A Quantum Algorithm to Simulate Open Quantum Systems

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Motivation for open quantum systems (OQS)

Characterization

- No real quantum system is ever truly closed!
- Quantify (non) Markovian environments in NISQ devices, tailored error-correction codes

Engineering

- Preparation of entangled or topological states
- Decoherence-free subspaces

Simulation

- OQS as powerful as closed QC
- Non-equilibrium steady states, transport in strongly correlated systems, quantum biology

Prior work

Engineer existing open dynamics (physically non-unitary, i.e. analog)

- Barreiro et al. “An open-system quantum simulator with trapped ions”. *Nature* **470**, (2011)
- Marshall et al. “Modular quantum-information processing by dissipation”. *PRA* **94**, 052339 (2016)

Create open dynamics (operationally non-unitary, i.e. digital)

- Richard Cleve and Chunhao Wang. “Efficient Quantum Algorithms for Simulating Lindblad Evolution.” 44^{th} ICALP (2017)
- Shen et al. “Quantum channel construction with circuit quantum electrodynamics”. *PRB* **95**, 134501 (2017)
- Han et al. “Experimental Simulation of Open Quantum System Dynamics via Trotterization”. *PRL* **127**, 020504 (2021)

Parallelize (embed each Kraus operator in a larger unitary)

- Hu et al. “A quantum algorithm for evolving open quantum dynamics on quantum computing devices”. *Sci Rep* **10**, 3301. (2021)
- Schlimgen, et al. “Quantum Simulation of Open Quantum Systems Using a Unitary Decomposition of Operators”. *PRL* **127**, 270503 (2021)

New direction - parallel simulation of OQS

- These methods block-encode each Kraus matrix A in its own unitary, which are measured separately
- We must rely on post-selection, and thus fail with some probability
- This probability depends both on the initial state and the embedded operator

$$U_A = \begin{bmatrix} A & \cdots \\ \vdots & \ddots \end{bmatrix}$$

$$U_A|0\rangle|\psi\rangle = |0\rangle \otimes A|\psi\rangle + |\psi_\perp\rangle$$

$$\text{if } |0\rangle \rightarrow \frac{A}{\sqrt{p_{succ}}}|\psi\rangle$$

$$p_{succ} = \langle\psi|A^\dagger A|\psi\rangle$$

TWO-UNITARY DECOMPOSITION ALGORITHM (TUD)

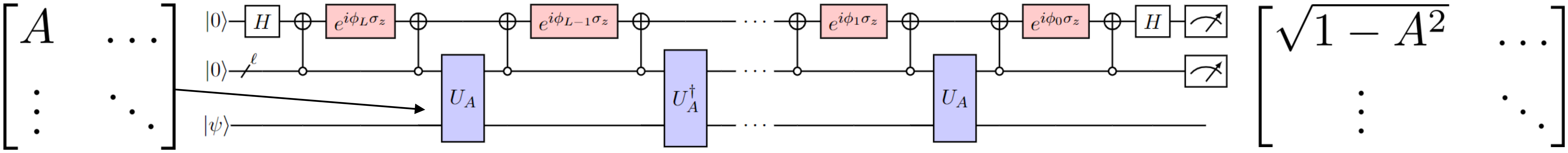
We use the quantum singular value transform (QSVT) to implement A as two unitaries, where each unitary can be obtained with $p_{succ} \approx 1$

Algorithm

- Kraus operators are *contractions*: $\sum_k A_k^\dagger A_k = 1 \rightarrow |A| \leq 1$
- Any contraction A admits a *two-unitary decomposition*[1]: $A = (U_1 + U_2)/2$

