Numerical phase space optics methods and applications to the analysis of fiber coupling efficiency in atmospheric turbulence

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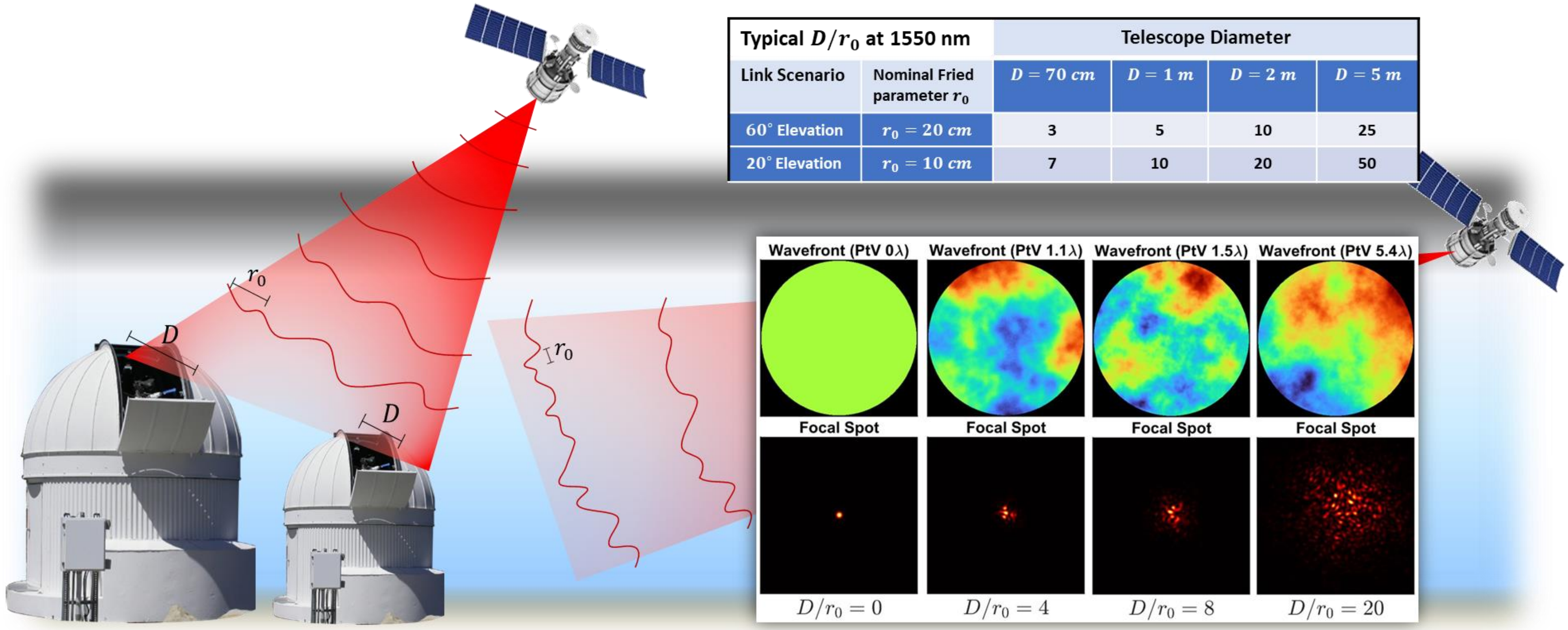
Motivation

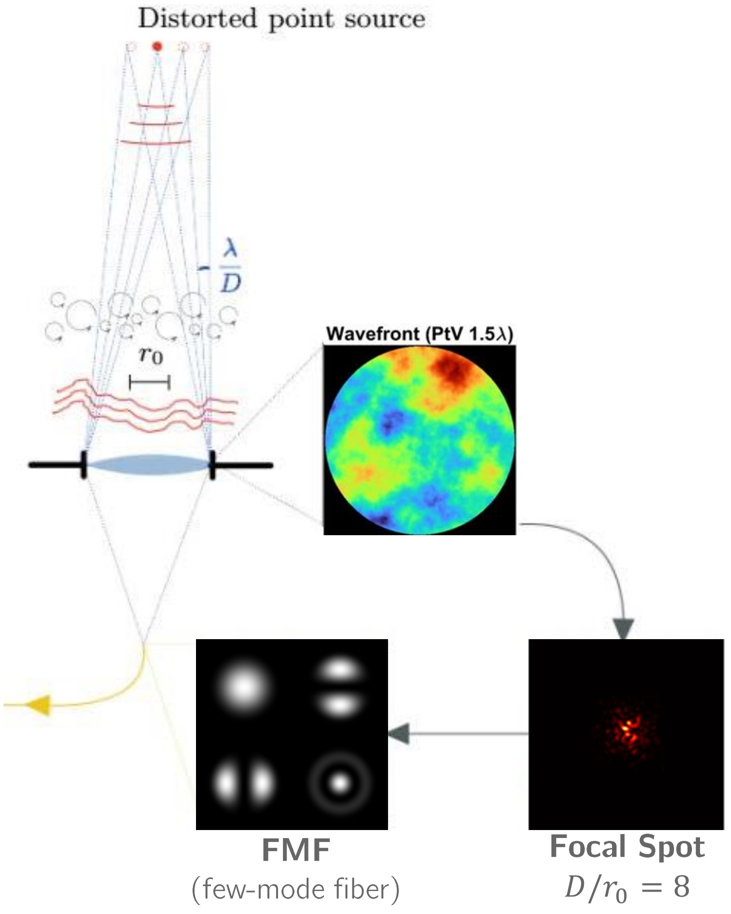
Typical D/r_0 at 1550 nm		Telescope Diameter			
Link Scenario	Nominal Fried parameter r_0	$D = 70\ cm$	$D = 1\ m$	$D = 2\ m$	$D = 5\ m$
60° Elevation	$r_0 = 20\ cm$	3	5	10	25
20° Elevation	$r_0 = 10\ cm$	7	10	20	50



- **Goal:** Development of low-cost photon counting optical ground receiver for space-to-ground links.
- **Approach:** Explore passive (no adaptive optics) fiber-coupled receiver designs capable of operating at peak efficiency in a range of atmospheric conditions.

Motivation

- Light arriving at a ground-based telescope is distorted by refractive index fluctuations in the atmosphere.
- **Blurring of the focal spot limits the coupling efficiency for fiber-coupled systems which do not support many spatial modes.**
- More spatial modes → Higher cost/complexity for photon-counting detection
- Spatial mode constraints not unique to FSOC (applications to astrophotonics/remote sensing).
- Adaptive optics (AO) systems to correct distortions can be expensive.
- AO not strictly necessary for pulse-position modulation (PPM) links.



- What are the fundamental and practical limits to coupling efficiency for passive mode-limited optical receivers over turbulent optical channels?
- How to design a passive mode-limited optical system optimized to approach peak efficiency in a range of optical turbulence conditions?

Outline

- Introduction
  - Phase space optics
  - Wigner distribution function
- Numerical phase space optics
- Phase space analysis of fiber coupling with optical turbulence
  - Waveguide Wigner distributions
  - Long-exposure/short-exposure Wigner distributions
  - Optimal efficiency and coupling geometry

Introduction

Why phase space optics?

- Pulse-position modulation (PPM) optical channels do not require coherent spatial optical processing (i.e. imaging). An imaging system is a receiver for a spatially multiplexed channel.
- Methods of imaging optics are nevertheless useful for efficient power collection, particularly to single-mode fiber (SMF) which benefit from the spatial mode processing that imaging systems and adaptive optics provide.
- Going from SMF-coupling to few-mode fiber (FMF), objectives for imaging and power collection separate.
- Without AO, optical comm. over atmospheric channels must deal with partially coherent optical fields.
- Passive receivers for PPM links over atmospheric channels have much in common with radiometry and non-imaging optics (power transfer and concentration of light from incoherent/partially-coherent sources) which make extensive use of phase space methods.

The problem of efficient collection of light from partially coherent fields into optical waveguides is given a robust framework within the phase space formulation of paraxial wave optics (Wigner optics).

