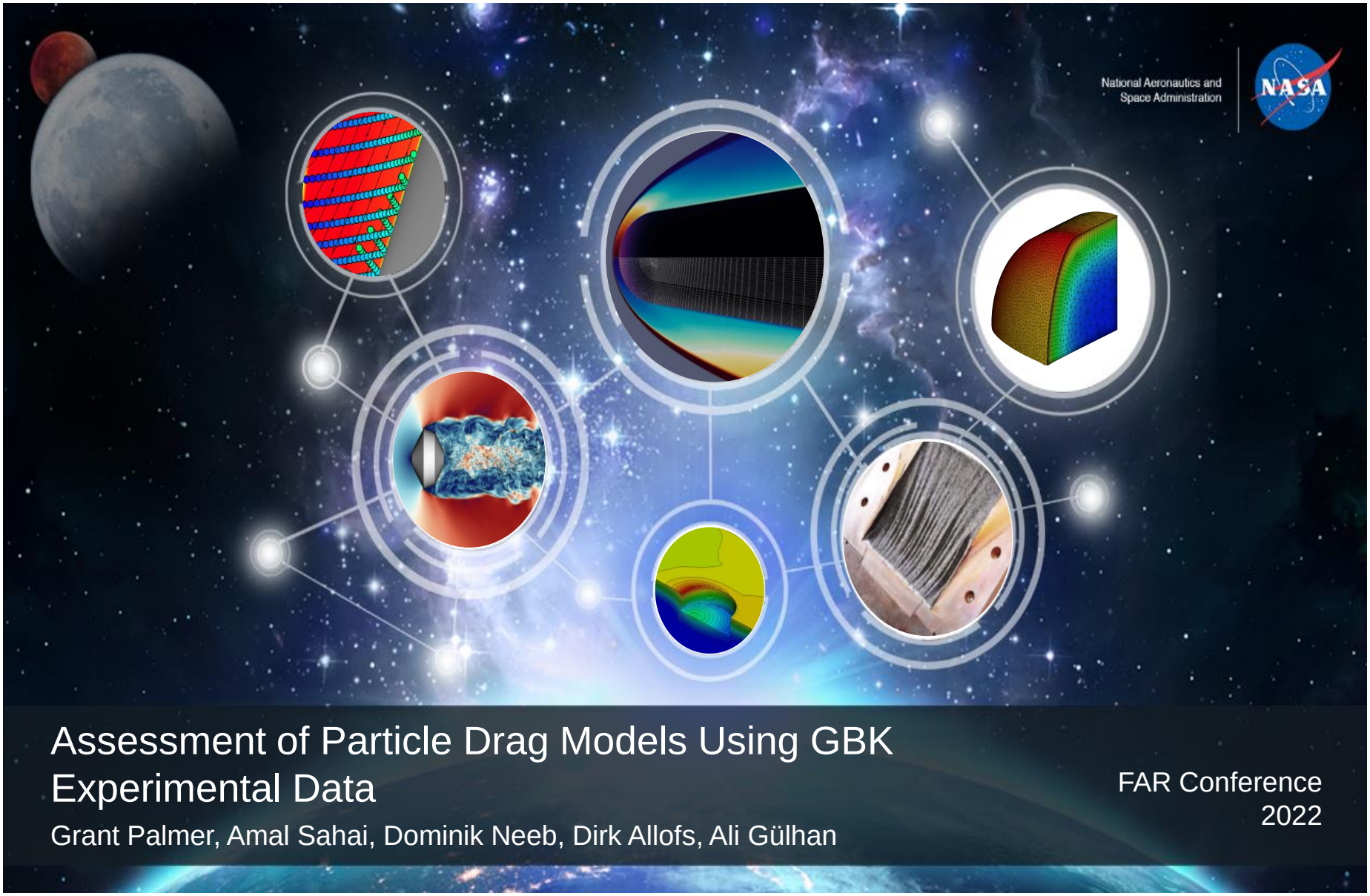


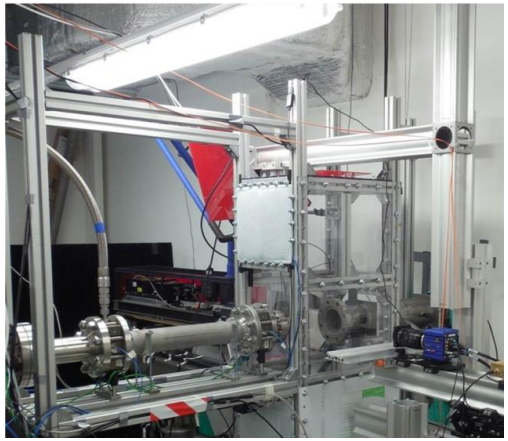
Assessment of Particle Drag Models Using GBK Experimental Data

