

Aeroservoelastic Flight Dynamic Modeling of UAM Conceptual Vehicles

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This paper presents an aeroservoelastic flight dynamic modeling approach for NASA Lift+Cruise eVTOL aircraft in forward flight. A conceptual aerodynamic analysis for forward flight is performed in CHARM, a rotorcraft aeromechanics unsteady vortex-lattice flow solver. A trim analysis is performed to obtain the steady-state aerodynamic information at a trim point to be used for the aeroelastic analysis. A NASTRAN finite-element model for the Lift+Cruise vehicle is available. The structural dynamic mode shapes are extracted from the NASTRAN for the aeroelastic analysis. An aeroservoelastic model is developed using the Theodorsen's theory. A stability and control derivative analysis is performed to formulate a flight dynamic model of the Lift+Cruise vehicle. An aeroservoelastic flight dynamic model is formulated by coupling these two models together. Flight dynamic and flutter analyses are performed to determine vehicle stability and flutter-free operation.

I. Introduction

Interests in urban air mobility (UAM) have greatly surged in recent times as UAM is envisioned to revolutionize the transportation and mobility in the urban setting. The potential promise of UAM has stimulated many companies to design and build a new class of UAM aircraft powered by electric distributed propulsion (DEP) with eVTOL capabilities. NASA Revolutionary Vertical Lift Technology (RVLT) project has developed several UAM conceptual aircraft as reference vehicles.

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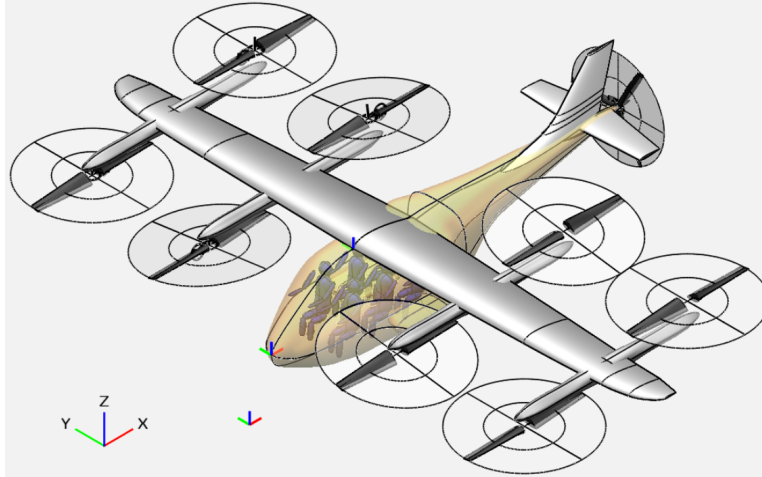


Figure 1. Lift+Cruise Vehicle (Credit: Silva et al.¹)

In this paper, we present an aeroservoelastic flight dynamic model development for the NASA Lift+Cruise reference vehicle. Figure 1 illustrates the Lift+Cruise vehicle,¹ a six-passenger aircraft. It employs 8 vertical lift rotors for VTOL and an aft push fan for propulsion. In forward flight, the Lift+Cruise vehicle operates like a conventional fixed-wing aircraft and the 8 vertical lift rotors are stopped and locked in place with the rotor blade axis aligned in the forward flight direction. The vehicle concept is sized and analyzed in NDARC² for performance. Two versions of the Lift+Cruise vehicle are conceptualized: a turboelectric version and an all-electric version.¹ The electric Lift+Cruise vehicle has a design gross weight of 8,210 lbs and cruises at an average 100 knots. This average airspeed is between the best endurance airspeed of 88 knots and the best range airspeed of 112 knots at 6,000 ft.

II. Conceptual Aerodynamic Analysis

A conceptual aerodynamic analysis of the Lift+Cruise vehicle in forward flight is performed in CHARM,³ a rotorcraft aeromechanics flow solver developed by Continuum Dynamics, Inc. CHARM uses an unsteady vortex lattice method in conjunction with two-dimensional (2D) airfoil look-up tables to determine aerodynamic forces and moments. For aeroelastic analysis, the lift curve slope $c_{l\alpha}$ is an important parameter. A trim analysis is performed to obtain the steady-state aerodynamic information at a trim point to be used for the aeroelastic analysis.

Figure 2 illustrates the Lift+Cruise CHARM model. In this model the fuselage is not present. The loads produced by the fuselage are obtained from a look-up table. The lift rotors and booms are modeled as fixed aerodynamic surfaces as the lift rotors are locked in the streamwise direction in forward flight. The lift rotor blade pitch angles are set at zero to prevent flow reversal. The trim solutions are obtained by iterating on the angle of attack, horizontal tail incidence angle, and push rotor pitch angle to match the Lift+Cruise vehicle gross weight and drag at forward flight speed of 120 knots or 200 ft/sec and altitude of 4,000 ft. The push fan operates with a nominal tip speed of 550 ft/sec. The rotor radius is 4.5 ft which gives a nominal rotor speed of 1,167 rpm. Before the simulations using CHARM, the 2D airfoil look-up tables need to be generated first. The wing, horizontal tail, vertical tail, lift rotor, and push rotor airfoils 2D aerodynamics are shown in Figs. 3-6 which are provided by Advanced Rotorcraft Technology, Inc.⁴

