

Quantum Limits of Optical Communication

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References

V. Giovannetti, S. Guha, S. Lloyd, L. Maccone, J. H. Shapiro, and H. P. Yuen, Phys. Rev. Lett. 92, 027902 (2004)

V. Giovannetti, S. Guha, S. Lloyd, L. Maccone, and J. H. Shapiro, Phys. Rev. A (2004)

V. Giovannetti, R. García-Patrón, N. J. Cerf, and A. S. Holevo, Nat. Photonics 8, 796 (2014)

Guha, Phys. Rev. Lett. 106, 240502 (2011)

Chung, Guha, Zheng, PRA 96, 012320 (2017)

Chen, Habib, Dutton, Lazarus and Guha, Nature Photonics 6, 374-379 (2012)

Narayanan, Seshadreesan, Guha, Pfister, arXiv:2003.04356 (2020)

Delaney, Seshadreesan, MacCormack, Galda, Guha, Narang, arXiv:2102.13052 (2021)

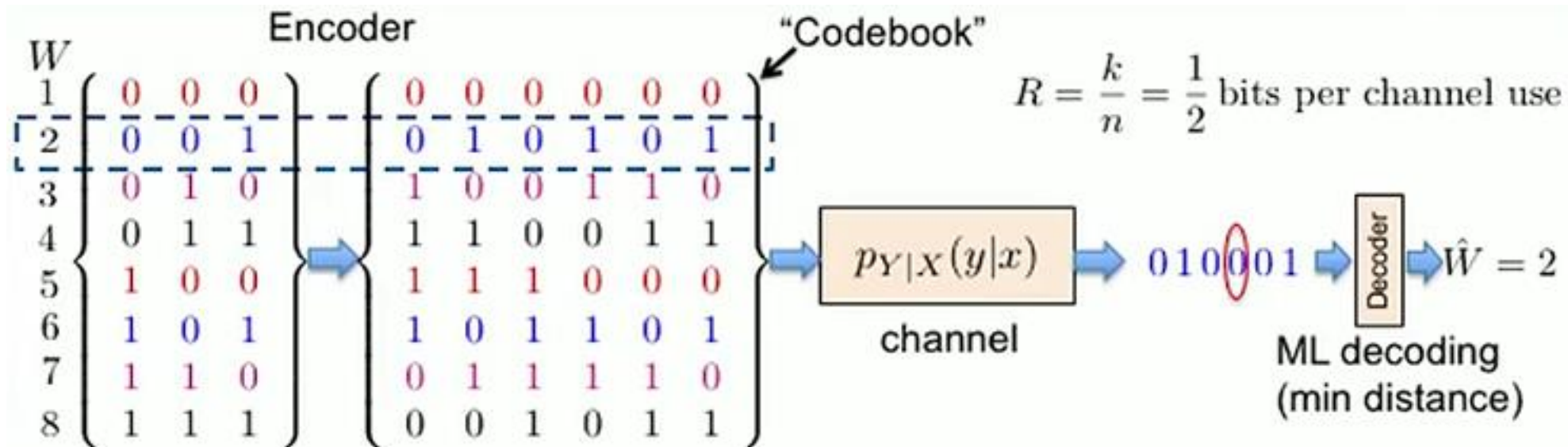
Giovannetti, Guha, Lloyd, Maccone, Shapiro, & Yuen, PRL (2004)

Takeoka and Guha, Physical Review A 89, 042309 (2014)

BLUF: Bottom Line Up Front

The two Metrics in classical communication, Channel Capacity (Holevo's Limit) and State Discriminations (Helstrom's limit) **Can be Enhanced** by Quantum Enabled supports, (1) classical encoding and measurement, (2) classical encoding and quantum measurement, (3) quantum encoding and classical measurement, and (4) quantum encoding and measurement.