Grant Palmer, Amal Sahai, Dominik Neeb, Dirk Allofs, Ali Gülhan

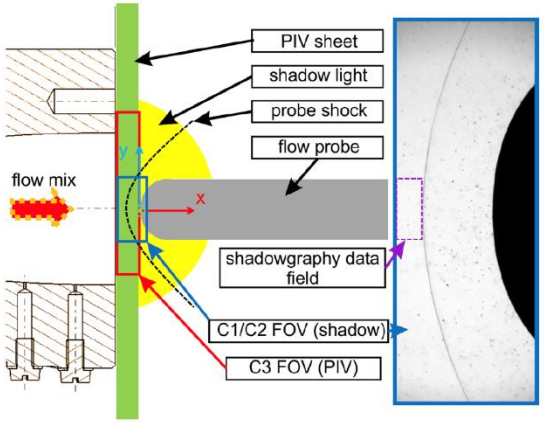
FAR Conference
2022

Objective

- The objective of this study is to use recent dusty-flow experiments in the German Aerospace Center (DLR) GBK facility to verify, validate, and gain insight into particle drag models.

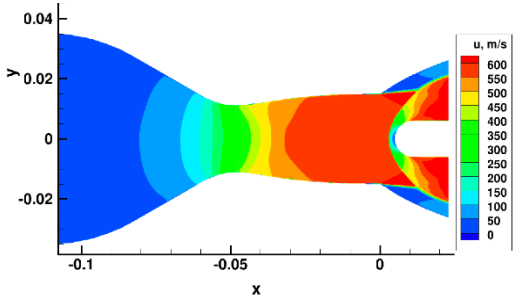


DLR GBK facility.



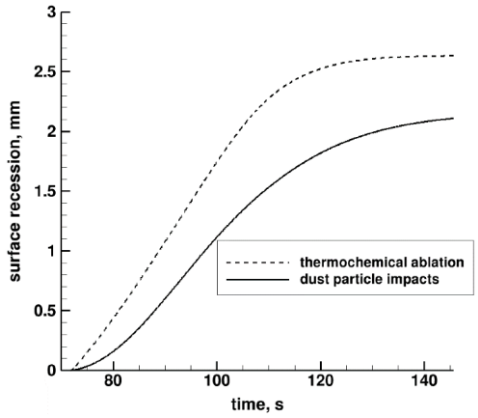
GBK instrumentation set-up.

CFD solution, GBK nozzle.



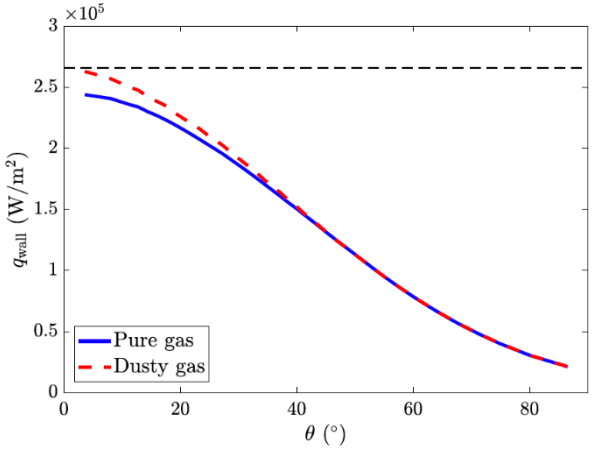
Background – Dusty Flow Simulations

- Simulations of dust-laden flows can be an important element of spacecraft design.
  - Dust particle impacts may cause heatshield erosion and surface heating rate augmentation during Martian entry.
  - Plume-surface interaction (PSI) effects of a spacecraft landing on the surface of the Moon or Mars.
- The primary force that determines particle trajectories through a shock layer or nozzle plume is the drag force experienced by the particle.



Comparing surface recession from particle impacts and thermochemical ablation, Schiaparelli entry capsule.

Heating rate augmentation with dusty flow



Drag Force on a Particle

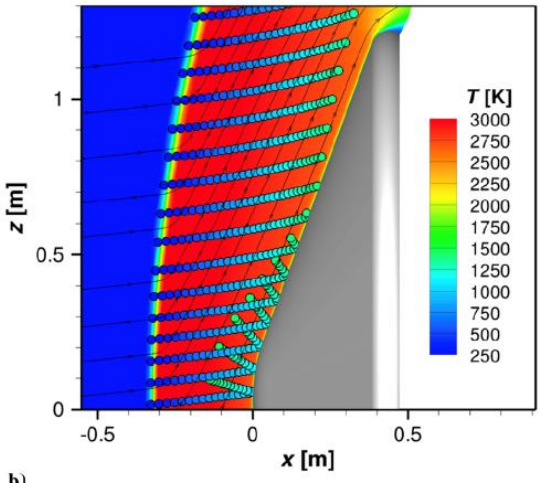
- The drag force on a particle is typically expressed in terms of the relative particle velocity, the frontal area of the particle, the density of the surrounding fluid, and a non-dimensional drag coefficient , C_D.

$$\overrightarrow{F_D} = m_p \frac{d\overrightarrow{V_p}}{dt} = \frac{1}{2}\rho_g \left|\Delta \vec{V}\right| \Delta \vec{V} C_D A$$

- Research into developing drag models for spherical particles has been going on for more than 170 years since the early work of Stokes.

