

# The Additive Manufacturing Moment Measure – A Parallel Computation Technique for Determining Build Variance in the Laser Powder Bed Fusion Process

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# Motivation

Several challenges have motivated many process modeling approaches for laser powder bed fusion (L-PBF) additive manufacturing (AM):

- ❖ Quality control of L-PBF AM parts
  - ❖ Ensure integrity of build instructions
  - ❖ Determine and measure variance from build instructions (digital twin)
  - ❖ Develop in-situ process monitoring techniques
- ❖ Reduce development cost for build strategies
  - ❖ Identify localized defect risk based on processing conditions





# Additive Manufacturing (AM): Process Models

Many groups recognize the need for additive manufacturing process models

## ❖ High fidelity models

- ❖ Khairallah et al., <https://doi.org/10.1016/j.actamat.2016.02.014>.
- ❖ Martin et al., <https://doi.org/10.1038/s41467-019-10009-2>.
- ❖ ....

Physical correctness

Highest computational cost

Part scales are not yet feasible

part scales:  $1\text{cm}^3 - 20\text{cm}^3$

## ❖ Analytical and semi-analytical models

- ❖ Pinkerton et al., <https://doi.org/10.1088/0022-3727/37/14/003>.
- ❖ Tapia et al. <https://doi.org/10.1007/s00170-017-1045-z>.
- ❖ Groeber et al., <https://doi.org/10.1088/1757-899x/219/1/012002>.
- ❖ Tang et al., <https://doi.org/10.1016/j.addma.2016.12.001>
- ❖ Promopatum et al., <https://doi.org/10.1016/J.ENG.2017.05.023>
- ❖ Tang et al., <https://doi.org/10.1007/s11665-018-3647-5>
- ❖ Stump et al., <https://doi.org/10.1016/j.apm.2019.07.008>.
- ❖ Schwalbach et al., <https://doi.org/10.1016/j.addma.2018.12.004>.
- ❖ Gordon et al., <https://doi.org/10.1016/j.addma.2020.101552>
- ❖ Zagade et al., <https://doi.org/10.1016/j.addma.2021.102222>
- ❖ Wang et al., <https://doi.org/10.3390/cryst11121568>
- ❖ ...

Lowered computational costs

Layer-wise thermal history

Mesh generation & optimization

Time-step approach

Localized build conditions:

- Not addressed, or
- Memory bounded for part scales

## ❖ Graph theory-based models

- ❖ Rao et al., <https://doi.org/10.1115/msec2016-8516>.
- ❖ Fruggiero et al., <https://doi.org/10.1016/j.promfg.2019.09.022>.
- ❖ Cole et al., <https://doi.org/10.1016/j.ijthermalsci.2020.106383>.
- ❖ Yavari et al., <https://doi.org/10.1115/1.4043648>
- ❖ Yavari et al., <https://doi.org/10.1115/1.4047619>.
- ❖ ...

Lowered computational costs

Point-wise approach

Layer-wise thermal history

Time step approach

Memory bounded for part scales



# The Additive Manufacturing Moment Measure (AM3) Approach



## Additive Manufacturing

- ❖ *“Process of joining materials to make parts from 3D model data, usually layer upon layer, as opposed to subtractive manufacturing and formative manufacturing methodologies”*
  - ❖ ISO/ASTM 52900:2021, Additive manufacturing - General principles - Terminology

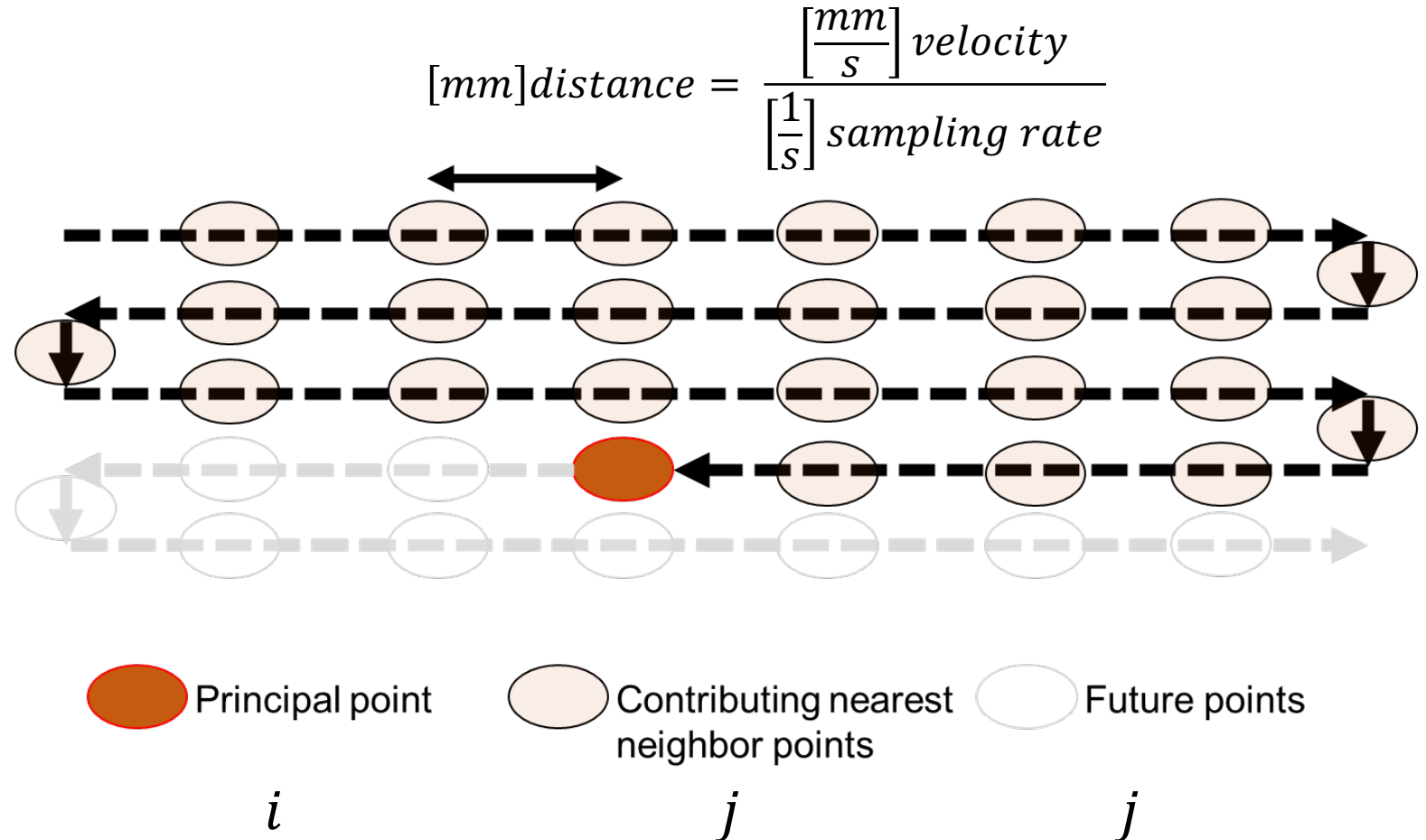
## Moment Measure

- ❖ Motivated from physics, a moment measure is the generalization of the concept of a moment to a discrete point field of observables
- ❖ Combined = Additive Manufacturing Moment Measure (AM3)
  - ❖ Rapid part scale modeling of process conditions
  - ❖ Compare modeling against in-situ monitoring
  - ❖ Evaluate of build instructions
  - ❖ Measure variances from build instructions



# AM3 Approach: Concept Illustration

- ❖ Calculate the moment measure at “principal point” as a sum of conditional intensity functions applied to neighbor points





# AM3 Approach: Point Field

point ( <i>i, j</i> )	x	y	z	power (P)	speed (V)	Time (t)
0	8.89	6.12	3.6	347	509	5504.9752
1	9.08	5.75	3.6	347	1249	5504.9756
2	9.34	5.24	3.6	343	1260	5504.9762
3	9.59	4.77	3.6	345	1330	5504.9766
4	9.85	4.26	3.6	9.4	1263	5504.9772
...	...	...	...	...	...	...
11647	10.66	-0.69	3.6	54	731	5511.2567
11648	10.68	-0.53	3.6	57	234	5511.2572
11649	10.81	-0.5	3.6	57	505	5511.2577
11650	11.06	-0.55	3.6	54	289	5511.2582
11651	10.94	-0.49	3.6	57	720	5511.2587

In-situ process measured point field data for build layer at 3.6 [mm]



# AM3 Approach: Build Illustration

## ❖ Ti-6Al-4V L-PBF part with complex process conditions

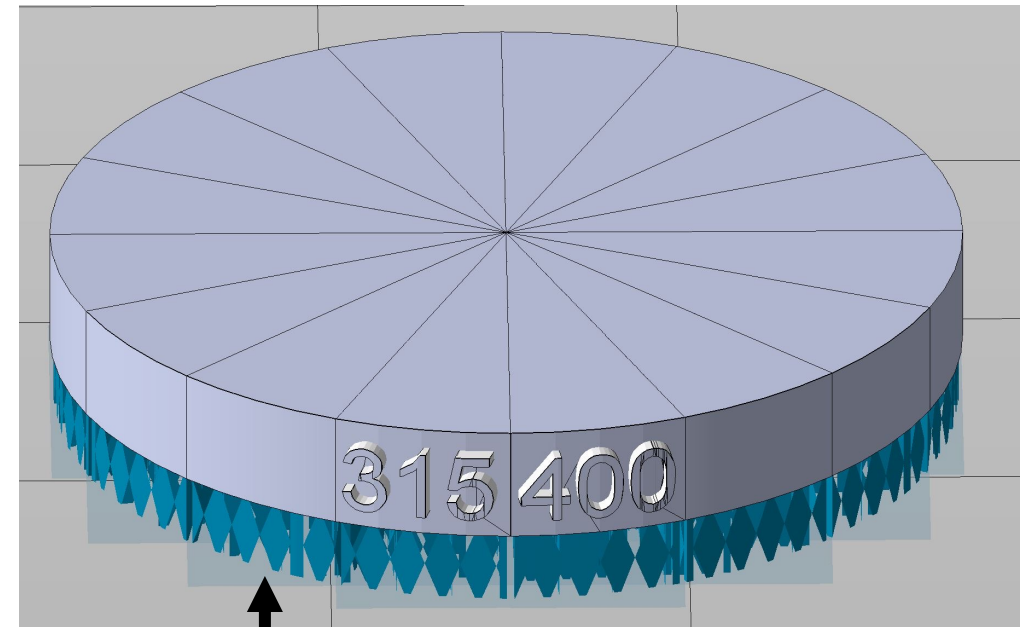
Build meta-data

- ❖ Speeds: 825, 940, 1060, 1175, 1300, 1400, 1530, 1650 [mm/s]
- ❖ Powers: 350, 275 [W]
- ❖ Contours: 130 [W], 825 [mm/s]
- ❖ Hatch spacing: 0.1 [mm]