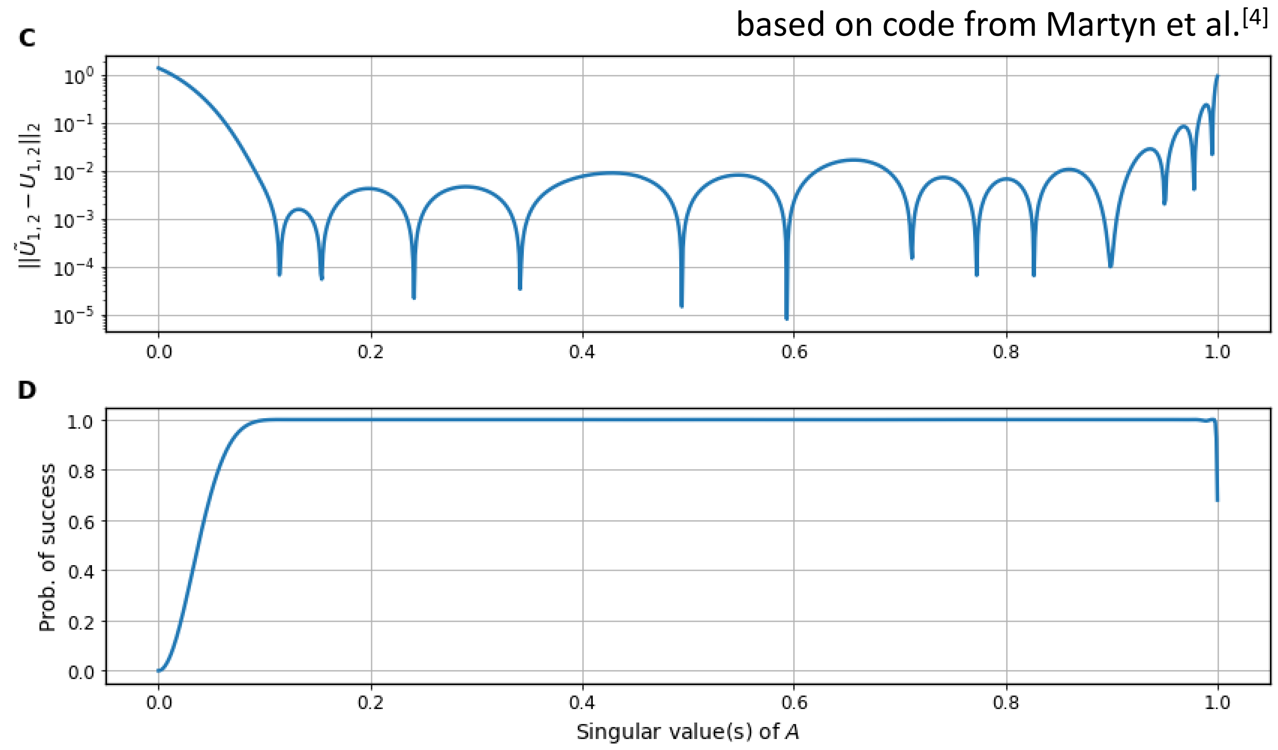
$$U_{1,2} = \oplus_i \; e^{i \arccos \sigma_i} |w_i\rangle\langle v_i| = (A \pm i\sqrt{1 - A^2})/2$$

- We can use QSVT[2,3] to process a block encoding of A into $\sqrt{1 - A^2}$ without explicitly performing SVD



- We use LCU to add/subtract A and i$\sqrt{1 - A^2}$ to produce $U_{1,2}/2$
- With 2 extra calls to the algorithm, oblivious amplitude amplification[5] (OAA) can boost this to $p_{succ} \approx 1$

