Flow Field Reconstruction for Inhomogeneous Turbulence Using Data and Physics Driven Models

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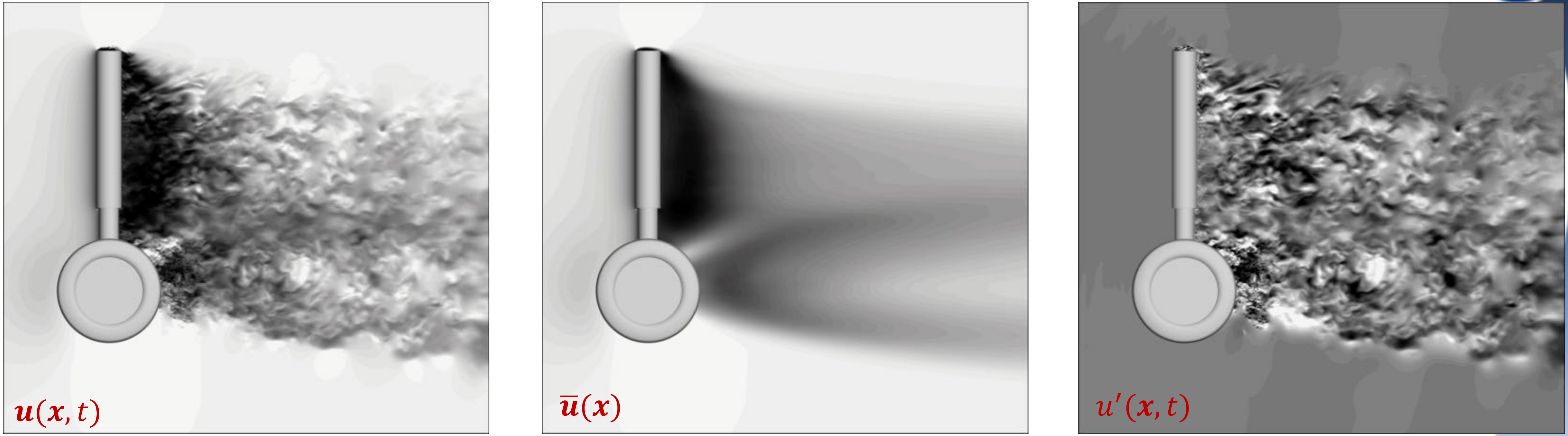
What is flow field reconstruction?



Immersed Boundary WMLES of LAGOON Landing Gear. Figure generated by Man-Long Wong. See Wong et al., AIAA 2022-2850 for further details.

Consider a flow field decomposition:

$$\boldsymbol{u}(\boldsymbol{x},t) = \bar{\boldsymbol{u}}(\boldsymbol{x},t) + \boldsymbol{u}'(\boldsymbol{x},t)$$

TOTAL

MEAN

FLUCTUATION (focus of present work)

What is flow field reconstruction?

Problem Description: Given a field, $\boldsymbol{u}'(\boldsymbol{x},t)$ generate a field, $\tilde{\boldsymbol{u}}'(\boldsymbol{x},t)$ such that

1. Is purely vortical (discretely divergence free):

$$\nabla \cdot \tilde{\boldsymbol{u}}' = 0; \qquad \tilde{\boldsymbol{u}}'(\boldsymbol{x},t) = \nabla \times \boldsymbol{A}_\omega$$

2. Estimates the second order, two-point statistics accurately

$$\langle u_i'(\boldsymbol{x}+\boldsymbol{y},t+\tau)u_j'(\boldsymbol{y},\tau)\rangle = \langle \tilde{u}_i'(\boldsymbol{x}+\boldsymbol{y},t+\tau)\tilde{u}_j'(\boldsymbol{y},\tau)\rangle$$

Important Note: Equivalence between original and reconstructed fields is required to be purely statistical and not pointwise (LES accuracy must be described statistically)

$$\boldsymbol{u}'(\boldsymbol{x},t) \neq \tilde{\boldsymbol{u}}'(\boldsymbol{x},t)$$

Why is it relevant?

Reduced order model (sparse representation) for flows with a **vast range of dynamically active scales (high Reynolds numbers)** needed in many applications

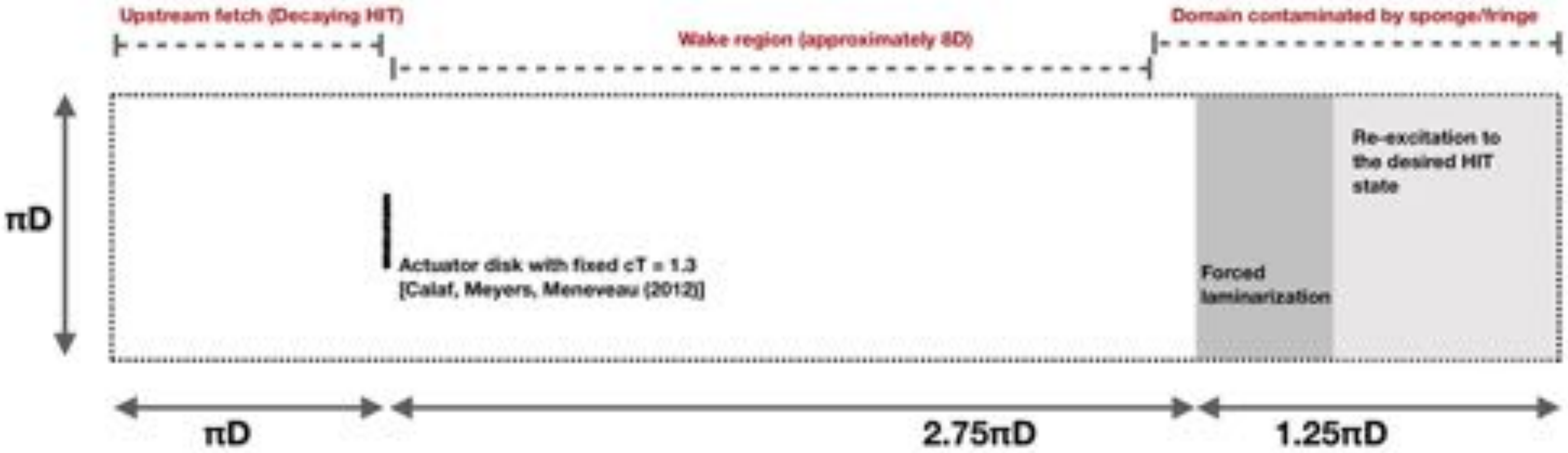
1. Atmospheric Science: Synoptic (Days) + Meso (Hours) + Microscale (Seconds/Minutes) fluctuations - Scalar transport, Wind Energy, etc.
2. Aeroacoustics: Air-frame noise, fan noise, jet noise
3. Aero-structural loading: Unsteady pressure loading and vibrations
4. Hybrid RANS-LES: fluctuation generation at interfaces

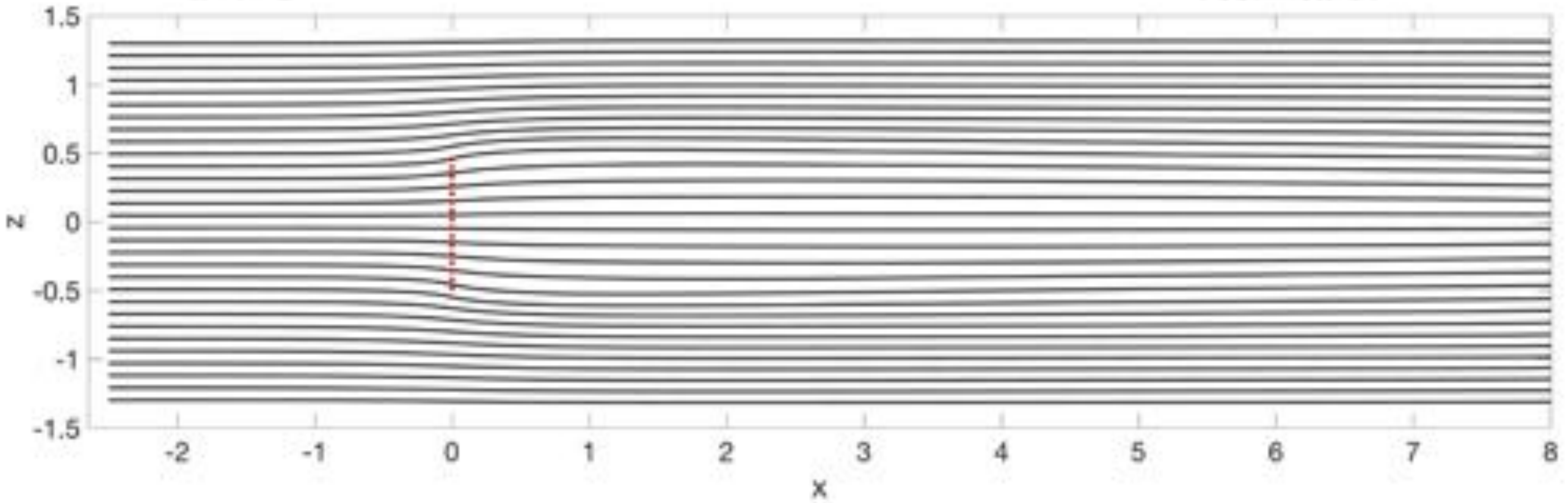
Bottom Line: Vast array of applications where fluctuations (as opposed to steady state) make up for nearly the entire figure of merit.

Outline

- Simple Test problem: Turbulent wake with turbulent co-flow
- Is a sparse representation possible?
- Model for Large-Scales: Truncated modal expansion
- A physics driven model for small scales
- Evaluation of the the combined model
- Conclusions and Outlook