- Some definitions
 - **Modulation:** electrical (classical) $\rightarrow$ optical or RF (quantum)
 - **Detection:** optical or RF (quantum) $\rightarrow$ electrical (classical)
 - **Receiver:** pre-processing (optical), detection, post-processing (electrical)
- Noise in detection
 - Noise is unavoidable (imposed by physics). The statistics of noise, $p_{Y|X}$ is a function of detector, e.g., direct intensity, homodyne, and heterodyne
 - When we know the information processing task, pre-processing the information-bearing light—while still in the quantum domain—can present itself to inevitable detection noise in a more information-favorable way
- Quantum information (and estimation) theory
- Quantum optics
 - Tools to understand most general transformations of light and “circuit synthesis” into known optical components



$k = 3$ bit message

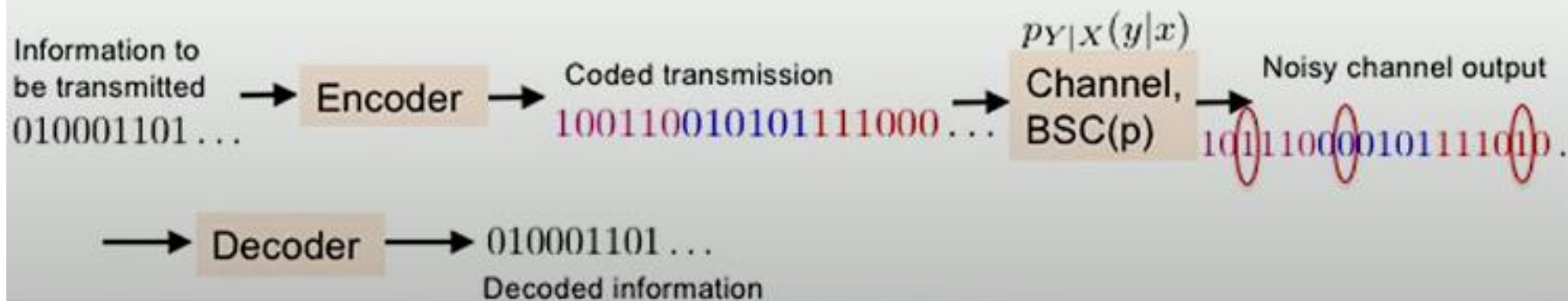
$(6, 3, 3)$ code

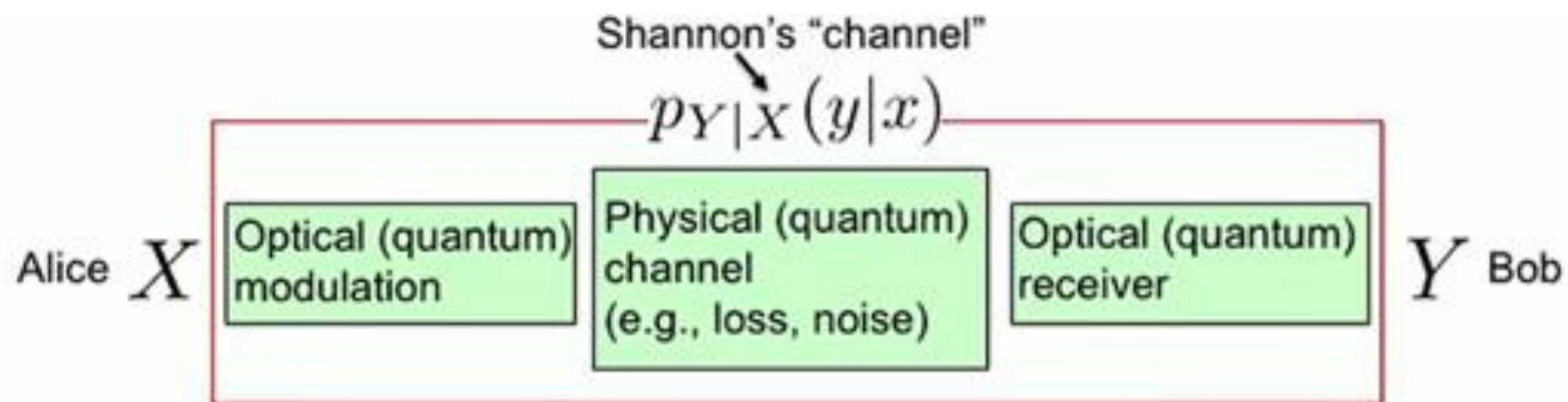
$(d-1)/2$ errors can be deterministically corrected

$K = 2^k = 8$ codewords

(n, k, d)

$P_e^{(n)} = P[\hat{W} \neq W]$





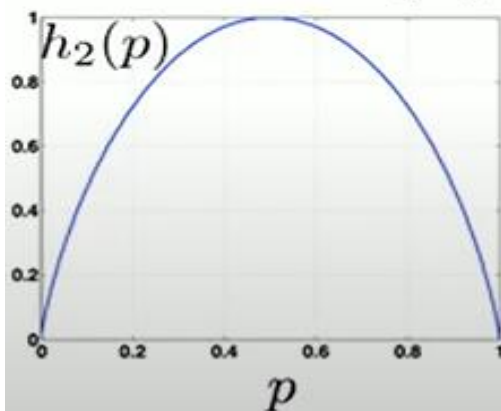
- Shannon entropy of a random variable X

