An ODE-fitting approach to estimate critical aircraft performance parameters for trajectory prediction

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***Abstract*—Ground-based decision support tools (DST) in air traffic management (ATM) typically perform trajectory prediction based on aircraft performance model (APM) parameters, but some or all of these parameters might not be readily available. In particular, the three critical parameters required for trajectory prediction are the thrust setting, drag coefficients, and takeoff weight of the aircraft. Unfortunately, these parameters are coupled and appear together in physics-based kinetic models. Past approaches utilize data from a specific phase of flight (climb, level flight or descent), where one or more of the parameters are assumed to be known and estimate the remaining unknown parameters. This approach introduces bias/errors and also does not extend to scenarios where all of the above parameters are not known with sufficient accuracy.**

This paper is the *first of its kind* to propose a generalized framework for simultaneous estimation of all three critical APM parameters (thrust, drag, and mass). The proposed approach utilizes data from both the climb and descent phases and fits the ordinary differential equation for altitude in each phase using historical trajectory data available from radar tracks or ADS-B. The approach yields a set of optimized APM parameters that are best suited to fit each historical flight record. The methodology is applied on on sample flights from three different aircraft types, and the results demonstrate low fit error and consequently will yield a high level of prediction accuracy.

***Index Terms*—trajectory prediction, aircraft performance model, ODE fitting, air traffic management**

NOMENCLATURE

h	Altitude of aircraft above mean sea level
C_L	Lift coefficient
C_{D0}	Parasite drag coefficient
κ	Induced drag coefficient
δ_{cl}	Thrust setting coefficient for climb
δ_{des}	Thrust setting coefficient for descent
m_0	Starting (take off) weight of aircraft
m	Weight of the aircraft at time t
m_f	Cumulative weight of fuel consumed at time t
V_t	True airspeed
V_w	Wind speed
T_{max}	Maximum climb thrust
AF	Acceleration Factor
g	Acceleration due to gravity
S	Wing reference area
ρ	Density of air
f	Fuel flow rate
ψ_w	Wind direction
ψ_a	Aircraft heading

I. INTRODUCTION

Air Traffic Management (ATM) is defined by the International Civil Aviation Organization (ICAO) as "The dynamic, integrated management of air traffic and airspace - safely, economically and efficiently - through the provision of facilities and seamless services in collaboration with all parties" [1]. At the core of ATM lie Decision Support Tools (DST) that require accurate aircraft position prediction to determine airspace usage and provide better recommendations.

A key tenet of the next generation air transportation system (NextGen) is deploying automation capabilities to enable Trajectory Based Operations (TBO) in the National Airspace System (NAS). In order to increase the safety and efficiency of air traffic operations in the NAS, predicting aircraft trajectories with sufficient accuracy is essential for ground-based automation applications. Ground-based DSTs utilize trajectory prediction in order to provide recommendations on decisions related to, e.g., scheduling of arrivals, traffic flow management and conflict resolution, and inaccurate trajectory predictions can undermine the safety and efficiency of these operations. DSTs perform trajectory prediction based on aircraft performance model (APM) parameters, but some or all of these parameters might not be readily available.

While the physics of trajectory prediction is well-established for traditional aircraft, it requires knowledge of aircraft performance parameters (e.g., drag coefficients) and operating procedures (e.g., descent speed, flap schedule) for the flight being predicted. With improvements in data availability and computing power, trajectory prediction accuracy has improved over the years, however, challenges still persist with availability of certain parameters (due to proprietary reasons). Accurate prediction of the aircraft's trajectory is essential because mismatch between the predicted and actual trajectory can lead to scheduling errors and inefficiencies. In particular, errors in trajectory prediction lead to a shift towards tactical decision making as strategic decisions are ineffective or incorrect. This tactical shift leads to system inefficiencies and workload-intensive tactical actions [2]. Trajectory prediction inaccuracies can stem from various sources including inaccurate input parameters, misinterpreted pilot intent, inaccurate aircraft performance model parameters, inaccurate data on atmospheric conditions, invalid assumptions, etc. Several of these causes have been shown to have a significant impact on the prediction in the literature [2], [3]. This work focuses on improving the estimation of aircraft performance model parameters—specifically thrust setting, drag coefficients and takeoff weight—that are critical for accurate prediction by ground-based DST.

