Introduction to Orbits

Rice/Envision Aerospace and Aviation Academy

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June 2022

Video Credit: DSCOVR: EPIC Team

Lesson Contents

Acknowledgements

Information and image sources are cited on each slide.

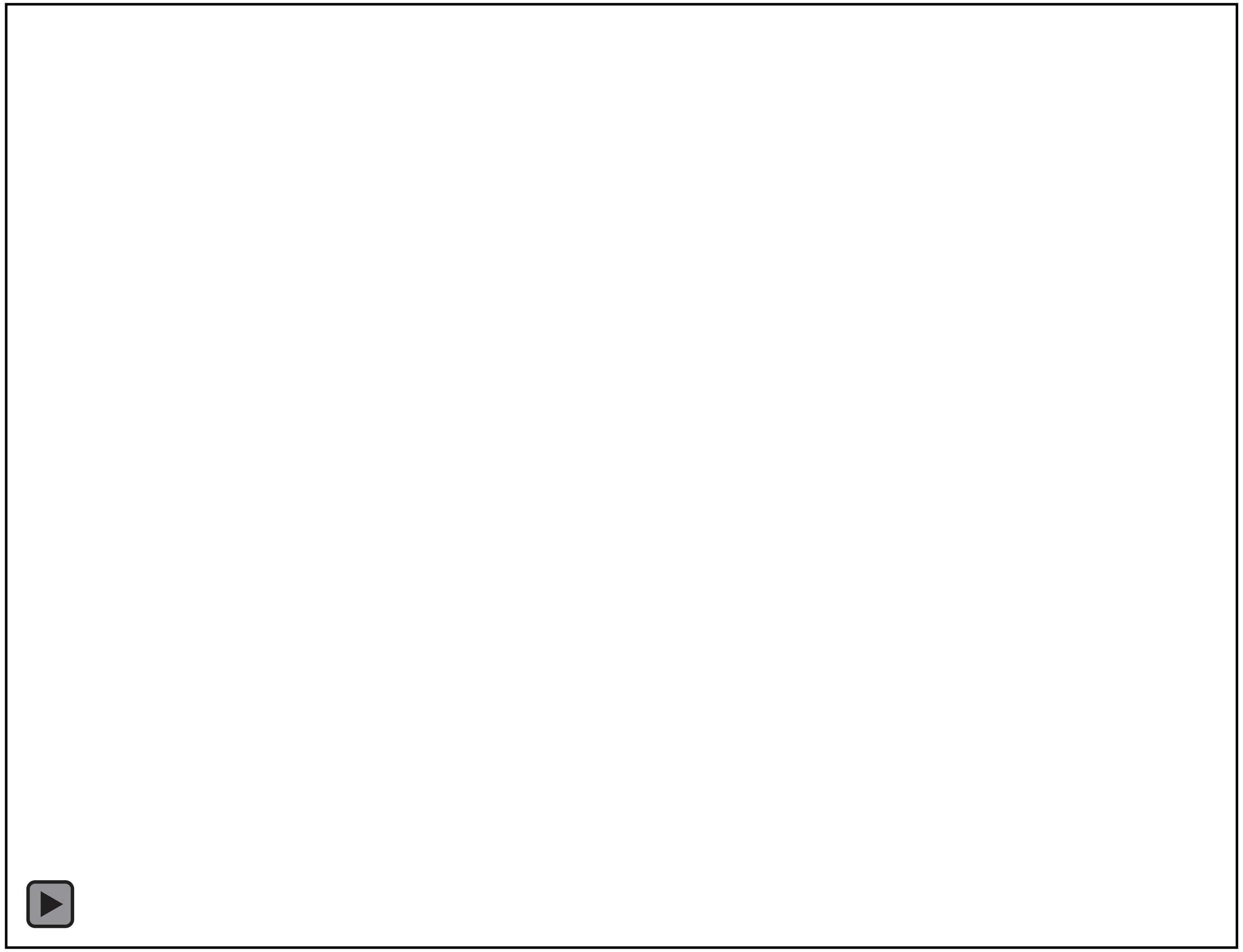
Unless otherwise credited, animations were developed by the author using Copernicus 4.6.

Microsoft® Clip Art was used in the presentation.

Some content from Bate, R. R., Mueller, D. D., and White, J. E., *Fundamentals of Astrodynamics*, Dover Publications, New York, 1971.

What is an Orbit?

An orbit is “the curved path, usually elliptical, described by a planet, satellite, spaceship, etc.”



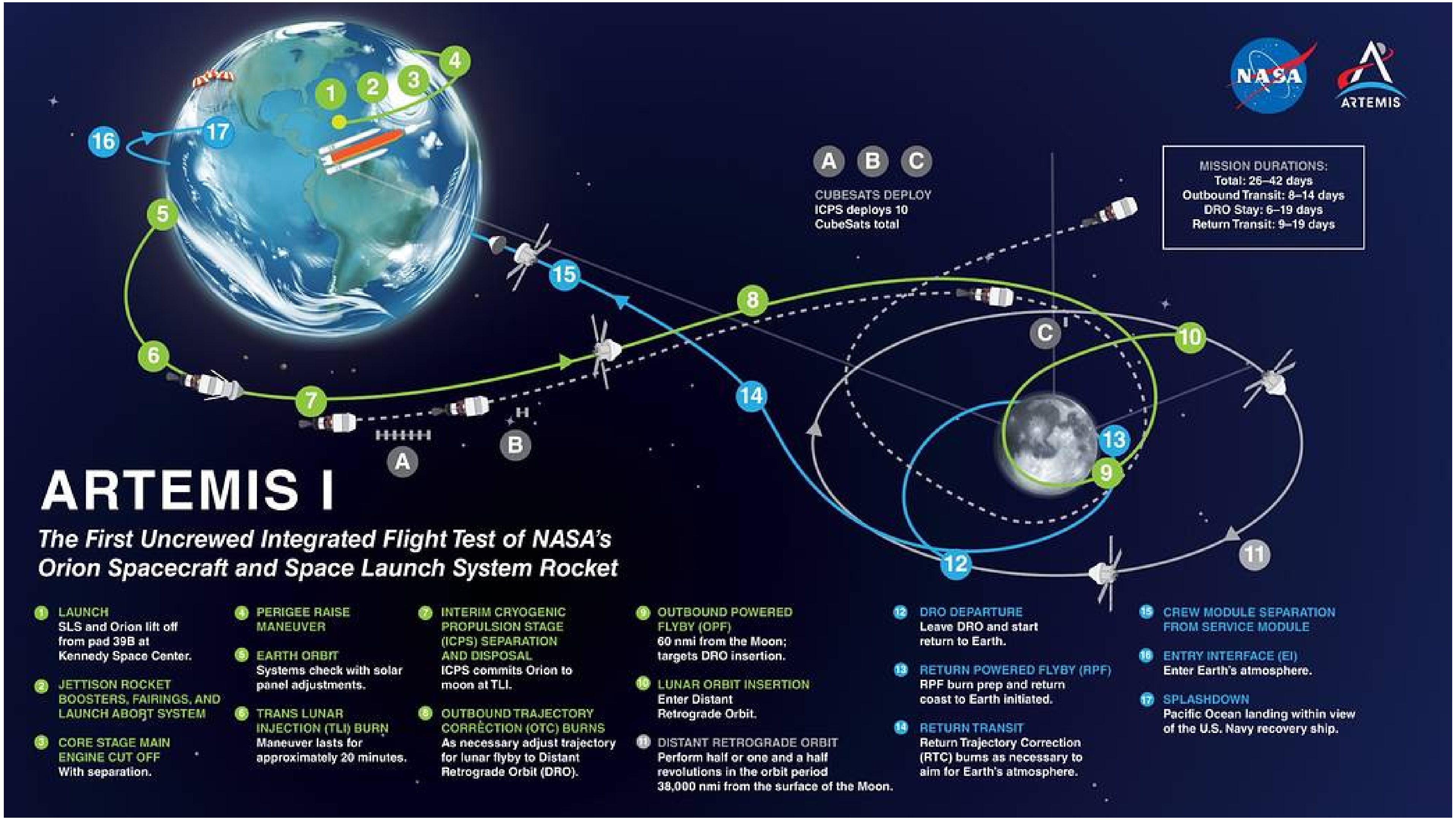
Circular Orbit About the Earth

Source: https://www.dictionary.com/browse/orbit

Why Study Orbits?

An orbit is the path a spacecraft takes in space;

Orbits are used to go from one location in space to another, for example, from the Earth to the Moon, and back.

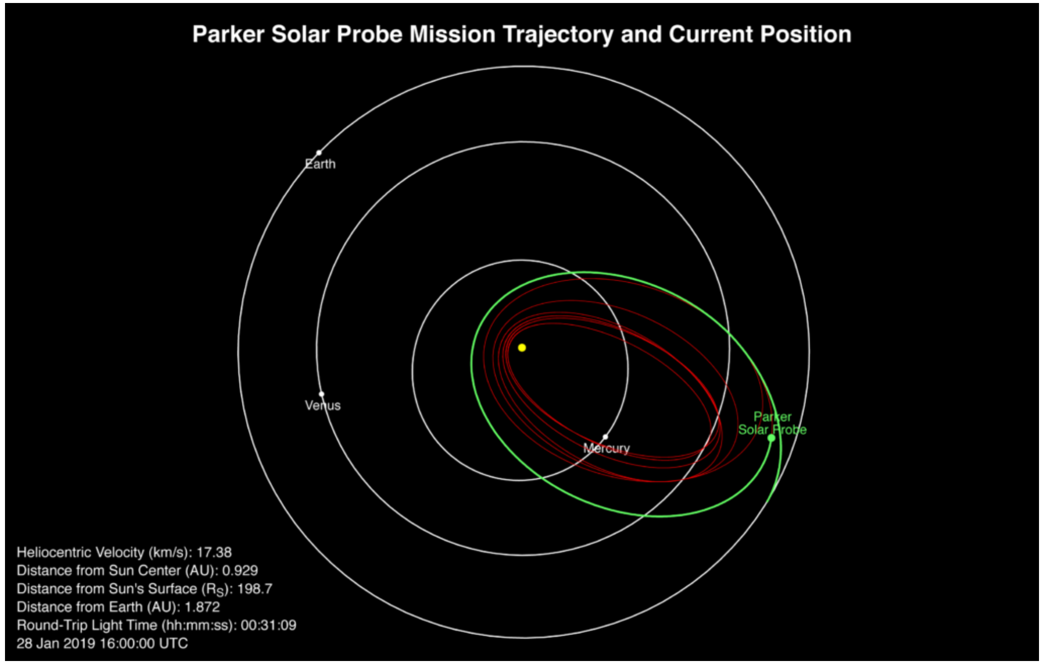


Artemis 1

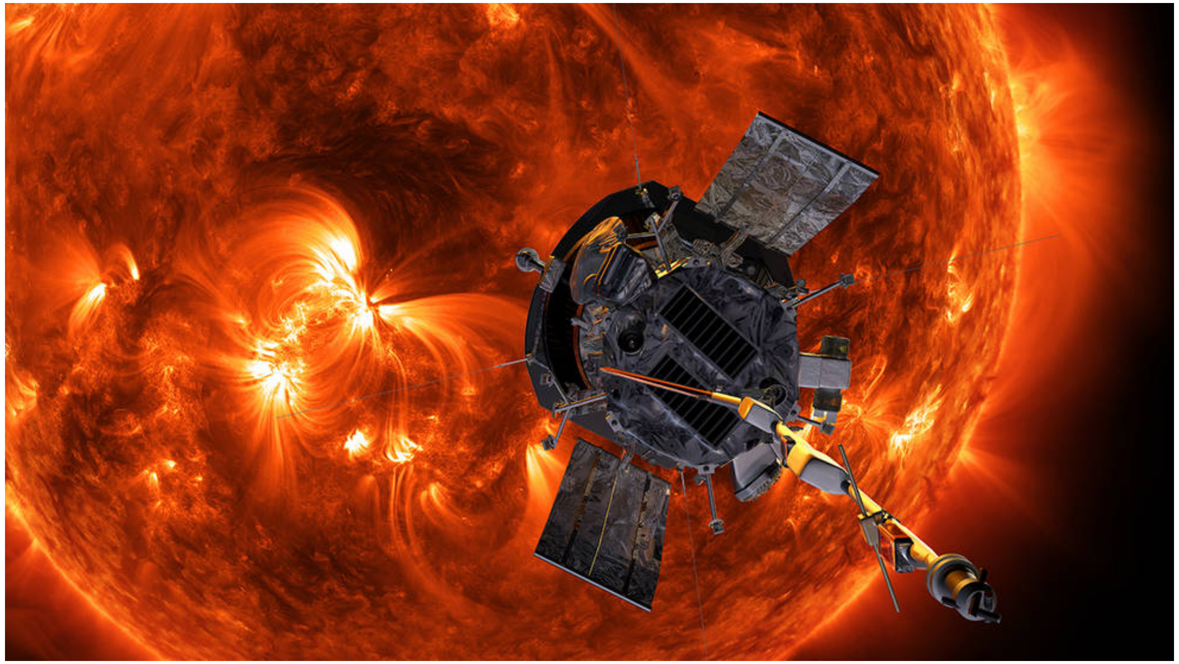
Source: https://www.nasa.gov/image-feature/artemis-i-map

Why Study Orbits?

