NASA White Paper - Terramechanics for LTV Modeling and Simulation

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Simulating the interaction between wheel and soil is critical to the overall rover dynamics. This paper presented simple models for wheel soil interaction including the rolling resistances on the wheel due to soil compression and bulldozing and the maximum tractive force between wheel and soil. Summary of typical lunar soil properties were presented in this paper and the wheel resistance model implementation and integration were also discussed. Integrating the wheel resistance model with the rover simulation will improve its dynamics and wheel slip models and enable the capability simulate the situation where wheel got stuck in the soil.

I. Lunar Regolith

The thick fragmental and unconsolidated layer of debris that covers the lunar surface is referred to regolith. This regolith can range in size from small dust like particles to large boulders and is between five to fifteen meters thick. Understanding how it is created and its composition allows us to characterize its properties and its affects in the operation of a lunar rover.

A. Major Constituents

Lunar regolith consists primarily of larger pieces that have been broken down over time. These major constituents include rock fragments, mineral fragments, and glassy particles. The primary elements that compose the lunar dust are oxygen, silicon, iron, calcium, aluminum, and magnesium in addition to various other trace compounds.

B. Creation Process

Due to the fact that the Moon lacks an atmosphere, the only "weathering" phenomenon that exists is the exposure to the harsh space environment. This "space weathering" involves micrometeorite bombardment, the irradiation of cosmic and solar rays, and solar wind. The impact of micrometeorites causes vaporization, melting, and the formation of agglutinates, which are mineral fragments that are fused together with glass. As the lunar soil matures over time it becomes finer grained with more agglutinates and the finer particles contain mostly iron-bearing glass.

II. Soil Mechanics[1]

When a rover wheel is in contact with the soil, there are two basic stress acting on the soil:

$$\text{Shear Stress: } \tau = \frac{T}{A} \tag{1}$$

$$\text{Normal Stress: } \sigma = \frac{W}{A} \tag{2}$$

Where T is the shear force produce by the wheel propulsion motor, W is the weight of the vehicle on that wheel, and A is the contact surface between the wheel and soil.

The maximum shear stress τ is linearly proportion to the normal stress apply on the soil as illustrated in Eq. (3) and Fig. 1. Where angle of internal friction, ϕ, and cohesion, c, are soil properties that define the resistance of the soil again shear stree.

$$\tau = c + \sigma \tan \phi \tag{3}$$

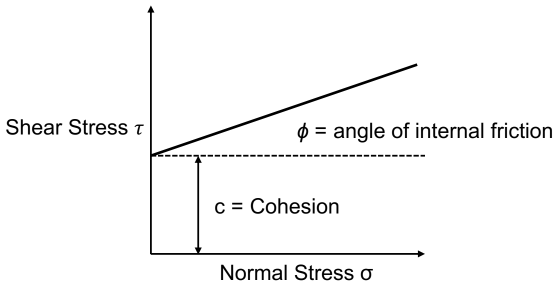

Figure 1: Relationship Between Shear Stress and Normal Stress

Modeling the soil reaction as a nonlinear spring:

$$P = \frac{W}{A} = kz^n \tag{4}$$

$$\text{Where, } k = \frac{k_c}{b} + k_\phi \tag{5}$$

$$\text{Therefore: } P = \left(\frac{k_c}{b} + k_\phi\right) z^n \tag{6}$$

Where P is the applied pressure, z is the soil compression depth, k_c is modulus of cohesion, k_ϕ is modulus of friction, and n is the exponent of sinkage for the soil. These basic properties of the soil mechanics are important for understanding the wheel-soil interaction in the later sections.

