

# A Multivibrational Mode Relaxation Scheme for Two-Temperature Thermochemical State Models

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# Contents

- Preliminary
- Objective
- Introducing multivibrational relaxation scheme
- Verification of the proposed approach
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- Contributions

# Preliminary

Consider compressible Navier-Stokes equations for two-temperature reacting flows:

$$\partial_t \mathbf{u} + \nabla \cdot \mathbf{f}(\mathbf{u}) = \mathbf{S}, \quad \mathbf{u} = \begin{pmatrix} \rho_s \\ \rho \mathbf{v} \\ \rho E \\ \rho e_V \end{pmatrix}, \quad \mathbf{f} \cdot \mathbf{n} = \begin{pmatrix} \rho U c_s \\ \rho U \mathbf{v} + p \mathbf{n} \\ \rho U H \\ \rho U e_V \end{pmatrix}, \quad U = \mathbf{v} \cdot \mathbf{n},$$

$$\mathbf{S} = \begin{pmatrix} \dot{w}_s \\ \dot{w}_m \\ \dot{w}_E \\ \dot{w}_V := \dots + \underbrace{\left( Src^{T \leftrightarrow V} \equiv \frac{dE_V}{dt} \right)}_{\text{Translational-vibrational energy exchange source term}} + \dots \end{pmatrix}$$

: the **Landau-Teller** expression

Consider the Landau-Teller Equation:

$$Src^{T \leftrightarrow V} \equiv \frac{dE_v}{dt} = \sum_{s = mol.} \rho_s \frac{ev_s^0(T) - ev_s(T)}{\tilde{\tau}_s},$$

where

averaged vibrational  
relaxation time for species  $s$

an averaging function

vibrational relaxation of species  $s$   
in a medium species  $r$ .

$$\tilde{\tau}_s = f(\tau_{sr}).$$

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From Milikan-White [J. Chem. Phys., 1963]:

$$p\tau_{sr}^{\text{MW}} = \exp [a_{sr}(T^{-1/3} - b_{sr}) - 18.42],$$

where

**single** characteristic  
vibrational temperature

reduced mass

$$a_{sr} := g(\mu_{sr}, \theta_{v,s}), \quad b_{sr} := h(\mu_{sr}).$$

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● Park et al. [JTHT, 1994] suggested a series of **tailored**  $a_{sr}$  and  $b_{sr}$  values for  $CO_2 - N_2$ . Camac [Cornell U. Press, 1966] values were used for  $CO_2 - CO_2$ .

● **No other values are available for general polyatomic molecules.**

# Landau-Teller Expression in CFD

Consider again the Landau-Teller equation:

$$\begin{aligned}
 Src^{T \leftrightarrow V} &\equiv \frac{dE_v}{dt} = \sum_{s=mol.} \rho_s \frac{ev_s^0(T) - ev_s(T)}{\tilde{\tau}_s} \\
 &\approx \sum_{s=mol.} \rho_s \frac{ev_s^0(T) - ev_s(T_v)}{\tilde{\tau}_s} \quad \text{introducing } T_v \\
 &\approx \sum_{s=mol.} \rho_s \frac{C_{vv,s}(T - T_v)}{\tilde{\tau}_s} = \frac{\rho C_{vv}(T - T_v)}{\bar{\tau}}
 \end{aligned}$$

$$\bar{\tau} = \frac{\sum_{s=mol.} \rho_s C_{vv,s}}{\sum_{s=mol.} \frac{\rho_s C_{vv,s}}{\tilde{\tau}_s}} \approx \frac{\sum_{s=mol.} \frac{\rho_s}{MW_s}}{\sum_{s=mol.} \frac{\rho_s}{MW_s \tilde{\tau}_s}}$$

to account for elastic collisions

$$\Rightarrow : f(\tau_{sr}^{Millikan-White}) + \tilde{\tau}_s^{Park}(T)$$



# Objective

To develop an effective vibrational relaxation time  $\bar{\tau}$  for *generic* molecules that directly depends on species **vibrational frequencies**.



# Multivibrational Mode Approach

Consider again the Landau-Teller equation:

$$Src^{T \leftrightarrow V} \equiv \frac{dE_v}{dt} \simeq \sum_{s = mol.} \rho_s \frac{ev_s^0(T) - ev_s(T_v)}{\tilde{\tau}_s} \simeq \frac{\rho C_{vv}(T - T_v)}{\bar{\tau}}$$

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 &\quad \downarrow \\
 &\sum_{s = mol.} \rho_s \left( \sum_{m = mode} \frac{ev_{s,m}^0(T) - ev_{s,m}(T_v)}{\tilde{\tau}_s^{(MM)}(T, \theta_{v,m})} \right)
 \end{aligned}$$



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$$\tilde{\tau}_s^{(MM)} = \frac{\sum_{m=mode} |ev_{s,m}^0(T, \theta_{v,m}) - ev_{s,m}(T_v, \theta_{v,m})|}{\sum_{m=mode} \frac{|ev_{s,m}^0(T, \theta_{v,m}) - ev_{s,m}(T_v, \theta_{v,m})|}{\tilde{\tau}_s(T, \theta_{v,m})}},$$

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$\nearrow = g_m \frac{R_s \theta_{v,m}}{e^{\frac{\theta_{v,m}}{T}} - 1}$

$\longrightarrow : f(\tau_{sr}^{Millikan-White}) + \tilde{\tau}_s^{Park}(T)$

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$\begin{matrix} = g_m \frac{R_s \theta_{v,m}}{e^{\frac{\theta_{v,m}}{T}} - 1} & & = g_m \frac{R_s \theta_{v,m}}{e^{\frac{\theta_{v,m}}{T_v}} - 1} \end{matrix}$

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$$\tilde{\tau}_s^{(MM)} = \frac{\sum_{m=mode} |ev_{s,m}^0(T, \theta_{v,m}) - ev_{s,m}(T_v, \theta_{v,m})|}{\sum_{m=mode} \frac{|ev_{s,m}^0(T, \theta_{v,m}) - ev_{s,m}(T_v, \theta_{v,m})|}{\tilde{\tau}_s(T, \theta_{v,m})}}$$

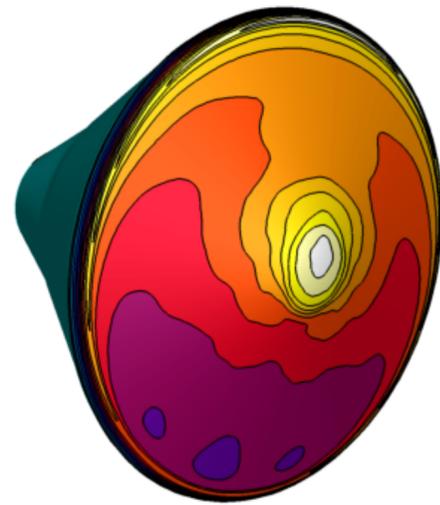
$\xrightarrow{\text{diagonal lines}} g_m \frac{R_s \theta_{v,m}}{e^{\frac{\theta_{v,m}}{T}} - 1} = g_m \frac{R_s \theta_{v,m}}{e^{\frac{\theta_{v,m}}{T_v}} - 1}$

$\rightarrow : f(\tau_{sr}^{Millikan-White}) + \tilde{\tau}_s^{Park}(T)$

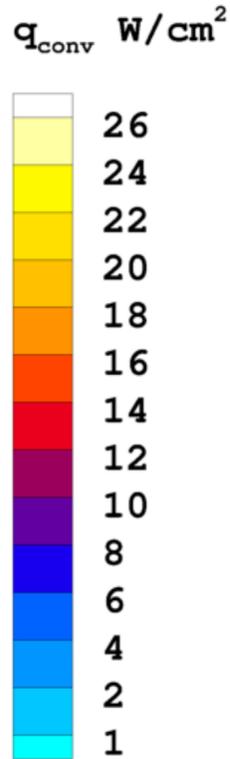
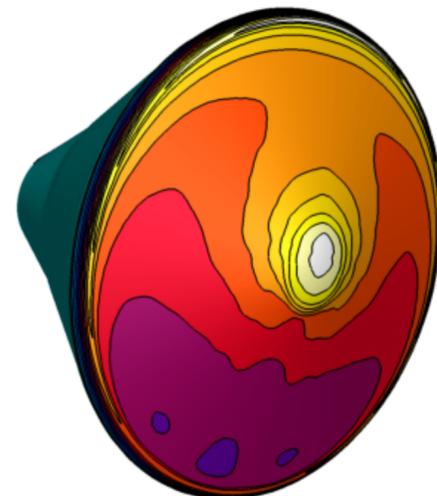
For molecules with single vibrational mode:  $\tilde{\tau}_s^{MM} = \tilde{\tau}_s$

# Application on CO<sub>2</sub> Mixture

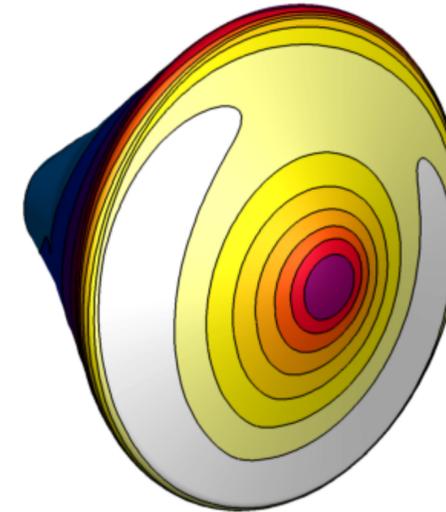
Relaxation:  
Park et al. [JTHT, 1994]



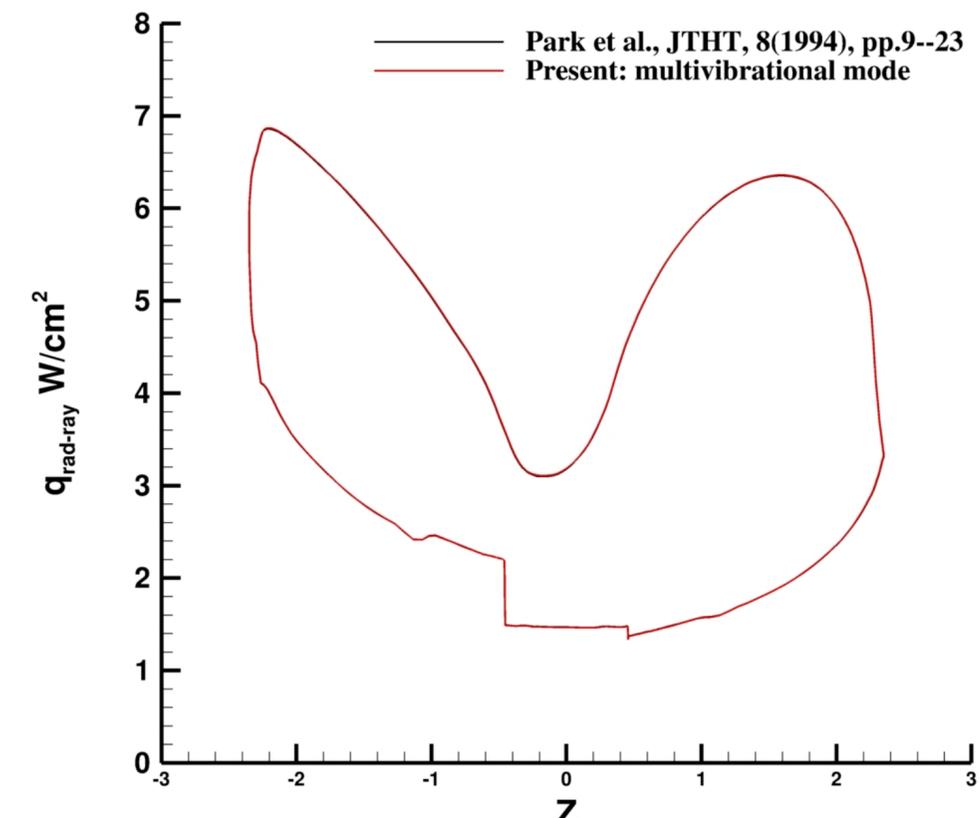
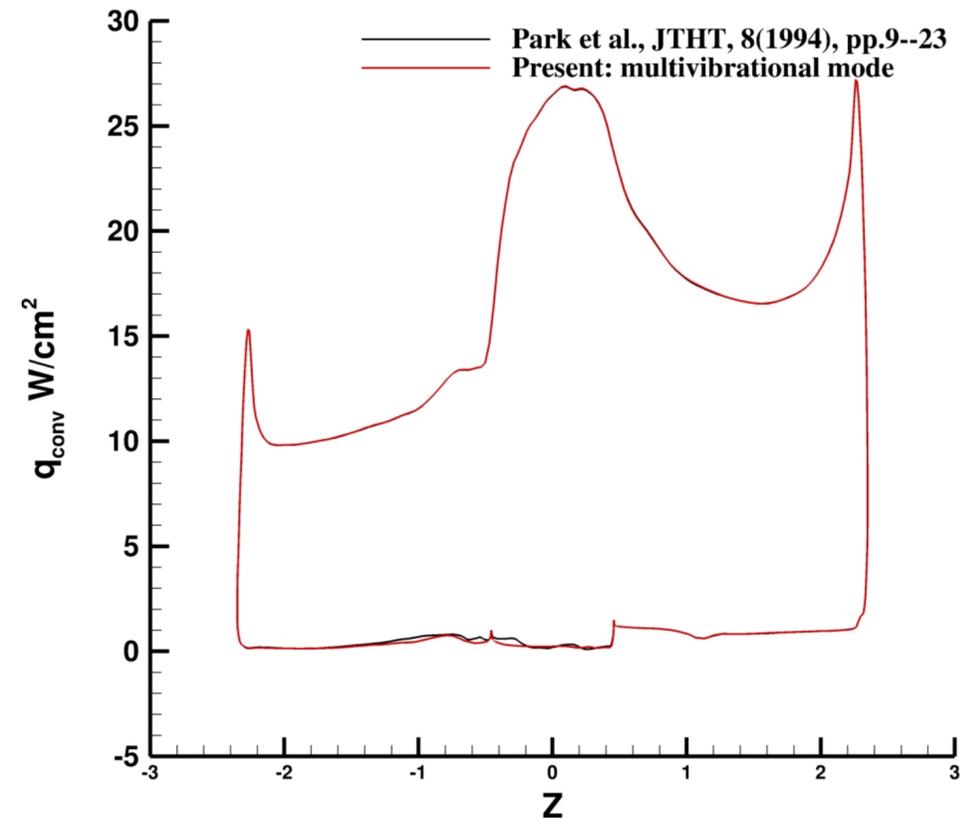
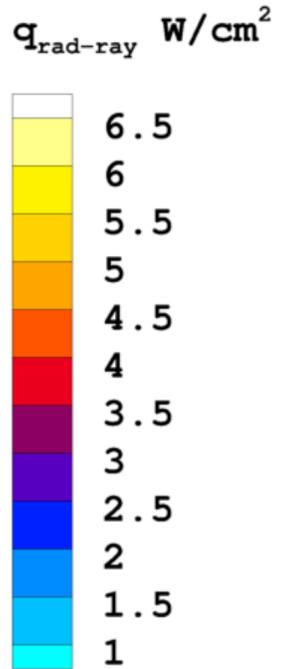
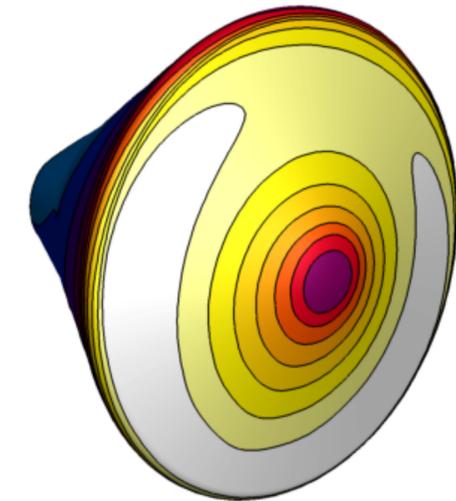
Relaxation:  
**present**



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Relaxation:  
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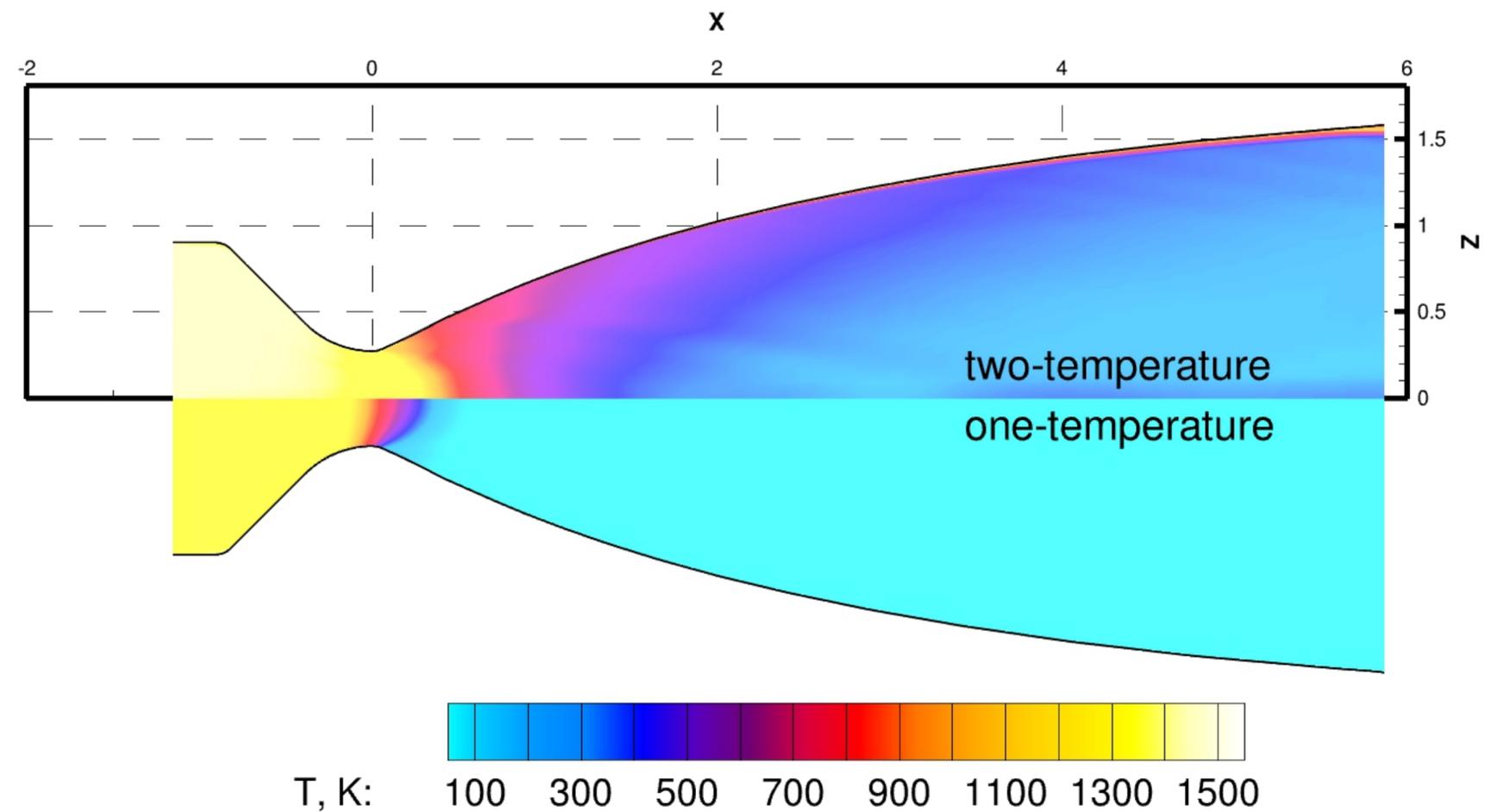


# Application on non-CO<sub>2</sub> Mixture



*NH<sub>3</sub> – N<sub>2</sub> gas mixture*

Accounting for thermal nonequilibrium could drastically change the temperature profile.