Turbulent Wake of a Dragging Disk





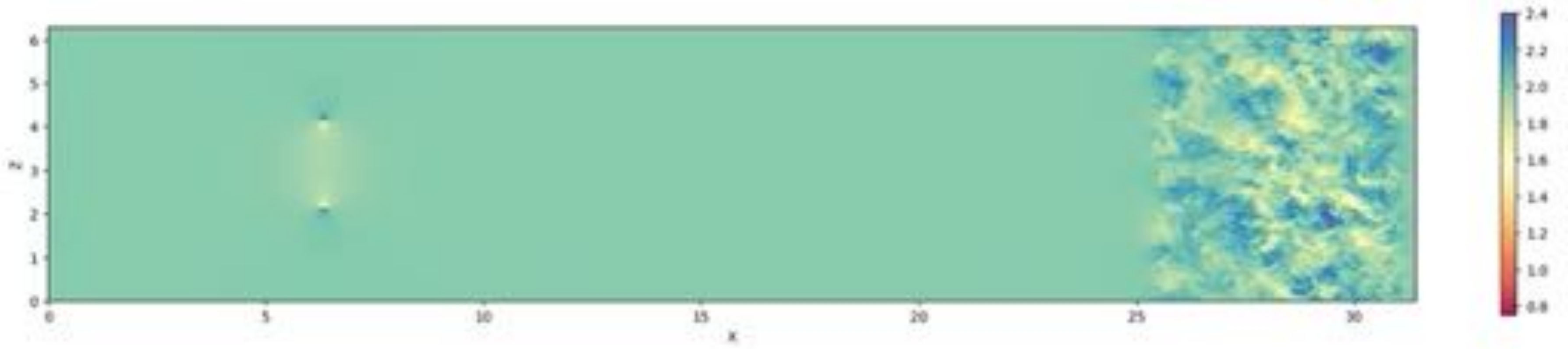
Incompressible Flow Formulation

Fully spectral spatial discretization

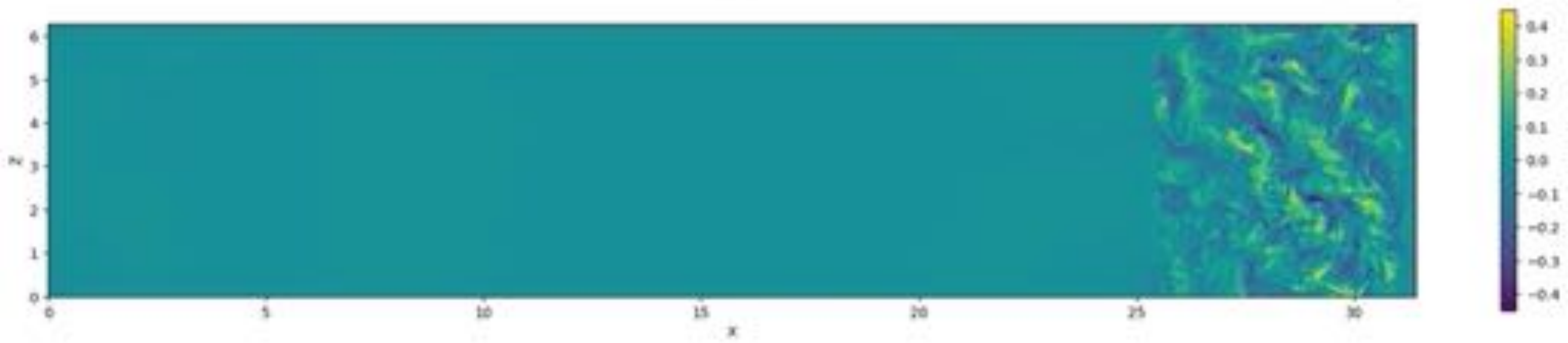
RK4 time discretization

Sigma SGS model; $Re \rightarrow \infty$

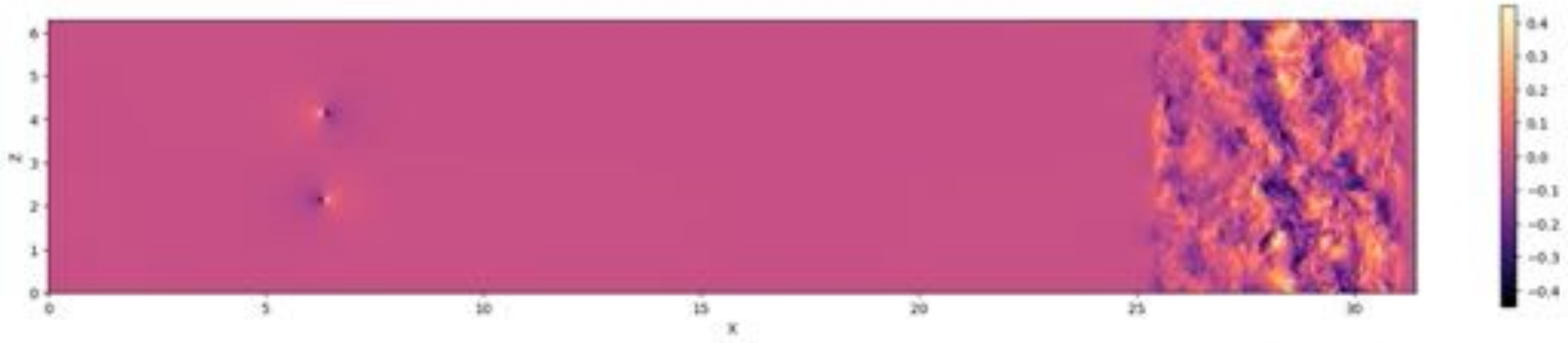
Turbulent Wake of a Dragging Disk



U component

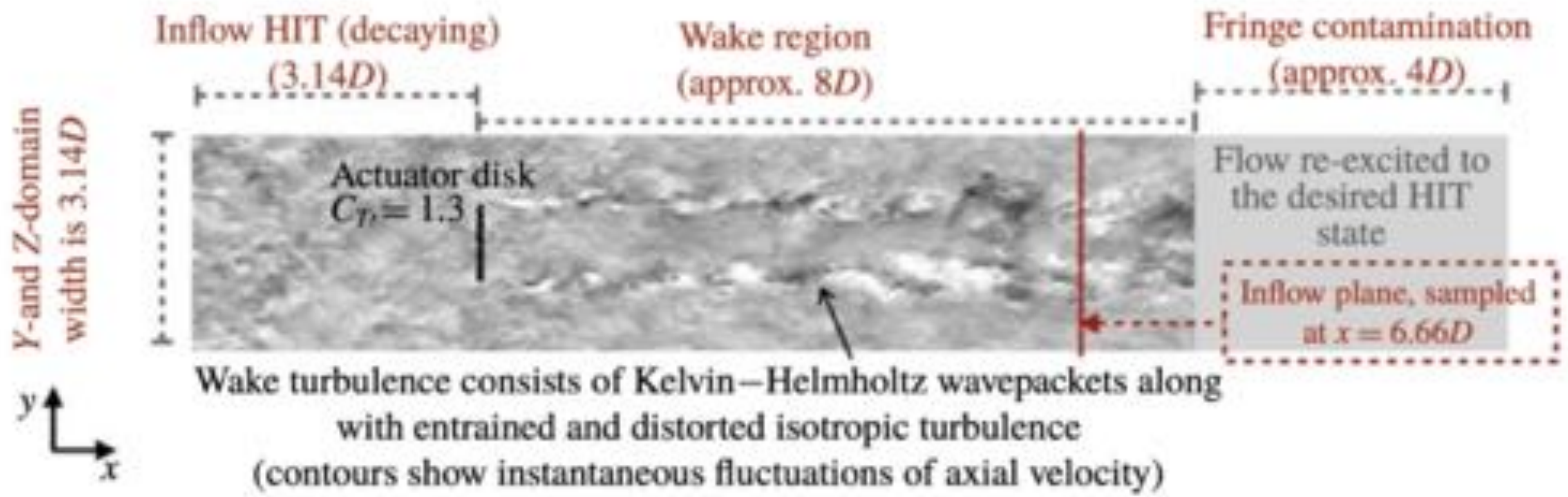


V component

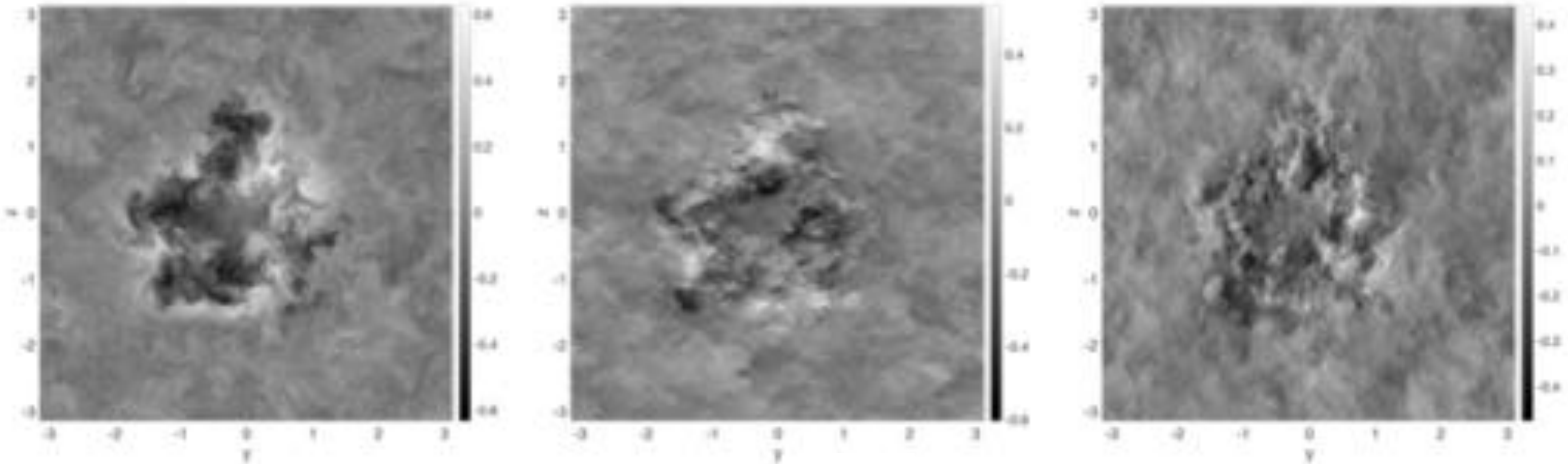


W component

Turbulent Wake of a Dragging Disk



Wake turbulence consists of Kelvin–Helmholtz wavepackets along with entrained and distorted isotropic turbulence (contours show instantaneous fluctuations of axial velocity)



(d) u (fluct)   (e) v (fluct)   (f) w (fluc n)

See Ghate, Towne & Lele, J. Fluid Mech., 2019

Seeking a modal expansion

Since the flow is temporally stationary, we need to find the **principal components** of the 2-pt cross-spectral density tensor defined as:

$$\boldsymbol{S}(\boldsymbol{x},\boldsymbol{x}',f) = \int_{\infty}^{\infty} \boldsymbol{C}(\boldsymbol{x},\boldsymbol{x}',\tau)e^{-i2\pi f\tau}d\tau = \sum_{j=1}^{\infty} \lambda_j(f)\boldsymbol{\psi}_j(\boldsymbol{x},f)\boldsymbol{\psi}_j^{\star}(\boldsymbol{x}',f)$$

$\tau = t - t'$ (due to stationarity) $\quad \boldsymbol{C}(\boldsymbol{x},\boldsymbol{x}',\tau) = \overline{\boldsymbol{q}(\boldsymbol{x},t)\boldsymbol{q}(\boldsymbol{x}',t-\tau)}$

Further leveraging azimuthal homogeneity – simplifies the SVD substantially (POD modes are Fourier modes in azimuthal direction following Lumley (1970)

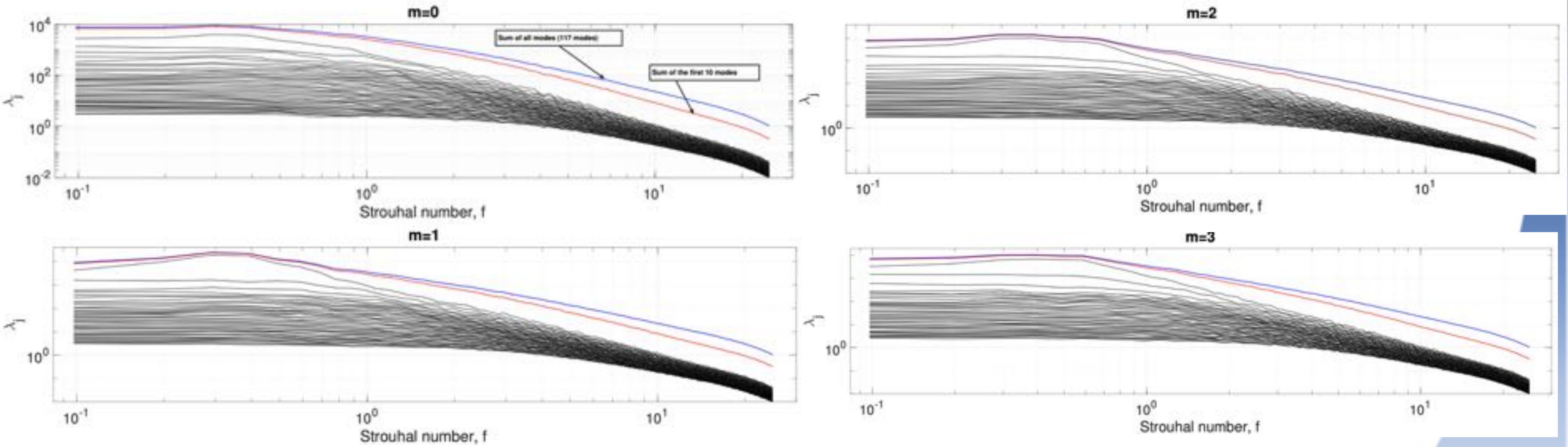
$$\hat{\boldsymbol{u}}(r,m,f) = \int_{t=0}^{T}\int_{\theta=0}^{2\pi} \boldsymbol{u}(r,\theta,t)e^{i(m\theta+ft)}d\theta dt = \sum_{j=1}^{J} a_j(m,f)\boldsymbol{\Psi}_j(r,m,f)$$

where, $a_j(m,f) = \langle \hat{u}(r,m,f), \hat{\psi}_j(r,m,f)\rangle_r$ is the modal energy with the property $|a_j a_m^{\star}| = \delta_{jm}\lambda_j$

