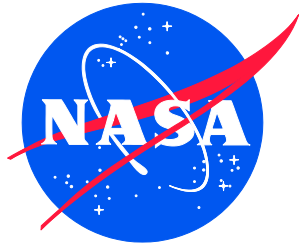


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Uncertainty Models for the Hybrid Parametric Variation Method of Uncertainty Quantification; Analysis

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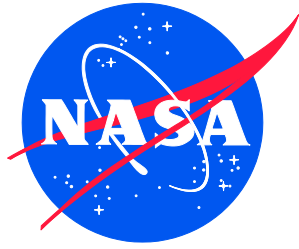
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Uncertainty Models for the Hybrid Parametric Variation Method of Uncertainty Quantification

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**Dexter Johnson – NESC Lead
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June 23, 2022

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1.0 Signatures

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2.0 Introduction

There is some level of uncertainty in every finite element model (FEM), which flows to a level of uncertainty in predicted results. The purpose of uncertainty quantification (UQ) is to provide statistical bounds on prediction accuracy based on model uncertainty. This is distinct from model updating, which attempts to modify models to improve their accuracy. UQ does not improve the accuracy of models, but accepts that the models are inherently inaccurate and attempts to quantify the impact of that inaccuracy on predicted results. Previously, an alternate method for UQ, called the Hybrid Parametric Variation (HPV) method, was applied to Space Launch System (SLS) Hurty/Craig-Bampton (HCB) components to predict system-level statistics for launch vehicle attitude control transfer functions and core stage section loads due to buffet [refs. 1 and 2]. The HPV method combines a parametric variation of the HCB fixed-interface (FI) modal frequencies with a nonparametric variation (NPV) method that randomly varies the HCB mass and stiffness matrices as Wishart [ref. 3] random matrix distributions using random matrix theory (RMT).

Alternatively, the most common method for modeling uncertainty in the structural dynamics community is a parametric approach, which varies physical parameters in the model. However, there are several disadvantages associated with the parametric method. Determining a reduced set of parameters that have a significant impact on the system response can be time consuming, and the selected parameter probability distributions are rarely reliably known. Therefore, in practice, the parameters are surrogates for the actual errors, and the link to parameter uncertainty is unknown. Another major drawback is that the uncertainty that can be represented is limited to the form of the nominal FEM. It is the experience of the authors that based on numerous aerospace programs, almost all FEM errors are in form rather than parameter values. This hypothesis is supported by the observation of the authors that it is almost never possible to ‘tune’ a FEM to match modal test results by only modifying model parameters. Model-form uncertainty cannot be directly represented by FEM input parameters nor included in a parametric approach. However, model-form uncertainty can be modeled using RMT, where a probability distribution is developed for the matrix ensemble of interest. The major advantage of the NPV method is that it covers errors in model form.

The HPV method anchors uncertainty at the HCB [ref. 4] component level to component modal test results by matching the HCB and test modes based on mode descriptions or other methods, and then applying differing levels of frequency variation. The specific variations depend on the confidence to which a component FEM has been validated through modal testing. The NPV method is layered on the frequency variation to match modal test self-orthogonality and cross-orthogonality (XO) results. Once the component uncertainty models are identified, they are assembled, and the uncertainty is propagated to the system level using a Monte Carlo (MC) analysis approach that generates statistics for system-level predictions. This provides a UQ method that can be traced to test data, which can be updated as additional data and improved correlated models become available.

The purpose of this paper is to collect and present all of the theory for HPV that has been previously published in reports and papers [ref. 1, 2, 5, and 6] and to present examples of its application. Specifically, component uncertainty models based on the dispersion of corresponding mass and stiffness matrices using proposed test/analysis correlation metrics are investigated. The first example is purely academic so that the true answers are known, and the validity of the HPV method and the corresponding uncertainty models can be determined. The

second example is an application to a component that is design specific to the SLS. Based on this work and other assessments [refs. 1, 2, 5, and 6], the HPV method provides another tool to the toolset used for complex system UQ analysis. From experience gathered to date using the HPV method, additional design specific applications must be investigated to provide further confidence in the validity of the HPV method of UQ analysis.