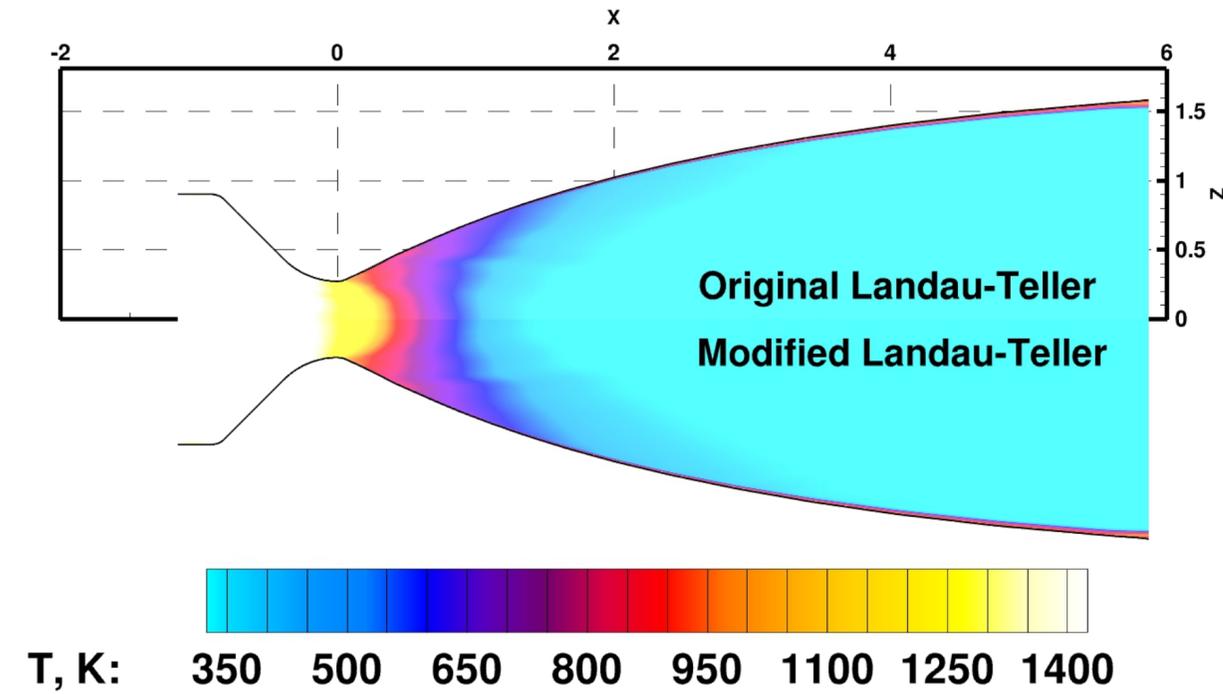


- Relaxation time derived from the Landau-Teller equation is either underestimated ( $T \ll T_{ve}$ ) or overestimated ( $T_{ve} \ll T$ ).

- Adopting Kustova & Oblapenko's work [Phys. Rev. E., 2019]:

$$\bar{\tau}_s^{\text{modified}} = \bar{\tau}_s \frac{T_{ve}}{T}.$$

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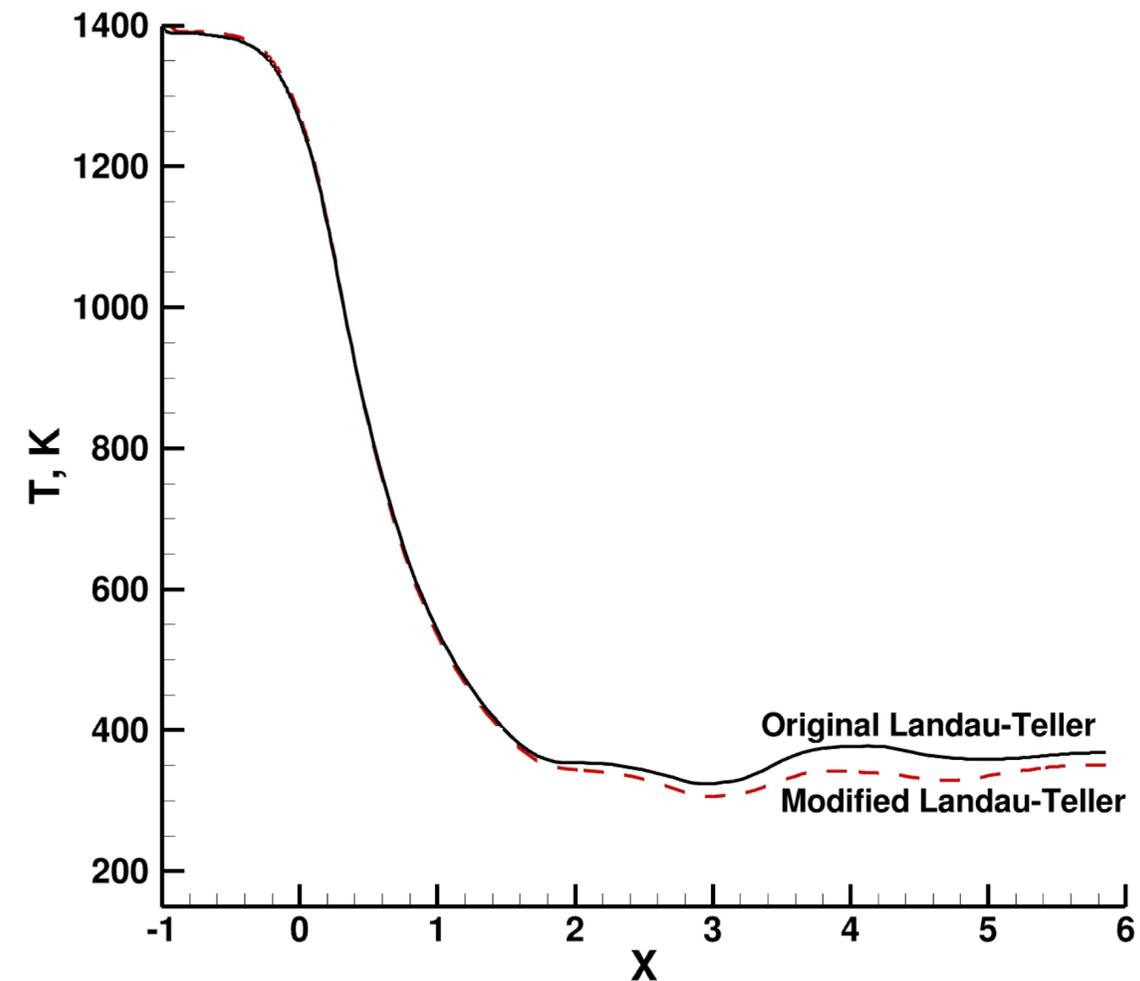
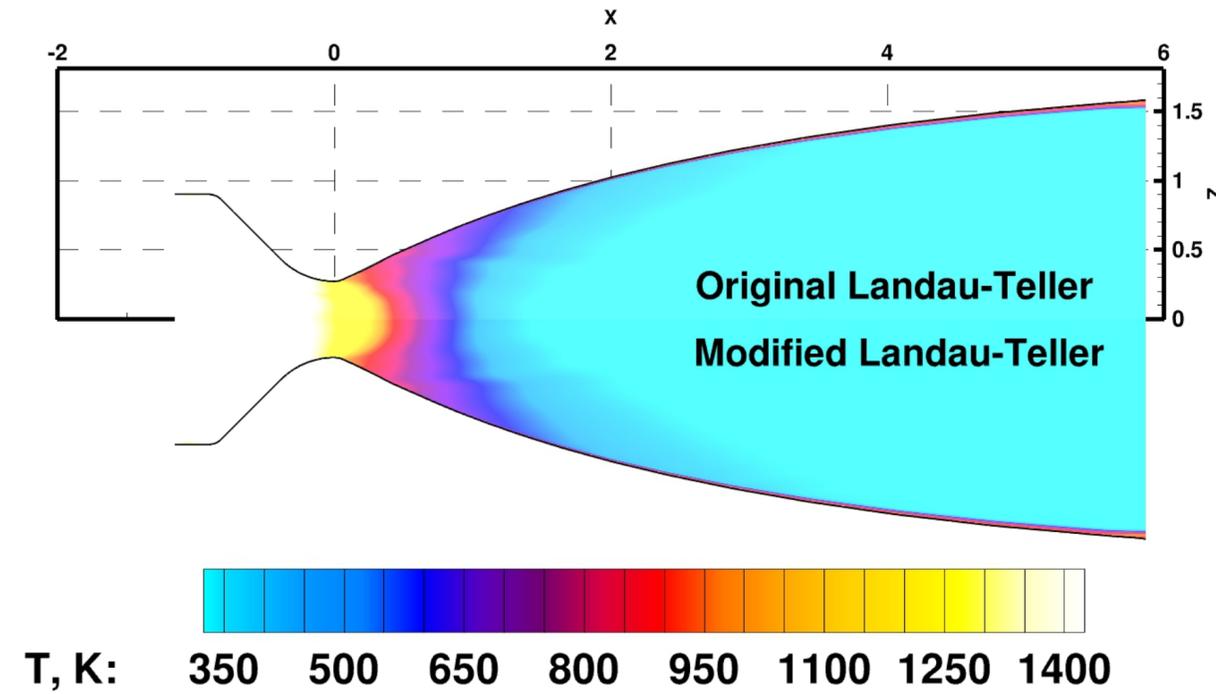
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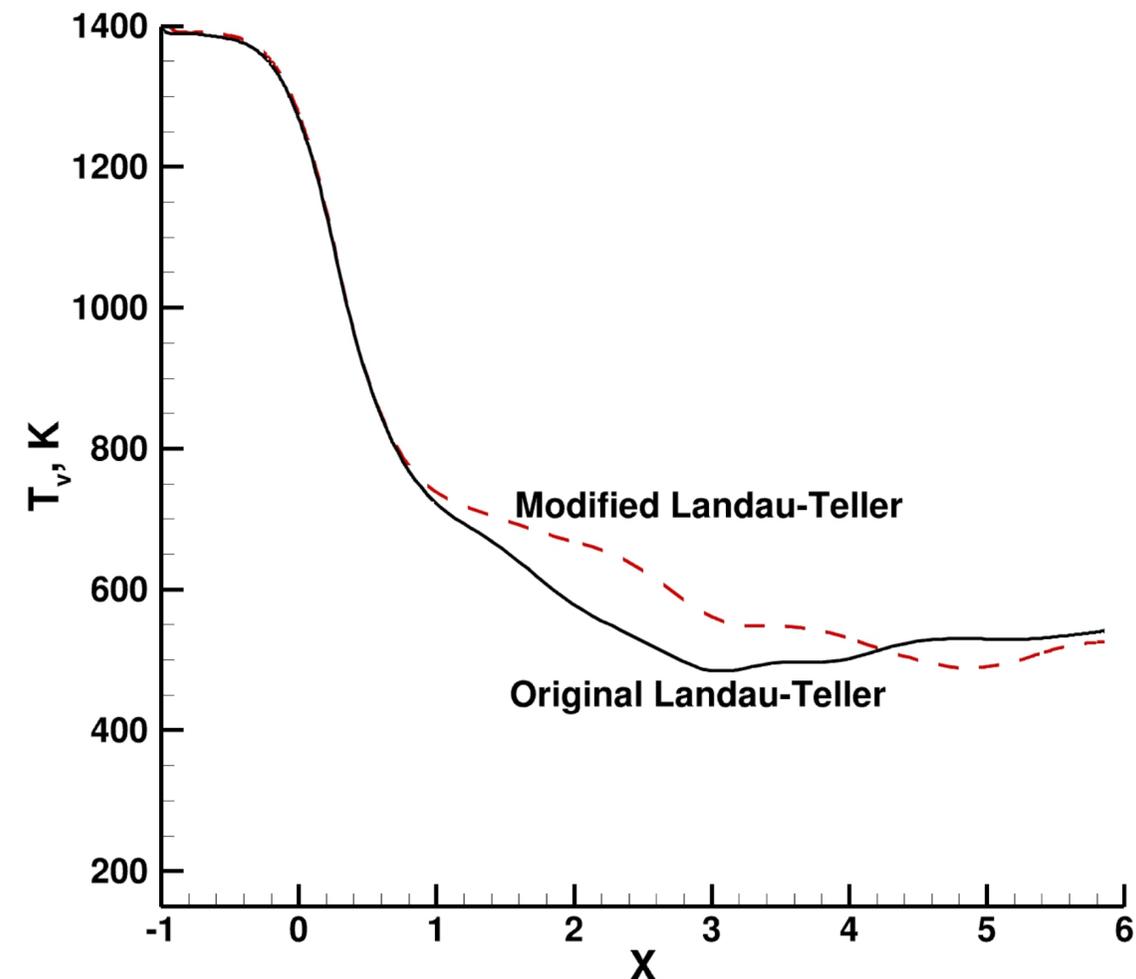
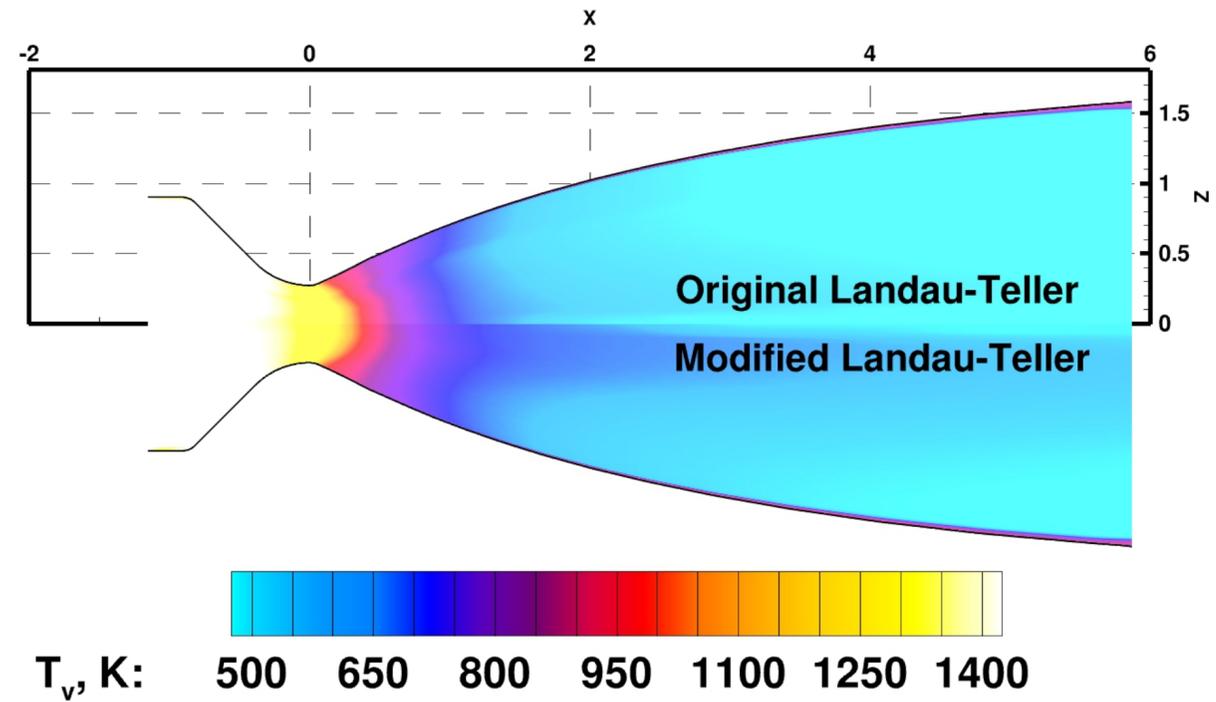
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# Contributions

- A new multivibrational relaxation procedure was developed
- The new scheme was shown to be in excellent agreement against the Park's 1994 model for the Martian atmosphere.
- The proposed scheme is applicable to generic molecules that have multiple vibrational frequencies.



# Backups

# Multivibrational Mode Approach

Consider again the Landau-Teller equation:

$$Src^{T \leftrightarrow V} \equiv \frac{dE_v}{dt} \approx \sum_{s = mol.} \rho_s \frac{ev_s^0(T) - ev_s(T_v)}{\tilde{\tau}_s} \approx \frac{\rho C_{vv}(T - T_v)}{\bar{\tau}},$$

$$Src^{T \leftrightarrow V} = \sum_{s = mol.} \rho_s \left( \sum_{m = mode} \frac{ev_{s,m}^0(T) - ev_{s,m}(T_v)}{\tilde{\tau}_s^{(MM)}(T, \theta_{v,m})} \right) = \frac{\rho C_{vv}(T - T_v)}{\bar{\tau}}.$$

$$= g_m \frac{R_s \theta_{v,m}}{e^{\frac{\theta_{v,m}}{T}} - 1}$$

$$= g_m \frac{R_s \theta_{v,m}}{e^{\frac{\theta_{v,m}}{T_v}} - 1}$$

$$\tilde{\tau}_s^{(MM)} = \frac{\sum_{m = mode} |ev_{s,m}^0(T, \theta_{v,m}) - ev_{s,m}(T_v, \theta_{v,m})|}{\sum_{m = mode} \frac{|ev_{s,m}^0(T, \theta_{v,m}) - ev_{s,m}(T_v, \theta_{v,m})|}{\tilde{\tau}_s(T, \theta_{v,m})}}$$

$$\rightarrow : f(\tau_{sr}^{MW}) + \tilde{\tau}_s^{Park}(T)$$

Consider now the Landau-Teller Equation:

$$Src^{T \leftrightarrow V} \equiv \frac{dE_v}{dt} = \sum_{s = mol.} \rho_s \frac{ev_s^0(T) - ev_s(T)}{\tilde{\tau}_s},$$

where

averaged vibrational relaxation time  
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$$\tilde{\tau}_s = f(\tau_{sr}).$$

The landmark correlation for  $\tau_{sr}$  is provided by Milikan-White [J. Chem. Phys., 1963]:

$$p\tau_{sr}^{MW} = \exp [a_{sr}(T^{-1/3} - b_{sr}) - 18.42],$$

where

single characteristic  
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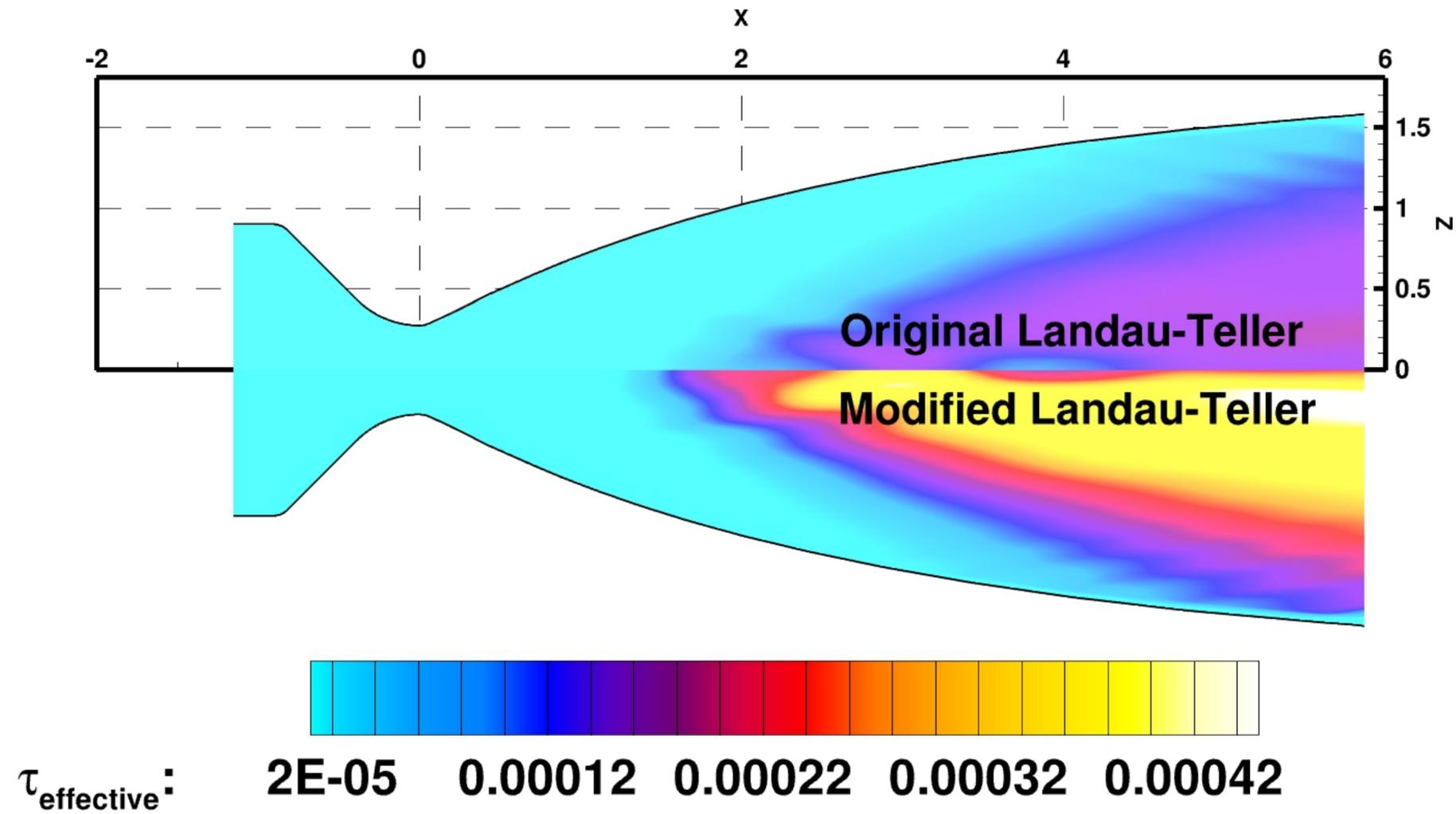
reduced mass

$$a_{sr} := g(\mu_{sr}, \theta_{v,s}), \quad b_{sr} := h(\mu_{sr}).$$

Some polyatomic molecules, such as  $CO_2$  and  $NH_3$ , has **multiple** vibrational frequencies and characteristic vibrational temperatures. **Thus, the above formula is not directly applicable to those molecules.**

For  $CO_2 - N_2$ , Park et al. [JTHT, 1994] suggested a series of **tailored** values for  $a_{sr}$  and  $b_{sr}$ .