Once $\lambda_j(m,f)$ and $\Psi_{\mathrm{j}}(r,m,f)$ are "learned" using data, **stochastic fields can be trivially generated**

$$\tilde{\boldsymbol{u}}(x,t) = IFFT_{m,f}\left\{\sum_{j=1}^{J} \tilde{a}(m,f)\boldsymbol{\Psi}_{\mathrm{j}}(\mathrm{r},\mathrm{m},\mathrm{f})\right\} \text{ where } \tilde{a}_j = a_j e^{i\xi} \text{ and } \xi \in N(0,2\pi)$$

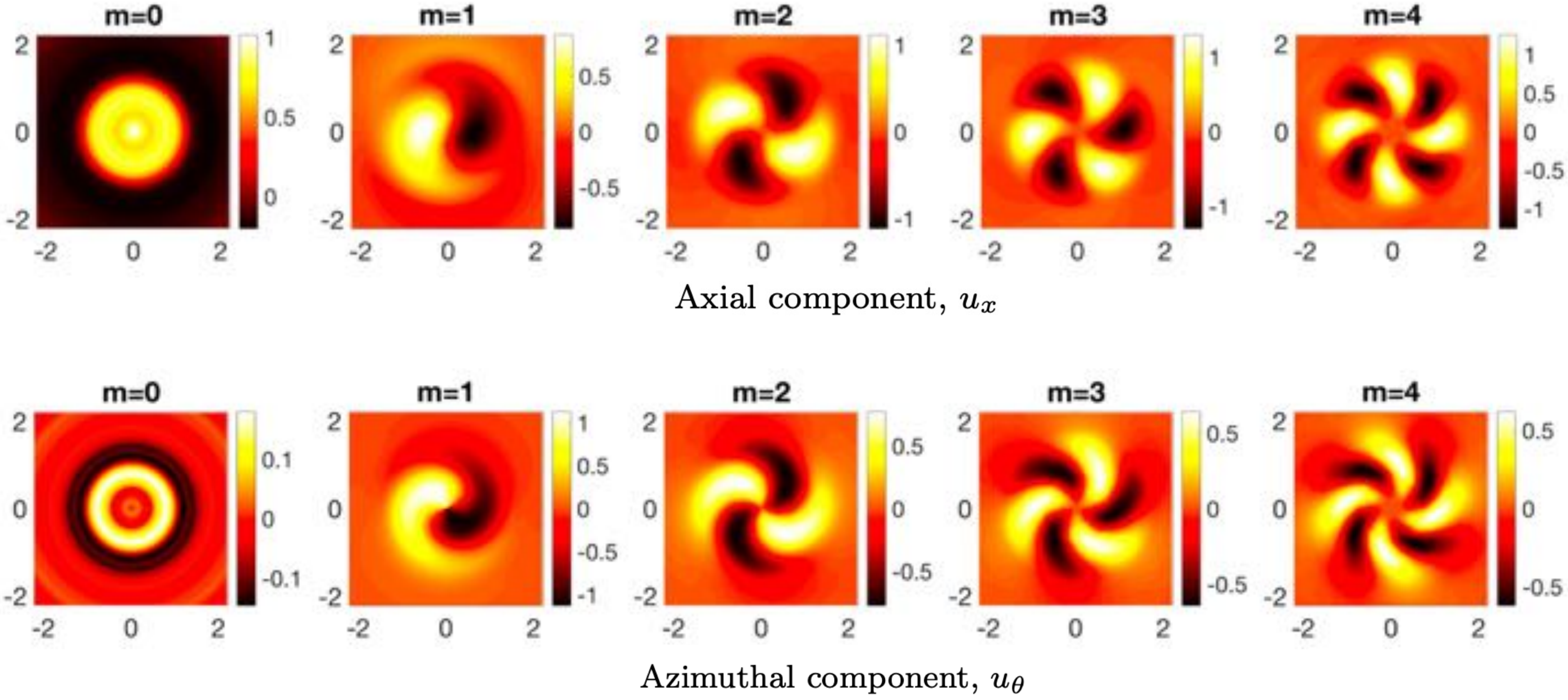
Is the flow low-rank?



Low-rank expansions likely to work only for low Strouhal numbers, significant loss of energy would occur at high Strouhal numbers

What have we “learned” from data?

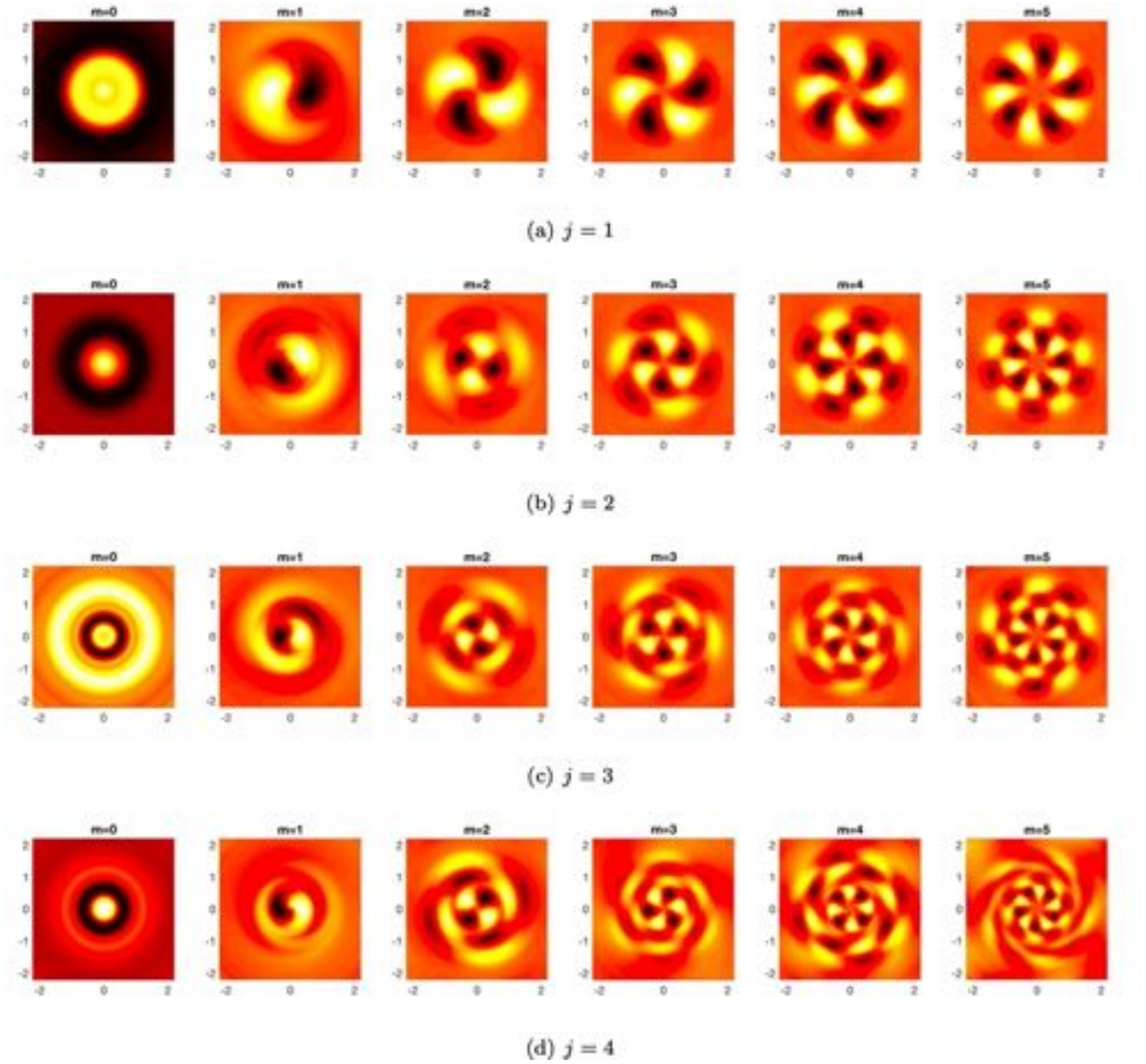
Consider the most energetic (j=1) mode at St = 0.4



Most energetic modes appear to pick up on shear-layer instabilities (linear-processes such as K-H)

What have we “learned” from data?

Now consider higher rank modes



- Increasing “uncertainty” with increasing mode order and azimuthal wavenumber
- Ambient co-flow and non-linear interactions responsible for lack of low-rank
- Would require a tremendous amount of data to ”learn” small-scale content (higher values of j, m and f) – statistical convergence is very slow $\sqrt{N_{sample}}$ (Welch, 1967)

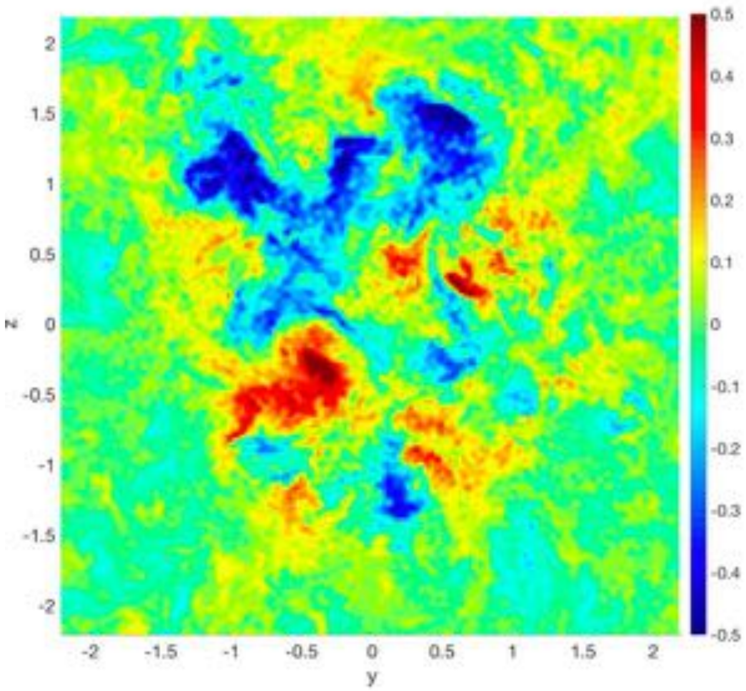
Truncated modal expansion: ROM for "Large Scales"

$$\boldsymbol{u}^{\text{trunc}}(y,z,t) = \boldsymbol{\mathcal{I}}_{(r,\theta)\rightarrow(y,z)}\left\{\sum_{|f|<F_{co}}\left(\sum_{|m|<M_{co}}\left(\sum_{j<j_{max}} a_j(m,f)\boldsymbol{\Psi}_j(m,f,r)e^{-i(m\theta+ft)}\right)\right)\right\}$$