3.0 HPV Theory

The basis for the NPV component of the HPV method is to replace the HCB matrices representing each system component with an ensemble of random matrices, based on RMT. Each matrix in the ensemble must be close to the nominal matrix in the sense of some matrix norm and must meet certain requirements (e.g., symmetry and correct sign definiteness). However, the matrices are otherwise free and are not tied to any particular set of parameters in the FEM. Soize [ref. 7] used the maximum entropy principle to derive the positive definite and positive semidefinite ensembles SE^+ and SE^{+0} that follow the matrix variate gamma distribution and are capable of representing random structural matrices. This means the matrices in the ensembles are real, symmetric, and possess the appropriate sign definiteness to represent structural mass, stiffness, or damping matrices. As the dimension of the random matrix n increases, the matrix variate gamma distribution converges to a matrix variate Wishart distribution. In applications involving structural dynamics, the matrix dimensions are usually sufficient to give a negligible difference between the two distributions. Letting ensemble member random matrix G be any of the random mass, stiffness, or damping matrices, it is assumed that G follows a matrix variate Wishart distribution, where $G \sim W_n(p, \Sigma)$ is assumed in the remainder of this paper.

In general, a Wishart distribution with parameters p and Σ can be thought of as the sum of the outer product of p independent random vectors S_i having a multivariate normal distribution with zero mean and covariance matrix Σ . Parameter p is the number of random vectors used to construct the matrices and is sometimes identified as the shape parameter, p . Using this approach, the random matrix G can be written as:

$$G = \sum_{i=1}^p S_i S_i^T \quad S \sim N_n(0, \Sigma) \quad (1)$$

where the expected value is:

$$E(G) = \bar{G} = p\Sigma \quad (2)$$

The dispersion or normalized standard deviation of the random matrix G is defined by the relation:

$$\delta_G^2 = \frac{E(\|G - \bar{G}\|_F^2)}{E(\|\bar{G}\|_F^2)} \quad (3)$$

in which $\|*\|_F^2$ is the Frobenius norm squared.¹ It can be shown that Eq. (3) reduces to the expression:

$$\delta_G^2 = \frac{1}{p} \left[1 + \frac{(tr(\bar{G}))^2}{tr(\bar{G}^T \bar{G})} \right] = \frac{1}{p} [1 + \gamma_G] \quad (4)$$

where $\gamma_G = \frac{(tr(\bar{G}))^2}{tr(\bar{G}^T \bar{G})}$. The uncertainty in the random matrix G is dictated by the shape parameter p , which is the number of inner products in Eq. (1). The larger the value of p , the smaller the dispersion δ_G , although Eq. (4) shows that a fixed value of p does not imply a fixed dispersion. Suppose G_1 and G_2 represent structural matrices (e.g., stiffness) from two different system components. There are instances when it is desired to have the same amount of uncertainty in each of the substructures. Eq. (4) shows that if $p_1 = p_2 = p$, then the dispersion values are not the same in general, $\delta_{G_1}^2 \neq \delta_{G_2}^2$, unless $\gamma_1 = \gamma_2$. To have the shape parameters between two matrices G_1 and G_2 be equal, p_1 must be set equal to p_2 and eliminated between the two relations, yielding:

$$\delta_{G_2} = \delta_{G_1} \sqrt{\frac{1+\gamma_2}{1+\gamma_1}} \quad (5)$$

This expression leads to the definition of a normalized dispersion that is a function of the shape parameter:

$$\delta_{G_n} = \frac{\delta_{G_1}}{\sqrt{1+\gamma_1}} = \frac{\delta_{G_2}}{\sqrt{1+\gamma_2}} = \frac{1}{\sqrt{p}} \quad (6)$$

It is important to realize that just because two components have the same normalized dispersion, this does not mean that they will have the same uncertainty relative to a specific metric used to quantify component uncertainty. This is primarily due to the nonlinear dependence of the metric on the level of uncertainty.

In the random matrix method developed by Soize [refs. 7 and 8] (referred to as method 1 by Adhikari [ref. 9]), the Wishart parameters are selected as p and $\Sigma = G_o/p$, where G_o is the nominal value of G , and the mean of the distribution is given by Eq. (2) as

$\bar{G} = p\Sigma = p(G_o/p) = G_o$. Therefore, method 1 preserves the nominal matrix as the mean of the ensemble. In general, the nominal matrix can be decomposed as:

$$G_o = LL^T \quad (7)$$

In the case of a positive definite matrix, this would be the Cholesky decomposition [ref. 10].