## ❖ Configurable Architecture Additive Testbed

- ❖ Instrumented for LPBF diagnostic research
- ❖ 1070 [nm] laser, IPG Photonics

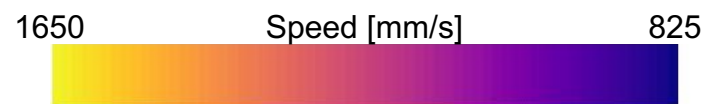
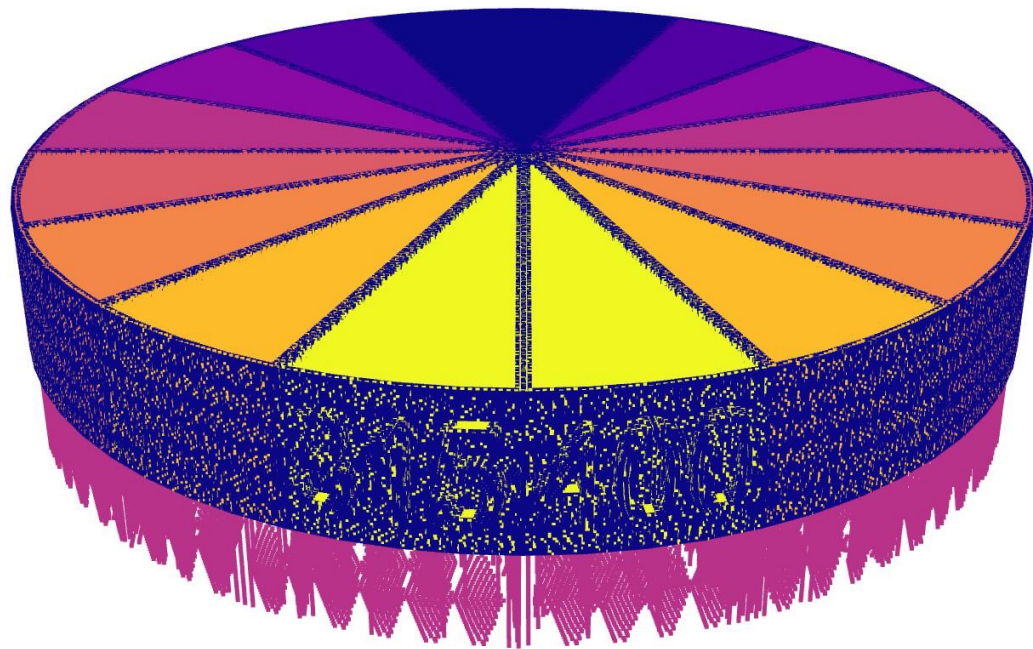
Visualized PBF Part that will be built



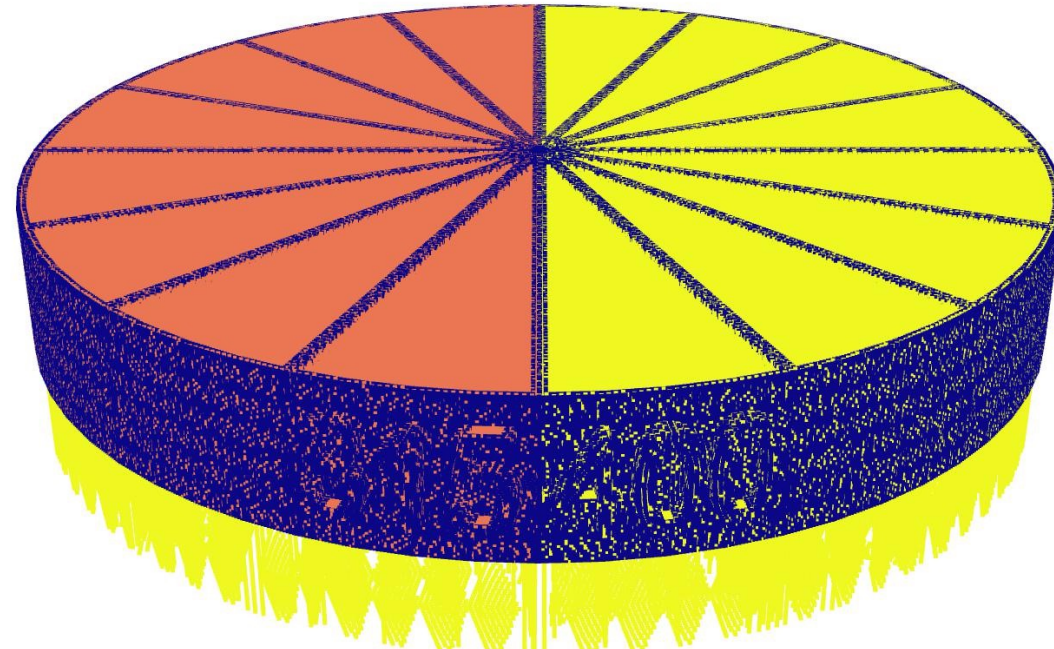
Support material

# AM3 Approach: Build Illustration

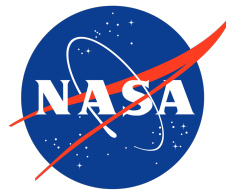
❖ Speed



❖ Power







# AM3 Approach: Build Instructions

Small subset of total instructions...

\$\$LAYER/3600

\$\$POWER/400.0

\$\$SPEED/1300.0

\$\$HATCHES/1,106,10435,7243,11688,4835,11577,4788,9983,7850,9619,8291,11466,4742,11354,4696,9301,8642,8985,8987,11243,4650,11132,4604,8893,8905,804,8816,11021,4558,10909,4512,8715,8728,8626,8639,10798,4466,10687,4419,8537,8550,8448,8461,10576,4373,10464,4327,8359,8372,8270,8283,10353,4281,10242,4235,8181,8194,8092,8105,10130,4189,10019,4143,8003,8016,7914,7927,9908,4096,9797,4050,7825,7838,7736,7749,9685,4004,9574,3958,7647,7660,7558,7571,9463,3912,9352,3866,7469,7482,7380,7393,9240,3820,9129,3774,7291,7304,7202,7215,9018,3727,8906,3681,7113,7126,7024,7037,8795,3635,8684,3589,6935,6948,6846,6859,8573,3543,8461,3497,6757,6770,6668,6681,8350,3451,8239,3405,6579,6592,6491,6503,81