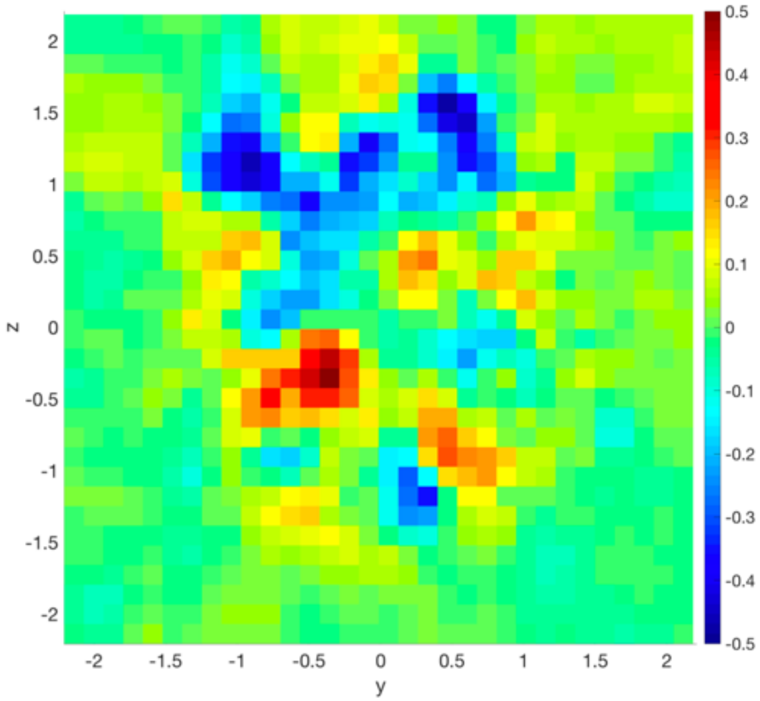
$$\boldsymbol{u}^{\text{res}}(y,z,t) = \boldsymbol{u}(y,z,t) - \boldsymbol{u}^{\text{trunc}}(y,z,t)$$

DATA – DRIVEN ROM FEASIBLE

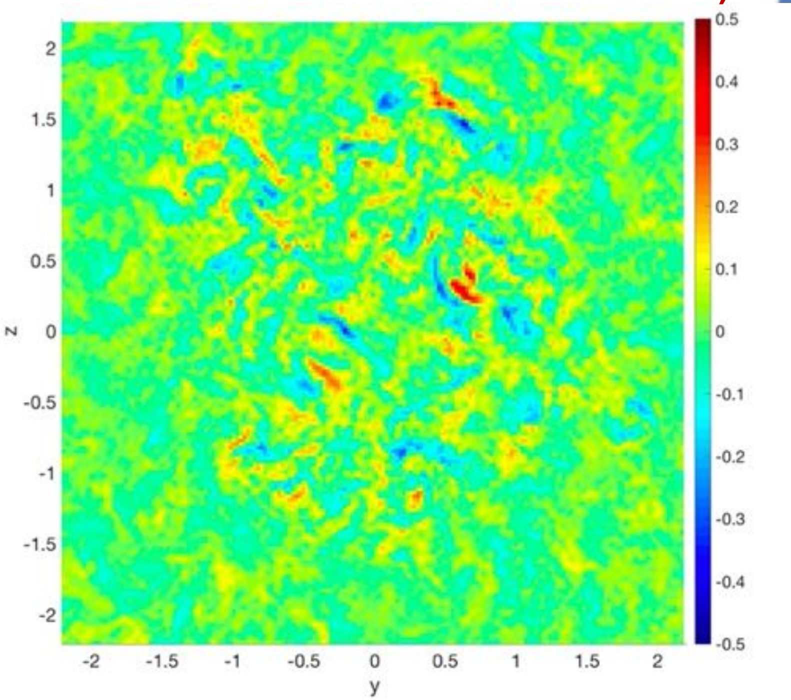
DATA-DRIVEN ROM DIFFICULT
(UNFORTUNATELY ALSO THE RESPONSIBLE OF HIGH CFD COST)



$u(y,z)$

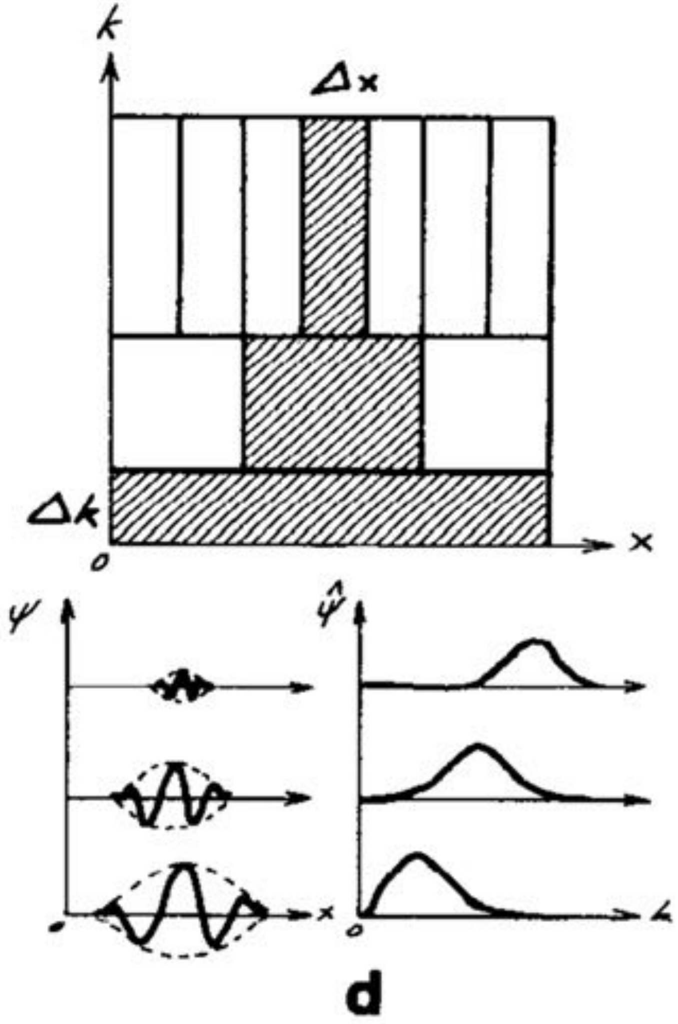


$u^{\text{trunc}}(y,z)$



$u^{\text{res}}(y,z)$

Wavelets – A potential solution to representation problem?



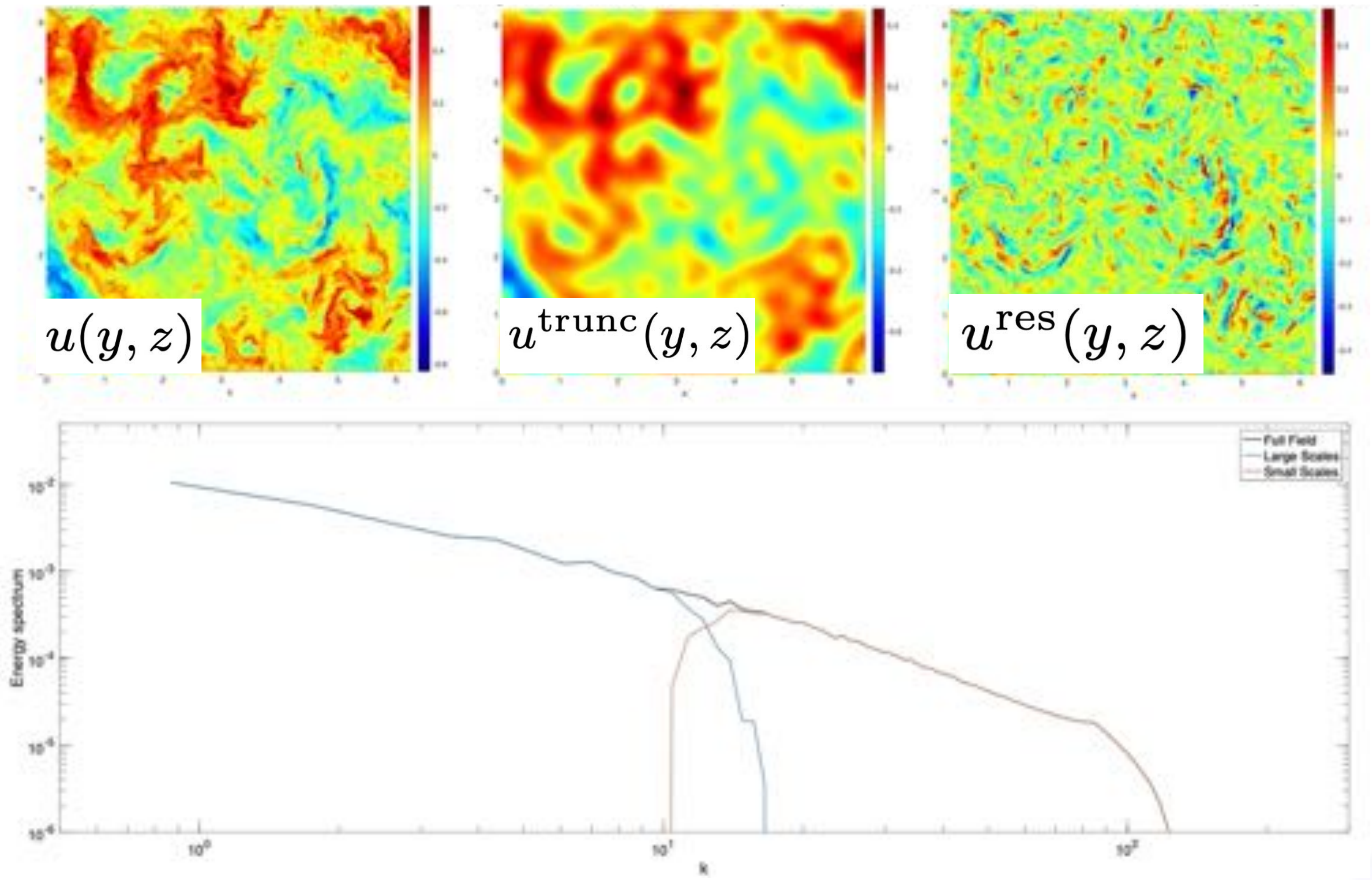
Uncertainty Principle
(Fourier Duality)
requires: $\Delta_X \Delta_k \geq 2\pi$

Wavelets can in principle provide **optimal spatio-spectral localization**

Could solve the representation issue – but most wavelet NS algorithms are **not much cheaper** than traditional CFD algorithms (See Farge, Annual Rev. Fluid Mech., 1992)