Results – arbitrary A

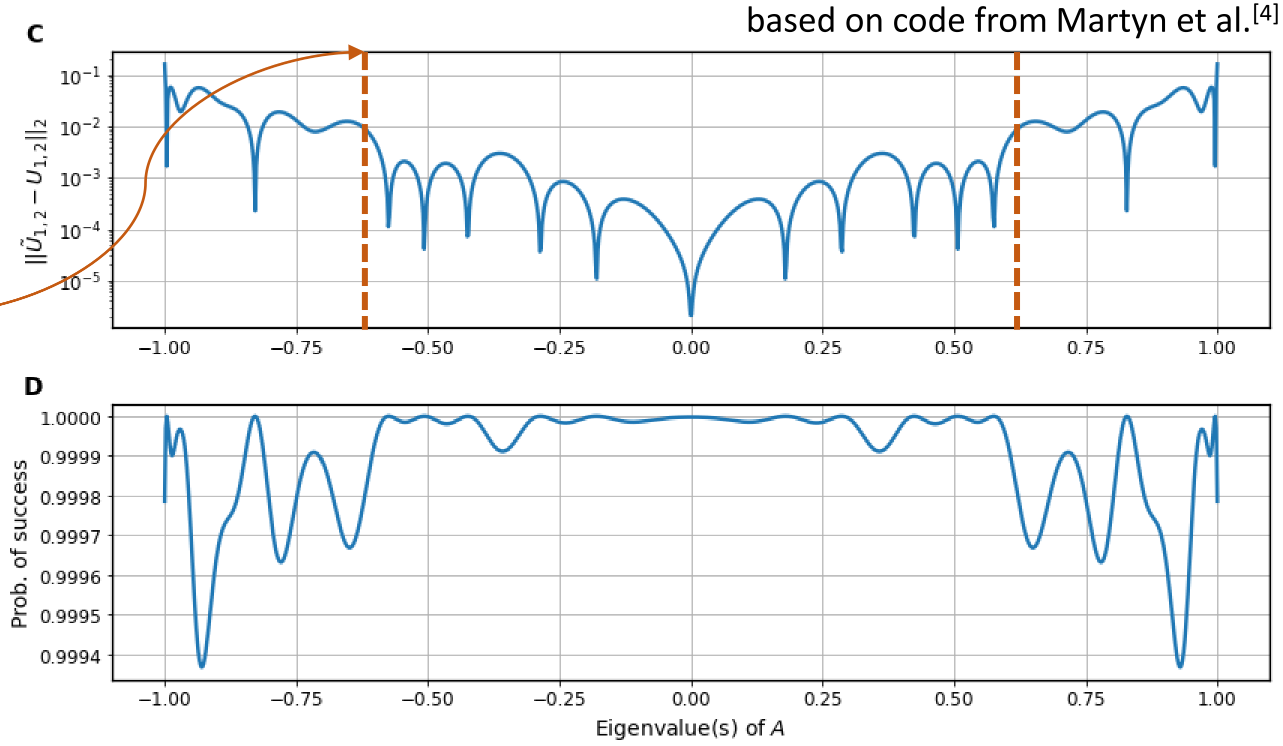


- Issue: we incur large error near 0 due to the TUD polynomial
- Solution: we can avoid this with Hermitian inputs, so we process $\mathrm{H}_1 = (A + A^\dagger)/2$ and $H_2 = i(A - A^\dagger)/2$ instead

Results – Hermitian A

based on code from Martyn et al.[4]

Caveat: the variance of the final expectation values scales like $\mathcal{O}(\alpha^4)$