III. Wheel-Soil Interactions

The total rolling resistance of a vehicle are sum of the following resistance:[1]

- Compression Resistance, R_c, on all the wheels
- Rolling Resistance, R_r, on the vehicle
- Bulldozing Resistance, R_b, on the front wheels
- Gravitational Resistance, R_g, on the vehicle

A. Compression Resistance[1]

Akin derived the compression resistance of a wheel on soil based on the nonlinear spring soil model:

$$R_c = (k_c + k_\phi b) \frac{z^{n+1}}{n+1} \tag{7}$$

$$z = \left(\frac{3}{3-n} \frac{W}{(k_c + bk_\phi)\sqrt{D}} \right)^{\frac{2}{2n+1}} \tag{8}$$

$$R_c = \frac{1}{n+1} \left(\frac{3}{3-n} \frac{W}{\sqrt{D}} \right)^{\frac{2(n+1)}{2n+1}} \left(\frac{1}{k_c + bk_\phi} \right)^{\frac{1}{2n+1}} \tag{9}$$

Where R_c is the compression resistance, D is the wheel diameter, b is the wheel width, and W is the weight of the vehicle on the wheel. For Lunar soil with $n = 1.0$, the equation become:

$$R_c \approx 0.85854 \left(\frac{W^4}{(k_c + bk_\phi) D^2} \right)^{\frac{1}{3}} \tag{10}$$

The soil bearing capacity can be determine using the Terzaghi's Theory. The safe wheel weight on the soil can be define as:

$$W_s = A \left(cN_c + \gamma z N_q + \frac{1}{2} \gamma b N_\gamma \right) \tag{11}$$

Where c is the soil cohesion that has unit of N/m^2 and γ is weight density of the soil that has unit of N/m^3. N_c, N_q, and N_γ are the Terzaghi bearing capacity factors and they are define as:

$$N_q = \frac{exp\left[\left(\frac{3\pi}{2} - \phi\right)\tan\phi\right]}{2\cos^2\left(\frac{\pi}{4} + \frac{\phi}{2}\right)} \tag{12}$$

$$N_c = \frac{N_q - 1}{\tan\phi} \tag{13}$$

$$N_\gamma = \frac{2(N_q + 1)\tan\phi}{1 + 0.4\sin(4\phi)} \tag{14}$$

1. Modeling Compression Resistance in Simulation

Based on Equation (9) the following vehicle design parameter are needed, which will be define in the simulation configuration file:

- D: vehicle wheel diameter
- b: vehicle wheel width

The following soil parameter are needed, which are define by the simulation user for the soil. In reality these parameters various from one location to another as the vehicle traverse through different terrains. These parameters will be variables in the sim so that we can drive these soil parameters with some soil data or model in the future. However, they are currently set as constant in the simulation configuration file.

- n: exponent of sinkage of the soil ($n = 1$ for lunar soil)
- k_c: modulus of cohesion of soil ($k_c = 1400 N/m^2$ for lunar soil)
- k_ϕ: modulus of friction of soil ($k_\phi = 830000 N/m^3$ for lunar soil)

The following dynamic variable is needed. This variable is changing during the simulation as vehicle traverser through the surface.

- W: Weight of the vehicle on each wheel. (already calculating in the simulation)

Equation (11) can be use to determine the safe wheel weight on the soil. This can be add to the simulation to provide an indication on wether the wheel weight excess this safe weight value although the consequence on the soil are not currently modeled in the simulation. The following vehicle design and performance parameters are needed and they are defined in the simulation configuration file:

- A: wheel contact surface with the soil.
- b: wheel width.
- z: wheel sinkage depth.

The following soil parameters are needed for W_s, which will be define in the simulation configuration file. Again, in reality these parameters various across the surface. These parameters will be variables in the sim so that they can be drive with soil data or model in the future, but for now they are set as constant.

- c: soil cohesion ($c = 170N/m^2$ for lunar soil)
- γ: weight density of the soil ($\gamma = 2470N/m^3$ for lunar soil)
- N_c: Terzaghi's bearing capacity cohesion factor ($N_c = 48.09$ for lunar soil)
- N_q: Terzaghi's bearing capacity friction factor ($N_q = 32.23$ for lunar soil)
- N_γ: Terzaghi's bearing capacity density factor ($N_c = 33.27$ for lunar soil)

N_c, N_q, N_γ can also be calculate using the angle of inertial resistance of the soil, ϕ.