- Quantifies the “amount of uncertainty” in X

$$H(X) = - \sum_{x=1}^{|\mathcal{X}|} p_X(x) \log_2 p_X(x) \text{ bits}$$

- Binary entropy, $p_X(0) = p, p_X(1) = 1 - p$

$$H(X) = -p \log_2 p - (1 - p) \log_2 (1 - p) \triangleq h_2(p)$$



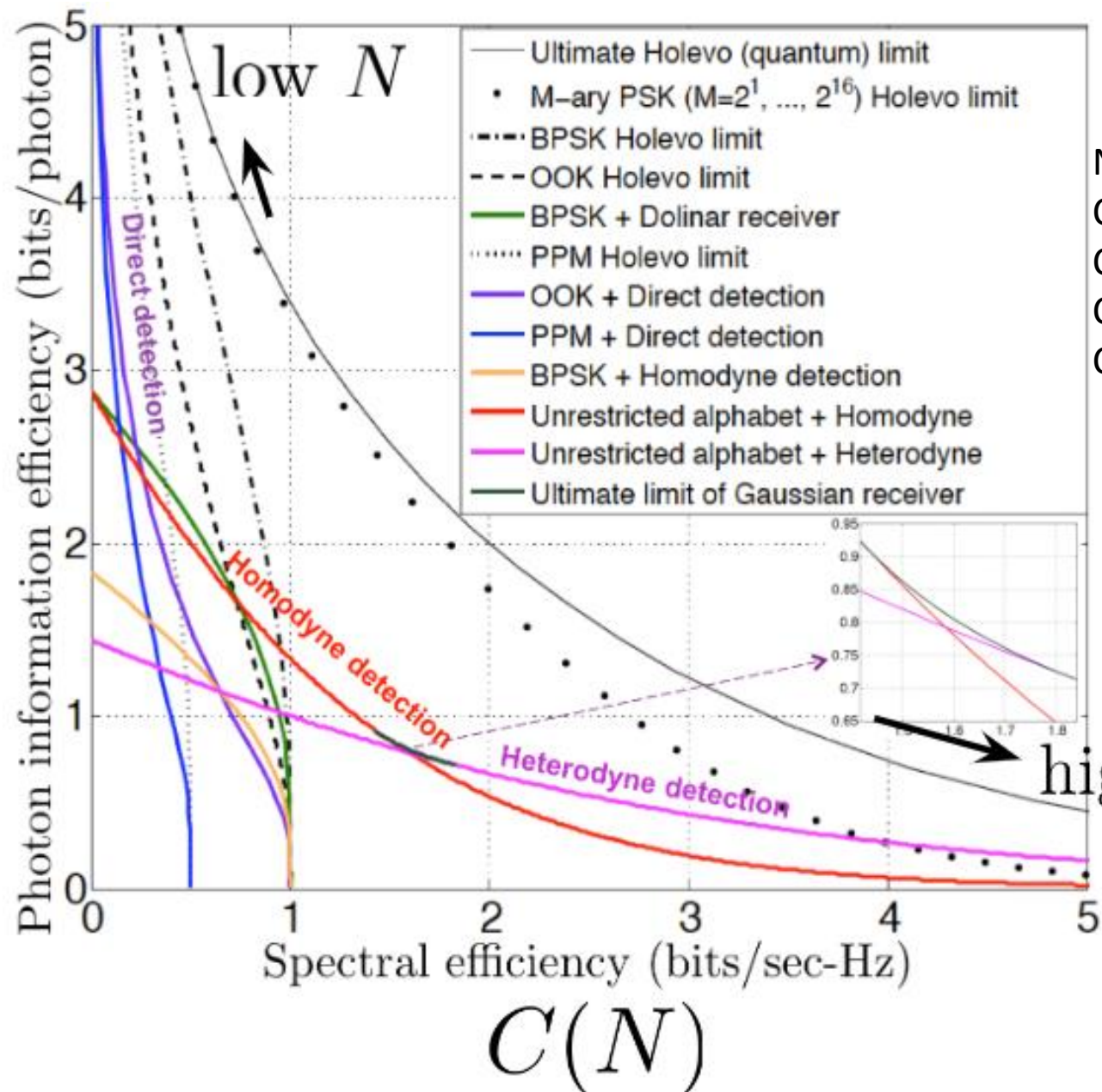
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- Mutual information