It is important to understand orbits to determine the environment a spacecraft experiences, how long it takes to travel to the destination, etc.



Parker Solar Probe Trajectory



Parker Solar Probe Spacecraft

Image Credits: https://blogs.nasa.gov/parkersolarprobe/
Source: https://www.nasa.gov/press-release/goddard/2018/scientist-to-discuss-heliophysics-parker-solar-probe

Anatomy of an Orbit

Periapsis -- the location of minimum orbit altitude

Argument of Periapsis -- the angle, measured in the orbit plane, from the ascending node to the periapsis

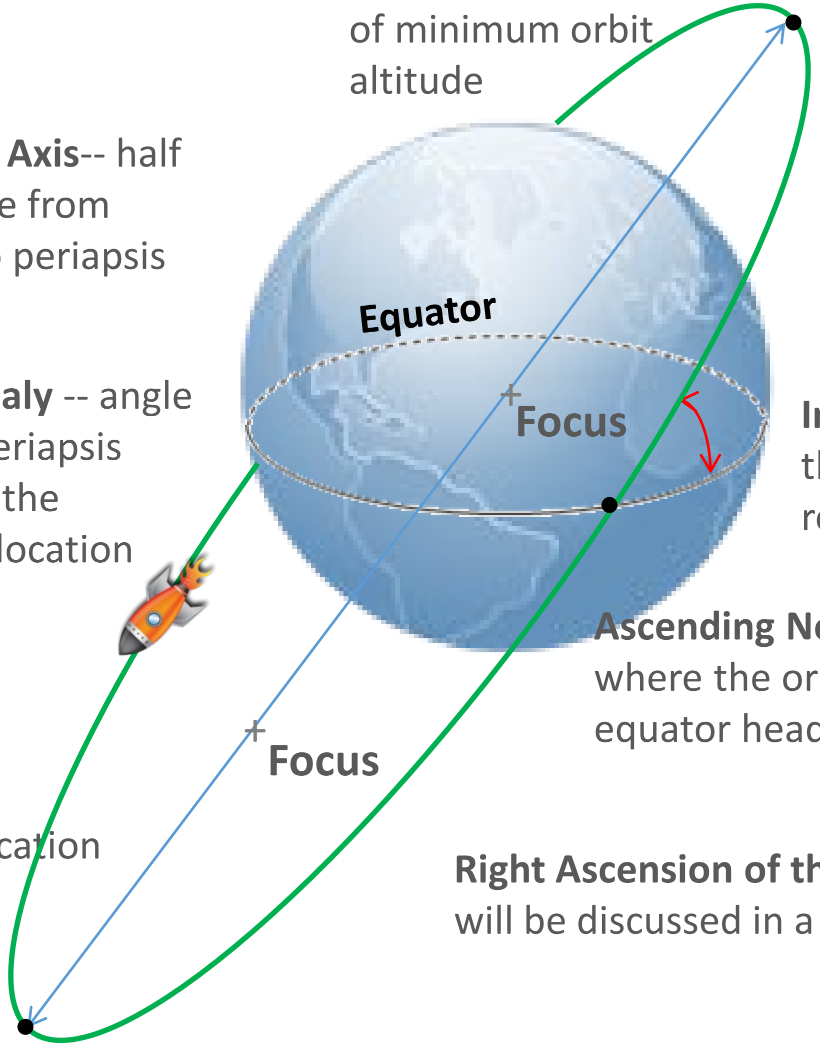
Semimajor Axis-- half the distance from apoapsis to periapsis

True Anomaly -- angle from the periapsis location to the spacecraft location

Inclination -- the tilt of the orbit plane with respect to the equator

Eccentricity – is a measure of the shape of an orbit – circular, elliptical, parabolic, or hyperbolic

Ascending Node -- the location where the orbit crosses the equator headed south to north

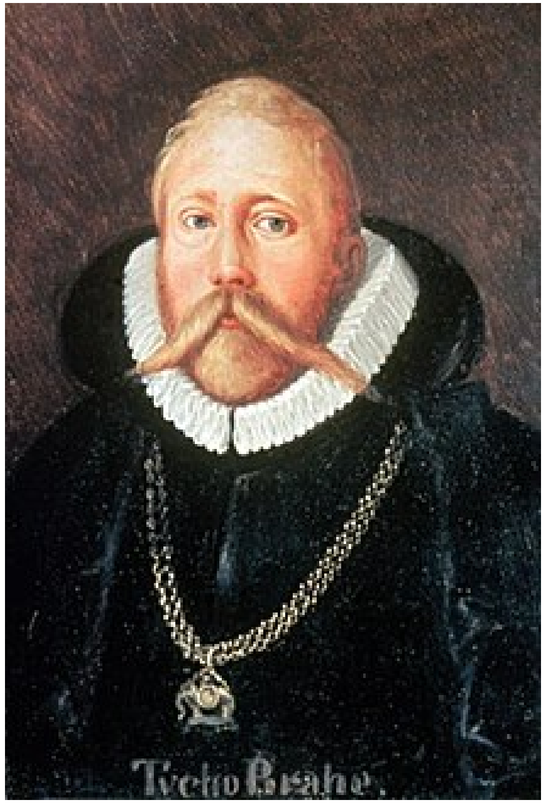


Apoapsis -- the location of maximum orbit altitude

Right Ascension of the Ascending Node will be discussed in a subsequent section.

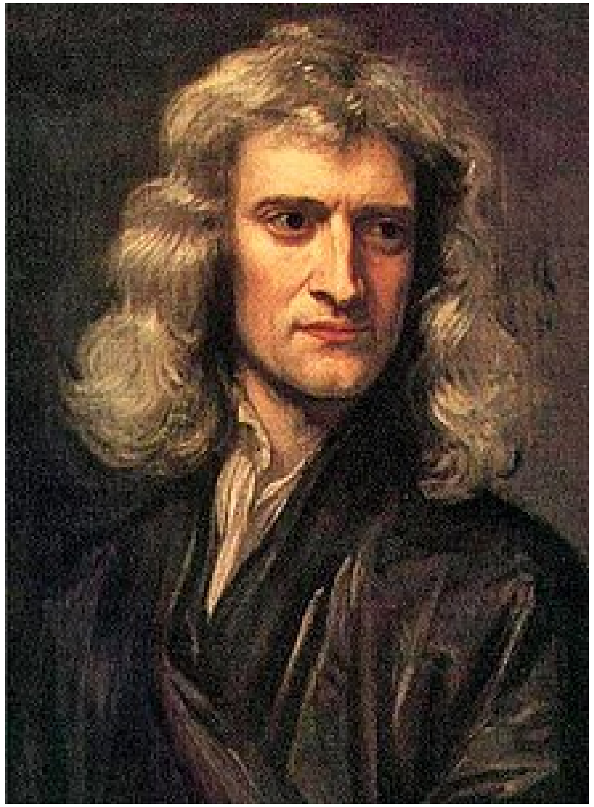
Orbit Elements

Johannes Kepler, Tycho Brahe, and Isaac Newton



Brahe, 1546-1601

Kepler, 1571-1630



Newton, 1643 (1642 O.S.) - 1727

Image Credits: https://en.wikipedia.org/wiki/Johannes_Kepler
Source: https://en.wikipedia.org/wiki/Tycho_Brahe
Source: https://en.wikipedia.org/wiki/Isaac_Newton

Kepler’s First Law

The orbit of each planet is an ellipse*, with the sun as a focus.

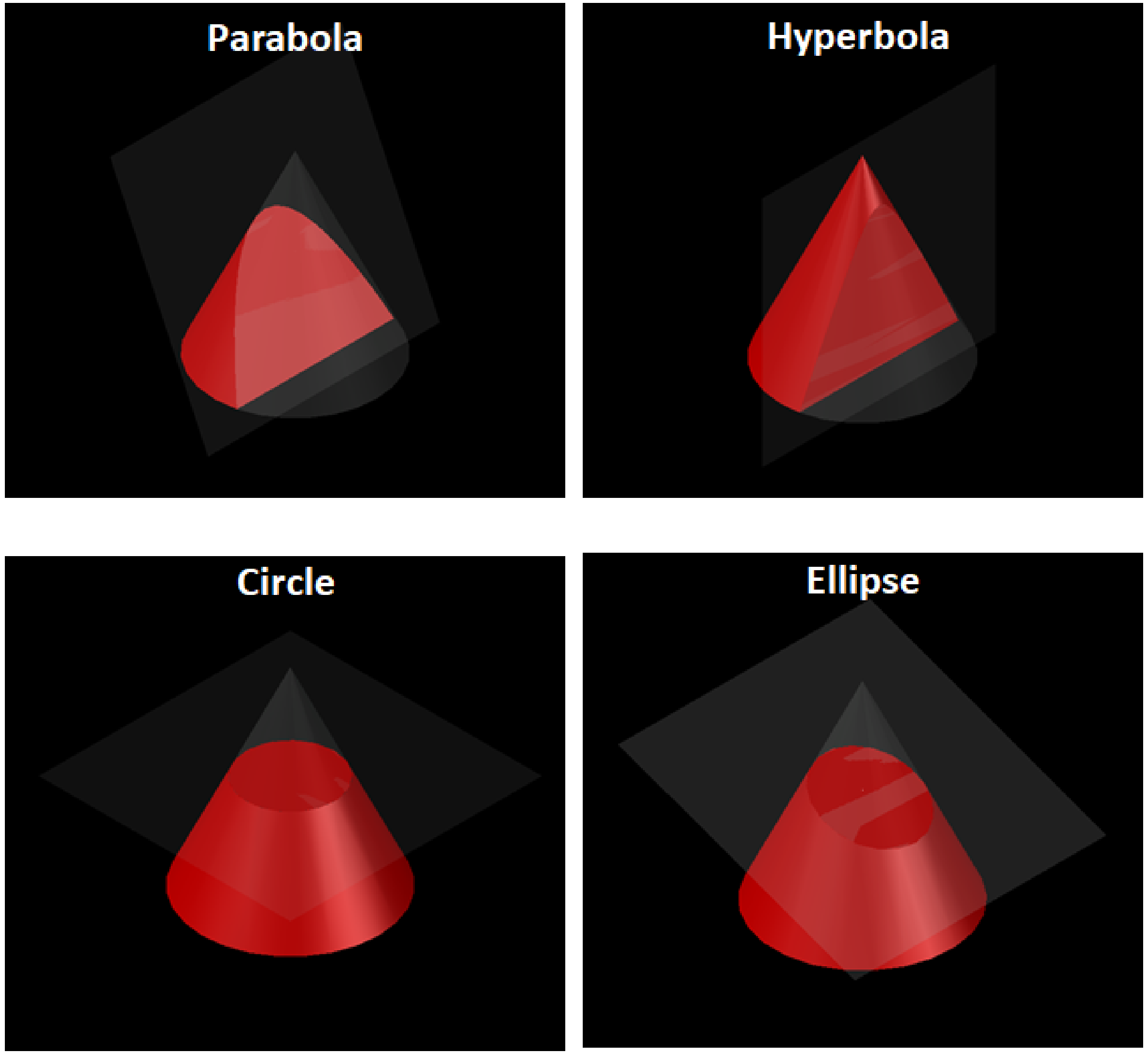
*Actually, other orbit shapes are possible and are described by the “conic sections.”