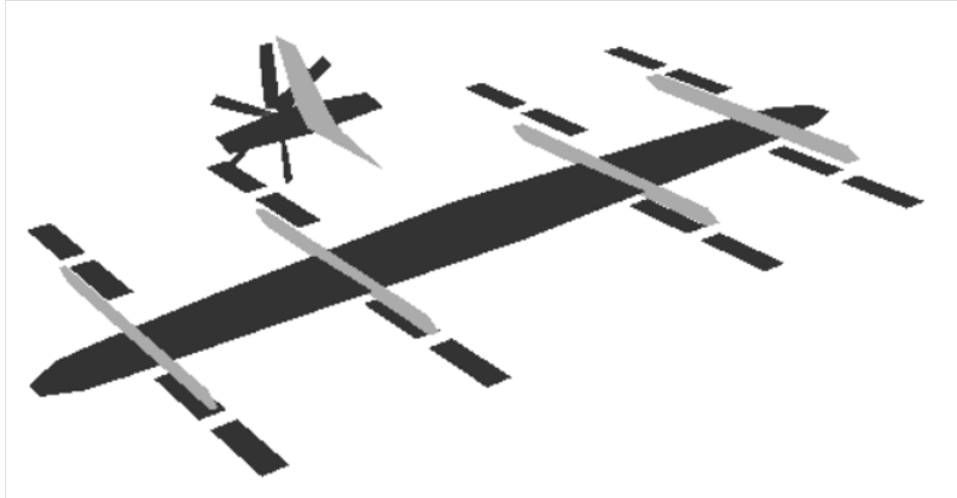


Figure 2. Lift+Cruise CHARM Model

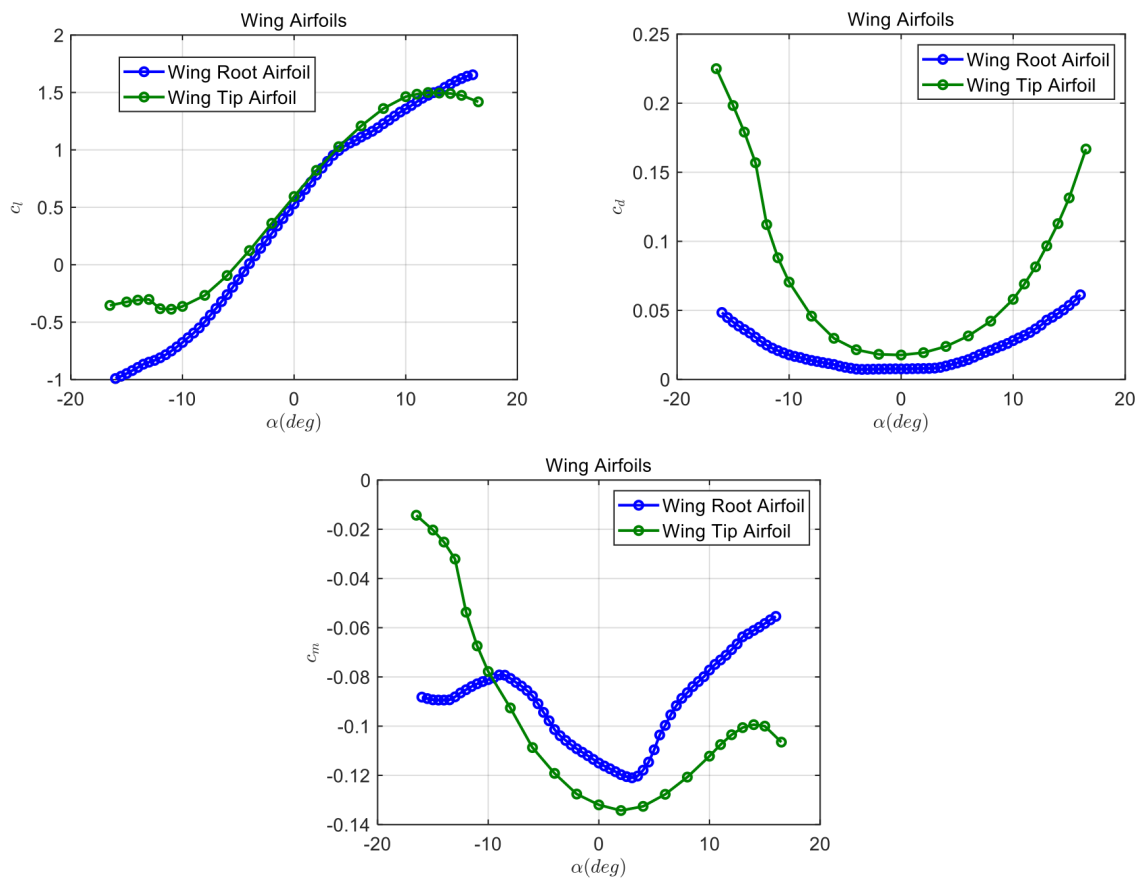


Figure 3. Wing Airfoil Aerodynamics

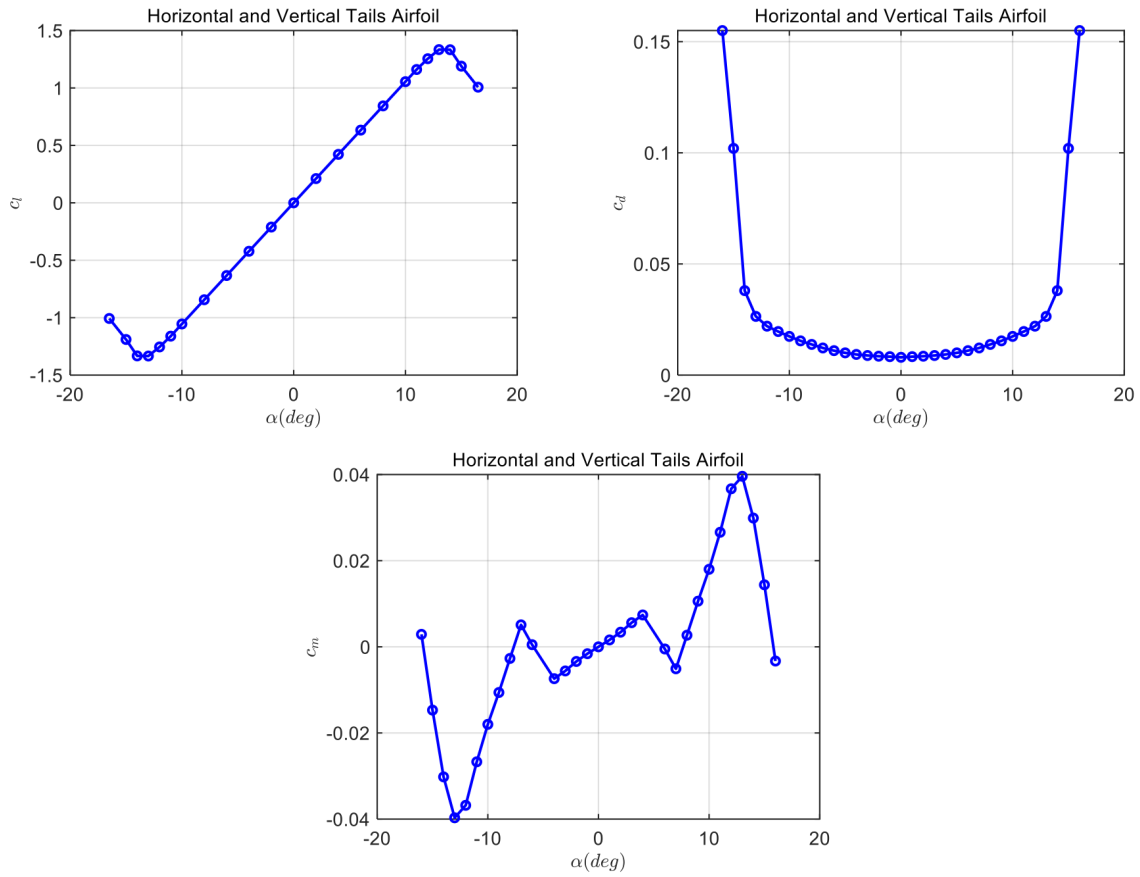


Figure 4. Horizontal and Vertical Airfoil Aerodynamics

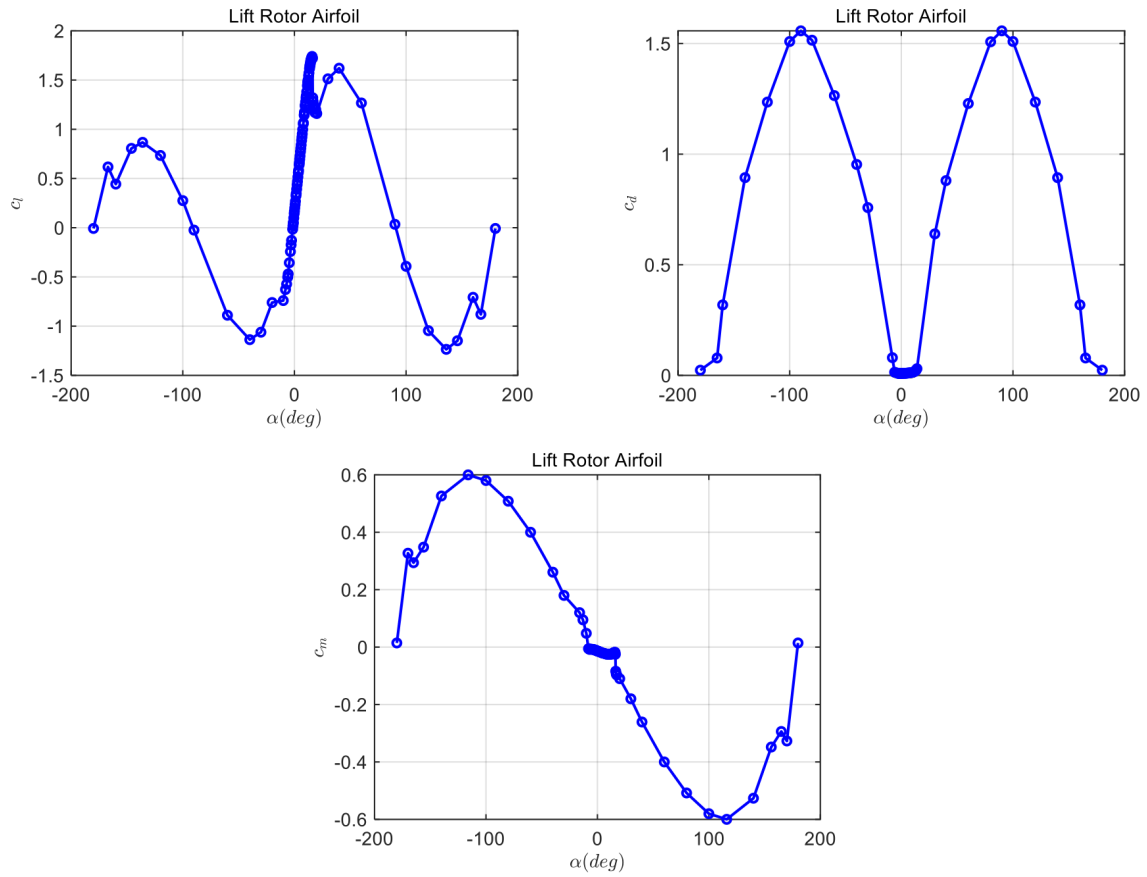


Figure 5. Lift Rotor Airfoil Aerodynamics

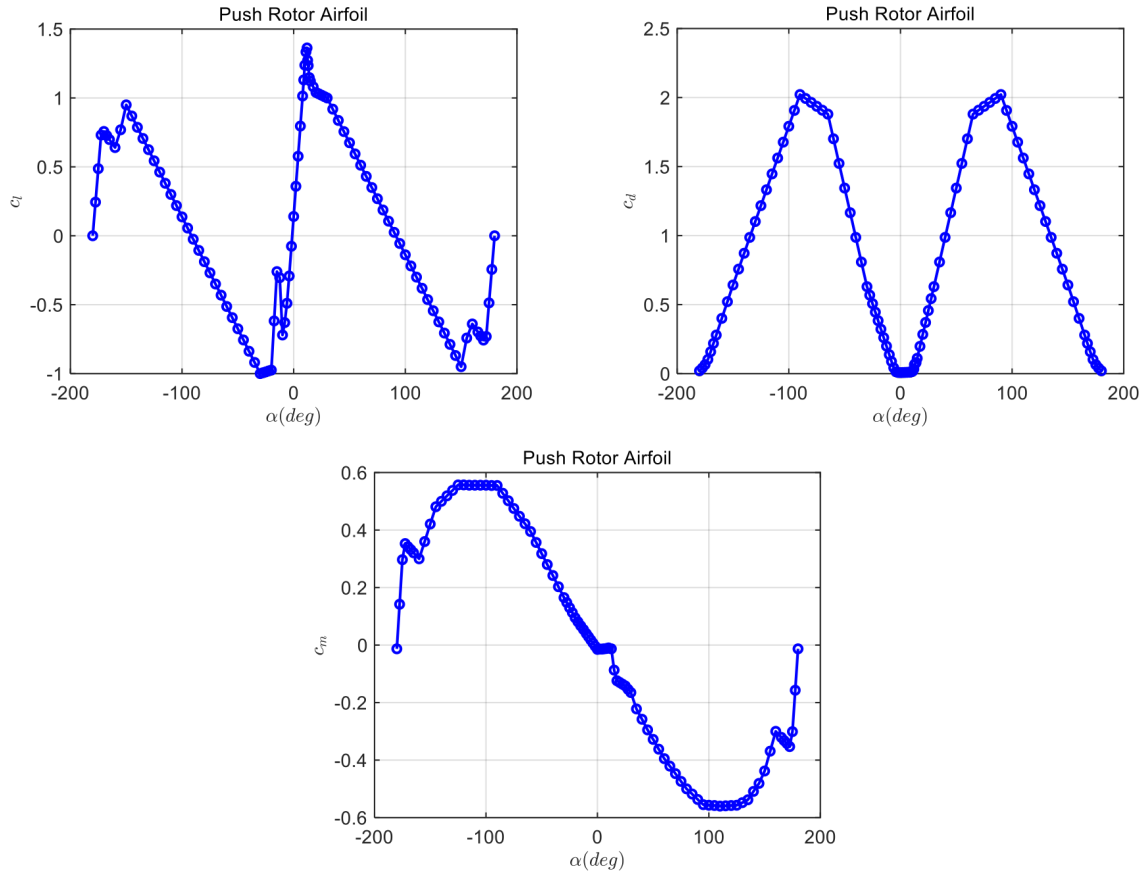


Figure 6. Push Rotor Airfoil Aerodynamics

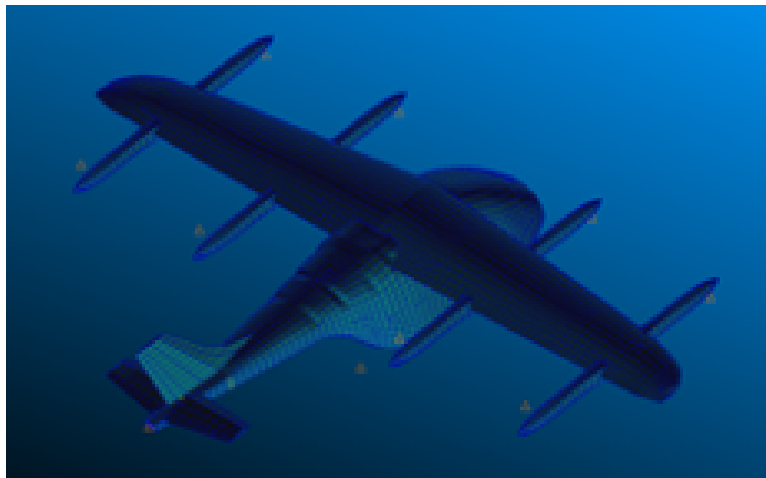


Figure 7. Lift+Cruise NASTRAN Model

To conduct the trim analysis, the gross weight of 8,395 lbs obtained from a NASTRAN model as shown in Fig. 7 is used. An airspeed 200 ft/sec is selected at 4,000 ft. The trim lift coefficient is computed to be $C_L = 0.854$. The push rotor pitch angle is increased to 29.07° in order for the thrust to balance the drag. The trim angle of attack is 4.52° . Figure 8 shows the CHARM vortex trim solution. The plots of the