National Aeronautics and Space Administration

Simulated Band 13 - 10.3 µm - Clean Longwave Window - IR [C]

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2022-03-04 12:00Z
2022 Mar 04
07:00am EST Friday

012 Forecast Hours
INIT: 20220304_00z
GEOS f5294_fp

The Relationship Between Two Methods for Estimating Uncertainties in Data Assimilation

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Predictability, and Uncertainty Quantification in Geosciences using Data Assimilation

EGU Genera Assemb y 2022

23 27 May 2022

1 ECMWF; 2 UCAR

Original Title: The Relationship Between Desroziers and Three-Cornered Hat Methods

OUTLINE

1. Motivation
2. Background
3. The relationship: 3CH & DBCP
4. Revisiting the Motivation
5. Additional Comparisons from ERA5 & GEOS
6. Closing Remarks

Motivation

In a recent work now published in J. Tech., Semane et al. (2022) compare estimates of observation uncertainty for radio occultation bending angle with the method of Desroziers et al. (2005; DBCP) and the Three-Cornered Hat (3CH) method of Gray & Allan (1974). The Fig. is a comparison.

A back of the envelop calculation during the review process (by the presenter) showed that, when things are ideal, the observation error standard deviation derived with 3CH should equal that derived with DBCP.

What is going on?

What does the mathematical analysis say?

Background: Cornered-Hat Methods

Atomic Timekeeping and the Statistics of Precision Signal Generators

JAMES A. BARNES

If three oscillators are used, it is possible to independently measure the three quantities σ_{12}, σ_{13}, and σ_{23}. Thus there exist three independent equations:

$$\left.\begin{aligned}\sigma_{12}^2 &= \sigma_1^2 + \sigma_2^2\\ \sigma_{13}^2 &= \sigma_1^2 + \sigma_3^2\\ \sigma_{23}^2 &= \sigma_2^2 + \sigma_3^2\end{aligned}\right\}. \qquad (18)$$

A METHOD FOR ESTIMATING THE FREQUENCY STABILITY OF AN INDIVIDUAL OSCILLATOR

James E. Gray and David W. Allan

Time and Frequency Division
National Bureau of Standards
Boulder

JOURNAL OF GEOPHYSICAL RESEARCH, VOL. 103, NO. C4, PAGES 7755–7766, APRIL 15, 1998

Toward the true near-surface wind speed: Error modeling and calibration using triple collocation

Ad Stoffelen
Royal Netherlands Meteorological Institute, de Bilt, Netherlands

MARCH 2021 SJOBERG ET AL. 555

The Three-Cornered Hat Method for Estimating Error Variances of Three or More Atmospheric Datasets. Part I: Overview and Evaluation

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The idea in so-called Cornered Hat Methods is to use more than one dataset of the same observable to try to estimate the uncertainty in their estimates obtained from them by taking the truth out of the way.

$$\begin{aligned}\mathbf{x} &= \mathbf{t} + \epsilon^{x}\\ \mathbf{y} &= \mathbf{t} + \epsilon^{y}\end{aligned}$$

From where it follows:

$$\begin{aligned}cov(\mathbf{t}) = & \frac{1}{4}[cov(\mathbf{x}+\mathbf{y}) - cov(\mathbf{x}-\mathbf{y})]\\ & -\{E[\mathbf{t}\odot(\epsilon^{x}+\epsilon^{y})] + E(\epsilon^{x}\odot\epsilon^{y})\}\end{aligned}$$

Assuming the datasets have uncorrelated errors, an estimate of the sought uncertainties can be shown to be:

$$\begin{aligned}\hat{\mathrm{X}} &= cov(\mathbf{x}) - \frac{1}{4}[cov(\mathbf{x}+\mathbf{y}) - cov(\mathbf{x}-\mathbf{y})]\\ \hat{\mathrm{Y}} &= cov(\mathbf{y}) - \frac{1}{4}[cov(\mathbf{x}+\mathbf{y}) - cov(\mathbf{x}-\mathbf{y})]\end{aligned}$$

This is the gist of the so-called 2CH – which by neglecting the cross-term between the truth and the errors turns out to have poor accuracy (Sjoberg et al. 2021). Higher order Cornered-Hat Methods only require there be no error correlation among the chosen datasets.

