = 1.24$
Raymer (general aviation)	23	16.8	$\chi = 1.31$
rotor group, blade			
Boeing	51	10.1	$\chi = 0.90$
GARTEUR	51	11.8	$\chi = 0.98$
Tishchenko	51	18.8	$\chi = 0.99$
rotor group, hub			
Boeing	45	27.5	$\chi = 1.07$
GARTEUR	45	27.6	$\chi = 1.34$
Tishchenko	51	22.8	$\chi = 0.89$
horizontal tail			
GARTEUR	13	37.3	$\chi = 0.65$
Torenbeek (transport)	13	24.7	$\chi = 1.80$
vertical tail			
GARTEUR	12	26.7	$\chi = 0.79$
Torenbeek (transport)	12	29.2	$\chi = 1.92$
tail rotor			
Boeing	18	28.5	$\chi = 0.64$
GARTEUR	18	18.0	$\chi = 0.67$
fuselage group			
GARTEUR (airplane)	28	13.0	$\chi = 0.89$
GARTEUR (helicopter)	28	21.5	$\chi = 0.94$
Tishchenko	28	15.0	$\chi = 0.90$
engine section or nacelle group			
Boeing (support)	14	24.9	$\chi = 1.30$
Boeing (cowl)	17	23.0	$\chi = 0.84$
Raymer (cargo/transport)	16	23.7	$\chi = 0.64$
propeller			
Boeing	13	27.9	$\chi = 1.13$
GARTEUR	13	30.0	$\chi = 1.18$
Torenbeek	13	24.4	$\chi = 0.77$
fuel system			
Raymer (cargo/transport)	34	24.8	$\chi = 1.75$
Raymer (general aviation)	34	27.2	$\chi = 1.10$
Torenbeek (integral tanks)	32	29.2	$\chi = 0.60$
Torenbeek (generic)	34	29.3	$\chi = 0.90$
drive system			
Boeing (alternate)	81	12.1	$\chi = 0.61$
GARTEUR (helicopter)	81	15.6	$\chi = 0.90$
GARTEUR (tiltrotor)	2	4.3	$\chi = 0.75$
Tishchenko	81	12.1	$\chi = 0.99$
tail rotor	24	19.5	$K_{trgb} = 1.00$
rotary wing flight controls			
Boeing	24	19.0	$\chi = 0.91$
GARTEUR	23	20.1	$\chi = 0.81$
Tishchenko	22	32.8	$\chi = 1.35$

Table 30-11. Statistics of parametric weight equations, Boeing data base.

model	number of aircraft	average error (%)	technology factor
rotor group, blade			
Boeing	20	6.0	$\chi = 1.05$
GARTEUR	20	12.8	$\chi = 0.93$
Tishchenko	20	16.3	$\chi = 0.90$
rotor group, hub			
Boeing	20	10.6	$\chi = 0.93$
GARTEUR	18	11.3	$\chi = 1.28$
Tishchenko	19	20.5	$\chi = 0.99$
fuselage group			
Boeing	26	12.9	$\chi = 1.04$
GARTEUR (airplane)	23	16.6	$\chi = 0.89$
GARTEUR (helicopter)	26	16.5	$\chi = 0.98$
Tishchenko	23	12.9	$\chi = 0.87$
propeller			
Boeing	12	19.1	$\chi = 0.95$
GARTEUR	12	18.3	$\chi = 0.97$
Torenbeek	12	21.0	$\chi = 0.51$
drive system			
Boeing	25	6.3	$\chi = 0.98$
Boeing (alternate)	27	10.2	$\chi = 0.73$
GARTEUR (helicopter)	27	14.4	$\chi = 1.07$
Tishchenko	27	10.2	$\chi = 1.18$

Small Aircraft Weight Models

Weight estimation models for small unmanned aircraft systems (UAS) are available from a number of sources (refs. 1 to 11). The weights are estimated from parametric equations, based on the weights of existing components. This chapter summarizes the weight models for electric motors, electric control systems, and small propellers.

The results from the original equations are in various units; equations are presented here also for English and SI units. Technology factors χ are included in the weight equations. The input usually includes a weight increment dW that can be added to the results of the weight model. Thus typically a component or element weight is obtained from $W = \chi w_{\mathrm{model}} + dW$.

