

Experimental Design for Composite Overwrapped Pressure Vessel Life Prediction

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NASA Strand and Vessel Testing

- NASA's Engineering Safety Center (NESC) project to assess safety of Composite Overwrapped Pressure Vessels (COPVs)
- COPVs
 - Transport gasses under high pressure
 - Metal Liner
 - Wrapped by a Series of Carbon Strands
- Research Question: **Reliability of COPVs at Use Conditions for the Expected Mission Life**
 - Primary Focus on Strands
 - Secondary Focus on Relationship to Vessels
 - Strands Less Expensive to Test
- [https://www.nasa.gov/offices/nesc/home/Feature COPVs Jan-2012.html](https://www.nasa.gov/offices/nesc/home/Feature_COPVs_Jan-2012.html)

NASA Strand and Vessel Testing

- Analyses Use Classic Weibull Model

$$R(t_i) = e^{-\left(\frac{t_i}{t_{ref}} SR^\rho\right)^\beta}$$

- Observed Life Time: t_i
- SR : Stress Ratio, ratio of stress level to strength scale parameter
- Critical Parameters:
 - ρ : Sensitivity to Stress Ratio
 - β : Shape parameter for time to Failure
 - t_{ref} : Reference time to Failure

NASA Strand Study

- Previous Strand Test
 - Relevant strand study conducted at a national lab
 - 57 strands at high loads for 10 years
 - Net information learned:
 - Strands either fail very early or
 - Last more than 10 years
 - Limited information based on 10 years of study!
- Estimates of Critical Parameters for Planning

NASA Strand Study

- Team's Initial Concept
 - Much larger study than the original 10 year study
 - Censor very early
 - Reduces time
 - Allows the larger study in a practical amount of time
- Proceed in phases
- Have detailed data records to track any problems

NASA Strand Study

Experimental Phases

- Phase A – During “shake-out” of tests rigs
- Phase B – “Gold Standard” Experiment for Strands
- Phase C – “Proof” Study
- In Parallel: Vessel Studies (Opportunistic)

Phase A

- Conducted During Shake-Out of Equipment
 - Small study (although bigger than the national lab study!)
 - Statistical goal: Determine if the parameters from the national lab study are valid as the basis for planning the larger study!
 - Note: Phase A gave the team an opportunity to re-plan the larger experiment, if necessary!

Phase B

- “Gold Standard” Experiment
 - Planned time required: 1 year
 - Used 4 “blocks” of equal numbers of strands
 - Allowed the team to correct for time effects
 - Allowed the team to mitigate problems, especially early
 - Study assumed the “classic” Weibull model
 - Size of the experiment assured ability to assess model

Observations

- Phase A: Surprisingly Similar to Initial Study
- Phase B:
 - Serious problem occurred with the gripping in the first block
 - Serious conversations with possibility of replacing!
 - Other three blocks well behaved and by themselves produced better than the planned precision for the estimates
- Final Decision: Drop the First Block

