Equations of Motion for a Generic Multibody Tilt-rotor Aircraft

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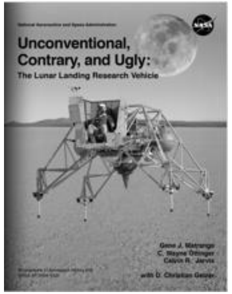
Motivation

LLRV to LLTV

Positive Training

Apollo

Crew must have a thorough understanding of control response, control power, and the unique physics of flight in the lunar environment of vacuum and 1/6 gravity, primarily the relationship between flight deck angle (thrust vector) and linear acceleration.



Neil Armstrong: "I felt very comfortable – I felt at home. I felt like I was flying something I was used to and it was doing the things that it ought to be doing…"

Donald "Deke" Slayton, then NASA's astronaut chief, *"said there was no other way to simulate a moon landing except by flying the LLTV"*.

eVTOL Production Aircraft

Joby Image credit: [1]

[1]: https://evtol.com/news/joby-aviation-reveals-s4-toyota-investment/

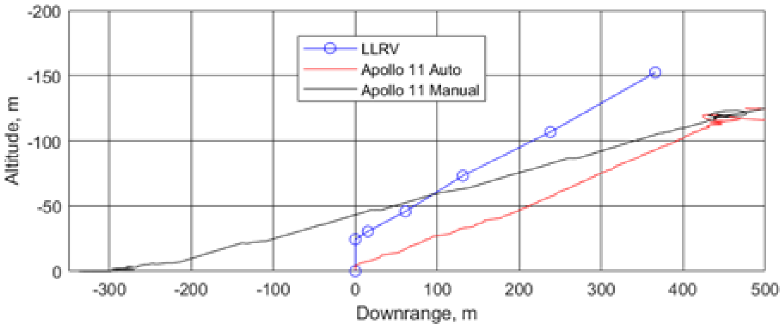
Tilt-rotor configurations allows decoupling between vehicle flight path and attitude. Able to replicate the ratio of tilt angle to linear acceleration as what a vehicle would experience on the lunar surface

Artemis

Motivation

- **Vertical Take-off and Landing (VTOL) Vehicles draw upon advantages from fix-wing and rotorcraft**
  - Longer endurance, better efficiency, operations at higher speeds
  - Ability to take off and land vertically, hover, and maneuver in confined spaces
  - Two configurations: **Tilt-rotor** vs. Tilt-wing
- **Often flight dynamics simulations treat the vehicle as a single rigid body**
  - Rotors are treated as thrust application points
  - Provide reasonably accurate results if the mass of the appendages (rotors, nacelles, wing sections) and their motion/displacement relative to the main body are small
- **Lunar landing trajectories stress the operation boundaries of these aircrafts**
  - Coupling of multi-body dynamics with complex effects such as vortex ring state, aero-propulsive interactions, flutter, etc. is not well understood

V-22 Image credit: [2]



Apollo 11 and LLRV touchdown trajectories

[2]: https://en.wikipedia.org/wiki/Bell_Boeing_V-22_Osprey

Background: Dynamics Modeling

- **1) Analytical single rigid body approach**
  - Treat the vehicle as a single rigid body
  - Effects like rotor aerodynamics and blade flapping can be incorporated with various levels of fidelity
  - Worked well for the XV-15 aircraft
- **2) Multibody approach via commercial software**
  - Detail models of the wing, rotors, nacelle, etc. (as many as 800 states)
  - Difficult to gain insight into the underlying vehicle dynamics

XV-15 Image credit: [3]

- **3) Analytical multibody approach**
  - Where this paper resides
  - Previous literature in this category leaves out portions of the final set of equations
  - Su 2019 provides a complete derivation and equations for a two-rotor configuration, but the two nacelles were assumed to tilt synchronously and the rotor spin DoF is ignored

[3]: NASA monograph “The History of the XV-15 Tilt Rotor Research Aircraft From Concept to Flight”.