B. Rolling Resistance[1]

Akin defines rolling resistance due to other factors including tire compression, bearing friction, and motor friction using a simple linear function with coefficient of friction:

$$R_r = W_v c_f \tag{15}$$

Where W_v is weight of the vehicle on a level surface and c_f is coefficient of friction.

1. Modeling Rolling Resistance in Simulation

The rolling resistance in Eq. (15) is currently modeled in the rover simulation. Both W_v and c_f are defined in the simulation configuration file. There are two different values of c_f currently used in the sim depending on whether or not the rover's brakes are engaged.

C. Gravitational Resistance[1]

When vehicle is on an incline, the gravitational resistance can be model using the following equation:

$$R_g = W_v \sin\theta_{slope} \tag{16}$$

Where θ_{slope} is the angle of the incline (positive for uphill and negative for downhill)

1. Modeling Gravitational Resistance in Simulation

The gravitational resistance is currently modeled in the rover simulation as part of the rover dynamics. In rover simulation, the gravitational effect is classified as an external force from the environment instead of a resistance. Resistances are in the opposite direction of the motion and do not actually accelerate the vehicle when there is no motion, which is not the case for gravitational effect.

D. Bulldozing Resistance[1]

Akin derived the Bulldozing resistance of a wheel as it roll over a distance on a soil as:

$$R_b = \frac{b\sin(\alpha+\phi)}{2\sin\alpha\cos\phi}\left(2zcK_c+\gamma z^2K_\gamma\right)+\frac{l_0^3\gamma}{3}\left(\frac{\pi}{2}-\phi\right)+cl_0^2\left[1+\tan\left(\frac{\pi}{4}+\frac{\phi}{2}\right)\right] \tag{17}$$

Where α is the angle of attack of the wheel on soil, l_0 is the length of soil ruptured by compression, and K_c and K_γ are cohesive and density modulus of soil deformation. These parameters are defined as:

$$\alpha = \cos^{-1}\left(1-\frac{2z}{D}\right) \tag{18}$$

$$l_0 = z\tan^2\left(\frac{\pi}{4}-\frac{\phi}{2}\right) \tag{19}$$

$$K_c = (N_c-\tan\phi)\cos^2\phi \tag{20}$$

$$K_\gamma = \left(\frac{2N_\gamma}{\tan\phi}+1\right)\cos^2\phi \tag{21}$$

1. Modeling Bulldozing Resistance in Simulation

Starting from the parameters for compression resistance calculation, the additional parameters needed to calculate the Bulldozing resistance are:

- K_c: cohesive modulus of soil deformation ($K_c = 33.37$ for lunar soil)
- K_γ: density modulus of soil deformation ($K_\gamma = 72.77$ for lunar soil)

These parameters can also be calculate using the basic soil properties.

E. Wheel Slip and Tractive Force of Wheel[2]

Wheel slip occurs when wheel circumferential speed wr does not equal to the wheel translational speed V_w. wheel slip can be describe by slip factor:

$$s = 1-\frac{V_w}{wr} \tag{22}$$

Where V_w is the wheel translational speed, w is the wheel rotational rate, and r is wheel radius. Relationship between s, V_w, and wr are described below:

- If $wr = V_w$ then $s = 0$
- if $wr > V_w$ then $0 < s < 1$
- if $wr \gg V_w$ (or $V = 0$) then $s = 1$
- if $wr < V_w$ (slipping while deceleration) then $-1 < s < 0$

When slip occurred ($s \neq 0$), tractive force for smooth wheel can be determine using following equation:

$$H_{smooth} = (Ac + W \tan \phi) \left[1 - \frac{K}{sl} \left(1 - e^{\frac{-sl}{K}} \right) \right] \tag{23}$$