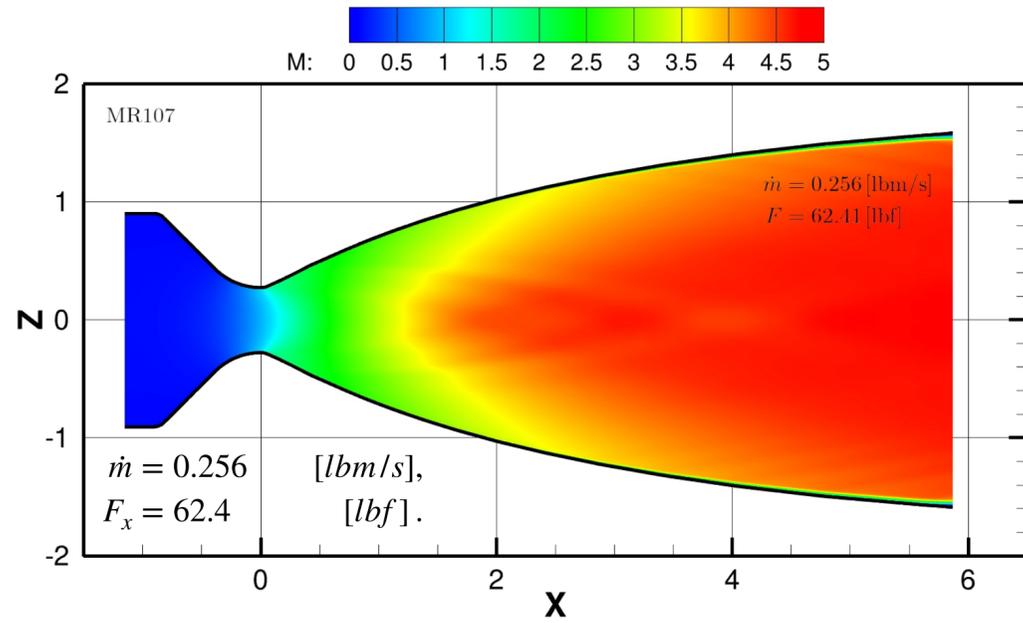
# $NH_3 - N_2$ gas mixture



# Test Case 1: MR107

## One-Temperature

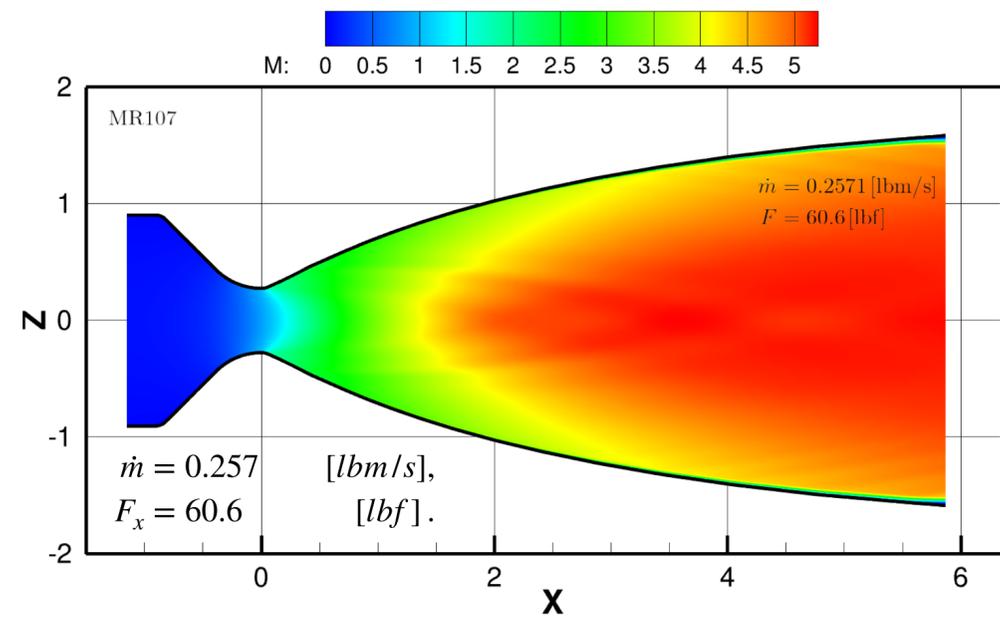
$P_0 = 147$  [psia],  $T_0 = 1365$  [K],  $c_{NH_3} = 0.427$ ,  $c_{N_2} = 0.525$ ,  $c_{H_2} = 0.048$   
 $T_w$ : Radiative Equilibrium,  $\epsilon = 0.35$



## Two-Temperature w/lump sum relaxation time

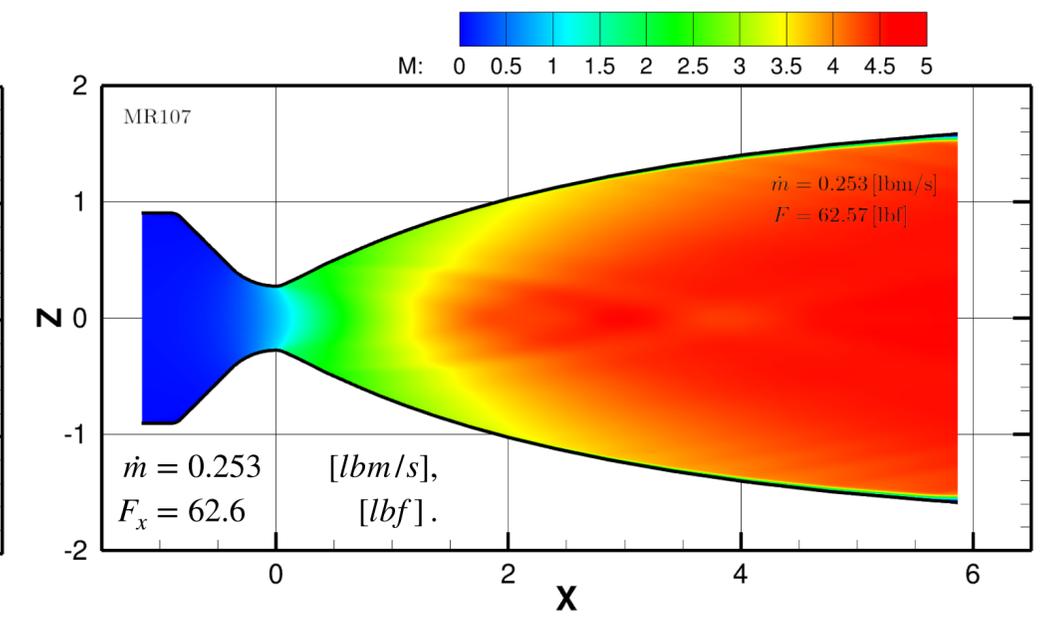
assuming:  $\bar{\theta}_v = 3500$  [K]

$P_0 = 147$  [psia],  $T_0 = 1365$  [K],  $c_{NH_3} = 0.427$ ,  $c_{N_2} = 0.525$ ,  $c_{H_2} = 0.048$   
 $T_w$ : Radiative Equilibrium,  $\epsilon = 0.35$ ,  $2T$  w/lump sum relaxation time



## Two-Temperature w/multimode relaxation time

$P_0 = 147$  [psia],  $T_0 = 1365$  [K],  $c_{NH_3} = 0.427$ ,  $c_{N_2} = 0.525$ ,  $c_{H_2} = 0.048$   
 $T_w$ : Radiative Equilibrium,  $\epsilon = 0.35$ ,  $2T$  w/multivibrational mode relaxation time

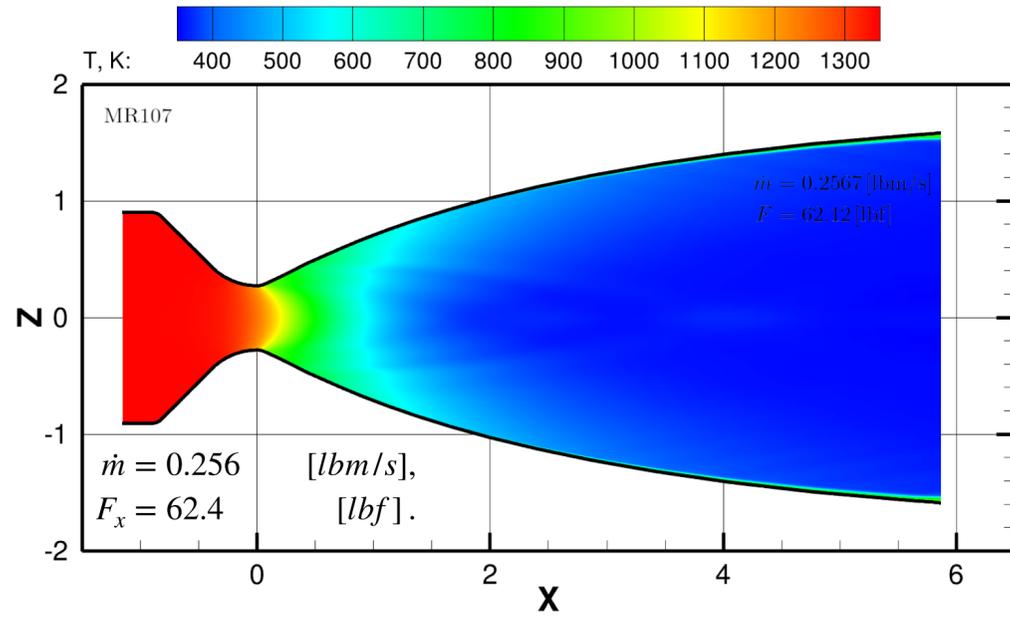


- The lump sum approach with single characteristic temperature of 3500K underpredicts the RCS thrust.
- One-temperature and two-temperature multimode approaches predict comparable RCS thrust.

# Test Case 1: MR107

## One-Temperature

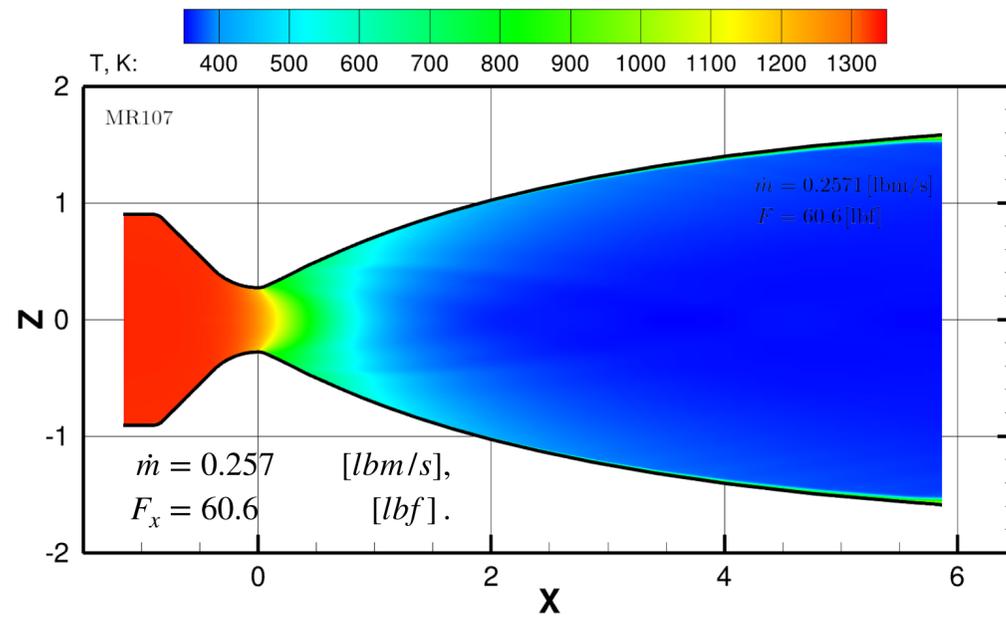
$P_0 = 147$  [psia],  $T_0 = 1365$  [K],  $c_{NH_3} = 0.427$ ,  $c_{N_2} = 0.525$ ,  $c_{H_2} = 0.048$   
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## Two-Temperature w/lump sum relaxation time

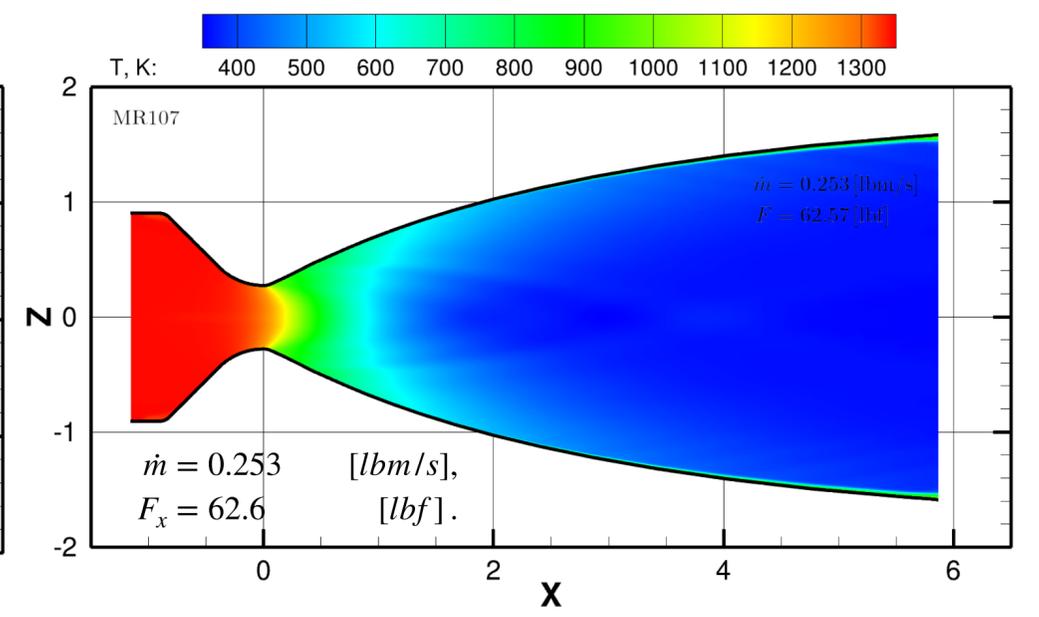
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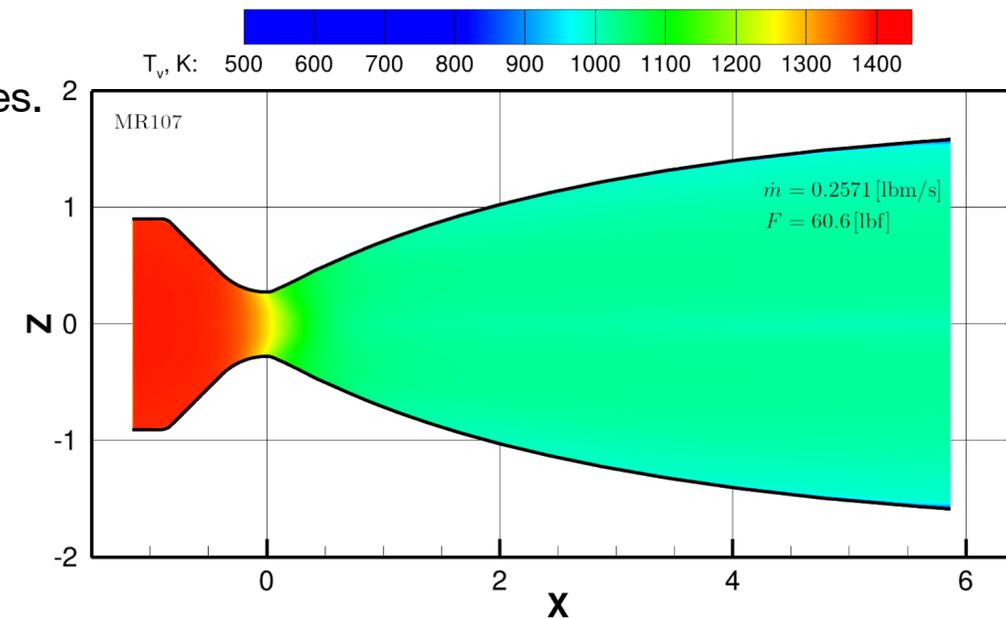