31–1   Electric Motors

The motor or generator weight can be a fixed input value, or calculated as a function of power or torque. As a function of power, the weight of one engine is:

$$W_{\mathrm{one\ eng}} = K_{0\mathrm{motor}} + K_{1\mathrm{motor}}P + K_{2\mathrm{motor}}P^{X_{\mathrm{motor}}}Q^{X_{q\mathrm{motor}}} + K_{\mathrm{ESC}}P^{X_{\mathrm{ESC}}}$$

where $P = P_{\mathrm{eng}}$ (hp or kW), and $Q = P_{\mathrm{peak}R}/N_{\mathrm{spec}}$ (ft-lb or m-N) for takeoff rating R. Note then $P^X Q^{X_q} = P^{X+X_q}N^{-X_q} = N^X Q^{X+X_q}$. The last term accounts for the weight of the electronic speed control system. The equations in SI and English units are summarized in table 31-1, and compared in figures 31-1 and 31-2.

Noth (2008): Brushless model aircraft motors

$$W_{\mathrm{one-eng}} = 0.294P$$

for maximum continuous power P in kW, weight in kg. Based on 784 motors, power from 10 W to 10 kW, the average error is about 30%. Also, for commercial brushed motors

$$W_{\mathrm{one-eng}} = 11.1P$$

Based on 1272 motors, power from 0.1 W to 100 W, the average error is about 50%.

Moodie (2017): Consumer market motors

$$W_{\mathrm{one-eng}} = 1.339 \left(\frac{\Omega}{1000} \right)^{-0.2933} P^{0.8272}(L/D)^{0.3363} = 5.239(P/Q)^{-0.2933}P^{0.8272}(L/D)^{0.3363}$$

for maximum motor speed Ω in rpm, peak power P in W, motor length-to-diameter ratio L/D (average 1.6), weight in g. Based on 605 motors, power from 15 W to 3 kW, the average error is 27.9%. Based on the same data set

$$W_{\text{one-eng}} = 0.971 \left(\frac{\Omega}{1000} \right)^{-0.1867} P^{0.8541} = 2.314 (P/Q)^{-0.1867} P^{0.8541}$$

with average error 29.0%; or

$$W_{\text{one-eng}} = 0.593 P^{0.8443}$$

with average error 31.4%.

Strom (2017): Brushless DC motors

$$W_{\text{one-eng}} = 0.0.00084659 P^{0.98831}$$

for power in ft-lb/sec, weight in lb. Based on 244 motors, power up to 12 hp, the average error is about 20%.

Russell (2018): Small motors

$$W_{\text{one-eng}} = 0.3256 Q^{0.5017}$$

for maximum torque Q in ft-lb, weight in lb. Based on 5 motors, torque from 0.4 to 1.1 ft-lb and power from 400 W to 1200 W, the average error is 5%. Based on the same data set

$$W_{\text{one-eng}} = 0.01169 P^{0.4915}$$

for maximum power P in W, weight in lb; the average error is 5%.

Ng (2018): AC permanent magnet synchronous motors for aeronautical applications

$$W_{\text{one-eng}} = 0.4025 Q^{0.71}$$

for maximum continuous torque Q in Nm, weight in kg. Based on 17 motors, torque from 3 Nm to 1000 Nm and power from 4 kW to 260 kW, the average error is about 20%.

$$W_{\text{one-eng}} = 0.4107 P^{0.89}$$

for maximum continuous power P in kW; the average error is about 20%.

Winslow (2018): From database of brushless DC motors

$$W_{\text{one-eng}} = 0.3930 P^{0.9424}$$

for power P in W, weight in g. Based on 296 motors, power from 20 W to 9800 W, the average error is 22.9%.

Lim (2018); Brushless DC motors

$$W_{\text{one-eng}} = 0.2259P - 2.236$$

for maximum continuous power P in W, weight in g. Based on several hundred motors, power up to 4.5kW, the average error is about 30%.

Duffy (2018): Aviation motors

$$W_{\text{one-eng}} = 0.58P^{0.80}$$

for maximum continuous power P in kW, weight in kg. Based on 49 motors, power from 3 kW to 300 kW and torque from 5 Nm to 1800 Nm, the average error is about 35%.

$$W_{\text{one-eng}} = 0.33Q^{0.75}$$

for maximum continuous torque Q in Nm, weight in kg; the average error is about 25%.