II. Literature Review

The literature on trajectory prediction and associated challenges in ATM is vast, and the reader is referred to the comprehensive review paper by Mondoloni and Rozen [2]. The theoretical basis for aircraft trajectory prediction and aircraft performance models in ATM has been well established for many years [4], [5]. However, due to the limitations in available data for ATM applications and the inherent coupling, several variations are possible in the estimation of APM parameters using historical data. The variations typically stem from the choice of parameters being estimated and assumptions related to the other parameters (e.g., holding aircraft mass and thrust setting constant over the prediction window and estimate drag). In this section, we review trajectory prediction literature relevant to the estimation of aircraft performance parameters. This subset of the literature can be divided into three main categories: mass estimation, thrust setting (or thrust law) estimation, and drag coefficients estimation. In most cases, when a value for an APM parameter is unavailable or not estimated, authors have relied on Eurocontrol's Base of Aircraft Data (BADA) model [6] tables to fill that gap. BADA is a collection of ASCII files that specifies operation performance parameters, airline procedure parameters and performance summary tables for 405 aircraft types. This information is designed for use in trajectory simulation and prediction algorithms within the domain of ATM. A majority of the work on estimating APM parameters or input parameters to APM is dedicated to improving the estimate of the starting weight (or takeoff weight) of the aircraft. Aircraft weight is a fundamental parameter that has an impact on the climb and descent profile of the flight. However, data concerning the mass of almost all modern commercial flights are treated as confidential information by the airlines. In a study by Sun et al. [7], Bayesian inference is used to estimate the mass of the aircraft by combining data from five different ways of estimating takeoff weight. Thrust and drag values, when required, are obtained from the BADA model. Lee and Chatterji [8] have used flight plan information and aircraft performance data from BADA to come up with a closed form solution for the takeoff weight. They have relied on BADA for both the thrust and drag coefficients. Sun et al. [9] have estimated the thrust setting and mass for flights and validated the results using Cessna Citation II flights. The drag polar from BADA is utilized in this work. Schultz et al. [10] provide a method for weight estimation by dynamically adjusting the weight during the climb phase using thrust and drag from BADA model. Finally, some other work in literature such as Dalmau et al. [11] estimate fuel consumption from radar tracks in the descent phase rather than the starting mass of the aircraft.

For studies on thrust or thrust law estimation, Alligier et al. [12] modify the BADA thrust model to obtain an equivalent weight and equivalent thrust setting profile during the climb phase of flights. They then use it to predict the energy rate of the aircraft. The maximum thrust and drag from BADA are utilized in their process. In other work, Alligier et al. [13] learn the thrust law and mass based on historical flight data using the BADA drag polar model.

Sun et al. [14] have proposed a stochastic total energy model formulation for estimation of aircraft drag parameters. Their formulation is among the most generic, which incorporates thrust setting and mass estimation in the learning process. Maximum thrust information from BADA is utilized and scaled through the incorporation of a thrust setting parameter with a uniform distribution. Mass is allowed to vary between empty and maximum weight. However, the work considers only the climbing phase of flights, and the trajectory points are fitted without explicitly accounting for the temporal aspect.

Among the literature reviewed and described, none of the methods *simultaneously* estimate mass, thrust, and drag parameters. Furthermore, most of the work is conducted on climb phase with limited attention given to the descent phase. In many of the approaches that use least-square fitting, the temporal aspect of the data is not explicitly considered and errors are minimized for trajectory points as standalone samples. Finally, several past methods estimate the starting mass but assume it to be constant during the sections that are being used to fit to the data. In reality, the mass is continuously decreasing due to fuel burn and needs to be accounted as such. Considering the limitations in current practice, we identify three key contributions of the approach described in this paper:

1) The approach simultaneously accounts for thrust, drag, and starting mass estimation in the optimization process.
2) The approach fits aircraft altitude, as opposed to directly fitting a noisy (and/or derived) rate of climb (ROC), via an ordinary differential equation (ODE) fitting approach, thereby accounting for the temporal aspect of the flight.
3) The approach combines data from all segments of the flight that have non-zero ROC, as opposed to just the initial climb or final descent phase.

III. Theoretical Background

In order to predict the trajectory of the aircraft, the flight's dynamics need to be modeled. These are usually described by a six-degrees-of-freedom (6 dof) aircraft dynamics model. However, for ATM applications, information such as pitch, roll, yaw moments, initial attitude, or moments of inertia, is not available or even required. Simplifying assumptions motivated by energy transfer leads to a point-mass model known as the total energy model (TEM) that is typically used in ATM-related trajectory prediction applications. It involves a point-mass, steady- state representation of the aircraft dynamics using a kinetic or kinematic model and several simplifying assumptions such as small angle of attack, thrust acting in the direction of air velocity, negligible crab angle, etc. The kinetic model involves the estimation of the forces acting on the aircraft (thrust, drag, and weight), whereas the kinematic approach is developed without directly computing the forces causing the aircraft motion but focusing on estimating the aircraft's rate of climb and other states directly through look-up tables. Both models involve assumptions related to their parameters that can limit their applicability and result in uncertainty in predictions.

In typical trajectory prediction applications using TEM, the parameters for one or more of the core components in the kinetic model (thrust, drag, and weight) are assumed to be available (for example from BADA tables), and parameters for the remaining components are estimated using available data [14]. The model can then be used in conjunction with flight data, operating procedures and other data sources to predict trajectories. In such an approach, the errors introduced by the use of incorrect parameters (e.g., BADA APM values) propagate through the model and ultimately impact the prediction accuracy.