Background: 3CH & DBCP

3CH Method

Given three datasets $\{\mathcal{X}, \mathcal{Y}, \mathcal{Z}\}$, the practical 3CH, avoids the cross-terms:

$$\mathbf{X}^p = \frac{1}{2}\{cov(\mathbf{x}-\mathbf{y}) + cov(\mathbf{x}-\mathbf{z}) - cov(\mathbf{y}-\mathbf{z})\}$$
$$\mathbf{Y}^p = \frac{1}{2}\{cov(\mathbf{y}-\mathbf{z}) + cov(\mathbf{y}-\mathbf{x}) - cov(\mathbf{z}-\mathbf{x})\}$$
$$\mathbf{Z}^p = \frac{1}{2}\{cov(\mathbf{z}-\mathbf{x}) + cov(\mathbf{z}-\mathbf{y}) - cov(\mathbf{x}-\mathbf{y})\}$$

assuming the random errors can be neglect with a wise choice of uncorrelated datasets.

The full-version 3CH uncertainties are:

$$\begin{aligned}\hat{\mathbf{X}} &= \frac{1}{2}\{cov(\mathbf{x}-\mathbf{y}) + cov(\mathbf{x}-\mathbf{z}) - cov(\mathbf{y}-\mathbf{z})\} \\ &+ [E(\epsilon^x \odot \epsilon^y) + E(\epsilon^x \odot \epsilon^z) - E(\epsilon^y \odot \epsilon^z)] \\ \hat{\mathbf{Y}} &= \frac{1}{2}\{cov(\mathbf{y}-\mathbf{z}) + cov(\mathbf{y}-\mathbf{x}) - cov(\mathbf{z}-\mathbf{x})\} \\ &+ [E(\epsilon^y \odot \epsilon^z) + E(\epsilon^y \odot \epsilon^x) - E(\epsilon^z \odot \epsilon^x)] \\ \hat{\mathbf{Z}} &= \frac{1}{2}\{cov(\mathbf{z}-\mathbf{x}) + cov(\mathbf{z}-\mathbf{y}) - cov(\mathbf{x}-\mathbf{y})\} \\ &+ [E(\epsilon^z \odot \epsilon^x) + E(\epsilon^z \odot \epsilon^y) - E(\epsilon^x \odot \epsilon^y)]\end{aligned}$$

where $cov(\mathbf{u},\mathbf{v}) = E(\mathbf{u}\mathbf{v}^T) - E(\mathbf{u})E(\mathbf{v}^T)$ for arbitrary $\mathbf{u}$ and $\mathbf{v}$.

$\mathbf{x} \Leftrightarrow \mathbf{o}$

$\mathbf{y} \Leftrightarrow \mathbf{b}$

$\mathbf{z} \Leftrightarrow \mathbf{a}$

DBCP Method

DBCP

$$\hat{\mathbf{R}} \equiv \frac{1}{2}[cov(\mathbf{o}-\mathbf{a}, \mathbf{o}-\mathbf{b}) + cov(\mathbf{o}-\mathbf{b}, \mathbf{o}-\mathbf{a})]$$
$$\hat{\mathbf{B}} \equiv \frac{1}{2}[cov(\mathbf{a}-\mathbf{b}, \mathbf{o}-\mathbf{b}) + cov(\mathbf{o}-\mathbf{b}, \mathbf{a}-\mathbf{b})]$$
$$\hat{\mathbf{A}} \equiv \frac{1}{2}[cov(\mathbf{a}-\mathbf{b}, \mathbf{o}-\mathbf{a}) + cov(\mathbf{o}-\mathbf{a}, \mathbf{a}-\mathbf{b})]$$

