

The Harmonic Linearized Navier-Stokes Equations for Transition Prediction in Three-Dimensional Flows

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Outline

1 Motivation

2 Theory

3 Results

- Transition Induced by Roughness Patch in Mach 3.5 Flow (ZPG)
- Roughness Effects on Nosedip Transition in Mach 6 Flow (FPG)

4 Concluding Remarks

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1 Motivation

2 Theory

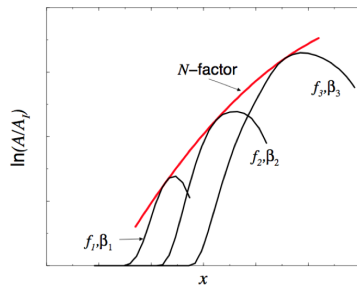
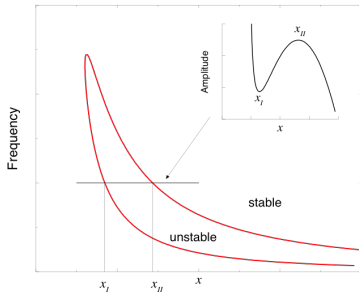
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Transition Prediction: e^N Method

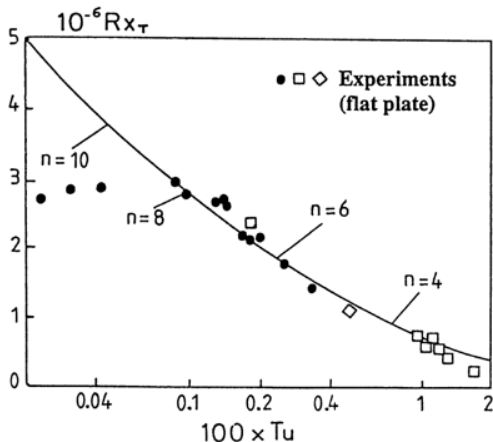
- Most used transition prediction method in aerodynamic design calculations
- Transition onset location predicted at the critical N -factor
- Critical N -factor obtained from correlations against experiments
- Neutral instability curve
- N -factor envelope



$$N = \max_{f, \beta} \left[\ln \left(\frac{A}{A_I} \right) \right]$$

Critical N -factor correlation: TS

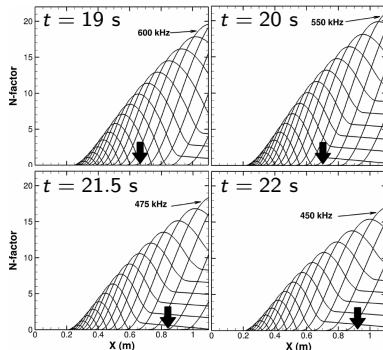
- Flat plate experiments used to correlated critical N -factor due to TS waves with freestream turbulence level



$$N = -8.43 - 2.4 \ln T_u. \text{ (Mack 1977)}$$

Critical N -factor correlation: Mack's second mode

- Nonablating, 7° half-angle cone with $r_n = 2.5$ mm
- Cold wall, $M_\infty = 3.8 - 5.5$ & $AoA = 0^\circ$: Mack's second-mode instability
- N -factor computed with Parabolized Stability Equations (PSE)¹
- Second-mode N -factors



- N -factor correlation and sensitivity

t, s	X_{tr}, m	N -factor	$\Delta X_{tr}/\Delta N, m$
19.0	0.667	10.87	0.041
20.0	0.703	13.05	0.034
21.0	0.780	14.15	0.064
21.5	0.841	14.51	0.058
22.0	0.922	14.41	0.096

- $t = 19$ s assumed to be influenced by step discontinuities
- Strong correlation for $N_{tr} \approx 13.5$

¹F. Li et al. "Transition analysis for the ascent phase of HIFiRE-1 flight experiment". In: *Journal of Spacecraft Rockets* 52.5 (2015), pp. 1283–1293.