Sir George Gabriel Stokes
1819 - 1903





Simulation of dust particles traveling through Martian shock layer

Particle Knudsen, Mach, and Reynolds Number

- Particle drag model equations are typically functions of the non-dimensional particle Reynolds, Mach, and Knudsen numbers.

Reynolds number $$Re = \frac{\rho_g \left|\Delta\vec{V}\right| d_p}{\mu_g}$$

Mach number $$M = \frac{\left|\Delta\vec{V}\right|}{c}$$

Knudsen number $$Kn = \sqrt{\frac{\pi\gamma}{2}}\left(\frac{M}{Re}\right)$$

- The particle non-dimensional numbers are similar to the standard definitions but are based on the relative velocity between the particles and surrounding fluid, $\Delta\vec{V}$.

Relative particle velocity: $$\Delta\vec{V} = \overrightarrow{v_g} - \overrightarrow{v_p}$$

Particle Knudsen, Mach, and Reynolds Number

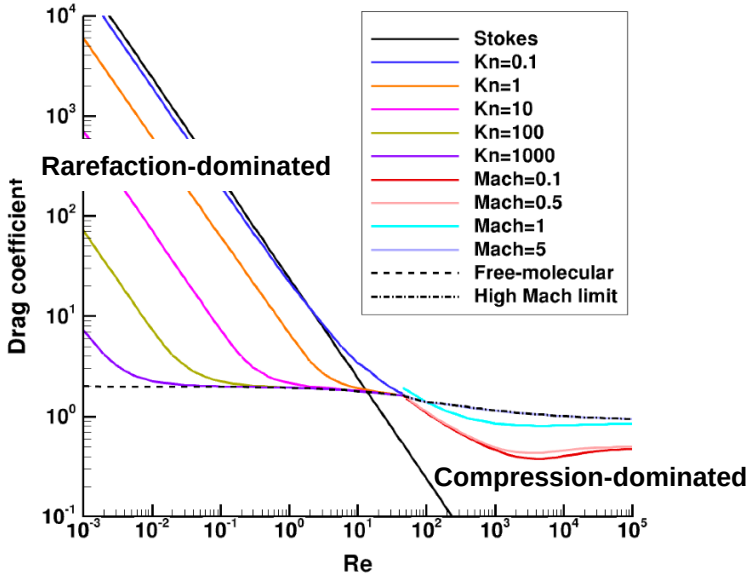
- Knudsen number is the ratio of the mean free path of the fluid surrounding the particle to the particle diameter. It defines the flow regime around the particle – from continuum to free-molecular flow.
- Mach number defines the compressibility effects experienced by the particle – from incompressible to hypersonic flow.

Flow Regime	Kn
Continuum	< 0.01
Slip	$0.01 – 0.1$
Transitional	$0.1 - 10$
Free-molecular	> 10

Flow Regime	M
Incompressible	< 0.1
Compressible, subsonic	$0.1 – 0.65$
Transonic	$0.65 – 1.2$
Supersonic	$1.2 – 5$
Hypersonic	> 5

Particle Drag Regimes

- Particle Reynolds number defines the drag regime experienced by the particle and the nature of the drag coefficient, C_D.
  - At Re ~ 45, the drag coefficient is relatively independent of both Knudsen and Mach numbers.
  - For Re < 45, the particle is in the rarefaction-dominated drag regime. C_D primarily influenced by Knudsen number.
  - For Re > 45 , the particle is in the compression-dominated drag regime. C_D primarily influenced by Mach number.
- Particles in GBK experiments will be in the compression-dominated drag regime.



Particle drag coefficient as a function of Reynolds number (Loth 2021 drag model)

Early Particle Drag Models

- One of the earliest and simplest drag models was developed by Stokes [1851] for incompressible, continuum, low Reynolds number flows.

$$C_D = \frac{24}{Re}$$

- Schiller and Naumann [1933] and later Clift and Gauvin [1970] extended the Stokes correlation to higher Reynolds numbers.

$$C_D = \frac{24}{Re}(1 + 0.15Re^{0.687}) + \frac{0.42}{1 + (42500/Re^{1.16})}$$

- The Clift-Gauvin correlation is still intended for incompressible, continuum flows and, like the Stokes correlation, is only a function of particle Reynolds number.

Henderson Drag Model

- A more ambitious spherical particle drag model was developed by Henderson [1976]. Until recently, it was the most widely-used and considered the most accurate particle drag model.
  - Applicable to all particle flow regimes: from continuum to free-molecular, from incompressible to Mach 6.
  - Reynolds numbers up to the laminar-turbulent transition (Re > 1500-2000).
  - Based primarily on the ballistic range experimental data of Bailey and Hiatt [1971] as well as other experimental data.
- The Henderson drag model does have some deficiencies:
  - Based on experimental data primarily in the continuum and slip flow regimes, very little data for transitional flow and none for free-molecular.
  - There is a gap in the Henderson correlations between 1.0 < Mach < 1.75 that Henderson “fills” by linear interpolation.

Henderson Drag Model Equations

- Because it is intended for all particle flow regimes, the Henderson correlations are complicated functions of particle Reynolds, Knudsen, and Mach numbers.