## Two-Temperature w/multimode relaxation time

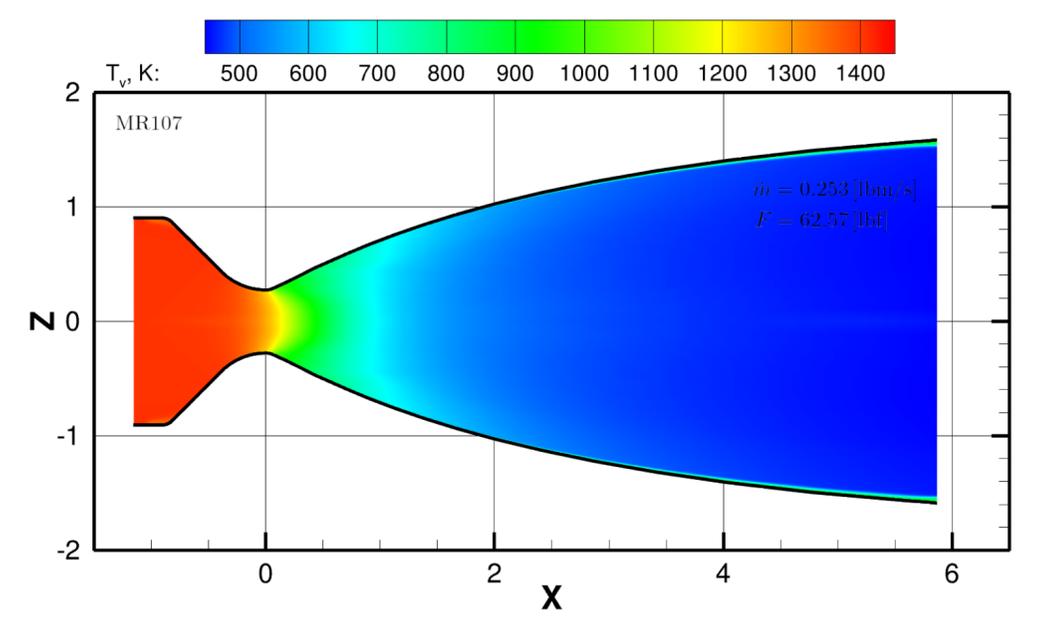
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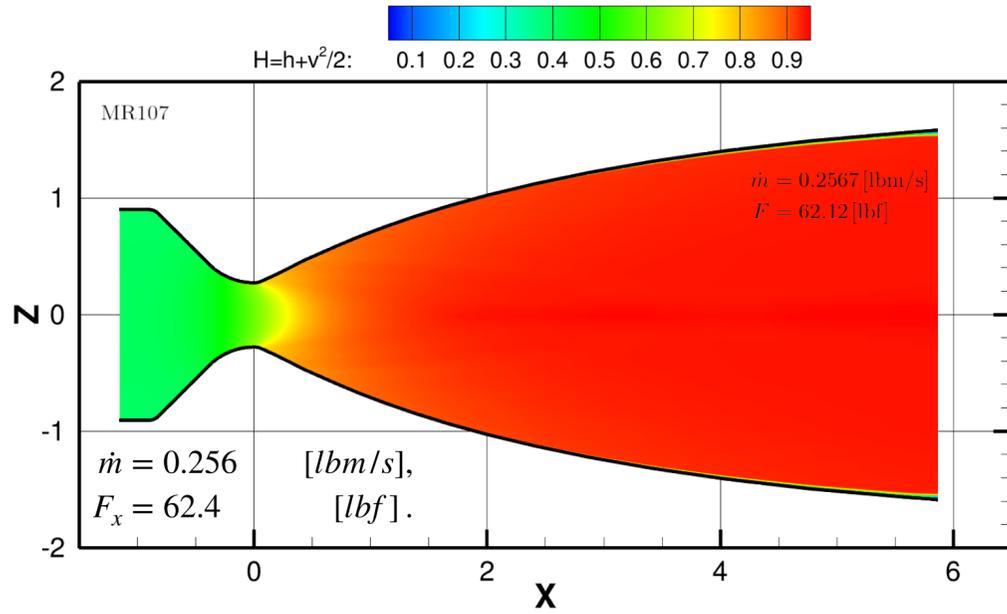


- Comparable nozzle temperature with either approaches.
- Cooler vibrational temperature with the multimode relaxation approach compared with a lump sum approach and single characteristic temperature of 3500K.
- Comparable enthalpy with either of the approaches.

# Test Case 1: MR107

## One-Temperature

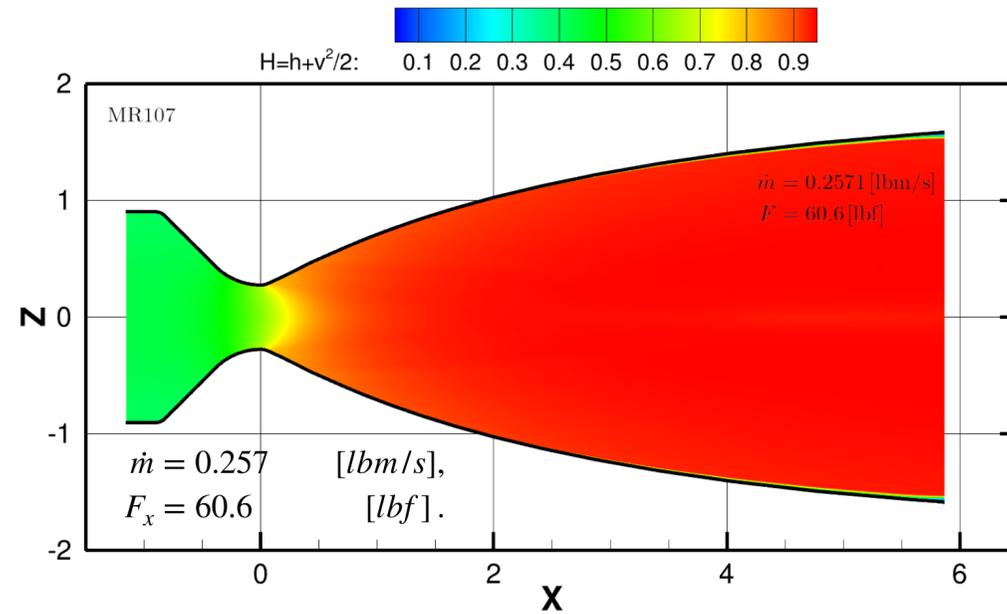
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## Two-Temperature w/lump sum relaxation time

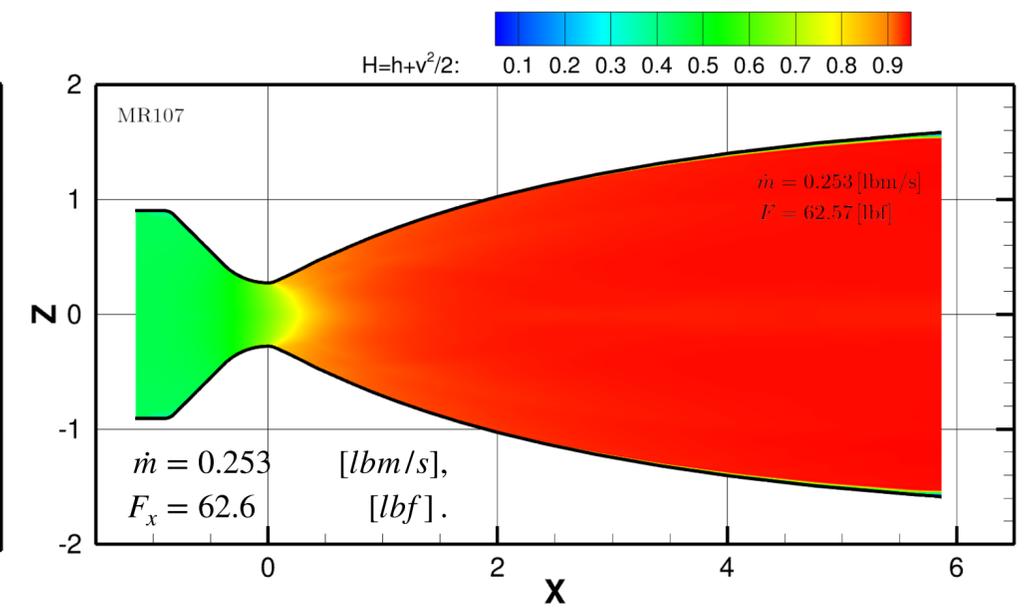
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$P_0 = 147$  [psia],  $T_0 = 1365$  [K],  $c_{NH_3} = 0.427$ ,  $c_{N_2} = 0.525$ ,  $c_{H_2} = 0.048$   
 $T_w$  : Radiative Equilibrium,  $\epsilon = 0.35$ ,  $2T$  w/lump sum relaxation time

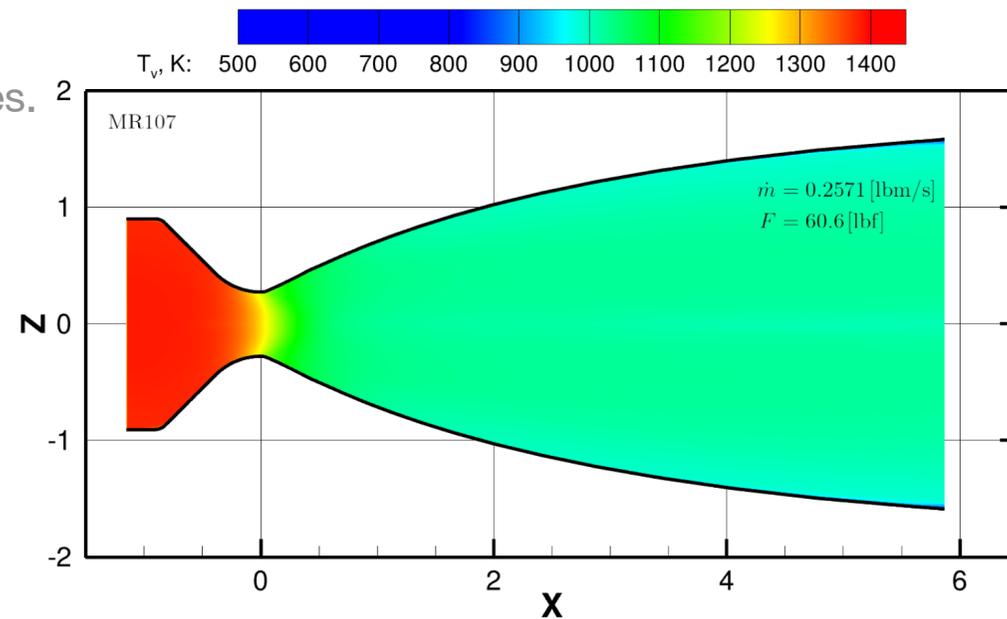


## Two-Temperature w/multimode relaxation time

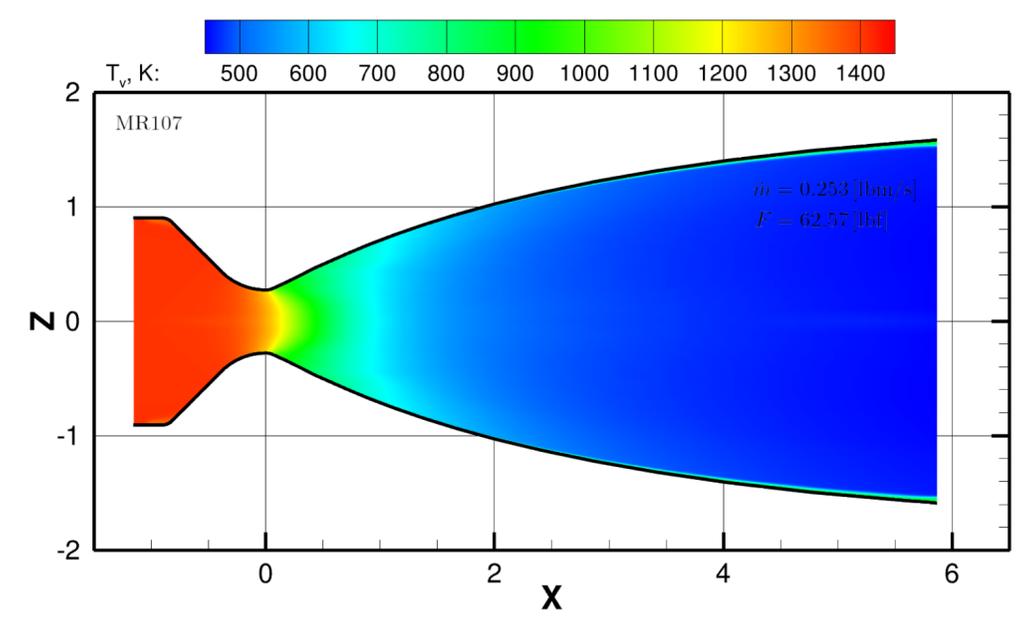
$P_0 = 147$  [psia],  $T_0 = 1365$  [K],  $c_{NH_3} = 0.427$ ,  $c_{N_2} = 0.525$ ,  $c_{H_2} = 0.048$   
 $T_w$  : Radiative Equilibrium,  $\epsilon = 0.35$ ,  $2T$  w/multivibrational mode relaxation time



$P_0 = 147$  [psia],  $T_0 = 1365$  [K],  $c_{NH_3} = 0.427$ ,  $c_{N_2} = 0.525$ ,  $c_{H_2} = 0.048$   
 $T_w$  : Radiative Equilibrium,  $\epsilon = 0.35$ ,  $2T$  w/lump sum relaxation time



$P_0 = 147$  [psia],  $T_0 = 1365$  [K],  $c_{NH_3} = 0.427$ ,  $c_{N_2} = 0.525$ ,  $c_{H_2} = 0.048$   
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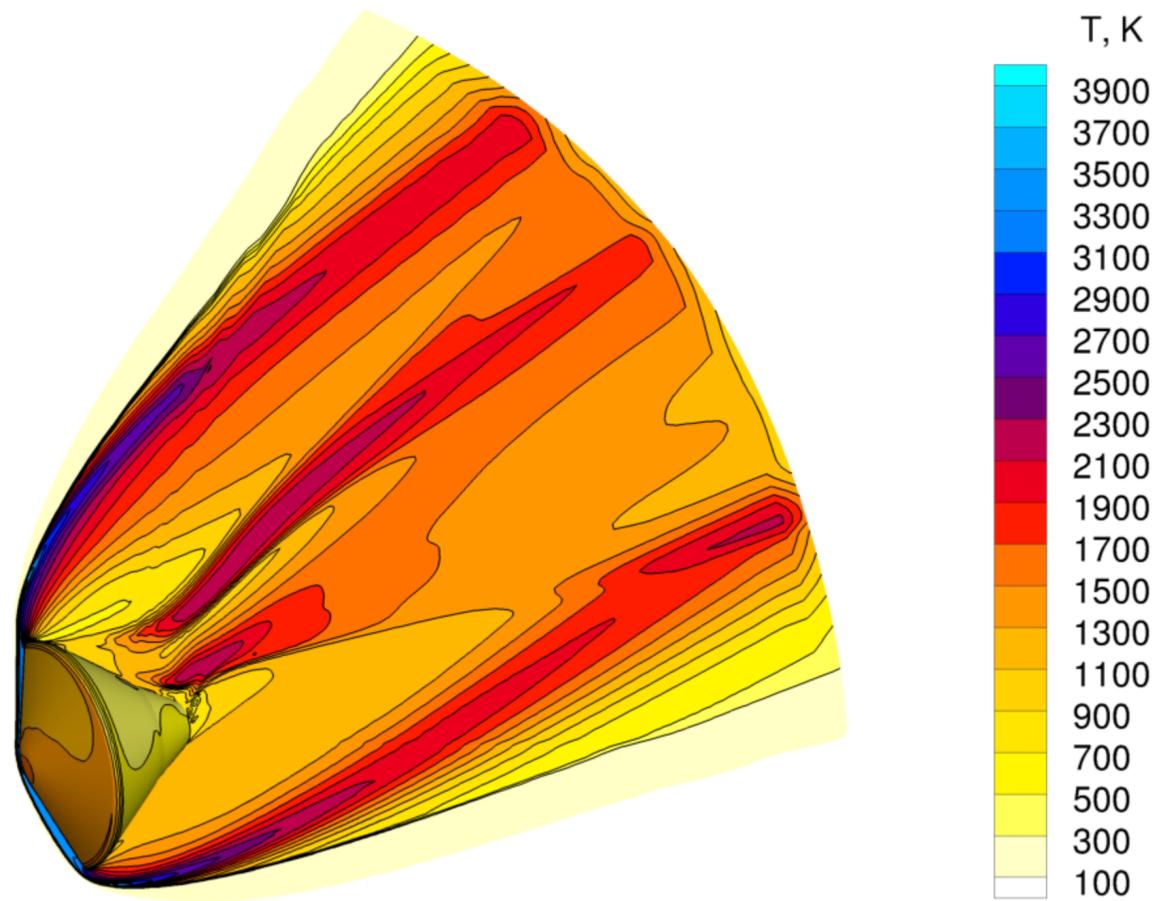
- Comparable nozzle temperature with either approaches.
- Cooler vibrational temperature with the multimode relaxation approach compared with a lump sum approach and single characteristic temperature of 3500K.
- Comparable enthalpy with either of the approaches.

# Test Case 2: SRL



### Park model

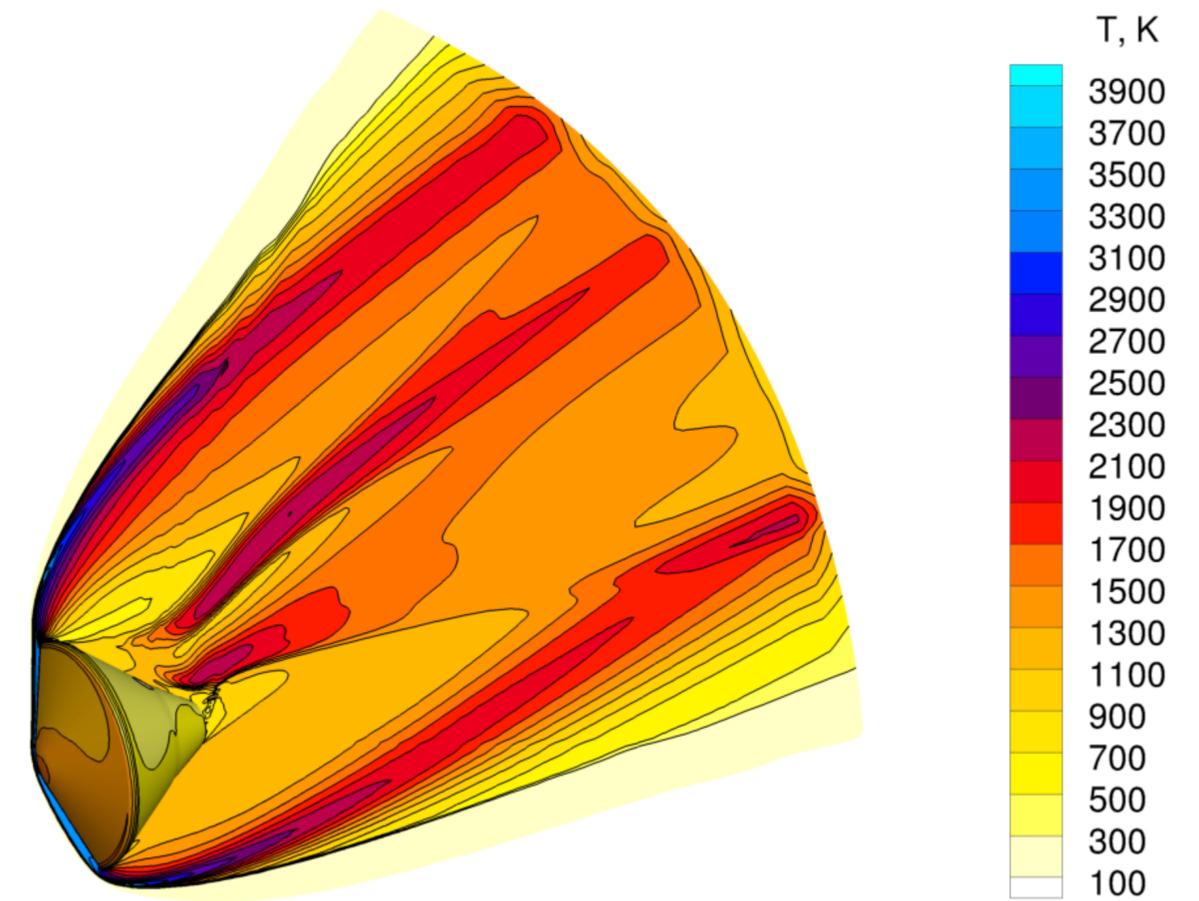
Laminar Half-Body Fully-Catalytic w/o CO2 Catalysis  
 $V_\infty = 4710.07 \text{ m/s}$ ;  $\rho_\infty = 1.2\text{e-}3 \text{ kg/m}^3$ ;  $T_\infty = 178.54 \text{ K}$ ;  $\alpha = -15.0^\circ$ ;  
 $\epsilon = 0.85$



