Gradient Based Optimization of Chaotic Panel Flutter

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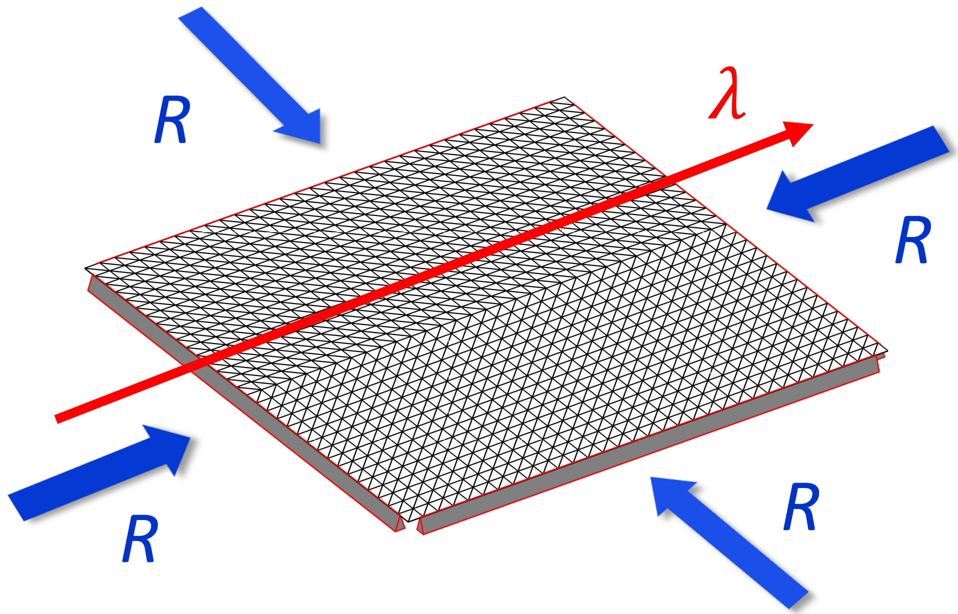
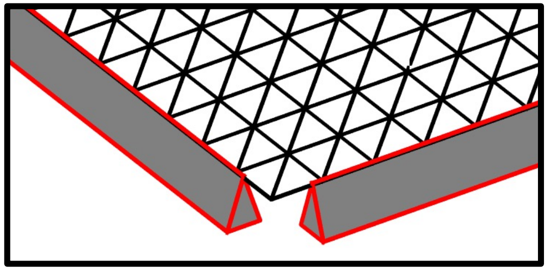
Bret Stanford
NASA Langley Research Center
Aeroelasticity Branch

History

- This work dates back to the summer of 2018:
  - I had a summer student from MSU, Christian Coker, who would return to MSU to start a Masters thesis with Manav after the summer was over
  - The research topic for that summer: sensitivity analysis of chaotic panel flutter
  - By the end of the summer, we were able to successfully compute and verify these sensitivities…
  - …and then everyone moved on to new things
- Goals of this presentation:
  - Finally present the work that was done in the summer of 2018
  - Extend the work: utilize the sensitivities for a simple design optimization demonstration