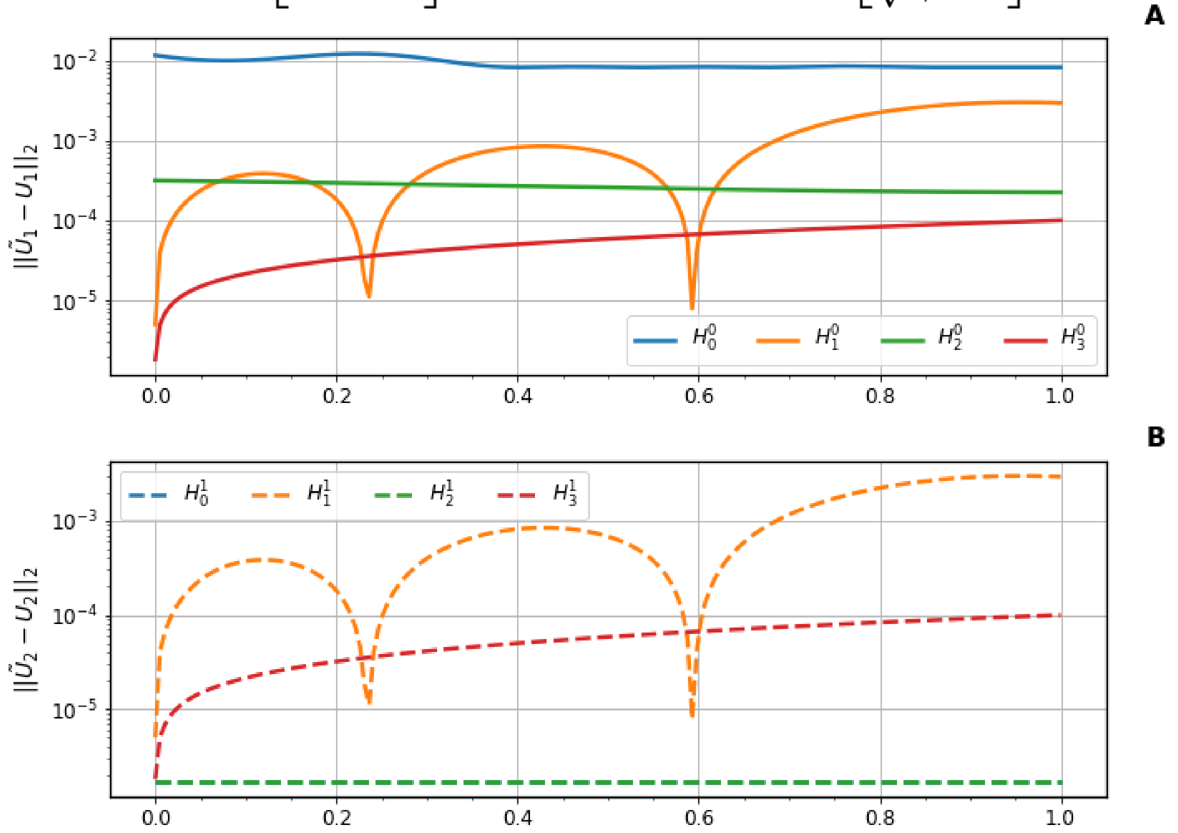


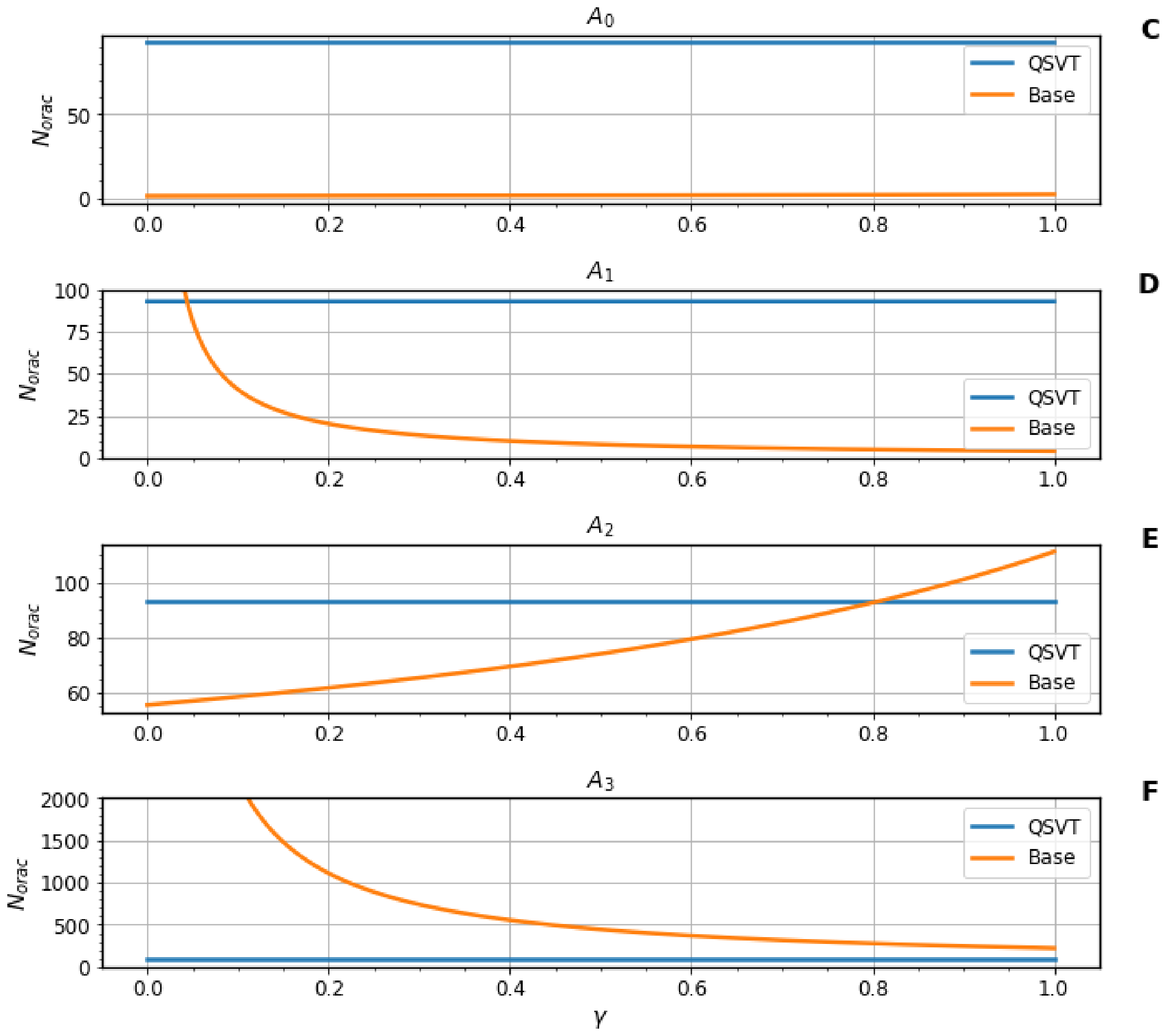
- We now obtain high error only near ± 1
- This can be mitigated by scaling the input by $1/\alpha < 1$
- All final statistics should be rescaled by α