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Linear Stability Analysis

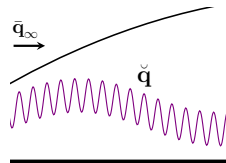
- Decomposition of flow variables: $\mathbf{q}(\mathbf{x}, t) = \bar{\mathbf{q}}(\mathbf{x}) + \tilde{\mathbf{q}}(\mathbf{x}, t)$; $\tilde{\mathbf{q}} \ll \bar{\mathbf{q}}$
- Harmonic LNSE (HLNSE): Exploit basic state independence w.r.t. time

▶ 2D: $\bar{\mathbf{q}}(\xi, \eta)$; $\tilde{\mathbf{q}}(\xi, \eta, \zeta, t) = \tilde{\mathbf{q}}(\xi, \eta) \exp[i(\beta\zeta - \omega t)]$

★ 2D linear system: $\mathbf{L}_{HLNSE} \tilde{\mathbf{q}}(\xi, \eta) = \tilde{\mathbf{f}}$

▶ 3D: $\bar{\mathbf{q}}(\xi, \eta, \zeta)$; $\tilde{\mathbf{q}}(\xi, \eta, \zeta, t) = \tilde{\mathbf{q}}(\xi, \eta, \zeta) \exp[-i(\omega t)]$

★ 3D linear system: $\mathbf{L}_{HLNSE} \tilde{\mathbf{q}}(\xi, \eta, \zeta) = \tilde{\mathbf{f}}$



- PSE: Exploit slow variations in streamwise direction

▶ 2D: $\bar{\mathbf{q}}(\xi, \eta, \zeta)$; $\tilde{\mathbf{q}}(\xi, \eta) = \hat{\mathbf{q}}(\xi, \eta) \exp\left[i \int_{\xi_0}^{\xi} \alpha(\xi') d\xi'\right]$

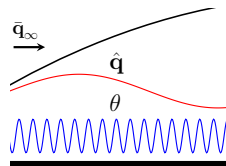
- ★ Parabolic integration of 1D systems:

$$\left(\mathbf{L}_{PSE}(\alpha) + \mathbf{M}_{PSE} \frac{1}{h_{\xi}} \frac{\partial}{\partial \xi} \right) \hat{\mathbf{q}}(\xi, \eta) = 0$$

▶ 3D: $\bar{\mathbf{q}}(\xi, \eta, \zeta)$; $\tilde{\mathbf{q}}(\xi, \eta, \zeta) = \hat{\mathbf{q}}(\xi, \eta, \zeta) \exp\left[i \int_{\xi_0}^{\xi} \alpha(\xi') d\xi'\right]$

- ★ Parabolic integration of 2D systems:

$$\left(\mathbf{L}_{PSE}(\alpha) + \mathbf{M}_{PSE} \frac{1}{h_{\xi}} \frac{\partial}{\partial \xi} \right) \hat{\mathbf{q}}(\xi, \eta, \zeta) = 0$$



The 3D HLNSE

- Solve linear evolution of disturbances in 3D nonparallel configurations, as roughness elements, steps, gaps, corners, ...
- 3D linear system of equations: $\mathbf{L}\mathbf{q} = \mathbf{f}$
- Typical resolution of 3D convective problems: $(n_\xi, n_\eta, n_\zeta) = \mathcal{O}(10^3, 10^2, 10^2)$
- Leading dimension: $m = 5n_\xi n_\eta n_\zeta = \mathcal{O}(10^7 - 10^8)$
- Block pentadiagonal system by using 4th-order FD in ξ
 - Block size: $n_b = 5n_\eta n_\zeta = \mathcal{O}(10^4 - 10^5)$

$$\begin{bmatrix}
 C_1 & D_1 & E_1 & 0 & & & & & & 0 \\
 B_2 & C_2 & D_2 & E_2 & 0 & & & & & 0 \\
 A_3 & B_3 & C_3 & D_3 & E_3 & 0 & & & & 0 \\
 0 & A_4 & B_4 & C_4 & D_4 & E_4 & 0 & & & 0 \\
 0 & 0 & A_5 & B_5 & C_5 & D_5 & E_5 & 0 & & 0 \\
 \vdots & & & & & & & & & \vdots \\
 0 & & \dots & 0 & A_{n-4} & B_{n-4} & C_{n-4} & D_{n-4} & E_{n-4} & 0 \\
 0 & & & \dots & 0 & A_{n-3} & B_{n-3} & C_{n-3} & D_{n-3} & E_{n-3} \\
 0 & & & & \dots & 0 & A_{n-2} & B_{n-2} & C_{n-2} & D_{n-2} \\
 0 & & & & & \dots & 0 & A_{n-1} & B_{n-1} & C_{n-1} \\
 0 & & & & & & \dots & 0 & A_n & B_n \\
 0 & & & & & & & & C_n & D_n
 \end{bmatrix}
 \begin{bmatrix}
 q_1 \\
 q_2 \\
 q_3 \\
 q_4 \\
 q_5 \\
 \vdots \\
 q_{n-4} \\
 q_{n-3} \\
 q_{n-2} \\
 q_{n-1} \\
 q_n
 \end{bmatrix}
 =
 \begin{bmatrix}
 f_1 \\
 f_2 \\
 f_3 \\
 f_4 \\
 f_5 \\
 \vdots \\
 f_{n-4} \\
 f_{n-3} \\
 f_{n-2} \\
 f_{n-1} \\
 f_n
 \end{bmatrix}$$