Mach < 1.0

$$C_D = 24\left[Re + S\left\{4.33 + \left(\frac{3.65 - 1.53\frac{T_p}{T_g}}{1 + 0.353\frac{T_p}{T_g}}\right) exp\left(-0.247\frac{Re}{S}\right)\right\}\right]^{-1} + exp\left(-0.5\frac{M}{\sqrt{Re}}\right)\left[\frac{4.5 + 0.38\left(0.03Re + 0.48\sqrt{Re}\right)}{1 + 0.03Re + 0.48\sqrt{Re}} + 0.1M^2 + 0.2M^8\right] + 0.6S\left[1 - exp\left(-\frac{M}{Re}\right)\right]$$

Mach > 1.75

$$C_D = 0.9 + \frac{0.34}{M^2} + 1.86\sqrt{\frac{M}{Re}}\left[2 + \frac{2}{S^2} + \frac{1.058}{S}\sqrt{\frac{T_p}{T_g}} - \frac{1}{S^4}\right] \Big/ \left(1 + 1.86\sqrt{\frac{M}{Re}}\right)$$

Loth Drag Model

- Loth [2008] and Loth, et al. [2021] developed a particle drag model with separate correlations for the rarefaction-dominated and compression-dominated drag regimes.
- It is claimed that the Loth drag model is currently the most accurate drag model for spherical particles.
  - Applicable to all particle flow regimes: from continuum to free-molecular, from incompressible to Mach 10,
  - Matches underlying experimental data for Reynolds numbers up to Re = 1.0×10^6
  - In addition to the Bailey and Hiatt ballistic range data, the Loth drag model is calibrated to additional experimental data including that of Hoerner [1958].
  - The 2021 update to the model included DSMC simulations for additional data in the slip and transitional flow regimes.

Loth Drag Model Equations

- The Loth drag model has separate correlations for the rarefaction-dominated and compression-dominated drag regimes.

Rarefaction-dominated drag regime, Re < 45

$$C_{D,Kn,Re} = \frac{24}{Re}(1 + 0.15Re^{0.687})\frac{1}{1 + Kn\left[2.514 + 0.8\exp(-\frac{0.55}{Kn})\right]}$$

$$C_{D,fm} = \frac{(1 + 2S^2)exp(-S^2)}{S^3\sqrt{\pi}} + \frac{(4S^4 + 4S^2 - 1)\mathrm{erf}(S)}{2S^4} + \frac{2}{3S}\sqrt{\pi}$$

$$C_{D,fm,Re} = \frac{C_{D,fm}}{1 + \left[\left(C_{D,fm}/J_M\right) - 1\right]\sqrt{Re/45}} \qquad C_D = \frac{C_{D,Kn,Re}}{1 + M^4} + \frac{M^4 C_{D,fm,Re}}{1 + M^4}$$

Compression-dominated drag regime, Re > 45

$$C_D = \frac{24}{Re}(1 + 0.15Re^{0.687})H_M + \frac{0.42C_M}{1 + \left({}^{42500}/_{Re^{1.16C_M}}\right) + \left({}^{G_M}/_{Re^{0.5}}\right)}$$

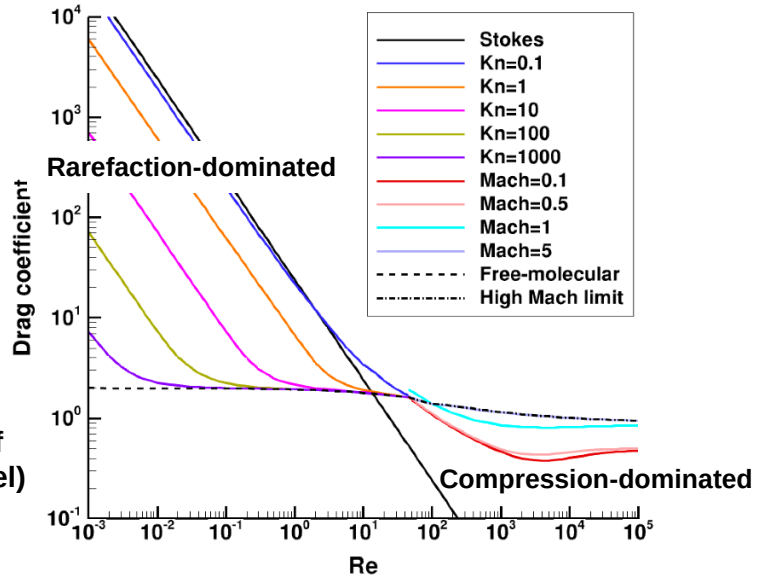
GBK particles will be in the compression-dominated drag regime.