$$I(X; Y) = H(Y) - H(Y|X) = H(X) - H(X|Y)$$

$$H(X|Y) = \sum_y p_Y(y) H(X|Y = y)$$

- Capacity, $C = \max_{p_X(x)} I(X^y; Y) = 1 - h_2(p)$ bits/use

$$\frac{C(N)}{N}$$



N =Average number of Photons per pulse

$C(N)$ =Number of bits per pulse

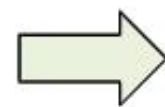
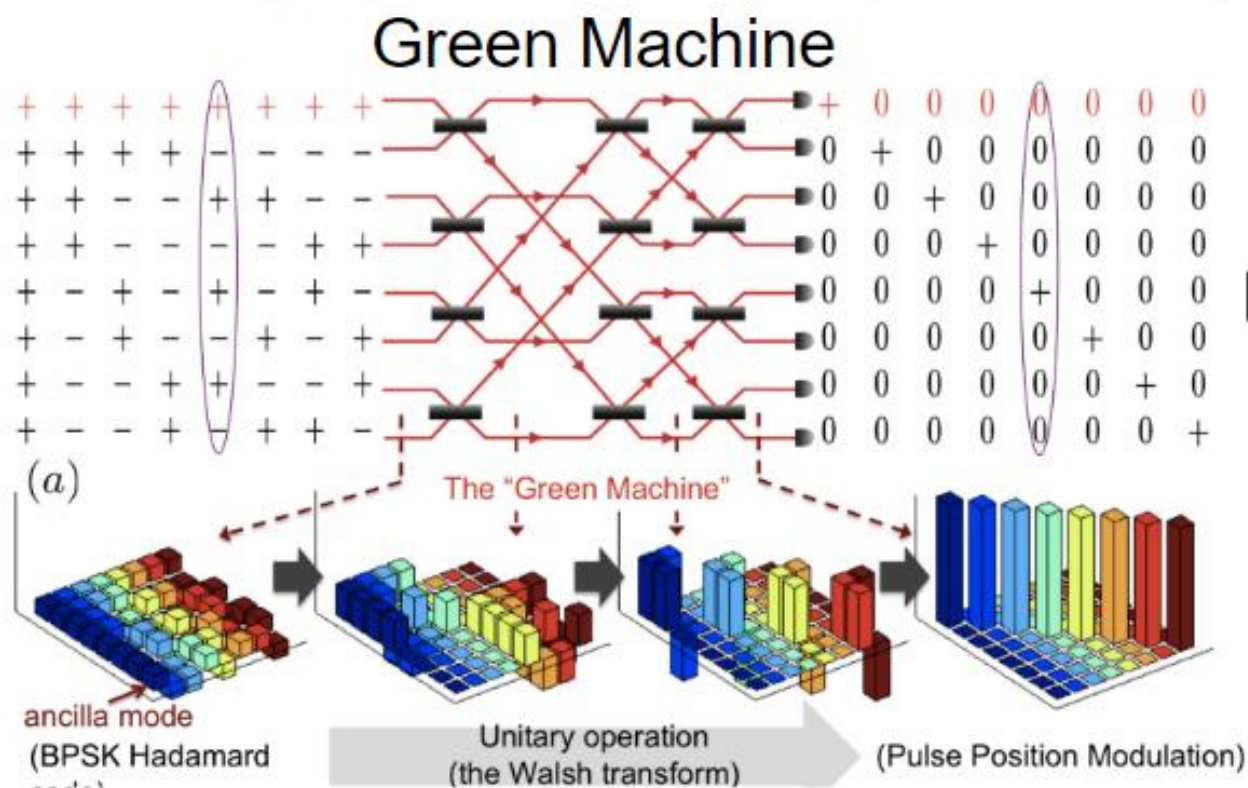
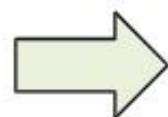
$C(N)/T=WC(N)$ =Number of bits per second

$C(N)$ =Number of bits per (Second*Hz)

$C(N)/N= (\text{Bits/Pulse})/(\text{photons/Pulse})=\text{Bits/photon}$

n-mode input, n-mode output (n=8 shown)

$$\begin{bmatrix} \alpha \\ -\alpha \\ \alpha \\ -\alpha \\ \alpha \\ -\alpha \\ \alpha \\ -\alpha \end{bmatrix}$$