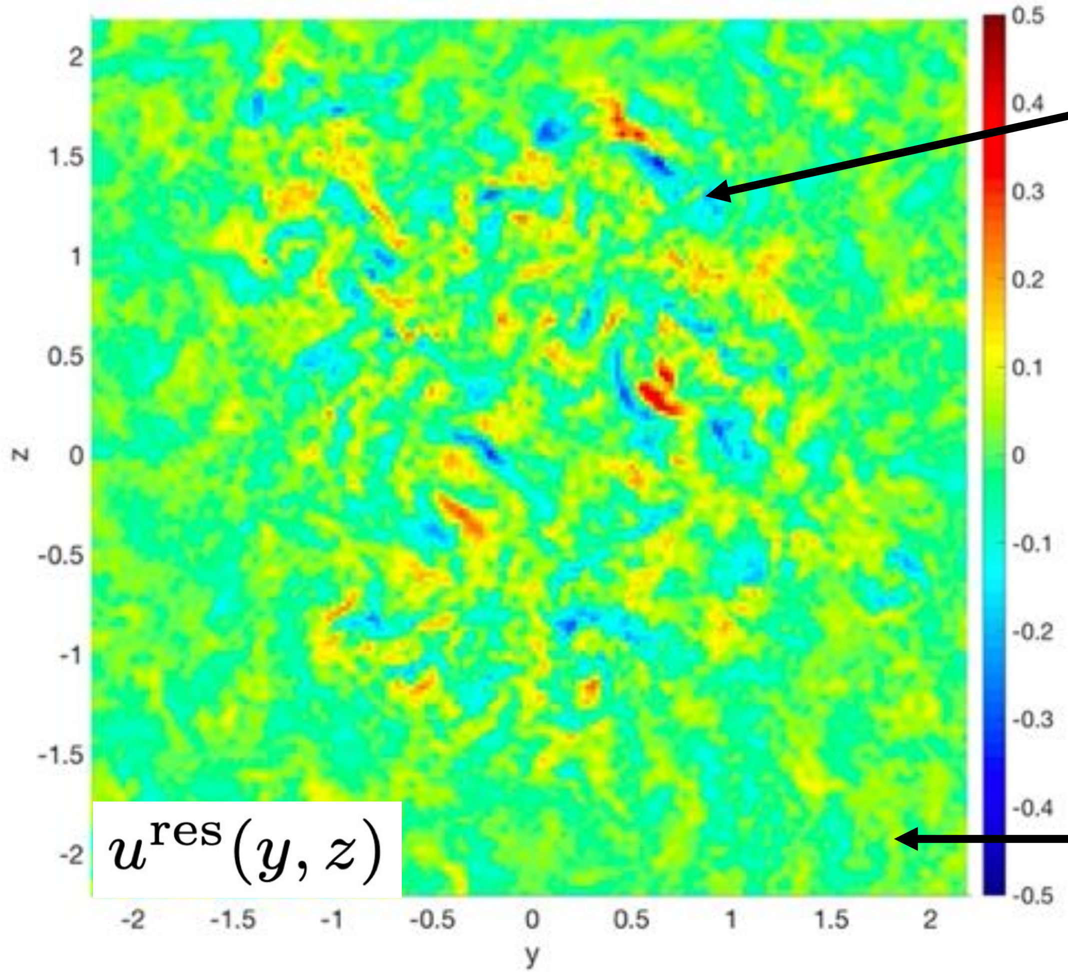
Requirements: Model for residual scales

Requirement 1: Spectral extrapolation – superresolution (obvious requirement)



Requirements: Model for residual scales

Requirement 2: Allow inhomogeneity (spatial or temporal)

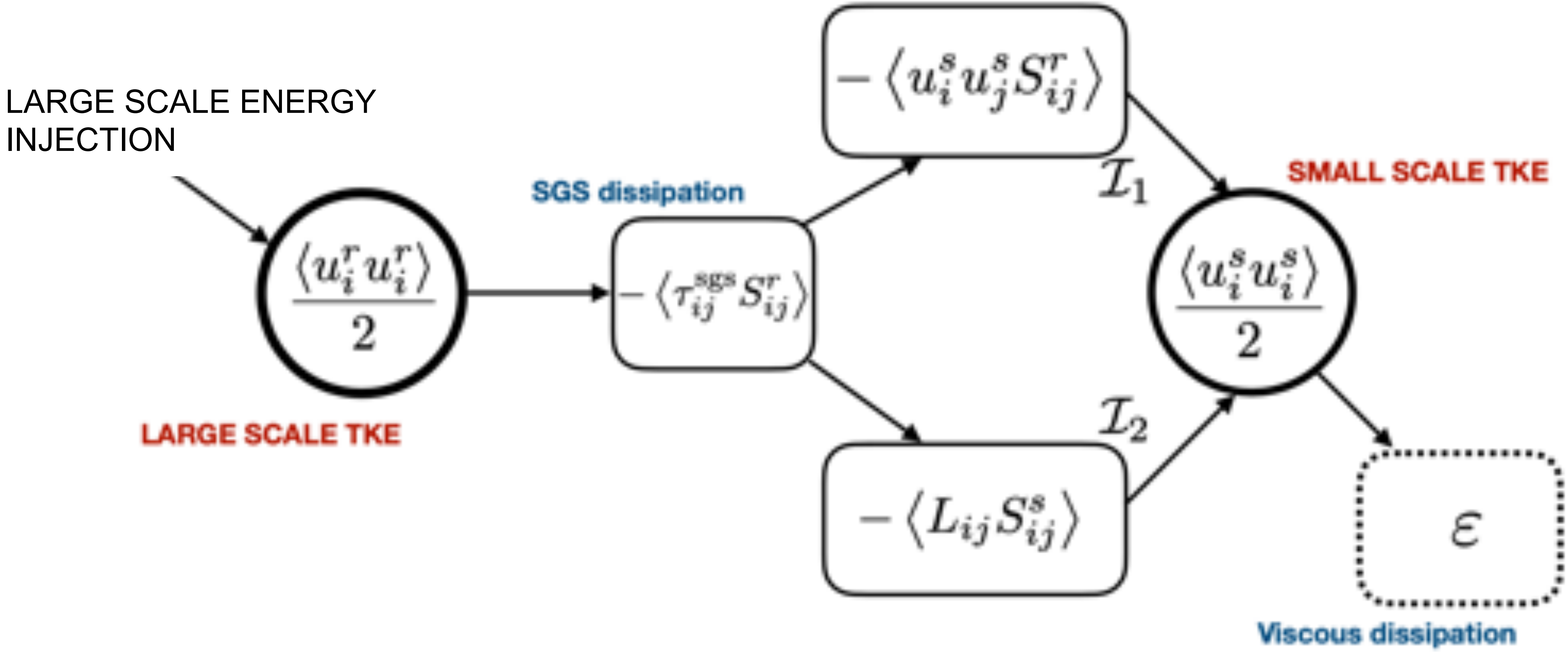


Intense fluctuations generated by shear from the mean velocity profile

Spatially decaying isotropic turbulence due to lack of shear

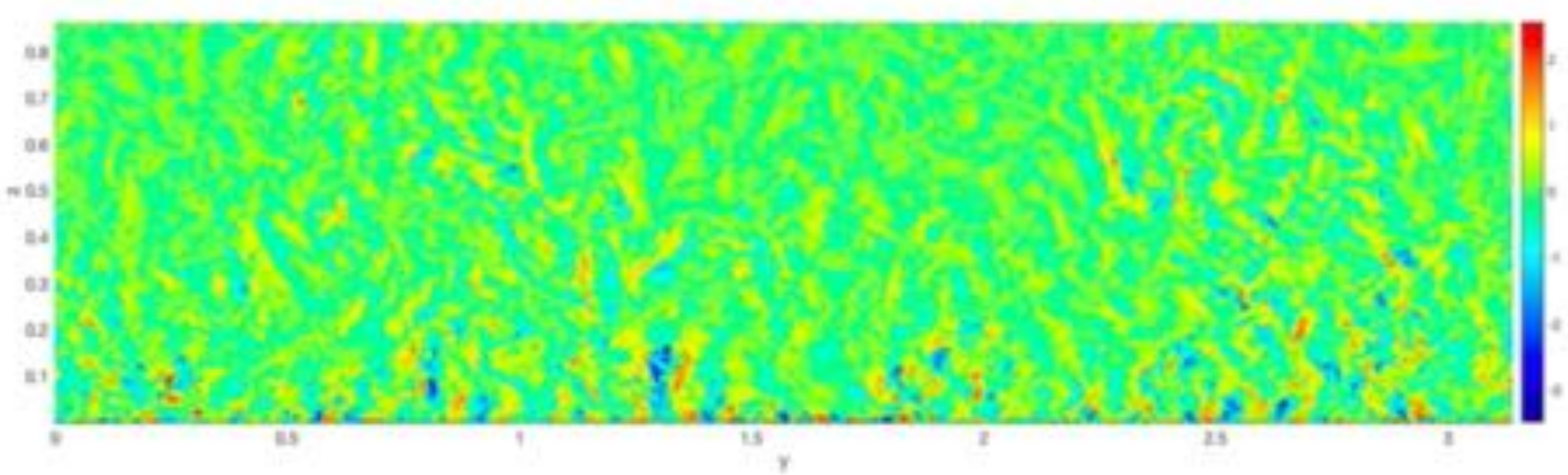
Requirements: Model for residual scales

Requirement 3: Capture correct energy transfers (even in lack of mean shear)

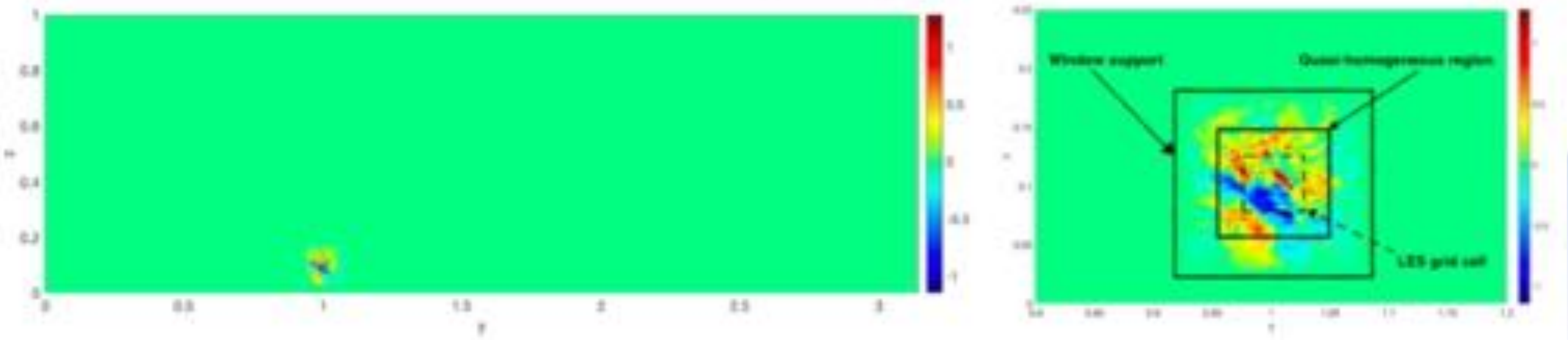


Seeking a model for small scales: Quasi-Homogeneity

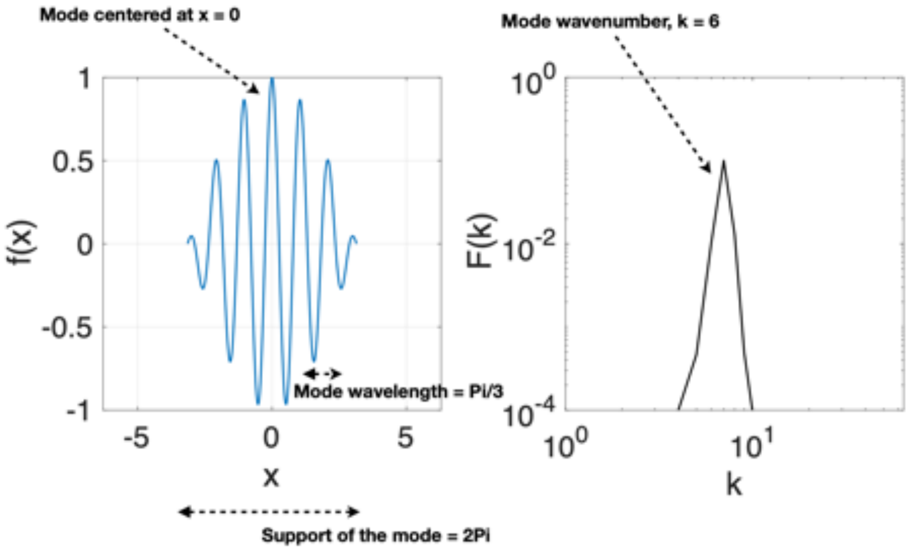
Consider u_{res} for a generic boundary layer (spanwise periodic) on the cross-plane