Phase space optics and Wigner distributions

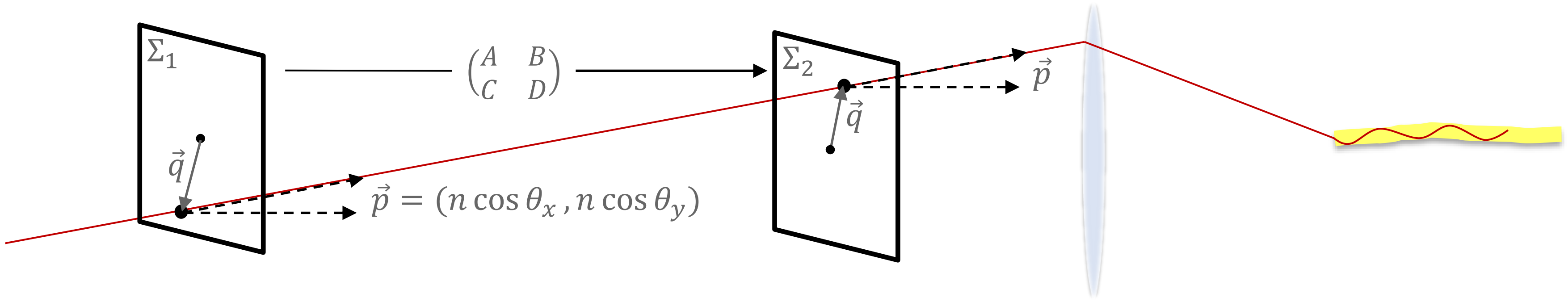
Geometrical optics in phase space:

- Each **point in phase space** corresponds to an **optical ray**.
- **State.** Beam represented by a phase space distribution ρ (e.g. the radiance).
- **Coordinates.** In any plane Σ_1 transverse to optical axis each ray gets canonical coordinates $(\vec{q}, \vec{p})$:
  - $\vec{q} =$ position of ray intersection
  - $\vec{p} =$ direction cosines $\times$ refractive index of surrounding medium

Conservation of étendue. Given a second plane Σ_2 parallel to Σ_1, the phase space volume element

$$dG = d^2\vec{q}d^2\vec{p}$$

is preserved by the change of coordinates from Σ_1 to Σ_2. Given a bundle of rays Ω, the volume $\mathrm{G} = vol(\Omega)$ of that bundle of rays is well-defined identical in any transverse plane (also yields *brightness theorem*).



Phase space optics and Wigner distributions

Wave optics in phase space (Wigner optics):

- Paraxial wave optics traditionally formulated in configuration (position) space $\vec{q} = (q_x, q_y)$.
- State represented by a complex optical field $\psi(\vec{q})$ (or a statistical ensemble $\{(\psi_i, \rho_i)\}$).
- A representation in phase space introduced by Wigner (1932).

Definition. The ***Wigner distribution function*** (WDF) for a complex optical field $\psi(\vec{q})$ is defined by

$$W_\psi(\vec{q},\vec{p}) = \frac{1}{\lambda^2}\iint \psi^*\left(\vec{q} - \frac{\vec{r}}{2}\right)\psi\left(\vec{q} + \frac{\vec{r}}{2}\right)e^{-ik\vec{p}\cdot\vec{r}}d^2\vec{r}.$$

- Joint space-frequency representation. Quasi-probability distribution on phase space $\mathbf{u} = (\vec{q}, \vec{p})$.
- For a statistical ensemble $\{\psi_i, \rho_i\}$ the WDF of the ensemble is the ensemble average $W = \sum_i \rho_i W_{\psi_i}$. Equivalently, given the MCF $\mu(\vec{q}_1, \vec{q}_2) = \langle \psi(\vec{q}_1)\psi(\vec{q}_2)^* \rangle$:

$$W(\vec{q},\vec{p}) = \frac{1}{\lambda^2}\iint \mu\left(\vec{q} + \frac{\vec{r}}{2}, \vec{q} - \frac{\vec{r}}{2}\right)e^{-ik\vec{p}\cdot\vec{r}}d^2\vec{r}.$$

- **Note**: The WDF depends **quadratically** on the optical field ψ.

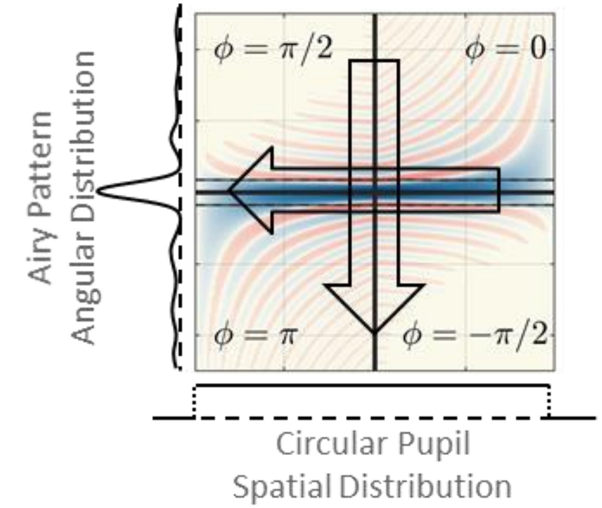
Properties of the Wigner distribution function

Marginals. Integrating WDF over all optical momenta (or all space) yields power density in the conjugate domain

$$|\boldsymbol{\psi}(\vec{\boldsymbol{q}})|^2 = \iint W_\psi(\vec{q},\vec{p})d^2\vec{p},$$

$$\left|\widetilde{\boldsymbol{\psi}}(\vec{\boldsymbol{p}})\right|^2 = \iint W_\psi(\vec{q},\vec{p})d^2\vec{q},$$

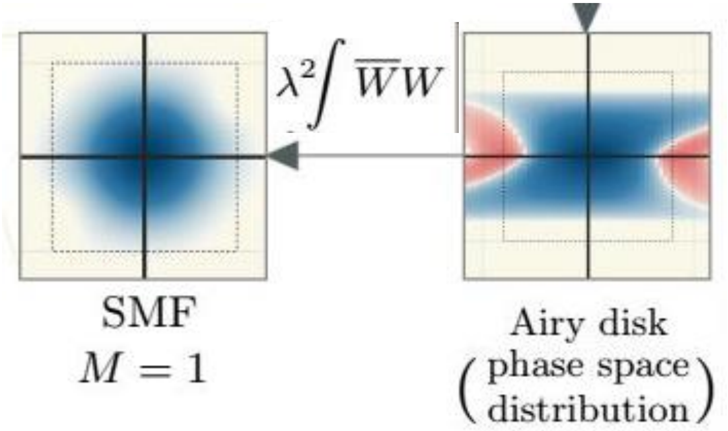
where $\widetilde{\boldsymbol{\psi}}$ is the Fourier transform rescaled by λ. Integrating over phase space gives the L^2-norm $|\langle\psi,\psi\rangle|^2$.



Mode coupling. Square-modulus of L^2 inner product given by overlap integral of WDFs:

$$|\langle\boldsymbol{\varphi},\boldsymbol{\psi}\rangle|^2 = \left(W_\varphi, W_\psi\right) = \lambda^2\iint W_\varphi(\vec{q},\vec{p})W_\psi(\vec{q},\vec{p})d^2\vec{q}d^2\vec{p}.$$

Square-modulus is a bilinear functional of Wigner distributions.



Propagation of the Wigner distribution function

First-order optical systems. Paraxial propagation through a first-order optical system with ray-transfer matrix $\boldsymbol{S}$:

$$W_{out}(\boldsymbol{u}) = W_{in}(\boldsymbol{S}^{-1}\boldsymbol{u}).$$

Wigner distribution is constant along optical rays in first-order optical systems.