The TEM model equates the work done by the forces acting on an aircraft to the change in total mechanical energy of the system. Equation (1) encapsulates this in terms of the quantities of interest available to this work. Readers are referred to Mondoloni and Rozen [2] for a detailed derivation of this form of the equation.

$$\frac{(T-D)V_t}{mg} = \dot{h} + \frac{V_t}{g}\Big[\dot{V}_t + \frac{d}{dt}\big(V_w \cos(\psi_a - \psi_w)\big)\Big] \qquad (1)$$

Here, T refers to the thrust produced by the aircraft, D is the drag force acting on the aircraft, m is the mass of the aircraft, γ is the flight path angle, V_t is the true airspeed of the aircraft, V_w is the wind speed, h is the altitude, and g is the acceleration due to gravity, ψ_a and ψ_w are the aircraft heading and wind direction respectively. The quantities on the left hand side of the equation indicate the kinetic components of the model that can be estimated using an appropriate APM; the quantities on the right side of the equation are related to the kinematics of the aircraft and can be estimated using recorded flight data and other data sources (e.g, meteorological data). We can substitute for T and D using standard thrust and clean drag models resulting in the parameterized version of equation (1) used in this work as shown in equation (2). We are assuming a clean drag configuration (i.e., incompressible air flow and low Mach speed flight) and also focusing on flight regimes with no additional resistance due to wake or landing gear.

$$\frac{\Big[T-D\Big]V_t}{mg} \leftrightarrow \frac{\Big[\delta T_{max} - 0.5\rho V_t^2 S(C_{D0} + \kappa C_L^2)\Big]V_t}{(m_0 - m_f)g} = \dot{h} + \frac{V_t}{g}\dot{V}_t \qquad (2)$$

Note that wind-related components are ignored in the current formulation, but in future work, we will perform sensitivity analysis to determine whether the results change significantly by including or excluding the wind-impacted terms. Using historical track and weather data, for a single flight, we estimate the quantities highlighted in blue in equation (2) i.e., the two drag coefficients (C_{D0} and κ), the thrust setting (δ), and the starting mass[1] (m_0). Equation (2) can be recast into the version shown in equation (3), which is more useful for setting up the ODE solver. The second ODE used for the mass variation is shown in equation (4). These two equations form the backbone of the ODE-fitting method used in this work.

Final ODE set

$$\dot{h} = \frac{\delta}{(m_0 - m_f)}\Big[\frac{T_{max}V_t}{g.AF}\Big] - \frac{C_{D0}}{(m_0 - m_f)}\Big[\frac{\rho V_t^2 S}{2g.AF}\Big] - \kappa(m_0 - m_f)\Big[\frac{2g.cos^2(\gamma)}{S\rho V_t.AF}\Big] \qquad (3)$$

$$\dot{m} = f \qquad (4)$$

Here T_{max} is the max climb thrust and f is the fuel flow rate, AF is the acceleration factor, and m_f is the cumulative weight of fuel consumed up to time t. The quantities in blue color are being fit in the optimization process, and those in black color are available using the historical flight data. The functional form of T_{max} and f depends on the phase of flight, height, mode, and other factors. For this work, the functional form from BADA version 3.11 [6] is used. It is noted that while the max climb thrust is inherited from BADA, the parameter δ allows scaling this up or down, thereby providing the means for controlling the total thrust injected into the TEM. Similarly, incorporating the fuel flow differential equation allows for a realistic mass reduction as the flight progresses compared to the constant mass assumption made in the literature.

IV. Technical Approach

The central idea is to "fit" the ODEs represented by equations (3) and (4) such that the altitude profile from a historical flight record is regenerated by our model with minimal error. In other words, we separate the known quantities (e.g., true air speed) as a function of time from the unknown parameters and

[1] 'Starting mass' has been used instead of "initial mass" to disambiguate from a similar term (initial value) typically used in ODE integration.

solve for the optimal model parameters that result in the least mean square of altitude (fit) error. Figure 1 provides a detailed flowchart of the ODE-fitting process followed for each flight record.

The main design variables in the optimization process are the APM parameters to be estimated for a historical flight record. In the present formulation, this is a set of **five parameters**: the parasite drag coefficient (C_{D0}), the induced drag coefficient (κ), the starting mass of the aircraft (m_0) and the thrust coefficients for climb and descent ($\delta_{cl}, \delta_{des}$). It is quite clear that we have to allow for different climb and descent thrust settings, since the former is typically close to 100% of the maximum climb thrust and the latter is close to 0 or idle thrust. In addition, if there are multiple climb or descent segments identified, then additional sets of thrust coefficients for those phases will also be added into the set of design variables. The formulation of the design variables is such that it allows any of the aforementioned APM parameters to be frozen while optimizing. In this way, if a specific decision support tool does not have the capability to alter any of these parameters, the proposed approach would still be able to provide APM parameters that are suited to that DST. This greatly enhances the applicability of this approach because it can effectively be adapted to a wide variety of DSTs in ATM applications with minimal modifications. The list of APM parameters, and their upper and lower bounds as provided to (and enforced by) the optimizer are shown in Table I. It is noted that the bounds on the takeoff weight (m_0) will be different for each aircraft type and limits corresponding to empty weight and maximum operational weight (from the BADA v3.x tables) are used.

TABLE I
APM PARAMETER BOUNDS.