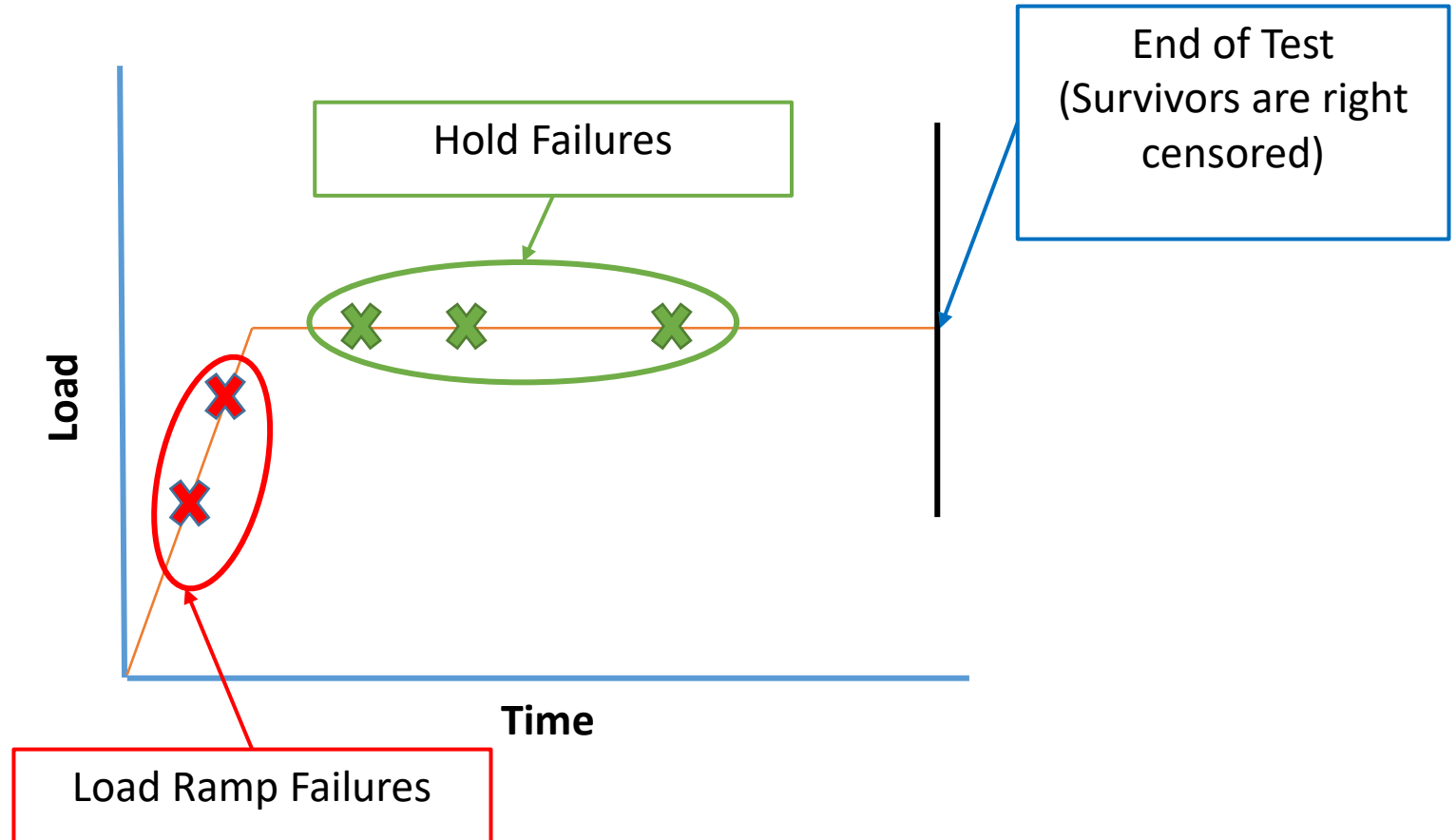
NASA Strand Study: Benefits

- Phase A:
 - Opportunity to Confirm Initial Study Parameter Estimates
 - Allowed opportunity to revise the experimental protocol if the estimates were significantly different
- Phase B:
 - Allowed opportunity to model changes in time over the year.
 - Mitigated the problem with the first block!
 - Provided simple mechanism for replacing the first block if needed!

Analysis: Stress Rupture Data

Description of Stress Rupture Test

- Stress Rupture
 - Failures occur after a period of time where there is no increase in load
- Failures are needed to determine reliability
- Must extrapolate from where test is performed versus where reliability predictions are made
- Test strands at higher loads and then extrapolate
- Need a model to make predictions