Optical misalignments. Decentering and tilt relative to optical axis represented by phase space translation

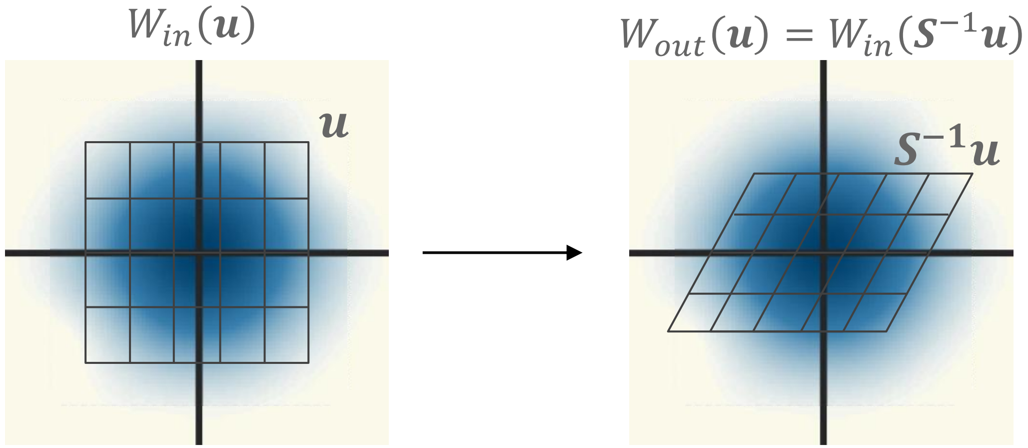
$$\boldsymbol{u} \mapsto \boldsymbol{u} + \boldsymbol{v}$$

where $\boldsymbol{v} = (\vec{\mathrm{q}}, \vec{\mathrm{p}})$ represents transverse shift of the axis $\vec{q}$ and tilt of the axis $\vec{p} = \left(\cos(\theta_x), \cos(\theta_y)\right)$.

Wave propagation through first-order optical systems represented by affine coordinate transformation:

$$A = \begin{pmatrix} \boldsymbol{S} & \boldsymbol{v} \\ 0 & 1 \end{pmatrix}$$

The same transformation also propagates the MCF.



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  - ➢ Phase space optics
  - ➢ Wigner distribution function
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  - ➢ Optimal efficiency and coupling geometry

Discrete Wigner Transform

Implementation

- Numerical calculation of the Wigner distribution function based on the fast Fourier transform (FFT).
- Known as the discrete Wigner-Ville distribution (DWVD) in time-frequency analysis.

Sampling

- Sampling of optical signal ψ for DWT determined by **FFT size $\boldsymbol{N}$** and spatial **grid spacing $\boldsymbol{\Delta q}$**
- Volume of discretized phase space element completely determined by FFT size N

$$\Delta q \Delta p = \frac{\lambda}{2N}$$

- Choice of sample spacing Δq determines balance between sampling in space or frequency domain.

Memory

- **Memory represents the primary constraint for a useful numerical implementation for 2D signals**.
- Naïve implementation requires $O(N^4)$ memory
- With truncation, memory requirement reduced to $O(N^2 S^2 B^2)$ where SB is space-bandwidth product of signal
- For signals/optical modes with axial symmetry, reduces to $O(N^2 (S\Delta \mathrm{q}) B^2)$ ⟵ **Significant reduction**

Algorithm for modeling free-space optical systems

1. Calculate the Wigner distributions W_1 and W_2 of the incident light and optical modes in the system, either via the coherent field ψ or via the MCF $\mu(\vec{q}_1, \vec{q}_2)$ for a partially coherent field.
   - This is done in a convenient reference plane where the field is known (e.g. at the transmitter, telescope aperture, or optical fiber).
2. Compute the affine change of coordinates A_1 and/or A_2 describing propagation to a common plane (e.g. focal plane), including any pointing offsets to be modeled.
3. Numerically approximate the coupling integral as a sum

$$\lambda^2(\Delta q)^2(\Delta p)^2 \sum_{ijkl} W_1\big(\boldsymbol{u}_{ijkl}\big) W_2(A_1^{-1} A_2 \boldsymbol{u}_{ijkl})$$

   by interpolating the Wigner distribution W_2 on the grid $\boldsymbol{u}'_{ijkl} = A_1^{-1} A_2 \boldsymbol{u}_{ijkl}$ determined by transforming the original grid $\boldsymbol{u}_{ijkl}$ used to sample the Wigner distribution W_1.

Algorithm for modeling free-space optical systems

Remarks:

- Propagation effects are modeled via **interpolation of the original Wigner distribution function**.
- After the initial computation of WDF, no need for Fresnel integrals or implementation of boundary conditions.
- **The same algorithm yields propagation of the MCF**.
- Coupling efficiency for a **partially coherent field** into a **multi-mode waveguide** given by a **single overlap integral**

$$\langle\eta\rangle = \iiiint W\overline{W}\, d^2\vec{q}\, d^2\vec{p}$$

where $\overline{W} = \sum_m W_{\varphi_m}$ is a **waveguide distribution** obtained as a sum of the individual mode distributions.

Caveats:

- Many efficient operations in traditional wave-optics simulations (WOS) are inefficient with WDF (e.g., phase screens).
- WDF increases computational requirements for coherent fields.

Numerical phase space optics most effective for modeling partially coherent fields and multi-mode waveguides.

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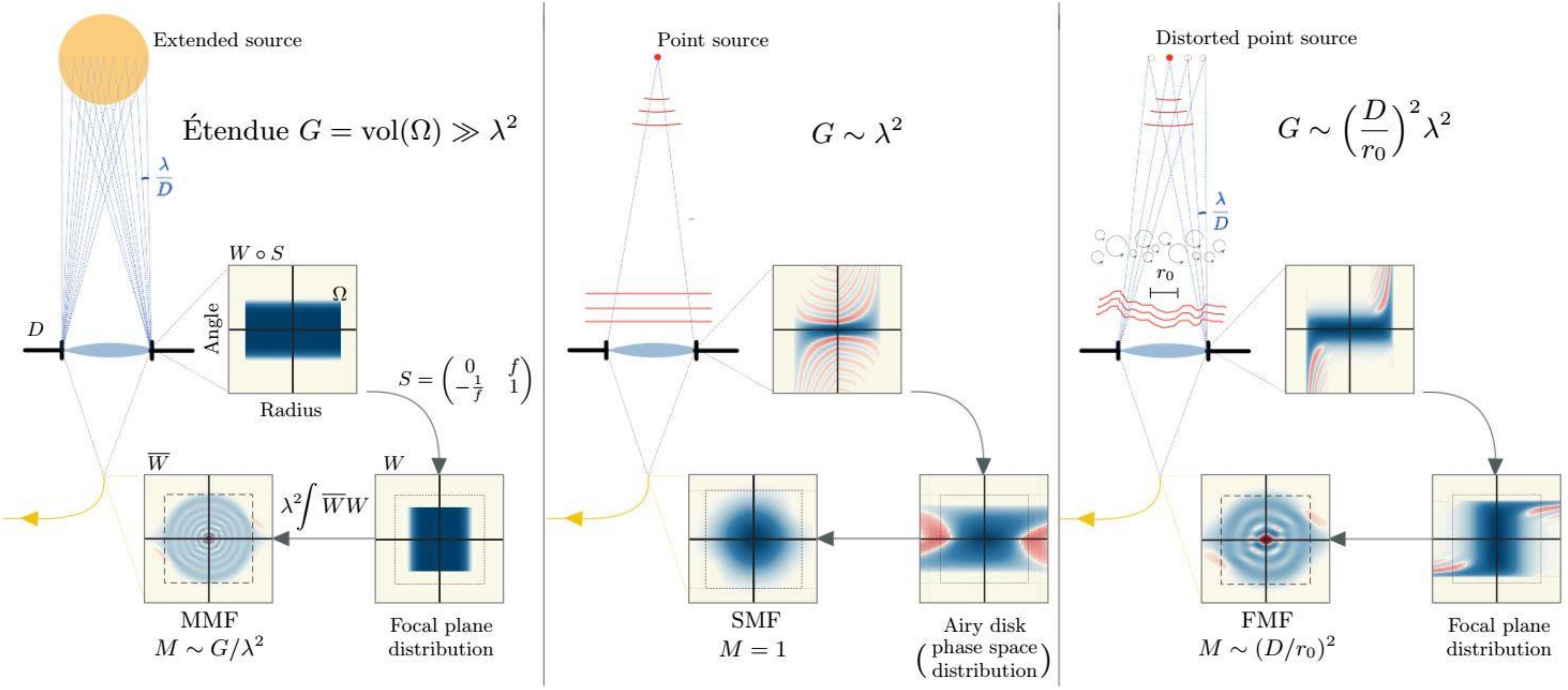
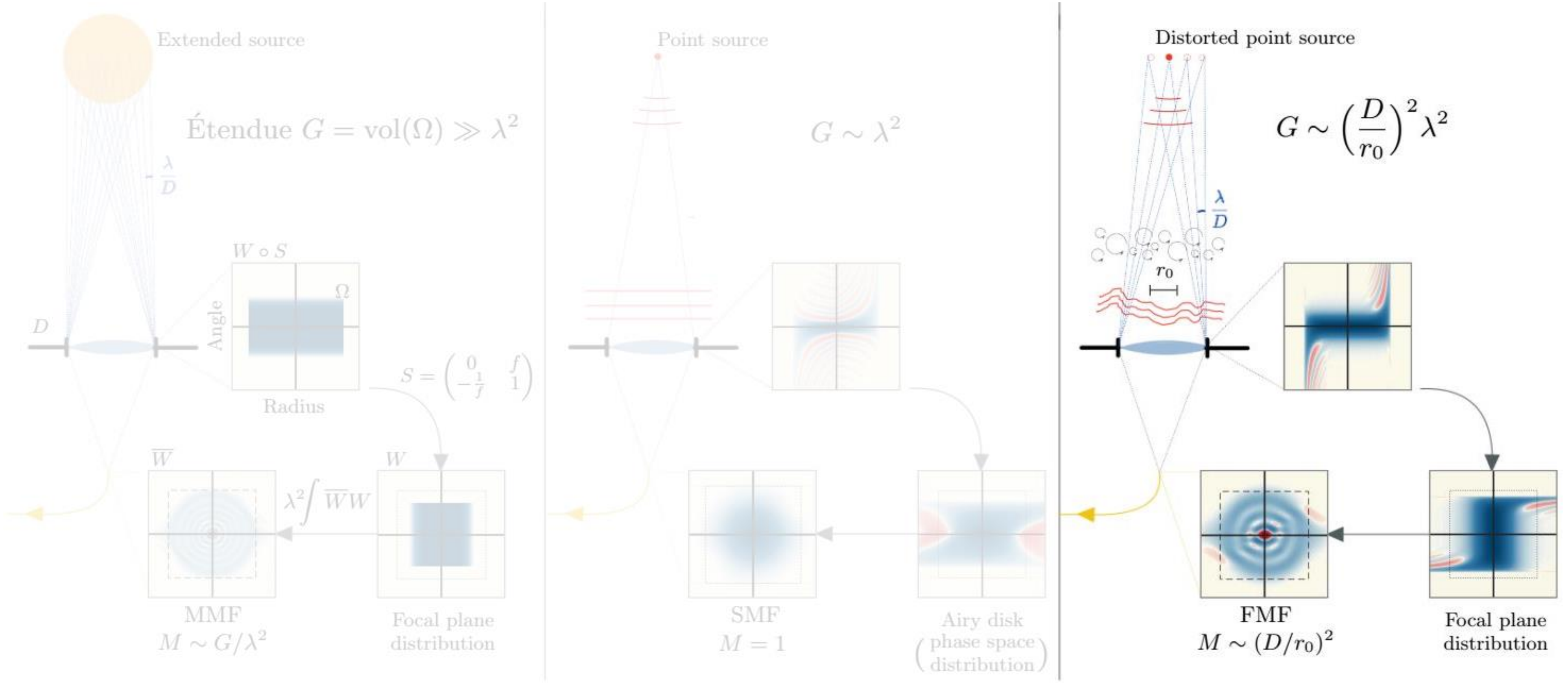
Free-space optical systems in phase space