\$\$POWER/150.0

\$\$SPEED/700.0

\$\$POLYLINE/1,2,9,11743,4858,11209,5992,10569,7061,9825,8063,8987,8987,8977,8990,-10,3,2,-10,11743,4858

\$\$POLYLINE/1,2,7,10459,6991,9726,7978,8976,8806,512,342,11571,4923,11097,5925,10459,6991

\$\$POWER/400.0

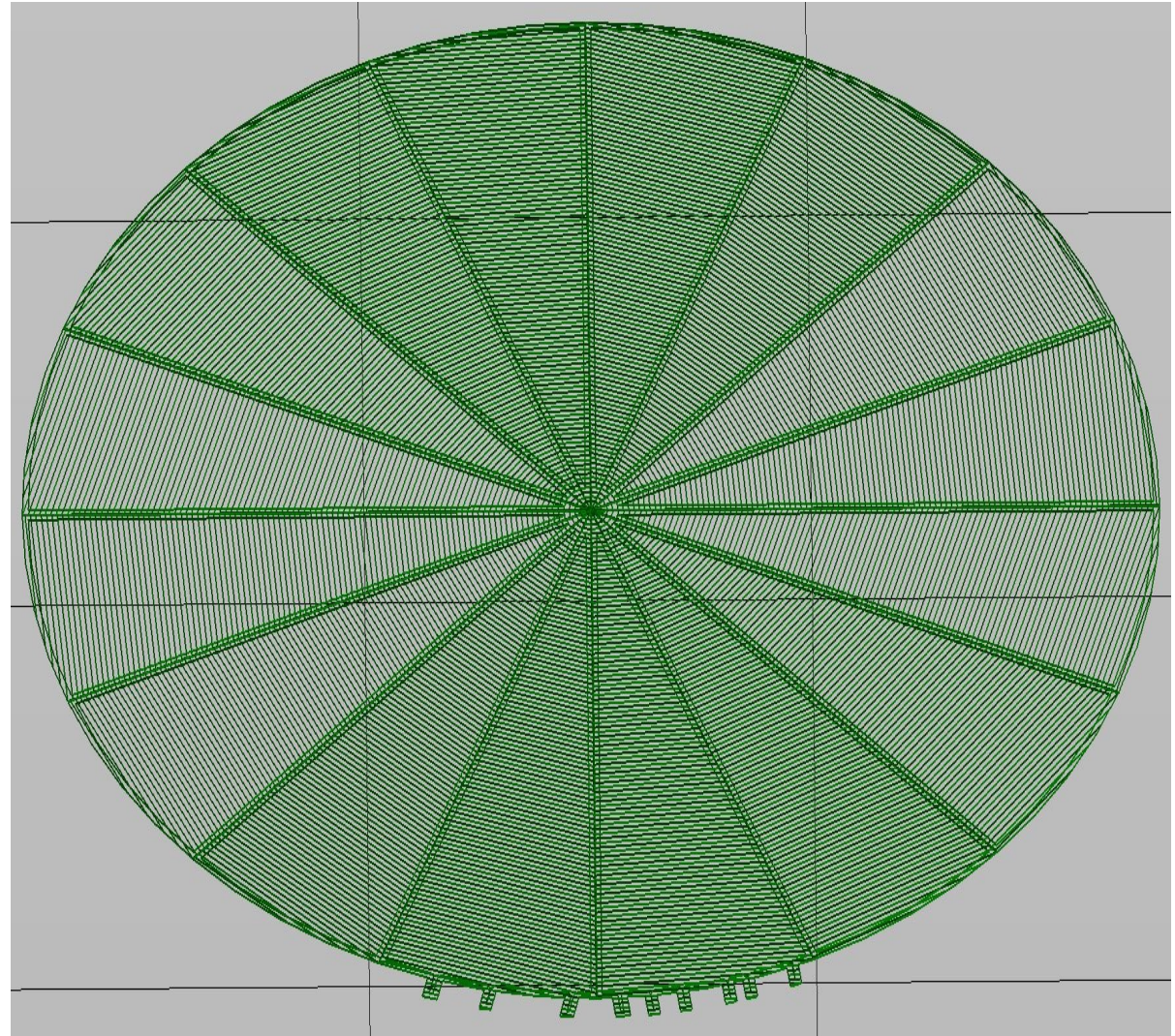
\$\$SPEED/1200.0

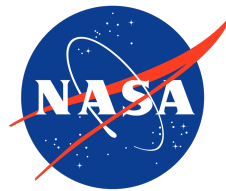
\$\$HATCHES/2,106,6867,10686,8951,8937,8866,8852,6216,11076,5712,11342,8781,8767,8695,8682,5284,11544,4862,11742,8610,8597,8525,8512,4808,11631,4759,11515,8440,8426,8355,8341,4711,11398,4663,11282,8269,8256,8184,8171,4615,11166,4567,11050,8099,8086,8014,8001,4519,10933,4471,10817,7929,7915,7843,7830,4422,10701,4374,10585,7758,7745,7673,7660,4326,10468,4278,10352,7588,7575,7503,7490,4230,10236,4182,10120,7417,7405,7332,7319,4134,10003,4085,9887,7247,7234,7162,7149,4037,9771,3989,9655,7076,7064,6991,6979,3941,9538,3893,9422,6906,6894,6821,6809,3845,9306,3797,9190,6736,6723,6650,6638,3748,9073,3700,8957,6565,6553,6480,6468,3652,8841,3604,8725,6395,6383,6310,6298,3556,8608,3508,8492,6224,6212,6139,6127,3460,8376,3411,8260,6054,6042,5969,5957,3363,8143,3315,8027,5884,5872,5798,5787,3267,7911,3219,7795,5713,5702,5628,5616,3171,7678,3122



# AM3 Approach: Build Layer

- ❖ Known process steps
- ❖ Complex patterns
- ❖ Localized processing conditions
- ❖ Previous points influence current points



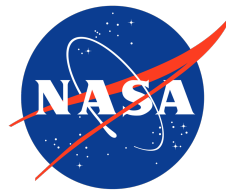


# AM3 Approach: Point Field

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In-situ process measured point field data for build layer at 3.6 [mm]





# AM3 Approach: Equations

$$(1) \quad AM3_i = \sum_j^N F_{i,j}$$

*$i, j \in$  process point field  
 $R =$  neighborhood radius*

$$(2) \quad F_{i,j} = \begin{cases} f_{i,j}, & r_{i,j} \leq n_{i,j} \\ 0, & r_{i,j} > n_{i,j} \end{cases}$$

$$(3) \quad r_{i,j} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2 + (z_i - z_j)^2}$$

$$(4) \quad n_{i,j} = \begin{cases} R, & 0 \leq \tau_{i,j} \\ -1, & 0 > \tau_{i,j} \end{cases}$$

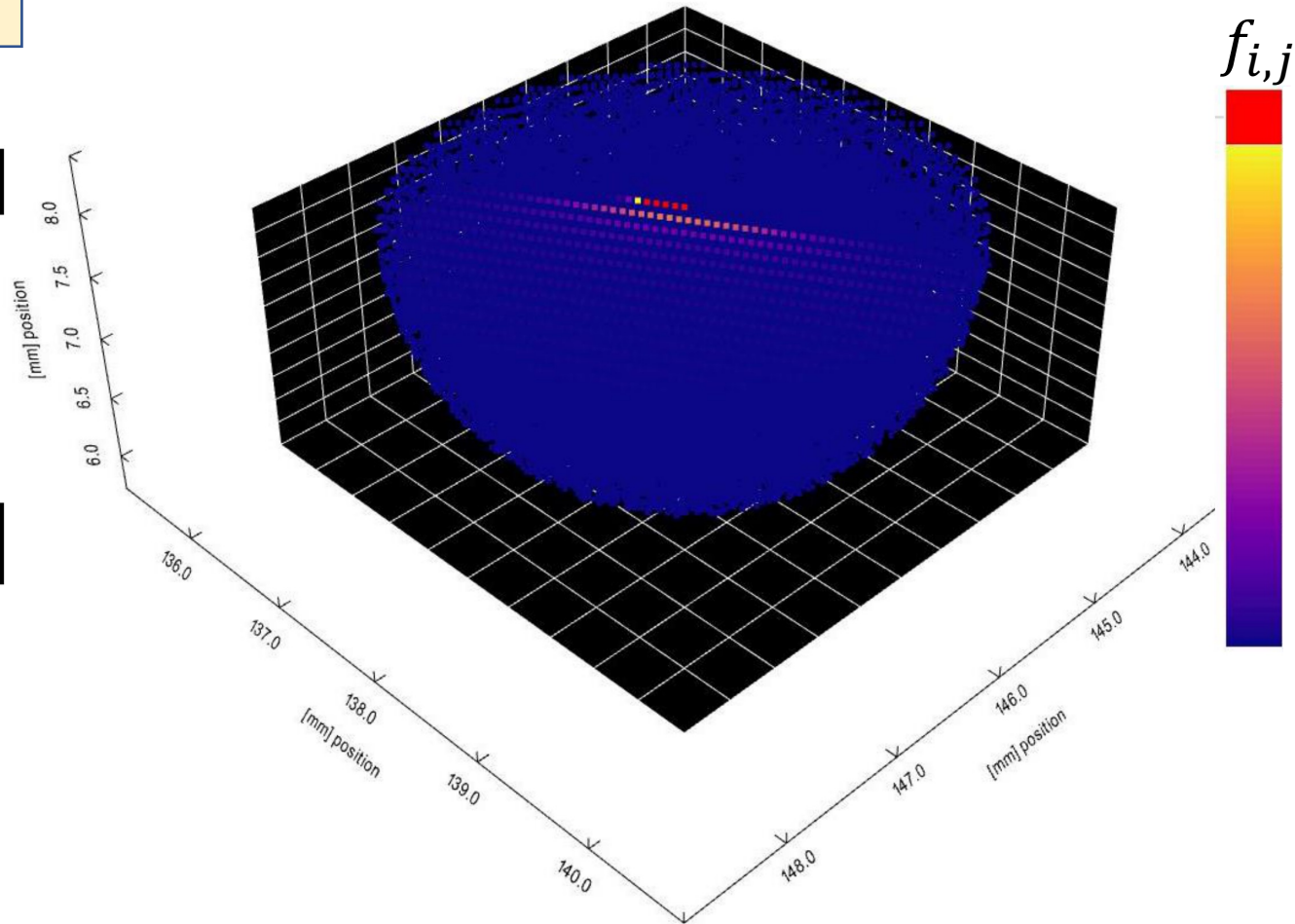
$$(5) \quad \tau_{i,j} = t_i - t_j$$



# AM3 Approach: Concept Illustration

$i, j \in \text{process point field}$

$$AM3_i = \sum_j^N$$

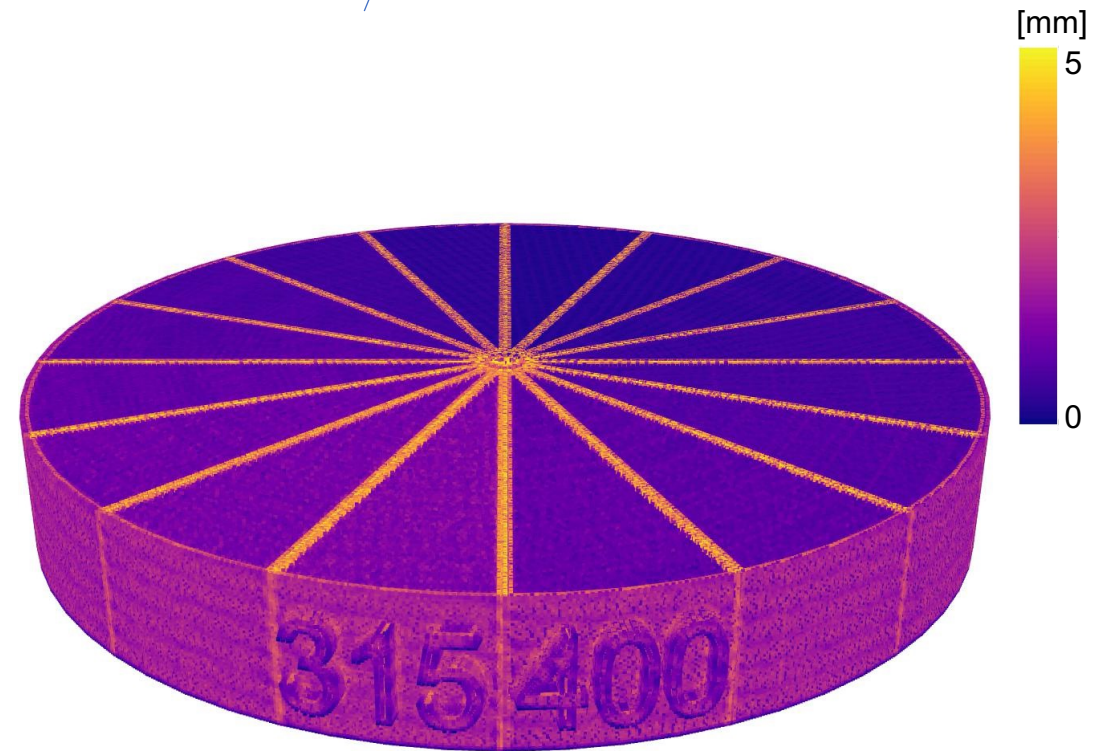


# AM3: LoF Moment Measure

$$f_{i,j} = \frac{V_j}{v_j}, R \propto 0.5 \sqrt{\frac{8}{\pi e} \frac{\lambda P_i}{\rho C_p V_i (T_m - T_0)}}$$

Lack of fusion indicator

- ❖ Melt pool width estimation
  - ❖ Derived from Rosenthal equation
    - ❖ Tang et. al. 2017
- ❖ Neighborhood criteria
  - ❖ Within specified radius
    - ❖ proportional to the melt pool width
  - ❖ Lower time-stamp than principal point (past)
  - ❖ Additional conditionals can be applied



Tang, M., Pistorius, P.C. and Beuth, J.L., 2017. "Prediction of lack-of-fusion porosity for powder bed fusion." Additive Manufacturing, 14, pp.39-48. <https://doi.org/10.1016/j.addma.2016.12.001>