NDARC NASA15: Motor weight depends primarily on the peak torque. Considering only the high torque-to-weight motors, the weight is

$$W_{\text{one eng}} = 0.3928Q^{0.8587}$$

where $Q = P_{\text{peak}R}/N_{\text{spec}}$ (ft-lb) for takeoff rating R. Based on 25 motors, power from 20 hp to 540 hp and torque from 15 ft-lb to 61000 ft-lb, the average error is 21.8%.

31–2   Electric Control Systems

The electric control system or electronic speed control weight can be estimated as a function of power or current. From the maximum current I and operating voltage V, the device power is $P = IV/1000$ kW or $P = 745.7IV/550$ hp. The equations in SI and English units are summarized in table 31-2, and compared in figurea 31-3 and 31-4.

Noth (2008): Motor control electronics for brushless motors

$$W_{\text{ESC}} = 0.0260P$$

for maximum power P in kW, weight in kg. Based on 170 devices, power from 10 W to 10 kW, the average error is about 50%. A technology factor of 2.3 is recommended (0.06 kg/kW).

Schömann (2013): Electronic speed control

$$W_{\text{ESC}} = 0.026P_{\text{ESC}} + 5.747$$

for maximum continuous power P_{ESC} in W, weight in g. Based on 74 devices, power from 50 W to 11 kW, the average error is 42.4%.

Bershadsky (2016): Electronic speed controller

$$W_{\mathrm{ESC}} = 0.8421 I_{\max}$$

for maximum current $I_{\max}$ in Amp, weight in g. Based on 16 devices, current from 7 A to 100 A, the average error is about 20%.

Moodie (2017): Electronic speed control for consumer market

$$W_{\mathrm{ESC}} = 2.998 I_{\max}^{0.863}$$

for maximum current $I_{\max}$ in Amp, weight in g. Based on 44 devices, current from 5 A to 180 A, the average error is 31.9%.

Strom (2017): Electronic speed controller

$$W_{\mathrm{ESC}} = 0.0030939 I_{\max}^{0.91428}$$

for maximum current $I_{\max}$ in Amp, weight in lb. Baed on 175 devices, current up to 200 A, the average error is about 30%.

Winslow (2018): From database of electronic speed controller

$$W_{\mathrm{ESC}} = 0.893 I_{\max}^{.9355}$$

for maximum current $I_{\max}$ in Amp, weight in g. Based on 56 devices, current from 7 A to 250 A, the average error is 22.1% (15.8% for weight less than 50 g).

Lim (2018): Electronic speed controller

$$W_{\mathrm{ESC}} = 1.1652 I_{\max} - 2$$

for maximum current $I_{\max}$ in Amp, weight in g. Based on about 100 devices, current from 5 A to 100 A, the average error is about 30%.

Duffy (2018): Inverters and motor controllers

$$W_{\mathrm{ESC}} = 0.125 P^{0.96}$$

for maximum continuous power P in kW, weight in kg. Based on 45 devices, power from 1 kW to 600 kW, the average error is 55%.

31–3   Small Propellers

The NDARC generic rotor and propeller weight models are

$$w_{\mathrm{blade}} = K_{\mathrm{blade}} N_{\mathrm{rotor}} N_{\mathrm{blade}}^{X_{\mathrm{bld}N}} R^{X_{\mathrm{bld}R}} c^{X_{\mathrm{bld}c}} V_{\mathrm{tip}}^{X_{\mathrm{bld}V}} \nu_{\mathrm{blade}}^{X_{\mathrm{bld}\nu}} (W_{MTO}/N_{\mathrm{rotor}})^{X_{\mathrm{bld}W}}$$

$$w_{\mathrm{hub}} = K_{\mathrm{hub}} N_{\mathrm{rotor}} N_{\mathrm{blade}}^{X_{\mathrm{hub}N}} R^{X_{\mathrm{hub}R}} c^{X_{\mathrm{hub}c}} V_{\mathrm{tip}}^{X_{\mathrm{hub}V}} \nu_{\mathrm{hub}}^{X_{\mathrm{hub}\nu}} (W_{MTO}/N_{\mathrm{rotor}})^{X_{\mathrm{hub}W}}$$

$$W_{\mathrm{at}} = \chi_{\mathrm{at}} K_{\mathrm{at}} N_{\mathrm{at}} N_{\mathrm{blade}}^{X_{\mathrm{at}N}} R^{X_{\mathrm{at}R}} c^{X_{\mathrm{at}c}} V_{\mathrm{tip}}^{X_{\mathrm{at}V}} P_{\mathrm{at}}^{X_{\mathrm{at}P}}$$

using system units. The equations in SI and English units are summarized in table 31-3, and compared in figure 31-5.