Chaotic Panel Flutter

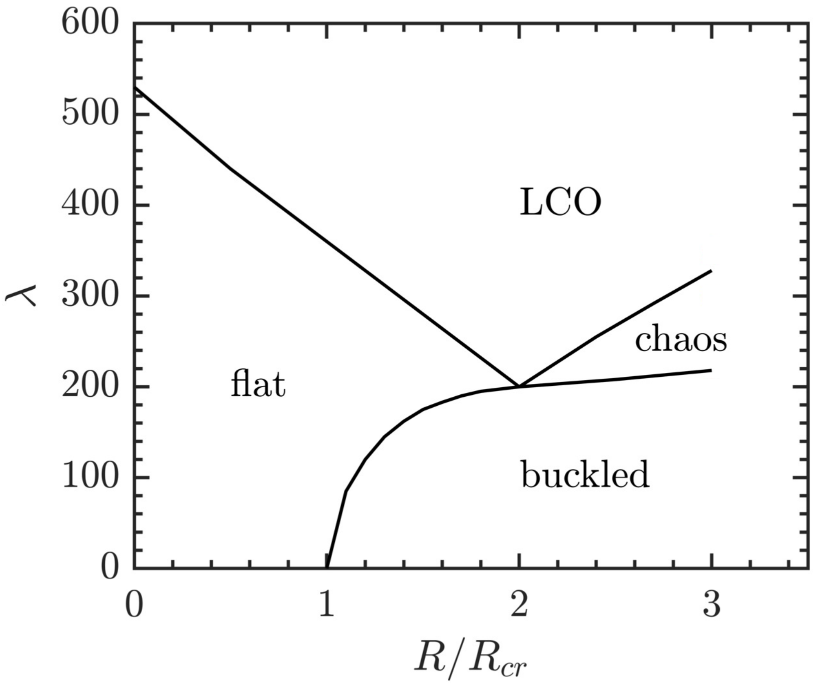


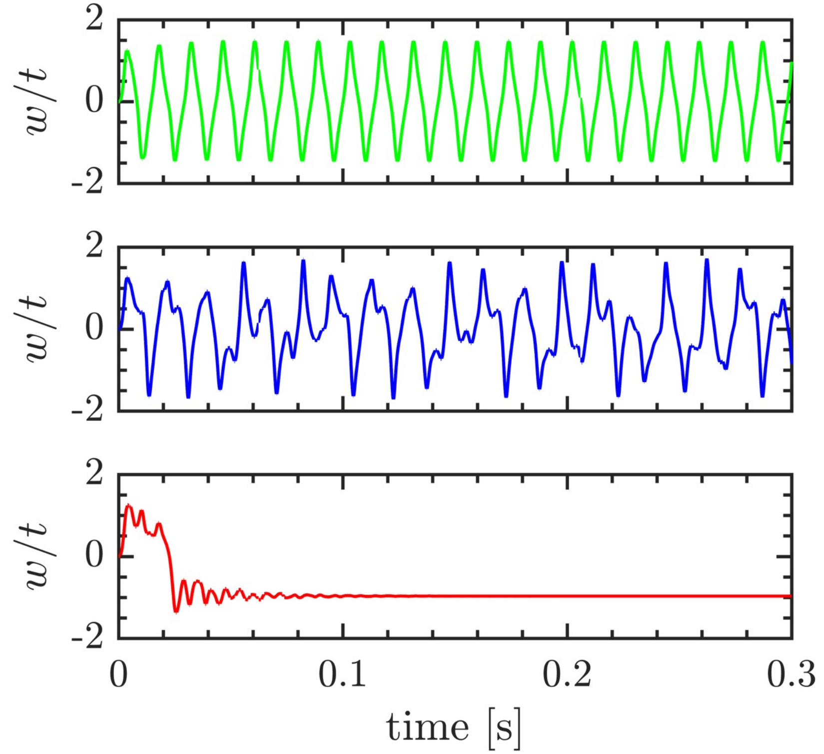


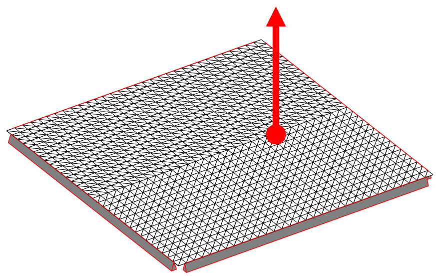
Simply-supported metallic panel:

- High-speed flow over the top surface (dynamic pressure parameter: λ)
- In-plane compressive loading (edge loading parameter: R)