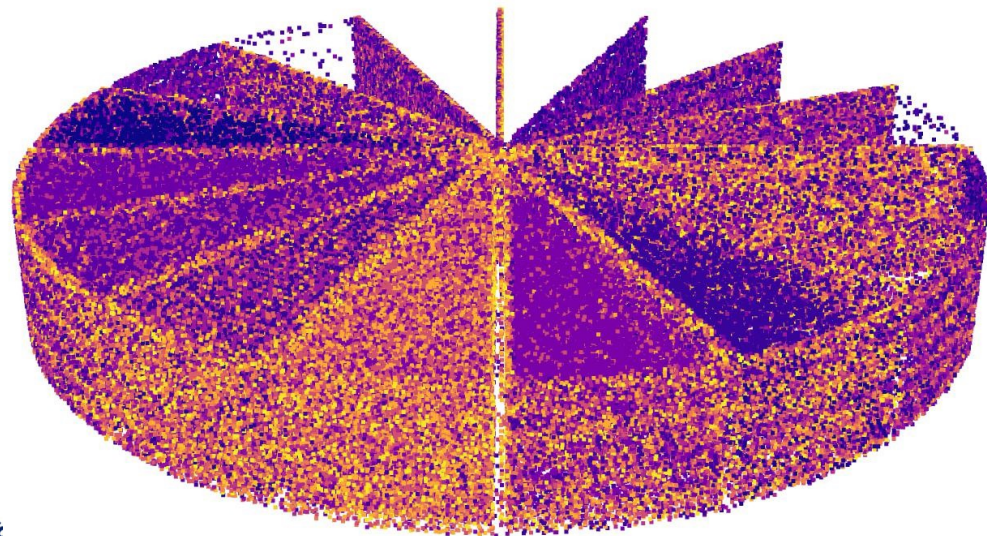
5 mm

# AM3: LoF Moment Measure

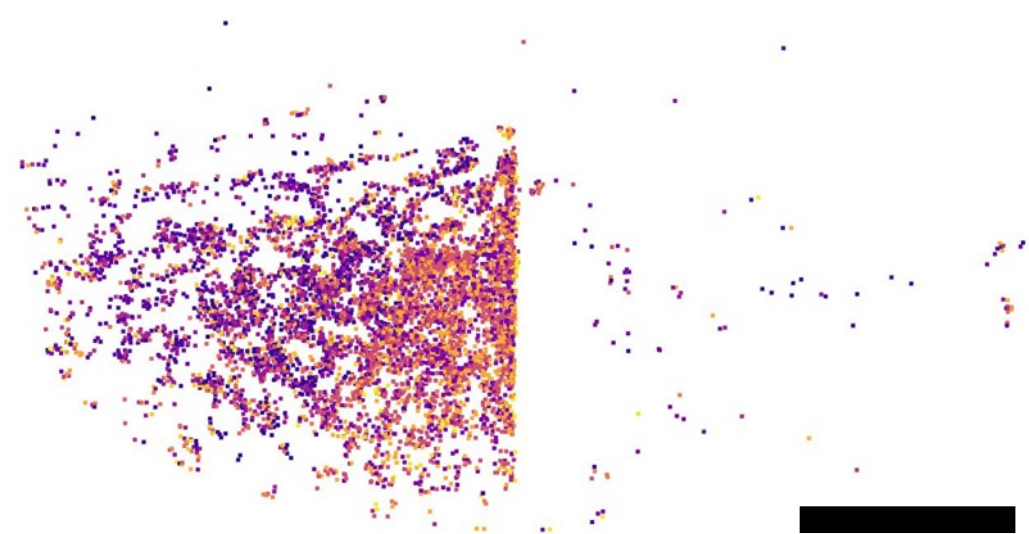
$$f_{i,j} = \frac{V_j}{v_j}, R \propto 0.5 \sqrt{\frac{8}{\pi e} \frac{\lambda P_i}{\rho C_p V_i (T_m - T_0)}}$$

Lack of fusion indicator

Map of lack of fusion indicator moment measures



Lack of fusion indicator moment measures mapped to porosity segments from x-ray CT, Otsu's threshold



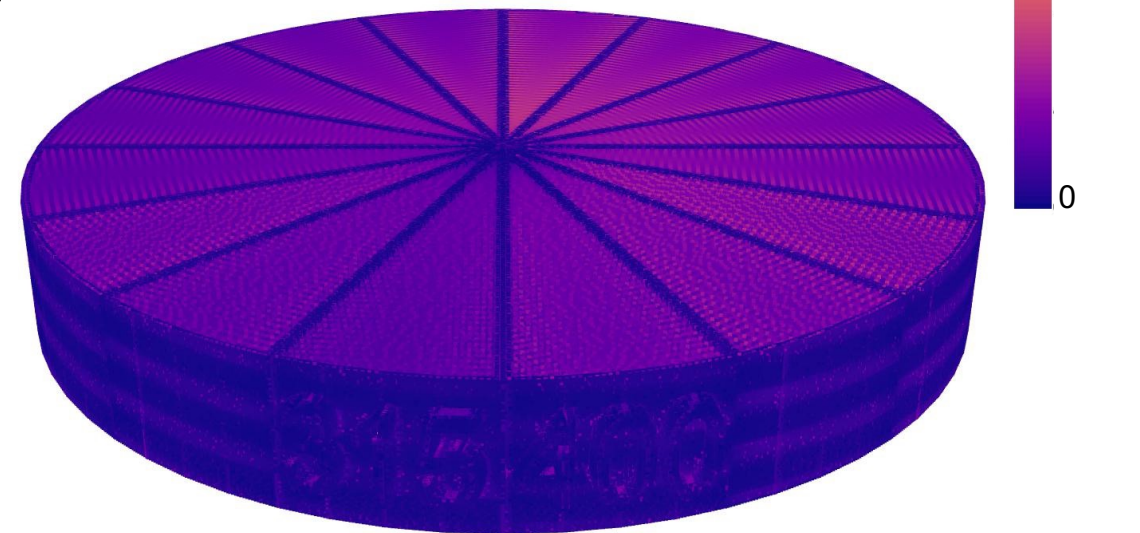
5 mm

# AM3: Thermal Rise Moment Measure

$$f_{i,j} = \frac{\frac{\lambda P_j}{v_j}}{\rho C_p \sqrt{2\pi}^{2/3} (\sigma_j^2 + 2\alpha\tau_{i,j})^{2/3}} \exp \left( -\frac{r_{i,j}^2}{2(\sigma_j^2 + 2\alpha\tau_{i,j})} \right)$$

Thermal rise:  
Localized heating

- ❖ Green's function heat equation
  - ❖ Adapted from Groeber et al. 2017
- ❖ Neighborhood criteria
  - ❖ Within specified radius
  - ❖ Lower time-stamp than principal point (past)
  - ❖ Additional conditionals can be applied

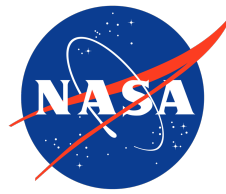


Groeber, M A, E Schwalbach, S Donegan, K Chaput, T Butler, and J Miller. 2017. "Application of Characterization, Modelling, and Analytics Towards Understanding Process-Structure Linkages in Metallic 3d Printing." IOP Conference Series: Materials Science and Engineering 219 (July): 012002. <https://doi.org/10.1088/1757-899x/219/1/012002>. Modified the 6<sup>th</sup> equation of the article to be compatible with the point field approach.

5 mm

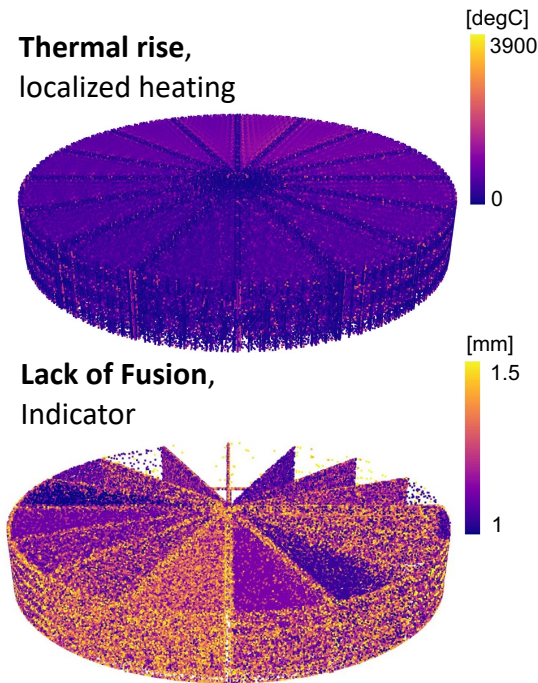


# AM3: Point Fields from Build Instructions OR In-situ Measurements

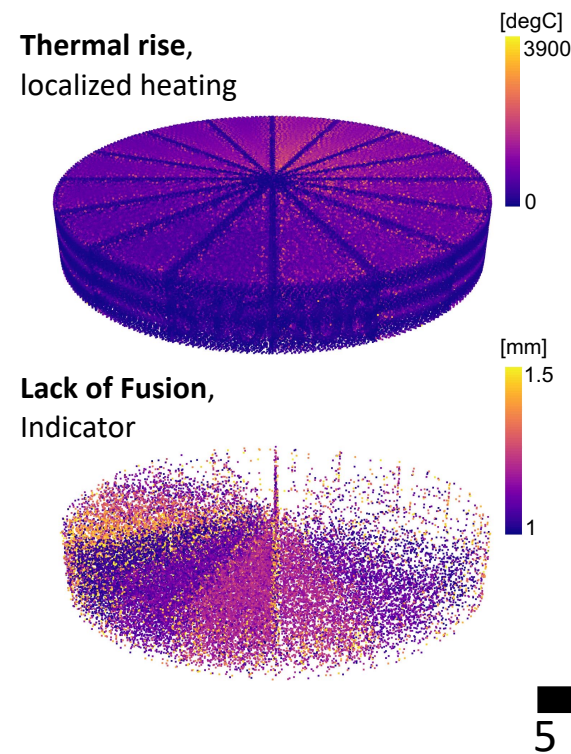


- ❖ AM3 approach is good for process to microstructure R&D
- ❖ Multiple moment measures calculated with a single pass
- ❖ Measures of build variance