Bershadsky (2016): Small propellers

$$w_{\text{prop}} = 0.08884D^2$$

for diameter D in inches, weight in g. Based on about 30 propellers, diameter from 4 in to 30 in, the average error is about 30%. Separate equations are given for wooden propellerss (to 11 in), plastic (13 in), nylon reinforced plastic (14 in), and carbon fiber (30 in), but there is too little data to support separate equations.

Moodie (2017): Small propellers

$$w_{\text{prop}} = 0.00177D^{2.57329}\phi^{0.48293}N_{\text{blade}}^{3.16944}$$

for diameter D in inches, the pitch ($\phi = 2\pi r * \tan\beta$, distance propeller moves without slip in one revolution), weight in g. Based on 150 small propellerss, diameter 1.1 in to 73.2 in, the average error is 27.8%. Or

$$w_{\text{prop}} = 0.00374D^{2.35705}N_{\text{blade}}^{3.40172}$$

with average error 33.0%.

Russell (2018): Small propellers

$$w_{\text{prop}} = 0.1413R^{2.1324}$$

for radius R in ft, weight in lb. Based on 6 propellers, diameter from 13 in to 30 in, the average error is 12%.

Winslow (2018): Small propellers

$$w_{\text{prop}} = 0.0195R^{2.0859}\sigma^{-0.2038}N_{\text{blade}}^{0.5344} = 0.0246R^{2.2897}c^{-0.2038}N_{\text{blade}}^{0.3306}$$

for radius R in cm, solidity $\sigma = N_{\text{blade}}c/\pi R$, number of blades N_{blade}, weight in g. Based on 19 propellers, diameter 7 cm to 38 cm, the average error is about 20%. From the same database

$$w_{\text{prop}} = 98.88(2R)^{2.2049}N_{\text{blade}}^{0.8717}$$

for diameter $2R$ in m, number of blades N_{blade}, weight in g. Based on 19 propellers, diameter 7 cm to 38 cm, the average error is 20.8%.

31–4   References

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Table 31-1. Summary of models for electric motor weights.