Failure Boundaries

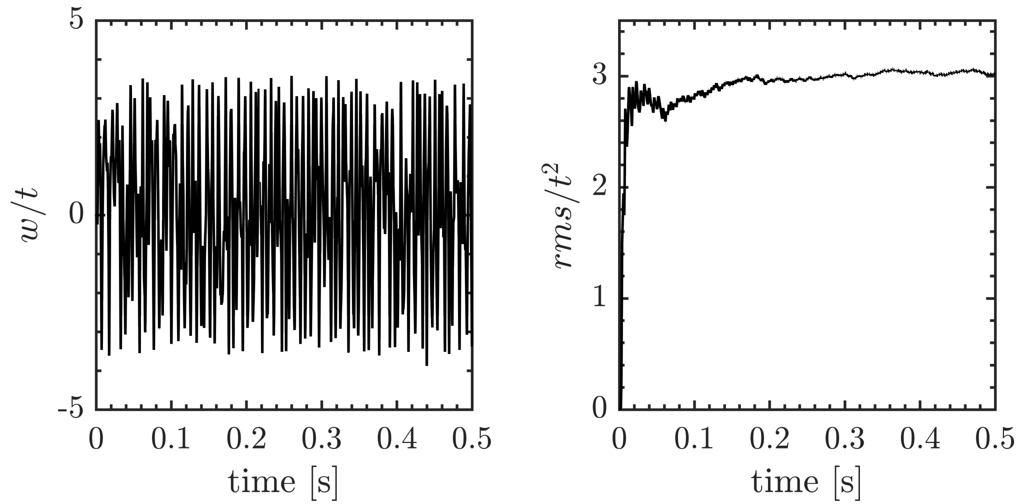






Goals

- Even if the chaotic response is messy from step to step…
  - A running RMS will converge in time



- We want to compute the derivative of that final time-converged RMS value, with respect to parameters:
  - Flow parameters, structural parameters, shape parameters, etc.
- Then use these derivatives in an optimization scheme, to alter the chaotic response in some way