Advantages of Kane's Method[4]

- **Kane's method permits the nonlinear equations of motion to be formulated with minimum labor in a systematic fashion and involves only the velocities and angular velocities, and their time derivatives**
  - Procedure can be automated via *MATLAB's symbolic toolbox*[5] while retaining insight into the various components
- **Constraint forces do not appear in Kane's equations of motion**
  - These forces appear when using Newton-Euler method and D'Alembert's Principle
  - Extra work is required to eliminate these constraint forces
  - Location where to form the angular momentum vector matters
- **Lagrange's method requires formulation of the system's kinetic energy and potential energy, partial derivatives w.r.t generalized coordinates and their time derivatives, etc.**
  - Results in unnecessarily lengthy equations

[4]: Kane, T., and Levinson, D., "Formulation of Equations of Motion for Complex Spacecraft." Journal of Guidance and Control. Vol. 3, March-April 1980
[5]: https://www.mathworks.com/products/symbolic.html

Procedure

1) Form expressions for velocities of mass centers and angular velocities of rigid bodies

2) Obtain partial velocities and angular velocities

r	1	2	3	4	5	6	$6+i$	$6+n+i$
${}^N\mathbf{v}_r^{B^*}$	$\hat{\mathbf{b}}_1$	$\hat{\mathbf{b}}_2$	$\hat{\mathbf{b}}_3$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$
${}^N\boldsymbol{\omega}_r^B$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\hat{\mathbf{b}}_1$	$\hat{\mathbf{b}}_2$	$\hat{\mathbf{b}}_3$	$\mathbf{0}$	$\mathbf{0}$
${}^N\boldsymbol{\omega}_r^{D_i}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\hat{\mathbf{b}}_1$	$\hat{\mathbf{b}}_2$	$\hat{\mathbf{b}}_3$	$\hat{\mathbf{b}}_2$	$\hat{\mathbf{c}}_1$
${}^N\mathbf{v}_r^{D_i^*}$	$\hat{\mathbf{b}}_1$	$\hat{\mathbf{b}}_2$	$\hat{\mathbf{b}}_3$	$-k_{3,i}\hat{\mathbf{b}}_2 + k_{2,i}\hat{\mathbf{b}}_3$	$k_{3,i}\hat{\mathbf{b}}_1 - k_{1,i}\hat{\mathbf{b}}_3$	$-k_{2,i}\hat{\mathbf{b}}_1 + k_{1,i}\hat{\mathbf{b}}_2$	$-l_i\hat{\mathbf{c}}_3$	$\mathbf{0}$

3a) Form accelerations of mass centers, angular momenta of rigid bodies (${}^N H^{B/B^*}$ and ${}^N H^{D/D^*}$), and their time derivatives in N

3b) Obtain generalized active forces (thrust, motor torque, aero, gravity, etc.)

4) Apply Kane's method

$$ {}^N\mathbf{v}_r^{B^*}\cdot\left(\mathbf{F}_B - m_B\,{}^N\mathbf{a}^{B^*}\right) + {}^N\boldsymbol{\omega}_r^{B}\cdot\left(\mathbf{T}_B - \sum_{i=1}^{n}\mathbf{M}_i - \frac{{}^Nd\,{}^N\mathbf{H}^{B/B^*}}{dt}\right) + \sum_{i=1}^{n}\left[{}^N\mathbf{v}_r^{D_i^*}\cdot\left(\mathbf{F}_i - m_D\,{}^N\mathbf{a}^{D_i^*}\right) + {}^N\boldsymbol{\omega}_r^{D_i}\cdot\left(\mathbf{T}_i + \mathbf{M}_i - \frac{{}^Nd\,{}^N\mathbf{H}^{D_i/D_i^*}}{dt}\right)\right] = 0 \quad (r = 1,\dots,6+2n) $$