Park et al., JTHT, 8(1994), pp.9-23

### Present

Laminar Half-Body Fully-Catalytic w/o CO2 Catalysis  
 $V_\infty = 4710.07 \text{ m/s}$ ;  $\rho_\infty = 1.2\text{e-}3 \text{ kg/m}^3$ ;  $T_\infty = 178.54 \text{ K}$ ;  $\alpha = -15.0^\circ$ ;  
 $\epsilon = 0.85$



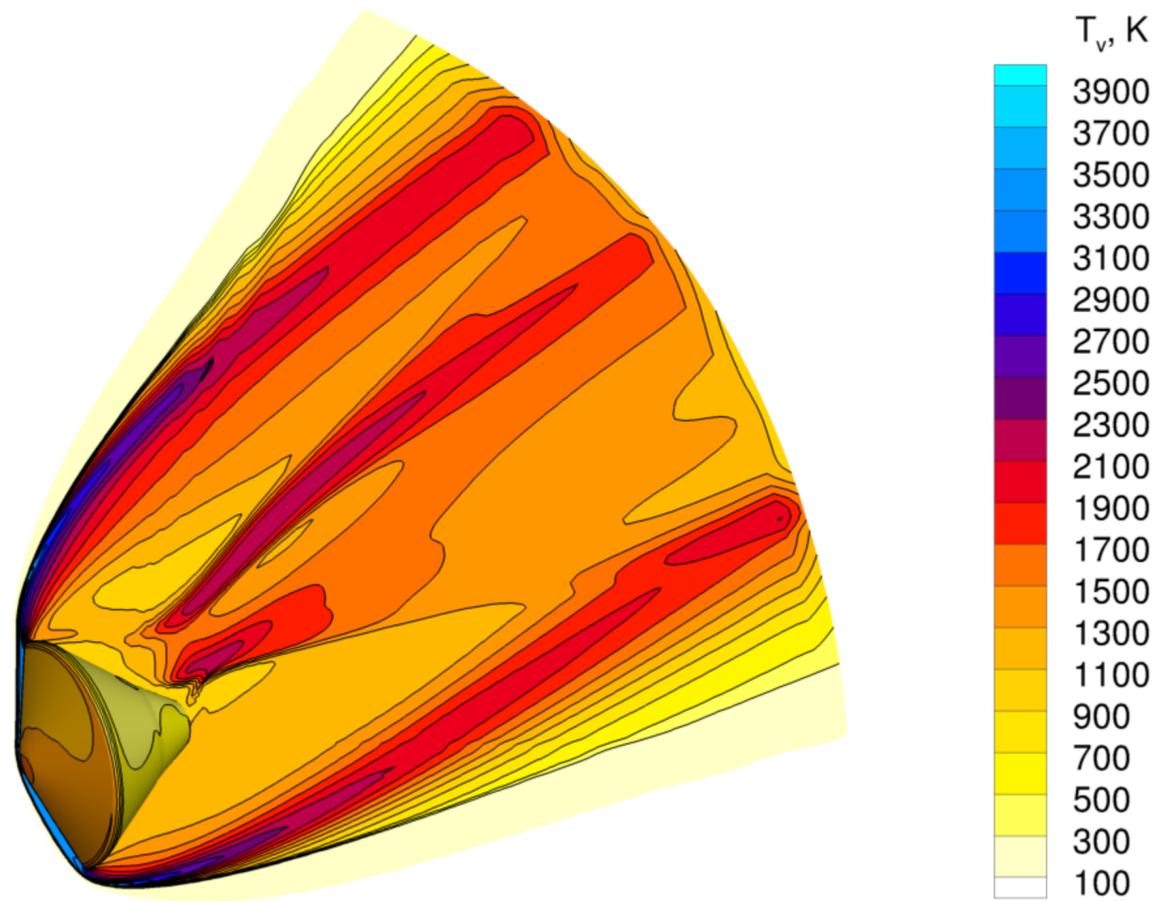
Present: multivibrational mode

# Test Case 2: SRL



### Park model

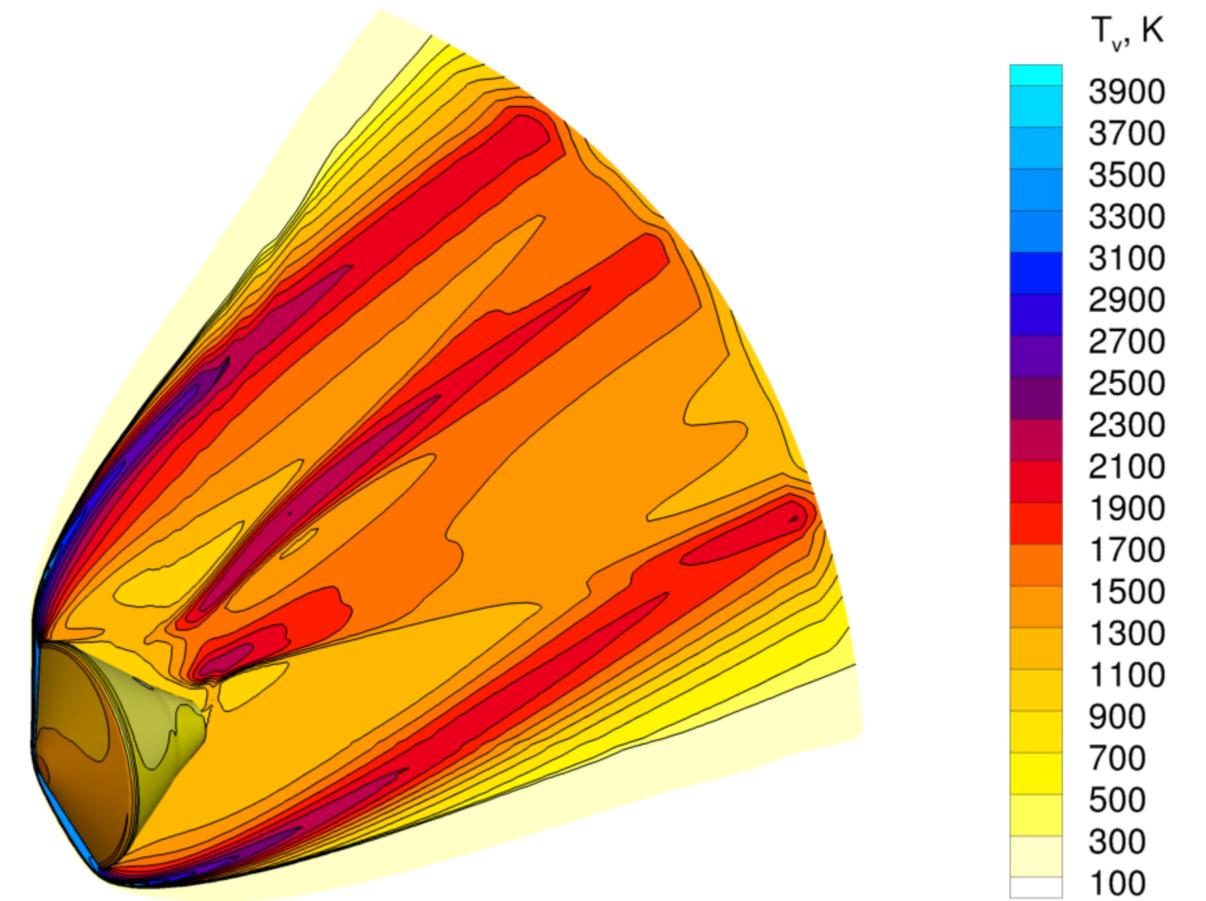
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 $V_\infty = 4710.07 \text{ m/s}$ ;  $\rho_\infty = 1.2\text{e-}3 \text{ kg/m}^3$ ;  $T_\infty = 178.54 \text{ K}$ ;  $\alpha = -15.0^\circ$ ;  
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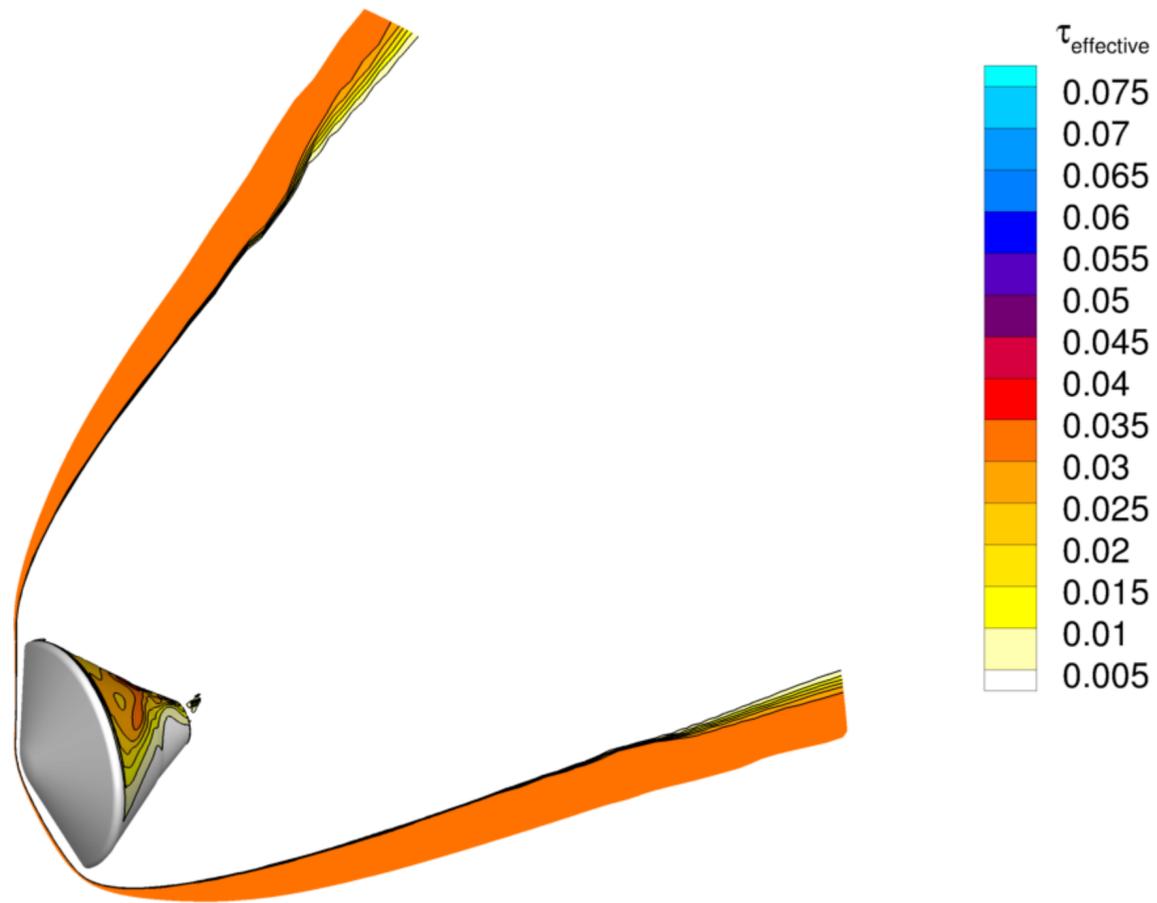
Present: multivibrational mode

# Test Case 2: SRL



### Park model

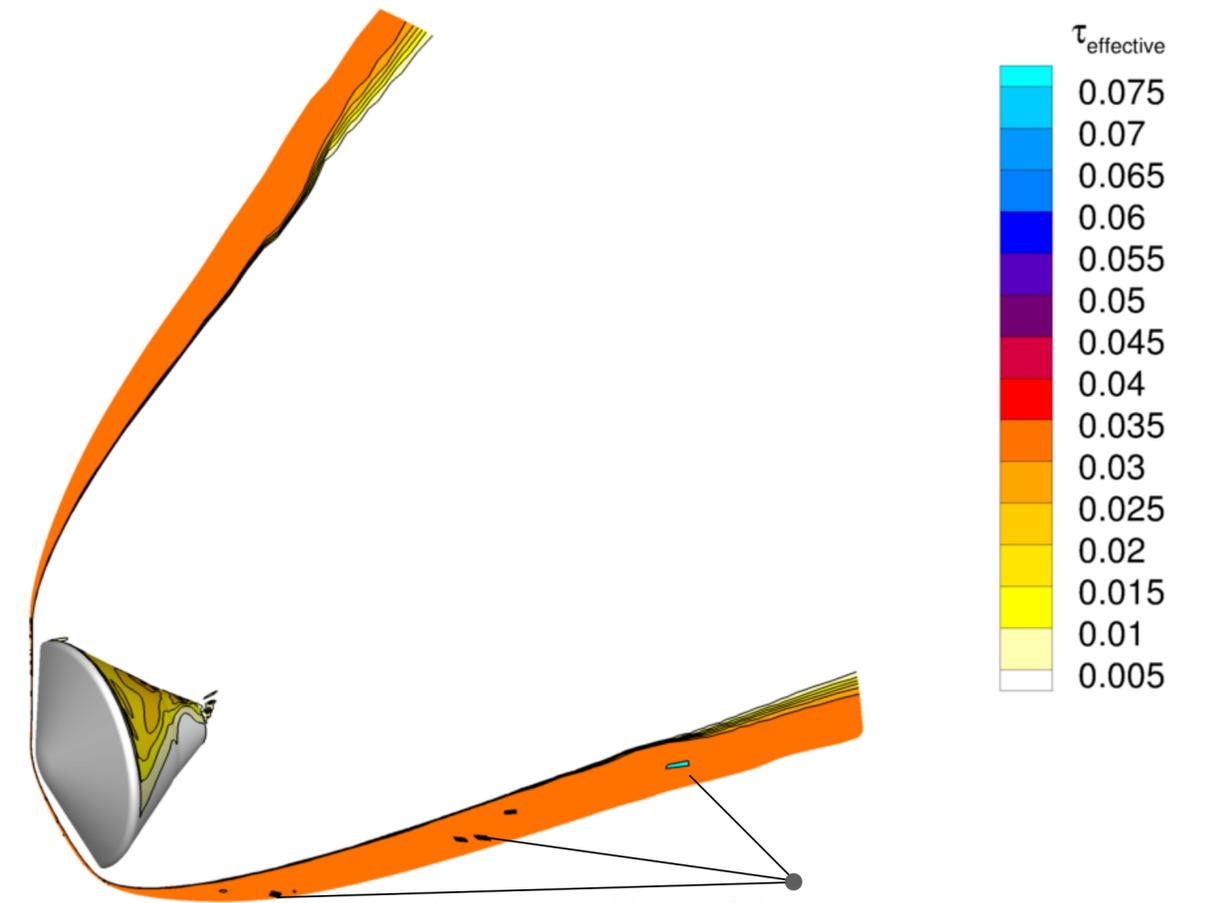
Laminar Half-Body Fully-Catalytic w/o CO2 Catalysis  
 $V_\infty = 4710.07 \text{ m/s}$ ;  $\rho_\infty = 1.2\text{e-}3 \text{ kg/m}^3$ ;  $T_\infty = 178.54 \text{ K}$ ;  $\alpha = -15.0^\circ$ ;  
 $\epsilon = 0.85$



Park et al., JTHT, 8(1994), pp.9-23

### Present

Laminar Half-Body Fully-Catalytic w/o CO2 Catalysis  
 $V_\infty = 4710.07 \text{ m/s}$ ;  $\rho_\infty = 1.2\text{e-}3 \text{ kg/m}^3$ ;  $T_\infty = 178.54 \text{ K}$ ;  $\alpha = -15.0^\circ$ ;  
 $\epsilon = 0.85$



due to sensitivity of the model to small values 1e-20 vs. 1e-21 vs. 1e-30. These have no effect on flow and surface properties.