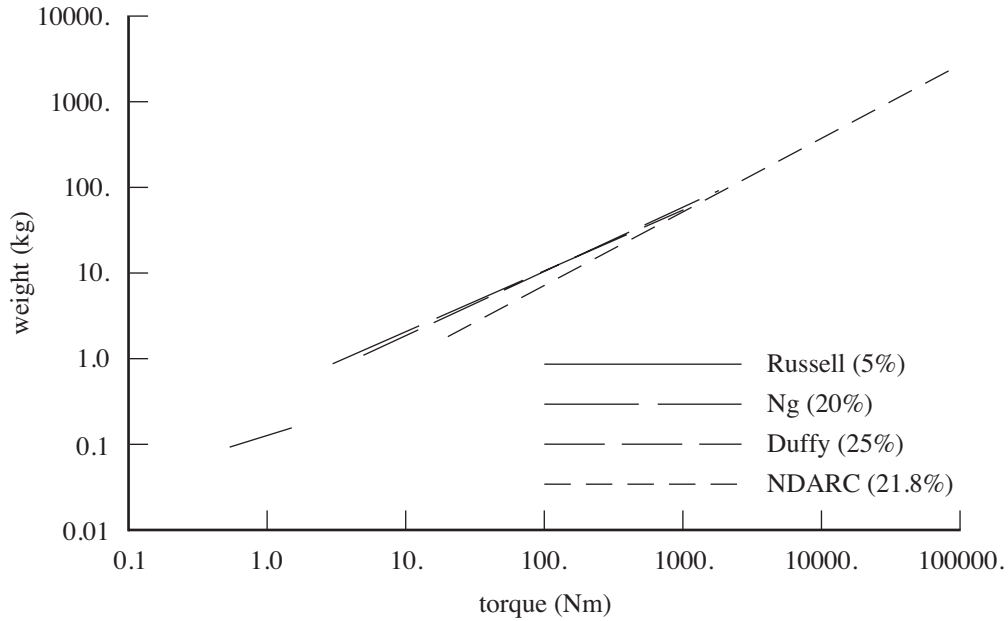|		SI units (kg, kW, Nm, m, Amp)	English units (lb, hp, ft-lb, ft, Amp)
Noth	$W_{\text{one-eng}} =$	$0.294P$	$0.48333P$
Moodie	$W_{\text{one-eng}} =$	$0.20228P^{0.8443}$	$0.34809P^{0.8443}$
Strom	$W_{\text{one-eng}} =$	$0.26219P^{0.98831}$	$0.43251P^{0.98831}$
Russell	$W_{\text{one-eng}} =$	$0.15812P^{0.4915}$	$0.30177P^{0.4915}$
Ng	$W_{\text{one-eng}} =$	$0.4107P^{0.89}$	$0.69733P^{0.89}$
Winslow	$W_{\text{one-eng}} =$	$0.26399P^{0.9424}$	$0.44140P^{0.9424}$
Lim	$W_{\text{one-eng}} =$	$0.2259P - 0.002236$	$0.49802P - 0.0049295$
Duffy	$W_{\text{one-eng}} =$	$0.58P^{0.8}$	$1.01115P^{0.8}$
Russell	$W_{\text{one-eng}} =$	$0.12677Q^{0.5017}$	$0.3256Q^{0.5017}$
Ng	$W_{\text{one-eng}} =$	$0.4025Q^{0.71}$	$1.10145Q^{0.71}$
Duffy	$W_{\text{one-eng}} =$	$0.33Q^{0.75}$	$0.91411Q^{0.75}$
NASA15	$W_{\text{one-eng}} =$	$0.13719Q^{0.8587}$	$0.3928Q^{0.8587}$

Figure 31-1. Comparison of models for motor weight.

Table 31-2. Summary of models for electric control system weights.

		SI units (kg, kW, Nm, m, Amp)	English units (lb, hp, ft-lb, ft, Amp)
Noth	$W_{\mathrm{ESC}} =$	$0.026P$	$0.042744P$
Schomann	$W_{\mathrm{ESC}} =$	$0.026P + 0.005747$	$0.042744P + 0.012670$
Duffy	$W_{\mathrm{ESC}} =$	$0.125P^{0.96}$	$0.20792P^{0.96}$
Bershadsky	$W_{\mathrm{ESC}} =$	$0.0008421I$	$0.0018565I$
Moodie	$W_{\mathrm{ESC}} =$	$0.002998I^{0.863}$	$0.0066095I^{0.863}$
Strom	$W_{\mathrm{ESC}} =$	$0.0014034I^{0.91428}$	$0.0030939I^{0.91428}$
Winslow	$W_{\mathrm{ESC}} =$	$0.000893I^{0.9355}$	$0.0019687I^{0.9355}$
Lim	$W_{\mathrm{ESC}} =$	$0.0011652I - 0.002$	$0.0025688I - 0.0044092$

Table 31-3. Summary of models for small propeller weights.