Loth Model High Mach Limit

- Particles in the GBK experiments will be in the compression-dominated drag regime.
- In the high Mach number limit, the coefficients in the Loth compression-dominated equation reduce to numerical values.

$$C_D = \frac{22.32}{Re}(1 + 0.15Re^{0.687}) + \frac{0.861}{1 + \left({}^{42500}/_{Re^{2.38}}\right) + \left({}^{5}/_{Re^{0.5}}\right)}$$

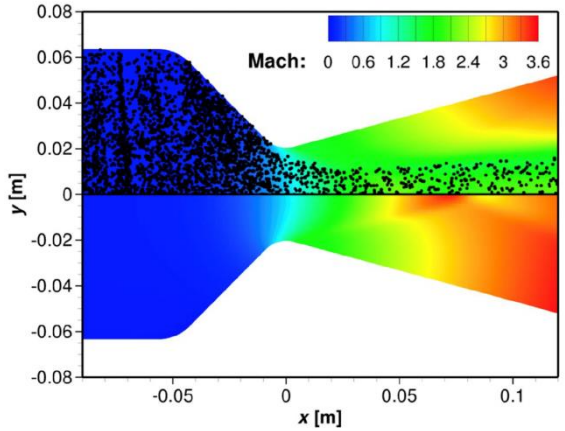
- This equation represents the "theoretical maximum" drag coefficient for spherical particles.



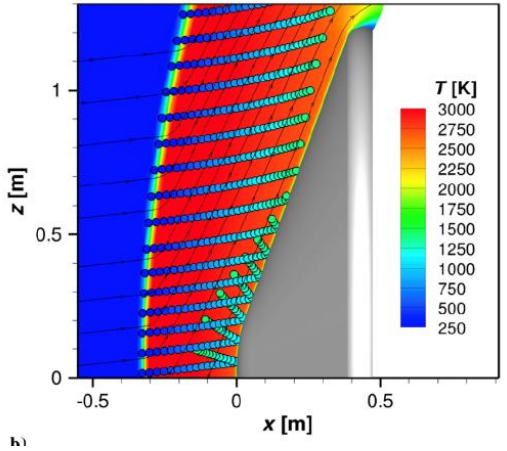
Particle drag coefficient as a function of Reynolds number (Loth 2021 drag model)

Integrated US3D-DUST Analysis Tool

- The integrated US3D-DUST code, developed under the NASA ESM project, is used to simulate GBK particle trajectories using various particle drag models.
- US3D – a 3-D unstructured Navier-Stokes flow solver developed at the University of Minnesota.
- The Dust Simulation and Tracking (DUST) code is a Lagrangian-based particle trajectory capability integrated into US3D as an external library.
- US3D-DUST has been used to simulate dusty-flow experiments and particle trajectories through Martian shock layers.



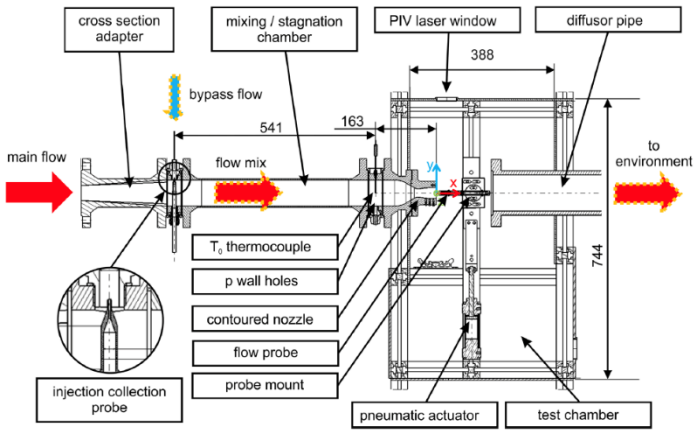
Simulation of dusty flow in JPL converging-diverging nozzle



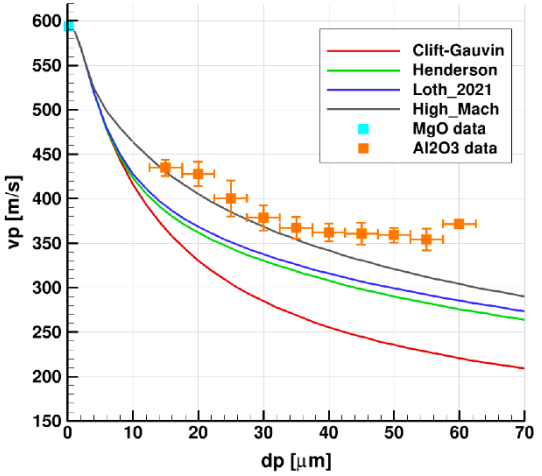
Simulation of dust particles traveling through Martian shock layer

DLR GBK Facility

- The DLR Multi-phase Flow Facility (GBK) is a small test facility that is part of the DLR supersonic wind tunnel complex and is well-suited to run experiments to validate particle drag models.
  - Advanced shadowgraphy system can simultaneously measure particle size and velocity. Data also taken using a Particle Image Velocimetery (PIV) system.
  - Particles of different compositions (e.g., MgO, Al_2O_3, SiO_2) can be injected into the test gas through the bypass flow tube.
  - Can detect small particles, 10 – 20 micron diameter.