Subfilter streamwise (x) velocity on YZ plane



Model for Small-Scales: Gabor modes



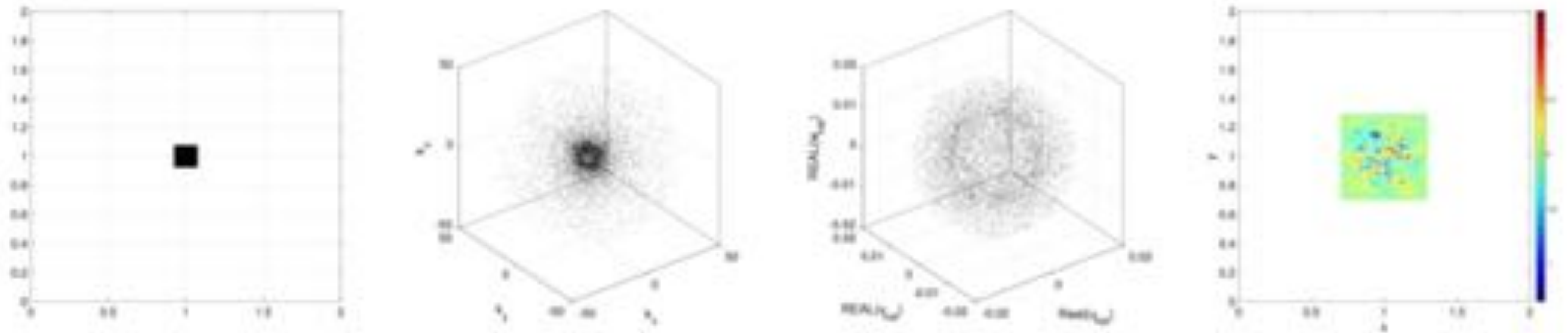
The scale separation parameter is defined as: $\varepsilon = \frac{\lambda}{\Delta}$

Typical compression in **Degrees of Freedom** > 95%

Physical space rendering can be performed using an $O(Nlog(N))$ algorithm (See Ghate & Lele, J. Fluid Mech., 2017)

Can be further improved to a $O(N)$ algorithm using wavelet properties

Multiple modes



Gabor Modes: Temporal Evolution

Governing equations for Gabor modes

1. Motion described in a *sweeping* frame:

$$\partial_t x_j = U_j^0$$

2. An Eikonal equation for evolution of a wavenumber:

$$\partial_t k_j = -k_m \partial_j U_m^0$$

3. A WKB-RDT approximation for evolution of complex amplitude

$$\partial_t \hat{u}_i = \left(\frac{2k_i k_m}{k^2} - \delta_{im}\right) \hat{u}_j \partial_j U_m^0 + \left(\frac{k_i k_j}{k^2} - \delta_{ij}\right) g_j \beta \hat{\theta} - (\nu + \nu_t) k^2 \hat{a}_i + \hat{f}_i^{\perp} - 2\epsilon_{ijk} \Omega_j \hat{u}_k$$

$$\partial_t \hat{\theta} = -\hat{u}_j \partial_j \Theta^0 - (\kappa + \kappa_t) k^2 \hat{\theta} + \hat{f}_\theta$$

where, $\hat{f}_i$ and $\hat{f}_\theta$ are Gabor projections of the Leonard stress terms $\partial_j L_{ij}$ and $\partial_j q_j$ respectively.

Important consideration: The ODEs governing evolution of the Gabor modes are only accurate up to leading order in ε; the proposed model is not a numerical method.

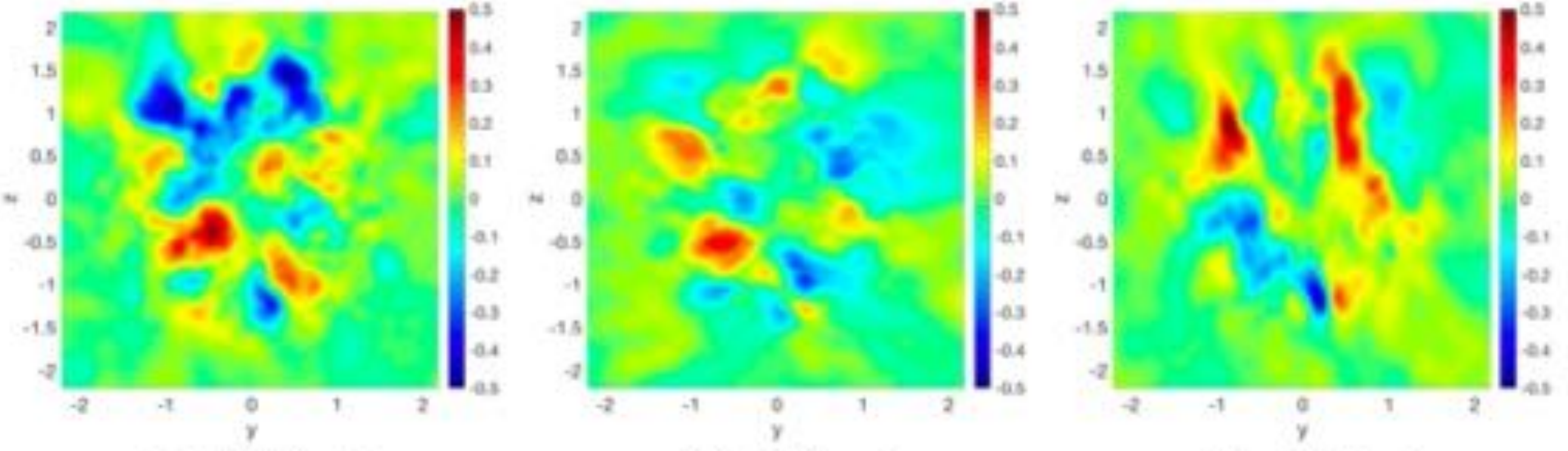
Model for the local (in scale space) convective non-linearity

The action of the convective non-linearity due to local triadic interactions will be modeled using a spectral viscosity based on Renormalization Group Theory (RNG) (see Canuto & Dubovikov, PoF, 1996)

$$\widehat{\partial_j h_{ij}}^{\perp} = -\nu_t(k) k^2 \hat{u}_i \quad , \quad \nu_t(k) = \left(\nu^2 + c_\nu \int_k^\infty q^{-2} E(q) dq\right)^{1/2} - \nu$$

See derivation in Ghate & Lele (J. Fluid Mech, 2020)

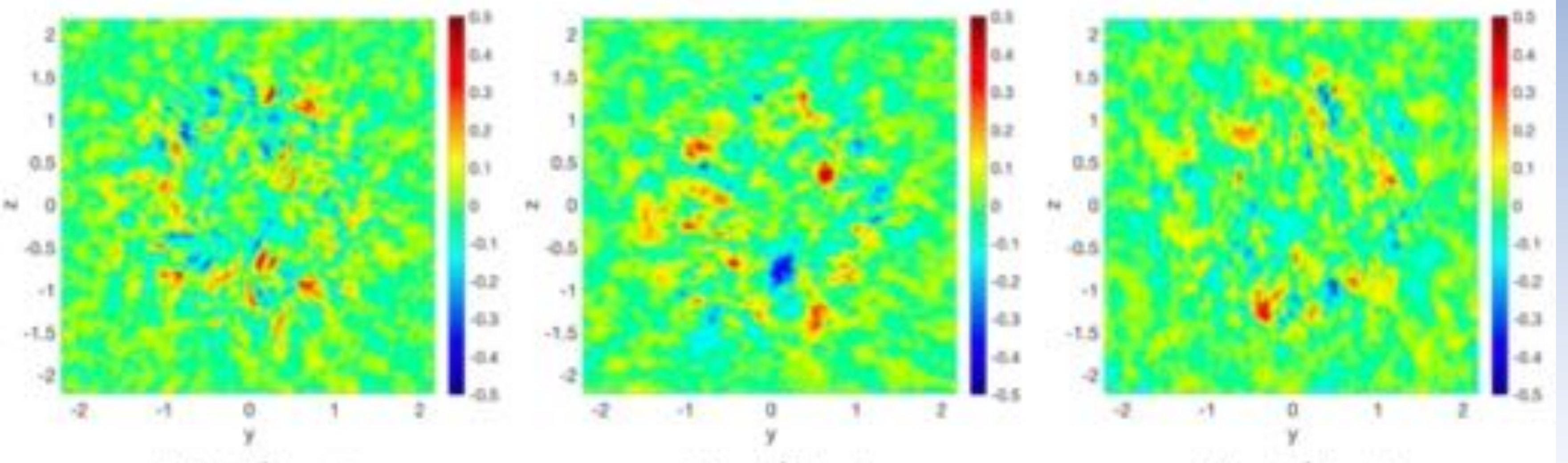
Evaluation of the combined model



(a) $u^{\mathrm{trunc}}(y,z)$ (b) $v^{\mathrm{trunc}}(y,z)$ (c) $w^{\mathrm{trunc}}(y,z)$