Generalized inertial forces

Generalized active forces

Final EOM: [M]u̇ = F

Generalized mass matrix

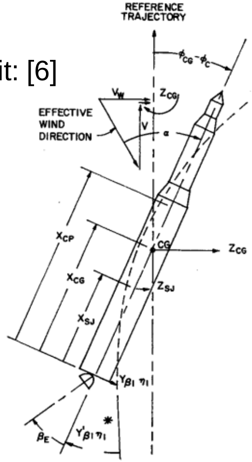
$$M = \begin{bmatrix} M_{R/R} & M_{R/\delta} & M_{R/\phi} \\ M_{\delta/R} & M_{\delta/\delta} & M_{\delta/\phi} \\ M_{\phi/R} & M_{\phi/\delta} & M_{\phi/\phi} \end{bmatrix}_{(6+2n)\times(6+2n)}$$

Terms in red: diagonal components

Terms in blue: off-diagonal components (inertial coupling effects such as “dog-wags-tail”, “tail-wags-dog”)

Image credit: [6]

Inertial reaction loads produced by Angular and linear momentum of the nozzle as it rotates about the gimbal



[6]: Frosh, J. Vallely, D., Saturn AS-501/S-IC Flight Control System. AIAA Journal of Spacecraft 1967.

Generalized F matrix (simplifies to Euler’s Eq. when rotor mass and inertia go to zero)

Rigid body portion (i = 1:6)

$$F_1 = -m_B(u_5u_3 - u_6u_2) - m_D\Sigma\left[-u_{6+i}(u_5 + u_{6+i})l_i\cos\delta_i + u_5\lambda_{3,i} - u_6\lambda_{2,i}\right] + \hat{\mathbf{b}}_1\cdot(\mathbf{F}_B + \Sigma\mathbf{F}_i)$$

$$\begin{aligned} F_6 = &-({}^N\omega^B \times \underline{\mathbf{I}}^{B/B^\star}\cdot{}^N\omega^B)\cdot\hat{\mathbf{b}}_3 - I_s\Sigma\sin\delta_i u_{6+i}(u_4\sin\delta_i + u_6\cos\delta_i) \\ &- I_t\Sigma\cos\delta_i u_{6+i}(u_4\cos\delta_i - u_6\sin\delta_i) + I_s\Sigma\cos\delta_i u_{6+n+i}(u_5 + u_{6+i}) \\ &- (I_t - I_s)\Sigma\cos\delta_i(u_5 + u_{6+i})(u_4\cos\delta_i - u_6\sin\delta_i) \\ &+ m_D\Sigma k_{2,i}\left[-u_{6+i}(u_5 + u_{6+i})l_i\cos\delta_i + u_5\lambda_{3,i} - u_6\lambda_{2,i}\right] \\ &- m_D\Sigma k_{1,i}\left[u_{6+i}l_i(u_4\cos\delta_i - u_6\sin\delta_i) - u_4\lambda_{3,i} + u_6\lambda_{1,i}\right] + \hat{\mathbf{b}}_3\cdot(\mathbf{T}_B + \Sigma\mathbf{T}_i) \\ &+ \Sigma(k_{1,i}\hat{\mathbf{b}}_2 - k_{2,i}\hat{\mathbf{b}}_1)\cdot\mathbf{F}_i \end{aligned}$$

Rotor portion (i = 7: 6+2n)

$$\begin{aligned} F_{6+i} = &-(u_4\sin\delta_i + u_6\cos\delta_i)\left[I_s u_{6+n+i} + (I_s - I_t)(u_4\cos\delta_i - u_6\sin\delta_i)\right] \\ &+ m_D l_i\sin\delta_i\,(u_5\lambda_{3,i} - u_6\lambda_{2,i}) + m_D l_i\cos\delta_i\,(u_4\lambda_{2,i} - u_5\lambda_{1,i}) + \tau_{6+i} + \hat{\mathbf{b}}_2\cdot\mathbf{T}_i \\ &- l_i\hat{\mathbf{c}}_3\cdot\mathbf{F}_i \end{aligned}$$

$$F_{6+n+i} = I_s u_{6+i}(u_4\sin\delta_i + u_6\cos\delta_i) + \tau_{6+n+i} + \hat{\mathbf{c}}_1\cdot\mathbf{T}_i$$