$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \sqrt{8}\alpha \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad n = 8$$

- Maximum capacity with the optimal symbol-by-symbol measurement

$$C_1 = 1 - h_2(p), \quad p = \frac{1}{2} [1 - \sqrt{1 - e^{-4N}}] \quad \text{bits per mode}$$

- Holevo capacity (joint detection over infinite-length modulated block)

$$C_\infty = S \left(\frac{|\alpha\rangle\langle\alpha| + |-\alpha\rangle\langle-\alpha|}{2} \right) = h_2 \left(\frac{1 + e^{-2N}}{2} \right) \quad \text{bits per mode}$$

- The joint-detection gain most pronounced at low photon number

$$\lim_{N \rightarrow 0} \frac{C_\infty}{C_1} = \infty$$

We do not know, $C_2, C_3, \dots, C_n, \dots$

$|\psi_1\rangle$ (hypothesis H_1) vs. $|\psi_2\rangle$ (hypothesis H_2)

– Assume equal priors: $p_1 = p_2 = \frac{1}{2}$

Inner product between
the two states

$$\langle\psi_1|\psi_2\rangle = \sigma$$

Consider a von Neumann
projective measurement:

$$\Pi_1 = |w_1\rangle\langle w_1|$$

$$\Pi_2 = |w_2\rangle\langle w_2|$$

$$|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$|w_1\rangle = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$|\psi_1\rangle = \frac{1}{\sqrt{2}} \begin{bmatrix} \sqrt{1+\sigma} \\ \sqrt{1-\sigma} \end{bmatrix}$$

$$|\psi_2\rangle = \frac{1}{\sqrt{2}} \begin{bmatrix} \sqrt{1+\sigma} \\ -\sqrt{1-\sigma} \end{bmatrix}$$

$$|w_2\rangle = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \rightarrow$$

$$P_e = P(H_1)P(H_2|H_1) + P(H_2)P(H_1|H_2)$$

$$= \frac{1}{2} |\langle w_2|\psi_1\rangle|^2 + \frac{1}{2} |\langle w_1|\psi_2\rangle|^2$$

This measurement
happens to be the one
that minimizes the
average error probability

$$P_e = \frac{1}{2} \left[1 - \sqrt{1 - |\sigma|^2} \right]$$

- M-ary ensemble $\{p_i, |\psi_i\rangle\}; i = 1, 2, \dots, M$
 - Pairwise inner products (Gram matrix), $\sigma_{ij} = \langle \psi_i | \psi_j \rangle$
- Yuen-Kennedy-Lax (YKL) [1975] conditions for MPE
 - Projective measurement, $\{|w_j\rangle\}, j = 1, 2, \dots, M$
 - Relative inner products (aligning measurement vectors in the M-dimensional space spanned by the M pure states):

$$x_{ij} = \langle w_i | \psi_j \rangle$$

- YKL conditions for minimum average probability of error

$$(1) \quad p_m x_{km} x_{mm}^* = p_k x_{kk} x_{mk}^*$$

$$(2) \quad \sum_{k=1}^M x_{kj} x_{ki}^* = \sigma_{ij}$$

$$(3) \quad \sum_{j=1}^M x_{jj} |w_j\rangle \langle \psi_j| \geq p_i |\psi_i\rangle \langle \psi_i|, \forall i$$

M(M+1)/2 non-linear simultaneous equations: solve for x_{ij}

$$P_{e,\min} = 1 - \sum_{i=1}^M p_i |x_{ii}|^2$$

(Check for uniqueness of solution)

Sometimes referred to as "Helstrom bound"

Coherent state binary-phase shift keyed (BPSK) codebook, with eight 5-mode codewords