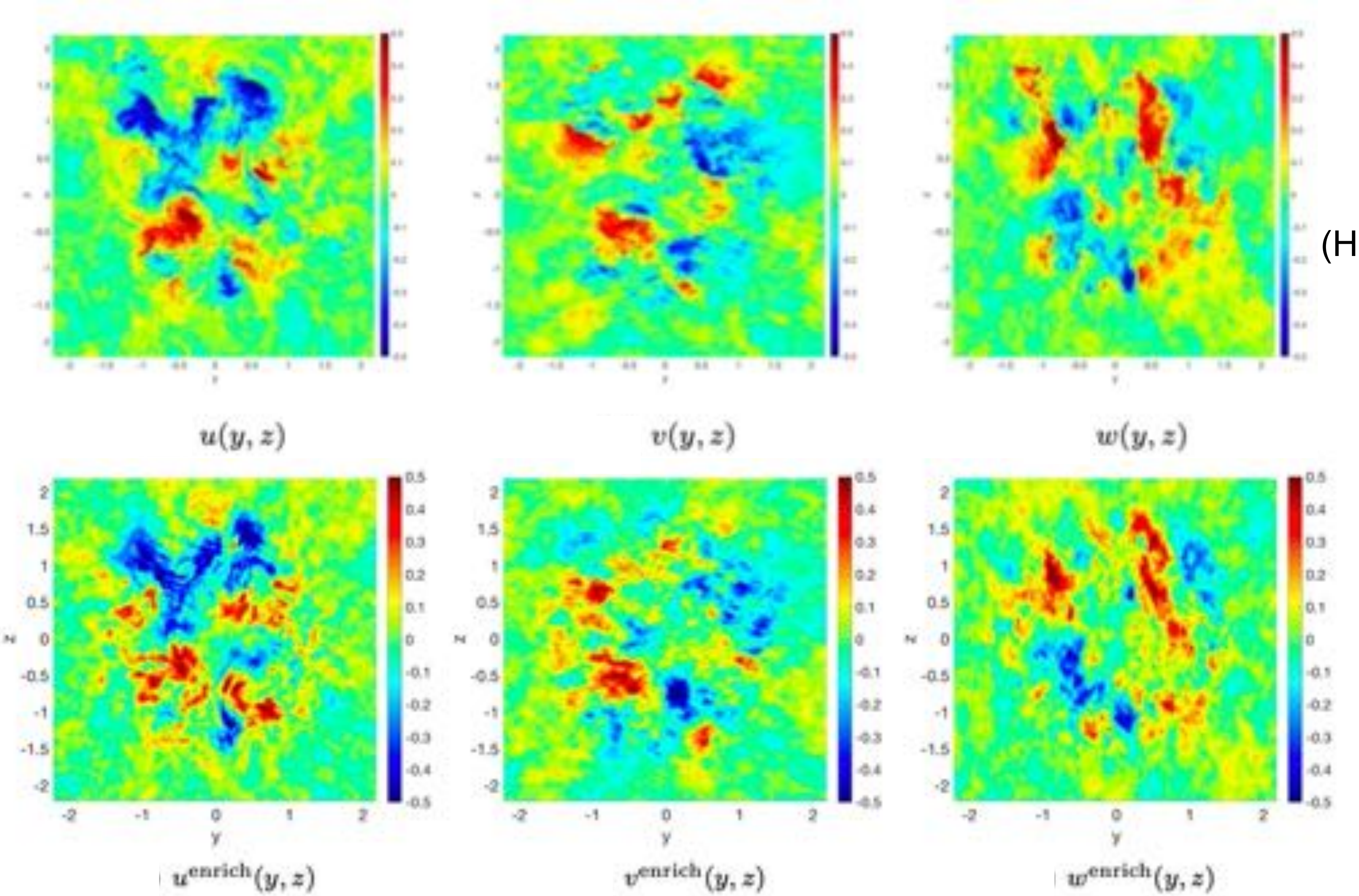
DATA DRIVEN
(Truncated SPOD)

PHYSICS DRIVEN
(Gabor Modes)



(d) $u^{\mathrm{gab}}(y,z)$ (e) $v^{\mathrm{gab}}(y,z)$ (f) $w^{\mathrm{gab}}(y,z)$

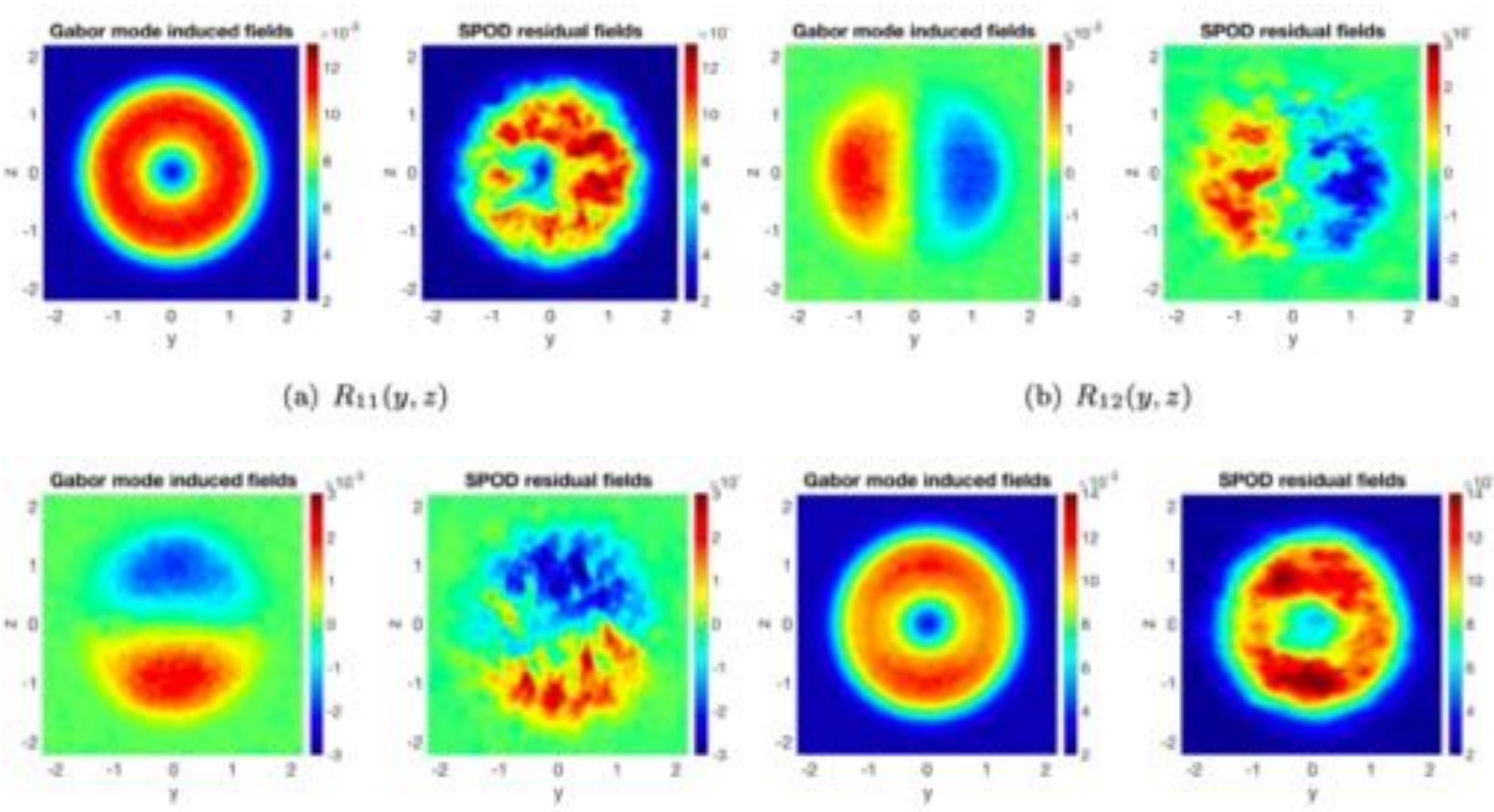
Evaluation of the combined model



REFERENCE
(HIGH RESOLUTION LES)

RECONSTRUCTED
(SPOD + GABOR
MODES)