	C_{D0}	κ	δ_{cl}	δ_{des}
Lower Bound	0.02	0.03	0.9	0.01
Upper Bound	0.04	0.055	1.0	0.15

The process begins with the historical flight record for which the APM parameters are to be estimated. The flight is partitioned into different climb and descent segments (level flight segments are ignored as the rate of climb is equal to zero). In the present work, the drag coefficients being estimated are from a *clean* drag configuration. This assumes that there are no flaps, slats, spoilers, or landing gear deployed in that configuration and that the aircraft is flying at lower Mach speeds (incompressible air flow). Based on the literature, a cut-off altitude of 10,000 feet above mean sea level (MSL) is chosen for fitting the APM coefficients. The adaptation of this framework for the prediction problem in the terminal area where *unclean* aerodynamic configuration might be observed is deferred for future work. Nevertheless, a large number of ATM decision support tools can still use a single drag polar for making the predictions, as the clean configuration is observed for a majority of the flight duration. Additionally, since the descent thrust setting's upper limit is set at 15%, it is necessary to remove certain descent segments that are performed at higher thrust settings (typically constant-Mach descents at higher altitudes).

Depending on the number of climb and descent segments above $10,000\ ft$ MSL, the parameter set is initialized within the bounds indicated in Table I. A pre-specified number of points is chosen within each phase as the actual data to which the optimizer will try to fit. With the parameters initialized and the flight segments identified, the rate of climb and fuel flow ODEs are integrated from the starting point of the phase until the end of the phase using the parameter set. Scipy's *odeint*[2] function is used for this purpose. The altitude profile obtained from the ODE solver is compared with the actual trajectory at the sampled/specified points, and a residual is calculated at each point. The residuals from all phases are combined to obtain a % root mean square error for the entire flight. An important implementation detail to note here is that, in the calculation of the error/discrepancy, only the altitude error is calculated against the actual altitude. The reason mass error is not calculated is because the actual mass is not available in the data. But because of the coupled nature of the two ODEs, the mass and $\dot{h}$ both are updated during the optimization because the parameters affect both those variables.

Once the problem has been set up as indicated, the optimization problem of minimizing RMS error from all the phases is solved. It is noted that several optimizers can be used for this purpose, and in the present work, an implementation of the classic pattern search algorithm is utilized [15]. The implementation provided in the python library pymoo [16] is used for this purpose. Further details on this implementation can be found on the pymoo website[3]. The main requirements of the algorithm are that it converges to the same minimum when started at multiple starting points and it reaches a solution in a reasonable amount of time as the approach needs to be scalable to thousands of flights. Both these requirements are satisfied by the pymoo Pattern Search implementation. In this case, the choice of the specific optimizer is not as important as the problem setup, and therefore, another similarly capable optimizer can be substituted for the one used in this work.

At the end of the optimization routine, the outputs are a set of best-fit APM parameters for that historical flight data record.

V. Data Utilized

The data sources used are track data (called IFF dataset obtained from the Sherlock data warehouse[4] and the features are flight ID, airline, aircraft type, flight route, latitude, longitude, altitude, ground speed, timestamp), Meteorological data (from National Oceanic and Atmospheric Administration (NOAA) forecasts [5] and is called Rapid Refresh (RAP). RAP contains Meteorological information for North America

[2]https://docs.scipy.org/doc/scipy/reference/generated/scipy.integrate.odeint.html

[3]https://pymoo.org/algorithms/soo/pattern.html

[4]https://sherlock.opendata.arc.nasa.gov/sherlock_open/

[5]https://rapidrefresh.noaa.gov/

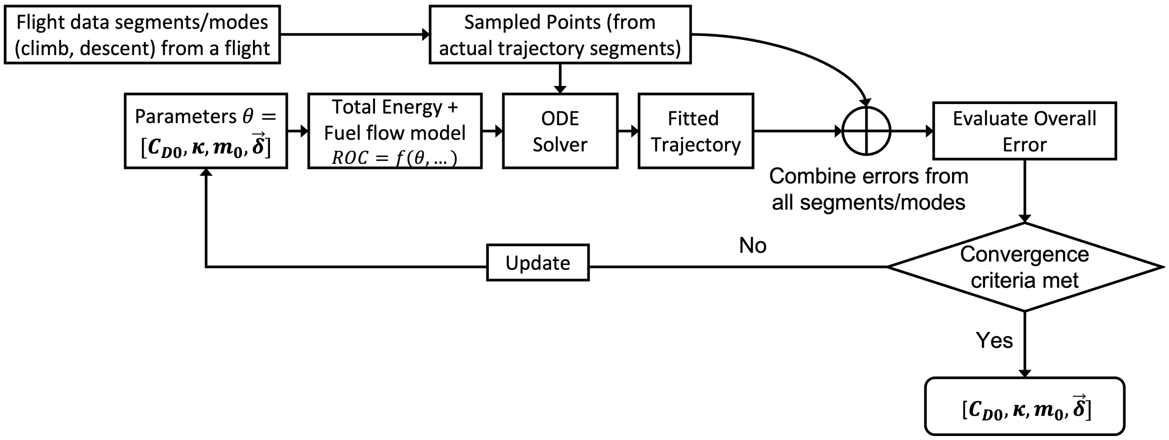

Fig. 1. Flowchart showing the various steps involved in ODE-fitting approach using historical flight data and total energy model.