Under $\tilde{\mathbf{B}} + \tilde{\mathbf{R}} \overset{icc}{=} \mathbf{B} + \mathbf{R}$	Full Optimality
$\hat{\mathbf{R}} \overset{icc}{=} \tilde{\mathbf{R}}$	$\hat{\mathbf{R}} \overset{opt}{=} \mathbf{R}$
$\hat{\mathbf{B}} \overset{icc}{=} \tilde{\mathbf{B}}$	$\hat{\mathbf{B}} \overset{opt}{=} \mathbf{B}$
$\hat{\mathbf{A}} \overset{icc}{=} \tilde{\mathbf{A}}$	$\hat{\mathbf{A}} \overset{opt}{=} \mathbf{A}$

where $\tilde{\mathbf{A}} = (\mathbf{I} - \tilde{\mathbf{B}}\tilde{\Gamma}^{-1})\tilde{\mathbf{B}}$, and $\tilde{\Gamma} = \tilde{\mathbf{B}} + \tilde{\mathbf{R}}$. $\tilde{\mathbf{B}}$ and $\tilde{\mathbf{R}}$ are the *prescribed* background and observation error covariances; $\tilde{\mathbf{A}}$ is the *perceived* analysis error covariance; and where $\mathbf{R}$, $\mathbf{B}$ and $\mathbf{A}$ are *true* error covariances.

The Relationship: 3CH & DBCP

With the association: $\{\mathcal{X}, \mathcal{Y}, \mathcal{Z}\} \to \{\mathcal{O}, \mathcal{B}, \mathcal{A}\}$:

Full 3CH

Suboptimal case:

$$\begin{aligned}\hat{\mathbf{X}} &= \tilde{\Gamma} - \mathbf{B} \\ \hat{\mathbf{Y}} &= \mathbf{B} \\ \hat{\mathbf{Z}} &= (\mathbf{I} - \tilde{\mathbf{G}})\mathbf{B}(\mathbf{I} - \tilde{\mathbf{G}})^T + \tilde{\mathbf{G}}\mathbf{R}\tilde{\mathbf{G}}^T\end{aligned}$$

Under Optimality: 3CH = DBCP.

Practical 3CH

Under Innovation Covariance Consistency:

$$\begin{aligned}\mathbf{X}^p &\overset{icc}{\equiv} \tilde{\mathbf{R}} \\ \mathbf{Y}^p &\overset{icc}{\equiv} \tilde{\mathbf{B}} \\ \mathbf{Z}^p &\overset{icc}{\equiv} -\tilde{\mathbf{A}}\end{aligned}$$

Under Optimality:

$$\begin{aligned}\mathbf{X}^p &\overset{opt}{\equiv} \mathbf{R} \\ \mathbf{Y}^p &\overset{opt}{\equiv} \mathbf{B} \\ \mathbf{Z}^p &\overset{opt}{\equiv} -\mathbf{A}\end{aligned}$$

Why do these results follow?

In addition to the fact that observation and background errors are assumed uncorrelated …

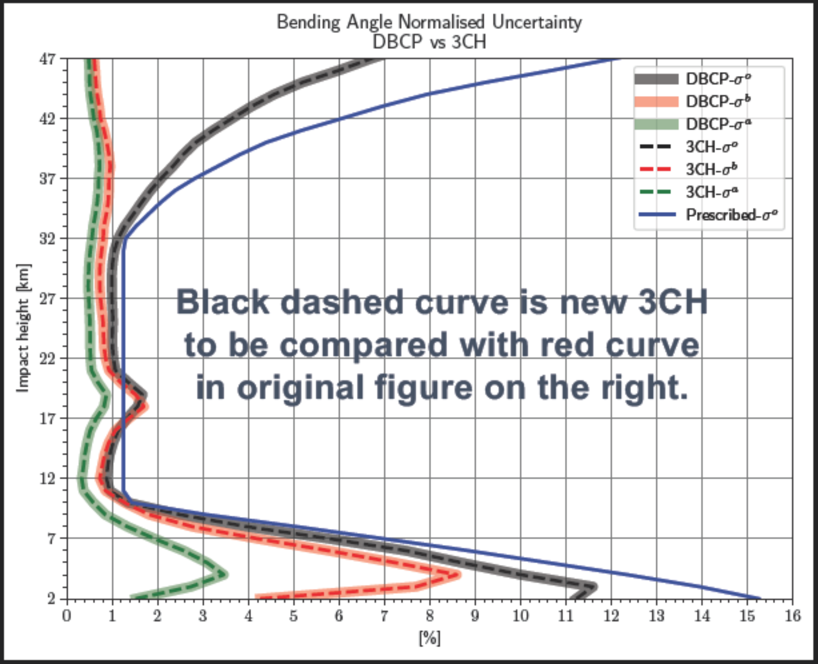
When it comes to the first two corners of 3CH:

- Random errors cancel out due to the orthogonality between the analysis error and the innovation vector.

When it comes to the third corner of 3CH:

- Random error add up to twice the analysis error covariance.

Revisiting the Motivation



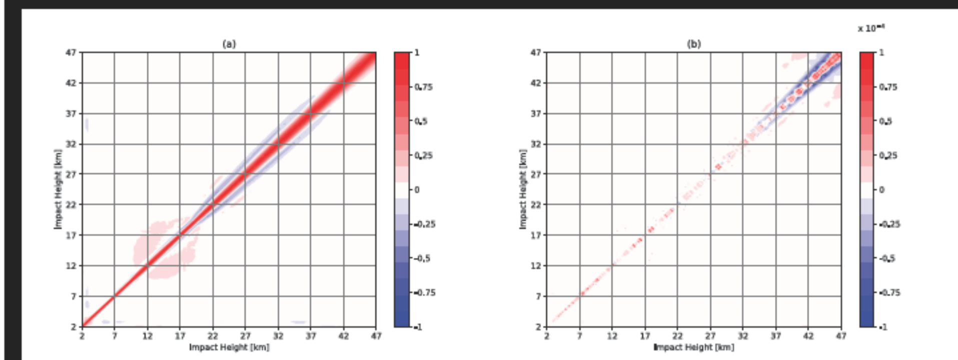
- Re-calculating 3CH using fully consistent corners as compared to DBCP residual vectors from ERA5.
- Diffs between 3CH & DBCP in Semane et al. (2022) are simply due to sampling strategy.

Note: only black and blue curves should be compared across plots; other curves are for different quantities.

Additional Comps: Observation Uncertainty Correlations

Results from ERA5 residuals for April 2021

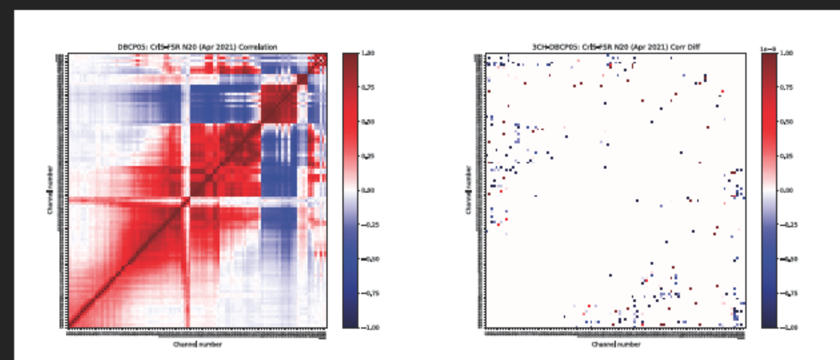
Vertical Correlations: RO Bending Angle



DBCP **Diff(DBCP,3CH) ~ 10^{-4}**

Results from NASA-GEOS residuals for April 2021

Interchannel Correlations for CrIS-FSR NOAA-20



DBCP **Diff(DBCP,3CH) ~ 10^{-9}**

The methods produce nearly identical results,
When the datasets used in 3CH are consistent with the residuals of DBCP.