Angular momentum and kinetic energy checks:

$${}^N\mathbf{H}^{S/S^\star} = \underline{\mathbf{I}}^{B/B^\star}\cdot{}^N\omega^B + m_B\mathbf{r}^{S^\star B^\star}\times{}^N\mathbf{v}^{B^\star} + \sum_{i=1}^{n}\left(\underline{\mathbf{I}}^{D_i/D_i^\star}\cdot{}^N\omega^{D_i} + m_D\mathbf{r}^{S^\star D_i^\star}\times{}^N\mathbf{v}^{D_i^\star}\right)$$

$$K = \frac{1}{2}\left[m_B\,{}^N\mathbf{v}^{B^\star}\cdot{}^N\mathbf{v}^{B^\star} + {}^N\omega^B\cdot\underline{\mathbf{I}}^{B/B^\star}\cdot{}^N\omega^B + \sum_{i=1}^{n}\left(m_D\,{}^N\mathbf{v}^{D_i^\star}\cdot{}^N\mathbf{v}^{D_i^\star} + {}^N\omega^{D_i}\cdot\underline{\mathbf{I}}^{D_i/D_i^\star}\cdot{}^N\omega^{D_i}\right)\right]$$

Simulation Results

- **Case 1: Response to initial conditions (no gravity, aero, motor torque, thrust)**
- **Case 2: Response to open loop gimbal commands**
  - Vehicle starts in hover
  - T = 5 sec, OL gimbal rate cmd of -2.86 deg/s for all rotors
  - T = 10 sec, OL gimbal rate cmd of +2.86 deg/s for all rotors
  - T = 15 sec, cmd to trimmed level flight with constant forward velocity

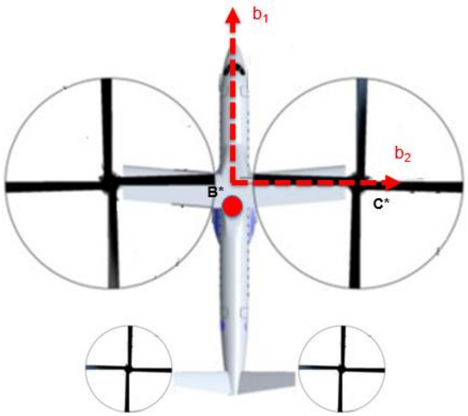
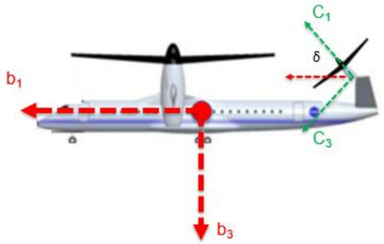


Image credit: [7]



Mass/Inertia

Parameter	Value
m_B	2176 kg
m_D	118 kg
l	1 m

$$I_B = \begin{bmatrix} 74110 & 0 & 0 \\ 0 & 6780 & 0 \\ 0 & 0 & 74529 \end{bmatrix} \text{kg-m}^2$$

$$I_D = \begin{bmatrix} 137 & 0 & 0 \\ 0 & 69 & 0 \\ 0 & 0 & 69 \end{bmatrix} \text{kg-m}^2$$

