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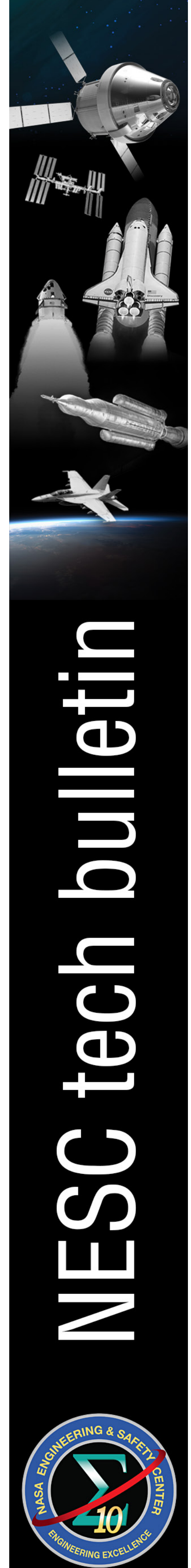

Uncertainty Quantification of Reduced Order Structural Dynamic Models

Uncertainty quantification (UQ) provides statistical bounds on prediction accuracy based on finite element model (FEM) uncertainty. An alternate method for UQ, called the Hybrid Parametric Variation (HPV) combines a parametric variation of the Hurty/Craig-Bampton (HCB) fixed-interface (FI) modal frequencies with a nonparametric variation (NPV) method. This provides a UQ method that can be traced to test data, which can be updated as additional data and improved correlated models become available.

Background

The purpose of uncertainty quantification (UQ) is to provide statistical bounds on prediction accuracy based on model uncertainty. This is distinct from model updating, which attempts to modify models to improve their accuracy. UQ does not improve the accuracy of models but accepts that the models are inherently inaccurate and attempts to quantify the impact of that inaccuracy on predicted results. The most common method for modeling uncertainty in the structural dynamics community is a parametric approach, which varies physical parameters in the model. However, there are several disadvantages associated with the parametric method. Determining a reduced set of parameters that have a significant impact on the system response can be time consuming, and the selected parameter probability distributions are rarely reliably known. Model-form uncertainty cannot be directly represented by FEM input parameters nor included in a parametric approach. However, model-form uncertainty can be modeled using random matrix theory (RMT), where a probability distribution is developed for the matrix ensemble of interest.

The HPV Method for UQ

An alternate method that has the potential to improve for UQ, called the HPV method, has been summarized in [ref 1]. The HPV method combines a parametric variation of the HCB FI modal frequencies with a NPV method that randomly varies the HCB mass and stiffness matrices as Wishart [ref. 2] random matrix distributions using RMT.

The basis for the NPV component of the HPV method is to replace the HCB matrices representing each system component with an ensemble of random matrices, based on RMT. Each matrix in the ensemble must be close to the nominal matrix in the sense of some matrix norm and must meet certain requirements (e.g., symmetry and correct sign definiteness). However, the matrices are otherwise free and are not tied to any particular set of parameters in the FEM. Soize [ref. 3] used the maximum entropy principle to derive the positive definite and positive semidefinite ensembles SE ^+ and SE ^(+0) that follow the matrix variate gamma distribution and are capable of representing random structural matrices. This means the matrices in the ensembles are real, symmetric, and possess the appropriate sign definiteness to represent structural mass, stiffness, or damping matrices. As the dimension of the random matrix n increases, the matrix variate gamma distribution converges to a matrix variate Wishart distribution.

The HPV method uses uncertainty models for HCB components based on component modal test/analysis correlation results. The NPV based dispersion of the HCB mass matrix is derived from the test self-orthogonality matrix. Two different test self-orthogonality metrics were considered, the root mean square (RMS) value of the off-diagonal terms, and the mean absolute value of the off-diagonal terms. FI mode eigenvalue uncertainty within the HCB stiffness matrix is based on frequency error between matching HCB FI modes and test modes. The NPV method is then applied to the HCB stiffness matrix by layering it on top of the FI eigenvalue variation. The stiffness matrix dispersion level is based upon the FEM/test XO matrix using the diagonal cross-generalized mass (DCGM) metric, which is the RMS value of the diagonal terms. The basis for the HCB component uncertainty model is shown in Figure 1.

Select HCB component dispersion values:

Fixed-interface eigenvalues - $\delta\lambda_{FI}$ Gaussian Process Model (GPM) - *Parametric*

HCB stiffness matrix - δK_{HCB} Wishart (ME) - *Non-parametric*

HCB mass matrix - δM_{HCB} Wishart (ME) - *Non-parametric*

No mass uncertainty assumed in SLS UQ work

Basis:

Component test/analysis correlations results:

Eigenvalue – test/analysis frequency error.

Stiffness – test/analysis cross-orthogonality (XO).

Mass – test mode self-orthogonality (SO).

$$\begin{bmatrix} M_{bb} & M_{bq} \\ M_{qb} & I \end{bmatrix} \begin{Bmatrix} \ddot{x}_b \\ \ddot{x}_q \end{Bmatrix} + \begin{bmatrix} K_{bb} & 0 \\ 0 & (\Omega^2) \end{bmatrix} \begin{Bmatrix} x_b \\ x_q \end{Bmatrix} = \begin{Bmatrix} f_b \\ \Phi_{iq}{}^T f_i \end{Bmatrix}$$

Figure 1. Basis for HCB Component Uncertainty Model

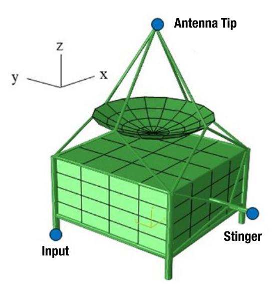

Figure 2. GSC Frequency Response Input and Output Locations

The validity and efficacy of the HPV method and corresponding component uncertainty model development procedure is examined by applying the approach to two examples in [ref 1]. The General Spacecraft (GSC) example (Figure 2) provides a representation where the test model was known. The uncertainty model developed for the GSC HCB component based on the test-configuration modal correlation results show the flight-configuration test frequencies and frequency response with the P98/90 probability enclosure coverage (Figure 3).

Based on this work and other assessments [refs. 4-7], the HPV method adds another device to the toolset used for complex system UQ analysis that accounts for both parametric and model-form uncertainty and is based on test data. From experience gathered to date using the HPV method, additional design specific applications must be investigated to determine which self-orthogonality metric provides the best mass matrix dispersion results, and to provide further confidence in the validity of the HPV method of UQ analysis.

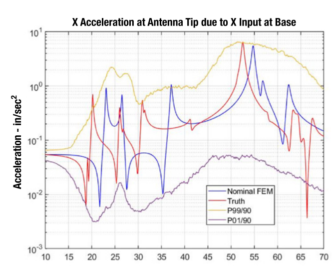

Figure 3. GSC Flight-Configuration

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