Where A is the area of contact ($A \cong bl$), c is soil cohesion, ϕ is soil angle of internal friction, K is coefficient of soil slip, and l is the length of contact patch. Where l can be determine using equation below:

$$l = \frac{D}{2} \cos^{-1} \left(1 - \frac{2z}{D} \right) \tag{24}$$

Note that when wheel is not rotating, Eq. (23) become ideal soil thrust:

$$H_{smooth,ideal} = Ac + W \tan \phi \tag{25}$$

Grouser on the wheel will provide additional traction. Akin derived the tractive force of grouser wheel as:

$$H_{grouser} = \left[Ac \left(1 + \frac{2h_g}{b} N_g \right) + W \tan \phi \left(1 + 0.64 \frac{h_g}{b} \tan^{-1} \frac{b}{h_g} \right) \right] \left[1 - \frac{K}{sl} \left(1 - e^{-\frac{sl}{K}} \right) \right] \tag{26}$$

Where h_g is the height of the grouser and N_g is the number of grouser in contact with the ground, which can be determine using following equation:

$$N_g = \frac{Nl}{D\pi} \tag{27}$$

Where N is the total number of grousers on the wheel.

1. Modeling Tractive Force

Starting from the parameters for compression and Bulldozing resistances, the additional parameters needed to calculate the tractive force of a smooth wheel is, K, coefficient of soil slip. Typical value of K for lunar soil is about 1.78 cm.[3]

Additional vehicle parameters needed to calculate tractive force of grouser wheel are:

- h_g grouser height of the wheel
- N total number of the grousers of the wheel

The maximum tractive force for the wheel can replace the current frictional resistance model in the contact dynamics to better simulate the wheel slip and potentially simulate situation where wheel got stuck in the soil.

IV. Modeling the Soil Interaction in Simulation in General

With wheel resistance torque, maximum traction torque, and motor torque, the following situation can be simulated in the rover simulation:

- if $T_{motor} < T_{traction}$ and $T_{net} > T_{resistance}$, then the wheel will rotate without slipping and propel the vehicle forward
- if $T_{motor} < T_{traction}$ and $T_{net} < T_{resistance}$, then the wheel will not propel the vehicle and vehicle will remains at rest
- if $T_{motor} > T_{traction}$ and $T_{net} > T_{resistance}$, then the wheel will rotate (slip) and propel the vehicle forward
- if $T_{motor} > T_{traction}$ and $T_{net} < T_{resistance}$, then the wheel will spin while $V = 0$ and the vehicle will remains at rest

Where T_{net} is the net torque from wheel motor and gravitational effect.

A. Parameters for wheel-soil resistance and tractive force

The following tables summarize the soil parameters, wheel parameters, and simulation input for wheel-soil parameters and tractive force calculation along with typical lunar soil property values.

Table 1: Soil Parameters

Symbol	Description	Lunar Soil Value	Used in
n	Exponent of sinkage	1.0	Everything
k_c	Cohesive modulus	$1400N/m^2$	R_c, R_b, H
k_ϕ	Frictional modulus	$820,000N/m^3$	R_c, R_b, H
ϕ	Angle of inertial friction	$30 - 40degree$	$N_q, N_c, N_\gamma, K_c, K_\gamma, R_b, H$
c	Cohesive strength of soil	$170N/m^2$	R_b, W_s, H
γ	Soil weight density	$2470N/m^3$	R_b, W_s, H
K	Coefficient of soil slip	$0.018m$	H
N_q	Terzaghi's bearing capacity factor	32.23	W_s
N_c	Cohesive bearing capacity factor	48.09	W_s, K_c
N_γ	Density bearing capacity factor	33.27	W_s, K_γ
K_c	Cohesive modulus of soil deformation	33.37	R_b
K_γ	Density modulus of soil deformation	72.77	R_b