Schematic for a phase space representation of a range of fiber-coupled free-space optical systems. The Wigner phase space distribution function is used to study the transition from single-mode to multi-mode systems.

Free-space optical systems in phase space



Schematic for a phase space representation of a range of fiber-coupled free-space optical systems. The Wigner phase space distribution function is used to study the transition from single-mode to multi-mode systems.

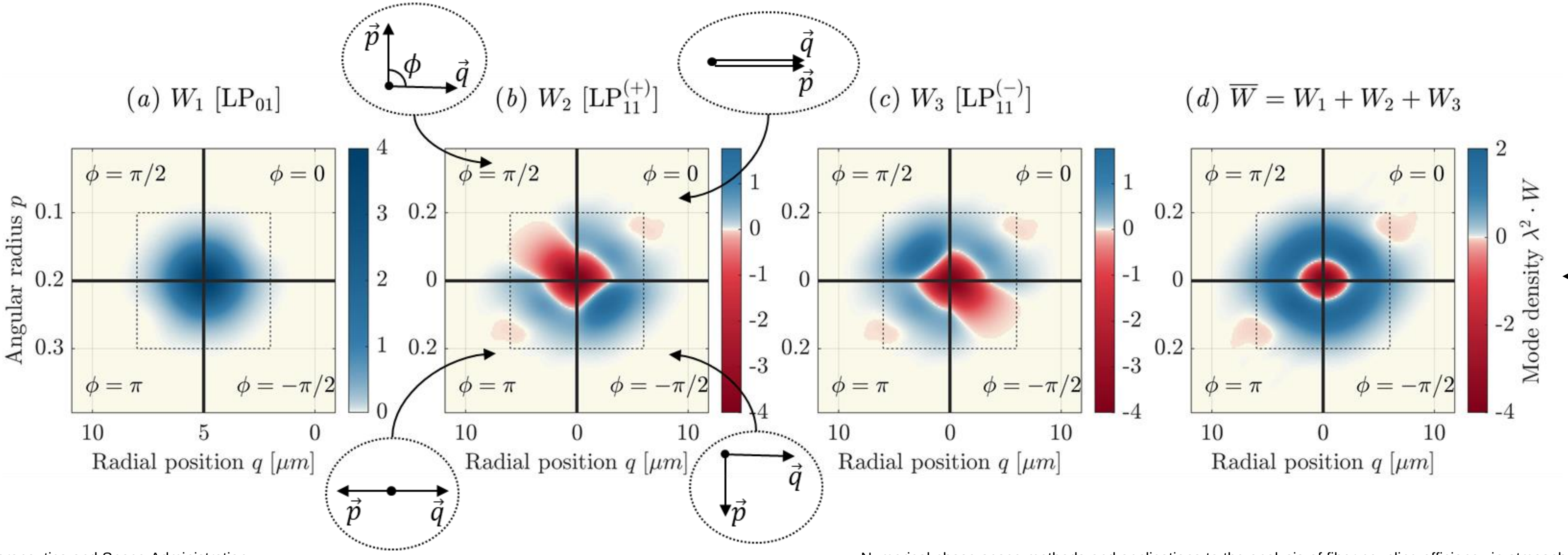
Waveguide distributions - Phase space diagrams

Definition. The ***waveguide distribution*** associated to a waveguide with guided modes φ_m with $m = 1,2, \ldots, M$ is defined as the sum of the individual mode distributions

$$\overline{W} = \sum_m W_{\varphi_m}$$

The waveguide distribution is invariant under a unitary change of basis of guided modes.

- Unlike in time-frequency analysis of 1D signals, cannot generate full phase space diagrams for 2D optical modes.
- Waveguide distributions inspected via four-quadrant **polar phase space diagrams** (PSDs):