Orbits of the Inner Planets

Conic Sections

Take a cone and cut it with a plane at different angles

The shapes appearing at the cutting plane are also the shapes of the orbits.



Circular Orbit

Circular orbits maintain a constant distance from their central body;

Orbit eccentricity, $e = 0$;

Many Earth satellites have circular orbits;

The best example is the International Space Station.

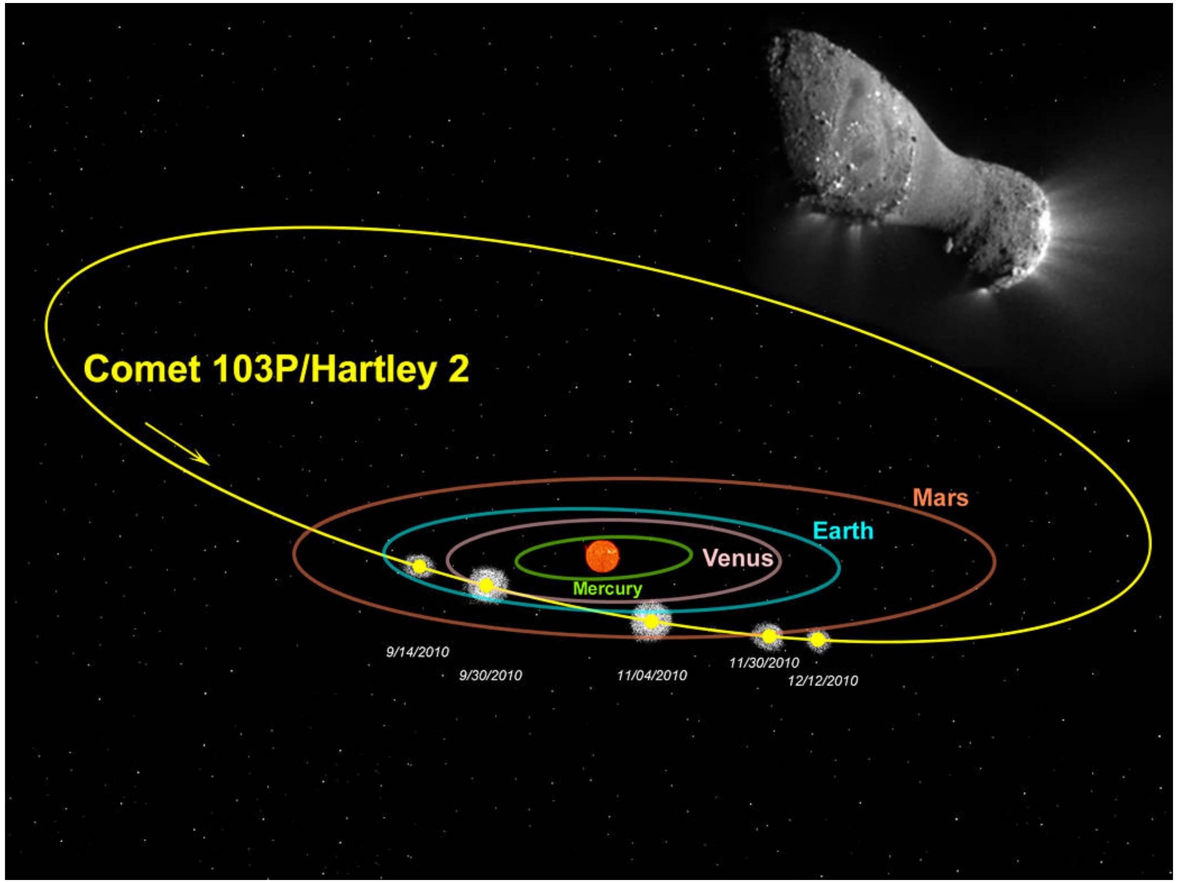
Example: International Space Station Orbit

Elliptical Orbit

Orbit eccentricity, $0 < e < 1$;

An elliptical orbit traces out an ellipse with the central body at one focus;

Comets such as 103P/Hartley 2 are in elliptical orbits with a period of 6.46 years ($e = 0.694$).



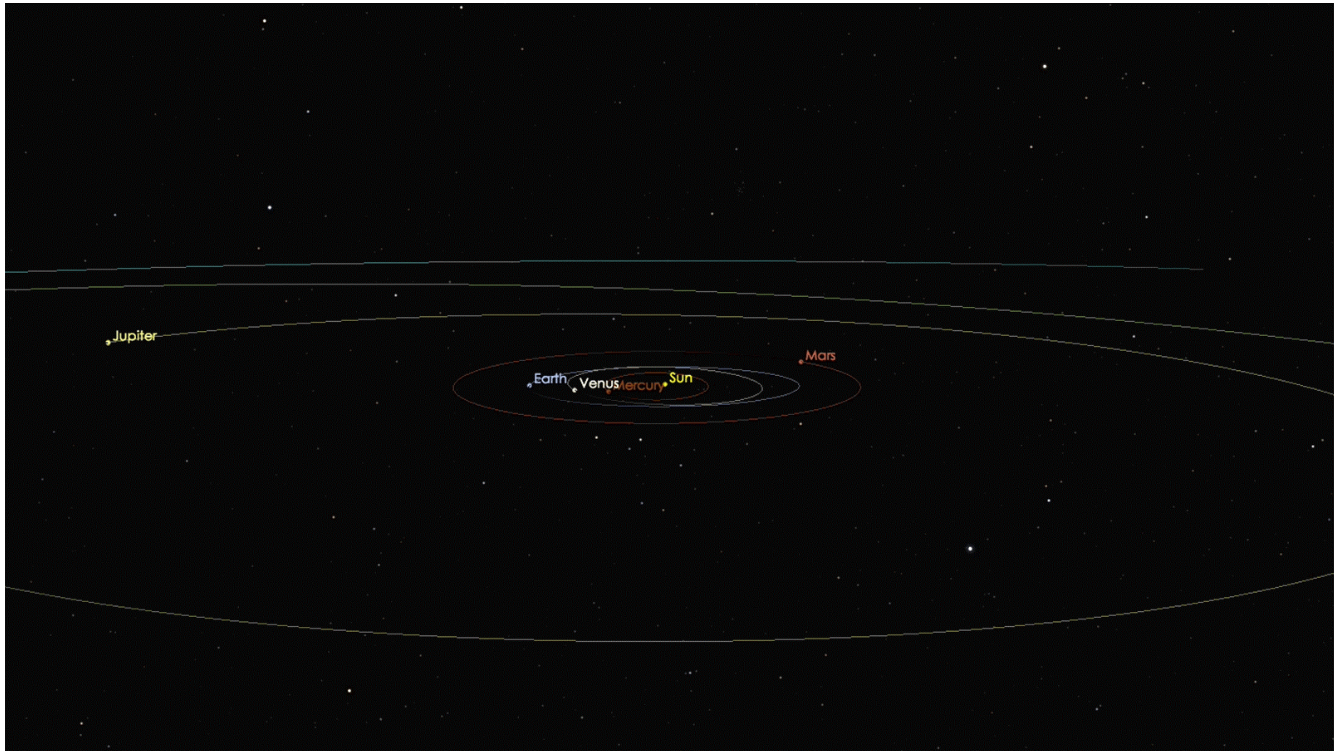
Example: Comet Hartley 2 Orbit

Image Credit: NASA/Steele Hill; Inset Image Credit: NASA/JPL-Caltech/UMD

Hyperbolic Orbit

Orbit eccentricity, $e > 1$;

For objects passing through the solar system, a hyperbolic orbit suggests an interstellar origin -- Asteroid Oumuamua was discovered in 2017 and is first and only object of this type ($e = 1.19951$).



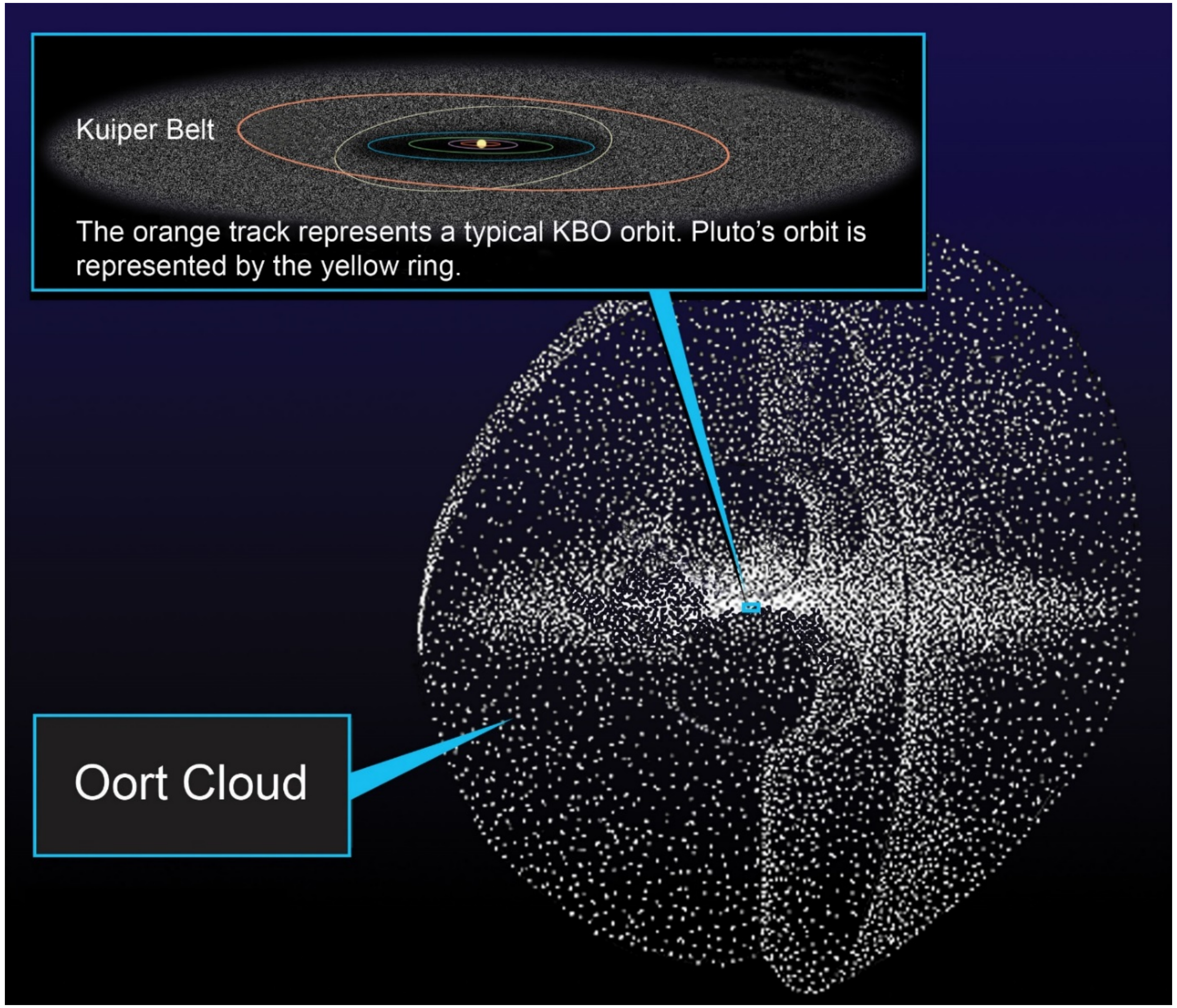
Example: Asteroid Oumuamua

Video Credit: https://solarsystem.nasa.gov/asteroids-comets-and-meteors/comets/oumuamua/in-depth/
Source: https://en.wikipedia.org/wiki/%CA%BBOumuamua

Parabolic Orbit

Orbit eccentricity, $e = 1$;

“Within observational uncertainty, long term comets all seem to have parabolic orbits. That suggests they are not truly interstellar, but are loosely attached to the Sun. They are generally classified as belonging the *Oort cloud* on the fringes of the solar system, at distances estimated at 100,000 AU.”