Table 2: Wheel Parameters

Symbol	Description	Used in
b	Wheel width	R_c, R_b, H
D	Wheel diameter	R_c, R_b, H
c_f	Coefficient of rolling friction	R_r
r	Wheel radius	s, H
N	Number of wheel grouser	H
h_g	Grouser height	H

Table 3: Simulation Inputs

Symbol	Description	Availability	Used in
W	Normal force on the wheel	Contact and dynamics model	Everything
V	Wheel velocity in rolling direction	Dynamics model	H
w	Wheel angular velocity	Dynamics model	H

B. Wheel Resistance Model

A wheel resistance model was implemented using the equations defined in this paper to calculate the basic parameters for wheel-soil contact, resistance due to wheel soil interaction, and maximum tractive force. The model did not implement the gravitational resistance defined in the reference since it is classified differently in the rover simulation and is automatically simulated as part of the rover dynamics model. Fig. 2 provides an overview of the wheel-soil resistance model implemented for the rover simulation.

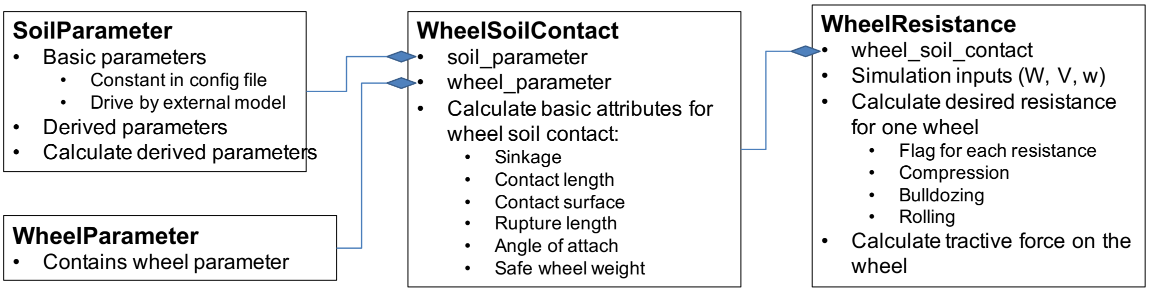

Figure 2: Wheel-Soil Resistance Model Overview

Where SoilParameter class defines basic soil properties and calculates the drived properties and WheelParameter class contains all the wheel parameters needed for the calculation. WheelSoilContact class contains instances of SoilParameter and WheelParameter, and it calculates basic parameter for wheel-soil interaction including sinkage, contact length, angle of attack, and safe wheel weight. The WheelResistance class contains an instance of WheelSoilContact, and it calculates compression, bulldozing, and rolling resistance as well as tractive force on the wheel.

C. Simulation Integration

To integrate the wheel resistance model with rover simulation, the simulation developer need to create WheelResistance instance for each rover wheel to model one contact point between the wheel and soil. Force on each wheel along with linear and angular wheel velocity are needed from the simulation. The output from the model are maximum resistant force and maximum tractive force, which can be use to simulate different situation including accelerating without slip, not accelerating due to insufficient drive torque, accelerating with slip, and slip but not accelerating (when wheel got stuck in the soil). The integration will require updates to the current rover controller class, torque collection for the rover dynamic model, and a new class that will handle the interactions between all the associated torques.

References

[1] Akin, D., "Planetary Surface Robotics: Terramechanics 1," `https://spacecraft.ssl.umd.edu/academics/788XF20/788XF20L06.terramechanics1x.pdf`, 2020, Accessed: 10/05/2020.

[2] Akin, D., "Planetary Surface Robotics: Terramechanics 2," `https://spacecraft.ssl.umd.edu/academics/788XF20/788XF20L07.terramechanics2x.pdf`, 2020, Accessed: 10/05/2020.

[3] III, W. D. C., Olhoeft, G. R., and Mendell, W., *Lunar Sourcebook*, Cambridge University Press, 1991.