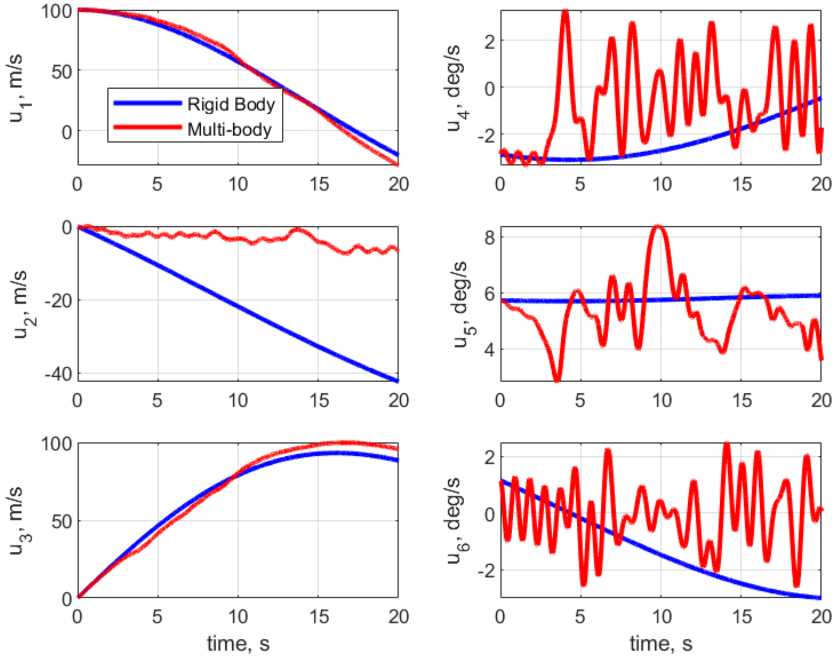
Rotor Locations

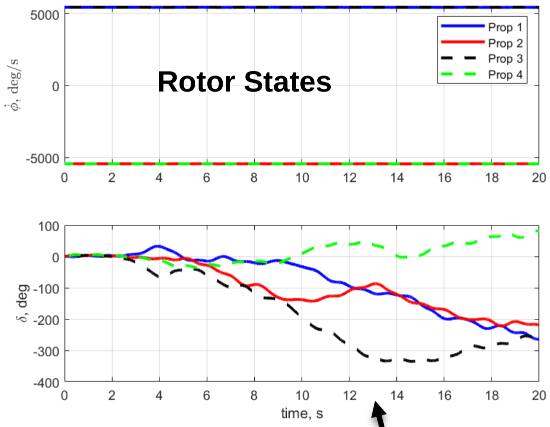
i	$L_{1,i}$ (m)	$L_{2,i}$ (m)	$L_{3,i}$ (m)
1	0.5	−5.5	−0.25
2	0.5	5.5	−0.25
3	−2.5	−2.5	−0.5
4	−2.5	2.5	−0.5

Case 1: Response to Initial Conditions

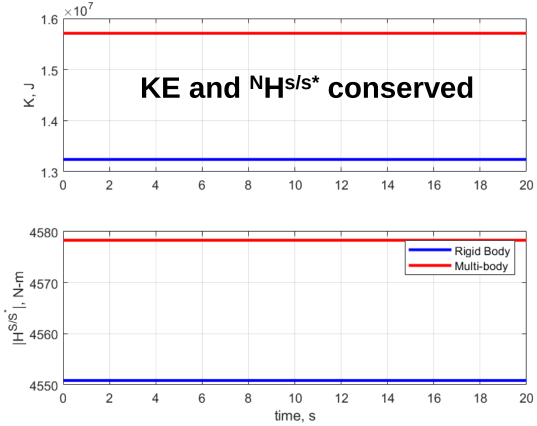
Intent is to check the validity of the EoM