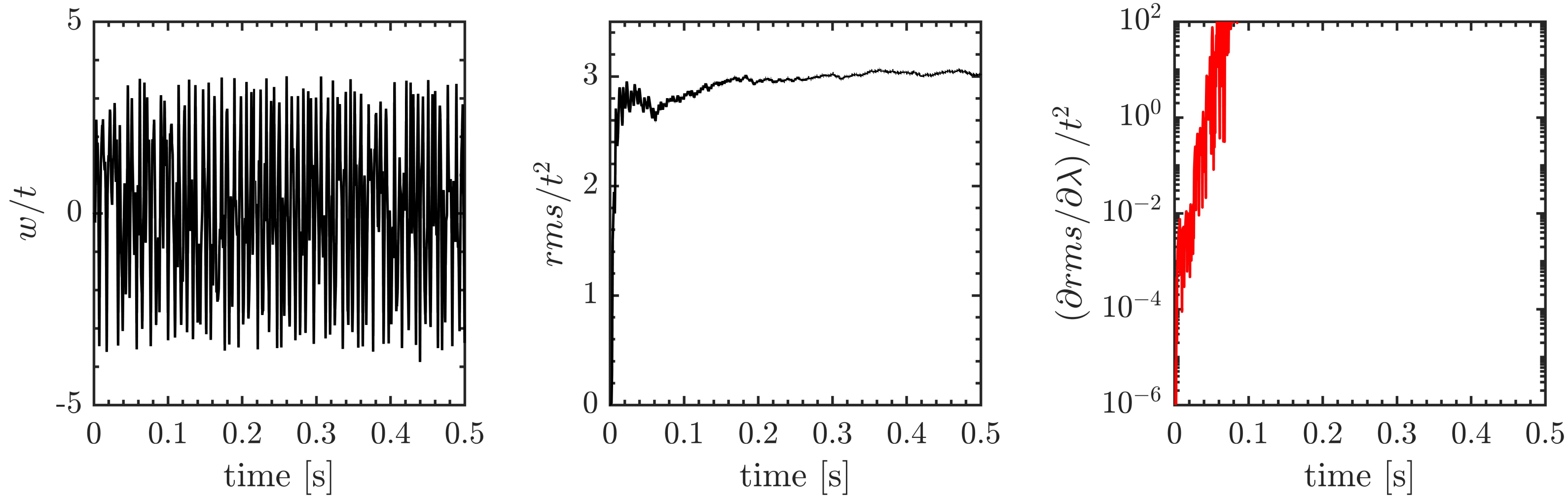
Some Math

- EOM for the panel response:
  - $\dot{\boldsymbol{x}} = \boldsymbol{R}(\boldsymbol{x}, \alpha)$
  - Geometric structural nonlinearities + piston theory aerodynamics
  - Jacobian ($\boldsymbol{J} = \partial \boldsymbol{R} / \partial \boldsymbol{x}$) is needed to drive the residual to 0 at each time step
- Compute a long-time average of some response of interest (i.e., RMS of the ¾-chord panel displacement):
  - $F(\alpha) = \lim_{T \to \infty} \left[\frac{1}{T} \cdot \int_0^T f(\boldsymbol{x}, \alpha) \cdot dt \right]$
- Want to compute $dF/d\alpha$

Some More Math

- Differentiate the EOM with respect to α:
  - EOM: $\dot{\boldsymbol{x}} = \boldsymbol{R}(\boldsymbol{x}, \alpha)$
  - Derivative of EOM: $\dot{\boldsymbol{x}}_\alpha = \boldsymbol{J} \cdot \boldsymbol{x}_\alpha + \boldsymbol{R}_\alpha$
- Final derivative we care about:
  - $F_\alpha = \lim_{T \to \infty} \left[\frac{1}{T} \cdot \int_0^T (f_\alpha + f_{\boldsymbol{x}} \cdot \boldsymbol{x}_\alpha) \cdot dt \right]$
- EOM derivative is a linear ODE, and its stability is governed by the Jacobian
- For chaos, sometimes the Jacobian is stable (negative eigenvalues), sometimes it's unstable (positive eigenvalues)
  - The equation for $\boldsymbol{x}_\alpha$ is not stable in time

Unstable Derivatives



derivative of RMS
with respect to λ

Time Stabilized Scheme

- Derivative of EOM: $\dot{\boldsymbol{x}}_\alpha = \boldsymbol{J} \cdot \boldsymbol{x}_\alpha + \boldsymbol{R}_\alpha$
- Utilize a variational analysis to re-write this equation in discrete-time:
  - $\boldsymbol{x}_\alpha^{n+1} = \left(\boldsymbol{I} - \boldsymbol{J}^{int}\right)^{-1} \cdot \left(\boldsymbol{x}_\alpha^n + \boldsymbol{R}^{int}\right)$
  - $\boldsymbol{J}^{int} = \int_{t_n}^{t_{n+1}} \boldsymbol{J} \cdot dt$
  - $\boldsymbol{R}^{int} = \int_{t_n}^{t_{n+1}} \boldsymbol{R}_\alpha \cdot dt$
- This discrete-time equation can give the same results as the continuous-time version