Basic Weibull Distribution

- Probability Density Function

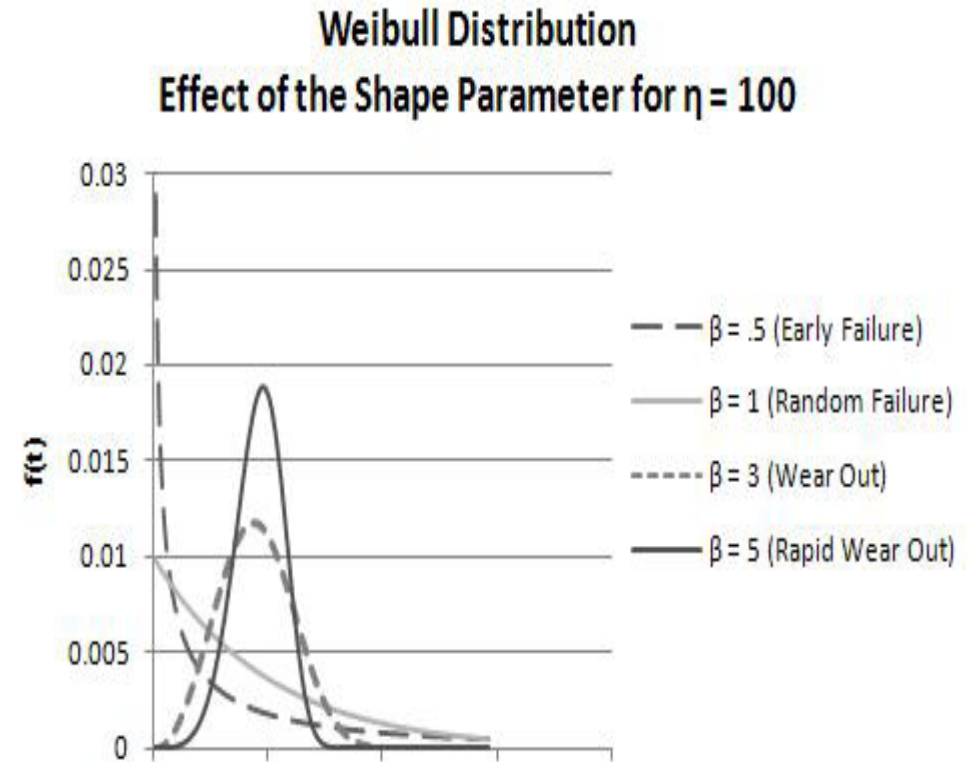
$$f(t, \beta, \eta) = \frac{\beta}{\eta} \left(\frac{t}{\eta} \right)^{\beta-1} e^{-\left(\frac{t}{\eta} \right)^{\beta}}$$

- Survivor Function = $1 - F(t)$

$$S(t) = e^{-\left(\frac{t}{\eta} \right)^{\beta}}$$

β : shape parameter

η : scale parameter (characteristic life, time where 63.2% of units will fail)



Smallest Extreme Value Distribution

- Smallest extreme value (SEV) distribution is an alternate parameterization of the Weibull distribution
- SEV represents Weibull as a log-location-scale distribution
 - If t_i is Weibull, then $\log(t_i)$ is SEV
- Weibull/SEV relationship mimics the Normal/Lognormal relationship
- Parameters
 - Log-location: $\mu = \log(\eta)$ Scale: $\sigma = \frac{1}{\beta}$
- Residuals
 - $e_i = \log t_i - \mu$
 - Scaled: $z_i = \beta e_i = \beta(\log t_i - \mu) = \frac{\log t_i - \mu}{\sigma}$
- Survivor Function
 - $S(t_i) = P(T > t_i) = e^{-e^{z_i}}$

Classic Stress Rupture Model: Weibull

- Classic Weibull Survival Function

$$S(t_i) = P(T > t_i) = e^{-\left(\frac{t_i}{t_{ref}} SR^\rho\right)^\beta} \quad \text{Note: } \eta = t_{ref} SR^{-\rho}$$

- Observed Life Time: t_i
- SR : Stress Ratio, ratio of stress level to strength scale parameter
- Critical Parameters:
 - ρ : controls the relationship between the failure time and stress ratio (SR)
 - β : Shape parameter for time to Failure
 - t_{ref} : Reference time to Failure

Classic Stress Rupture Model: SEV

- SEV Survival Function

$$S(t_i) = e^{-\left(\frac{t_i}{t_{ref}} SR^\rho\right)^\beta} = e^{-e^{\beta(\log t_i - \theta + \rho \ln(SR))}}$$