lift coefficient C_L , drag coefficient C_D , and L/D are shown in Fig. 9. The maximum L/D is about 12. The Lift+Cruise vehicle has a wing reference area of 231.486 ft^2

and a wing span of 47.72 ft which give a wing aspect ratio of 9.84. For comparison, the Boeing 757 has a wing aspect ratio of 7.8 and a maximum lift-to-drag ratio of 15.

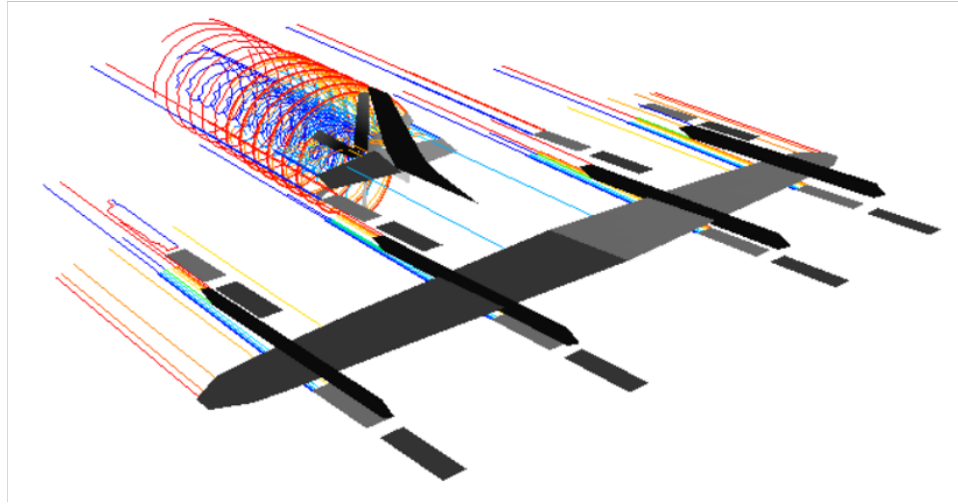


Figure 8. Lift+Cruise NASTRAN Model

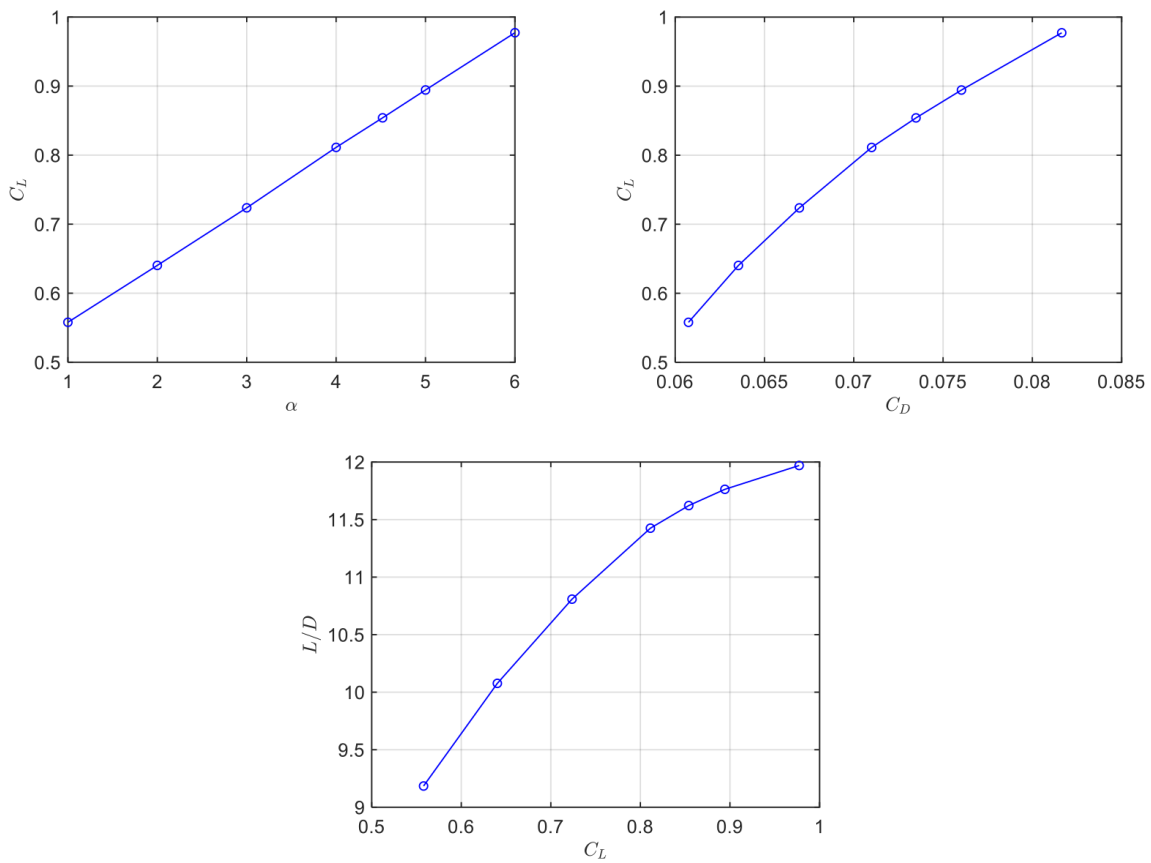


Figure 9. Lift and Drag Characteristics of Lift+Cruise

The wing lift coefficient c_l and drag coefficient c_d from the simulation are shown in Fig. 10. The effect of the

lift rotor-boom assembly can be seen in the wing c_l distribution. The wing lift curve slope c_{l_α} and pitching moment curve slope c_{m_α} about the aerodynamic center are an important quantities for aeroelastic analysis. These quantities are computed from the wing c_l and c_m distributions.