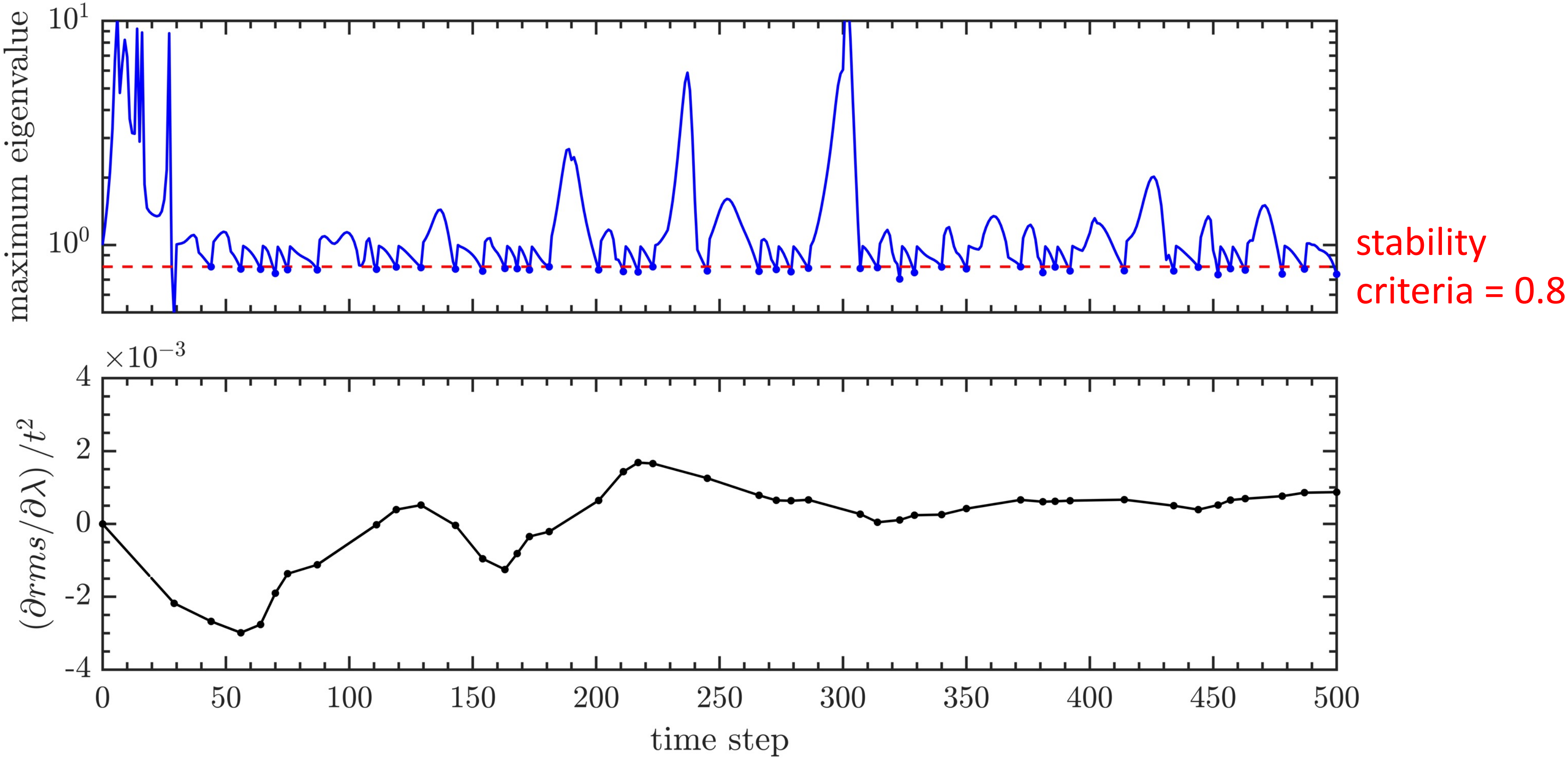
Time-Stabilized Scheme (2)

- $\boldsymbol{x}_\alpha^{n+1} = \left(\boldsymbol{I} - \boldsymbol{J}^{int}\right)^{-1} \cdot \left(\boldsymbol{x}_\alpha^{n} + \boldsymbol{R}^{int}\right)$
- The trick: don't update this equation at every time step
  - Only update it when the system becomes stable
  - Track the eigenvalues of $\left(\boldsymbol{I} - \boldsymbol{J}^{int}\right)^{-1}$ at every time step, as we compute the chaotic motion
  - Once all the eigenvalues are inside the unit circle, update the system, and re-set the 2 integrals to zero, and repeat
  - Adaptive time-stepping