Generalized amplitude damping channel

$$A_0 = \sqrt{p}\begin{bmatrix}1 & 0\\ 0 & \sqrt{1-\gamma}\end{bmatrix} \quad A_2 = \sqrt{1-p}\begin{bmatrix}\sqrt{1-\gamma} & 0\\ 0 & 1\end{bmatrix}$$

$$A_1 = \sqrt{p}\begin{bmatrix}0 & \sqrt{\gamma}\\ 0 & 0\end{bmatrix} \quad A_3 = \sqrt{1-p}\begin{bmatrix}0 & 0\\ \sqrt{\gamma} & 0\end{bmatrix}$$





Results – Expectation values

- The end goal is to estimate observables with respect to OQS

$$\langle O \rangle = Tr\Big(O\Lambda(\rho)\Big) = \sum_{k=1}^{m} \langle \psi | A_k^\dagger O A_k | \psi \rangle$$

- We decompose each term into unitaries

$$\begin{aligned}\langle A^\dagger O A \rangle &= \langle (H_1 - iH_2) O (H_1 + iH_2) \rangle \\ &= \langle H_1 O H_1 \rangle + \langle H_2 O H_2 \rangle - 2\mathrm{Im}\langle H_1 O H_2 \rangle\end{aligned}$$

↓

$$\frac{1}{2}(\langle U_1^\dagger O U_1 \rangle + \mathrm{Re}\langle U_1 O U_1 \rangle)$$

↓ Direct measurement ↓ Hadamard test

Results – Expectation values

- All constituent terms are estimated separately and summed
- We can achieve this with error that only depends on the block-encoding error

$$\|A-\tilde{A}\| \leq \beta \rightarrow |\langle A^\dagger O A\rangle - \langle \tilde{A}^\dagger O \tilde{A}\rangle| \leq 2\beta$$

- The query complexity to implement $U_{1,2}$ with $p_{succ} \approx 1$ is

$$\mathcal{O}\Big(\frac{1}{\delta}\log\left(1/\epsilon\right)\Big)$$

- This scales the number of trials needed to obtain a desired variance

Remarks and Conclusion

- The output unitaries only *approximate* $U_{1,2}$, but the result can be made arbitrarily close to unitary using higher-degree polynomials
- When given access to a block-encoding of the Sz-Nagy form (see paper), our method succeeds with $p_{succ} = 1$ *without* QSVT or OAA
- Our method applies to arbitrary operators if an upper bound on σ_{max} of A is known