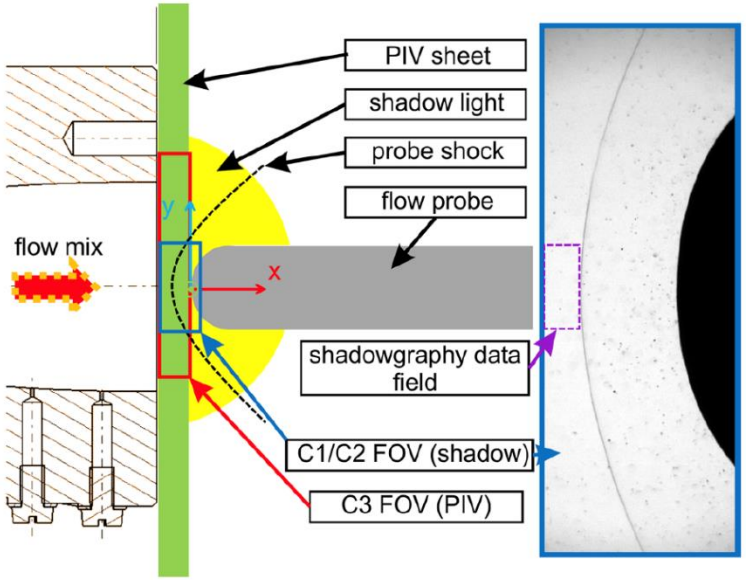
GBK facility



GBK particle velocity as a function of particle diameter.

GBK Dusty Flow Experiment

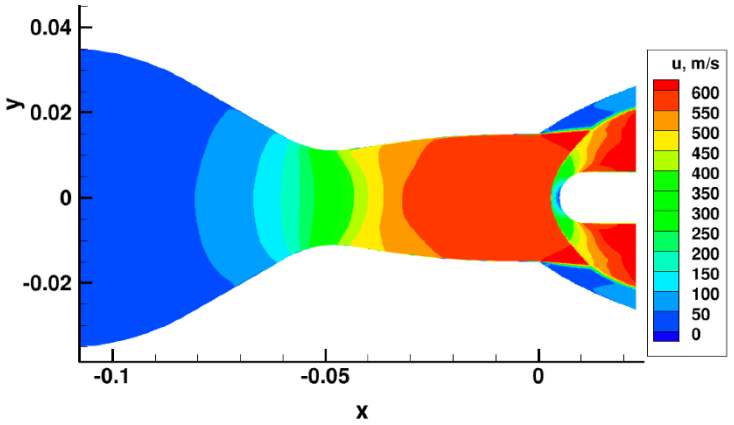
- Allofs, Neeb, and Gülhan [2022] published results of a dusty-flow experiment in the GBK.
  - MgO and Al_2O_3 particles were injected into the GBK flow through the bypass tube. Most particle diameters were between 15 – 60 microns.
  - GBK stagnation conditions: p_0 = 0.949 MPa, T_0 = 373.1 K.
  - Nozzle exit Mach number ~ 2.1.
  - 12-mm diameter hemispherical probe located 5 mm downstream of nozzle exit.
  - Shadowgraphy data taken 2.5 mm downstream of nozzle exit made simultaneous measurements of particle velocity and size.



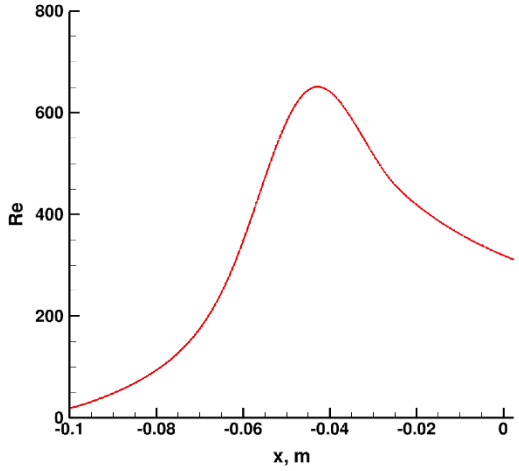
GBK experimental configuration

Results – Particle Reynolds and Knudsen Numbers

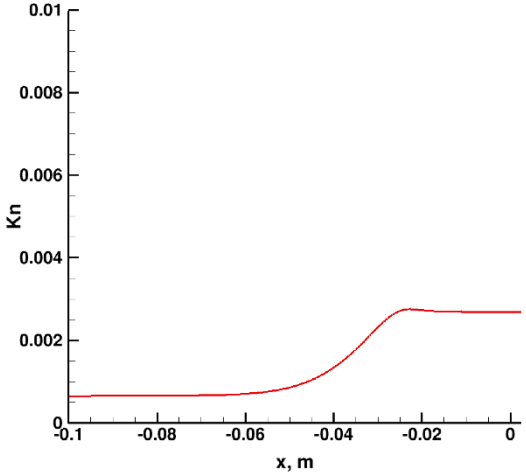
- Particle Reynolds and Knudsen number through GBK nozzle.
  - Chamber conditions: x = -0.1
  - Nozzle throat: x = -0.05
  - Nozzle exit plane: x = 0.0
  - Measurement location: x = 0.0025
- Re varies from 650 at nozzle throat) to 310 at Z=2.5 mm measurement location.
  - Compression dominated drag regime.
- Knudsen number less than 0.01 over entire range.
  - Continuum flow regime.