The Oort Cloud

Source: https://www-spof.gsfc.nasa.gov/stargaze/Scomets.htm
Image Credit: https://solarsystem.nasa.gov/resources/491/oort-cloud/

Kepler’s Second Law

The line joining the planet to the sun sweeps out equal areas, A in equal times, Δt.

$$\Delta t_1 = \Delta t_2$$

$$A_1 = A_2$$

Demonstration of Constant “Areal” Velocity

Kepler's Third Law

The square of the period, T of a planet is proportional to the cube of its mean distance, a to the sun (or its central body).

$$T^2 \propto a^3$$

For the orbits at the right:

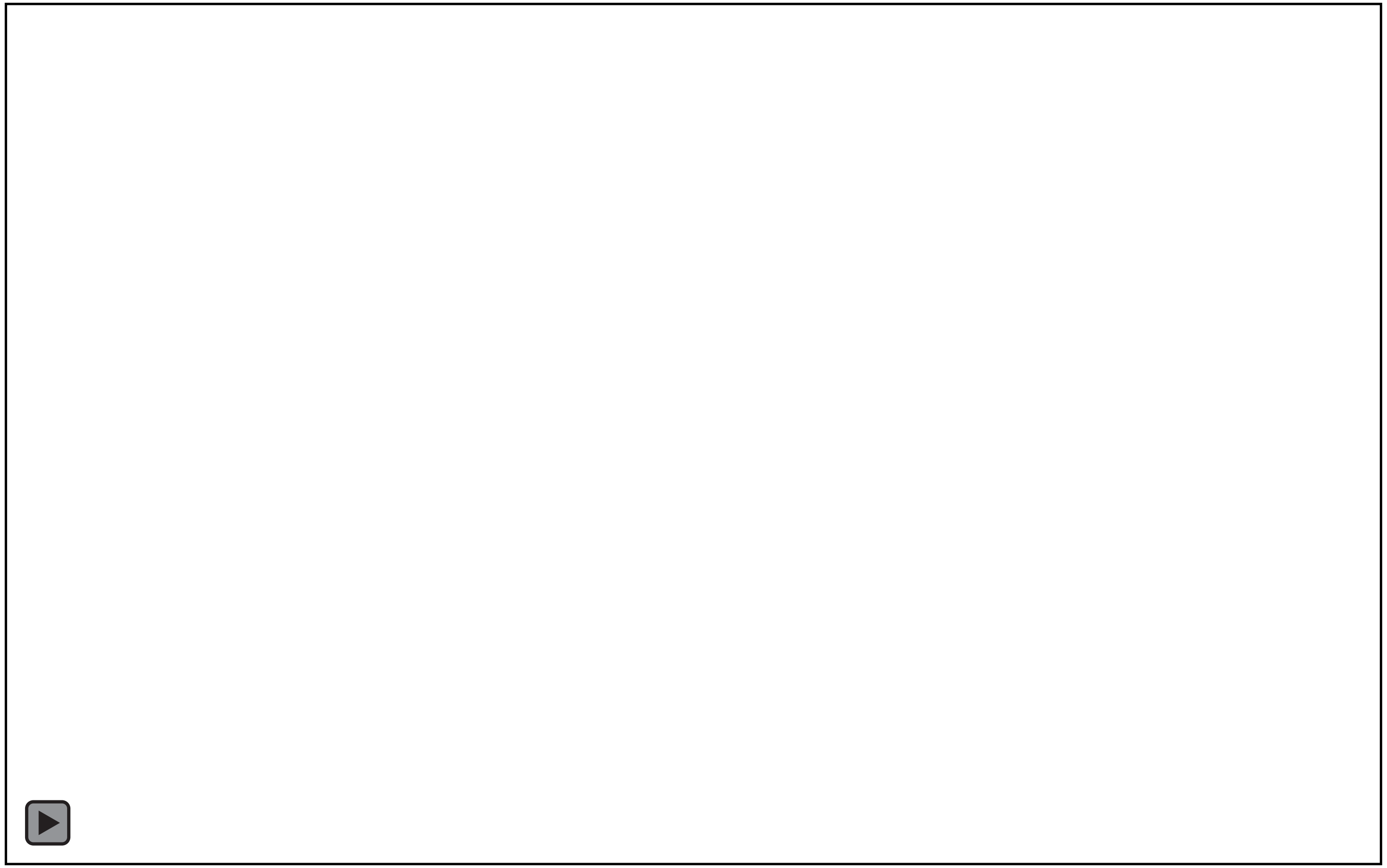
$$T_{outer\ orbit} = \sqrt{2^3}\, T_{inner\ orbit}$$

Orbits with Different Semimajor Axes

Kepler’s Third Law

As long as the semimajor axis, a is the same, the orbit period will be the same;

Each orbit has a different eccentricity, e but both orbits have the same a.



Two Orbits with the Same Period

Thank You, Sir Isaac Newton!

The governing differential equation for two body astrodynamics is derived from two laws originated by Sir Isaac Newton.

<u>Newton's Second Law</u>

$$\bar{\boldsymbol{F}} = m\bar{\boldsymbol{a}}$$

<u>Newton's Law of Gravitation</u>

$$\bar{\boldsymbol{F}} = \frac{-GMm}{r^2}\left(\frac{\bar{\boldsymbol{r}}}{r}\right)$$

Thank You, Sir Isaac Newton!

For two bodies that are moving only under gravitational force, we can equate both expressions:

$$\bar{\boldsymbol{F}} = m\bar{\boldsymbol{a}} = \frac{-GMm}{r^2}\left(\frac{\bar{\boldsymbol{r}}}{r}\right)$$

and we can express the acceleration, $\bar{\boldsymbol{a}}$ in terms of radial acceleration, $\ddot{\bar{\boldsymbol{r}}}$:

$$m\ddot{\bar{\boldsymbol{r}}} = \frac{-GMm}{r^2}\left(\frac{\bar{\boldsymbol{r}}}{r}\right)$$

Thank You, Sir Isaac Newton!

Rearrange…

$$m\ddot{\bar{\boldsymbol{r}}} + \frac{GMm}{r^2}\left(\frac{\bar{\boldsymbol{r}}}{r}\right) = \mathbf{0}$$

Simplify noting that m cancels, r combines in the denominator and $\mu = GM$:

$$\boxed{\ddot{\bar{\boldsymbol{r}}} + \frac{\mu}{r^3}\bar{\boldsymbol{r}} = \mathbf{0}}$$

Thank You, Sir Isaac Newton!

There's A LOT of information to be squeezed out of this equation including…

$$\ddot{\bar{\boldsymbol{r}}} + \frac{\mu}{r^3}\bar{\boldsymbol{r}} = 0$$

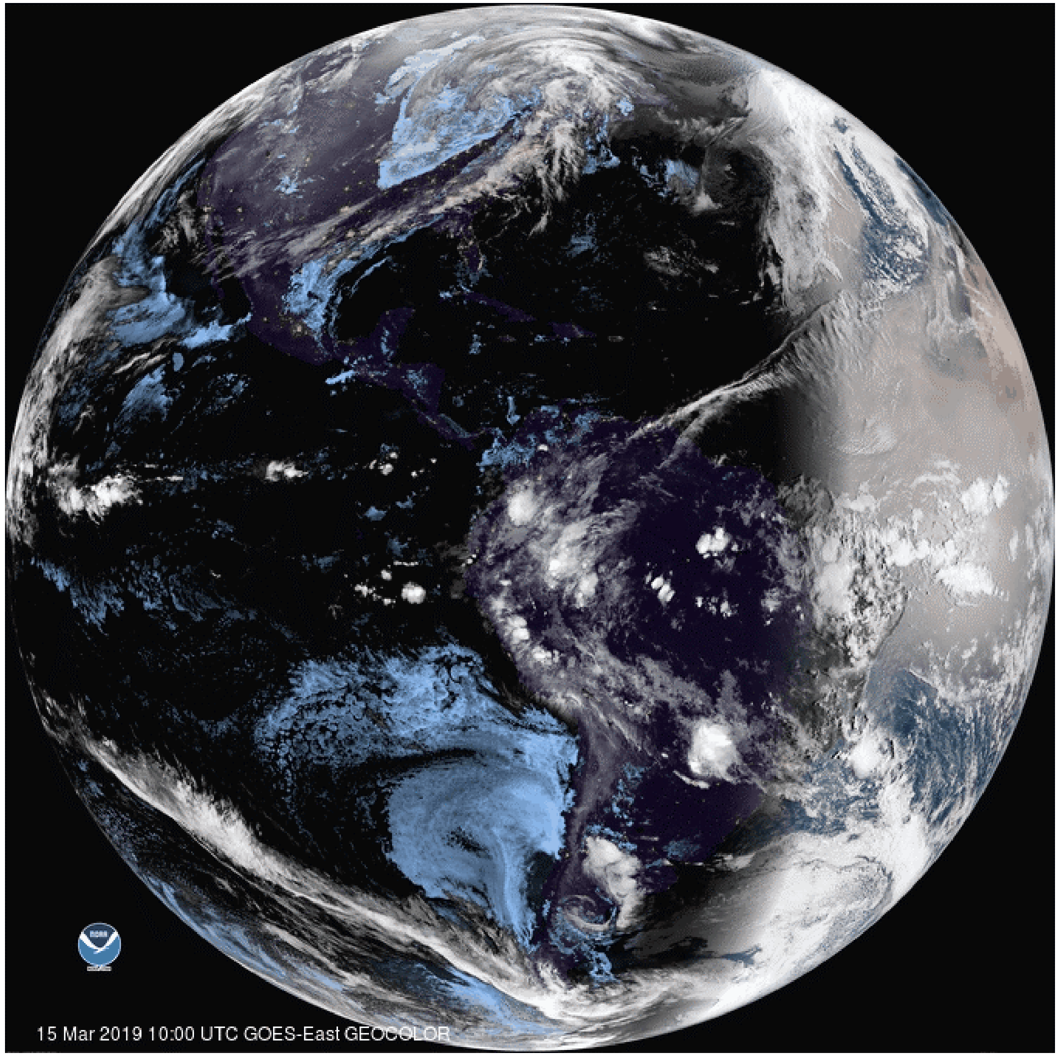
- Conservation of Specific Mechanical Energy $\quad \mathcal{E} = \frac{v^2}{2} + \left(c - \frac{\mu}{r}\right)$
- Conservation of Specific Angular Momentum $\quad \bar{\boldsymbol{h}} = \bar{\boldsymbol{r}} \times \bar{\boldsymbol{v}}$
- Kepler's 1st Law: Trajectory $\quad r = \frac{a(1-e^2)}{1 + e\cos\nu}$
- Kepler's 2nd Law: Constant "Areal" Velocity $\quad \frac{1}{2}r^2\dot{\theta} = constant$
- Kepler's 3rd Law: Orbit Period $\quad \mathrm{T} = 2\pi\sqrt{\frac{a^3}{\mu}}$

Example: Geostationary Orbit

A geostationary orbit has an orbit period of 24 hours with an orbit inclination of 0 degrees;

In this orbit, the spacecraft appears fixed over a specific location on Earth's equator;

Geostationary orbits are used for communications satellites and weather satellites.