Solution of 3D HLNSE

- Direct solution with combined multithread and multiprocessor approach
- Thomas algorithm and dual Schur complement method²
- Decouple streamwise from crossplane dependency
 - ▶ CPU time and memory requirements scale linearly with $n \equiv n_\xi$
- **Thomas algorithm** for block pentadiag. system with multithread BLAS/LAPACK

$$\begin{bmatrix}
 C_1 & D_1 & E_1 & 0 & & & \cdots & & 0 \\
 B_2 & C_2 & D_2 & E_2 & 0 & & & & 0 \\
 \vdots & & & & \ddots & & & & \vdots \\
 0 & \cdots & 0 & A_i & B_i & C_i & D_i & E_i & 0 & \cdots & 0 \\
 \vdots & & & & \ddots & & & & \vdots \\
 0 & & \cdots & & & 0 & A_{n-1} & B_{n-1} & C_{n-1} & D_{n-1} \\
 0 & & & \cdots & & 0 & A_n & B_n & C_n & D_n
 \end{bmatrix}
 \begin{bmatrix}
 q_1 \\
 q_2 \\
 \vdots \\
 q_i \\
 \vdots \\
 q_{n-1} \\
 q_n
 \end{bmatrix}
 =
 \begin{bmatrix}
 f_1 \\
 f_2 \\
 \vdots \\
 f_i \\
 \vdots \\
 f_{n-1} \\
 f_n
 \end{bmatrix}$$

- LU decomposition ($i = 1 : 1 : n$)

$$K_i = B_i - A_i Y_{i-2}$$

$$G_i = C_i - K_i Y_{i-1} - A_i Z_{i-2}$$

$$Y_i = G_i^{-1}(D_i - K_i Z_{i-1})$$

$$Z_i = G_i^{-1} E_i$$

- Forward solution ($i = 1 : 1 : n$)

$$r_i = G_i^{-1}(f_i - K_i r_{i-1} - A_i r_{i-2})$$

- Backward solution ($i = n : -1 : 1$)

$$q_i = r_i - Y_i q_{i+1} - Z_i q_{i+2}$$

²M. Carpenter et al. *Excitation of Crossflow Instabilities in a Swept Wing Boundary Layer*. AIAA Paper 2010-0378. 2010.

Solution of 3D HLNSE

- Direct solution with combined multithread and multiprocessor approach
- Thomas algorithm and dual Schur complement method²
- Decouple streamwise from crossplane dependency
 - ▶ CPU time and memory requirements scale linearly with $n \equiv n_\xi$
- **Dual Schur complement method** example with $n_p = 2$ and $n_\xi = 12$

$$\begin{bmatrix}
 C_1 & D_1 & E_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 B_2 & C_2 & D_2 & E_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 A_3 & B_3 & C_3 & D_3 & E_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & A_4 & B_4 & C_4 & D_4 & 0 & 0 & 0 & 0 & 0 & E_4 & 0 \\
 0 & 0 & A_5 & B_5 & C_5 & 0 & 0 & 0 & 0 & 0 & D_5 & E_5 \\
 \hline
 0 & 0 & 0 & 0 & 0 & C_8 & D_8 & E_8 & 0 & 0 & A_8 & B_8 \\
 0 & 0 & 0 & 0 & 0 & B_9 & C_9 & D_9 & E_9 & 0 & 0 & A_9 \\
 0 & 0 & 0 & 0 & 0 & A_{10} & B_{10} & C_{10} & D_{10} & E_{10} & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & A_{11} & B_{11} & C_{11} & E_{11} & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & A_{12} & B_{12} & C_{12} & 0 & 0 \\
 \hline
 0 & 0 & 0 & A_6 & B_6 & E_6 & D_6 & 0 & 0 & 0 & C_6 & D_6 \\
 0 & 0 & 0 & A_7 & D_7 & E_7 & 0 & 0 & 0 & 0 & B_7 & C_7
 \end{bmatrix}
 \begin{bmatrix}
 q_1 \\ q_2 \\ q_3 \\ q_4 \\ q_5 \\ \hline q_8 \\ q_9 \\ q_{10} \\ q_{11} \\ q_{12} \\ \hline q_6 \\ q_7
 \end{bmatrix}
 =
 \begin{bmatrix}
 f_1 \\ f_2 \\ f_3 \\ f_4 \\ f_5 \\ \hline f_8 \\ f_9 \\ f_{10} \\ f_{11} \\ f_{12} \\ \hline f_6 \\ f_7
 \end{bmatrix}$$