where $\theta = \log(t_{ref})$ and $\mu = \log(\eta) = \theta - \rho \ln(SR)$

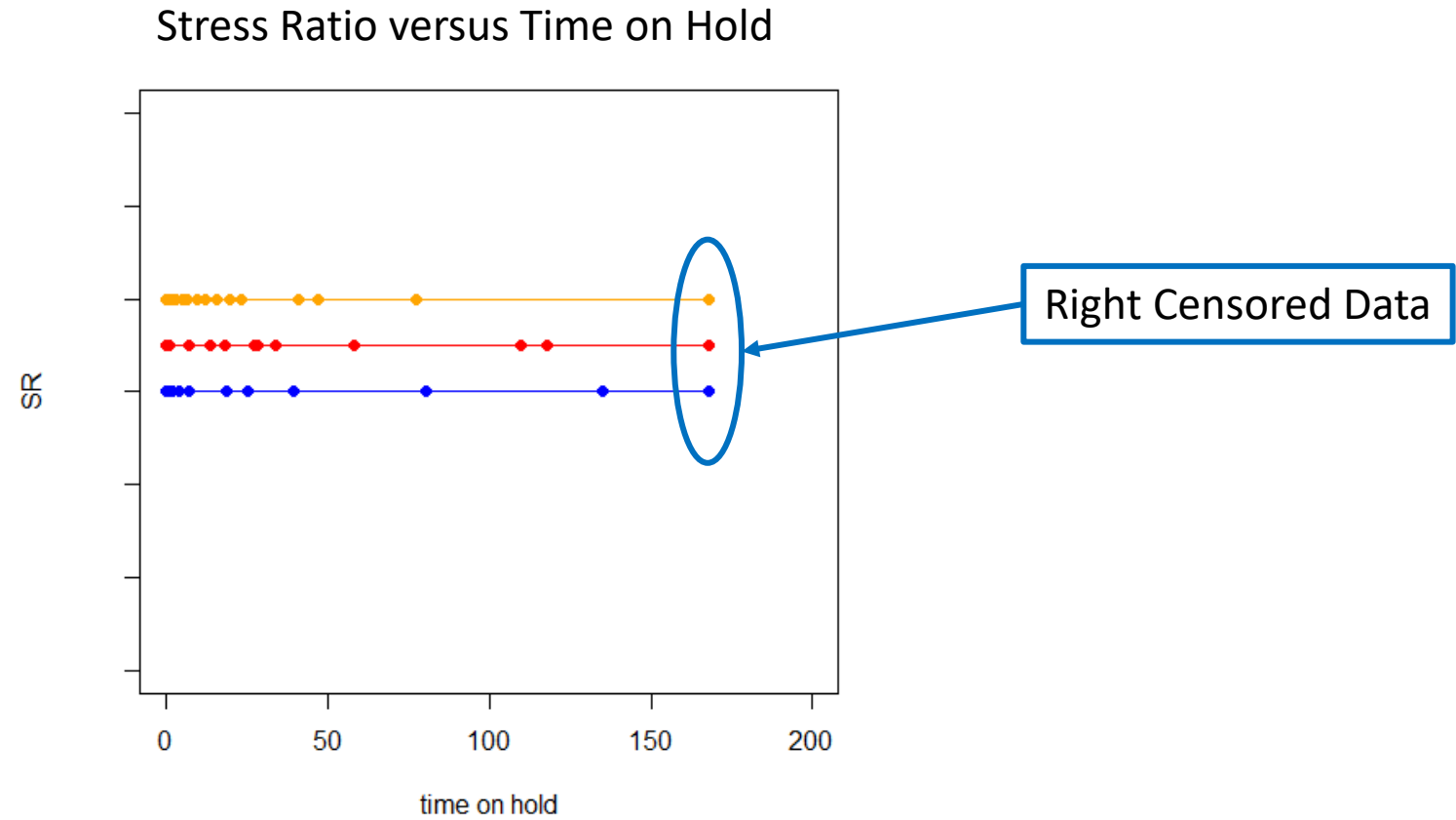
Now working with a linear model, similar to simple linear regression

- Scaled Residuals

- $z_i = \beta e_i = \beta(\log t_i - \mu) = \beta(\log t_i - \theta + \rho \ln(SR))$
- Used for predictions of the log probability for specific observations