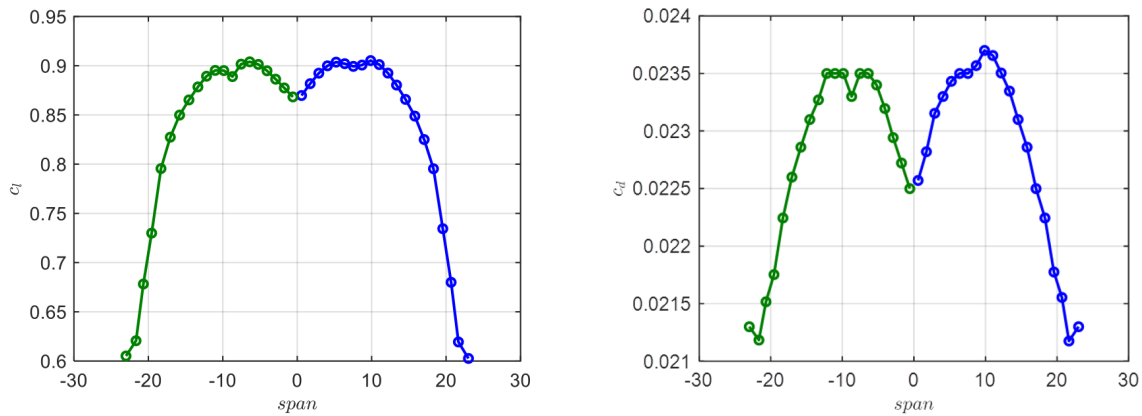


Figure 10. Wing c_l and c_d Distributions

III. Aeroservoelastic Flight Dynamic Model Development

A conceptual structural sizing has been performed for the Lift+Cruise vehicle. A NASTRAN finite-element model (FEM) has been developed.⁵ The frequencies of the structural dynamic modes are shown in Table 1. The mode shapes of the first six structural dynamic modes (modes 1-6 starting at index 7) are shown in Figure 11.

MODE NO.	EIGENVALUE	FREQUENCY RAD/S	FREQUENCY HZ	GENERALIZED MASS	GENERALIZED STIFFNESS
1	0.0	0.0	0.0	1.6795E+02	0.0
2	0.0	0.0	0.0	4.0037E+01	0.0
3	0.0	0.0	0.0	1.9112E+02	0.0
4	0.0	0.0	0.0	6.3491E+01	0.0
5	0.0	0.0	0.0	6.0104E+01	0.0
6	0.0	0.0	0.0	4.0233E+01	0.0
7	7.7460E+02	2.7832E+01	4.4295E+00	1.7886E+01	1.3855E+04
8	9.2943E+02	3.0487E+01	4.8521E+00	1.6978E+01	1.5780E+04
9	1.0684E+03	3.2687E+01	5.2023E+00	1.3628E+01	1.4561E+04
10	2.4543E+03	4.9541E+01	7.8846E+00	1.5860E+01	3.8925E+04
11	2.5168E+03	5.0168E+01	7.9845E+00	1.4342E+01	3.6096E+04
12	2.6978E+03	5.1941E+01	8.2666E+00	2.8445E+01	7.6740E+04
13	2.7814E+03	5.2739E+01	8.3937E+00	1.9590E+01	5.4489E+04
14	3.2202E+03	5.6747E+01	9.0316E+00	2.5244E+01	8.1293E+04
15	3.6364E+03	6.0303E+01	9.5975E+00	1.4101E+01	5.1277E+04
16	3.7013E+03	6.0838E+01	9.6827E+00	1.6076E+01	5.9504E+04
17	4.4168E+03	6.6459E+01	1.0577E+01	1.2656E+01	5.5901E+04
18	4.6238E+03	6.7998E+01	1.0822E+01	1.6832E+01	7.7825E+04
19	4.6501E+03	6.8191E+01	1.0853E+01	3.1833E+01	1.4803E+05
20	5.0233E+03	7.0875E+01	1.1280E+01	1.6266E+01	8.1709E+04

Table 1. Frequencies of Lift+Cruise Vehicle

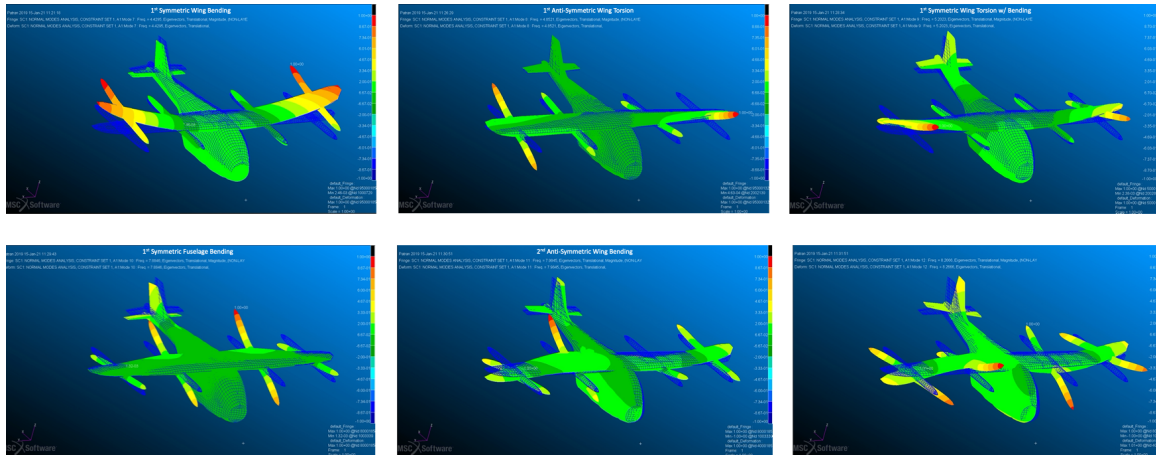


Figure 11. Lift+Cruise Vehicle NASTRAN Mode Shapes

A mode shape extraction is performed to obtain the bending displacement and torsional twist at the elastic axes of the wing, horizontal tail, and vertical tail. The bending and torsional modes shapes for the wing are plotted in Figs. 12 and 13. These mode shapes are used for the aeroservoelastic flight dynamic model development.