$$\begin{bmatrix}
 \mathbf{L}_{I,I} & 0 & \mathbf{L}_{I,S} \\
 \hline 0 & \mathbf{L}_{II,II} & \mathbf{L}_{II,S} \\
 \hline \mathbf{L}_{S,I} & \mathbf{L}_{S,II} & \mathbf{L}_{S,S}
 \end{bmatrix}
 \begin{bmatrix}
 q_I \\ \hline q_{II} \\ \hline q_S
 \end{bmatrix}
 =
 \begin{bmatrix}
 f_I \\ \hline f_{II} \\ \hline f_S
 \end{bmatrix};$$

$$q_s = \tilde{\mathbf{L}}_{s,s}^{-1} \tilde{f}_s \left\{ \begin{array}{l} \tilde{\mathbf{L}}_{s,s} = \mathbf{L}_{s,s} - \sum_{p=I}^{II} \mathbf{L}_{s,p} \mathbf{L}_{p,p}^{-1} \mathbf{L}_{p,s} \\ \tilde{f}_s = f_s - \sum_{p=I}^{II} \mathbf{L}_{s,p} \mathbf{L}_{p,p}^{-1} f_p \end{array} \right.$$

$$q_p = \mathbf{L}_{p,p}^{-1} f_p - \mathbf{L}_{p,s} q_s; p = I, II$$

²M. Carpenter et al. *Excitation of Crossflow Instabilities in a Swept Wing Boundary Layer*. AIAA Paper 2010-0378. 2010.

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Transition Induced by Roughness Patch in Mach 3.5 Flow (ZPG)

Configuration⁵

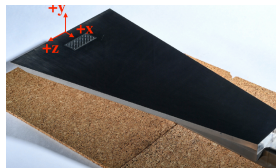
● Unperturbed, 2D flow

- ▶ Computed with the NASA VULCAN-CFD solver³
- ▶ Polished flat plate ($< 0.1 \mu\text{m}$ RMS roughness) with $100 \mu\text{m}$ thickness leading edge
- ▶ Flow conditions (NASA LaRC SLDT Mach 3.5 quiet facility)

- ★ $M_\infty = 3.5$, $Re_\infty = 12.6 \times 10^6 \text{ m}^{-1}$
- ★ $T_\infty = 92.52 \text{ K}$, Adiabatic wall

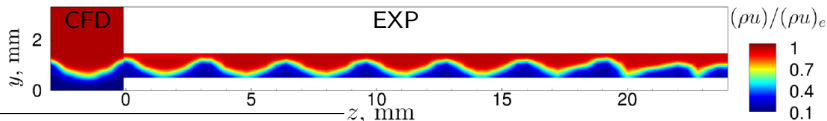
● Perturbed, 3D flow

- ▶ Computed with high-order DNS solver⁴
- ▶ Sinusoidal roughness patch with $3.5\lambda_x$ and $7.5\lambda_z$
 - ★ $x_0 = 28.8 \text{ mm}$, $k = 272 \mu\text{m}$, $\lambda_x = \lambda_z = 6.25 \text{ mm}$



● Experimental measurements

- ▶ Hotwire measurements of mass-flux mean (ρu) and fluctuations $((\rho u)')$
- ▶ Mean mass-flux contours at $x = 110.6 \text{ mm}$



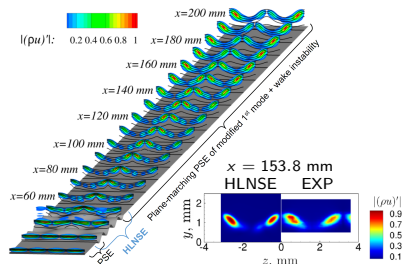
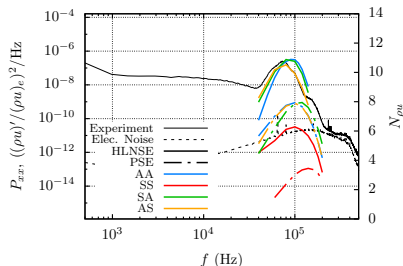
³R. Baurle et al. *VULCAN-CFD Theory Manual: Ver. 7.1.0*. NASA TM-2020-5000766. 2020.