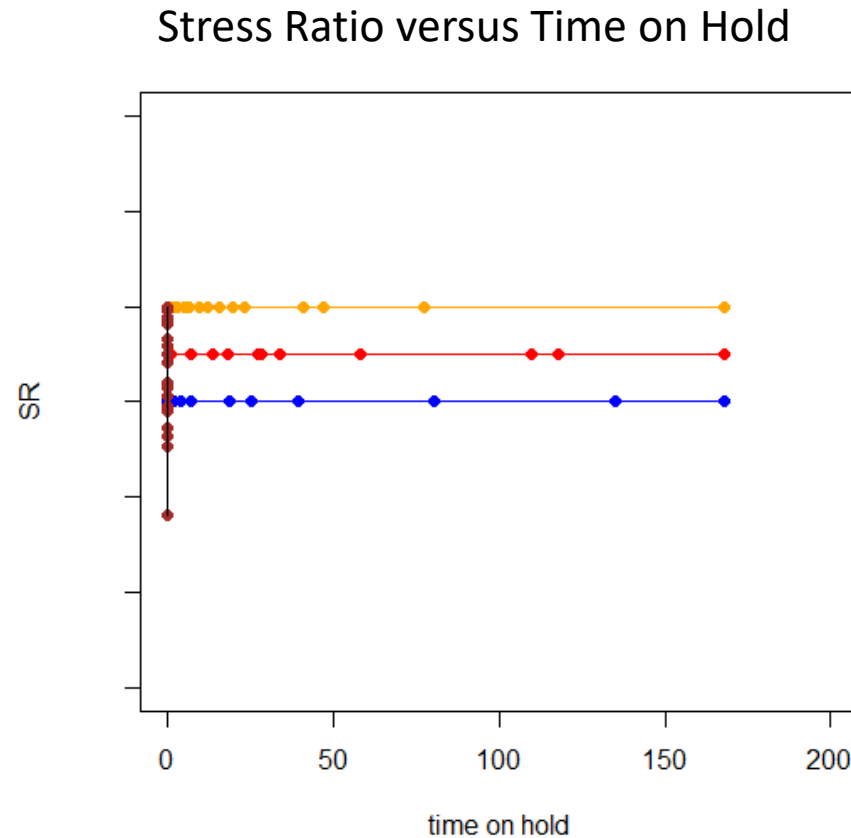
Weibull Regression: Fitting the Model

Stress Rupture Data



Stress Rupture Data with Ramp Failures

- Statisticians on team suggest a conditional analysis using only data that makes it to stress rupture hold.
- All ramp failures are dropped.
- In the literature, a procedure termed “effective time” has been suggested that incorporate ramp data as stress rupture data
- Recent work has shown the errors and limitations of this “effective time” approach



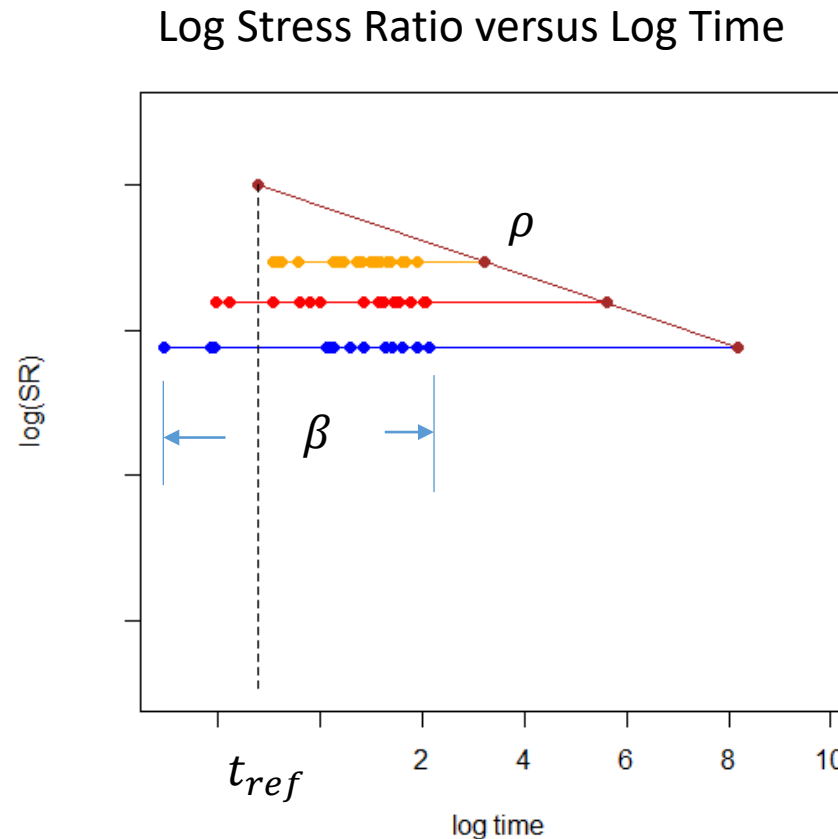
Note: X-Axis is ***Time on Hold***

Failures on Ramp Have No Time On Hold!