Vehicle States



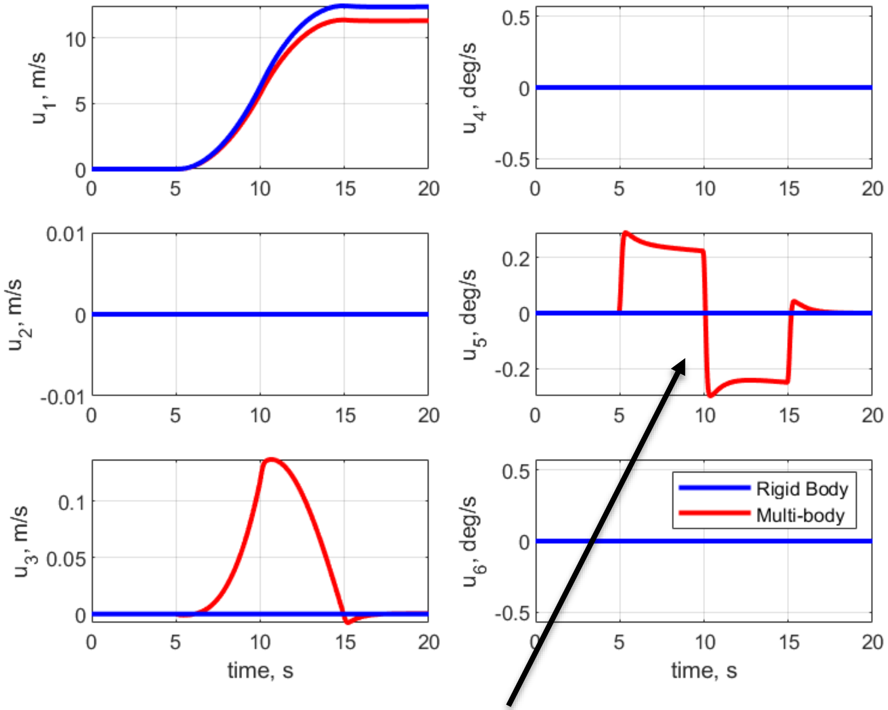


Rotor gimbal drift due to zero motor torque

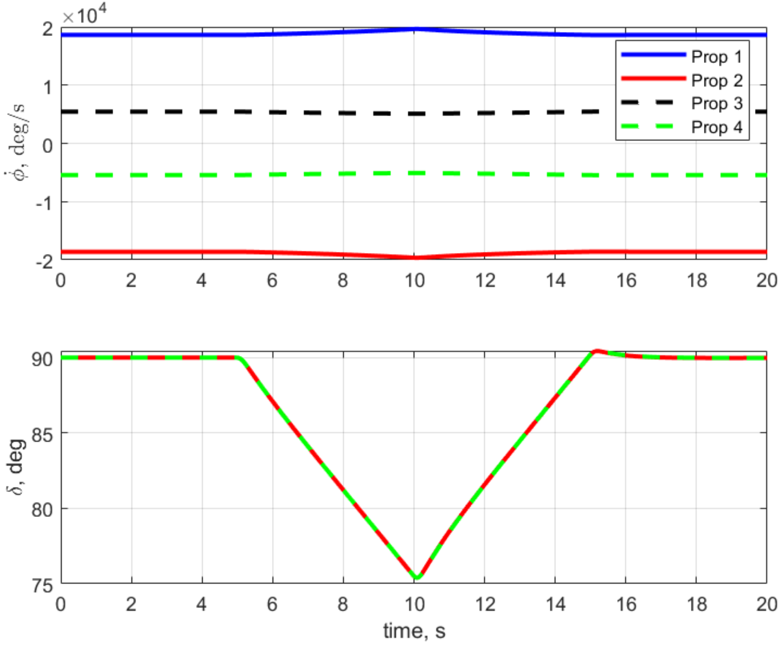


Direction of $^{N}H^{s/s*}$ also conserved (see paper)

Case 2. Open Loop Gimbal Commands



Rotor thrusts adjusted at each time step to ensure constant altitude



Law of action and reaction. Fuselage nose pitches up and down as the nacelles pitch in the opposite direction

Discussion

- **Kane's method is used to derive analytical multibody dynamical equations of motion for a generic tilt-rotor aircraft**
  - Final EOM is in a matrix format that can be readily implemented
- **Multibody approach recommended as the mass and motion of the rotors relative to the main body are significant**
- **Methodology can be readily extended to rotors with dual-gimbal capability or tilt-wing configurations**
  - Procedure can be automated via *MATLAB's symbolic toolbox* while retaining insight into the various components
- **Possible Future work:**
  - Linear analysis to yield further insight into the dynamic coupling
  - Controller performance with single-rigid body model vs. multibody model