Present: multivibrational mode

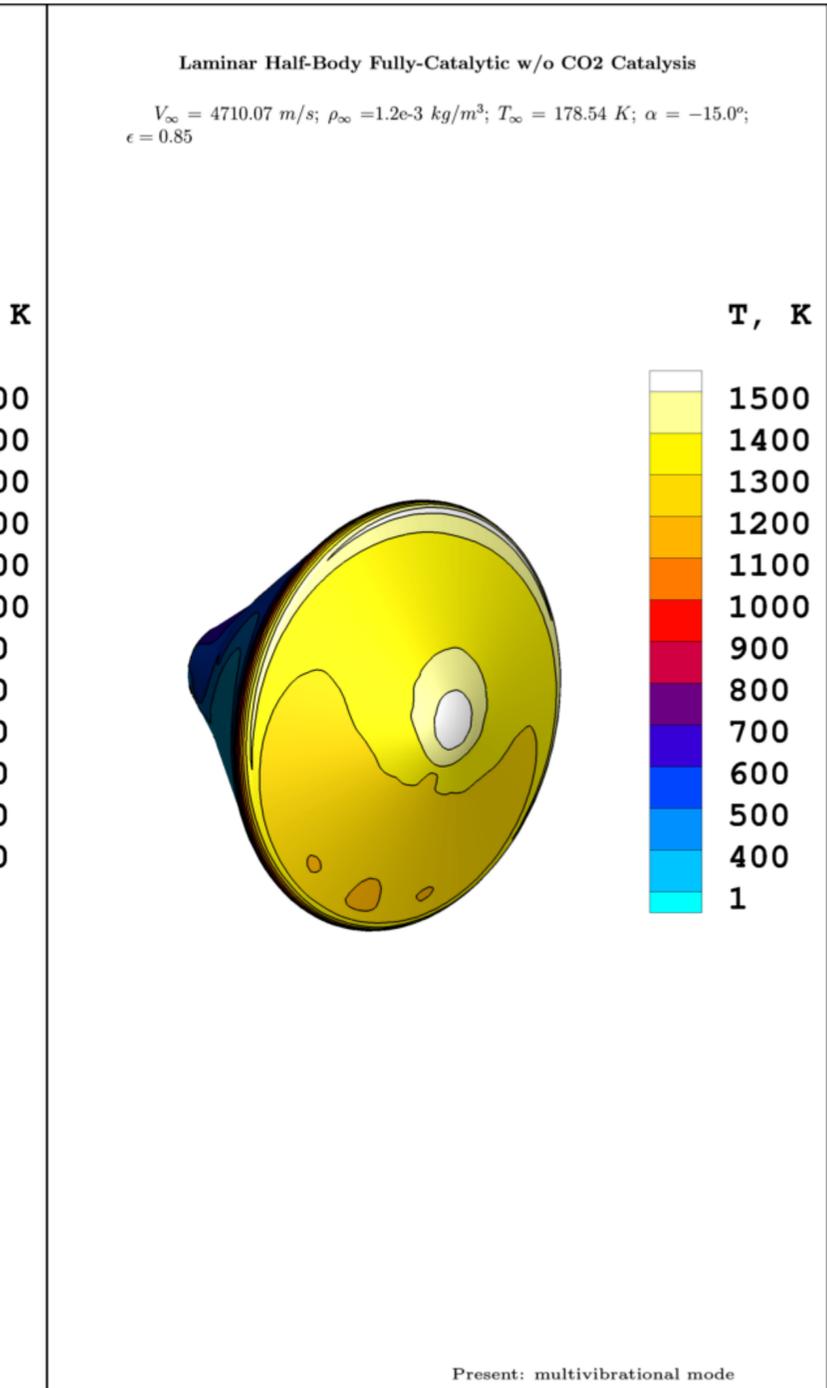
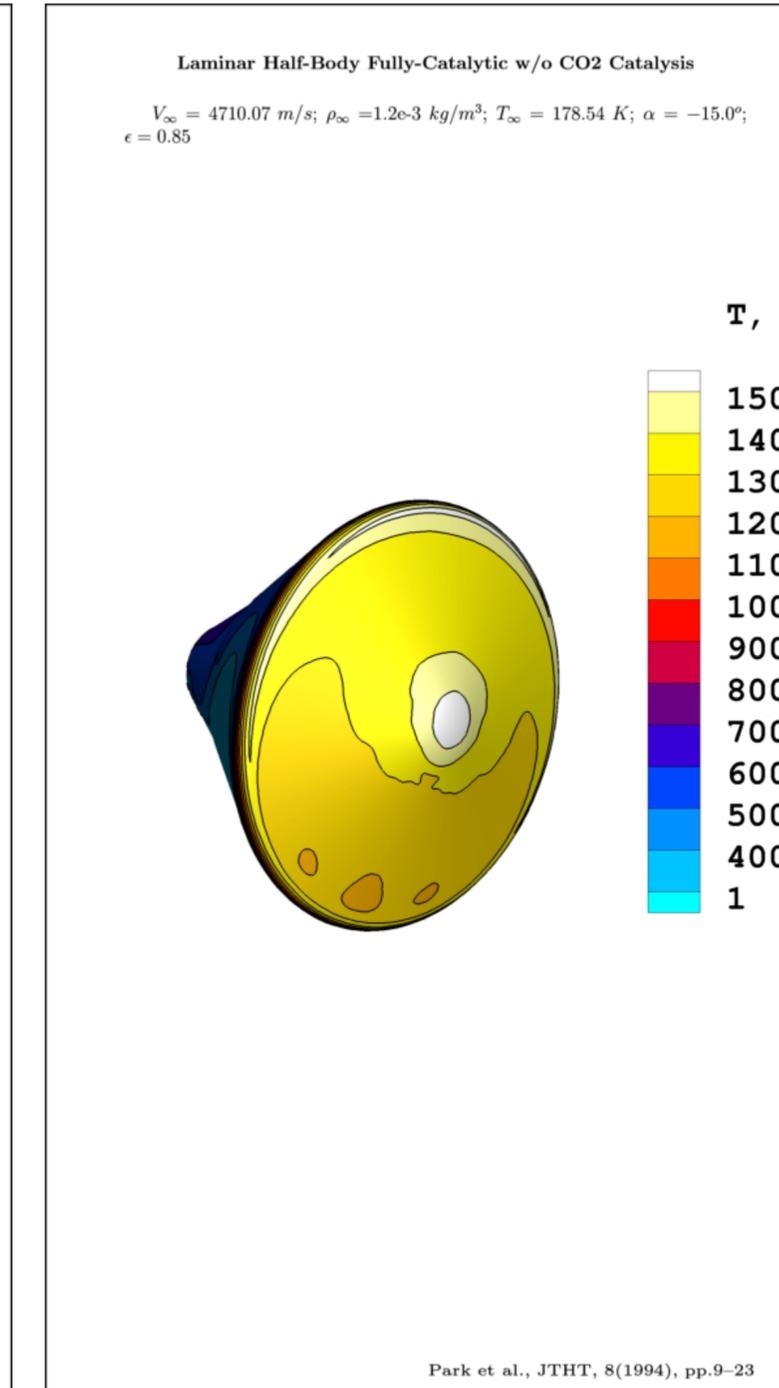
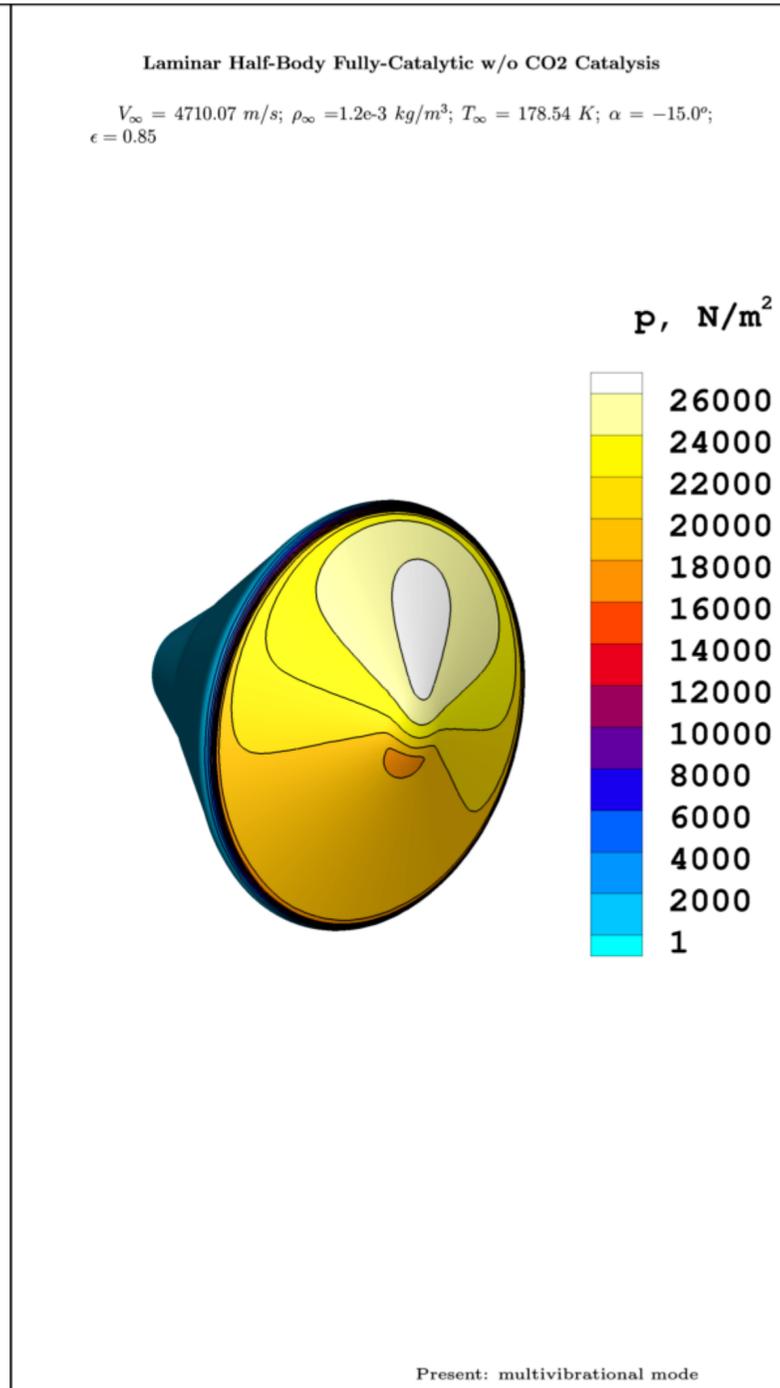
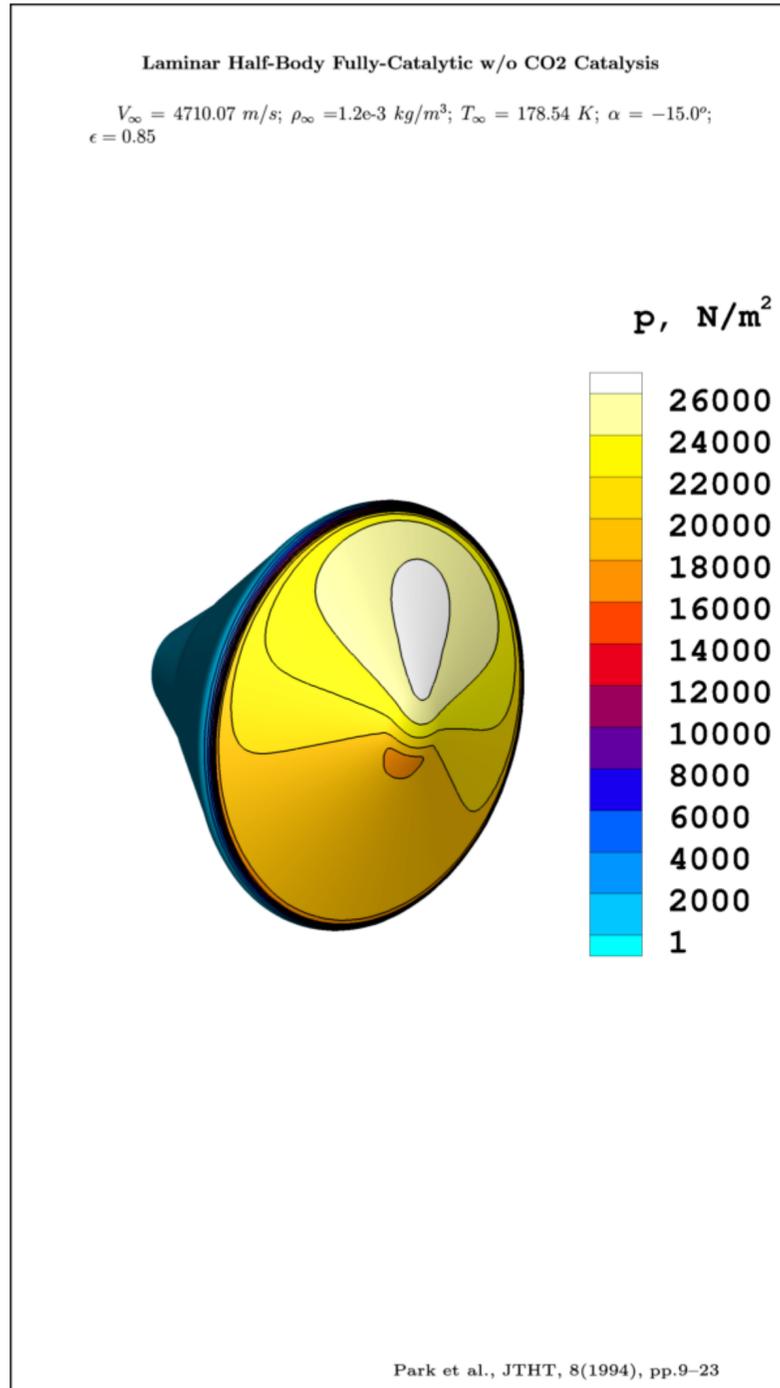
# Test Case 2: SRL

**Park model**

**Present**

**Park model**

**Present**



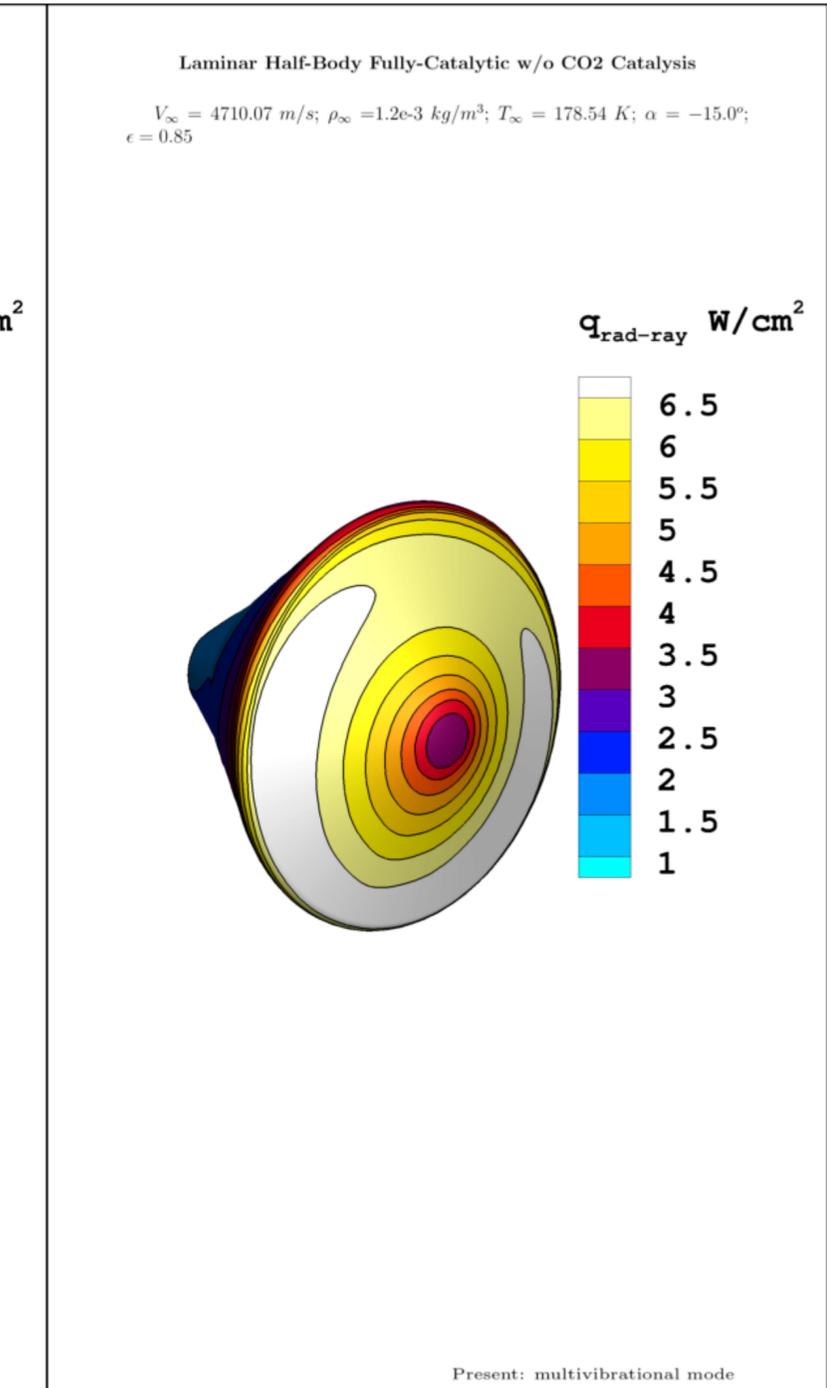
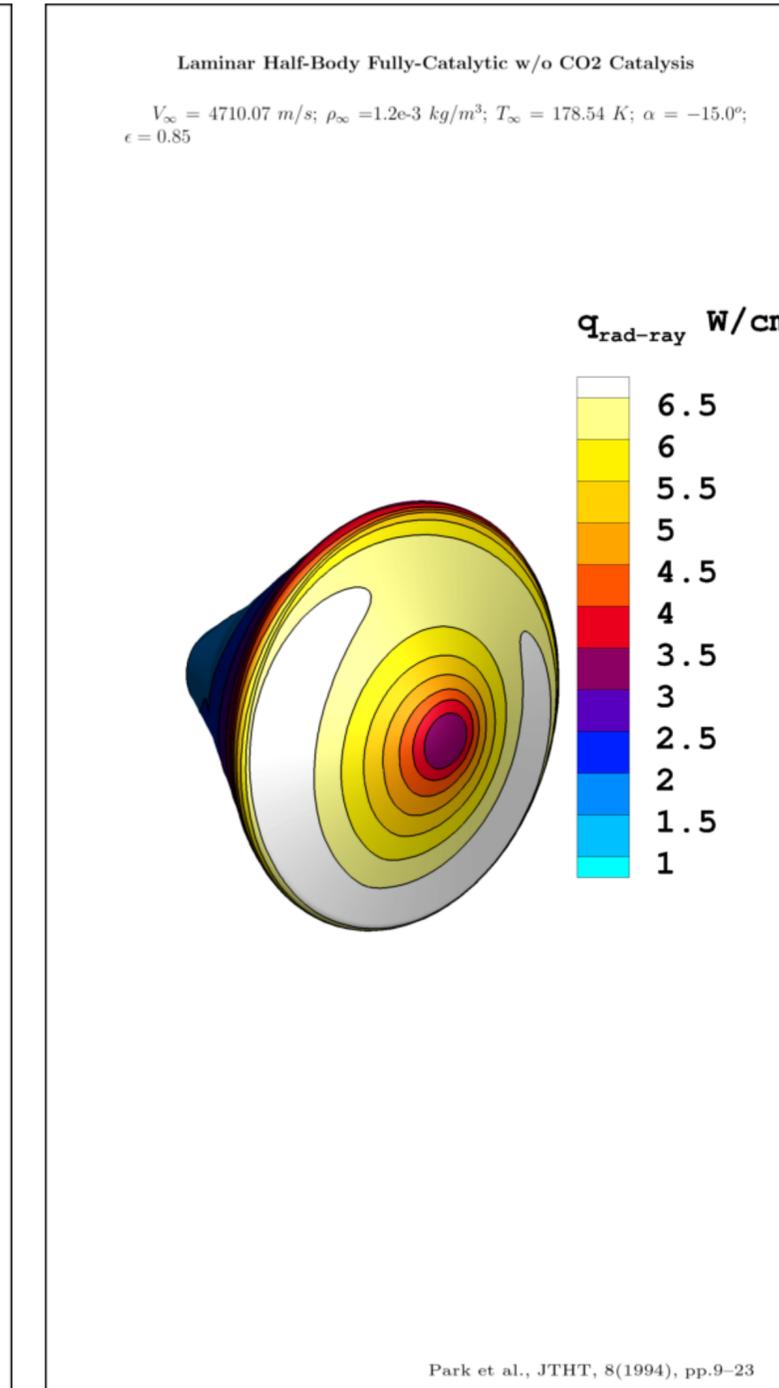
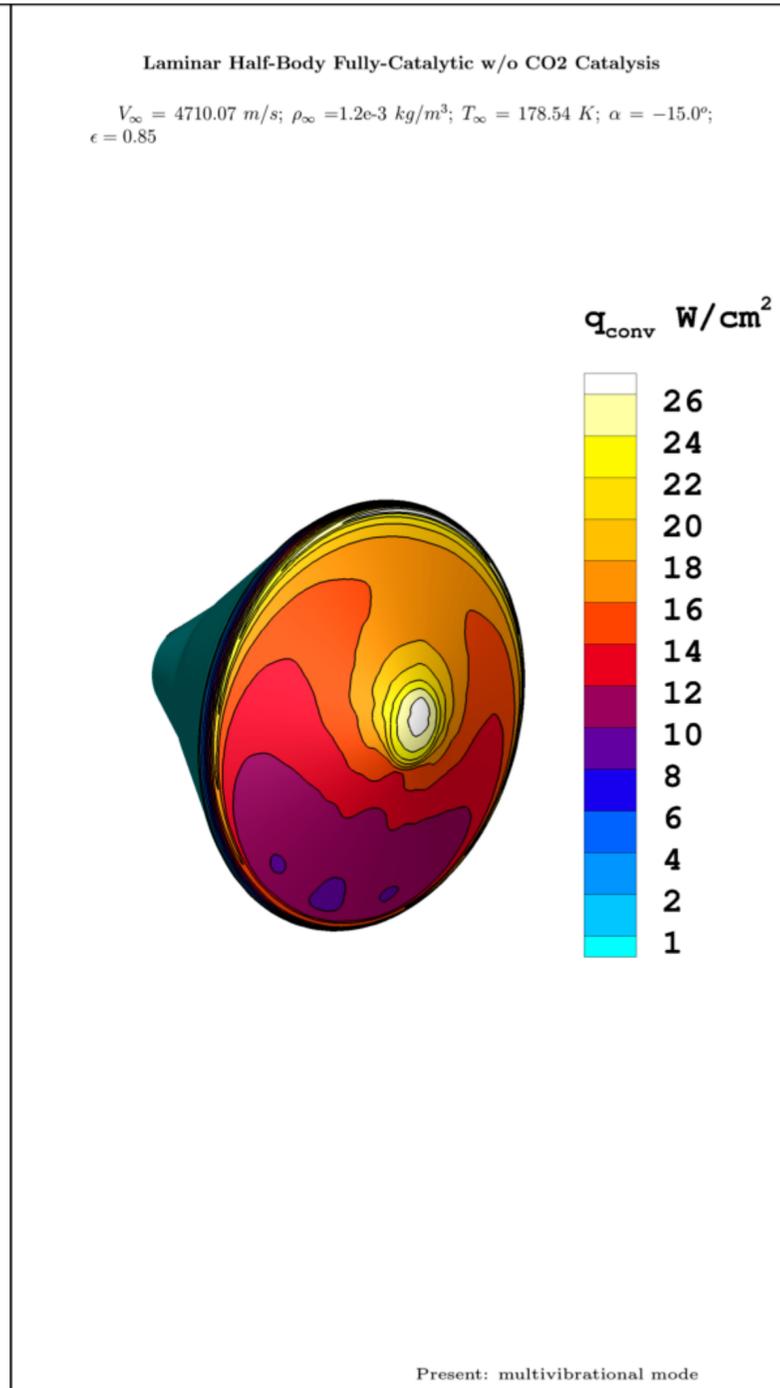
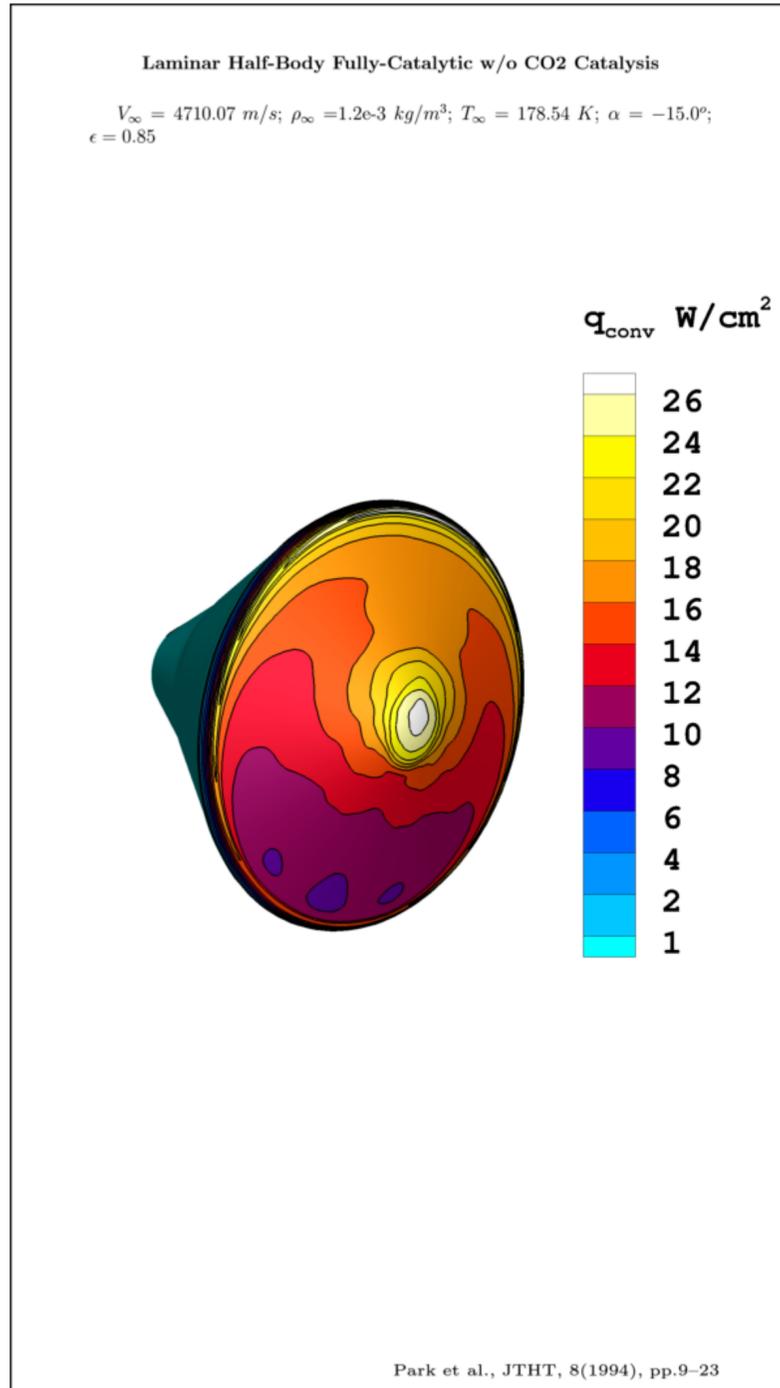
# Test Case 2: SRL