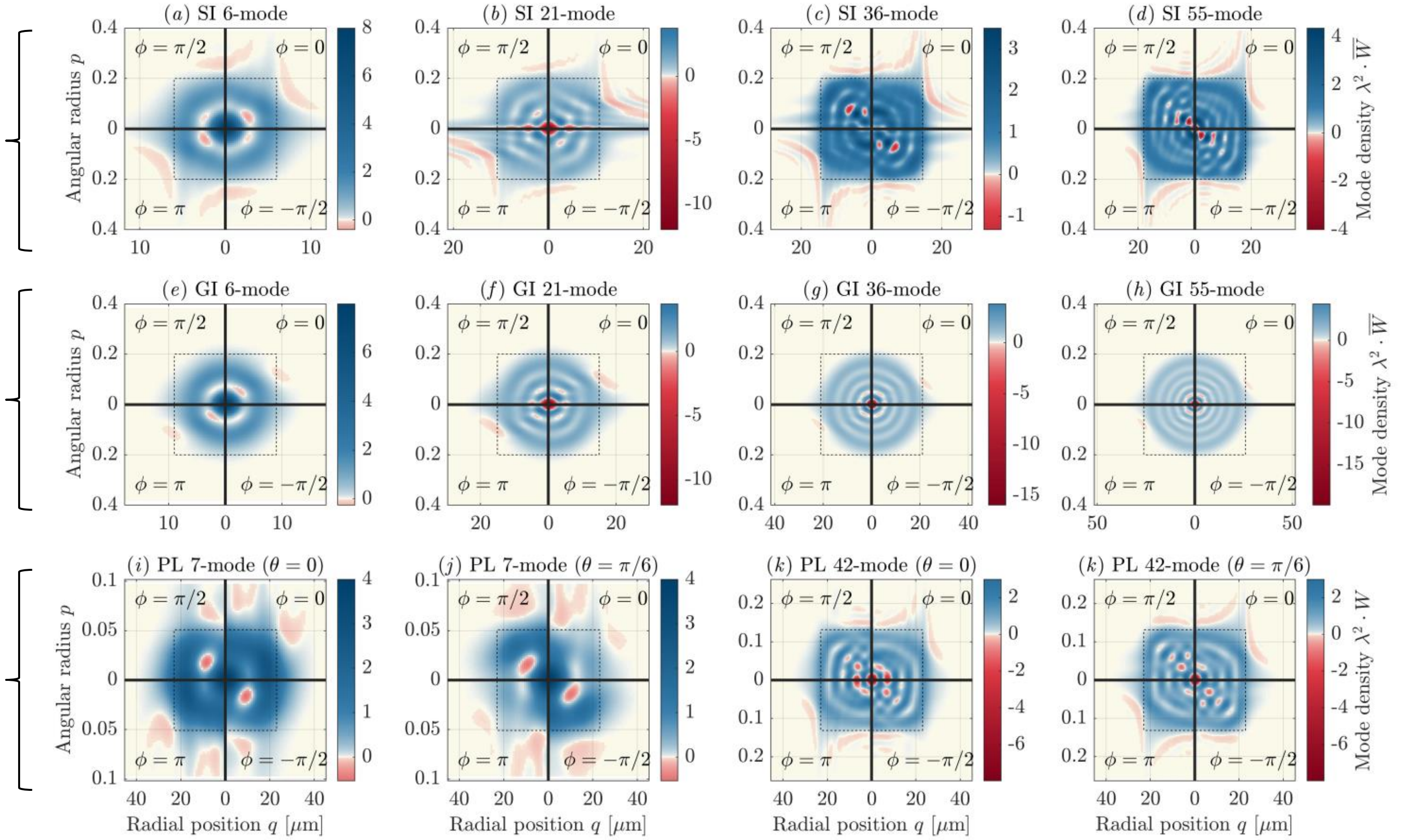
Waveguide distribution for 3-mode fiber

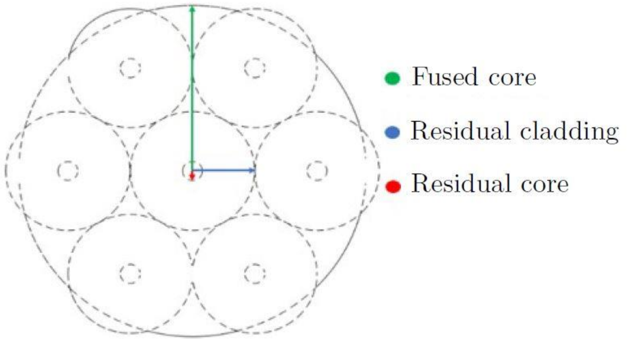
Waveguide distributions - Phase space diagrams

- Calculated waveguide distributions for step-index fibers, graded-index fibers, and photonic lantern structures with at most 78 guided modes.

Step-index fiber distributions

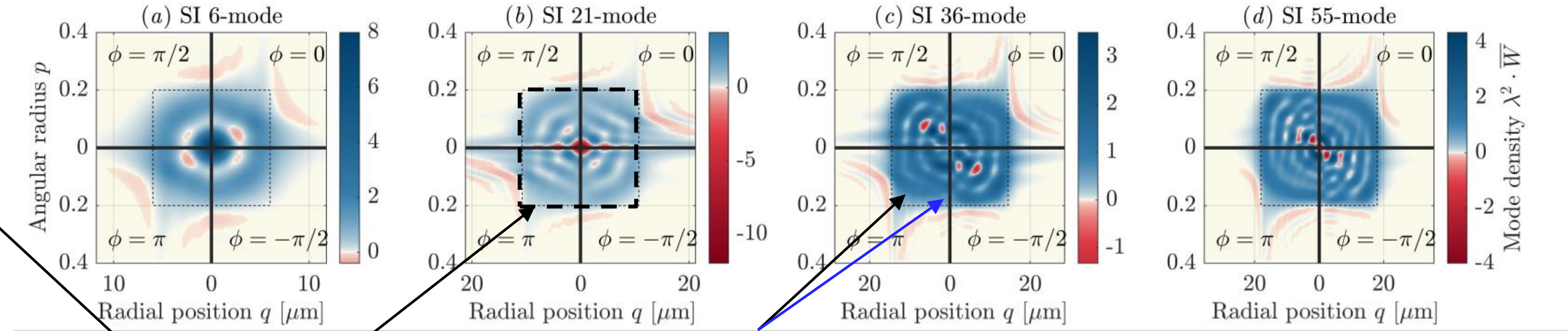
Graded-index fiber distributions





1x7 photonic lantern distributions

Waveguide distributions - Phase space diagrams



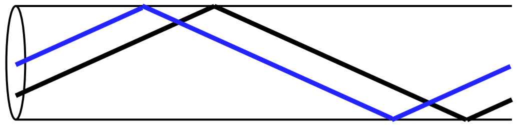
Step-index GO approximation

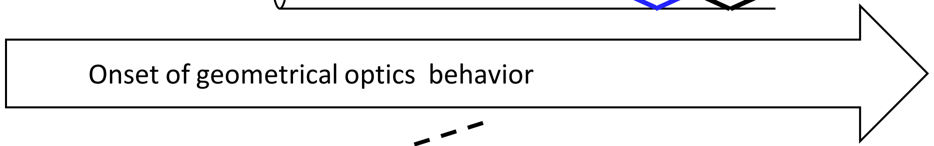
$$\overline{W}_{s.i.}(\vec{q},\vec{p}) = \begin{cases} \frac{1}{\lambda^2}, & q < a \text{ and } p < NA \\ 0, & \text{otherwise} \end{cases}$$

As the number of guided modes M increases the waveguide distribution can be used to quantify the trend toward geometrical optics (GO) behavior.

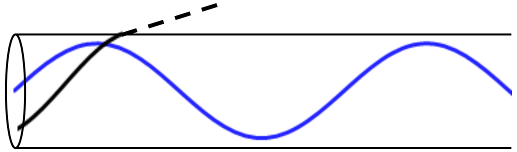
Geometrical phase space acceptance

Angles coupled at fiber center (below NA) coupled at any position in fiber core



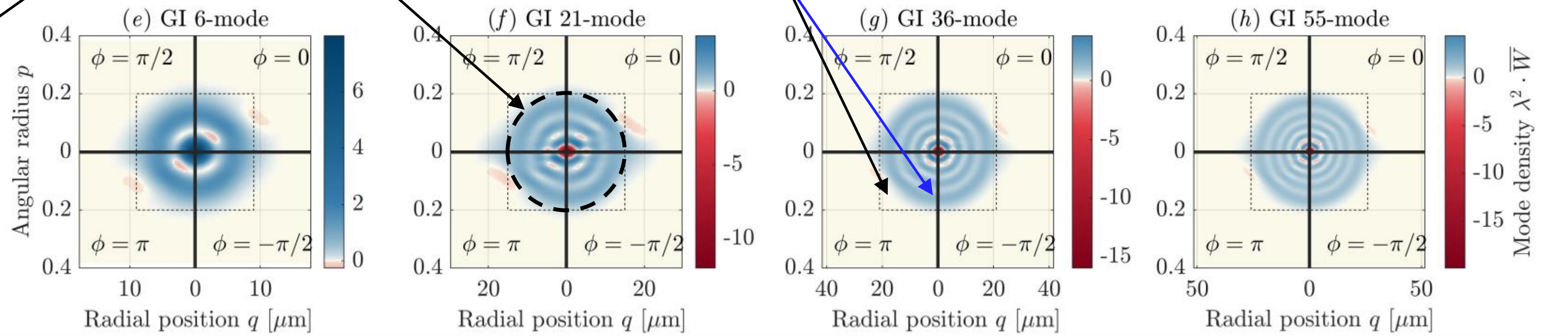


Higher angles coupled at fiber center not coupled near edge of graded-index fiber