True Structure of the Stress Rupture Model

$$\mu = \log(\eta) = \theta - \rho \ln(SR)$$

- ρ : controls the relationship between the failure time and stress ratio (SR)
- β : Shape parameter for time to Failure
- t_{ref} : Reference time to Failure



Stress Rupture model explains the behavior of the items *on hold*.

- Weibull regression gives us estimates for ρ , β and t_{ref}

Weibull Regression: Model Building

Full Model

- Separate individual models to each stress ratio
 - Two parameters for the SR=1 data: η_1 and α_1
 - Two parameters for the SR=2 data: η_2 and α_2
 - Two parameters for the SR=3 data: η_3 and α_3
- Largest possible Weibull model for the data
 - Has the largest log-likelihood
- Will compare to the Full Model to subset models to determine whether the improvement in log-likelihood justifies the extra parameters

Fit-to-Hold (subset model)

- Model the data that have achieved the target load as defined by the experimental protocol (no ramp data)
- Defines that the time at the sustained constant load begins the moment the test item achieves the target load
- Assumes a Weibull distribution to describe the time to failure under the sustained constant load
- Experimental protocol uses right-censoring at a nominal time

Fit-to-Hold (subset model)

- Ramp and Hold data are modeled separately
 - Three parameters to explain the hold data: ρ , β , and $\theta = \log t_{ref}$
- Model assumes

$$\alpha_1 = \alpha_2 = \alpha_3 = \beta$$

Comparisons

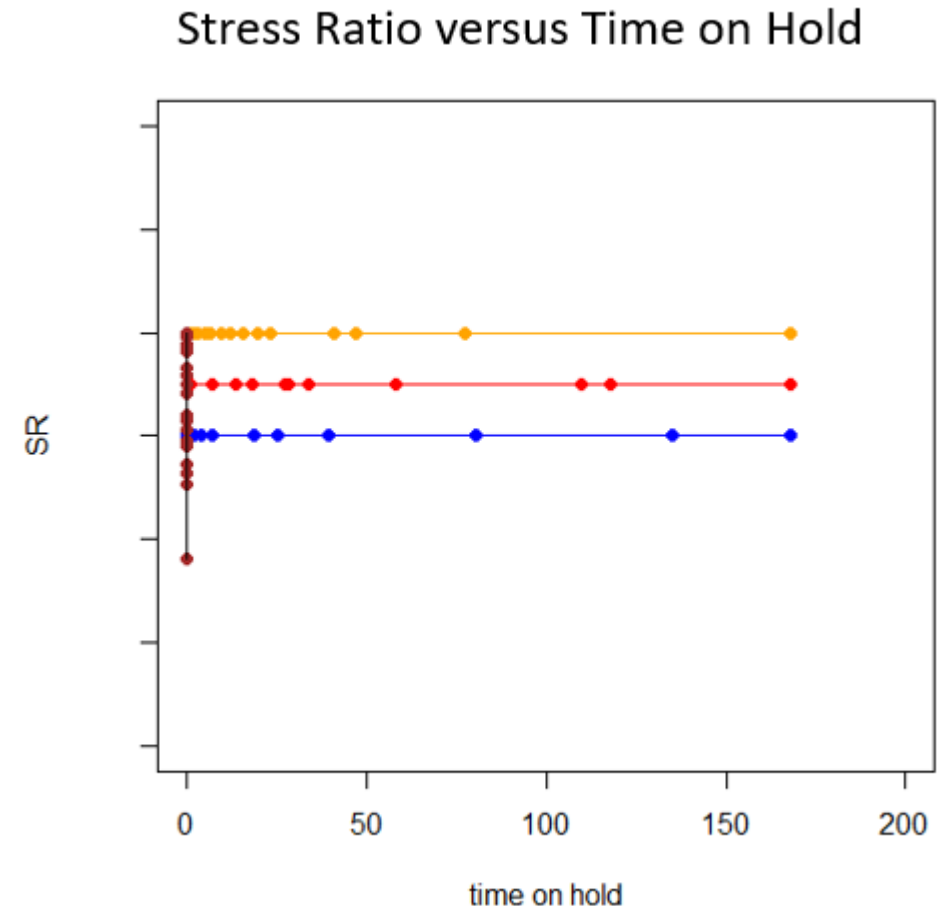
Model:	Fit-to-Hold	Full Model
Number of Observations:	708	708
Parameter Estimates:		
True Log-Likelihood:	−306.411	−305.900
Log-Likelihood Statistic:	612.822	611.800
<i>AIC</i> :	618.822	623.800