Build instructions generated point field



In-situ measurement point field

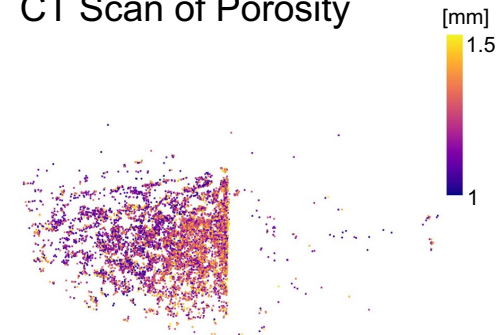


Experimental

Optical microscopy of top surface



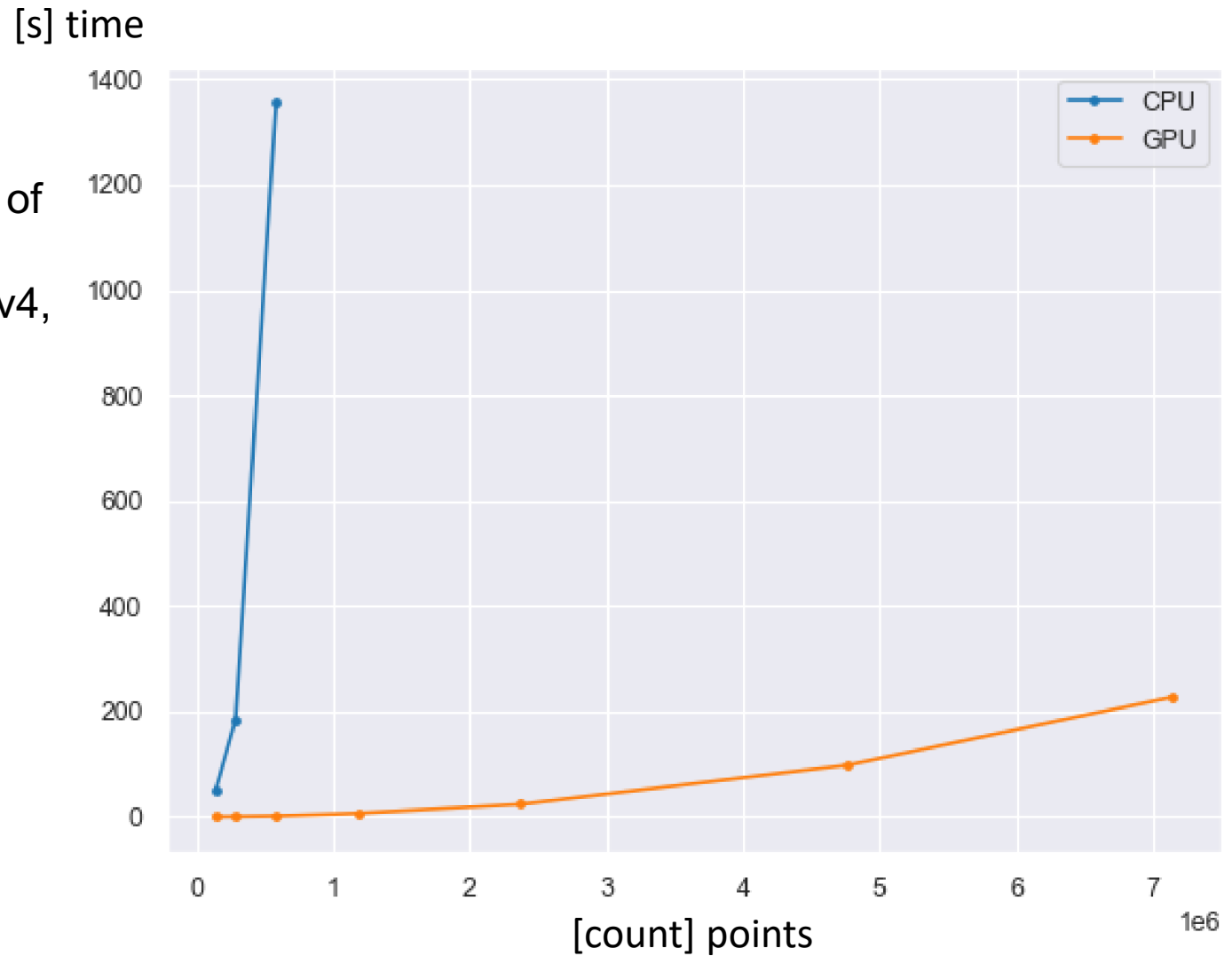
CT Scan of Porosity

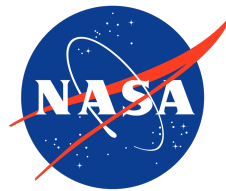


# AM3 Approach: Fully Parallel Performance

## ❖ AM3 calculation performance:

- ❖ CPU vs GPU per number of points
- ❖ CPU: Intel Xeon E5-2667v4, 128 Gb RAM
- ❖ GPU: Nvidia Titan V

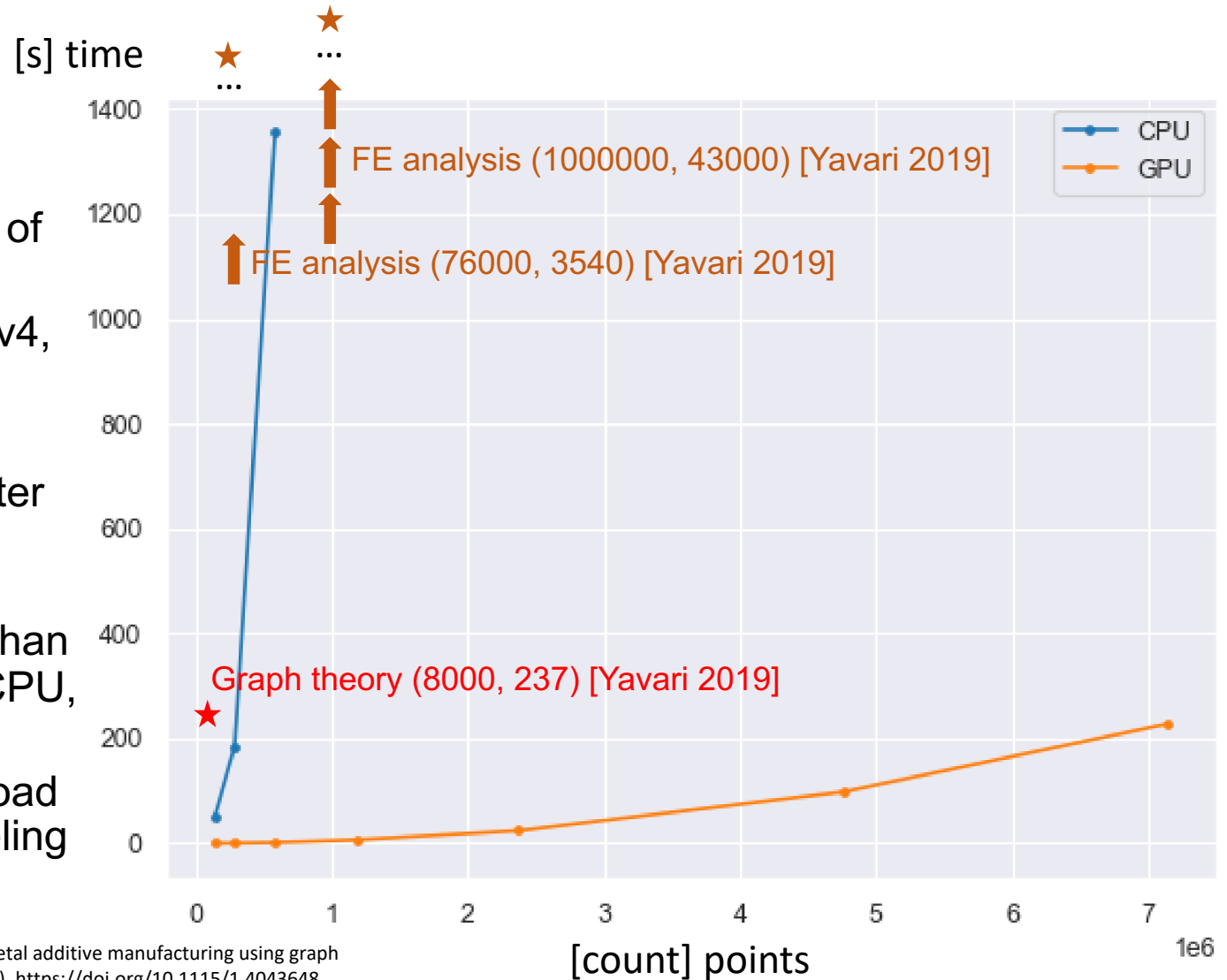




# AM3 Approach: Fully Parallel Performance

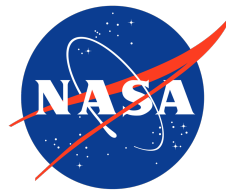
## ❖ AM3 calculation performance:

- ❖ CPU vs GPU per number of points
- ❖ CPU: Intel Xeon E5-2667v4, 128 Gb RAM
- ❖ GPU: Nvidia Titan V
- ❖ AM3-GPU is 10,000x faster than equivalent CPU-FE analysis
- ❖ AM3-CPU is ~10x faster than equivalent graph theory-CPU, 4,000x faster on the GPU
- ❖ Minimization of memory load is key for part scale modeling



Yavari, M.R., Cole, K.D. and Rao, P., 2019. "Thermal modeling in metal additive manufacturing using graph theory." Journal of Manufacturing Science and Engineering, 141 (7). <https://doi.org/10.1115/1.4043648>

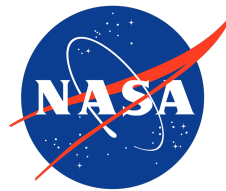




# AM3: Conclusions and Future Work

- ❖ AM3 is a process point field approach
  - ❖ Low-cost process condition models for AM (fast)
  - ❖ Quantify localized build conditions
  - ❖ Evaluate build quality – instructed vs measured
  - ❖ Achieve multiple moment measures with a single pass
- ❖ Future Work
  - ❖ Determine statistical relationships between AM3 values, in-situ sensor measurements, and the resulting microstructures
  - ❖ Improve “physical correctness” by developing multi-pass and boundary informed approaches





# Acknowledgements

- ❖ NASA, Transformational Tools and Technologies project
- ❖ NASA Langley, Advanced Materials and Processing Branch
  - ❖ Joel Alexa, Peter Messick, James Thornton, Harold Claytor
    - ❖ Analytical Mechanics Associates, Hampton VA
- ❖ NASA Langley, Nondestructive Evaluation Sciences Branch
  - ❖ William Sommer
    - ❖ Analytical Services & Materials, Hampton VA