|		SI units (kg, kW, Nm, m, Amp)	English units (lb, hp, ft-lb, ft, Amp)
Bershadsky	$w_{\mathrm{prop}} =$	$0.550810R^2$	$0.11281R^2$
Moodie	$w_{\mathrm{prop}} =$	$0.110230R^{2.35705}N_{\mathrm{blade}}^{3.40172}$	$0.014772R^{2.35705}N_{\mathrm{blade}}^{3.40172}$
Russell	$w_{\mathrm{prop}} =$	$0.807411R^{2.1324}$	$0.1413R^{2.1324}$
Winslow	$w_{\mathrm{prop}} =$	$0.365724R^{2.2897}c^{-0.2038}N_{\mathrm{blade}}^{0.3306}$	$0.067639R^{2.2897}c^{-0.2038}N_{\mathrm{blade}}^{0.3306}$
Winslow	$w_{\mathrm{prop}} =$	$0.455879R^{2.2049}N_{\mathrm{blade}}^{0.8717}$	$0.073196R^{2.2049}N_{\mathrm{blade}}^{0.8717}$
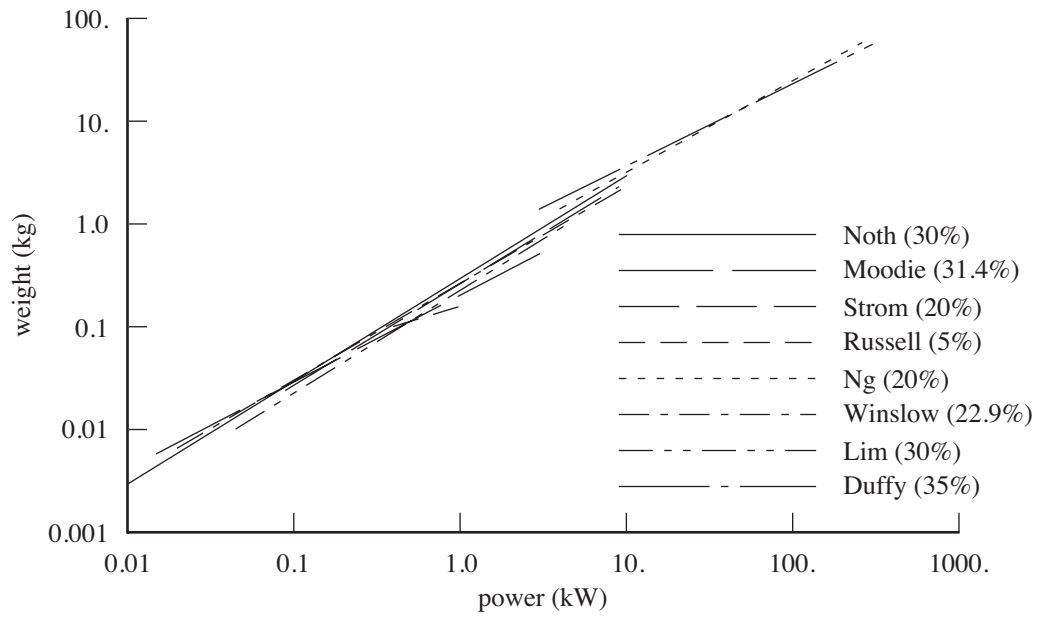
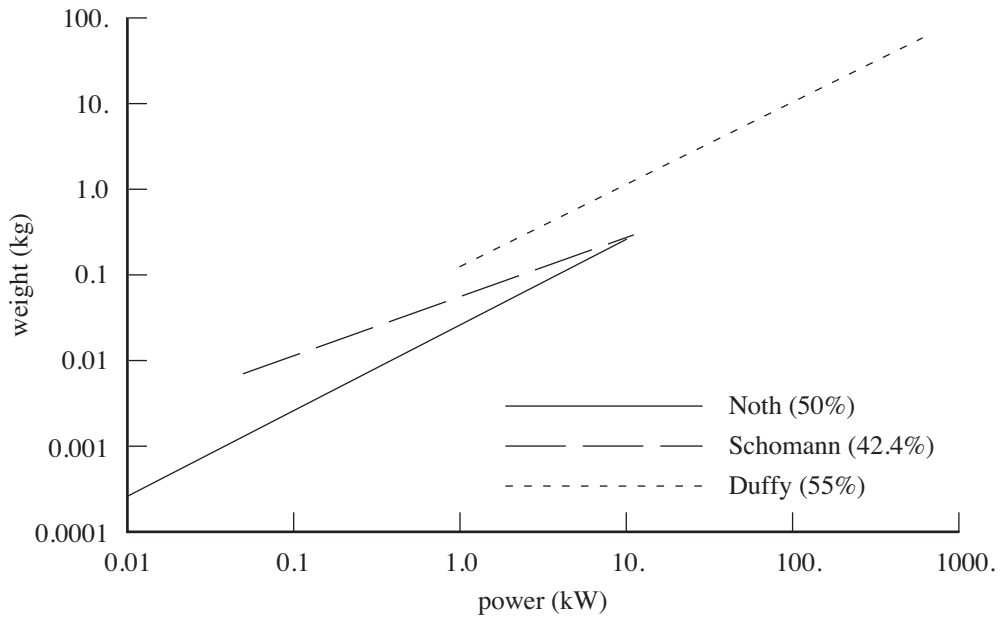Figure 31-2. Comparison of models for motor weight.