Parabolic-index GO approximation

$$\overline{W}_{p.i.}(\vec{q},\vec{p}) = \begin{cases} \frac{1}{\lambda^2}, & \left(\frac{q}{a}\right)^2 + \left(\frac{p}{NA}\right)^2 < 1 \\ 0, & otherwise \end{cases}$$



Waveguide distributions – Analysis

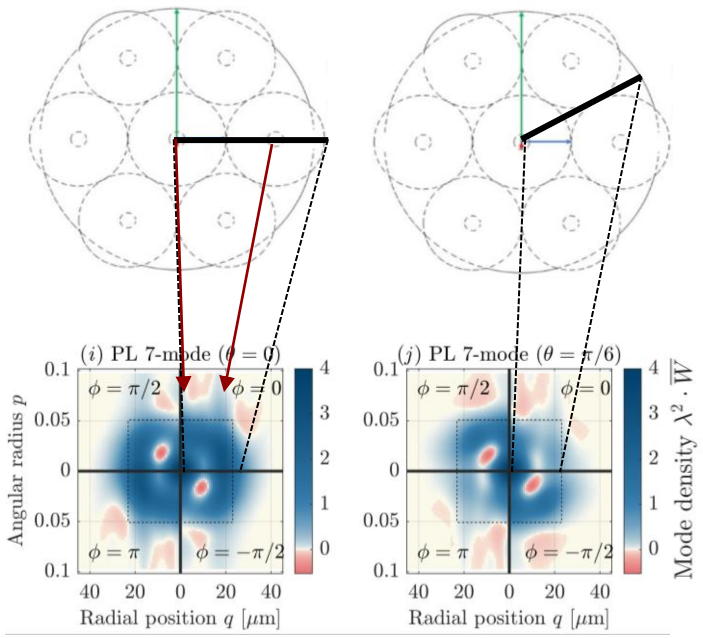
Trend toward geometrical optics behavior quantified by calculating a mode density

$$\delta(\Omega) = \frac{\lambda^2}{vol(\Omega)} \int_{\Omega} \overline{W}\ d^2\vec{q}\, d^2\vec{p}$$

associated to a phase space domain Ω.

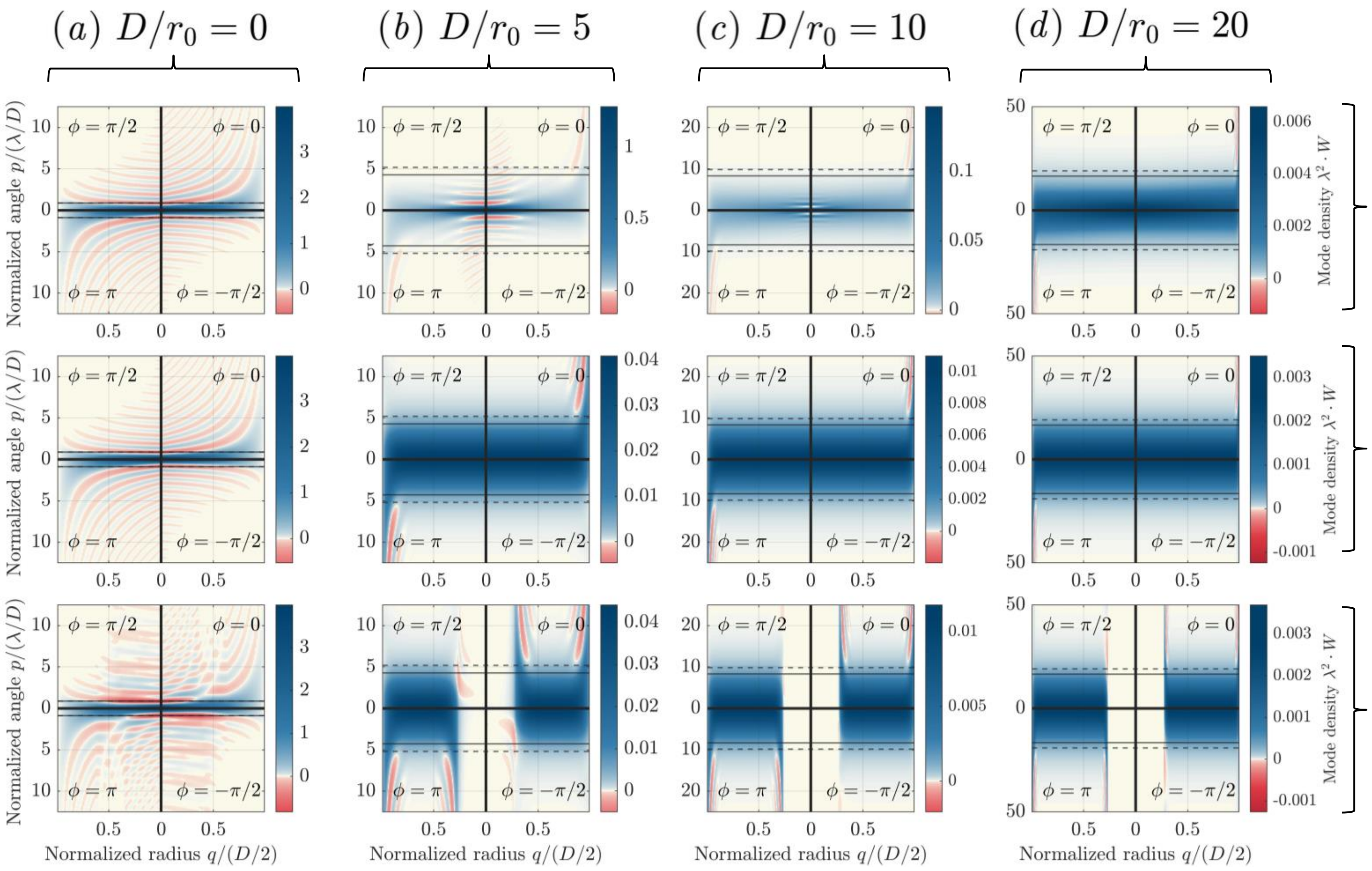
#Guided modes		1	3	6	7	10	15	21	30	36	42	51	55	78
Mode density within geometrical acceptance	SI	0.55	0.71	0.8	-	0.88	0.88	0.89	0.88	0.91	-	0.91	0.93	0.93
	GI	0.63	0.84	0.81	-	0.89	0.94	0.98	0.89	0.92	-	0.94	0.93	0.97
	PL	-	-	-	0.92	-	-	-	-	-	0.94	-	-	-

- If $vol(\Omega) \gg \lambda^2$, represents efficiency of coupling incoherent light with étendue $G = vol(\Omega)$.
- Mode density within geometrical acceptance increases with M
- The impact of higher NA residual cores in the photonic lantern quantified by mode-density with respect to step-index acceptance



Long- and short-exposure Wigner distributions

Wigner distributions associated to a plane wave after propagation through atmosphere calculated based on “long-exposure” and “short-exposure” statistics (MCF) in a circular pupil.



Short-exposure (**tilt compensated**) Wigner distribution

Long-exposure Wigner distribution

$$\mu(r) = \exp\left(-3.44\left(\frac{r}{r_0}\right)^{5/3}\right) \times \text{Pupil function}$$

Long-exposure Wigner distribution (with central obscuration)

Passive coupling with simple focusing systems

- Partially coherent WDF propagated to focal plane assuming simple focusing system

$$S = \begin{pmatrix} 0 & 0 & f & 0 \\ 0 & 0 & 0 & f \\ -1/f & 0 & 1 & 0 \\ 0 & -1/f & 0 & 1 \end{pmatrix}$$

- Optimal efficiency and optimal NA for each fiber calculated by sweeping over focal length f.