⁴M. Wu and M.P. Martin. "Direct numerical simulation of supersonic boundary layer over a compression ramp". In: *AIAA Journal* 45.4 (2007), pp. 879–889. DOI: 10.2514/1.27021.

⁵A. Chou et al. "Transition Induced by an Egg-Crate Roughness on a Flat Plate in Supersonic Flow". In: *Journal of Fluid Mechanics* (in review) (2022).

PSE vs. HLNSE Analysis⁶

- PSE: Wake instabilities downstream of roughness patch
- HLNSE: PSE of Mack's first mode upstream, HLNSE over patch, PSE downstream
- N -factor calculation for symmetric (SS, AS) and antisymmetric (AA, SA) modes
- Spectra at $x = 153.8$ mm
- AA, 80 kHz



- PSE along the wake misses $\Delta N \approx 3$ and predicts higher frequency peak ($f \approx 100\text{kHz}$)
- HLNSE analysis closely matches experimental frequency bandwidth and peak ($f \approx 80\text{kHz}$)
- Confirms significance of disturbance growth both upstream of and above a finite length roughness patch and the effect on the growth of instabilities in the wake⁵

⁵M. Choudhari et al. *Effect of 3D Roughness Patch on Instability Amplification in a Supersonic Boundary Layer*. AIAA Paper 2019-0877. 2019.

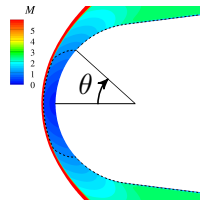
⁶A. Chou et al. "Transition Induced by an Egg-Crate Roughness on a Flat Plate in Supersonic Flow". In: *Journal of Fluid Mechanics* (in review) (2022).

Results

Roughness Effects on Nosedip Transition in Mach 6 Flow (FPG)

Configuration¹⁰

- Unperturbed, axisymmetric flow
 - ▶ Computed with the NASA VULCAN-CFD solver⁷
 - ▶ 7° half-angle cone with $R_N = 15.24$ mm at zero AoA
 - ▶ Flow conditions (AFRL Mach 6 High Re facility)
 - ★ $M_\infty = 5.9$, $Re_\infty = 75.8 \times 10^6 \text{ m}^{-1}$
 - ★ $T_\infty = 76.74 \text{ K}$, $T_w/T_{w,ad} = 0.57$



- Perturbed, 3D flow
 - ▶ Computed with high-order DNS solver⁸
 - ▶ Roughness elements designed based on experiments⁹
 - ★ $\theta = 45^\circ$ ($Me = 1.1$), $m = 420$, $k = 20 \text{ } \mu\text{m}$
 - ★ Cubic and prismatic elements (50% blockage)



- Roughness parameters supercritical based on both the ballistics-range data (Reda, 2002) and transient growth correlation (Reshotko & Tumin, 2004)

$k \text{ } [\mu\text{m}]$	k/δ_h	k/Θ	Re_k	Re_{kk}	$Re_\Theta(k/\Theta T_e/T_w)^{1.3}$	$Re_\Theta(k/\Theta)(T_e/T_w)^{1.27}$
			250	250	574	434
20	0.277	2.68	616	482	1, 213	889

⁷R. Baurle et al. *VULCAN-CFD Theory Manual: Ver. 7.1.0*. NASA TM-2020-5000766. 2020.

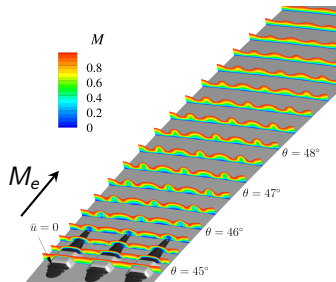
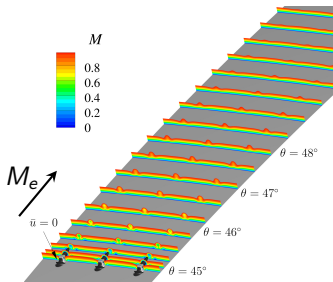
⁸M. Wu and M.P. Martin. "Direct numerical simulation of supersonic boundary layer over a compression ramp". In: *AIAA Journal* 45.4 (2007), pp. 879–889. DOI: 10.2514/1.27021.