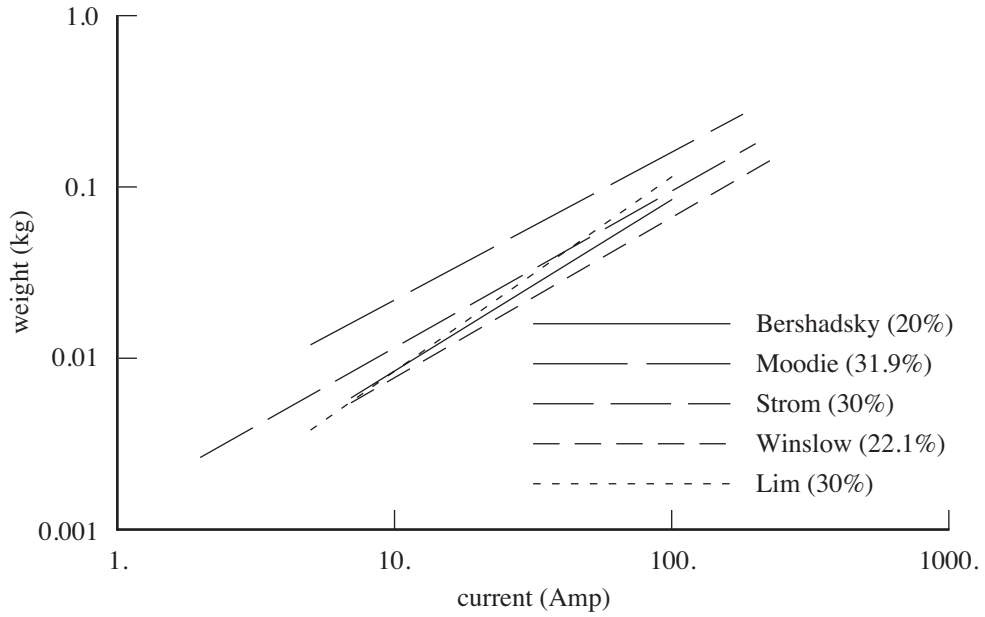Figure 31-3. Comparison of models for electric control system weight.

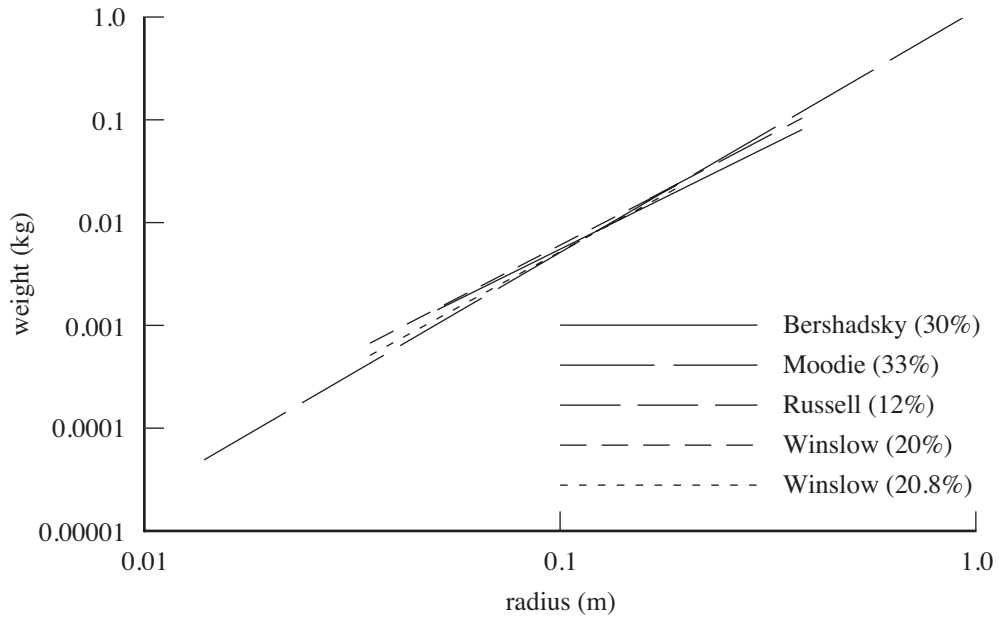Figure 31-4. Comparison of models for electric control system weight.

Figure 31-5. Comparison of models for propeller weight.