Earth as Seen from GOES 16

Animation Source: https://www.star.nesdis.noaa.gov/GOES/fulldisk_band.php?sat=G16&band=GEOCOLOR&length=12

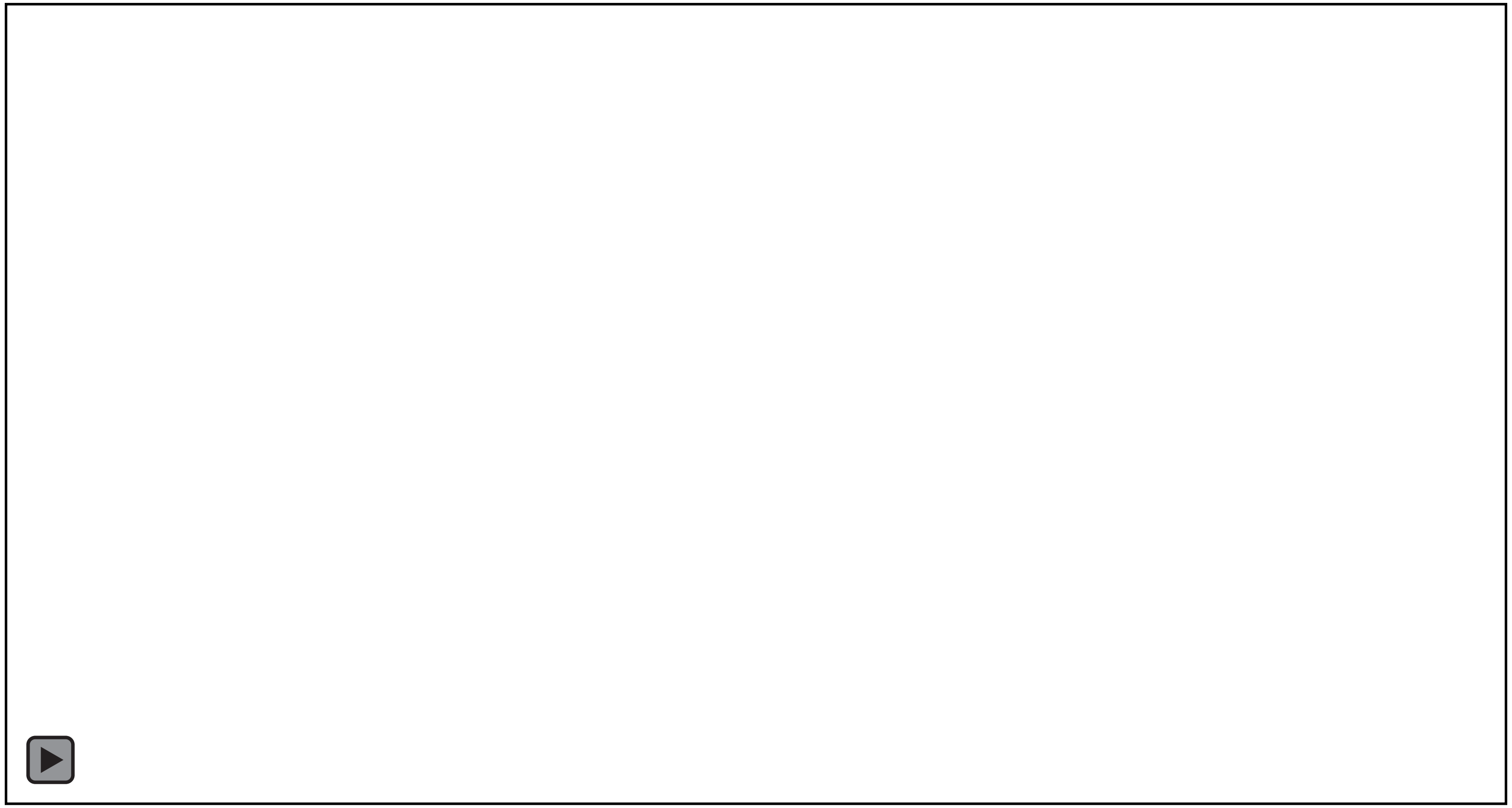
Example: Geostationary Orbit

At what altitude must the satellite be positioned to be geostationary?

$$\mathrm{T} = 2\pi\sqrt{\frac{a^3}{\mu}}$$

$$a = \sqrt[3]{\frac{\mu \mathrm{T}^2}{4\pi^2}}$$

$$h = a - r_e = {\sim}35786\ km$$



Geostationary Orbit

Perturbed Orbits

Recall our previous equation was derived for a body moving under the influence of ONLY the gravity of a central body:

$$\ddot{\bar{\boldsymbol{r}}} + \frac{\mu}{r^3}\bar{\boldsymbol{r}} = \mathbf{0}$$

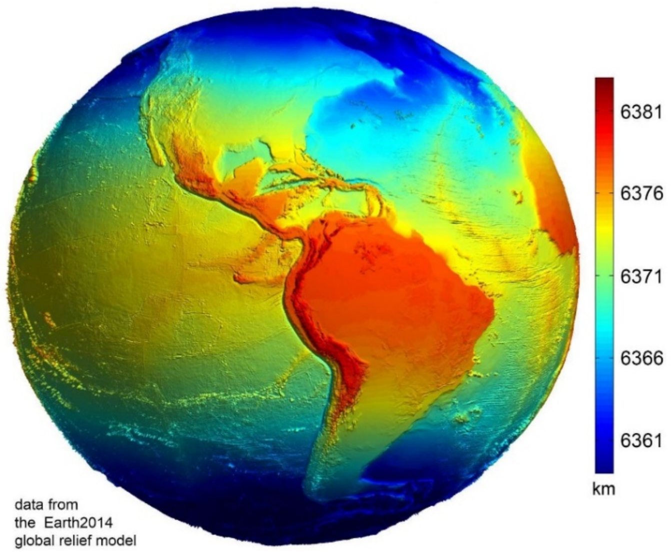
Some interesting things happen when there is a perturbing force such that:

$$\ddot{\bar{\boldsymbol{r}}} + \frac{\mu}{r^3}\bar{\boldsymbol{r}} \neq \mathbf{0}$$

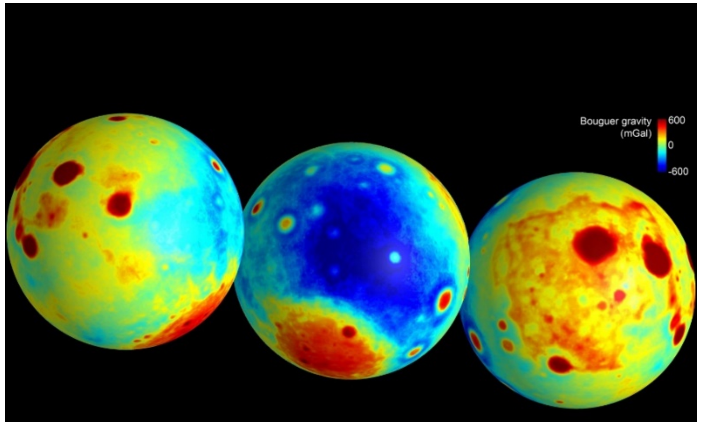
Perturbed Orbits

There are many forces that can perturb an orbit including:

- Spherical harmonics
- Drag
- Radiation pressure
- Other celestial bodies
- Tides
- Mass concentrations
- etc.



Equatorial Bulge from Earth 2014 Global Relief Model



Lunar Gravity Anomalies Measured by NASA's GRAIL Mission

Image Credits: Earth 2014 Global Relief Model, C. Hirt, used with permission
Source: http://www.ngs.noaa.gov/PUBS_LIB/Geodesy4Layman/80003051.GIF
Image credit: NASA/JPL-Caltech/CSM

Perturbations

Earth is not a perfect sphere -- it is oblate and has a slight bulge in the equatorial region and this imperfection gives rise to some major orbit perturbations;

Precession of the Ascending Node:

$$\frac{d\Omega}{dt} = \dot{\Omega} = \frac{-3J_2 n r_e^2 \cos i}{2a^2(1-e^2)^2}$$

Precession of the Periapsis:

$$\frac{d\omega}{dt} = \dot{\omega} = \frac{3J_2 n r_e^2}{4a^2(1-e^2)^2}(4 - 5\sin^2 i)$$

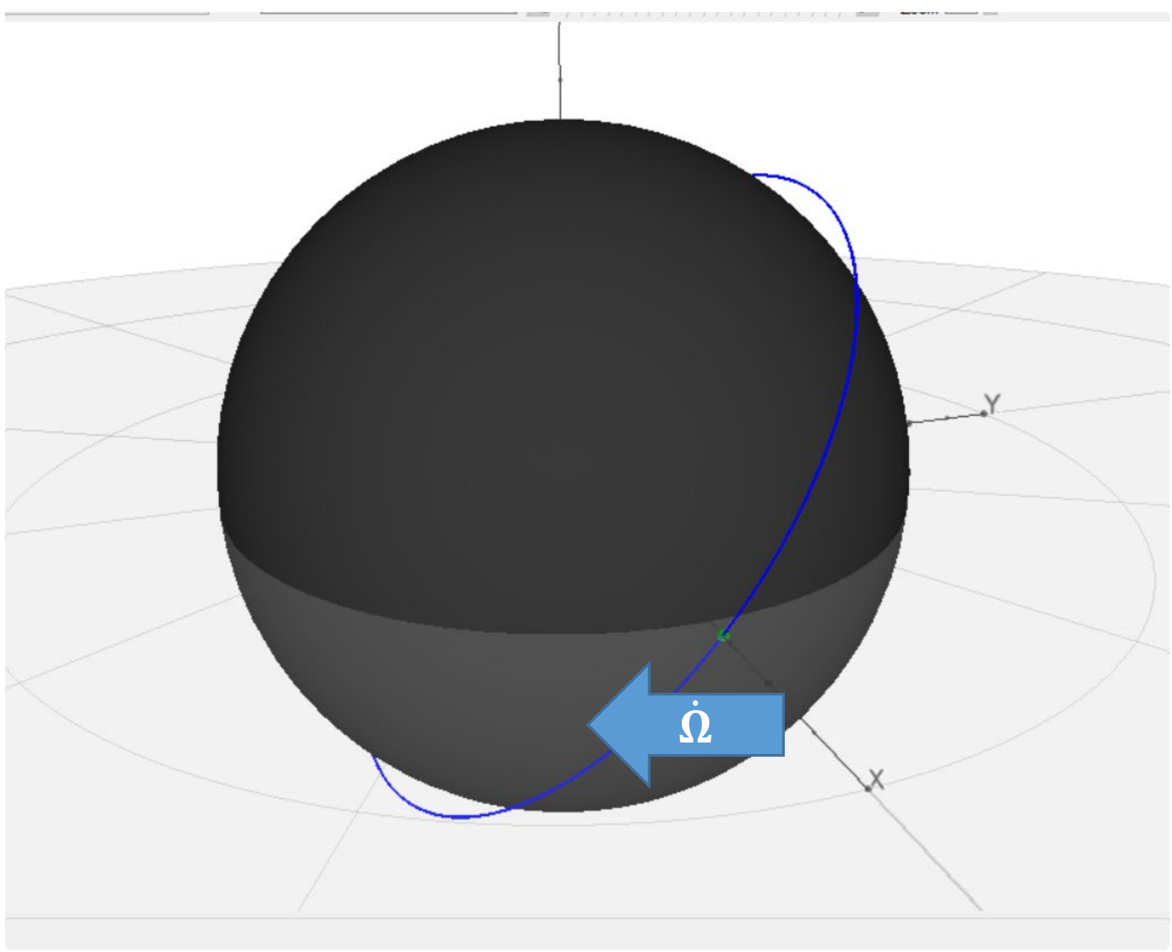
Note: For Earth, $J_2 = 1.082626683 \times 10^{-3}$

Perturbations: Precession of the Ascending Node

The oblateness perturbation causes the ascending orbit ascending node to precess at the rate:

$$\frac{d\Omega}{dt} = \dot{\Omega} = \frac{-3J_2 n r_e^2 \cos i}{2a^2(1-e^2)^2}$$