with features such as wind speed, temperature, air pressure, geopotential height, etc.), and event datasets—all accessed from Sherlock Data Warehouse—in this work. This raw track data is comprised of one position report every 6 to 12 seconds. This makes the raw trajectory data features such as speed and altitude very noisy. Therefore, it is crucial to smooth the trajectories using splines, derive new features using the current datasets, and detect flight modes using speed profile, altitude, etc. Finally, engine information for each flight is obtained from FAA. The steps in the data processing are outlined here:

- Merging the RAP, IFF, and engine datasets.
- Removing the trajectories with many missing values.
- Smoothing using spline interpolation algorithm.
- Deriving important variables such as Mach, true airspeed, etc. from the raw data.

VI. Results

A. Individual Flight Results

The ODE-fitting process is applied to several historical flight data records and the results are reported in this section. For the purpose of illustration, detailed plots are shown for a representative flight of several aircraft types. For the plots associated with each of the two flights, the actual flight data is shown in black color, the best-fit ODE model is shown in red colored dashed line, and the predictions based on baseline (BADA) APM parameters is shown in green colored dashed line. In order for it to be a reasonably fair comparison, the BADA model also utilizes the same fuel flow model as the ODE fit solution and uses the same values obtained from the flight for the rate-of-climb ODE. The only difference between the green and red lines is, thus, the different set of APM parameters.

The first flight for which the results are presented is an Airbus A320 aircraft flying between Los Angeles (LAX) and Cincinnati (CVG) airports. Figure 2 shows the results of the application of ODE-fitting on this flight record. The top row of the figure contains two plots that show the altitude profile of the flight—above mean sea level (MSL)—during the climb and descent phases, respectively. In each plot, there are three lines: solid black for the actual flight data, dashed red for the ODE- fit solution, and dashed green for the prediction obtained based on BADA default values. The bottom row contains the final values of each of the APM parameters produced from the optimization in red color and the default BADA value for the aircraft type in green color.

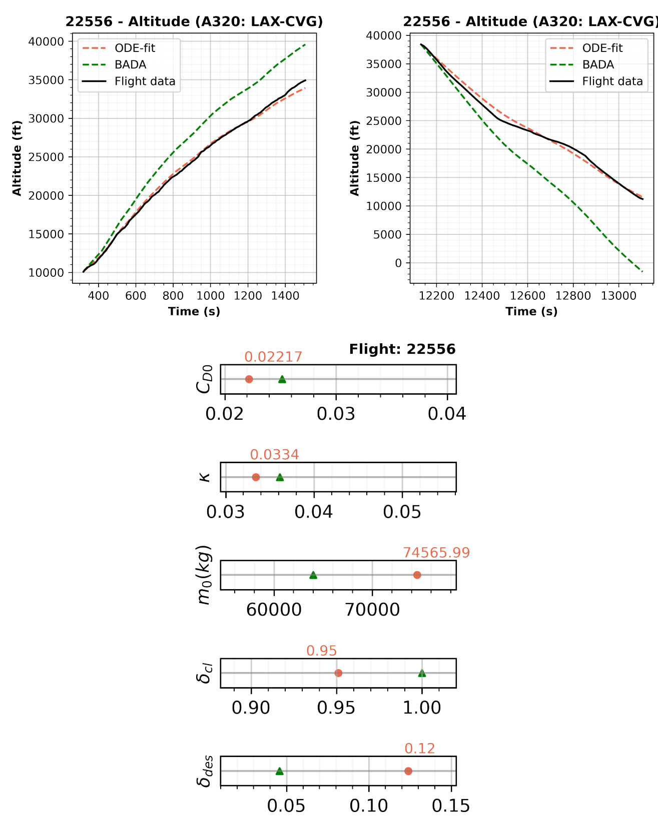

Fig. 2. Plot showing the altitude variation and comparison between the real trajectory, ODE fit trajectory, and BADA trajectory for climb and descent phase of the A320 flight number 22556.

The results from Figure 2 indicate that the ODE-fit optimal solution performs a much better job at recreating the history of the flight's altitude profile than the default BADA parameters. It is observed that all the optimal APM parameters differ from the default values by varying amounts. Both the drag and thrust setting coefficient are lower than the BADA default, whereas the starting mass is higher than the default (which is anticipated since this is a long flight of around 1600 nautical miles). The average thrust setting is slightly higher for the descent phase indicating that the flight probably flew the descent at a slightly higher thrust value than the BADA default. Indeed, the descent thrust seems to have a fair impact on the descent trajectory in this case as the BADA APM parameters fail to match the real flight.