# Back-ups

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# AM3 Approach: Summary

- ❖ Qualitative part-scale process model:
  - ❖ Physically informed process indicators
- ❖ Minimized Memory Requirements
  - ❖ Maximum part scales
  - ❖ “Moment” approach avoids “time-stepping”
- ❖ Calculation of localized build conditions
  - ❖ Thermal rise (localized heating)
    - ❖ Keyhole risk & thermal uniformity
  - ❖ Neighborhood population
    - ❖ Meta-data extraction
    - ❖ Lack of fusion indicator
    - ❖ Boundary proximity
      - ❖ Can be used for boundary conditions
- ❖ Point field approach
  - ❖ Generated from build file or in-situ measurements
    - ❖ “Digital twin”
  - ❖ Point wise process analysis
    - ❖ Build file integrity
    - ❖ Instructed vs measured
    - ❖ Build meta-data can be extracted

part scales:  $1\text{cm}^3 - 20\text{cm}^3$

A single time stamp per point in the field results in minimal memory and computation cost

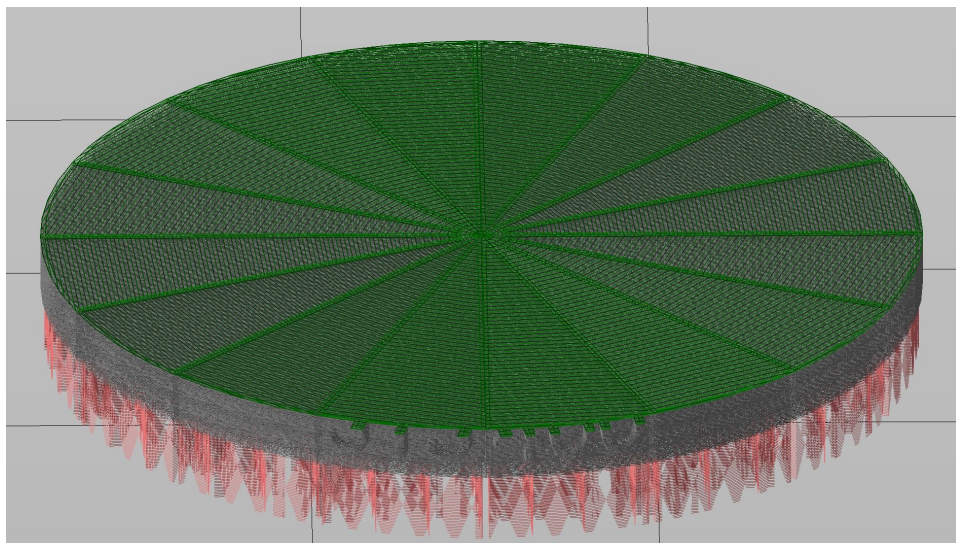
Single pass calculation provides quantifiable & physically informed localized process indicators

No other process modeling approach has been shown to calculate directly from the full-scale build instructions

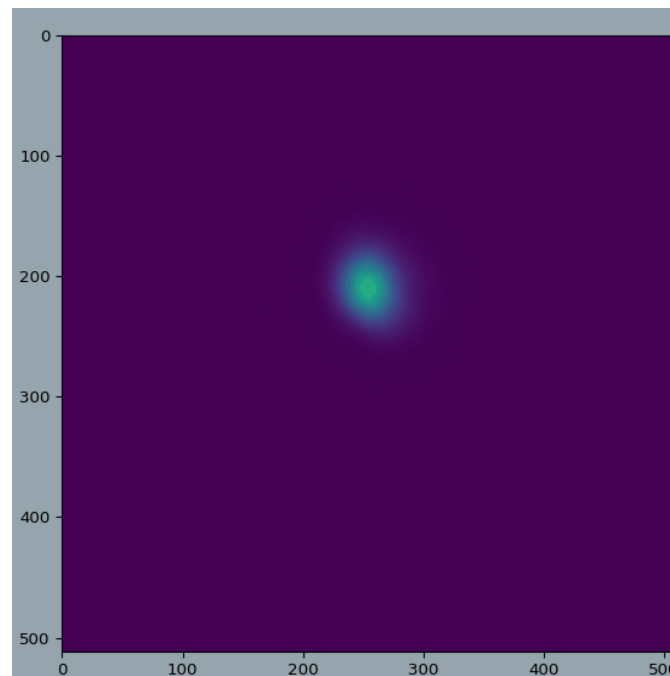


# Additive Manufacturing: Build Measures

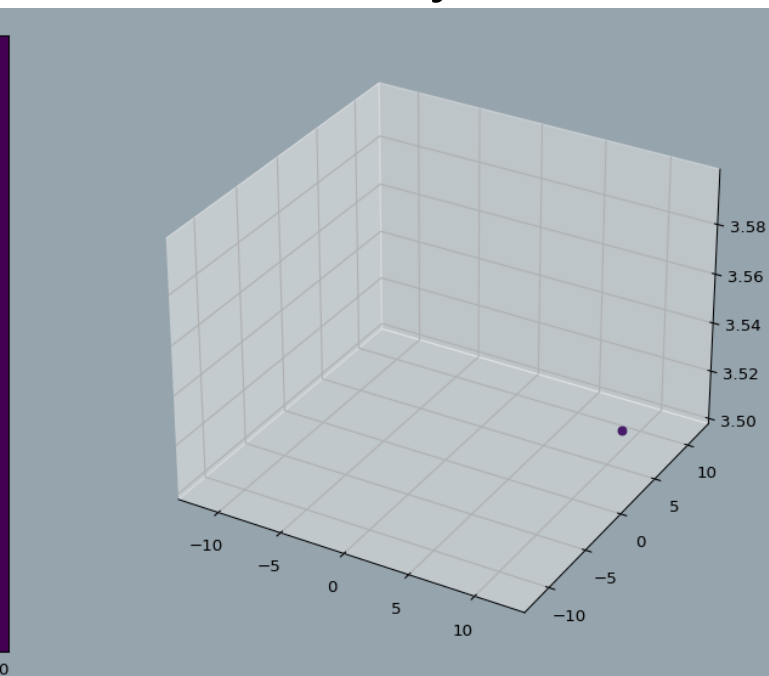
- ❖ Example build measures were taken from near the center of the build height



- ❖ Coaxial imagery of the melt pool during the build



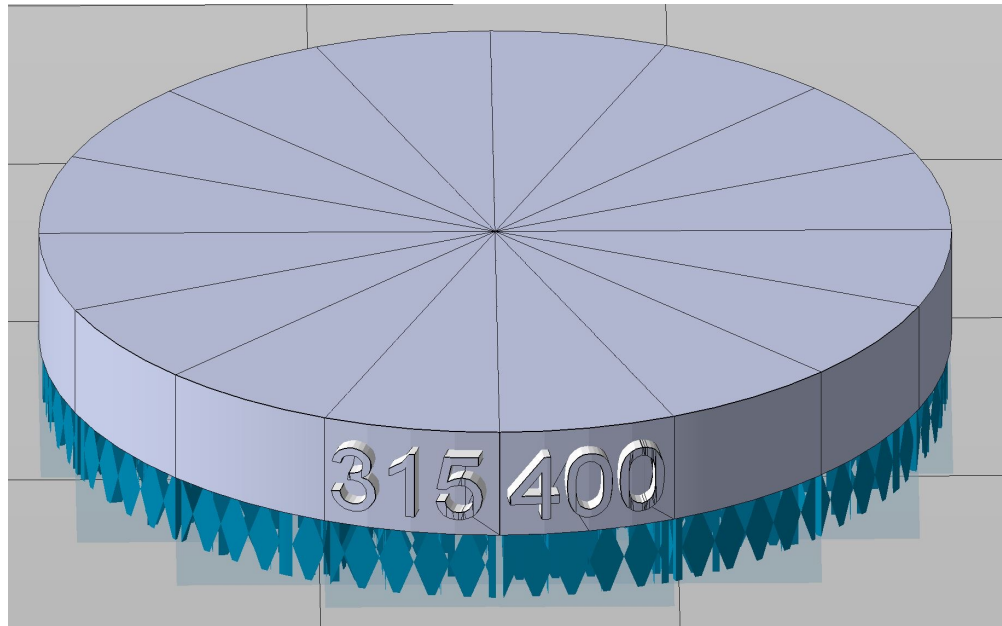
- ❖ Build measure: average coaxial image pixel intensity