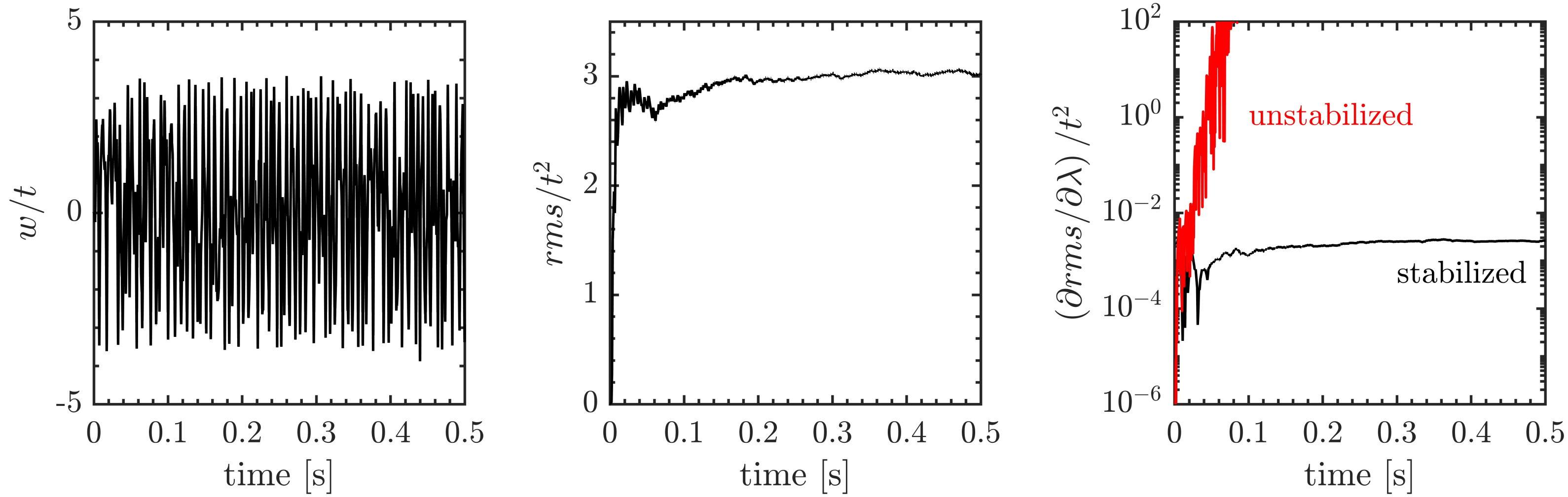
Time-Stabilized Scheme (3)

- Two problems:
  1. Need to compute the largest eigenvalue of $\left(\boldsymbol{I} - \boldsymbol{J}^{int}\right)^{-1}$ at every time step
  2. Stability criterion: in theory eigenvalues less than 1 should work
     - But numerical tests have found that values 0.8-0.9 work better
     - Lower values force larger adaptive time steps, and more stabilization of the derivatives
     - Best choice is ad-hoc and problem-dependent

Time-Stabilized Scheme (4)



Time-Stabilized Scheme (5)



- Verification of the sensitivities is in the paper
  - Paper also has a demonstration of the impact of the stability criterion (0.7, 0.8, 0.9, and 1.0)