The second flight, presented in Figure 3, is a Boeing B737 aircraft flying between Phoenix (PHX) and San Diego (SAN) airports. The plots presented in Figure 3 are formatted the same as the previous example. The drag coefficients are once again lower than the default BADA parameters, and the thrust coefficients are either identical (climb) or very close (descent). The starting mass is higher for the ODE-fit optimal solution. Once again, it is evident that the ODE-fitting process results in a better set of APM parameters that matches the real flight data better in both phases of flight. However, there are certain differences for this flight that are worth noting.

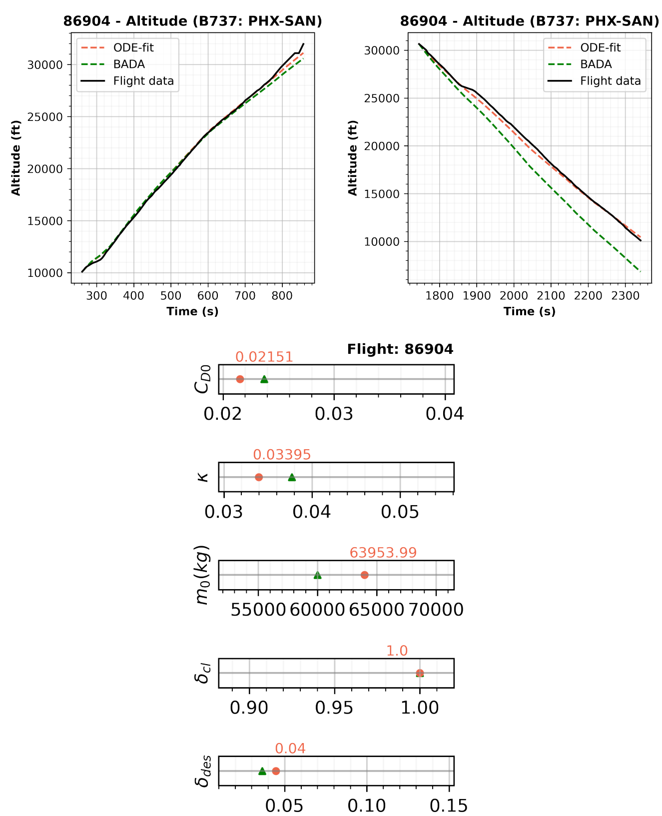

Fig. 3. Plot showing the altitude variation and comparison between the real trajectory, ODE fit trajectory, and BADA trajectory for climb and descent phase of the B737 flight number 86904.

One of the important differences between the second and first flights presented is seen in the climb phase of the flight. In the second flight (flight data item 86904), the fit for the climb phase is nearly identical between the default BADA and ODE-fit optimal solutions. Both are very close to the actual flight data. However, their APM parameter set is quite different. This same set of parameters, however, performs differently in the descent phase, whereas the ODE fit optimal outperforms the default BADA set. This illustrates the fact that multiple sets of parameters can potentially produce similar results for the altitude profile because it is an under-constrained problem. However, these similarities may not hold when data from multiple phases are considered. This is an aspect of APM parameter estimation that is often overlooked in previous work in the literature and highlights the importance of using data from both climbs and descents to obtain better overall results.

Finally, the results for the third flight are shown in Figure 4 using plots with the same formatting as prior examples. The drag coefficients are lower than the default, and the weight is higher than the default for this flight. There are marginal differences between the thrust coefficients. Overall, the ODE-fit optimal APM parameters perform slightly better than the default APM parameters.

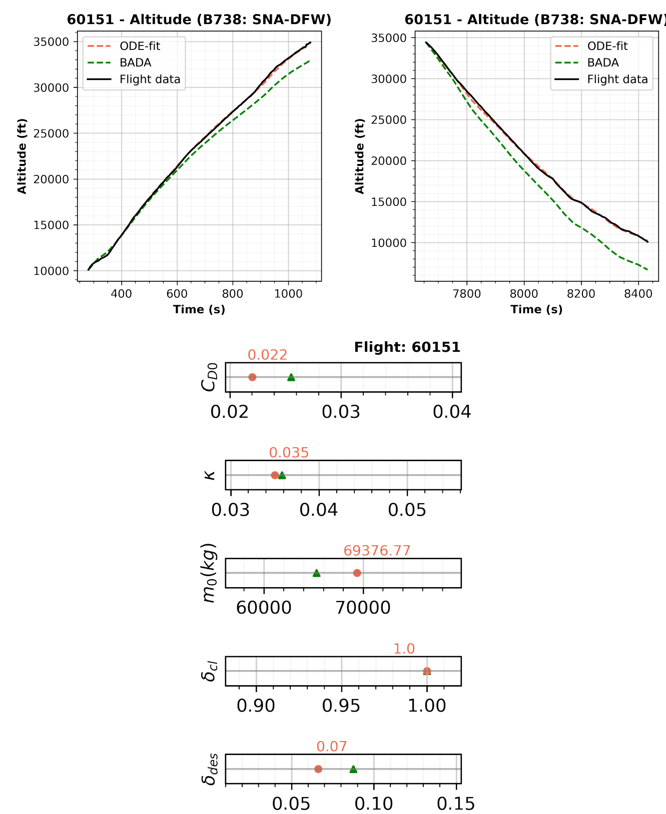

Fig. 4. Plot showing the altitude variation and comparison between the real trajectory, ODE fit trajectory, and BADA trajectory for climb and descent phase of the B738 flight number 60151.