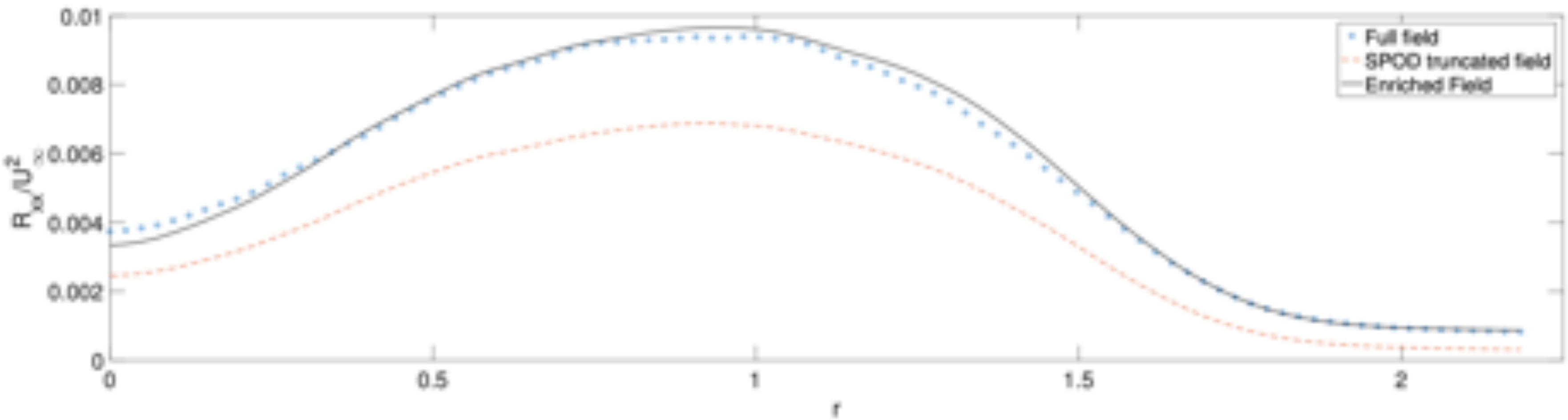
Evaluation of the combined model



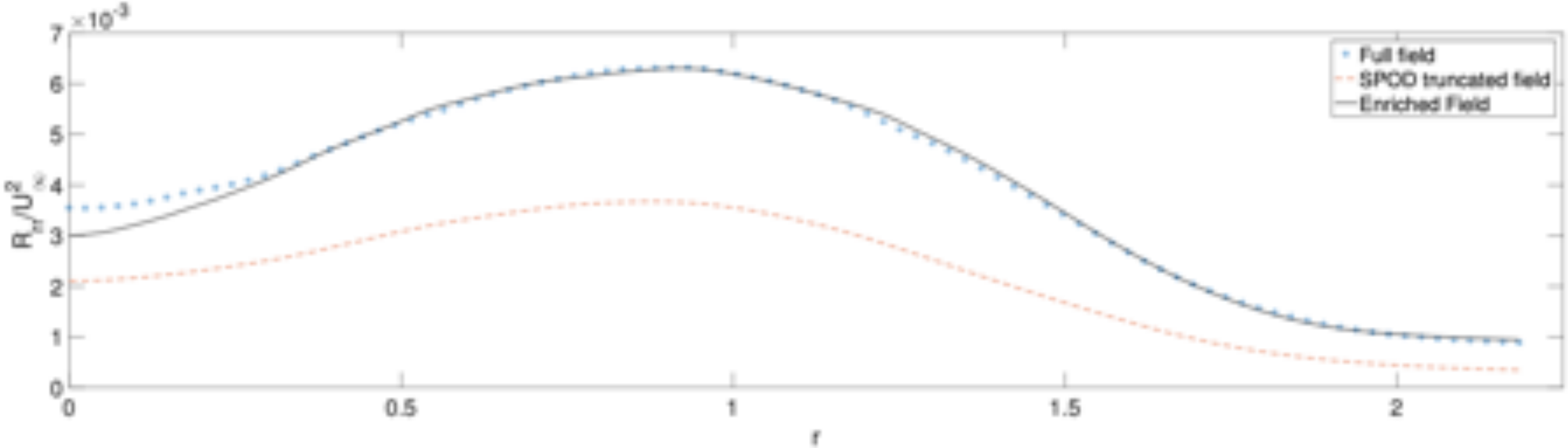
(a) $R_{11}(y,z)$ (b) $R_{12}(y,z)$

(c) $R_{13}(y,z)$ (d) $R_{22}(y,z)$

Evaluation of the combined model

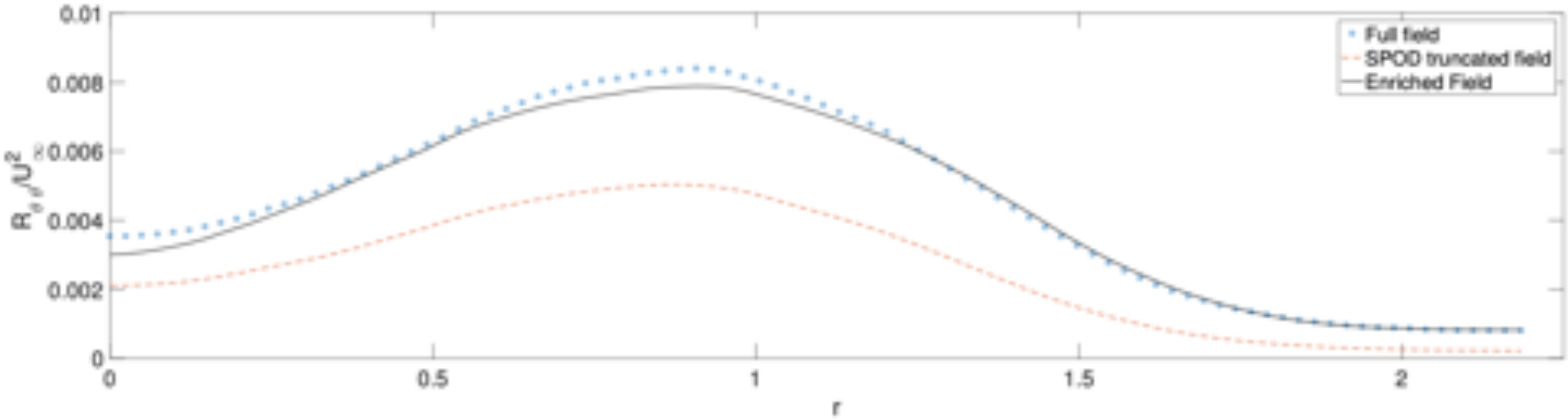


(a) $\langle u_x u_x \rangle$

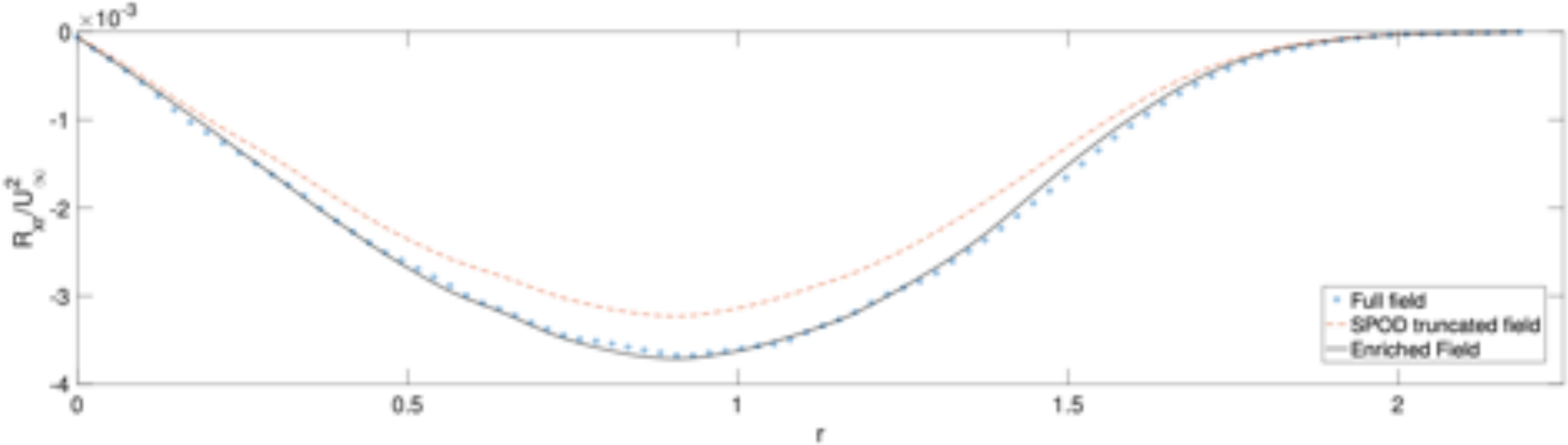


(b) $\langle u_r u_r \rangle$

Evaluation of the combined model

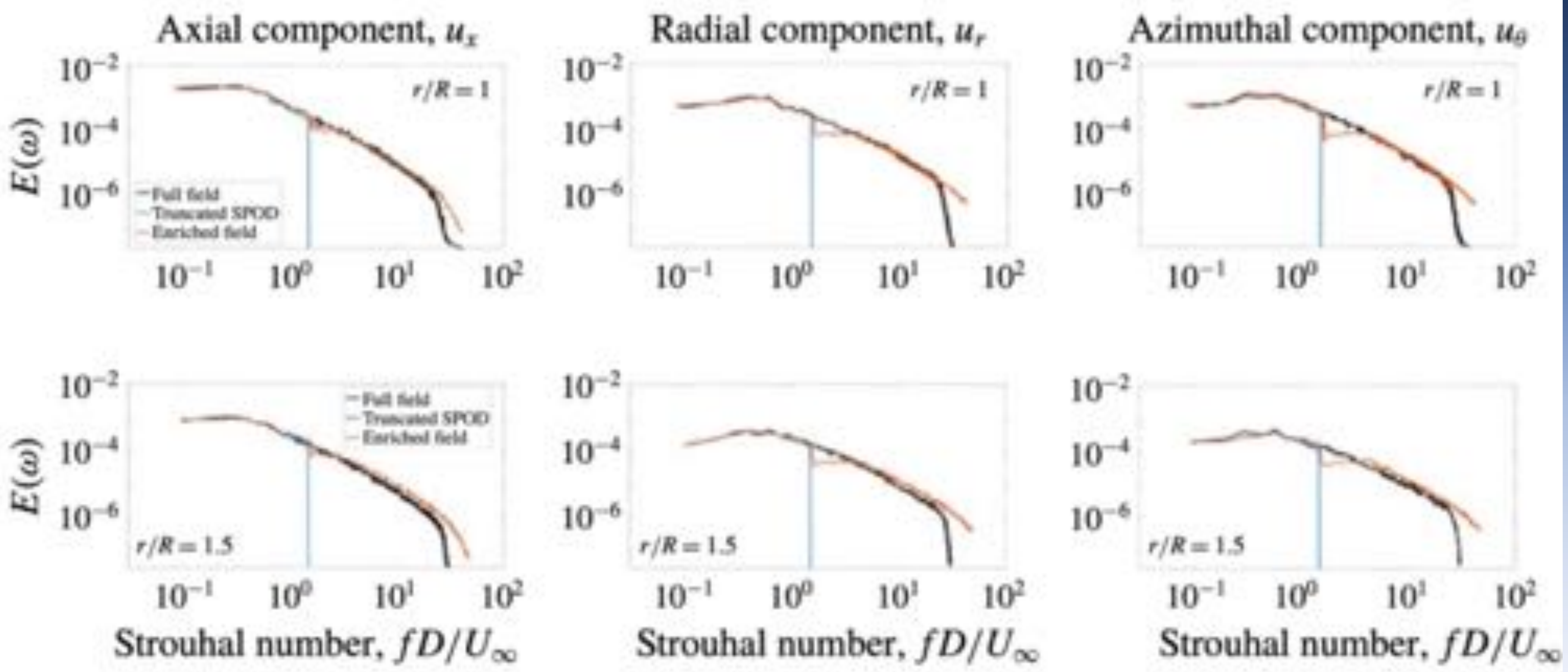


(c) $\langle u_\theta u_\theta \rangle$

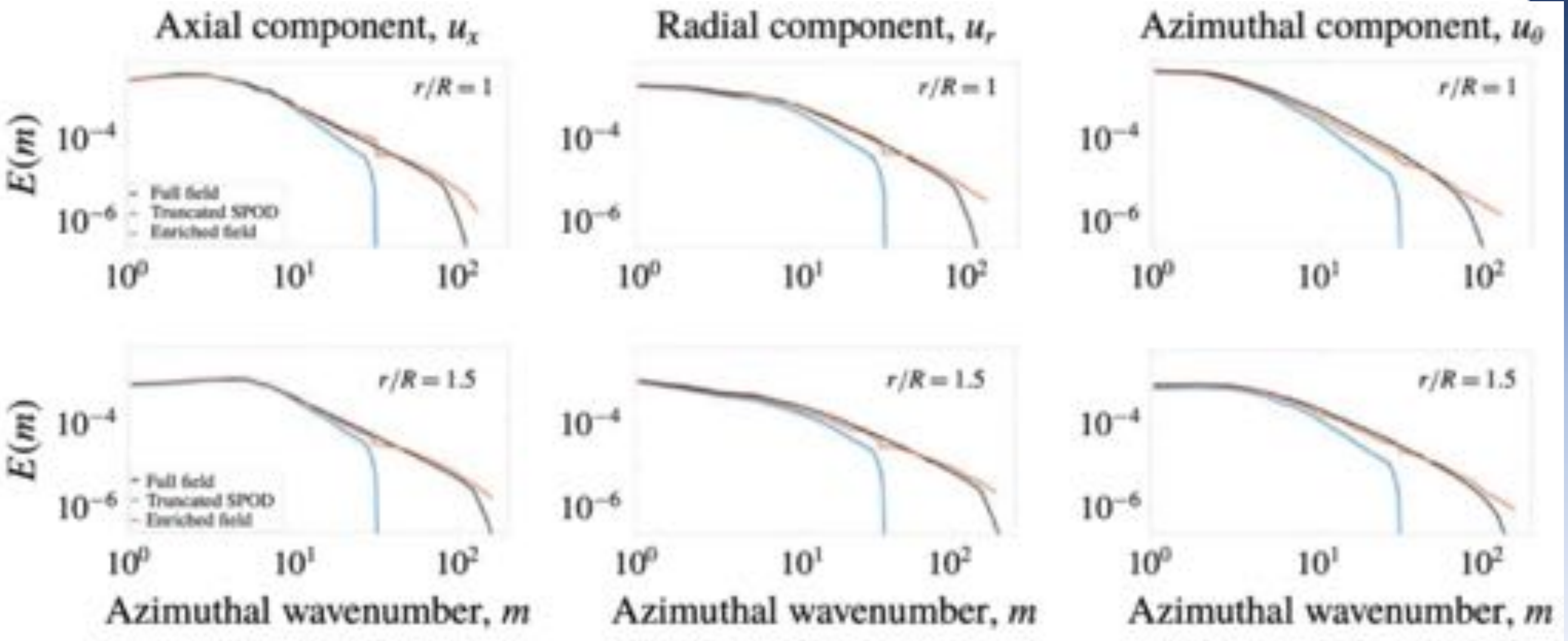


(d) $\langle u_x u_r \rangle$

Evaluation of the combined model



Evaluation of the combined model



Conclusions

- Data-driven modeling may not always be the only and complete solution!
- Room for physics driven-modeling still exists; models naturally inherit fundamental properties such as Galilean and Rotational invariance
- Combined model enables full-scale generation of stochastic solenoidal turbulent fluctuations with accurate second order 2-pt correlations
- POD type representations are highly limiting – lots of potential for NNs for more generalized representation
- Further development of the method for more complex flow configurations continues … (Ryan Hass, PhD student at Stanford University)

Acknowledgements

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