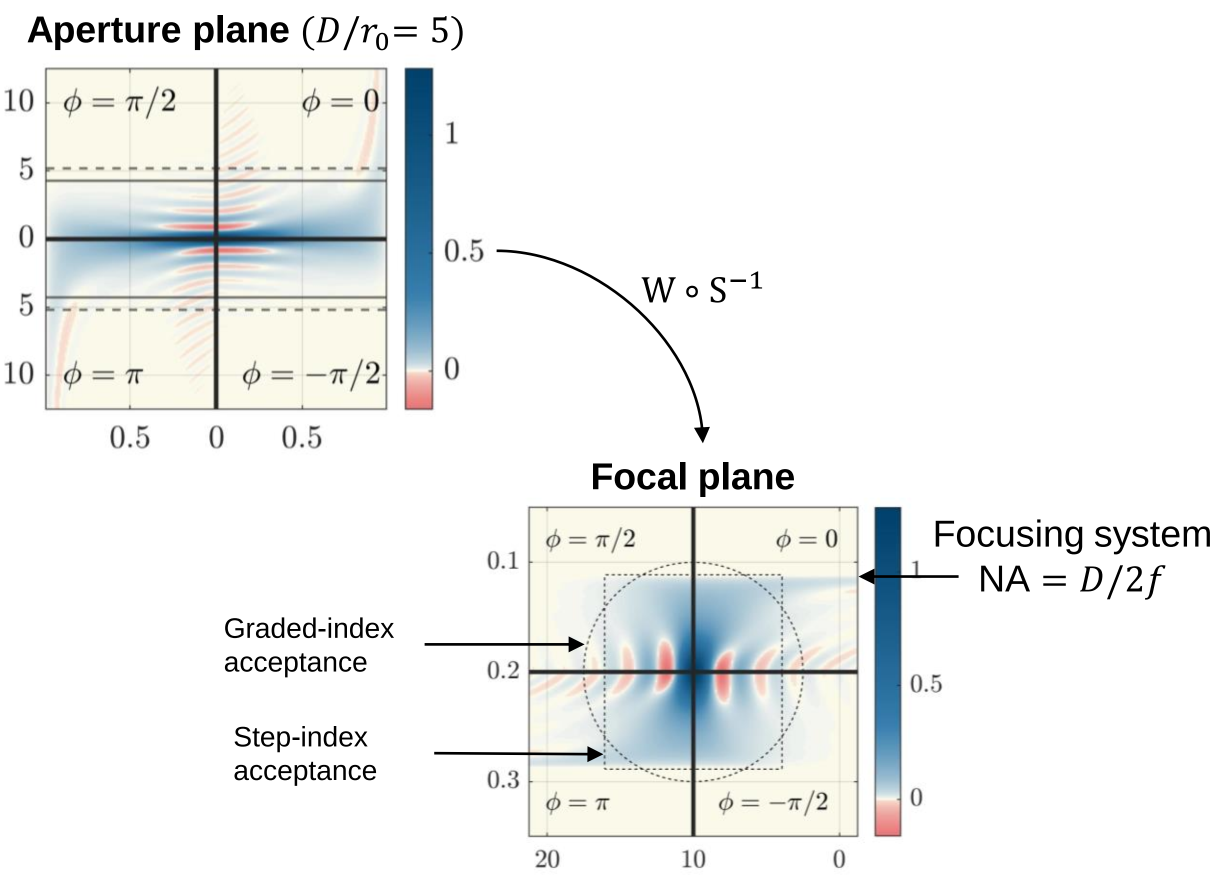


Figure. Focal plane distribution using optimal f-number for 21-mode graded-index fiber

Passive coupling without obscurations

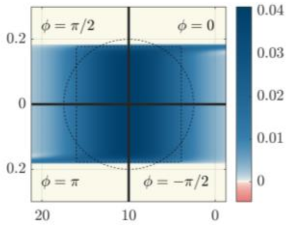
Analytical approximation for coupling efficiency to mode-limited fibers from **long-exposure WDF****

$$\eta \simeq \Sigma^4/M^8 + 0.822\left(1 - e^{-3\Sigma^2}\right) + 0.103(1 - \Gamma(4, 3\Sigma^2)/3!) + 0.028(1 - \Gamma(10, 3\Sigma^2)/9!),$$

depending on a single efficiency parameter $\Sigma^2 = (4/\pi^2)MS_{LE}$ where $S_{LE} = \left(1 + (D/r_0)^{5/3}\right)^{-6/5}$.

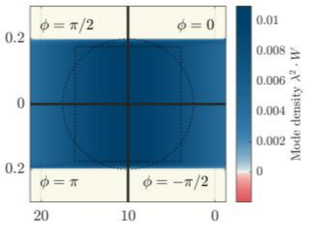
- Long-exposure and short-exposure efficiency for GI fibers same as SI fibers within 0.13 dB ($1 \leq M \leq 78$ and $D/r_0 \leq 20$).
- Step-index/Graded-index/Photonic lantern efficiency agrees with analytical approximation within 0.4 dB.

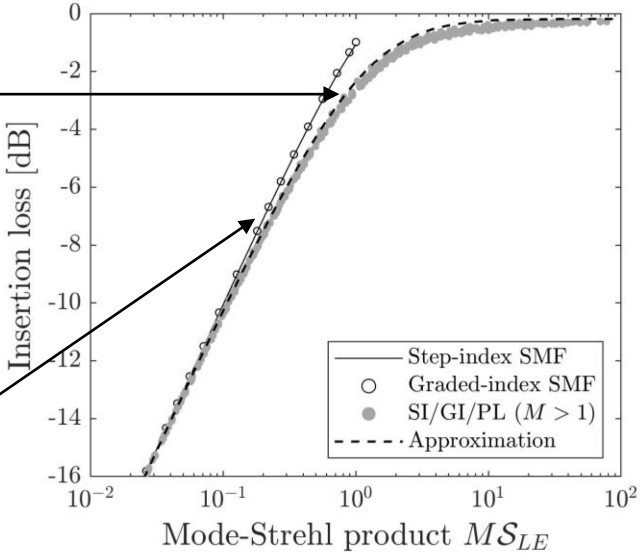
3 dB efficiency threshold corresponds to a Mode-Strehl product $MS_{LE} \simeq 0.77$. (No tilt compensation)

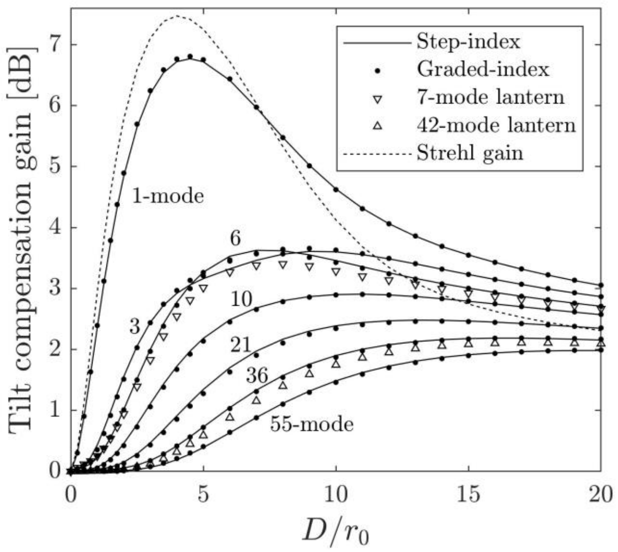


Low-efficiency regime. WDF constant over geometrical acceptance. Efficiency reduces to Mode-Strehl product

