Center of Mass and Center of Figure of Moon, Gravity and Inertia

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Definition

The center of the mass of the Moon is displaced from the center of figure. A model of the Moon in which the core is offset does account for the offset of the center of gravity from the center of figure, the moments of inertia of the Moon, the mascons and localization of Maria basins, and the igneous nature of rocks.

Calculations of Offset

The center of mass of the Moon is displaced from the center of figure. Earlier measurements had this displacement be 198 ± 0.06 km in a direction 14° ± 1° to the east of the Earth-pointing vector (Bartlett & Van Buren 1986). The simplest model assumes a two-component Moon with a spherical mantle (density = 3.35 g/cm^3) offset ~10 km from the center of figure by a crust (density = 2.9 g/cm^3) (Bartlett & Van Buren 1986 and references therein).

Smith et al. (1997) computed this offset of the center of figure from the center of mass to be (-1.74, -0.75, 0.27) km in the (x, y, z) directions, respectively. On the far side of the Moon, this offset is displaced ~25° toward the western limb and slightly north of the equator (Zuber et al. 1994). Displacements that result from the irregular shape of the Moon are thus isostatically compensated, perhaps by variations in crustal thickness (Figure 1; Zuber et al. 1994). Gravity fields were also measured, observing that the major axis of the lunar gravity field is aligned in the Earth-Moon direction, the major axis of topography is displaced from this line by ~10° to the east and intersects the far side 24° north from the equator (Smith et al. 1997). Apollo laser altimetry data were used to determine that the lunar mean radius of 1,737.7 km and the offset of the center of figure from the center of mass of -2.55 km in the direction of 25°E (Kaula et al. 1974; Smith et al. 1997).

The moments of inertia for this offset are described in Ransford and Sjogren (1972), where they determine an outer crust density of 3.315 g/cm^3 and a center of the core 466 km from the center of mass. If a difference in density of 0.4 g/cm^3 is assumed, a core of 700 km in diameter is calculated. This in turn implies an outer shell of 572-km thickness towards the Earth and a 1,504-km thickness on the far side (Ransford & Sjogren 1972).

On the lunar nearside, the cooling rate of a unit area perpendicular to the line joining the center of the Moon and the center of the Earth is:

$$\Delta q_f = -\sigma\left({T_M}^4 - \frac{{R_E}^2}{{R_{EM}}^2}{T_E}^4\right)$$

Where σ = Stefan-Boltzmann constant, T_M=temperature of the Moon, T_E=temperature of the Earth, R_E=radius of the Earth, and R_{EM}=distance between the Earth and the Moon.

For the lunar far side, this equation is:

$$\Delta q_b = -\sigma {T_M}^4$$

Note that the negative sign indicates that heat is radiated, and positive sign is heat absorbed.

The ratio of these show:

$$\frac{\Delta q_f}{\Delta q_b} = \frac{{T_M}^4 - \frac{{R_E}^2}{{R_{EM}}^2}{T_E}^4}{T_M^4}$$

If the ratio remains constant, it must represent the ratio of the crust build-up on the near side to the rate of far-side crustal build-up. For primordial Earth temperatures >6,000 K and Moon temperatures of 1,200 – 2,000 K, significant differences in the crust building will occur (Ransford & Sjogren 1972).

Lunar Topography Evolution and Surface Materials

Range measurements carried on multiple spacecraft, such as LOLA or lidar aboard the Clementine, have been used to produce accurate global topographic models of the Moon, and thus define the shape of the Moon. The Apollo 15, 16, and 17 missions carried laser altimeters (Kaula et al. 1973, 1974), which provided the first information on the shape of the Moon in a center of mass reference frame.

Such high-resolution maps describe the Moon as a sphere with a maximum positive and negative deviations of ~8 km, both occurring on the far side (e.g., Korolev and the South Pole - Aitken Basins). Such an offset of the core would be formed if the Moon were entirely molten early in its geologic history, and on solidification was synchronous with the Earth. A molten lunar surface is implied by samples returned from the Apollo missions (Team 1970; Swann et al. 1971; Roberson & Kaula 1972). The Apollo instruments, such as the surface magnetometers and seismometers, also indicated a layered lunar crust.

The early-forming molten Moon would mainly cool by radiation into space, while the Earth-facing side would be heated by Earth (Ransford & Sjogren 1972). The Sun's heat would also heat the outer layers but would be lost during the lunar night. Considering the equilibrium shape of a fluid body acted on by its own gravitational rotation and outside pull, there will be a small distortion in the system in the direction of this external pull (Ransford & Sjogren 1972). So long as the body remains fluid, this distortion would maintain its alignment in the pull direction, and when such a body solidifies, this distortion would be essentially "frozen" in place. The newly formed crust would have such a low heat conductivity and therefore the cooling rate would diminish. Further evidence for a molten lunar core comes from the samples from the Apollo missions, where remnant magnetization suggests that their formation was during a shrinking central core of molten material early in the Moon's geologic history (Runcorn 1977; Williams et al. 2001).

Redistribution of crustal material from major impacts (i.e., South Pole - Aitken Basin) has significantly influenced the overall lunar shape, with the magnitude of such major impact basin topography exceeding the magnitudes of long-wavelength deviations from sphericity (Smith et al. 1997). There is also an observed asymmetry in the composition of the Moon, namely Fe and Al, where the iron-rich basaltic Maria which mark the Earth-facing side of the Moon are absent from the far side surface, which is almost entirely of aluminum-rich anorthositic highlands (Bartlett & Van Buren 1986).

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Figures and Tables:

Figure 1: Geometrical view of lunar core offset in equatorial projection (y-axis). Adapted from Ransford & Sjogren (1972).