⁹P. Paredes et al. "Nosetip bluntness effects on transition at hypersonic speeds: experimental and numerical analysis". In: *Journal of Spacecraft Rockets* 56.2 (2019). DOI: 10.2514/1.A34277.

¹⁰P. Paredes et al. *Combined Bluntness and Roughness Effects on Cones at Hypersonic Speeds*. AIAA 2022-0000. 2022.

Boundary-Layer Flows with Roughness Elements¹²

- Roughness elements at $\theta = 45^\circ$ with $m = 420$ on $R_N = 15.24$ mm cone
- Three-dimensional view of 3 azimuthal wavelengths with Mach number contours at selected streamwise locations
- $k = 20$ μm , Cubic
- $k = 20$ μm , Prismatic



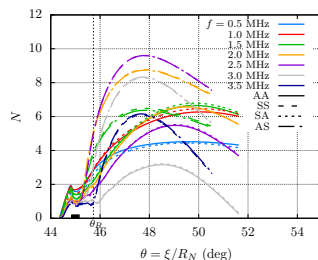
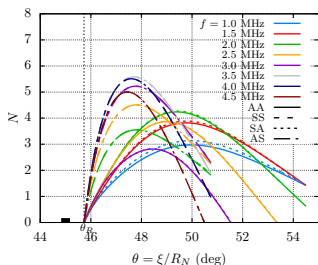
- Cubic: azimuthal variations remain concentrated behind roughness elements (similar to isolated roughness)¹¹
- Prismatic with planform size of 80.1 μm : azimuthal variations spread across the azimuthal wavelength and have a much longer streamwise extent

¹¹A. Theiss et al. Wake flow instability studies behind discrete roughness elements on a generic re-entry capsule. AIAA Paper 2016-4382. 2016.

¹²P. Paredes et al. Combined Bluntness and Roughness Effects on Cones at Hypersonic Speeds. AIAA 2022-0000. 2022.

PSE vs. Inflow-Resolvent HLNSE Analysis¹²

- Prismatic roughness elements with $k = 20 \mu\text{m}$ height and planform size of $80.1 \mu\text{m}$
- HLNSE: Inflow set at $\theta_0 = 44.36^\circ$ and optimization location at $\theta_1 = 47.4^\circ$
- PSE: Stable and robust integration only downstream of separation region
- N -factor curves for symmetric (SS, AS) and antisymmetric (AA, SA) modes
- Plane-marching PSE
- Inflow-resolvent HLNSE

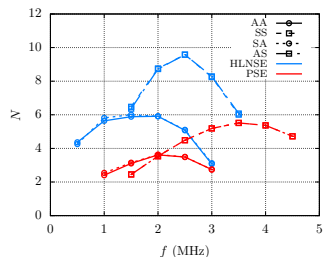
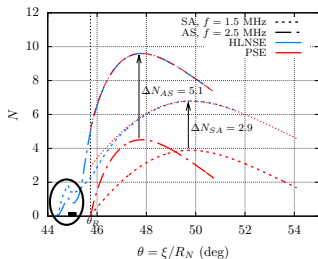


- Short region of amplification upstream of roughness elements due to transient growth
- Slight decay at roughness leading edge and grow again downstream of trailing edge
- Symmetric disturbances (SS, AS) experience earlier and stronger growth over separation
- $N_{max} \approx 9.6$ (8.6 without upstream TG) for AS/SS with $f = 2.5 \text{ MHz}$

¹²P. Paredes et al. Combined Bluntness and Roughness Effects on Cones at Hypersonic Speeds. AIAA 2022-0000. 2022.