Closing Remarks

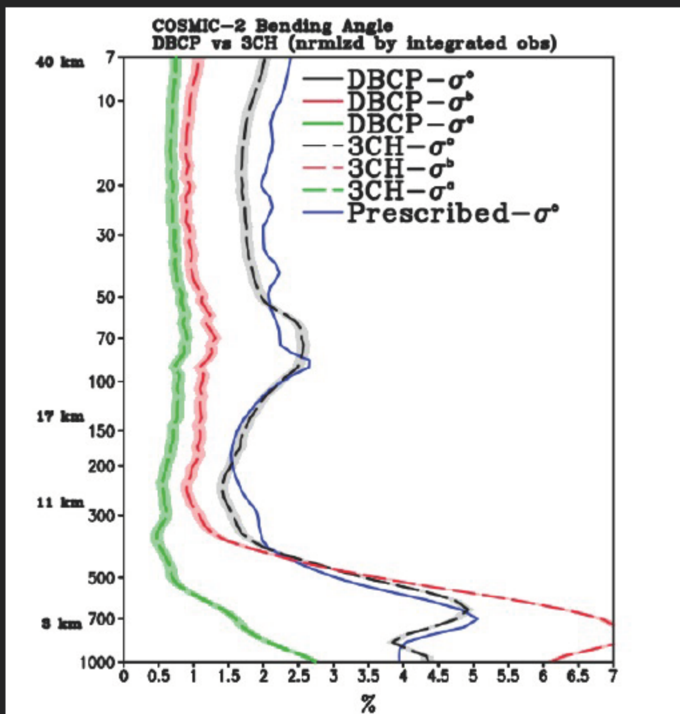
- This work shows that the full version of Three-Cornered Hat Method (3CH) is identical to the method of Desroziers et al. (2005; DBCP).

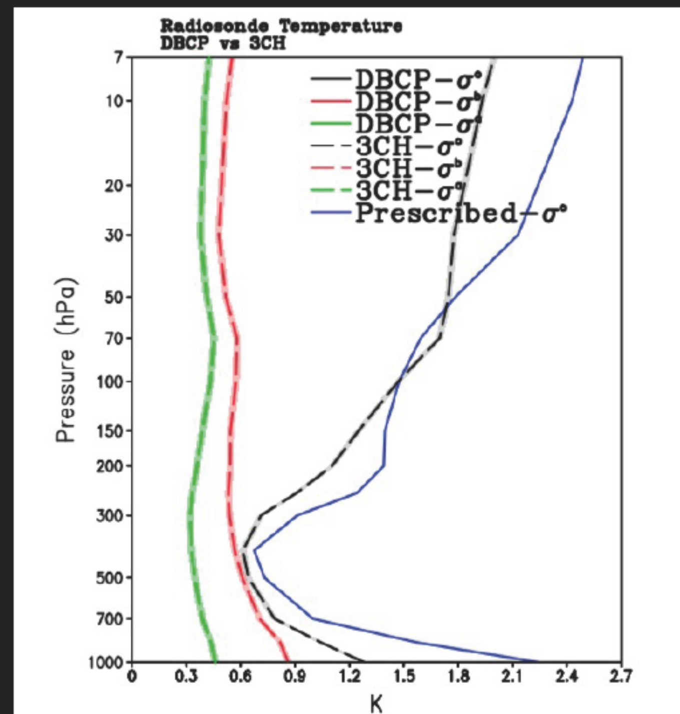
- The full version of 3CH requires knowledge of random errors not known in practice.

- The *practical* version of 3CH recovers identical results as DBCP for the observation and background error covariances, a consequence of such errors: (i) being uncorrelated; and (ii) the analysis errors being orthogonal to the innovation vector.

- Surprisingly, the *practical* version of 3CH recovers the *negative* of the analysis error covariance (a consequence of random errors adding up to twice the analysis error covariance).

Illustrations: Observation Uncertainty Standard Deviations





Results obtained using NASA-GEOS residuals for April 2021