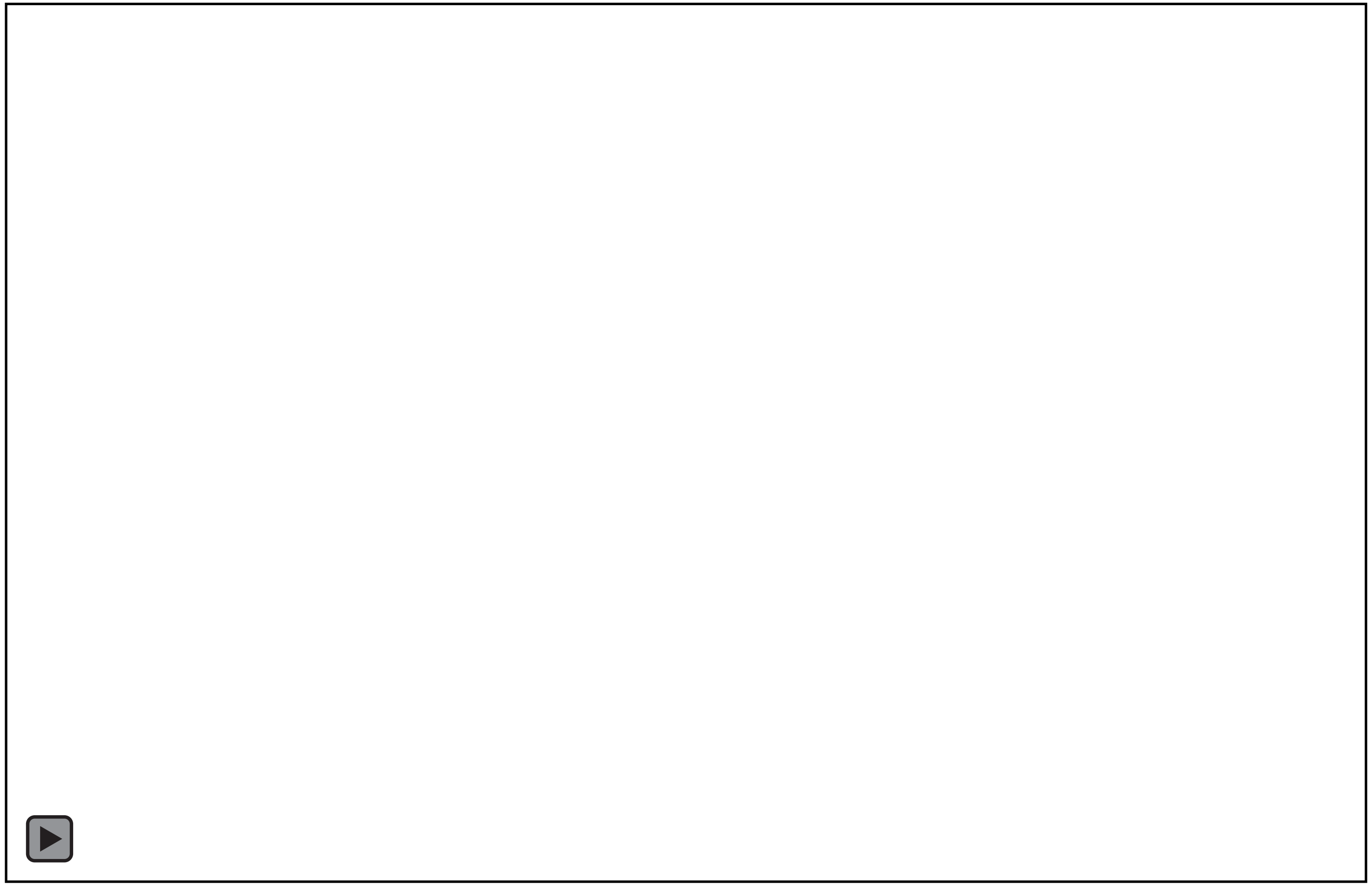
For orbit inclinations, $i < 90°$, precession is westward – when $i > 90°$, precession is eastward.



Precession of the Orbit Ascending Node

Example: Sun Synchronous Orbit

Sun synchronous orbits are useful for Earth observation spacecraft because they are designed to pass over sunlit portions of the planet at the same “local solar” time – this results in consistent illumination conditions for observations.

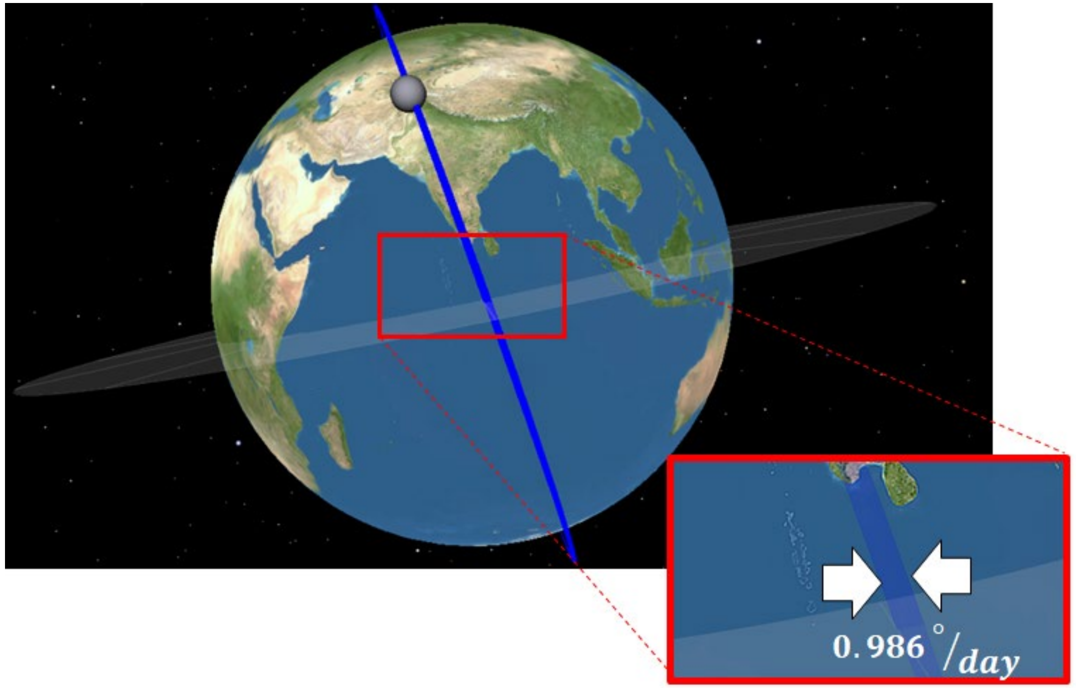


Sun Synchronous Orbit

Example: Sun Synchronous Orbit

To achieve this, the orbit ascending node needs to maintain a consistent offset from the orbit subsolar point – this is accomplished by moving the orbit ascending node at the same rate the sun appears to move across the celestial sphere --to meet this condition:

$$\dot{\Omega} \approx 0.986^{\circ}/_{day} \ EASTWARD$$



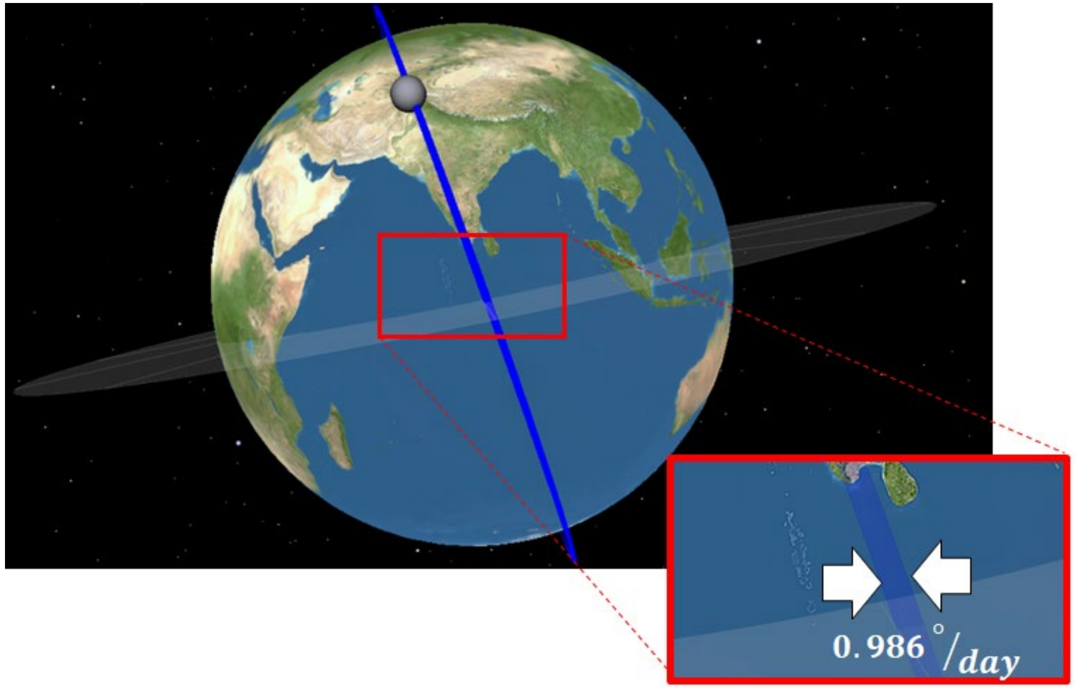
Daily Precession of the Ascending Node

Example: Sun Synchronous Orbit

Assuming a circular orbit ($e = 0$), we see that combinations of i and a may be used to specify the desired orbit.

$$\dot{\Omega} = \frac{-3J_2 n r_e^2 \cos i}{2a^2(1-e^2)^2}$$

One such combination is $i = 98.2°$ and $a = 7083\ km\ (altitude = 705\ km)$



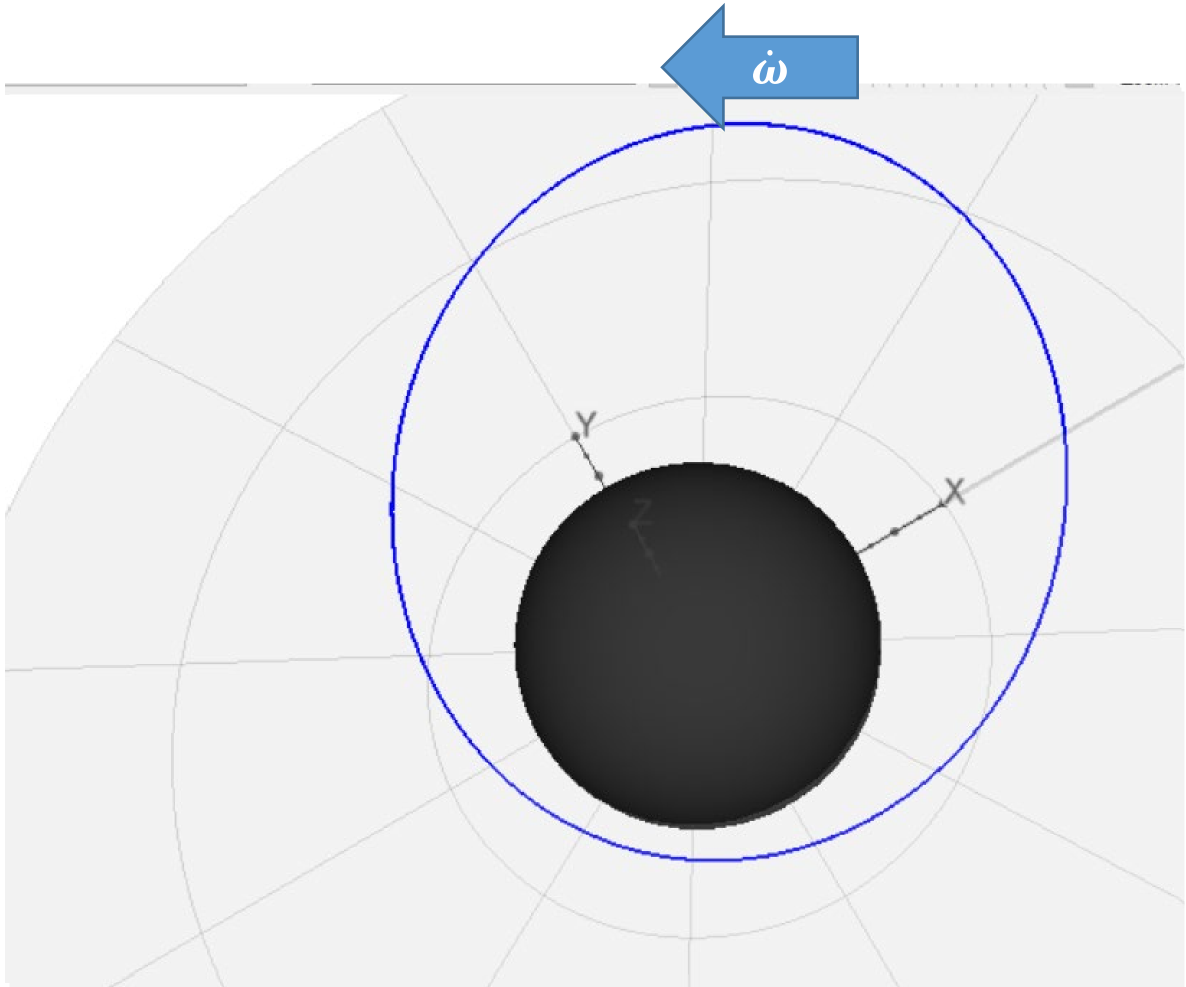
Daily Precession of the Ascending Node

Perturbations: Precession of the Periapsis

The oblateness perturbation also causes the periapsis and apoapsis to precess at the rate:

$$\dot{\omega} = \frac{3J_2 n r_e^2}{4a^2(1-e^2)^2}(4 - 5\sin^2 i)$$

Precession is positive when $(4 - 5\sin^2 i) < 0$ and negative when $(4 - 5\sin^2 i) > 0$.



Precession of the Orbit Periapsis

Example: Molniya Orbit

Communication satellites in geostationary orbits over the equator are of little use to those living at higher latitudes because they appear low in the sky;

A satellite orbiting at a higher inclination is desired;

However, it won’t appear to remain over the same point on the ground;

A Molniya orbit may be used to cause the spacecraft to dwell at nearly the same point for long periods of time.

Example: Molniya Orbit

In order to "lock" the location of the apoapsis and periapsis in place, we desire an orbit where the rate of movement of the periapsis goes to zero:

$$\frac{d\omega}{dt} = \dot{\omega} = \frac{3J_2 n r_e^2}{4a^2(1-e^2)^2}(4 - 5\sin^2 i) = 0$$

We see from the equation that this happens when:

$$(4 - 5\sin^2 i) = 0$$

This is true when the inclination is $i = 63.4°$

Example: Molniya Orbit

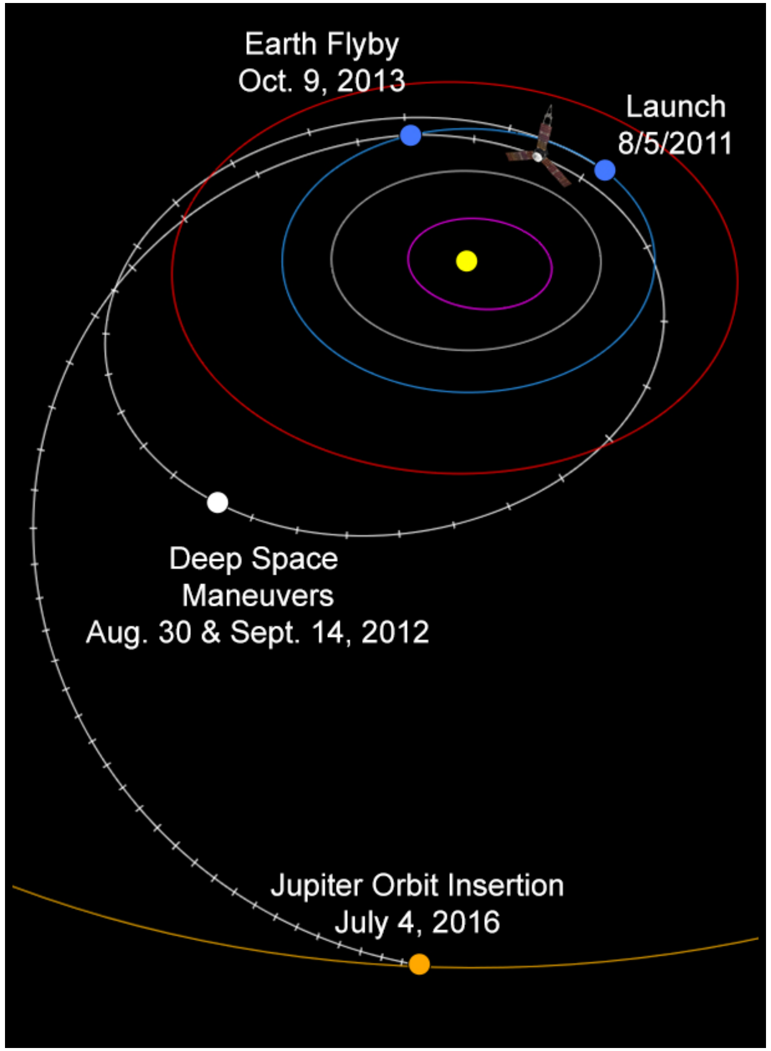
The spacecraft spends much of its orbit at high altitude, at high latitude, moving slowly -- appearing nearly stationary when near apoapsis;

Orbit is designed so that apoapsis stays “locked” into the same position over time.

Molniya Type Orbit (Time points in red are 10 minutes apart)

Other Orbits: Gravity Assist Orbit

Spacecraft orbits may be redirected using gravity assist maneuvers where a close fly-by of a planet is be used to change the direction of and orbit and add energy to it.



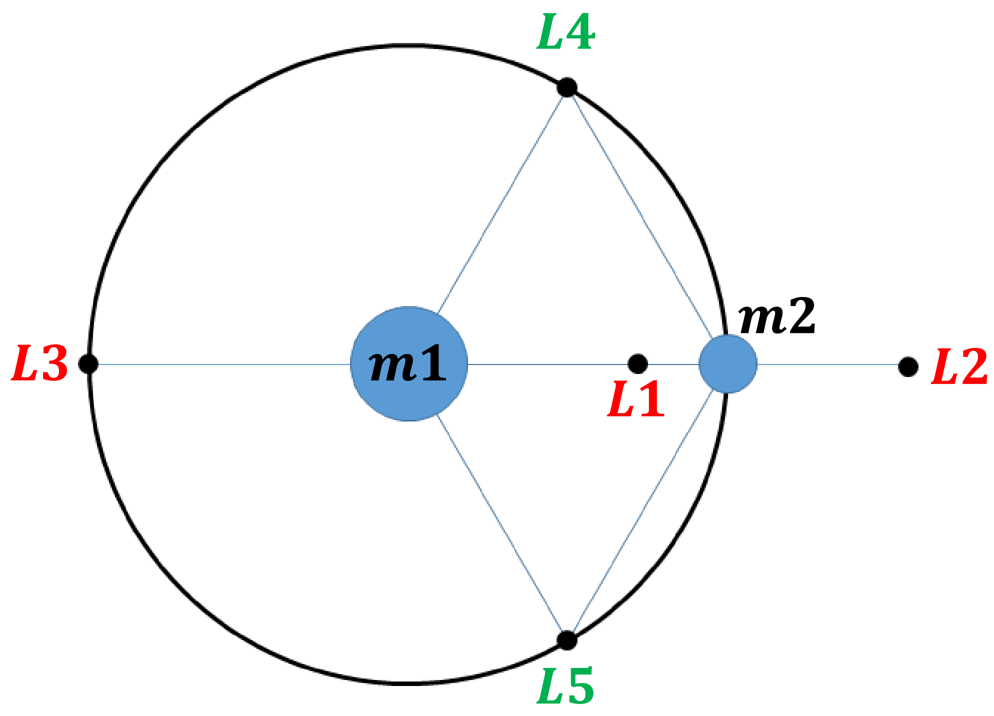
Juno Spacecraft Trajectory

Image Credit: NASA/JPL/Caltech
Source: https://www.jpl.nasa.gov/images/juno/earthflyby/JunoCruiseTraj_20130815.jpg

Other Orbits: The Restricted Three-Body Problem

When two large masses, $m1$ and $m2$, are orbiting one another, regions in space can serve as gravitational nodes where spacecraft or other celestial bodies can collect – these are called Lagrange points;

$L1$, $L2$ and $L3$ and unstable.
$L4$ and $L5$ are stable.

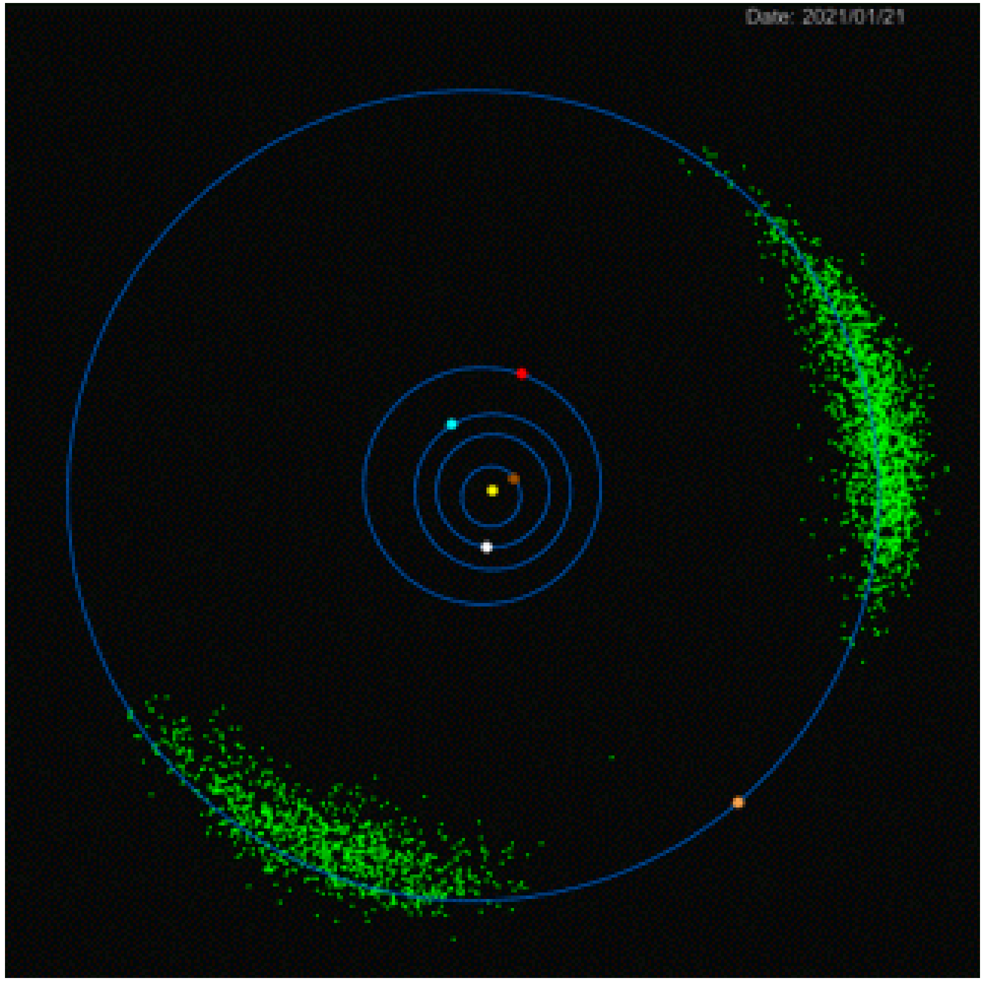


Dynamics of the Three-Body System

Other Orbits: Three-Body Orbits

In 1772, using three-body assumptions, Joseph-Louis Lagrange believed that, because the L4 and L5 points are stable, asteroids might be trapped near these points;

The first confirmed observation of a Jupiter Trojan was made by Max Wolf in 1906.