$$\eta \sim MS_{LE}.$$







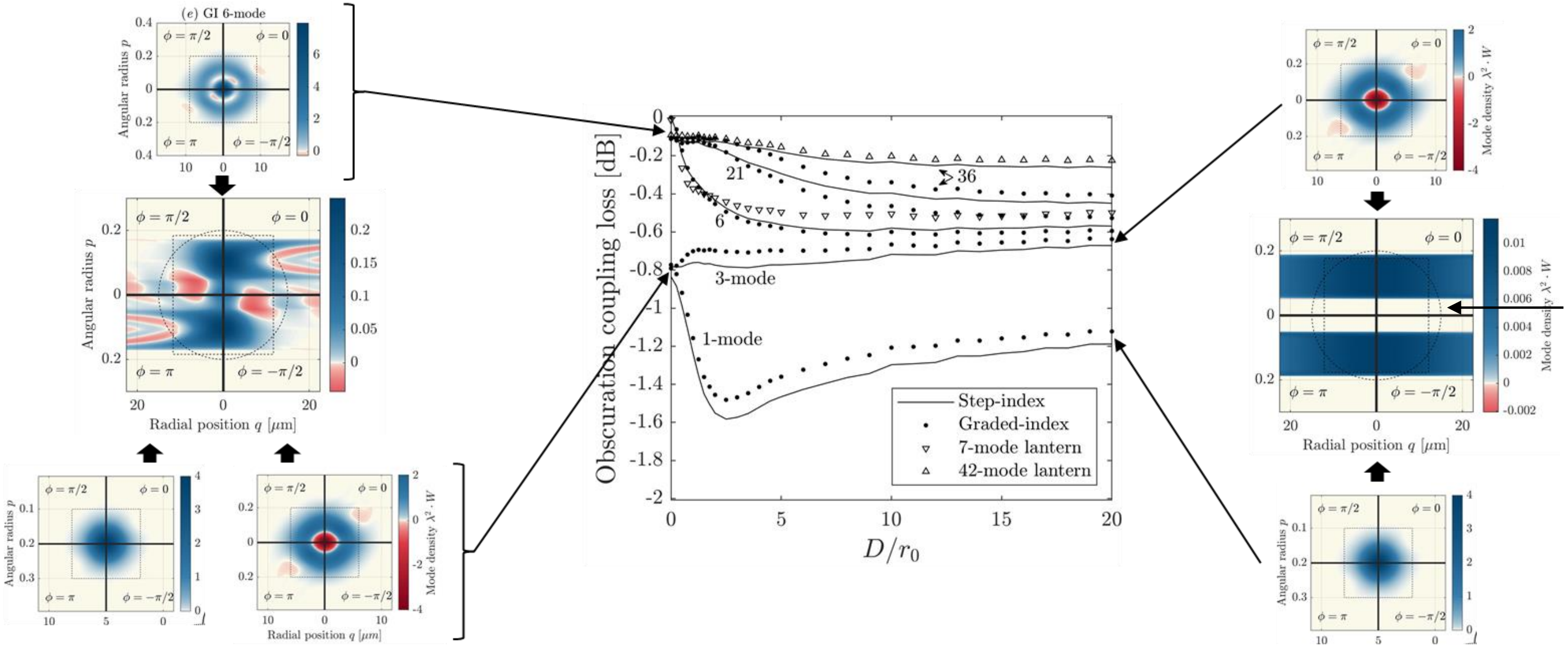
Chahine, Y.K., Tedder, S.A., Staffa, J., Vyhnalek, B.E., "Optimal efficiency for passively coupling partially coherent light into mode-limited optical waveguides," *J. Opt. Soc. Am. A* **38, 1732-1743 (Dec 2021).

Passive coupling with central obscuration

Unlike unobscured case, obscuration-induced coupling loss exhibits subtle dependence on mode structure and coupling geometry.

In diffraction-limit, significant mitigation for fibers supporting guided modes LP_{lm} with radial order $m \geq 2$.

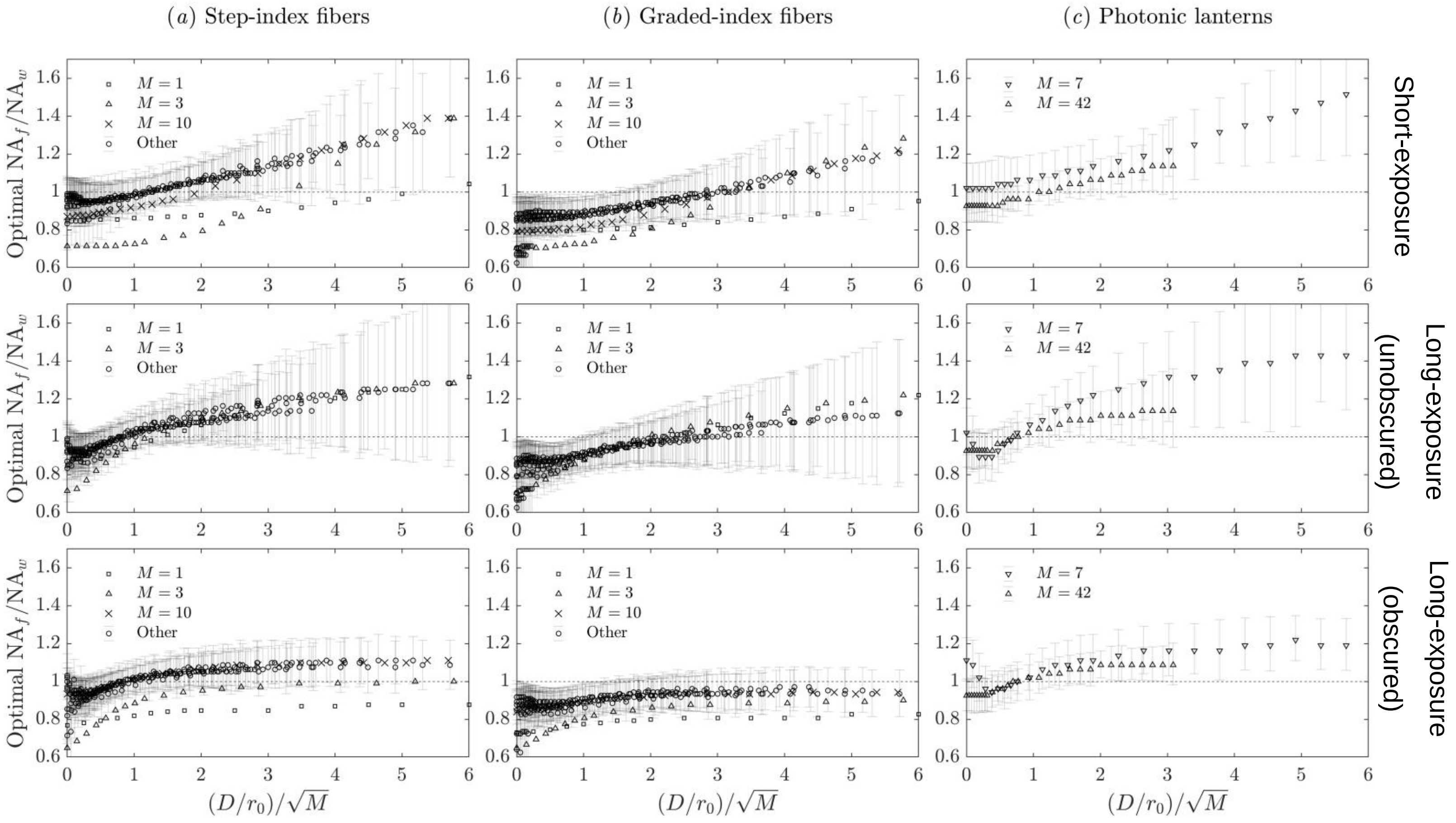
In severe turbulence, partial mitigation already for 3-mode fiber.



For $M \geq 6$, graded-index fibers suffer worse coupling due to bias of parabolic-index geometrical acceptance for low-angle light.

Optimal input NA with atmospheric turbulence

- Optimal input NA determined with 0.1 dB tolerance for fibers with $1 \leq M \leq 78$ as a function of increasing turbulence $D/r_0 \leq 20$.
- For $M > 10$, optimal NA found to depend almost solely on ratio $(D/r_0)/\sqrt{M}$.
- In low-efficiency regime $(D/r_0) > 3\sqrt{M}$ optimal input NA increases with turbulence with high tolerance (no obscuration).
- With obscuration, optimal input NA in high turbulence limits to strict balance with fiber NA depending on geometrical acceptance and obscuration size. Tighter tolerance than for unobscured apertures.



Conclusions

- Numerical method based on phase space representations outlined to study free-space optical systems
- **In absence of obscurations**, coupling efficiency found to be largely independent of mode structure (step-index/graded-index/photonic lantern), **depending only on the number of modes $\boldsymbol{M}$ and turbulence $\boldsymbol{D/r_0}$**.
- For fibers with more than 6-10 guided modes, **optimal coupling geometry** found to be dependent primarily on the geometrical phase space acceptance and efficiency regime **characterized by $(\boldsymbol{D/r_0})/\sqrt{\boldsymbol{M}}$**.
- With central obscuration, efficiency shown to have a dependence on the mode structure as characterized by the geometrical acceptance of the fiber.
- Phase space diagrams employed to interpret numerical results and provide a robust method for analyzing optical systems in the transition from single-mode to multi-mode behavior.