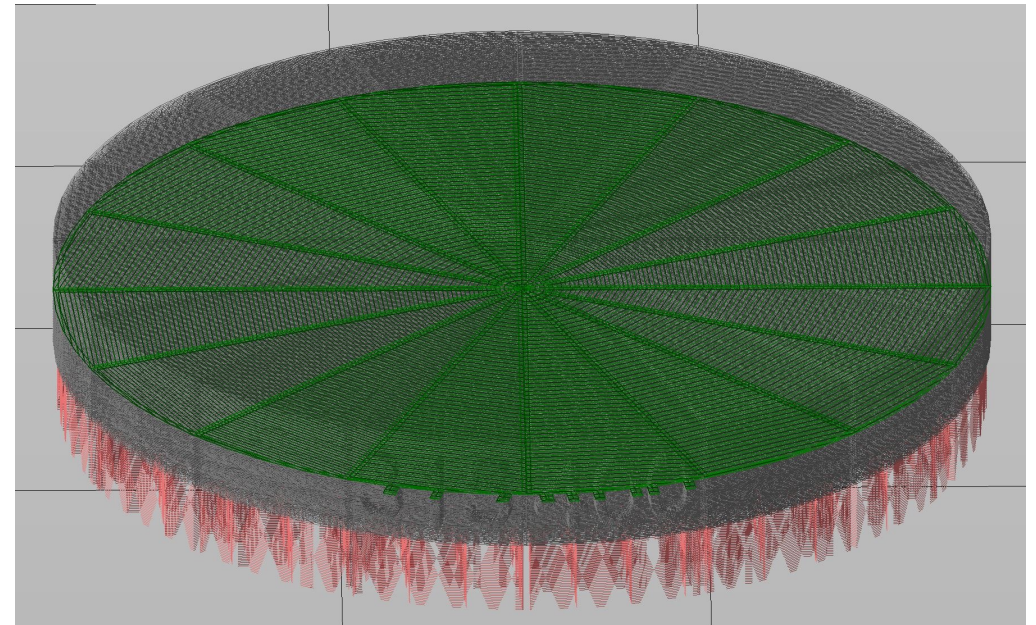


# AM3 Approach: Build Illustration

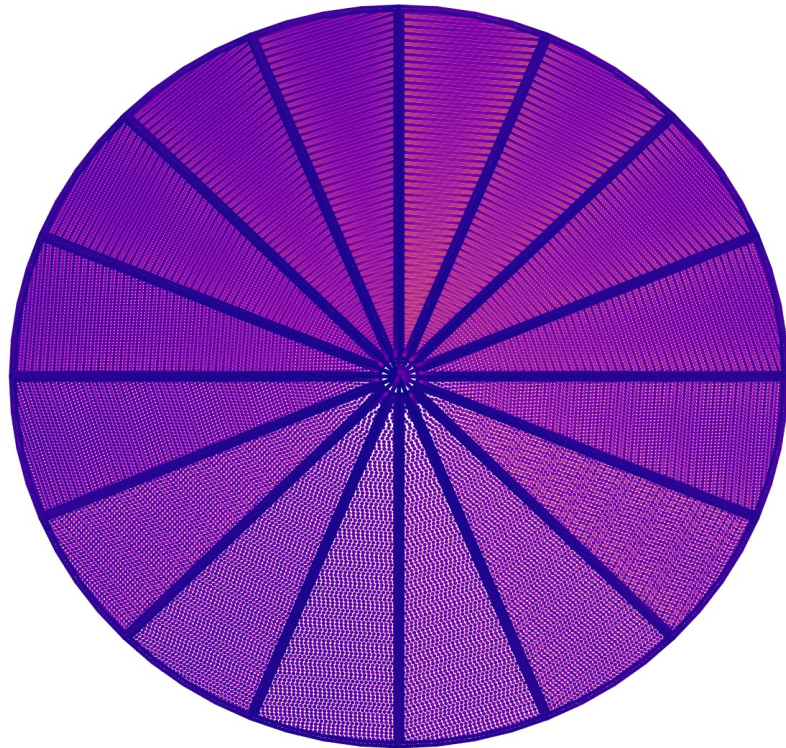
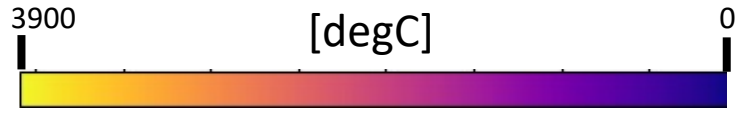
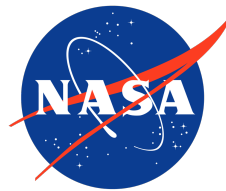
- ❖ Define the geometry of the part and assign a defined processing strategy



- ❖ Slice the part based on the geometry and build strategy: "Build Instructions"



# AM3 vs Surface Features vs Sensor Signal

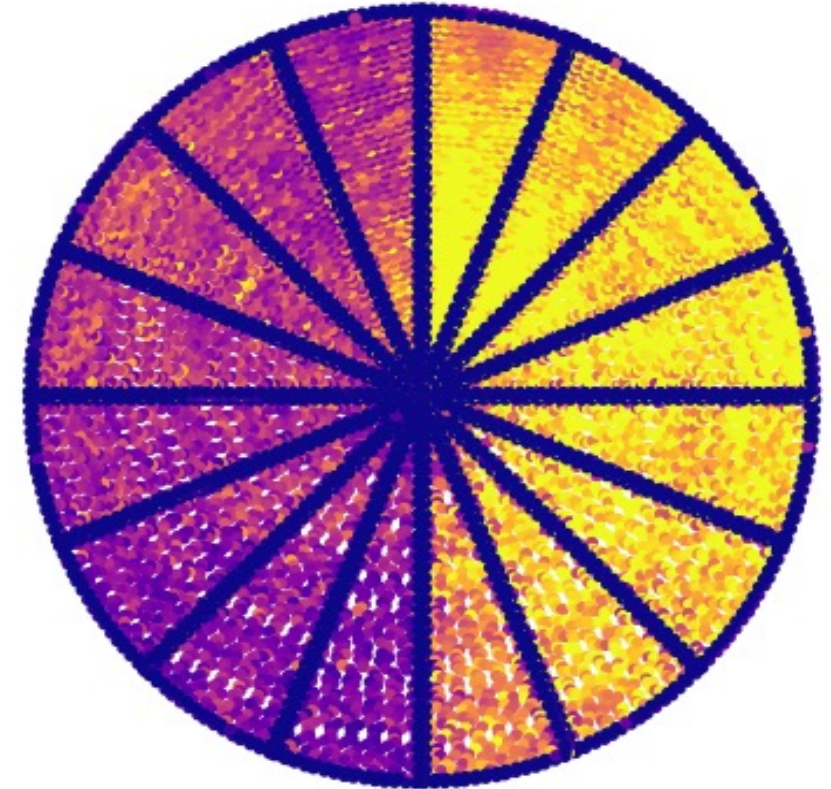


5 mm

AM3 thermal rise



Microscope Image of the as printed surface



Max intensity from coaxial images

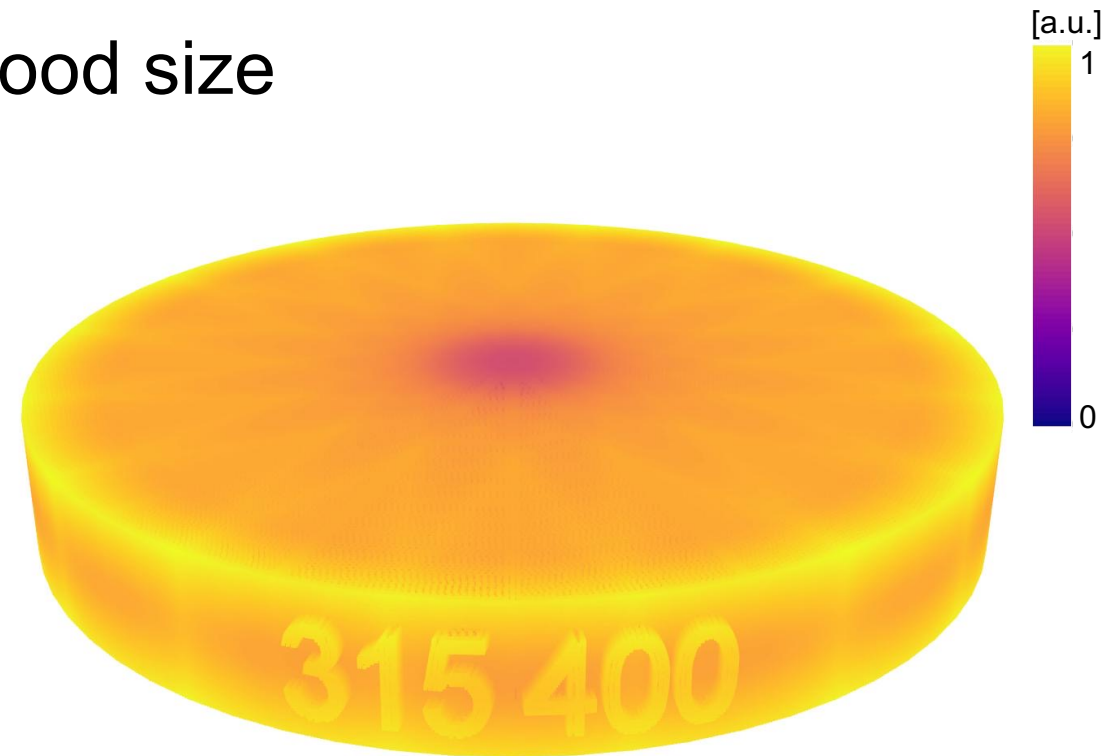


# AM3 Process Functions

$$f_{i,j} = \frac{V_j}{v_j}, n_{i,j} = R$$

Boundary proximity, process insensitive

- ❖ Moment measure of neighborhood size
- ❖ Neighborhood criteria
  - ❖ Within specified radius
- ❖ Maximum value normalized



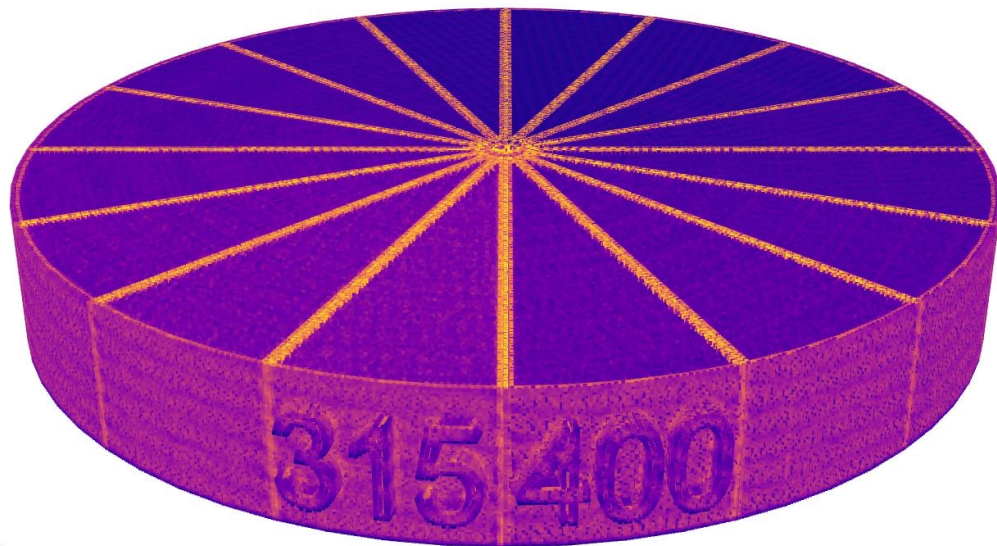
5 mm

# AM3 Process Functions

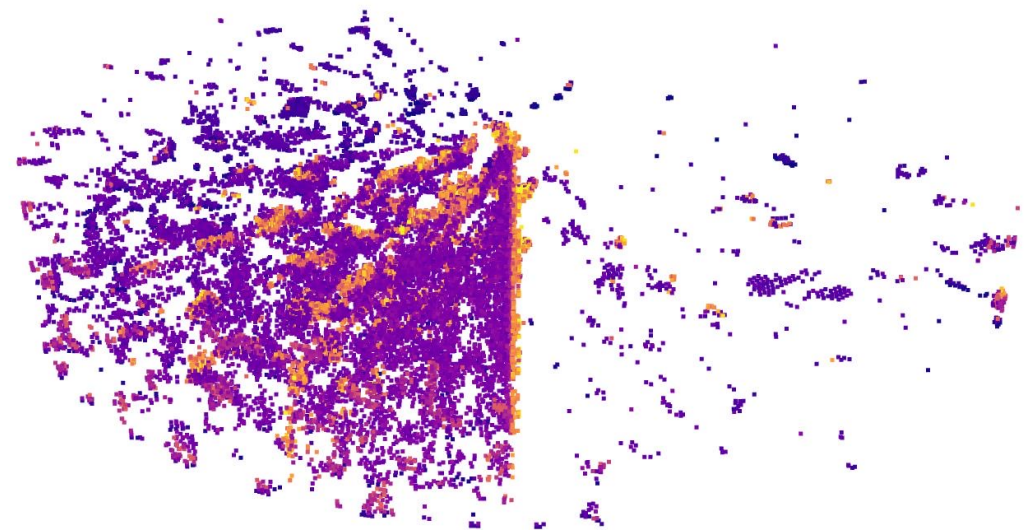
$$f_{i,j} = \frac{V_j}{v_j}, R \propto 0.5 \sqrt{\frac{8}{\pi e} \frac{\lambda P_i}{\rho C_p V_i (T_m - T_0)}}$$