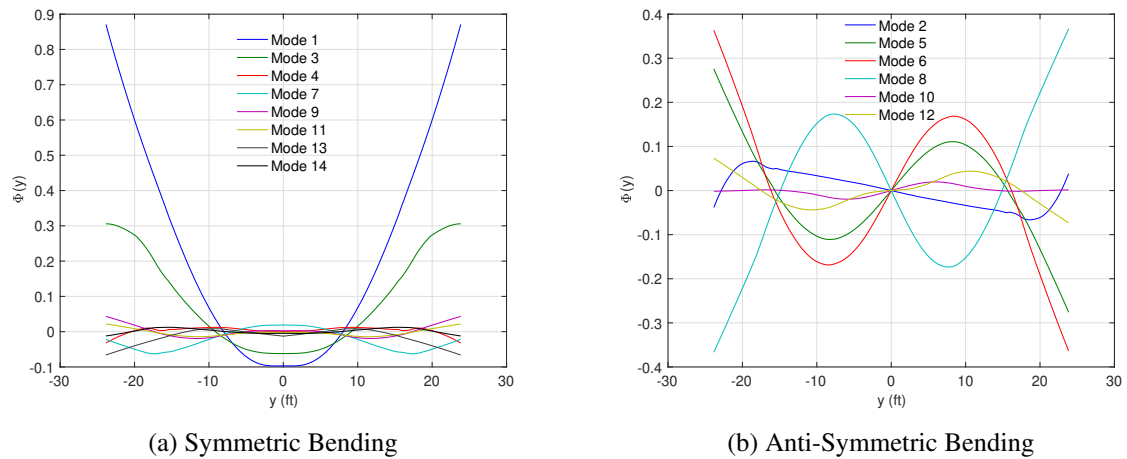


Figure 12. Bending Mode Shapes of Lift+Cruise

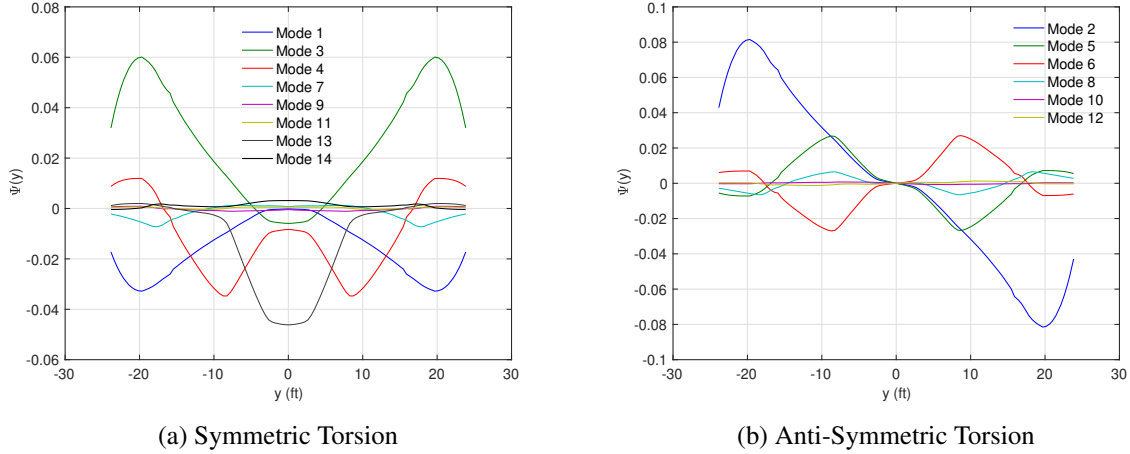


Figure 13. Torsional Mode Shapes of Lift+Cruise

The lift and pitching moment equations for a swept wing in bending and torsion are given by

$$l = c_{l_0} q_\infty c + c_{l_\alpha} q_\infty c \alpha_r + C(k) c_{L_\alpha} q_\infty c \left(\theta \cos \Lambda - \frac{\partial w}{\partial x} \sin \Lambda + \frac{e_c \cos \Lambda}{V_\infty} \frac{\partial \theta}{\partial t} - \frac{1}{V_\infty} \frac{\partial w}{\partial t} \right) + \rho_\infty \frac{\pi c^2}{4} \left(V_\infty \cos \Lambda \frac{\partial \theta}{\partial t} + e_m \cos \Lambda \frac{\partial^2 \theta}{\partial t^2} - \frac{\partial^2 w}{\partial t^2} \right) \quad (1)$$

$$m = c_{m_{ac}} q_\infty c^2 + c_{l_\alpha} q_\infty c e \alpha_r + C(k) c_{L_\alpha} q_\infty c e \left(\theta \cos \Lambda - \frac{\partial w}{\partial x} \sin \Lambda + \frac{e_c \cos \Lambda}{V_\infty} \frac{\partial \theta}{\partial t} - \frac{1}{V_\infty} \frac{\partial w}{\partial t} \right) - \rho_\infty \frac{\pi}{4} c^2 e_m \left(e_m \cos \Lambda \frac{\partial^2 \theta}{\partial t^2} - \frac{\partial^2 w}{\partial t^2} \right) - \rho_\infty \frac{\pi c^2}{4} e_c V_\infty \cos \Lambda \frac{\partial \theta}{\partial t} - \rho_\infty \frac{\pi c^4}{128} \cos \Lambda \frac{\partial^2 \theta}{\partial t^2} \quad (2)$$

where $C(k)$ is the complex-valued Theodorsen's function of the reduced frequency $k = \frac{\omega c}{2V_\infty}$,⁶ c_{l_0} is the sectional lift coefficient at zero angle of attack, c_{l_α} is the sectional lift curve slope, $c_{m_{ac}}$ is the pitching moment coefficient about the quarter chord, α_r is the rigid-body sectional angle of attack, θ is the torsional twist angle about the elastic axis denoted as x as shown in Fig. 14, w is the bending displacement, Λ is the elastic axis sweep angle, c is the chord length, e is the distance from the pitch center to the quarter-chord point which is positive when the pitch center lies aft of the quarter-chord point, e_c is the distance from the pitch center to the three-quarter chord point which is positive when the pitch center lies forward of the three-quarter chord point, and e_m is the distance from the pitch center to the mid-chord point which is positive when the pitch center lies forward of the mid-chord point.

The torsional twist angle and bending displacement can be expressed as

$$\theta(y, t) = \mathbf{\Psi}(y) \mathbf{q}(t) \quad (3)$$

$$w(y, t) = \mathbf{\Phi}(y) \mathbf{q}(t) \quad (4)$$

where $\mathbf{\Psi}(y) = \begin{bmatrix} \Psi_1(y) & \dots & \Psi_n(y) \end{bmatrix}$ is the torsional mode shape vector as a function of the aircraft pitch axis denoted as y which is related to the elastic axis by the transformation $y = x \cos \Lambda$, $\mathbf{\Phi}(y) = \begin{bmatrix} \Phi_1(y) & \dots & \Phi_n(y) \end{bmatrix}$ is the bending mode shape vector, and $\mathbf{q}(t) = \begin{bmatrix} q_1(t) & \dots & q_n(t) \end{bmatrix}^\top$ is the generalized displacement vector as a function of time.

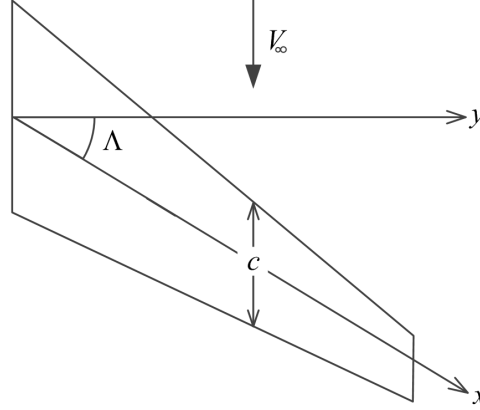


Figure 14. Swept Wing Elastic Axis

Theodorsen shows that the unsteady lift and pitching moment equations can be separated into their circulatory and non-circulatory parts.⁶ The circulatory lift and pitching moment contributions come from the aerodynamic circulation around the airfoil, while the non-circulatory contributions come from apparent mass effects of the volume of air being moved around the airfoil. These contributions are shown here,

$$l_c = C(k) c_{l\alpha} q_\infty c \left(\Psi \dot{\mathbf{q}} \cos \Lambda - \frac{\partial \Phi}{\partial x} \mathbf{q} \sin \Lambda + \frac{e_c}{V_\infty} \Psi \ddot{\mathbf{q}} \cos \Lambda - \frac{1}{V_\infty} \Phi \ddot{\mathbf{q}} \right) \quad (5)$$

$$l_{nc} = \rho_\infty \frac{\pi}{4} c^2 [V_\infty \Psi \dot{\mathbf{q}} \cos \Lambda + e_m \Psi \ddot{\mathbf{q}} \cos \Lambda - \Phi \ddot{\mathbf{q}}] \quad (6)$$

$$m_c = C(k) c_{l\alpha} q_\infty c e l_c \quad (7)$$

$$m_{nc} = -\rho_\infty \frac{\pi}{4} c^2 \left[e_c V_\infty \Psi \dot{\mathbf{q}} \cos \Lambda + \left(e_m^2 + \frac{c^2}{32} \right) \Psi \ddot{\mathbf{q}} \cos \Lambda - e_m \Phi \ddot{\mathbf{q}} \right] \quad (8)$$

The rigid-body aerodynamic contributions to the lift and pitching moment are expressed as

$$l_r(y, t) = \mathbf{l}_{\mathbf{x}_r}(y) \mathbf{x}_r(t) + \mathbf{l}_{\dot{\mathbf{x}}_r}(y) \dot{\mathbf{x}}_r(t) + \mathbf{l}_{\mathbf{u}}(t) \mathbf{u}(t) \quad (9)$$

$$m_r(y, t) = \mathbf{m}_{\mathbf{x}_r}(y) \mathbf{x}_r(t) + \mathbf{m}_{\dot{\mathbf{x}}_r}(y) \dot{\mathbf{x}}_r(t) + \mathbf{m}_{\mathbf{u}}(y) \mathbf{u}(t) \quad (10)$$

where $\mathbf{x}_r(t) = \begin{bmatrix} \mathbf{x}_{lon}^\top(t) & \mathbf{x}_{lat}^\top(t) \end{bmatrix}^\top$ is a vector of aircraft states for longitudinal motion described by $\mathbf{x}_{lon}(t) = \begin{bmatrix} h(t) & u(t) & \alpha(t) & \theta(t) & q(t) \end{bmatrix}^\top$ and lateral-directional motion described by $\mathbf{x}_{lat}(t) = \begin{bmatrix} \phi(t) & p(t) & \beta(t) & r(t) \end{bmatrix}^\top$, $\mathbf{u}(t) = \begin{bmatrix} \mathbf{u}_{lon}^\top(t) & \mathbf{u}_{lat}^\top(t) \end{bmatrix}^\top$ is a vector of control variables for longitudinal motion described by $\mathbf{u}_{lon}(t) = \begin{bmatrix} \delta_e(t) & \theta_p(t) \end{bmatrix}^\top$ and lateral-directional motion described by $\mathbf{u}_{lat}(t) = \begin{bmatrix} \delta_a(t) & \delta_r(t) \end{bmatrix}^\top$. The total lift and pitching moment are the sums of the circulatory and non-circulatory contributions as well as the rigid-body contribution.