We can obtain an exponential reduction in query complexity to achieve the same variance when compared to the direct use of the block-encoding

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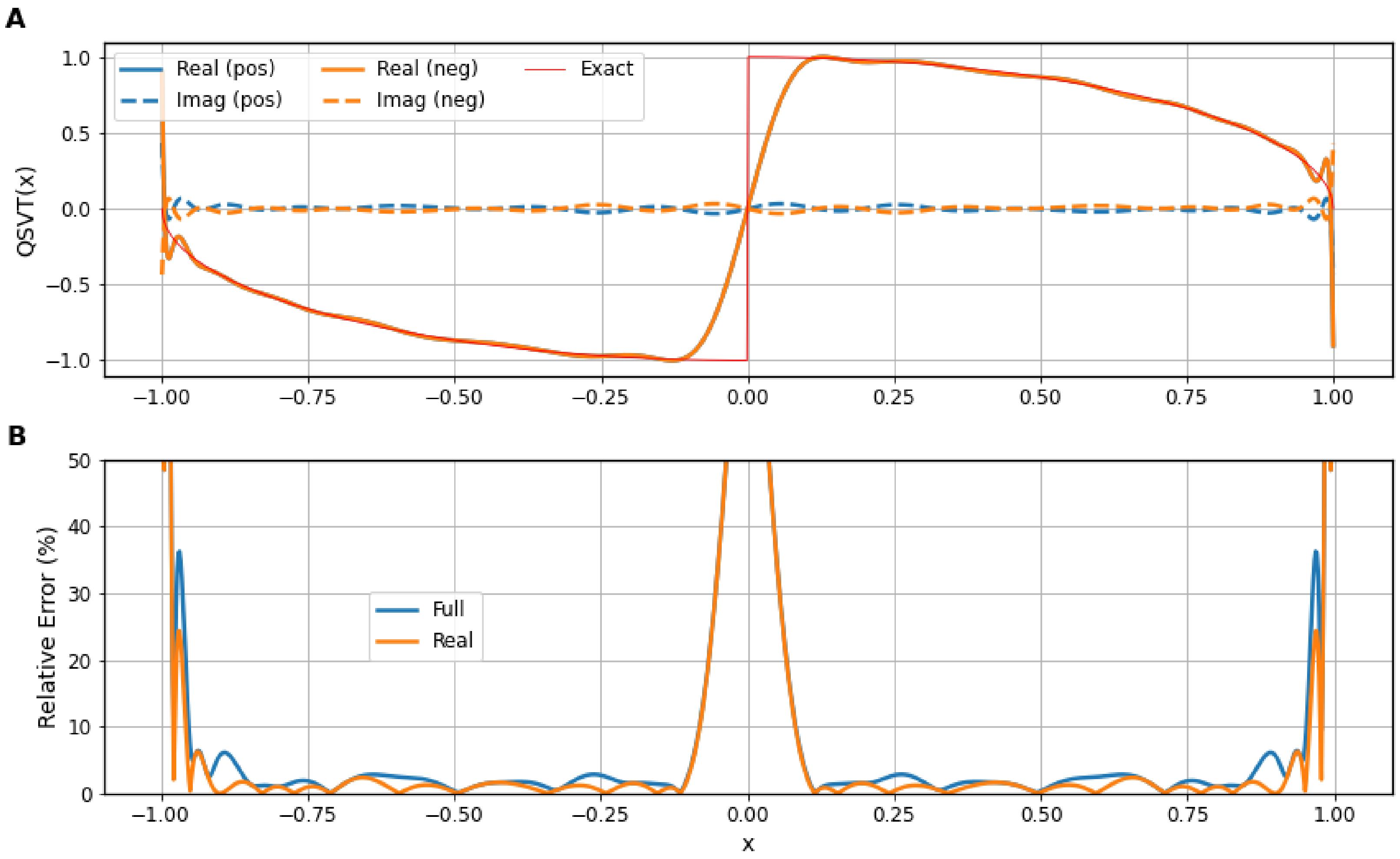
QSVT - odd polynomial



QSVT - even polynomial