Velocity contours through GBK nozzle



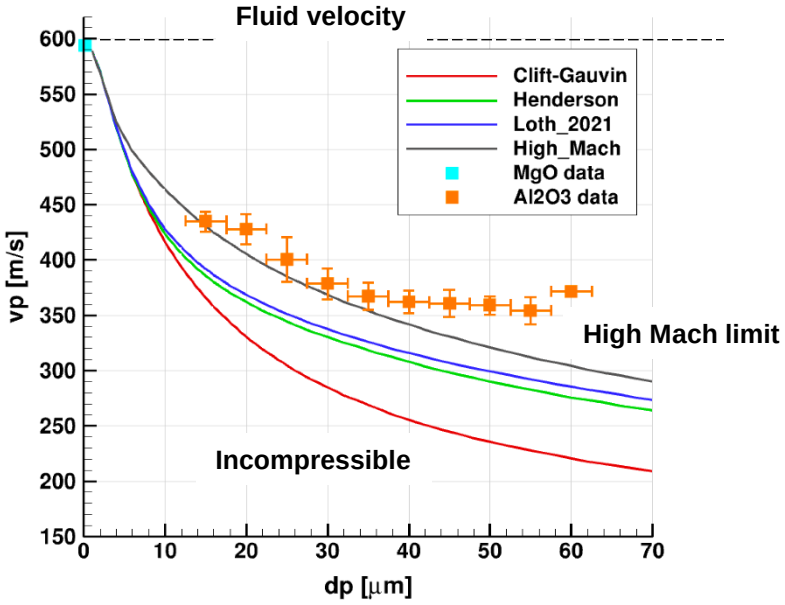
Particle Reynolds number



Particle Knudsen number

Results - General Expectations

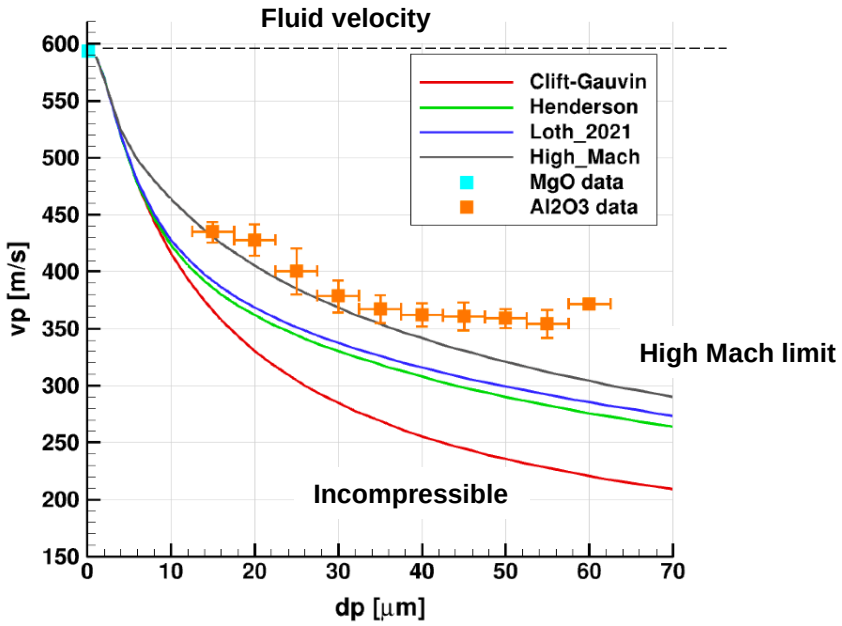
- Plot shows experimental and computed values of particle velocity as a function of particle diameter taken 2.5 mm downstream of nozzle exit.
- In the converging-diverging nozzle, particle velocity will always lag that of the surrounding fluid. Drag force on the particles will cause them to accelerate, i.e., pull the particles forward.
- Larger particles accelerate more slowly than smaller ones. Particle velocity should decrease with increasing particle size.
  - True for US3D-DUST results
  - True for experiment up to 40-micron but not for larger particles.
- Experimental velocities are higher than the “theoretical maximum” high Mach number Loth profile and the experimental data the Loth correlation is based on.



Velocity contours through GBK nozzle

Results

- Experimental velocities are higher than the “theoretical maximum” high Mach number Loth profile – what does that mean?.
- Higher velocity means the GBK particles experienced a higher drag force than predicted by the US3D-DUST code and the spherical particle drag models.
- The GBK particles may be sufficiently non-spherical to invalidate existing spherical drag models.
- Particle agglomeration effects may reduce the effective particle material density.



Velocity contours through GBK nozzle

Concluding Remarks

- US3D-DUST simulations of GBK experiments under-predict experimental particle velocity values 2.5 mm downstream of the nozzle exit.
- Experimental particle velocities are higher than the “theoretical maximum” for spherical particles.
  - GBK particles experienced higher drag than predicted by the spherical drag models.
- Possible reasons include the effects of non-spherical particles and particle agglomeration.
- Additional dusty-flow experiments should be performed in the GBK to reduce measurement uncertainties and provide additional data for particle drag model verification and validation.