The distributed aerodynamic forces and moments acting on an aircraft at the center of gravity (CG) can be expressed generally as

$$\begin{aligned} \mathbf{f}(y, t) = & \mathbf{f}_{\mathbf{x}_r}(y) \mathbf{x}_r(t) + \mathbf{f}_{\dot{\mathbf{x}}_r}(y) \dot{\mathbf{x}}_r(t) + \mathbf{f}_{\mathbf{u}}(t) \mathbf{u}(t) \\ & + \mathbf{f}_{l_c}(y) l_c(y, t) + \mathbf{f}_{l_{nc}}(y) l_{nc}(y, t) + \mathbf{f}_{m_c}(y) m_c(y, t) + \mathbf{f}_{m_{nc}}(y) m_{nc}(y, t) \end{aligned} \quad (11)$$

where $\mathbf{f}(y, t) = \begin{bmatrix} \mathbf{f}_{lon}(y, t) & \mathbf{f}_{lat}(y, t) \end{bmatrix}^\top$ is a vector of aircraft distributed forces and moments for longitudinal motion described by $\mathbf{f}_{lon}(y, t) = \begin{bmatrix} \frac{dL(y, t)}{dy} & \frac{dD(y, t)}{dy} & \frac{dm(y, t)}{dy} \end{bmatrix}$ and lateral-directional motion described by $\mathbf{f}_{lat}(t) =$

$\left[\frac{dY(y,t)}{dy} \quad \frac{dl(y,t)}{dy} \quad \frac{dn(y,t)}{dy} \right]$. Note that the generalized displacement $\mathbf{q}(t)$, velocity $\dot{\mathbf{q}}(t)$, and acceleration $\ddot{\mathbf{q}}(t)$ of the structural dynamic modes also contribute to the aircraft forces and moments.

The aeroservoelastic equation is formulated as

$$(\mathbf{M}_s + \mathbf{M}_a)\ddot{\mathbf{q}} + (\mathbf{C}_s + \mathbf{C}_a)\dot{\mathbf{q}} + (\mathbf{K}_s + \mathbf{K}_a)\mathbf{q} = \mathbf{Q}_{\mathbf{x}_r}\mathbf{x}_r + \mathbf{Q}_{\dot{\mathbf{x}}_r}\dot{\mathbf{x}}_r + \mathbf{Q}_{\mathbf{u}}\mathbf{u} \quad (12)$$

where $\mathbf{M}_s$, $\mathbf{C}_s$, and $\mathbf{K}_s$ are the diagonal structural mass, damping, and stiffness matrices; $\mathbf{M}_a$, $\mathbf{C}_a$, and $\mathbf{K}_a$ are the aerodynamic mass, damping, and stiffness matrices; and $\mathbf{Q}_{\mathbf{x}_r}$ and $\mathbf{Q}_{\dot{\mathbf{x}}_r}$ are the generalized aerodynamic force vectors due to the aircraft state vector $\mathbf{x}_r$ and its time derivative $\dot{\mathbf{x}}_r$ due to the apparent mass effect. These generalized force vectors are obtained by integrating the rigid-body lift l_r and pitching moment m_r .

$$\mathbf{Q}_{\mathbf{x}_r}\mathbf{x}_r + \mathbf{Q}_{\dot{\mathbf{x}}_r}\dot{\mathbf{x}}_r + \mathbf{Q}_{\mathbf{u}}\mathbf{u} = \int_{-\frac{b}{2}}^{\frac{b}{2}} \left[\Phi^\top(y) l_r(y,t) + \Psi^\top(y) m_r(y,t) \right] dy \quad (13)$$

The aerodynamic matrices are contributed by the circulatory lift and non-circulatory lift as

$$\mathbf{M}_a = \mathbf{M}_{nc} \quad (14)$$

$$\mathbf{C}_a = C(\bar{s}) \mathbf{C}_c + \mathbf{C}_{nc} \quad (15)$$

$$\mathbf{K}_a = C(\bar{s}) \mathbf{K}_c \quad (16)$$

where $\bar{s} = \frac{s\bar{c}}{2V_\infty}$ is the non-dimensional Laplace variables and the matrices $\mathbf{M}_{nc}$, $\mathbf{C}_{nc}$, $\mathbf{C}_c$, and $\mathbf{K}_c$ are evaluated as

$$\mathbf{M}_{nc} = \int_{-\frac{b}{2}}^{\frac{b}{2}} \left[\Phi^\top(y) \frac{\partial l_{nc}(y,t)}{\partial \ddot{\mathbf{q}}} + \Psi^\top(y) \frac{\partial m_{nc}(y,t)}{\partial \ddot{\mathbf{q}}} \right] dy \quad (17)$$

$$\mathbf{C}_{nc} = \int_{-\frac{b}{2}}^{\frac{b}{2}} \left[\Phi^\top(y) \frac{\partial l_{nc}(y,t)}{\partial \dot{\mathbf{q}}} + \Psi^\top(y) \frac{\partial m_{nc}(y,t)}{\partial \dot{\mathbf{q}}} \right] dy \quad (18)$$

$$\mathbf{C}_c = \int_{-\frac{b}{2}}^{\frac{b}{2}} \left[\Phi^\top(y) \frac{\partial l_c(y,t)}{\partial \dot{\mathbf{q}}} + \Psi^\top(y) \frac{\partial m_c(y,t)}{\partial \dot{\mathbf{q}}} \right] dy \quad (19)$$

$$\mathbf{K}_c = \int_{-\frac{b}{2}}^{\frac{b}{2}} \left[\Phi^\top(y) \frac{\partial l_c(y,t)}{\partial \mathbf{q}} + \Psi^\top(y) \frac{\partial m_c(y,t)}{\partial \mathbf{q}} \right] dy \quad (20)$$

The flight dynamic equation is formulated as

$$\mathbf{M}_r \dot{\mathbf{x}}_r = \mathbf{F} \quad (21)$$

where $\mathbf{F}(\bar{s}, t)$ is a vector of the aircraft forces and moments at the CG evaluated as

$$\mathbf{F} = \mathbf{F}_{\mathbf{x}_r}\mathbf{x}_r + \mathbf{F}_{\dot{\mathbf{x}}_r}\dot{\mathbf{x}}_r + \mathbf{F}_{\mathbf{u}}\mathbf{u} + \mathbf{F}_{\mathbf{q}}\mathbf{q} + \mathbf{F}_{\dot{\mathbf{q}}}\dot{\mathbf{q}} + \mathbf{F}_{\ddot{\mathbf{q}}}\ddot{\mathbf{q}} \quad (22)$$

The matrix $\mathbf{F}_{\mathbf{x}_r}$ is the stability matrix containing the aircraft stability derivatives. The matrix $\mathbf{F}_{\dot{\mathbf{x}}_r}$ is the apparent mass contribution. The matrix $\mathbf{F}_{\mathbf{u}}$ is the control matrix containing the aircraft control derivatives. The stability and control derivatives for the Lift+Cruise are computed by an analytical method from the aerodynamic perturbations about the trim solution computed by CHARM. Table 2 lists the stability derivatives of the Lift+Cruise in forward flight at 120 knots at 4,000 ft. Table 3 lists the apparent mass stability derivatives. Table 4 lists the control derivatives.

	alpha	u	beta	p	q	r
CL	4.8258	0.02878	-0.0020478	0.011495	10.506	-0.00023467
CD	0.28089	0.0039811	0.0020086	0.00038365	0.2514	-1.6089e-06
CY	0.00057942	0.00012533	-0.44849	-0.082138	0	0.36299
CL	0.0057476	5.6609e-06	-0.041069	-0.55005	-0.0083175	0.13667
Cm	-1.8545	-0.0049114	0.00034947	-0.0011433	-15.654	-5.3769e-05
Cn	-0.00046568	-8.7216e-05	0.16259	0.016793	0.00086014	-0.13551

Table 2. Lift+Cruise Stability Derivatives in Forward Flight at 120 Knots and 4,000 ft

	<u>alphadot</u>	<u>udot</u>	<u>betadot</u>	<u>pdot</u>	<u>qdot</u>	<u>rdot</u>
CL	2.3071	0	0	0.00051154	2.0178	1.3795e-05
CD	0.15537	0	0	-6.2884e-07	0.2514	6.3237e-07
CY	0	0	-0.018909	-0.003158	0	0.015129
CL	0.0057476	0	-0.001579	-0.025111	-0.0013789	-0.0035028
Cm	2.3071	0	0	-3.2815e-05	-3.72	8.8899e-07
Cn	-0.00019174	0	0.0066749	0.00062203	-0.0001128	-0.0052063