- The p -value associated with the χ^2 based on the difference in the log-likelihood statistics is 0.7959
 - The three extra parameters in the full model are not significant
- Smallest AIC value for Fit-to-Hold (adjustment for parameters)

Weibull Regression: Including Ramp

Adaptations to Include Ramp Failures

- Rigorous Approach
 - Add two additional parameters for a Weibull Distribution fit to only the ramp data along with the Fit-to-Hold Analysis
 - Two parameters for the ramp data: η and α
 - Three parameters to explain the hold data: ρ , β , and $\theta = \log t_{ref}$
- Left Censored Analysis
 - Assume that ALL data follow the same failure mechanism
 - Left censor all ramp failures and some early stress rupture failures
 - Three parameters to explain the hold data: ρ , β , and $\theta = \log t_{ref}$



Comparisons

	Rigorous	Left-Censored	Proper Mother
Overall log $\mathcal{L}$:	-251.103	-390.779	-250.592
Log-Like Stat:	502.206	781.558	501.184
AIC:	512.206	789.558	517.184
Ramp log $\mathcal{L}$:	55.308		55.308
Hold log $\mathcal{L}$:	-306.4114		-305.900

- All fit statistics indicate that the rigorous model is the superior fit to the data
 - Smallest AIC value (512.206) and log-likelihood statistic (502.206) and the largest overall log-likelihood value (-251.103) The three extra parameters in the full model are not significant
- The probability that the left censored analysis explains the data at least as well as the rigorous model is 1.03946 E-62
- Counter-intuitive to penalize the maximum likelihood fit to the data with left censoring especially when we know the precise time these items failed on the ramp

Weibull Regression: Residuals

Weibull Residuals

- The Maximum Likelihood Estimates of the Model Depends upon:

$$\begin{aligned} e_i &= \log t_i - \mu_i \\ &= \log t_i - \log t_{ref} + \rho \ln SR_i \end{aligned}$$

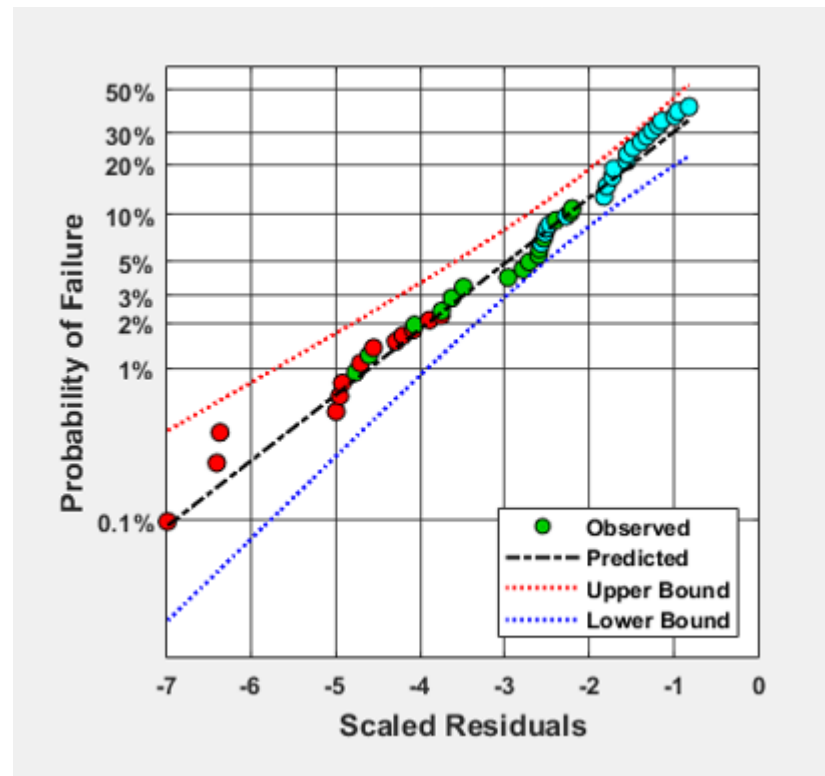
- e_i Is the “Raw Residual”
- Equally Important is the “Scaled Residual,” βe_i .
- These Residuals Contain All of the Relevant Information on t_{ref}, ρ, β .