A table showing the overall relative RMS errors for all three flights demonstrated earlier is presented in Table II. As seen from the table, for all three flights, the relative errors are less than 2.5% which indicate that the developed process is working well overall.

TABLE II
RELATIVE RMS ERRORS FOR DEMONSTRATED FLIGHTS.

Flight number	Aircraft Type	Relative RMSE
22556	A320	2.371%
86904	B737	1.704%
60151	B738	0.991%

B. Sensitivity Analysis

In order to ensure that the optimizer is always reaching the same minimum, the ODE fitting for each of the three flights presented earlier is repeated 30 times from random starting points in the APM parameters space. Each time, the optimizer is allowed to converge to the same tolerance of objective function (10^{-5}). The resulting APM parameter sets obtained from these experiments are plotted in Figure 5 as box-and-whisker plots.

As is evident from the figure, the optimizer converges to the same solution in almost all repetitions. The values of the individual parameters obtained as the optimal set and the RMS error produced by those parameters are close to each other in all the repetitions. A similar observation is made for the other two flights that are not shown in the figure. There are some outlier cases where the optimizer failed to converge or hit the boundary of the design space. This variability is seen more for flight 22556 (A320), which also had the highest fit error among the three flights shown (2.371% RMS).

C. Statistical Results

The approach presented in earlier subsections is applied on a larger scale to around 90 flights of the Boeing B738 airframe arriving at a single airport: Los Angeles International Airport (LAX). This section presents an overall statistical summary of the results obtained. During the fitting, there are some flights for which the optimizer is not able to find a good fit. The reason for this, in most cases, is the assumption on the bounds of the descent thrust coefficient (δ_{des}) being too low, due to which the optimizer hits that limit (15% of max climb thrust). This happens when the descent phase (or a part of it) is flown at a thrust setting that is significantly higher than the assumed idle thrust. Trajectory prediction algorithms in DSTs typically assume idle or near-idle thrust during the descent phase as it is difficult to accurately predict the conditions under which higher-than-normal thrust settings will be used (hence the 15% upper bound assumed in this work). Thus, those flights—the ones for which the optimizer hits this bound of the descent thrust coefficient (δ_{des})—are excluded from the analysis, as they would be of limited use to a trajectory predictor. Note that the developed framework will still be able to obtain an estimate of such a thrust setting by expanding the upper limit and breaking the descent into multiple parts, but that is beyond the scope of the present work. This initial set of fitted flights will be expanded in future work to include several thousand flights in order to validate the developed approach.

Figure 6 shows the distribution of the mean relative RMS error obtained from fitting the first set of 90 flights (excluding the ones mentioned earlier). The figure shows that over 75% of the % RMS errors are less than 3 % and all of them are within 6 % error range. This indicates that the fitting is doing a good job of finding the best APM parameters for the larger set of flights.

Figure 7 shows the distribution of the drag coefficients and starting mass for the fitted set of 90 flights. The distributions indicate a certain amount of variability in each of the three critical APM parameters despite the uniformity in the data for which they are fit (all B738 flights landing at LAX). Detailed studies and further analyses will be part of future work using the developed framework.

VII. DISCUSSION

The results shown in the previous section indicate that the proposed method performs well in being able to faithfully reproduce the altitude trajectory of multiple aircraft types. This is anticipated because the BADA coefficients were empirically obtained using only a handful of trajectories, whereas the current approach uses the specific flight data to derive the APM values. In order for the comparison to be fair, the BADA coefficients were also used with the same ODE setup and flight data. Therefore, the differences observed in the vertical trajectory are solely due to the differences in the five APM parameters optimized. One of the key observations from the flights shown in the paper was that, due to the under-constrained nature of the problem, there could be multiple sets of parameters that yield close altitude outputs and RMS errors for one phase of flight but fail to perform adequately in another phase of flight. It is therefore, critical to include as many data points as possible from different phases of flight in order to consistently obtain APM parameters that are best suited for a particular flight. The experiment conducted by varying the initialization of the optimizer with a different APM parameter set indicated that using flight data from both phases and performing ODE-fitting using the proposed approach consistently yielded the same optimal output.

There are several advantages of using the framework developed in this paper. First, it minimizes the number of assumptions made using an existing APM model such as BADA and allows the optimizer to search for the best set of APM parameters within a wide range that is consistent with the physics of the problem. Similarly, it can also be integrated into any existing trajectory prediction or ATM application by fixing the APM parameters that cannot be varied in that system and optimizing for the remaining ones. The solution obtained in such a case will be the one that matches historical trajectories (but might not necessarily match the actual thrust or drag coefficients).