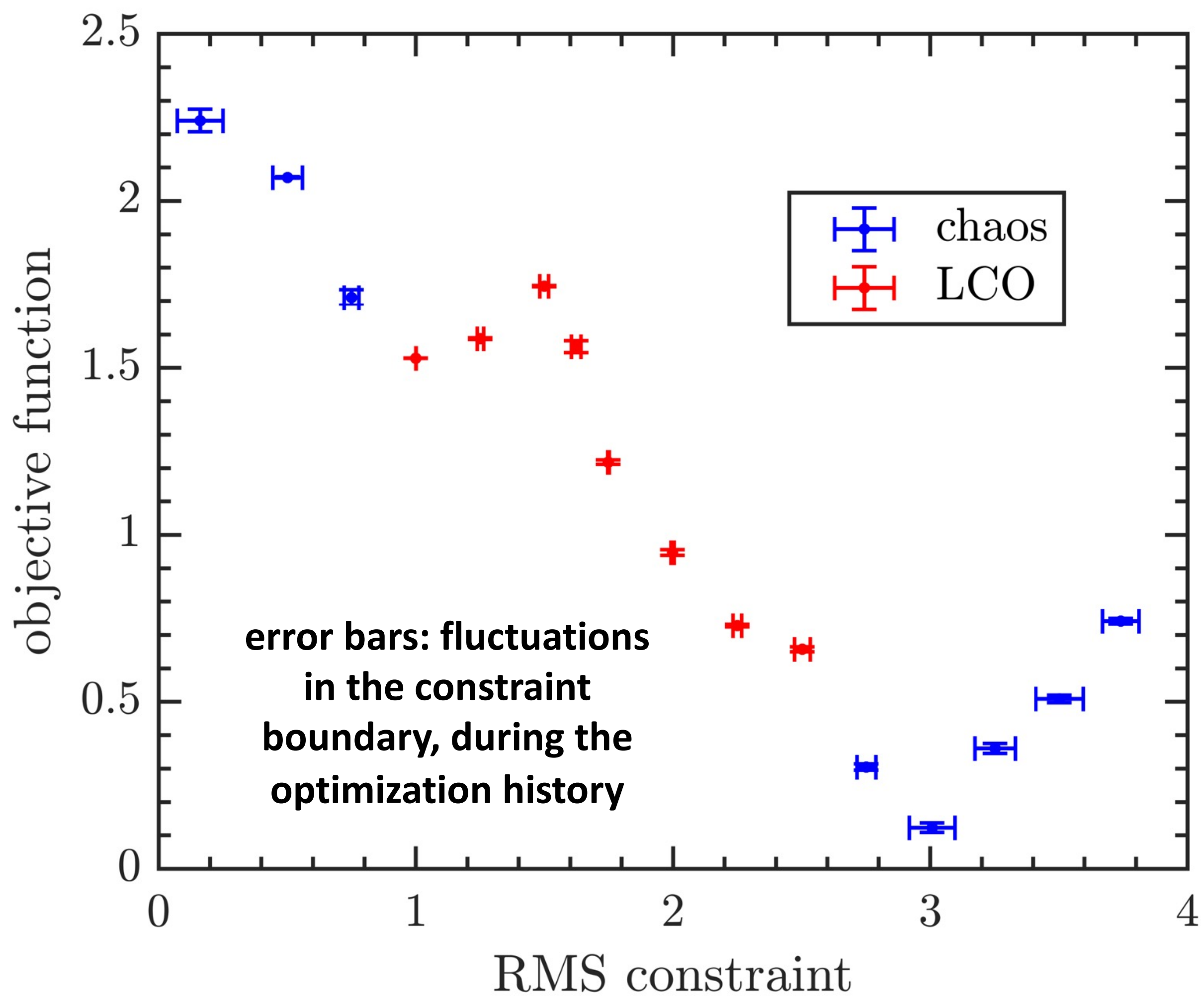
Design Optimization

- Find the thickness distribution of the panel that:
  - pushes the response RMS to a certain value (constraint)
  - with the smallest possible deviation from a baseline uniform thickness distribution (objective)