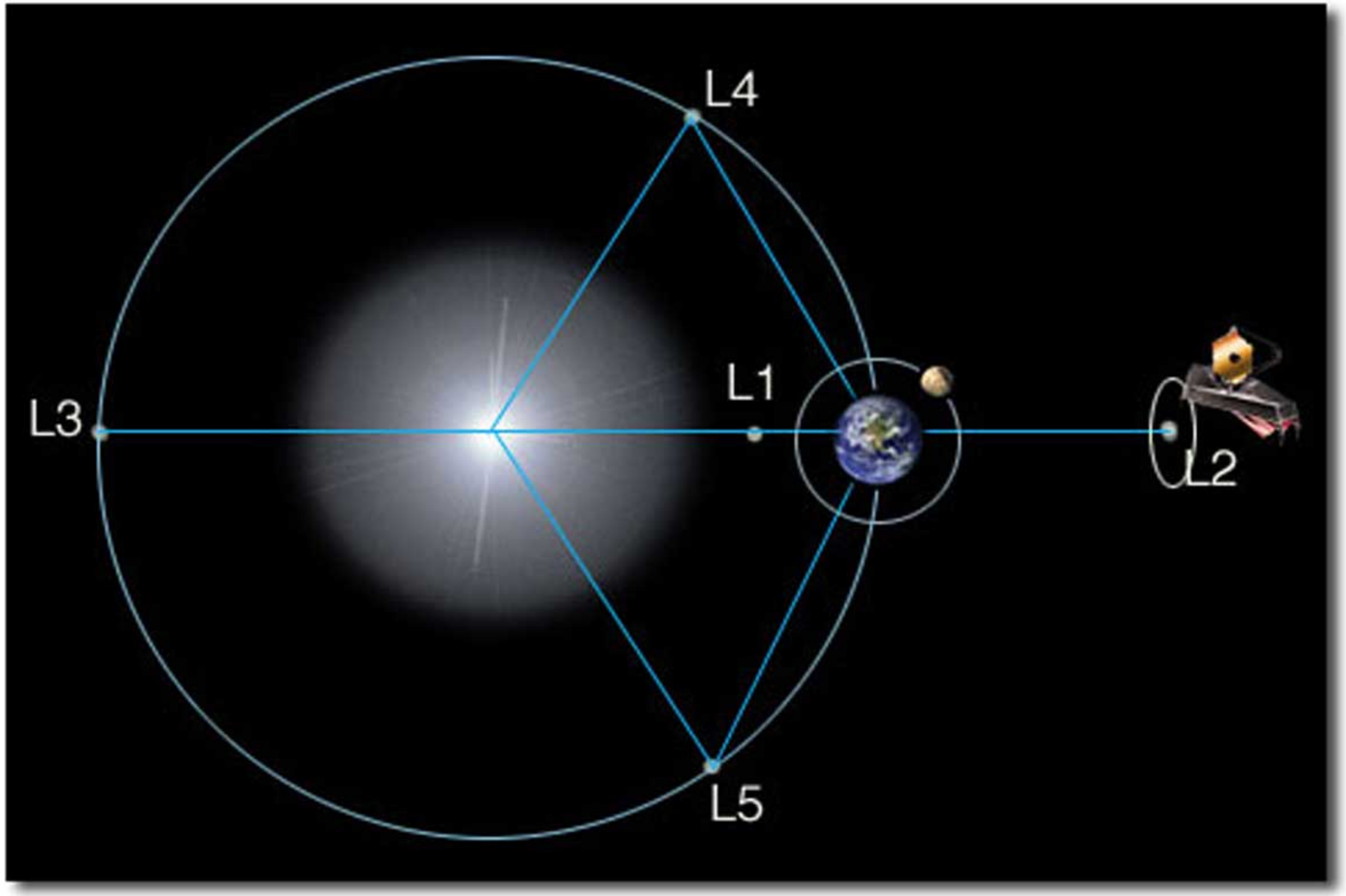
Jupiter's Trojan Asteroids

Animation Credit: CAS/Petr Scheirich. Used with permission.
Source: https://en.wikipedia.org/wiki/Jupiter_trojan

Other Orbits: Three-Body Orbits

Lagrange points are also used for spacecraft;

The James Webb Space Telescope (JWST) is located at the Earth-Sun L2 point.



JWST Orbit at the Earth-Moon L2 Point

Image Credit: NASA
Source: https://www.nasa.gov/images/content/463480main_lagrange_point_lg_1.jpg

Other Orbits: Three-Body Orbits

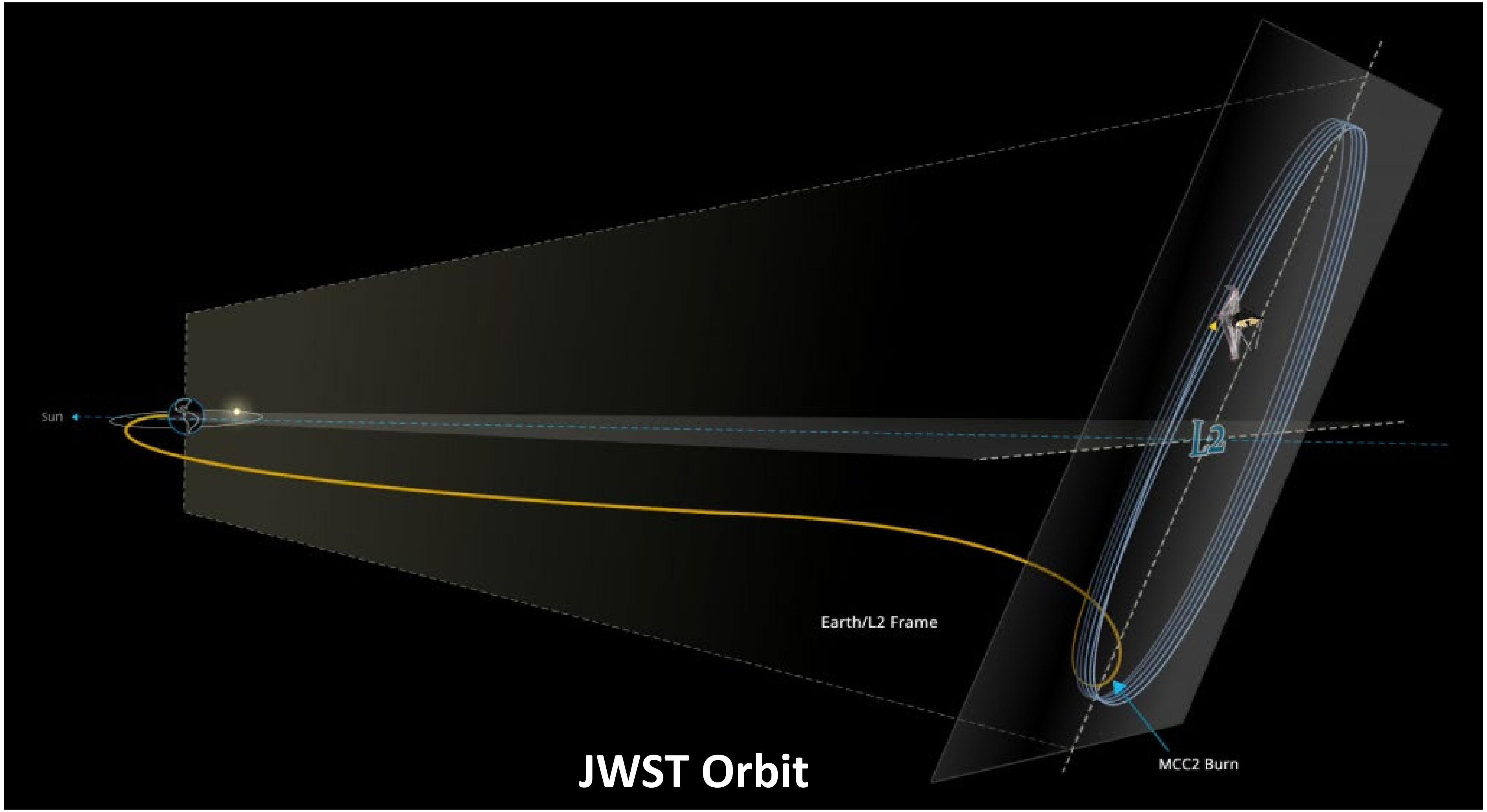


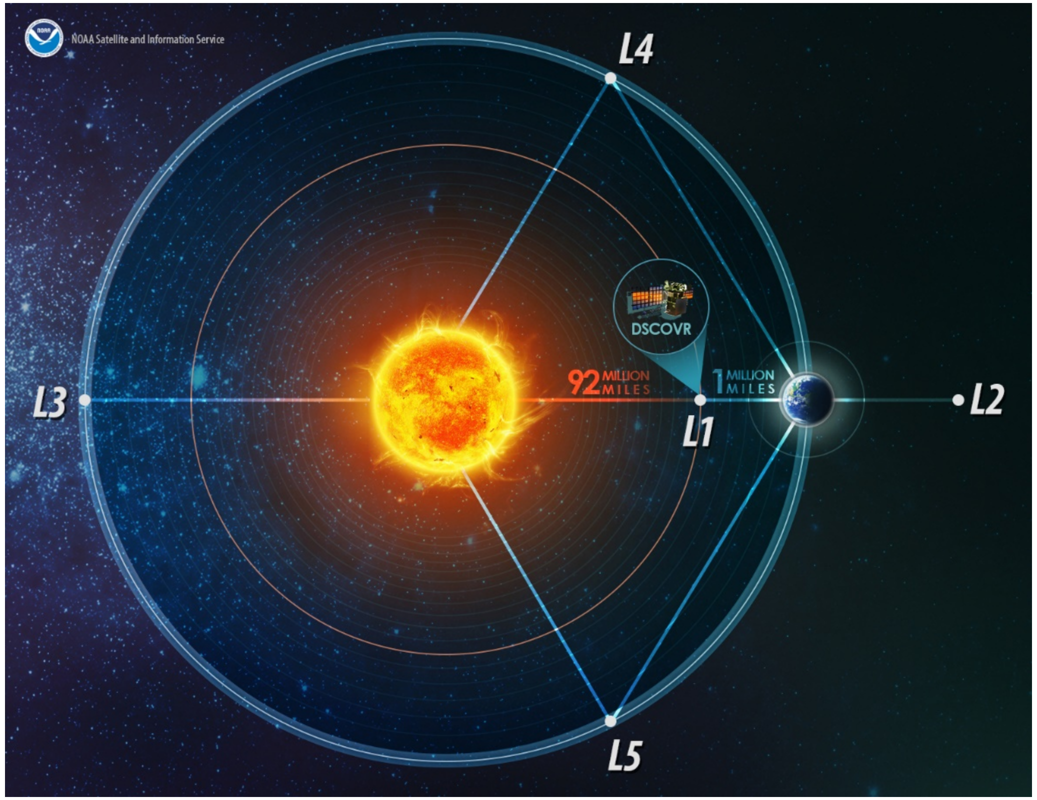
Image Credit: Steve Sabia/NASA Goddard
Source: https://blogs.nasa.gov/webb/2022/01/24/orbital-insertion-burn-a-success-webb-arrives-at-l2/

Other Orbits: Three-Body Orbits

Source: https://svs.gsfc.nasa.gov/13553

Other Orbits: Three-Body Orbits

The DSCOVR spacecraft is located at the Sun-Earth L1 point.



DSCOVR Spacecraft at the Sun-Earth L1 Point

The Earth-Moon System as Seen from the DSCOVR Spacecraft

Image Credit: NOAA
Video Credit: DSCOVR: EPIC Team

Questions?