**Lack of fusion indicator**

Map of lack of fusion indicator moment measures



Lack of fusion moment measures mapped to porosity segments from x-ray CT, Otsu's threshold



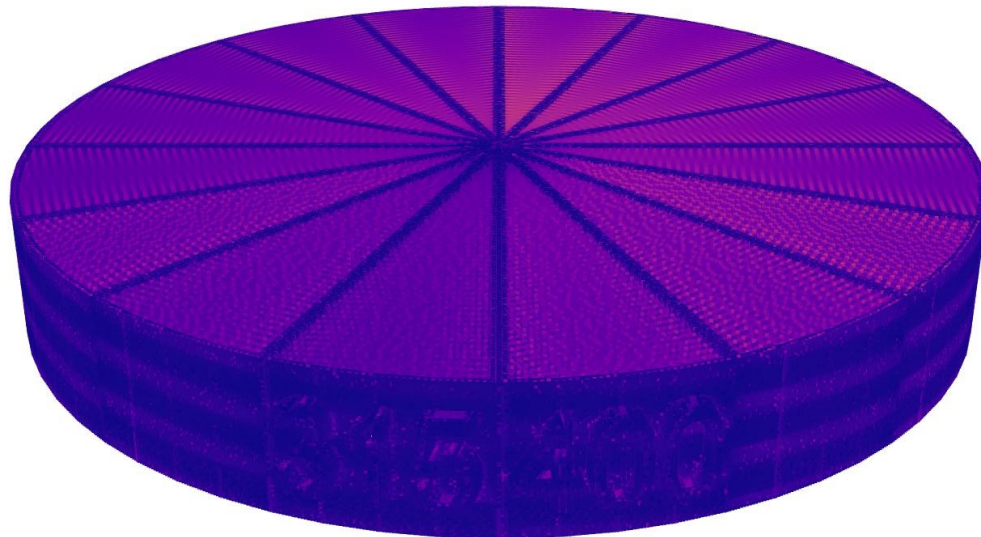
5 mm

# AM3 Process Functions

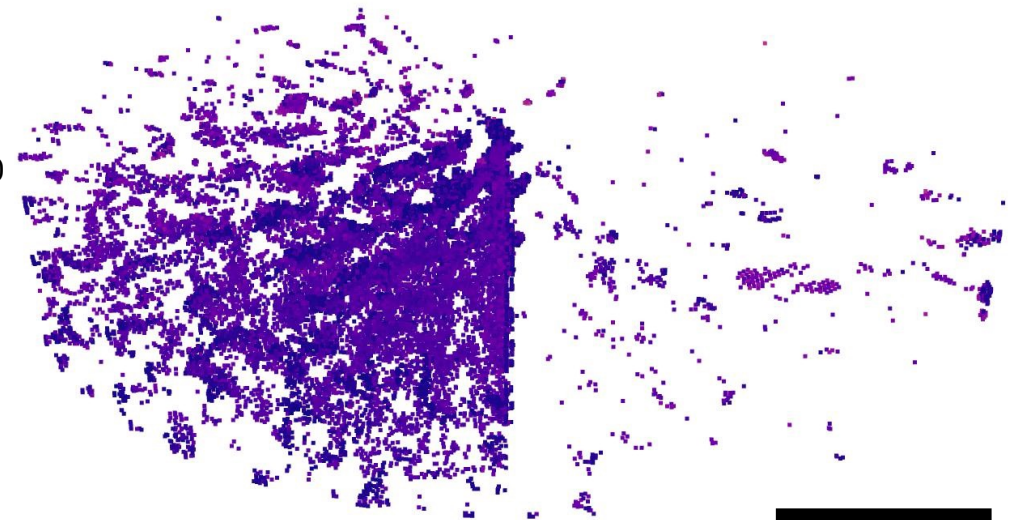
$$f_{i,j} = \frac{\frac{\lambda P_j}{\nu_j}}{\rho C_p \sqrt{2\pi}^{2/3} (\sigma_j^2 + 2\alpha\tau_{i,j})^{2/3}} \exp\left(-\frac{r_{i,j}^2}{2(\sigma_j^2 + 2\alpha\tau_{i,j})}\right)$$

**Thermal rise:**  
Localized heating

Map of thermal rise moment measures



Thermal rise moment measures mapped to porosity segments from x-ray computed tomography (CT), Otsu's threshold



5 mm

# Additive Manufacturing: Build Parameters

## ❖ Process enthalpy, Ti-6Al-4V

$$\text{enthalpy} = \frac{\lambda C_p P}{\rho \sqrt{\sigma \alpha V}}$$

$$\alpha = \frac{\kappa}{\rho C_p}$$

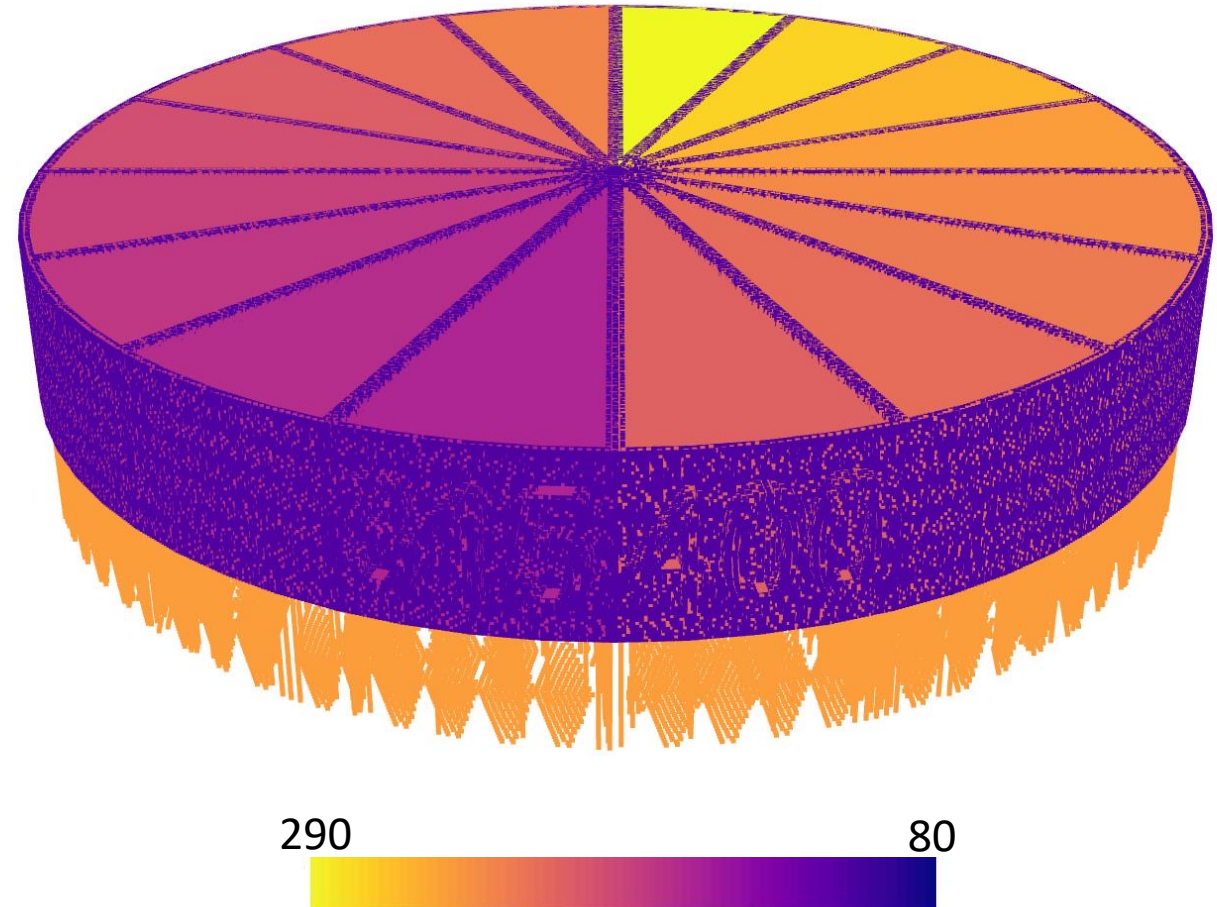
$$\kappa = 7.1 \text{ [W/mK]}$$

$$\rho = 4252 \text{ [kg/m}^3\text{]}$$

$$C_p = 553 \text{ [J/kgK]}$$

$$\lambda = 1 \text{ [a.u.]}$$

$$\sigma = 0.0004 \text{ [m]}$$





# Additive Manufacturing & Moment Measure

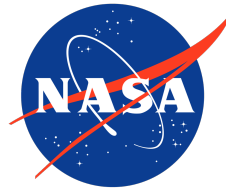
## Additive Manufacturing

- ❖ *“Process of joining materials to make parts from 3D model data, usually layer upon layer, as opposed to subtractive manufacturing and formative manufacturing methodologies”*
  - ❖ ISO/ASTM 52900:2021, Additive manufacturing - General principles - Terminology

## Moment Measure

- ❖ Motivated from physics, a moment measure is the generalization of the concept of a moment to a discrete point field of observables
- ❖ Combined = Additive Manufacturing Moment Measure (AM3)





# Moment Measure

- ❖ “In probability and statistics, a moment measure is a mathematical quantity, function or, more precisely, measure that is defined in relation to mathematical objects known as point processes, which are types of stochastic processes often used as mathematical models of physical phenomena representable as randomly positioned points in time, space, or both.”
  - ❖ [https://en.wikipedia.org/wiki/Moment\\_measure](https://en.wikipedia.org/wiki/Moment_measure)
  - ❖ D. J. Daley and D. Vere-Jones. *An introduction to the theory of point processes. Vol. {I}*. Probability and its Applications (New York). Springer, New York, second edition, 2008.