**Park model**

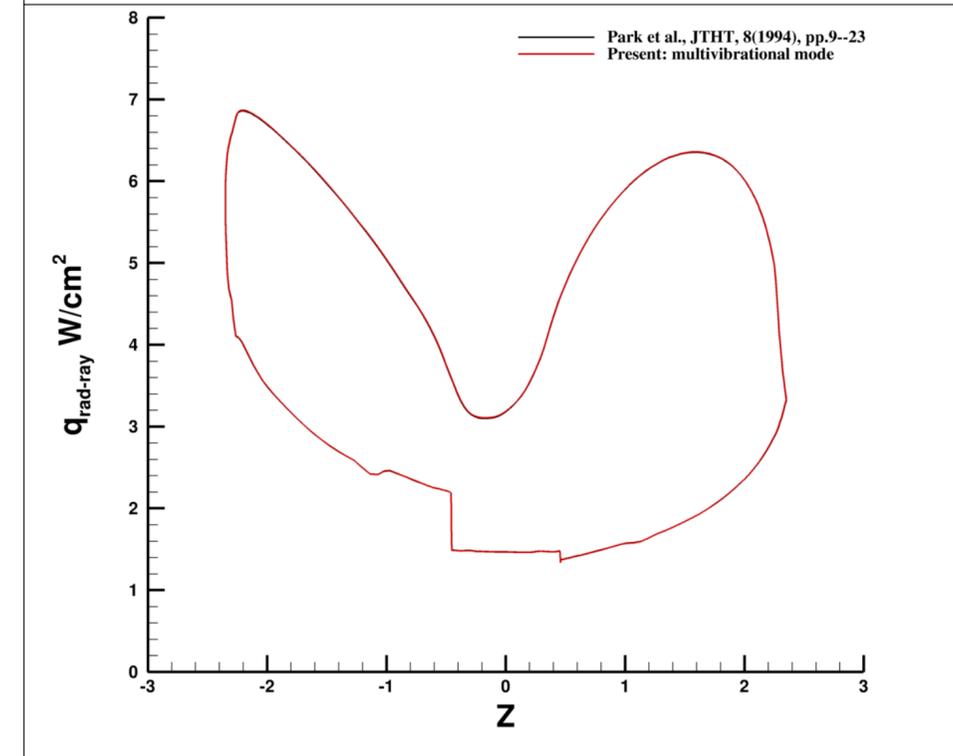
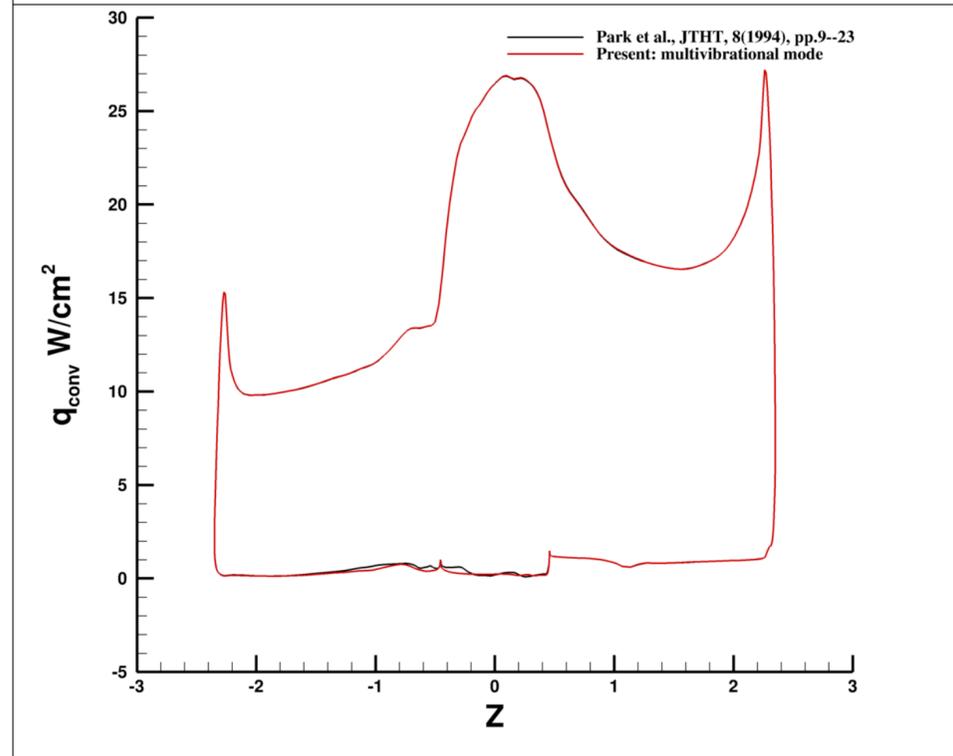
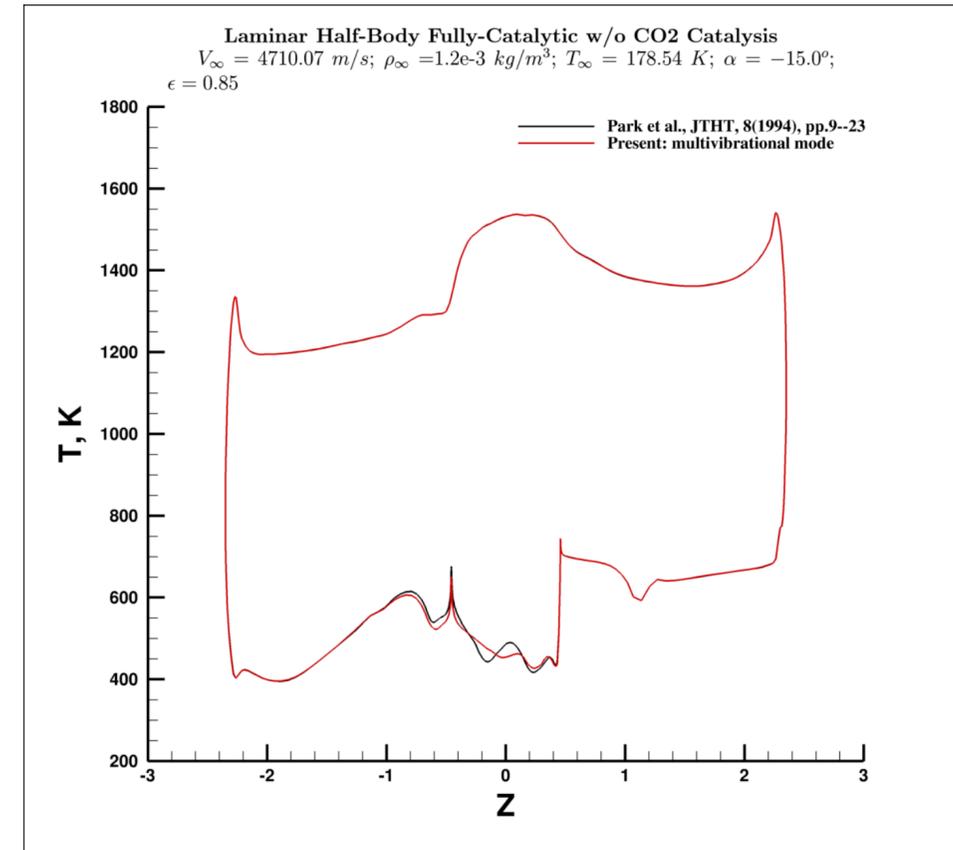
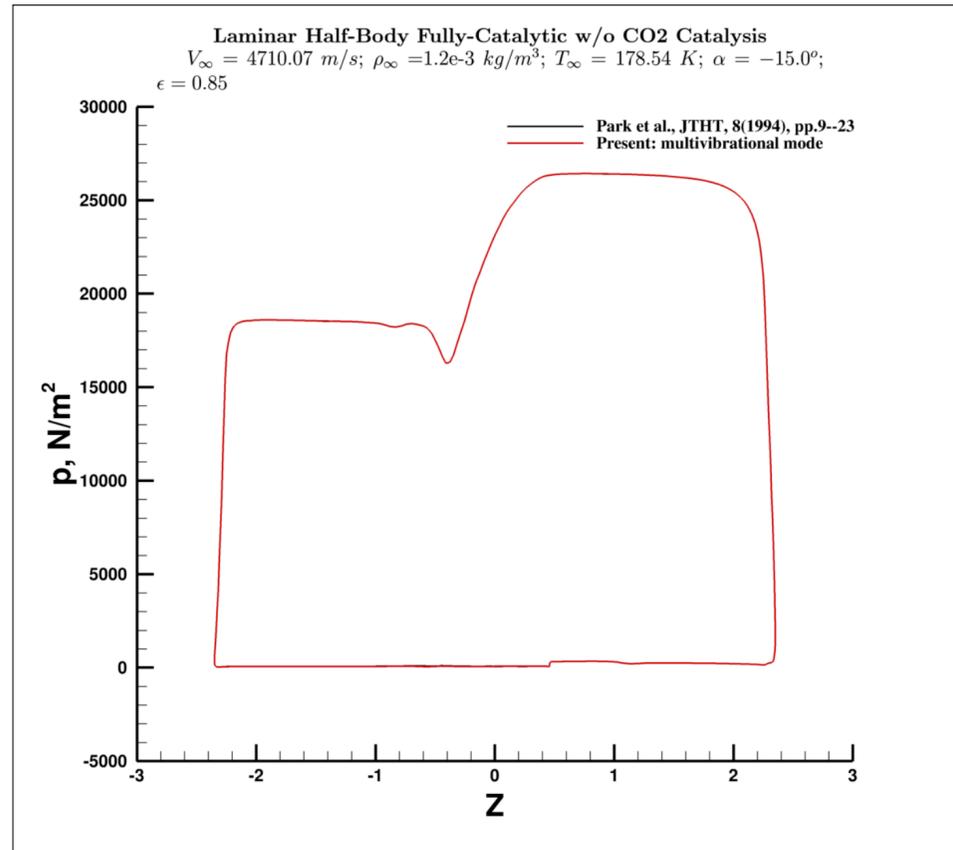
**Present**

**Park model**

**Present**



# Test Case 2: SRL





## A (revised) lump sum approach ( not suitable for multimode ) – continued

The  $\tau_s^{MW}$  relaxation time is a function of colliding species, and therefore must be weighted to arrive at a single valued relaxation time for molecule species  $s$ , before being added to  $\tau_s^{Park}$ . In LAURA v  $\geq 5$ , the weighted harmonic mean is used:

$$\tau_s^{MW} := \frac{\sum_{r \neq e^-} n_r}{\sum_{r \neq e^-} n_r / \tau_{sr}^{MW}} = \frac{\sum_{r \neq e^-} \frac{\rho_r}{Mw_r}}{\sum_{r \neq e^-} \frac{\rho_r}{Mw_r \tau_{sr}^{MW}}}.$$

This formulation does support a direct derivation from the Boltzmann equation and the Chapman-Enskog framework, which after applying the Landau-Teller's main assumption of weak deviations from the equilibrium, and further assuming that the specific heat is constant gives:

$$\frac{dE_v}{dt} = \sum_s \left( \rho_s (ev_s^0 - ev_s) \sum_r \frac{n_r}{n \tau_{rs}} \right).$$

In LAURA-4, LAURA-A, and at the birth of LAURA-5, a weighted mean, which is suggested by Gnoffo et al. [NASA-TP2867], was used:

$$\tau_s^{MW} := \frac{\sum_{r \neq e^-} n_r \tau_{sr}^{MW}}{n = \sum_{r \neq e^-} n_r} = \frac{\sum_{r \neq e^-} \frac{\rho_r}{Mw_r} \tau_{sr}^{MW}}{\sum_{r \neq e^-} \frac{\rho_r}{Mw_r}}.$$

The wighted mean is added to the current LAURA-5 as a user set option for comparison purposes\*. The following relation may be devised by comparing the two averaging formulations,

$$(\tau_s^{MW})^{LAURA5} < (\tau_s^{MW})^{LAURA4},$$

which suggests a larger translational-vibrational energy exchange source term with  $(\tau_s^{MW})^{LAURA5}$  averaging. The following inequality is used to arrive at the above conclusion:

$$\sum_i A_i B_i < \sum_i A_i \sum_i B_i.$$

\* : vib\_relax\_harmonic\_mean = .true. (default), which is a more correct way, may also lead to a more stiff numerical discretization.



Using zeroth-order approximation of the Chapman-Enskog, and assuming harmonic-oscillator and weak deviation from equilibrium, the following formulation is derived:

$$Src^{T \leftrightarrow V} \equiv \frac{dE_v}{dt} = \rho C_{vv} \frac{T}{T_v} \frac{(T - T_v)}{\bar{\tau}},$$

where the ratio may be used as a simple correction to the widely implemented translational-vibrational energy exchange source term.



The Millikan-White (M-W) relaxation time,  $\tau_s$ , of a molecule species  $s$  in a medium of  $r$  species gas at a total pressure of  $p$  is defined as

$$p\tau_s^{MW} = \exp [a_{sr}(T^{-1/3} - b_{sr}) - 18.42],$$

with the M-W coefficients given as

$$a_{sr} = 0.00116 \mu_{sr}^{1/2} \theta_{v,s}^{4/3} = \overbrace{0.00116 \mu_{ss}^{1/2} \theta_{v,s}^{4/3}}^{\text{laura: alantel}} \left( \frac{\mu_{sr}}{\mu_{ss}} \right)^{1/2},$$

$$b_{sr} = 0.015 \mu_{sr}^{1/4}$$

unless a more accurate set of coefficients is available; e.g., from Park [1994].

To account for the elastic collision cross section, Park [1994], proposed a correction,  $\tau_s^{Park}$ , to the M-W vibrational relaxation time as:

$$\begin{aligned} \tau_s^{Park} &= \frac{1}{\left( \sum_{r \neq e} n_r \right) \sqrt{8kT/\pi m_s} \underbrace{\sigma'(50000/T)^2}_{\sigma}} \\ &= \frac{1}{N_A \underbrace{\sqrt{8kN_A/\pi}}_{\sqrt{8\bar{R}/\pi}} \left( \sum_{r \neq e} \frac{\rho_r}{Mw_r} \right) \sqrt{T/Mw_s} \underbrace{\sigma'(50000/T)^2}_{\sigma}} \\ &= \frac{1.1412 \times 10^{-29}}{\left( \sum_{r \neq e} \frac{\rho_r}{Mw_r} \right) \sqrt{T/Mw_s} \underbrace{\sigma'(50000/T)^2}_{\sigma}} \end{aligned}$$



Requires an extra step calculation by user/developer. This is in fact a function of the molecule characteristic temperature.