Proper Basis for Constructing Probability Plots

- Estimate the Model
- Construct the Scaled Residuals
- Calculate the Median Ranks for these Residuals (Overall not by SR!)
- Plot $\ln[-\ln(1 - mr_i)]$ versus the βe_i
- Method Extends Easily to More Complicated Models

Standard Probability Plot

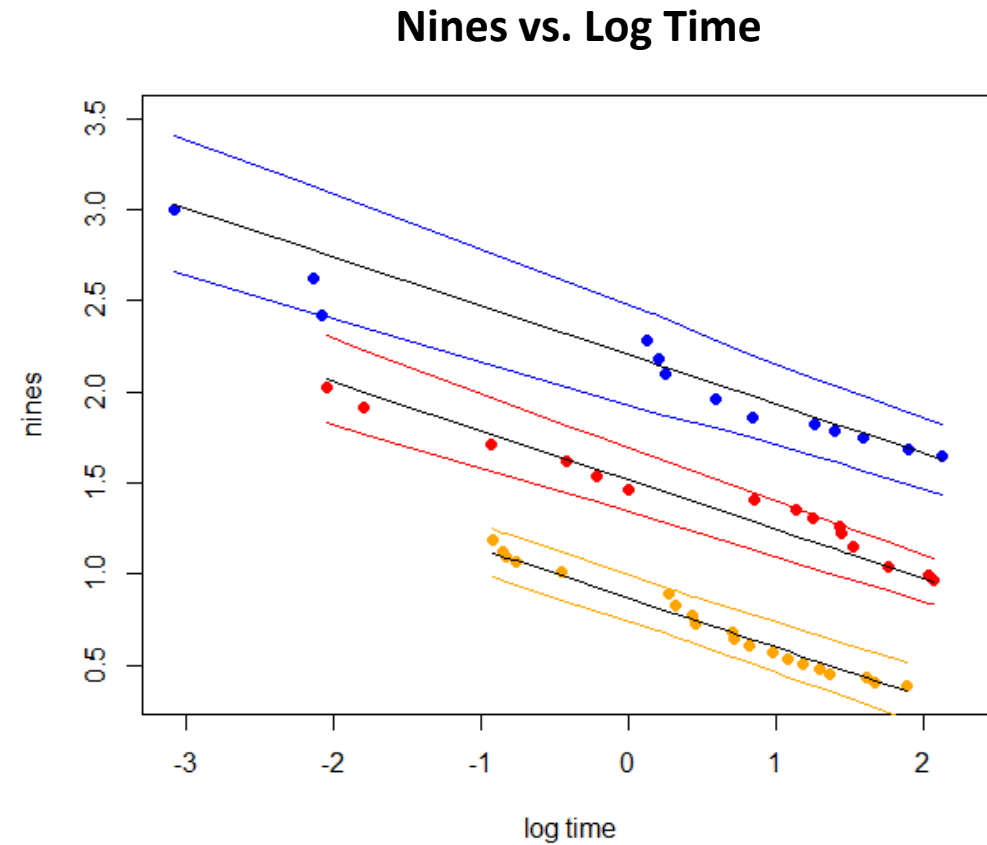
Weibull Probability Plot



Nines in Reliability

- A statistic in reliability that is just another way of reporting the reliability of a product, system, etc.
- Literally just a count of the number of nines for reliability
- Ex: 99% reliability is 2 nines of reliability
 - Interpretation: One in 100
- 99,999% is 5 nines of reliability
 - Interpretation: One in 100,000
- Nines Calculation: $-\log_{10}(1 - F(t))$

Nines versus Log Time



Note: Slope of Each
Prediction Line Is $-\beta$

Prediction Line Is $-\beta e_i$
(Negative Scaled Residual)

Where to Next...

- Working on Tutorials that cover best practices for analyzing Stress Rupture Data
- Advocating for the Use of the Fit-to-Hold Method for analysis
 - Model Building
 - Residual Analysis

Team

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