When the nominal matrix is positive semidefinite, an alternative decomposition approach can be used, as discussed in Section 3.3. Let $(n \times p)$ matrix X be given by

$$X = [X_1 \quad X_2 \cdots X_p] \quad (8)$$

¹ $\|*\|_F^2 = trace(*^T *)$

in which X_i is an $(n \times 1)$ column vector containing standard random normal variables such that $X_i \sim N_n(0, I_n)$. Note that $p \geq n$ must be satisfied for G to be full rank. An ensemble member $G \sim W_n(p, G_o/p)$ can then be generated for MC analysis using the expression:

$$G = \frac{1}{p} L X X^T L^T \quad (9)$$

It was noted by Adhikari [ref. 9] that method 1 does not maintain the inverse of the mean matrix as the mean of the inverse, where:

$$E(G^{-1}) \neq [E(G)]^{-1} = \bar{G}^{-1} \quad (10)$$

In some cases, $E(G^{-1})$ and $[E(G)]^{-1}$ can be vastly different, which is not physically realistic. Instead, Adhikari [ref. 9] proposed method 3², in which the Wishart parameters are selected as p and $\Sigma = G_o/\theta$ where:

$$\theta = \frac{1}{\delta_G^2} [1 + \gamma_G] - (n + 1) \quad (11)$$

An ensemble member $G \sim W_n(p, G_o/\theta)$ can be generated using the relation:

$$G = \frac{1}{\theta} L X X^T L^T \quad (12)$$

In this case, the inverse of the mean matrix is preserved as the mean of the ensemble inverses, where the mean matrix is given by:

$$\bar{G} = p\Sigma = p(G_o/\theta) = \frac{p}{\theta} G_o \quad (13)$$

Note that in method 3, the dispersion defined in Eq. (3) is calculated with respect to the mean given in Eq. (13), and Eqs. (5) and (6) are maintained. Note that the only difference between method 1 and method 3 is the normalization of the matrix (i.e., the matrices differ by a scale factor).

Experience has shown that the nonparametric method tends to skew frequencies up, in the sense that the mean frequencies from the random matrix ensemble are higher than the nominal. This effect was minimized by choosing method 1 (i.e., matching the matrix mean) for the mass matrix and method 3 (i.e., matching the matrix inverse mean) for the stiffness matrix. Therefore, the nonparametric portion of the HPV method is based on a method 1 randomization of a component mass matrix and a method 3 randomization of the component stiffness matrix. In general, structural components can be weighed and center of mass locations determined using computer software, so it is common practice in analytical model validation to assume that the error in a model is concentrated in the stiffness matrix rather than the mass matrix.

The Wishart matrix uncertainty model results in uncertainty in frequencies and mode shapes. However, an extensive amount of MC simulation and analysis performed during this and previous assessments has shown that, in comparison to modal frequencies, the corresponding component mode shapes tend to be more sensitive to the nonparametric matrix randomization provided by methods 1 and 3. Therefore, the HPV approach possesses a parametric component of uncertainty in which the FI mode eigenvalues in the component HCB representation are assumed to be random variables. The FI eigenvalues are random parameters within the HCB component stiffness matrix. During each iteration within a MC analysis, a random draw of HCB

² Adhikari changes method/criteria numbers depending on the reference. It is method 3 for the cited reference.

FI eigenvalues is selected to generate a random HCB component stiffness matrix. Note that the mean of this ensemble would be the nominal HCB stiffness matrix. However, for the current iteration, the parametrically randomized HCB stiffness is treated as the nominal matrix for NPV, and method 3 is applied to provide model-form uncertainty on top of the FI eigenvalue uncertainty. This is analogous to the approach proposed by Capiez-Lernout [ref. 11] for separating parametric and nonparametric uncertainty. In contrast to the nonparametric model-form uncertainty, the mode shapes are relatively insensitive to the parametric FI eigenvalue uncertainty. Therefore, the HPV approach provides the capability to independently adjust component frequency and mode shape uncertainty. However, this characteristic is problem dependent and should be confirmed during each application.

3.1 Dispersion of Mass

The HPV method has the capability of preserving rigid-body motion and rigid-body mass properties. It can preserve the certainty of subsets of component modes. If a component has slosh modes, then they are likely to have less uncertainty than the remaining elastic modes. In addition, there may be times when the component elastic body mass must be randomized while the rigid-body mass is preserved. This would be the case when the rigid body mass properties are known, but the elastic modal masses are not. Therefore, a special methodology has been developed to preserve the component slosh modes and rigid-body mass, when desired. This same methodology can be applied to any subset of the component modes.