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- HLNSE: Inflow set at $\theta_0 = 44.36^\circ$ and optimization location at $\theta_1 = 47.4^\circ$
- PSE: Stable and robust integration only downstream of separation region
- N -factor curves for symmetric (SS, AS) and antisymmetric (AA, SA) modes
- Comparison of N -factor curves
- Comparison of spectra at $\theta = 48^\circ$

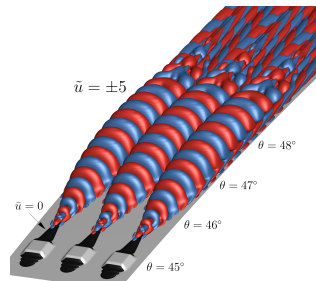
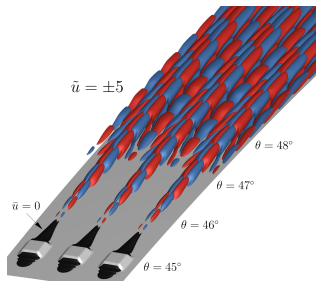


- Large growth over separated region, even after ignoring the transient growth upstream
- Overlap by upward shift confirms HLNSE disturbances become modal streak instabilities
- Rather significant shift in N -factor values and frequency bands for S and A modes
- Large difference between PSE and HLNSE predictions believed to be due to strong favorable pressure gradient (not observed in the roughness wake for ZPG case)

¹²P. Paredes et al. *Combined Bluntness and Roughness Effects on Cones at Hypersonic Speeds*. AIAA 2022-0000. 2022.

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- HLNSE: Inflow set at $\theta_0 = 44.36^\circ$ and optimization location at $\theta_1 = 47.4^\circ$
- PSE: Stable and robust integration only downstream of separation region
- Spatial structure of most amplified modes
 - SA, $f = 1.5 \text{ MHz}$
 - AS, $f = 2.5 \text{ MHz}$



- Isosurfaces of $\tilde{u} = \pm 5$ normalized such $E_0 = 1$ and separation region ($\tilde{u} = 0$)
- Shear layer over separated zone supports stronger growth of symmetric mode
- Amplification of symmetric mode terminates earlier
- Transition onset would occur within 3° downstream of roughness element

¹²P. Paredes et al. *Combined Bluntness and Roughness Effects on Cones at Hypersonic Speeds*. AIAA 2022-0000. 2022.

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Summary and Concluding Remarks

HLNSE

- 3D linear system of equations with complex arithmetic and $\mathcal{O}(10^7 - 10^8)$
- direct solution with combined multithread and multiprocessor algorithm based on Thomas algorithm and dual Schur complement method
- used to calculate linear evolution of disturbances in 3D nonparallel configurations

Roughness-induced transition in high-speed boundary-layer flows

- Flat-plate with roughness patch in Mach 3.5 flow (ZPG) - 10.5M DOF
 - ▶ Combination of PSE and HLNSE to study linear evolution of Mack's first modes over roughness patch and wake
 - ▶ PSE of streak instabilities along the wake misses $\Delta N \approx 3$ ($\approx 27\%$) and predicts higher frequency peak by $\Delta f \approx 20$ kHz ($\approx 25\%$) around transition onset location
 - ▶ Agreement with experiments confirms significance of disturbance growth upstream of and above finite length roughness patch and the effect on the growth of instabilities in the wake
- Blunt-cone with nosetip roughness elements in Mach 6 flow (FPG) - 8.8M DOF
 - ▶ Inflow-resolvent HLNSE used to capture disturbance amplification near the roughness element including flow separation
 - ▶ predicts $N_{max} = 9.6$ that represents an increase of nearly 71% with respect to PSE, 57% if transient growth upstream is not considered
 - ▶ most amplified frequency predicted at $f = 1.5$ MHz (29% lower than PSE analysis)
 - ▶ Demonstrated significant disturbance amplification over separated region

Thank you for your attention

- Acknowledgments

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- ▶ Resources supporting this work were provided by the the NASA High-End Computing (HEC) Program through the NASA Advanced Supercomputing (NAS) Division at Ames Research Center and the NASA K-Midrange Cluster at Langley Research Center.