Backup Slides

- Backup Slides

L2K CFD Grids

- L2K description and schematic received from Ali.

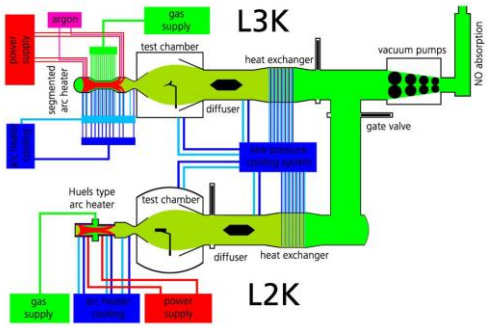


Fig. 1. Sketch of LBK facility

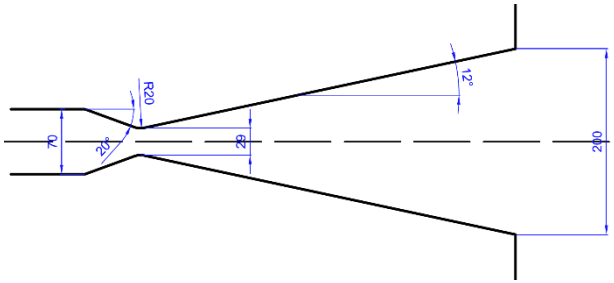
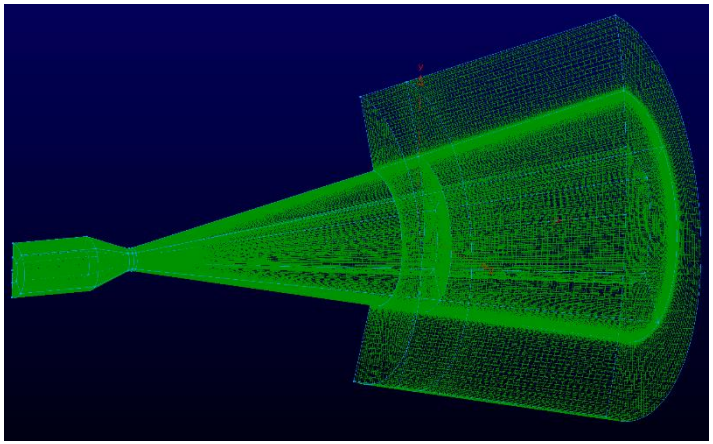


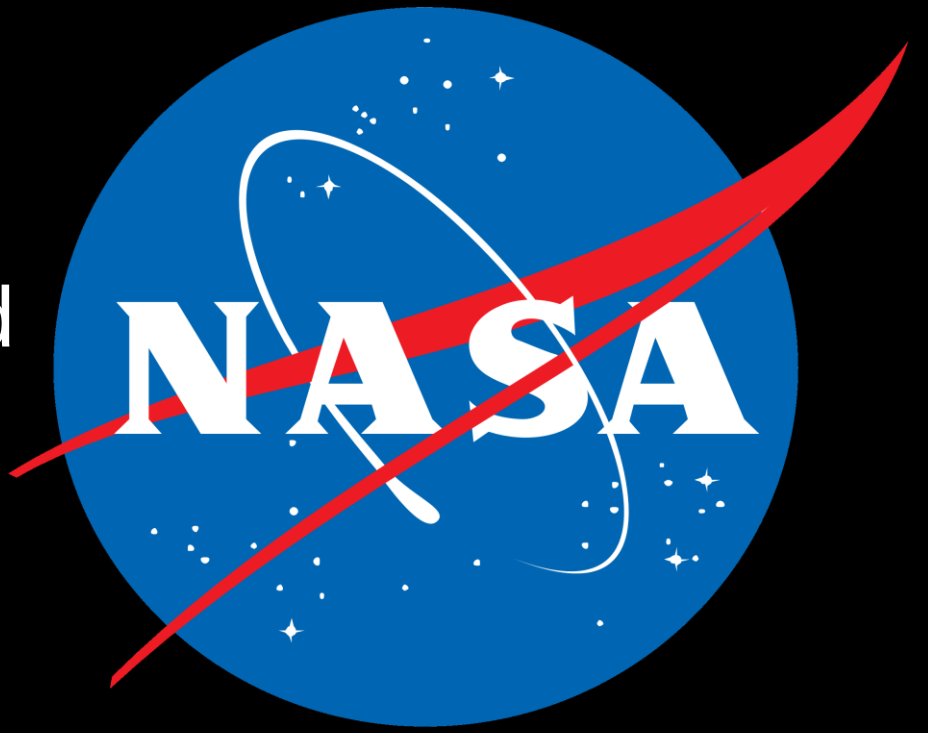
Fig. 2. Schematic of L2K nozzle.

- Two initial 3-D CFD grids generated: L2K nozzle only and nozzle with a 100 mm diameter flat face test article.



Nozzle-only grid

National Aeronautics and
Space Administration



Ames Research Center
Entry Systems and Technology Division