It can be adapted to run a large number of flights in parallel to obtain statistical distributions of APM parameters that can then be used according to the task. Alternatively, multiple flights of the same group (aircraft type and airline possibly) can be optimized together in order to obtain a single set of

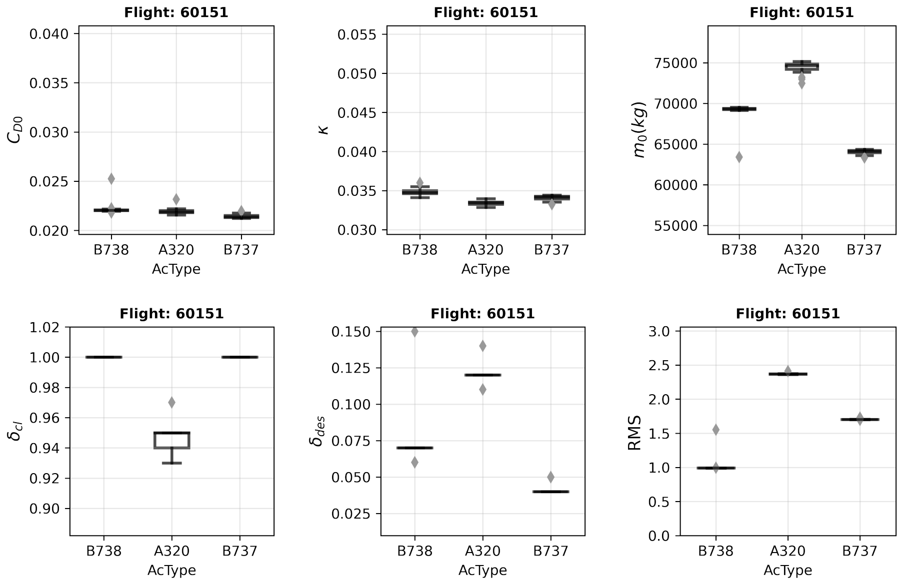

Fig. 5. Box-and-whisker plot of APM parameter minima and RMS error obtained from repeating the optimization process for each flight 30 times with random initialization.

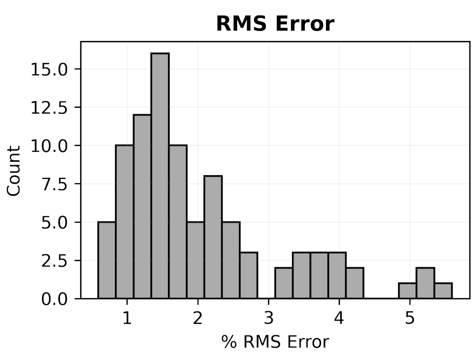

Fig. 6. Mean relative RMS error obtained from applying the developed framework on 90 B738 flights arriving at LAX airport.

APM coefficients per group that can be used to update the coefficients presently used in any ground-based automation system. For ground-based automation systems such as the FAA's Time-Based Flow Management (TBFM), which uses APMs to predict aircraft trajectory and estimated arrival times, the improvement offered by the current approach could make a measurable impact in improved scheduling of arrivals and reducing fuel burn.

While the developed framework allows more flexibility and generalization than some previous approaches, there are still some limitations that need to be acknowledged and addressed in future work. The framework still uses BADA's fuel flow and max thrust model and, even though the max thrust can get scaled based on the δ parameter, it is still constrained to be close to the initial value obtained from BADA.

VIII. Conclusion

This work presented a novel approach for the estimation of aircraft performance model parameters using ODE fitting and historical flight data. The approach consisted of fitting the ODE for rate of climb using historical flight data and an expanded set of APM parameters that included thrust, drag, and weight parameters simultaneously. Flight data from climb and descent phases were used in the optimization process. The formulated approach was demonstrated on three different aircraft types and showed good prediction capabilities on all three. A sensitivity analysis was conducted to ensure that the optimizer always converged to the same or similar solution. The proposed approach shows promise as an adaptable approach to obtain APM parameters for various ATM trajectory prediction applications.

Future work will include identifying the sensitivity of the obtained ODE-fit solution at the optimal point to assess its robustness. This is important in case the results need to be aggregated to provide a single APM set per aircraft type or other type of aggregation. The implementation of this approach on thousands of historical flights will also be conducted for more detailed statistical analysis. Investigation into using an open-source performance model such as Wrap [17] as the baseline for thrust and fuel flow will be investigated.

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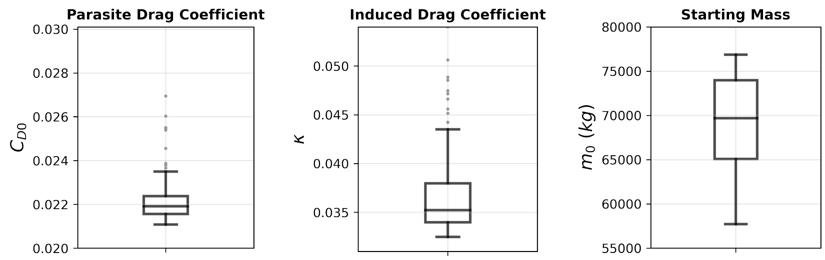

Fig. 7. Distribution of C_{D0}, κ, m_0 obtained from applying the developed framework on 90 Boeing B738 flights arriving at LAX airport.

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