Table 3. Lift+Cruise Apparent Mass Stability Derivatives in Forward Flight at 120 Knots and 4,000 ft

	<u>push_rotor_pitch</u>	<u>aileron</u>	<u>elevator</u>	<u>rudder</u>
CL	-0.008774	-0.008774	-0.00065606	0.2931
CD	0.002185	0.002185	0.03584	0.004629
CY	-0.006358	-0.006358	0	0
CL	0.03217	0.03217	0.36888	5.0061e-05
Cm	-0.1731	-0.1731	0.0034176	-1.0852
Cn	0.002335	0.002335	-0.029111	-4.4025e-06

Table 4. Lift+Cruise Control Derivatives in Forward Flight at 120 Knots and 4,000 ft

The contributions of the generalized displacement $\mathbf{q}(t)$, velocity $\dot{\mathbf{q}}(t)$, and acceleration $\ddot{\mathbf{q}}(t)$ to the aircraft forces and moments are obtained by integrating Eq. (11) which results in

$$\mathbf{F}_{\ddot{\mathbf{q}}} = \mathbf{F}_{nc,\ddot{\mathbf{q}}} \quad (23)$$

$$\mathbf{F}_{\dot{\mathbf{q}}} = C(\bar{s}) \mathbf{F}_{c,\dot{\mathbf{q}}} + \mathbf{F}_{nc,\dot{\mathbf{q}}} \quad (24)$$

$$\mathbf{F}_{\mathbf{q}} = C(\bar{s}) \mathbf{F}_{c,\mathbf{q}} \quad (25)$$

$$\mathbf{F}_{nc,\ddot{\mathbf{q}}} = \int_{-\frac{b}{2}}^{\frac{b}{2}} \left[\mathbf{f}_{lnc}(y) \frac{\partial l_{nc}(y,t)}{\partial \ddot{\mathbf{q}}} + \mathbf{f}_{mnc}(y) \frac{\partial m_{nc}(y,t)}{\partial \ddot{\mathbf{q}}} \right] dy \quad (26)$$

$$\mathbf{F}_{nc,\dot{\mathbf{q}}} = \int_{-\frac{b}{2}}^{\frac{b}{2}} \left[\mathbf{f}_{lnc}(y) \frac{\partial l_{nc}(y,t)}{\partial \dot{\mathbf{q}}} + \mathbf{f}_{mnc}(y) \frac{\partial m_{nc}(y,t)}{\partial \dot{\mathbf{q}}} \right] dy \quad (27)$$

$$\mathbf{F}_{c,\dot{\mathbf{q}}} = \int_{-\frac{b}{2}}^{\frac{b}{2}} \left[\mathbf{f}_{lc}(y) \frac{\partial l_c(y,t)}{\partial \dot{\mathbf{q}}} + \mathbf{f}_{mc}(y) \frac{\partial m_c(y,t)}{\partial \dot{\mathbf{q}}} \right] dy \quad (28)$$

$$\mathbf{F}_{c,\mathbf{q}} = \int_{-\frac{b}{2}}^{\frac{b}{2}} \left[\mathbf{f}_{lc}(y) \frac{\partial l_c(y,t)}{\partial \mathbf{q}} + \mathbf{f}_{mc}(y) \frac{\partial m_c(y,t)}{\partial \mathbf{q}} \right] dy \quad (29)$$

Thus, the aeroservoelastic equation and the aircraft flight dynamic equation are coupled together to form an aeroservoelastic system which is described by the following equation in the frequency domain:

$$\begin{bmatrix} (\mathbf{M}_s + \mathbf{M}_a) s^2 + [\mathbf{C}_s + \mathbf{C}_a(\bar{s})] s + \mathbf{K}_s + \mathbf{K}_a(\bar{s}) & -(\mathbf{Q}_{\dot{\mathbf{x}}_r} s + \mathbf{Q}_{\mathbf{x}_r}) \\ -[\mathbf{F}_{\ddot{\mathbf{q}}} s^2 + \mathbf{F}_{\dot{\mathbf{q}}}(\bar{s}) s + \mathbf{F}_{\mathbf{q}}(\bar{s})] & (\mathbf{M}_r - \mathbf{F}_{\dot{\mathbf{x}}_r}) s - \mathbf{F}_{\mathbf{x}_r} \end{bmatrix} \begin{bmatrix} \mathbf{q} \\ \mathbf{x}_r \end{bmatrix} = \begin{bmatrix} \mathbf{Q}_u \\ \mathbf{F}_u \end{bmatrix} \mathbf{u} \quad (30)$$

This equation is a function of the Theodorsen's function $C(\bar{s})$ which does not have a polynomial form. To convert the equation into a state-space form, the Theodorsen's function must be approximated by a rational fraction expression. A simple approximation has been established by R. T. Jones. It is given by

$$C(\bar{s}) \approx \hat{C}(\bar{s}) = \frac{0.5\bar{s}^2 + a_1\bar{s} + a_2}{\bar{s}^2 + a_3\bar{s} + a_2} \quad (31)$$

where $\bar{s} = \frac{s\bar{c}}{2V_\infty}$, $a_1 = 0.2808$, $a_2 = 0.01365$, and $a_3 = 0.3455$. The Theodorsen's function is only associated with the generalized displacement $\mathbf{q}(t)$ and velocity $\dot{\mathbf{q}}(t)$ due to the circulatory lift. We can therefore approximate these terms as

$$\hat{C}(\bar{s})\mathbf{q} = \frac{0.5\bar{s}^2 + a_1\bar{s} + a_2}{\bar{s}^2 + a_3\bar{s} + a_2}\mathbf{q} = 0.5\mathbf{q} + \frac{a_4\bar{s} - 0.5a_2}{\bar{s}^2 + a_3\bar{s} + a_2}\mathbf{q} \quad (32)$$

$$\hat{C}(\bar{s})\dot{\mathbf{q}} = \frac{0.5\bar{s}^2 + a_1\bar{s} + a_2}{\bar{s}^2 + a_3\bar{s} + a_2} \left(\frac{2V_\infty}{c} \right) \bar{s}\mathbf{q} = 0.5\dot{\mathbf{q}} + a_4 \left(\frac{2V_\infty}{c} \right) \mathbf{q} + \frac{a_5\bar{s} + a_6}{\bar{s}^2 + a_3\bar{s} + a_2} \left(\frac{2V_\infty}{c} \right) \mathbf{q} \quad (33)$$

where $a_4 = a_1 - 0.5a_3 = 0.10805$, $a_5 = 0.5a_2 + 0.5a_3^2 - a_1a_3 = -0.030506275$, and $a_6 = 0.5a_2a_3 - a_1a_2 = -0.0014748825$.

We introduce the inflow state vectors $\mathbf{y}(t)$ and $\mathbf{z}(t)$ as

$$\mathbf{y} = \frac{a_4\bar{s} - 0.5a_2}{\bar{s}^2 + a_3\bar{s} + a_2}\mathbf{q} \quad (34)$$

$$\mathbf{z} = \frac{a_5\bar{s} + a_6}{\bar{s}^2 + a_3\bar{s} + a_2}\mathbf{q} \quad (35)$$

These inflow states can be expressed in the time domain using the following relationships:

$$\dot{\mathbf{y}} + a_3 \left(\frac{2V_\infty}{c} \right) \mathbf{y} + a_2 \left(\frac{2V_\infty}{c} \right)^2 \mathbf{y} = a_4 \left(\frac{2V_\infty}{c} \right) \dot{\mathbf{q}} + 0.5a_2 \left(\frac{2V_\infty}{c} \right)^2 \mathbf{q} \quad (36)$$

$$\dot{\mathbf{z}} + a_3 \left(\frac{2V_\infty}{c} \right) \mathbf{z} + a_2 \left(\frac{2V_\infty}{c} \right)^2 \mathbf{z} = a_5 \left(\frac{2V_\infty}{c} \right) \dot{\mathbf{q}} + a_6 \left(\frac{2V_\infty}{c} \right)^2 \mathbf{q} \quad (37)$$