It is assumed that according to frequency, the component modes are ordered as:

$$\Phi = [\phi_r \quad \phi_s \quad \phi_e]$$

where ϕ_r are the n_r rigid-body modes, ϕ_s are the n_s slosh modes, and ϕ_e are the n_e elastic (non-slosh) modes. Assuming that the component modes are orthogonal and normalized with respect to the mass matrix, oblique projectors [ref. 12] onto the rigid body, slosh, and elastic modal spaces can be respectively generated as:

$$P_r = \phi_r \phi_r^T M_{CB} \quad (14)$$

$$P_s = \phi_s \phi_s^T M_{CB} \quad (15)$$

$$P_e = \phi_e \phi_e^T M_{CB} \quad (16)$$

The nominal component HCB mass matrix can be written as the sum:

$$M_{CB} = M_r + M_s + M_e$$

where the rigid body, slosh, and elastic mass components are:

$$M_r = P_r^T M_{CB} P_r = P_r^T M_{CB} = M_{CB} P_r \quad (17)$$

$$M_s = P_s^T M_{CB} P_s = P_s^T M_{CB} = M_{CB} P_s \quad (18)$$

$$M_e = P_e^T M_{CB} P_e = P_e^T M_{CB} = M_{CB} P_e. \quad (19)$$

3.1.1 Right Body Mass Not Preserved

Define $\Phi_{re} = [\phi_r \ \phi_e]$ and $M_{re} = M_r + M_e$, which is the combined rigid body and elastic mode component mass. Using Eqs. (17) and (18), the combined mass can be decomposed as:

$$M_{re} = M_{CB} \Phi_{re} \Phi_{re}^T M_{CB} = L_{mre} L_{mre}^T \quad (20)$$

where $L_{mre} = M_{CB} \Phi_{re}$ is an $(n \times n_{re})$ matrix, and $n_{re} = n_r + n_e$. Using method 1 to randomize the component mass matrix, following the discussion from the previous section, the shape parameter p_m can be expressed as:

$$p_m = \text{round} \left\{ \frac{1 + \gamma_{mre}}{\delta_m^2} \right\} \quad (21)$$

where

$$\gamma_{mre} = \frac{(\text{tr}(M_{re}))^2}{\text{tr}(M_{re}^T M_{re})} \quad (22)$$

$\text{round}(x)$ assigns the nearest integer, and the mass matrix dispersion δ_m is set by the user. Using Eq. (9), the randomized version of M_{re} is:

$$M_{rer} = \frac{1}{p_m} L_{mre} X X^T L_{mre}^T \quad (23)$$

where X is an $(n_{re} \times p_m)$ matrix of standard random normal variables described in Eq. (8). Note that n_{re} must be greater than p_m . Adding the unperturbed slosh mass, the randomized HCB mass matrix is:

$$M_{CBr} = M_s + M_{rer}. \quad (24)$$

As method 1 preserves the nominal mass matrix as the mean of the ensemble, $E(M_{CBr}) = M_{CB}$.

3.1.2 Rigid-Body Mass Preserved

In this situation, only the elastic component of the mass matrix M_e is randomized. Following the same approach, Eq. (19) can be used to decompose M_e as:

$$M_e = M_{CB} \phi_e \phi_e^T M_{CB} = L_{me} L_{me}^T. \quad (25)$$

where p_m is:

$$p_m = \text{round} \left\{ \frac{1 + \gamma_{me}}{\delta_m^2} \right\} \quad (26)$$

where

$$\gamma_{me} = \frac{(\text{tr}(M_e))^2}{\text{tr}(M_e^T M_e)}. \quad (27)$$

The randomized version of M_e is expressed as:

$$M_{er} = \frac{1}{p_m} L_{me} X X^T L_{me}^T \quad (28)$$

where X is $(n_e \times p_m)$. Note that n_e must be greater than p_m . Adding the unperturbed rigid body and slosh mass, the randomized HCB mass matrix is:

$$M_{CBr} = M_R + M_s + M_{er}. \quad (29)$$

As before, the nominal mass matrix is preserved as the mean of the ensemble, $E(M_{CBr}) = M_{CB}$.