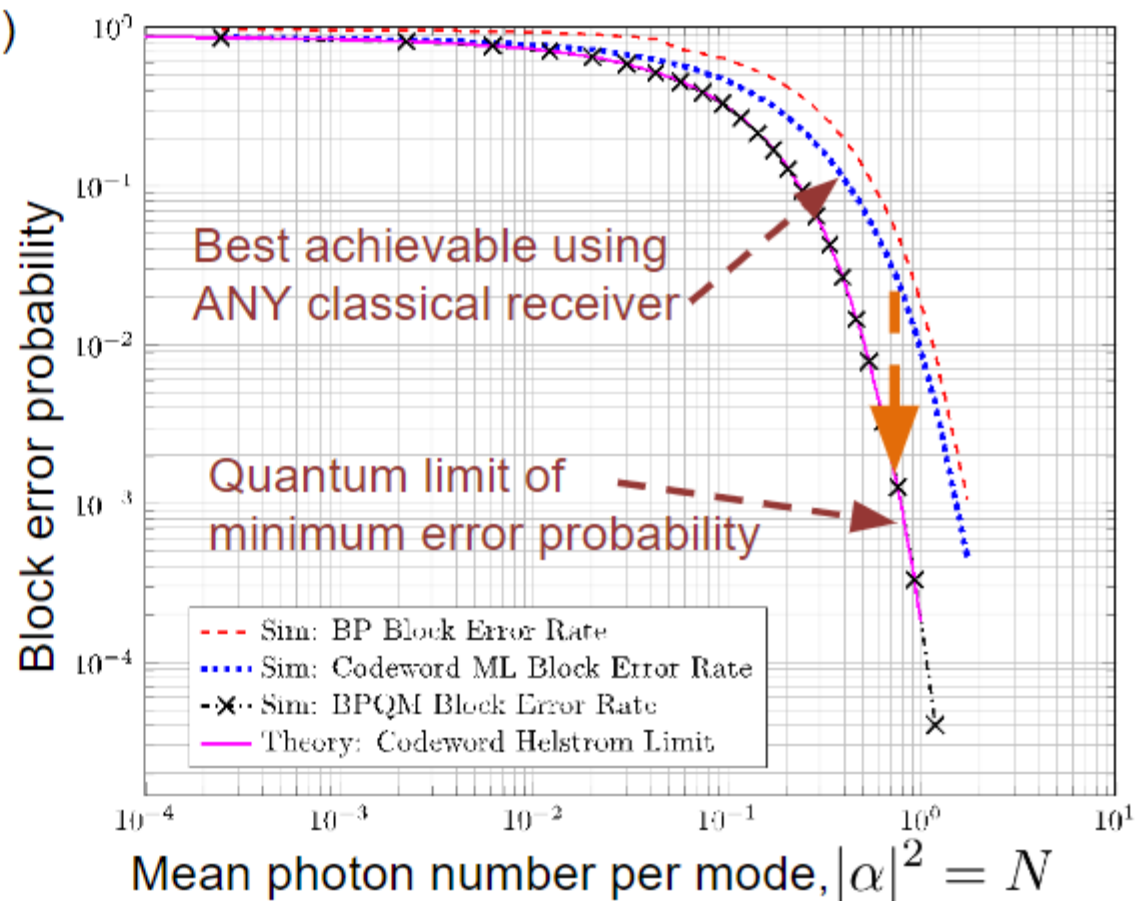
$$\left\{ \begin{array}{l} |-\alpha\rangle|-\alpha\rangle|\alpha\rangle|-\alpha\rangle|\alpha\rangle \\ |\alpha\rangle|\alpha\rangle|\alpha\rangle|\alpha\rangle|\alpha\rangle \\ |-\alpha\rangle|-\alpha\rangle|\alpha\rangle|\alpha\rangle|-\alpha\rangle \\ |\alpha\rangle|\alpha\rangle|\alpha\rangle|-\alpha\rangle|-\alpha\rangle \\ |\alpha\rangle|-\alpha\rangle|-\alpha\rangle|\alpha\rangle|\alpha\rangle \\ |\alpha\rangle|-\alpha\rangle|-\alpha\rangle|-\alpha\rangle|-\alpha\rangle \\ |-\alpha\rangle|\alpha\rangle|-\alpha\rangle|\alpha\rangle|-\alpha\rangle \\ |-\alpha\rangle|\alpha\rangle|-\alpha\rangle|-\alpha\rangle|\alpha\rangle \end{array} \right\}$$

$$\psi\rangle \longrightarrow \boxed{U} \longrightarrow |\psi_{\pm}\rangle = \sqrt{\frac{1+x}{2}}|0\rangle \pm \sqrt{\frac{1-x}{2}}|1\rangle$$

$$|\psi\rangle \in \{|\alpha\rangle, |-\alpha\rangle\}$$

$$\langle\alpha|-\alpha\rangle = e^{-2N} \equiv x$$

$$\langle\psi_+|\psi_-\rangle = x$$



Conclusion

Quantum Enhancements of Classical communication Potentially is in an order(s) of Magnitude.