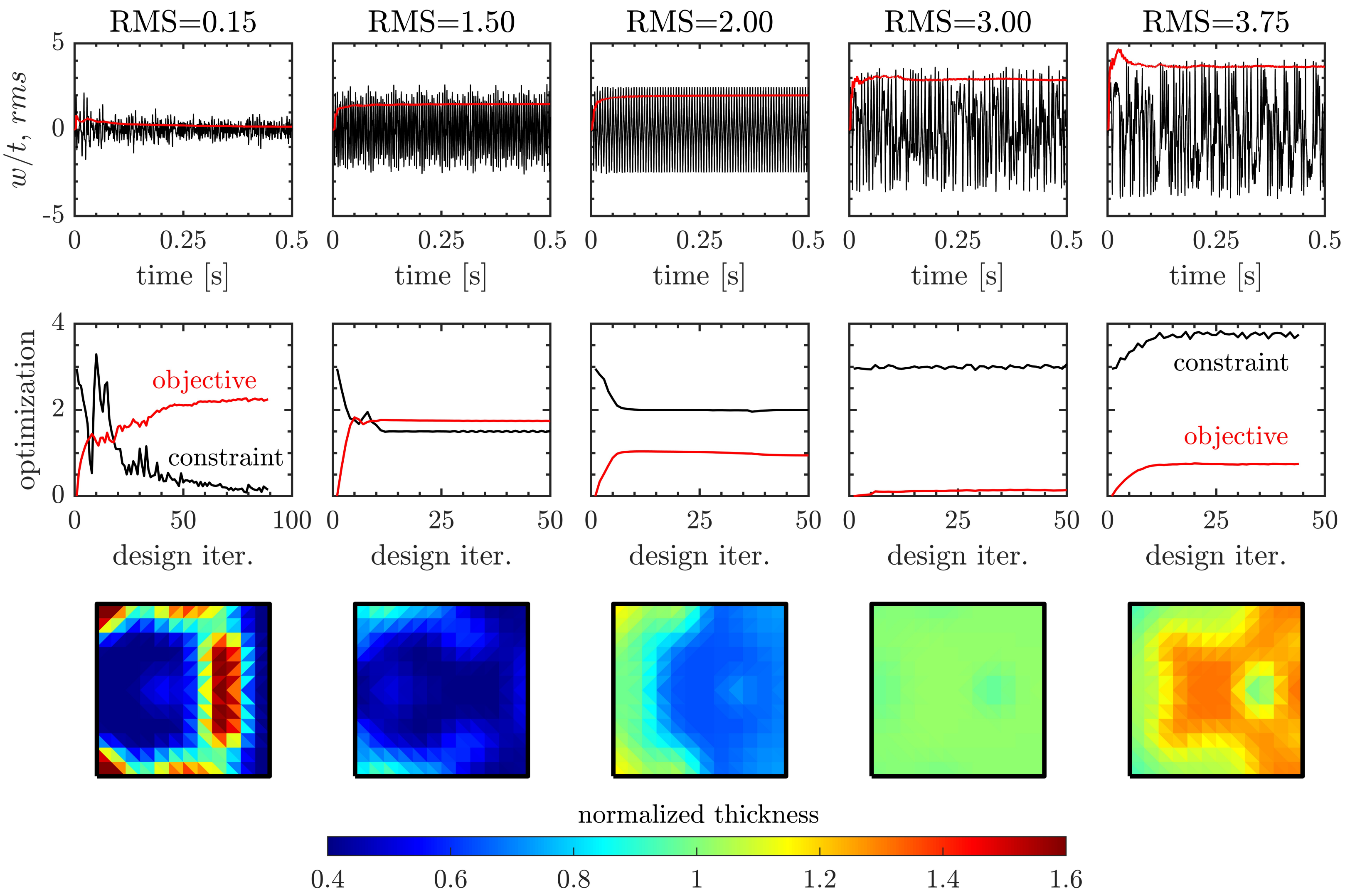
- Requires that we compute RMS derivatives with respect to the thickness of every finite element
  - Adjoint method
  - Stabilized time marching in reverse-time

Trade-off Curve

$$\frac{\|\boldsymbol{t}_{DV} - t_0\|}{t_0}$$



Optimal Designs



Conclusions

- Chaotic response of aeroelastic panels
- Can compute derivative of chaotic RMS with respect to parameters:
  - Stabilized time marching (adaptive time stepping) method developed by Manav and co-workers in 2017/2018
  - Have verified the accuracy of the sensitivities (in the paper)
- Can use those sensitivities to alter the thickness distribution so as to push the RMS output to different values

bret.k.stanford@nasa.gov