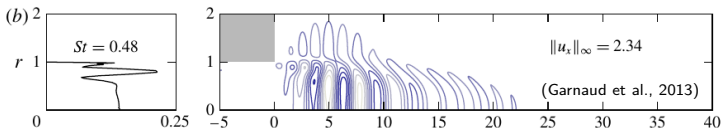


Outline

5 Backup slides

Inflow-Resolvent Analysis based on 3D HLNSE

- Inflow-resolvent analysis commonly used for 2D/axisymmetric configurations^{13,14}
- Optimal inflow disturbance, $\tilde{\mathbf{q}}_0 = \tilde{\mathbf{q}}(\xi_0)$ that maximizes J :
 - ▶ Outlet energy gain: $J = G_E^{out} = \frac{E(\xi_1)}{E(\xi_0)}$
- Energy norm: $E(\xi) = \int_{\eta} \tilde{\mathbf{q}}(\xi, \eta, \zeta)^* \mathbf{M}(\xi, \eta, \zeta) \tilde{\mathbf{q}}(\xi, \eta, \zeta) h_{\xi} h_{\zeta} d\eta$,
 $\mathbf{M} = \text{diag} \left[\frac{\bar{T}}{\gamma M^2 \bar{\rho}}, \bar{\rho}, \bar{\rho}, \bar{\rho}, \frac{\bar{p}(\xi)}{\gamma(\gamma-1)M^2 \bar{T}} \right]$
- Variational formulation using direct and adjoint eqs: $\mathcal{L}(\tilde{\mathbf{q}}, \tilde{\mathbf{q}}^{\dagger}) = J(\tilde{\mathbf{q}}) - \langle \tilde{\mathbf{q}}^{\dagger}, \mathbf{L}\tilde{\mathbf{q}} \rangle$
- Application to incompressible axisymmetric jet

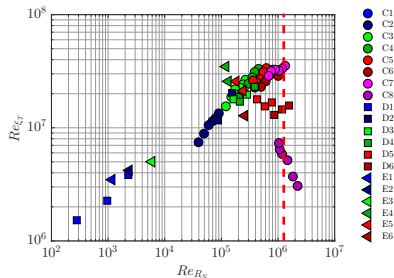
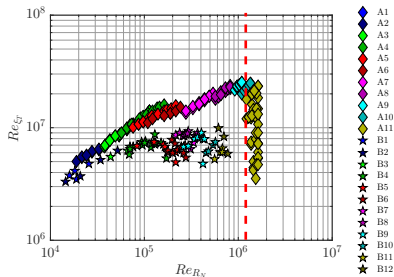


¹³X. Garnaud et al. "The preferred mode of incompressible jets: Linear frequency response analysis". In: *Journal of Fluid Mechanics* 716 (2013), pp. 189–202. DOI: 10.1017/jfm.2012.540.

¹⁴P. Paredes et al. "Nosetip bluntness effects on transition at hypersonic speeds: experimental and numerical analysis". In: *Journal of Spacecraft Rockets* 56.2 (2019). DOI: 10.2514/1.A34277.

Transition Measurements over Blunt Cones¹⁵

- Transition Reynolds number (Re_{ξ_T}) vs. nose Reynolds number (Re_{R_N})
- Mach 6. (A) Stetson (1983), (B) Aleksandrova et al. (2014).
- Mach 9 to 10. (C) Stetson (1983), (D) Marineau et al. (2014), (E) Softley et al. (1969).

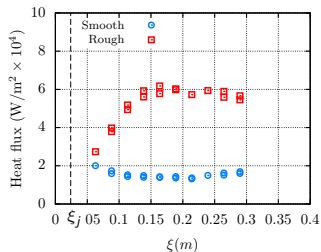
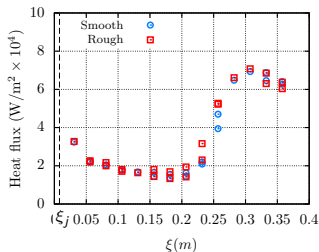


- **Small bluntness:** transition location moves downstream with increasing R_N
- **Large bluntness:** transition location rapidly moves upstream (Transition Reversal)
- Differences in critical Re_{R_N} attributed to uncontrolled nosetip roughness

¹⁵P. Paredes et al. "Nosetip bluntness effects on transition at hypersonic speeds: experimental and numerical analysis". In: *Journal of Spacecraft Rockets* 56.2 (2019). DOI: 10.2514/1.A34277.

Effect of nose bluntness at fixed $Re_{kk} \approx 300$

- AFRL Mach 6 High Reynolds number facility
- 7° half-angle cone at zero AoA; $R_N = 5.080, 15.24$ mm
- Array of $15\ \mu\text{m}$ cubical, laser-machined roughness elements at $\theta = 45^\circ$
- $R_N = 5.08$ mm, $m = 210$, $k/\delta_h = 0.381$
- $R_N = 15.24$ mm, $m = 420$, $k/\delta_h = 0.225$



- **Small bluntness:** Roughness has no effect on ξ_T
- **Large bluntness:** Rough tip moves ξ_T from the end of the cone to $\approx \xi_j$
- **Transition onset associated with combined effects of bluntness and roughness**