Thus, the time domain expressions of the aeroservoelastic flight dynamic equations are approximated by

$$(\mathbf{M}_s + \mathbf{M}_a)\ddot{\mathbf{q}} + (\mathbf{C}_s + 0.5\mathbf{C}_c + \mathbf{C}_{nc})\dot{\mathbf{q}} + \left[\mathbf{K}_s + 0.5\mathbf{K}_c + a_4 \left(\frac{2V_\infty}{c} \right) \mathbf{C}_c \right] \mathbf{q} + \mathbf{K}_c\mathbf{y} + \left(\frac{2V_\infty}{c} \right) \mathbf{C}_c\mathbf{z} = \mathbf{Q}_{x_r}\mathbf{x}_r + \mathbf{Q}_{\dot{x}_r}\dot{\mathbf{x}}_r + \mathbf{Q}_u\mathbf{u} \quad (38)$$

$$\mathbf{M}_r\dot{\mathbf{x}}_r = \mathbf{F}_{x_r}\mathbf{x}_r + \mathbf{F}_{\dot{x}_r}\dot{\mathbf{x}}_r + \mathbf{F}_u\mathbf{u} + \left[0.5\mathbf{F}_{c,\mathbf{q}} + a_4 \left(\frac{2V_\infty}{c} \right) \mathbf{F}_{c,\dot{\mathbf{q}}} \right] \mathbf{q} + (0.5\mathbf{F}_{c,\dot{\mathbf{q}}} + \mathbf{F}_{nc,\dot{\mathbf{q}}})\dot{\mathbf{q}} + \mathbf{F}_{\ddot{\mathbf{q}}}\ddot{\mathbf{q}} + \mathbf{F}_{c,\mathbf{q}}\mathbf{y} + \left(\frac{2V_\infty}{c} \right) \mathbf{F}_{c,\mathbf{q}}\mathbf{z} \quad (39)$$

IV. Flight Dynamic Stability and Flutter Analyses

A flight dynamic stability analysis is conducted by evaluating the eigenvalues of the aircraft modes without aeroelastic coupling. Table 5 lists the eigenvalues of the Lift+Cruise vehicle. The plunge mode due to altitude is slightly divergent but this is usually controlled in flight by the altitude-hold mode. The spiral mode is unstable. This needs to be further investigated.

	Eigenvalues	Frequency (rad/sec)	Damping
Short-Period	$-0.8327 \pm i2.8350$	2.9548	0.2818
Phugoid	$-0.1581 \pm i0.1571$	0.2228	0.7094
Plunge	0.000043749	0.000043749	-1
Dutch-Roll	$-0.2283 \pm i1.7682$	1.7829	0.1281
Roll	-1.9335	1.9335	1
Spiral	0.02884	0.02884	-1

Table 5. Eigenvalues of Lift+Cruise

Figure 15 is the root locus plot of the aeroservoelastic eigenvalues and the flight dynamic eigenvalues. The aeroelastic interaction causes an increase in the damping of the short-period mode but at the same time a decrease in the damping of the Dutch-roll mode. The eigenvalue of the roll mode is also increased. The aeroelastic effect on the phugoid mode, plunge mode, and spiral mode is not discernible.

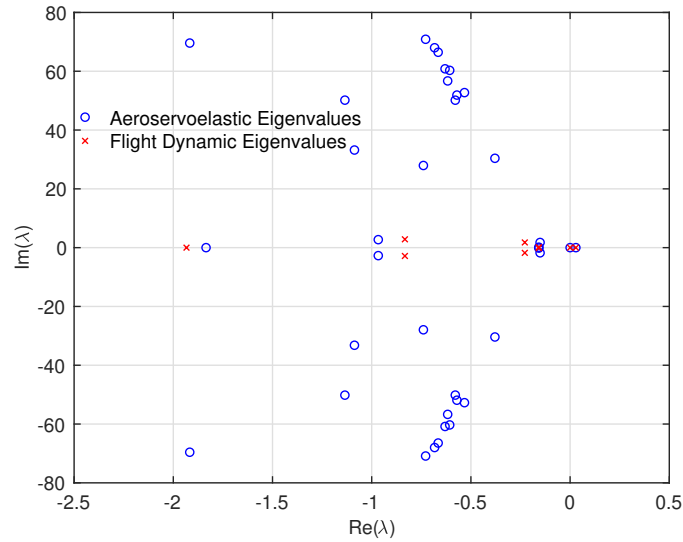
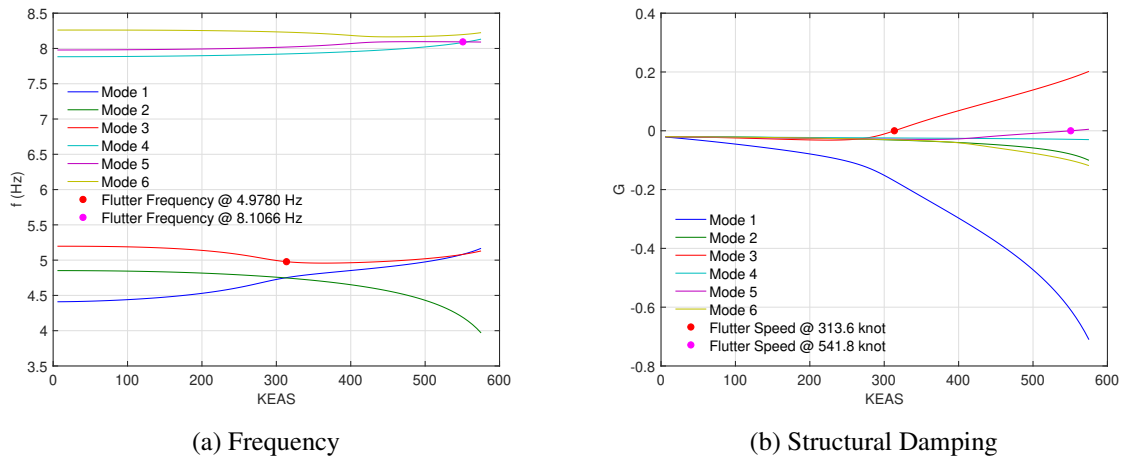


Figure 15. Aeroservoelastic and Flight Dynamic Eigenvalues of Lift+Cruise

A flutter analysis is conducted to determine the flutter speed. A 2% structural damping is assumed. The flutter speed search is swept over a range of airspeed from 5 knots to 575 knots. The first flutter speed occurs at an equivalent airspeed of 313.6 knots corresponding to mode 3 at 4.9780 Hz. The second flutter speed occurs at an equivalent airspeed of 541.8 knots corresponding to mode 5 at 8.1066 Hz. Both flutter speeds are well above the maximum forward flight airspeed of 120 knots, thereby ensuring flutter-free operation of the Lift+Cruise vehicle. Figures 16(a)-(b) are the plots of frequency and structural damping versus the equivalent airspeed.



(a) Frequency

(b) Structural Damping

Figure 16. Flutter V-G Diagram

V. Conclusion

This paper presents a development of an aeroservoelastic flight dynamic model of NASA Lift+Cruise eVTOL aircraft in forward flight. An aerodynamic trim analysis is performed using CHARM, a rotorcraft aeromechanics flow solver. The aeroelastic model is developed using Theodorsen's theory due to the low airspeed. NASTRAN mode shapes are extracted for use in the aeroelastic analysis. The aircraft stability and control derivatives are computed analytically using the aerodynamic information obtained from CHARM. These derivatives are used to develop a flight dynamic model. Both the aeroservoelastic and flight dynamic models are coupled together to capture the effects of structural dynamic interactions with the aircraft flight dynamics and control. The time-domain state-space formulation is obtained by the R. T. Jones approximation of the Theodorsen's function. A rigid-aircraft flight dynamic stability analysis is performed. The analysis indicates stable short-period, phugoid, Dutch-roll, and roll modes but unstable plunge and spiral modes. The aeroservoelastic eigenvalue analysis shows changes to the eigenvalues of the short-period mode, Dutch-roll, and roll modes due to aeroelastic interaction with the aircraft rigid-body modes. Future work will examine the aeroservoelastic flight control responses of the Lift+Cruise vehicle.

Acknowledgment

The authors wish to acknowledge the Revolutionary Vertical Lift Technology project for funding support of this work. The authors also would like to acknowledge Mr. Daniel Wachspress of Continuum Dynamics, Inc. for his support of CHARM.

References

- ¹Silva, C., Johnson, W., Antcliff, K., and Patterson, M., "VTOL Urban Air Mobility Concept Vehicles for Technology Development," AIAA Aviation Technology, Integration, and Operations Conference, AIAA 2018-3847, June 2018.
- ²Johnson, W. R., "NDARC NASA Design and Analysis of Rotorcraft," NASA/TP-2015-218751, NASA, Moffett Field, CA, 2015.
- ³Wachspress, D. A., Quackenbush, T. R., Boscchitsch, AA. H., Yu, M. K. and Whitehouse, G. R., "CHARM User's Manual Version 7.0," Continuum Dynamics, Inc., April 2022.
- ⁴Advanced Rotorcraft Technology, Inc., "Flight Control Models for a Concept Vehicle," Final Report NASA SBIR Phase III 80NSSC19C0003, March 2019.
- ⁵Winter, T., Robinson, J., Gamez, A., and Nascenzi, T., "Conceptual Design Structural Sizing for Urban Air Mobility," AIAA SciTech Forum, AIAA-2021-1722, January 2021.
- ⁶Theodorsen, T., "General Theory of Aerodynamic Instability and the mechanism of Flutter", NACA Report No. 496, 1949.
- ⁷Nguyen, N., Ting, E., and Chaparro, D., "Development of an Integrated Nonlinear Aeroservoelastic Flight Dynamic Model of the NASA Generic Transport Model," AIAA/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference, AIAA-2018-2210, January 2018.