**New:** to be used as legacy version.

**New:** number of **distinct** vibrational modes [integer]

```

C02
&species_properties
elec_impct_ion = 4.453 ! ?
disoc_ener = 2.96
alantel = 270.4
sigma = 5.e-20, 0., 0.
mol_wt = 44.0107
cprt0 = 3.50
multivibrational_mode_molecule = .true. !T: read multimode data
use_alantel = .true. !T: ignore multimode; F: ignore alantel
/
3
  0.49436505E+05 -0.62641160E+03  0.53017252E+01  0.25038138E-02
 -0.21273087E-06 -0.76899888E-09  0.28496778E-12  0.00000000E+00
 -0.45281985E+05 -0.70482794E+01  200.000  1000.000
  0.11769624E+06 -0.17887915E+04  0.82915232E+01 -0.92231568E-04
  0.48636769E-08 -0.18910533E-11  0.63300366E-15  0.00000000E+00
 -0.39083506E+05 -0.26526693E+02  1000.000  6000.000
 -0.15444233E+10  0.10168471E+07 -0.25614052E+03  0.33694011E-01
 -0.21811843E-05  0.69914208E-10 -0.88423515E-15  0.00000000E+00
 -0.80432145E+07  0.22541775E+04  6000.000  20000.000
3
2 932.109
1 1914.081
1 3373.804

```

**New:** for both single and multimode molecules. Millikan-White coefficients will be directly calculated via characteristic vibrational temperature

**New:** degeneracy [integer] & characteristic-vibrational-temperature [K]

## Limiting values in LAURA



$$\text{vib\_relax\_minT} = 200[K],$$

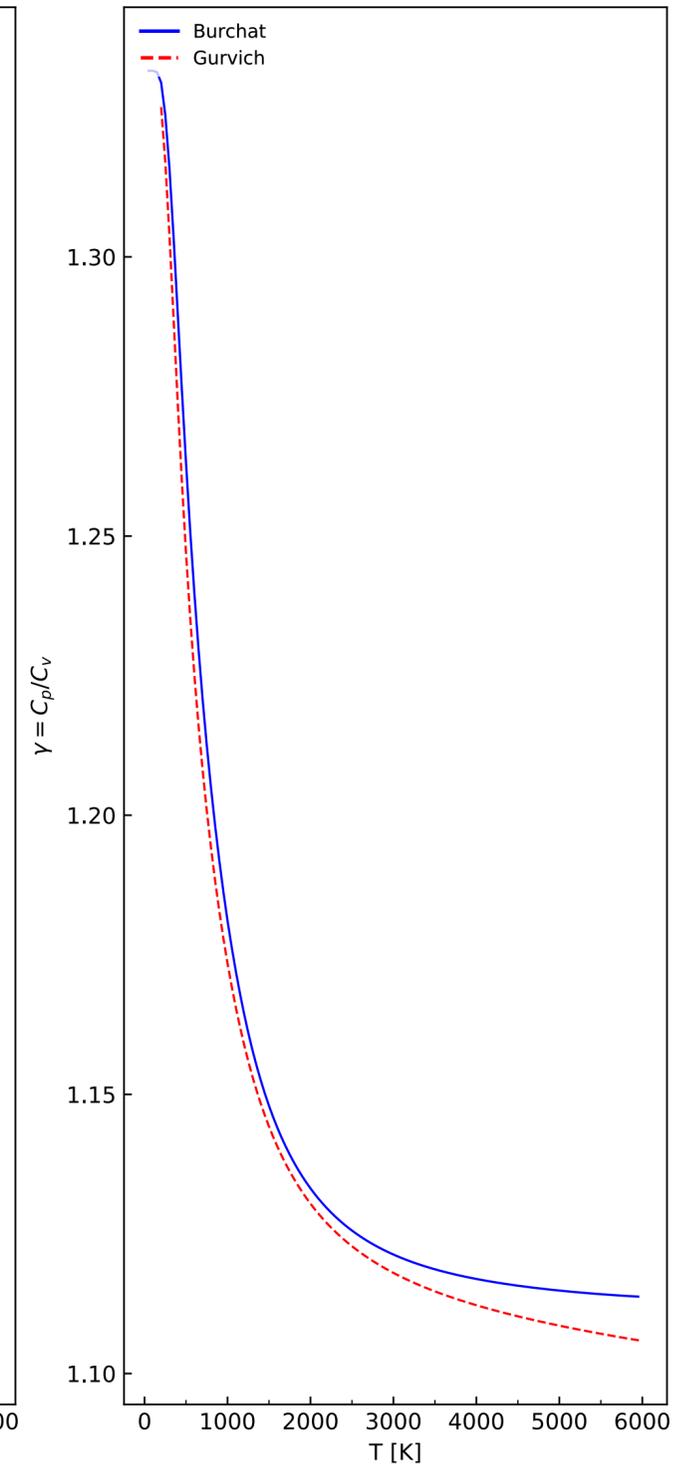
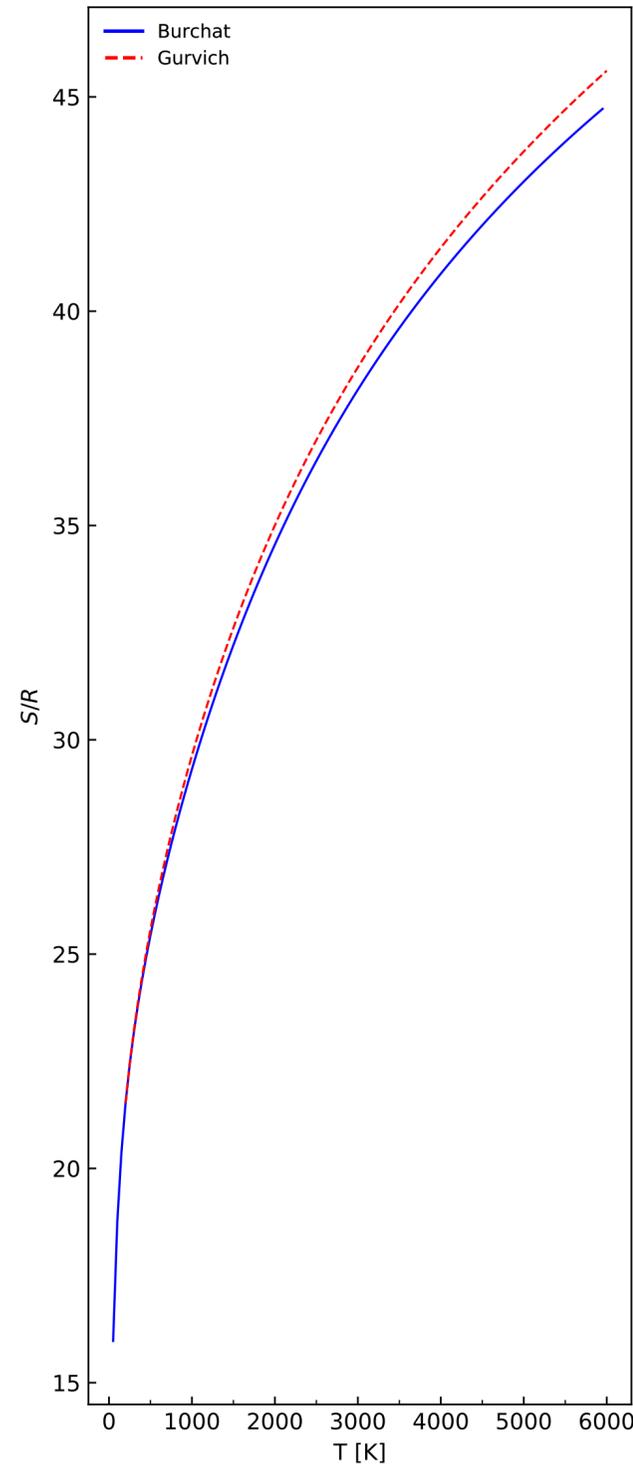
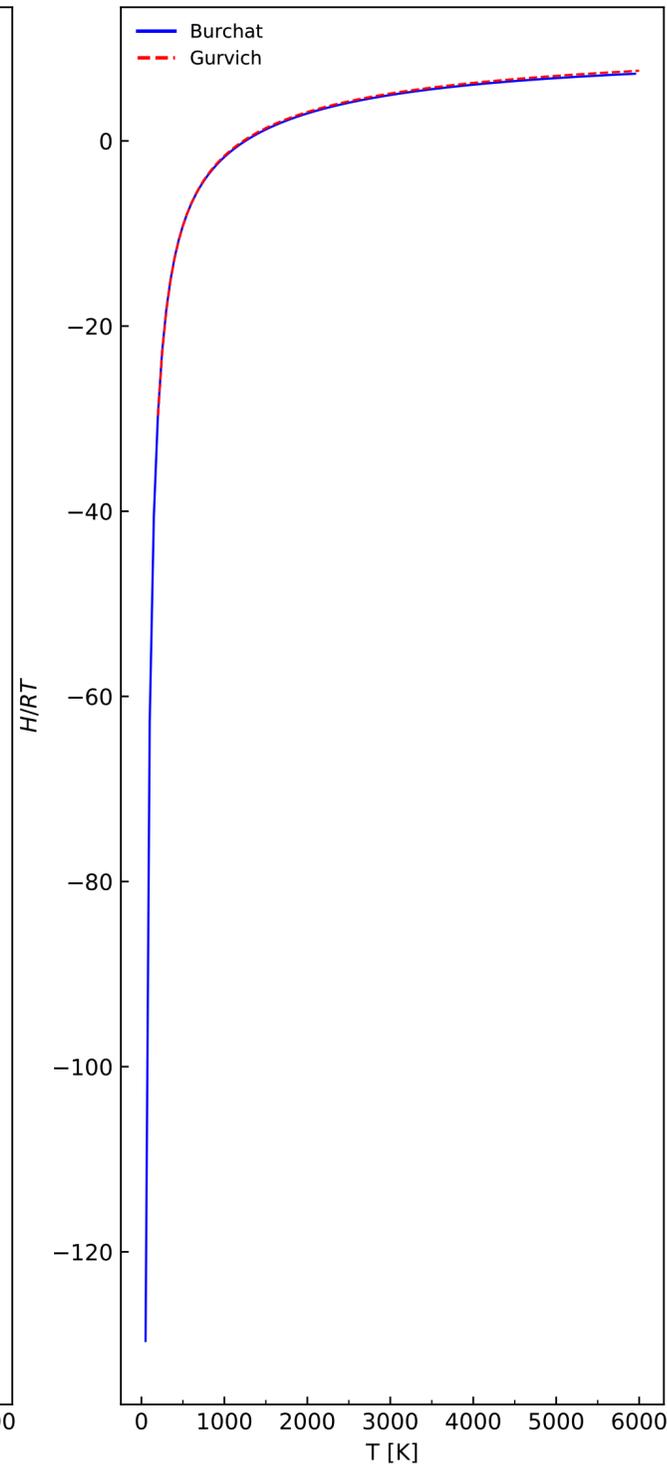
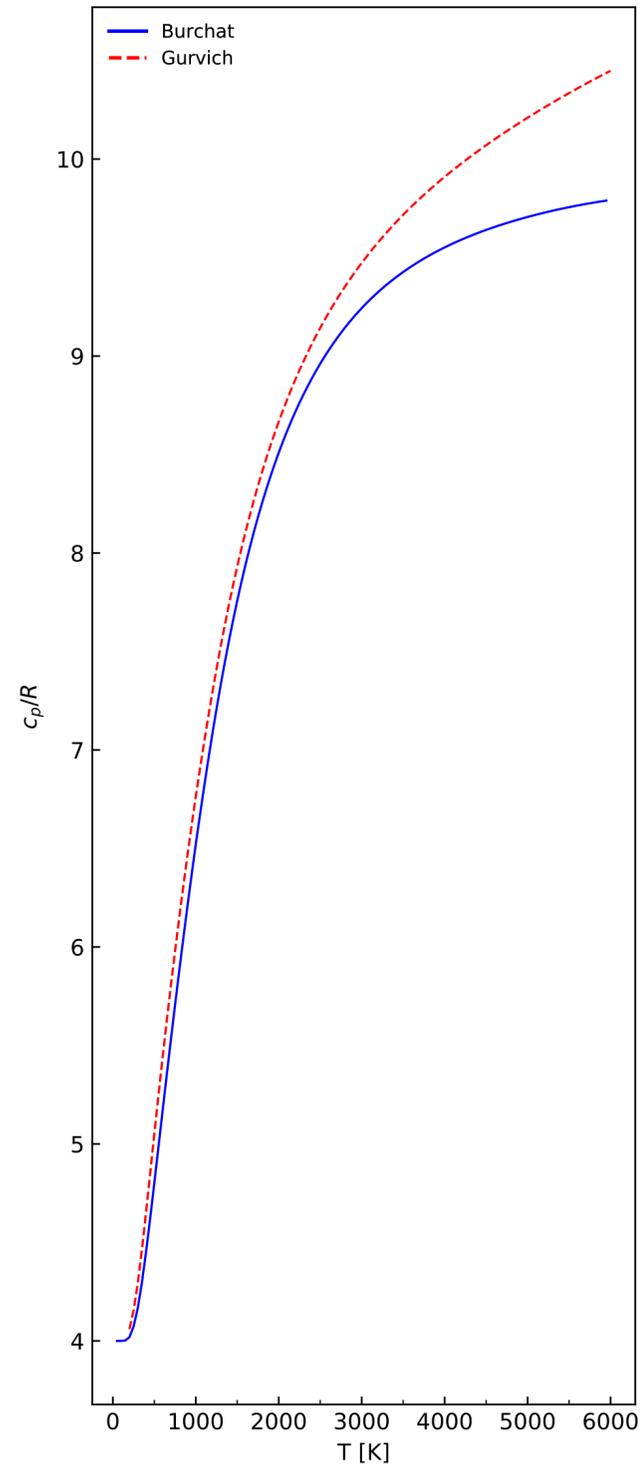
$$\text{vib\_relax\_minTv} = 200[K],$$

$$\sigma' = 10^{-21} [m^2],$$

$$\sigma'^{Park} = \begin{cases} 10^{-20} [m^2] & \text{Park 1994 - given arbitrarily;} \\ 10^{-20} [m^2] & \text{Park 1989: "the most appropriate value for N2 at T < 19000 [K]";} \\ 10^{-21} [m^2] & \text{Park 1989: for T > 19000 [K];} \end{cases}$$

$$\frac{\rho c_{vv}}{\tau} \leq 10^{20}$$

# Thermodynamic Properties of NH3



# Thermodynamic Properties Effects of NH3

The nozzle performance is nearly the same with NH3 thermodynamic properties suggested by either Gurvich or Burchat.

$P_0 = 147$  [psia],  $T_0 = 1365$  [K],  $c_{NH_3} = 0.427$ ,  $c_{N_2} = 0.525$ ,  $c_{H_2} = 0.048$

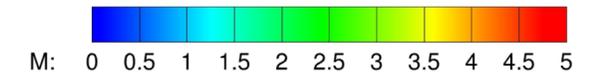
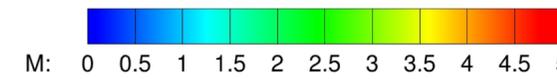
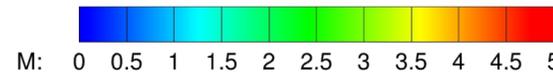
$T_w$  : Radiative Equilibrium,  $\epsilon = 0.35$

$P_0 = 147$  [psia],  $T_0 = 1365$  [K],  $c_{NH_3} = 0.427$ ,  $c_{N_2} = 0.525$ ,  $c_{H_2} = 0.048$

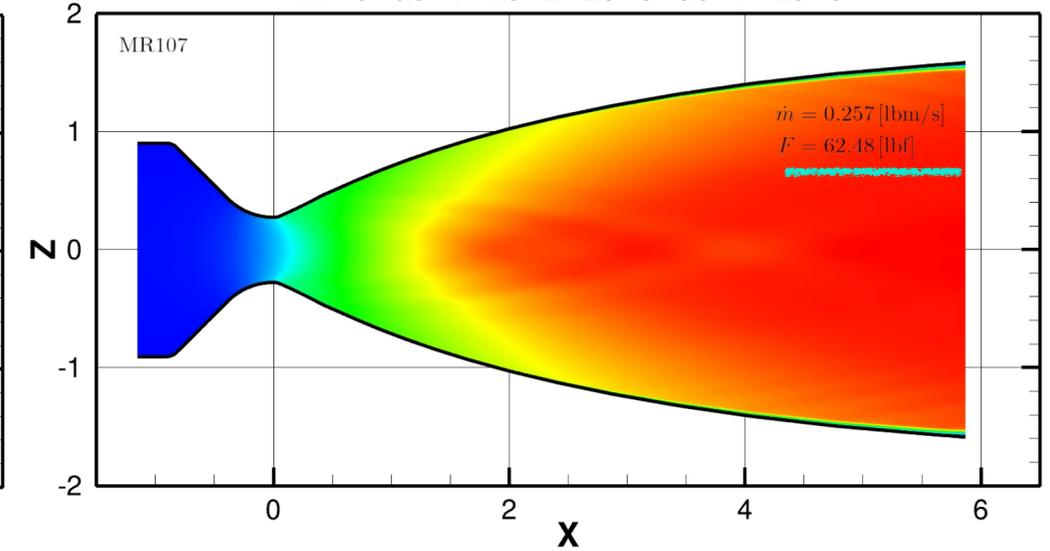
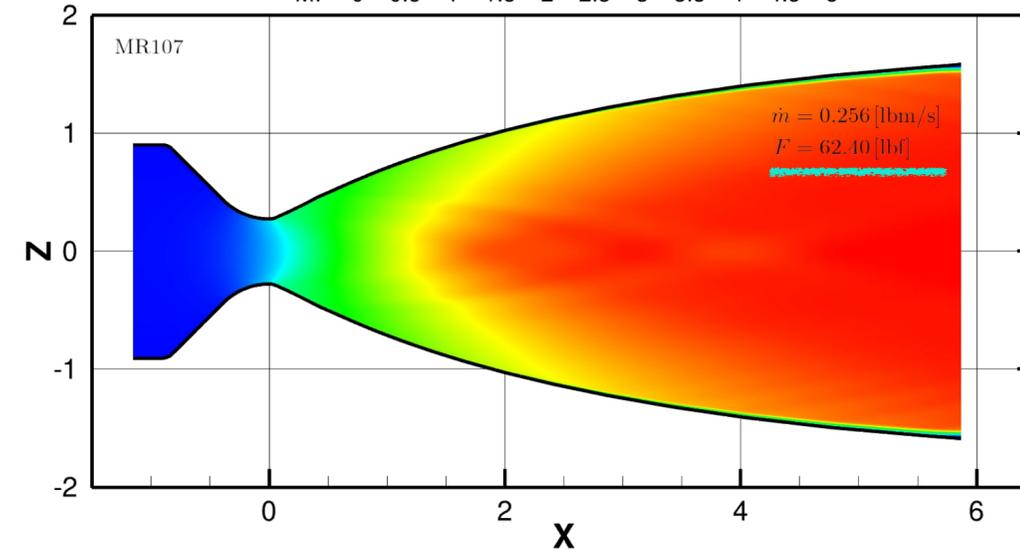
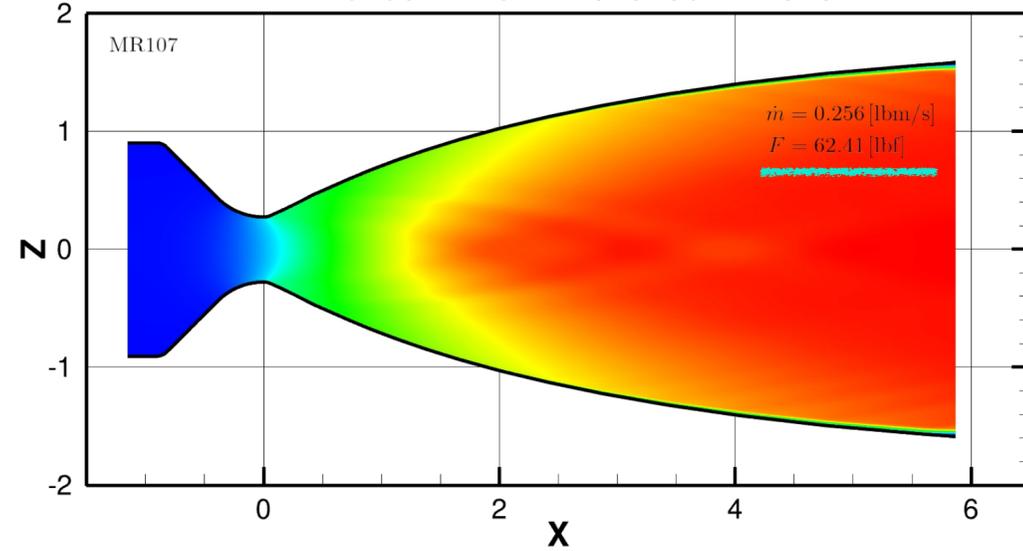
$T_w$  : Adiabatic

$P_0 = 147$  [psia],  $T_0 = 1365$  [K],  $c_{NH_3} = 0.427$ ,  $c_{N_2} = 0.525$ ,  $c_{H_2} = 0.048$

$T_w = 1000$  [K]



Gurvich



Burchat