The HPV approach for modeling component uncertainty initially requires the selection of dispersion value for the HCB mass matrix. Ideally, this dispersion value is selected based on component test results. Assuming that the test modes represent the true component, error or uncertainty in the test mode self-orthogonality matrix should be a function of the error or uncertainty in the corresponding FEM or static test analysis model (TAM) mass matrix. If the component FEM mass matrix is certain and there is no error in the TAM reduction, then there is no uncertainty in the test mode orthogonality matrix, regardless of the error that might be present in the stiffness matrix. The test mode orthogonality matrix can theoretically be used to determine the mass dispersion value using a selected metric.

One possibility would be to use the root-mean-square (RMS) off-diagonal value of the component test mode self-orthogonality matrix as an off-diagonal mass (*ODM*) metric to identify the degree of mass dispersion required. The identification process includes a series of MC analyses in which the HCB mass matrix dispersion value is swept over a range of values. The most probable value of *ODM* is computed for each MC analysis in the series. The goal is to select the mass dispersion value that gives a most probable *ODM* value that is equal to the component test value. Note that the diagonal values of the test and random self-orthogonality matrices are always unity, and there is no constraint between the diagonal and off-diagonal values. Therefore, the *ODM* metric is not sensitive to mode truncation and can be directly compared between test and random self-orthogonality matrices, where there are usually vastly differing numbers of modes involved. It has been found that the *ODM* metric is linear with respect to the mass matrix dispersion value [ref. 1].

In general, the NPV dispersion of the HCB mass matrix results in a random component self-orthogonality matrix in which the off-diagonal terms are distributed over all of the modes. In contrast, there can be situations where the off-diagonal terms of the test-based self-orthogonality matrix are dominated by just a few modes. In this case, the *ODM* metric can overestimate the mass uncertainty. An alternative self-orthogonality error metric is required, which is more sensitive to the situation where the off-diagonal terms of the test self-orthogonality matrix are dominated by a small number of modes. The mean off-diagonal value (*MOD*) is better at distinguishing between the uniform and dominant cases. When the test self-orthogonality is dominated by a few modes, the *MOD* metric value is less than the *ODM* metric value, indicating less mass uncertainty. In contrast, when the test self-orthogonality off-diagonal terms are evenly distributed over all of the modes, the *MOD* and *ODM* metrics are comparable. Both metrics give approximately the same amount of required mass matrix dispersion. As in the *ODM* metric, the *MOD* metric is independent of the number of random modes used in the MC analysis and is linearly dependent on the mass dispersion value. The efficacy of each metric will be investigated in subsequent sections.

3.1.3 Correction of Self-Orthogonality for Errors in TAM

As mentioned in the previous section, error in the test mode self-orthogonality matrix can be due to error in the FEM mass matrix or in the TAM reduction. The static reduction process is usually

used to generate a TAM. If the TAM is inaccurate, then it can contaminate the test-mode self-orthogonality matrix and indicate that there is error in the FEM mass matrix that is not present. However, the test self-orthogonality matrix using the TAM can be adjusted to approximately compensate for error introduced by the reduction process.

Assume that the TAM mass matrix can be written as:

$$M_{TAM} = M_F + \Delta M_{TAM} \quad (30)$$

where M_F is the correct reduced FEM mass matrix, and ΔM_{TAM} is the error in the TAM mass due to the inexact reduction process. Let ϕ_F be the FEM modes of interest at the sensor locations, while ϕ_T are the corresponding test modes, with both terms normalized with respect to unit TAM mass. The test self-orthogonality using the TAM is:

$$\hat{O}_T = \phi_T^T M_{TAM} \phi_T \quad (31)$$

or using Eq. (30) converts to:

$$\hat{O}_T = \phi_T^T M_F \phi_T + \phi_T^T \Delta M_{TAM} \phi_T = O_T + \phi_T^T \Delta M_{TAM} \phi_T \quad (32)$$

where O_T is the test self-orthogonality using an exact TAM. Note that the test modes are not normalized with respect to the exact TAM, so the diagonal of O_T is not unity. The FEM self-orthogonality with respect to the TAM is:

$$\hat{O}_F = \phi_F^T M_{TAM} \phi_F = \phi_F^T M_F \phi_F + \phi_F^T \Delta M_{TAM} \phi_F \quad (33)$$

which can be rewritten as:

$$\phi_F^T \Delta M_{TAM} \phi_F = \hat{O}_F - \phi_F^T M_F \phi_F = \hat{O}_F - m_F \quad (34)$$

where m_F is the diagonal generalized FEM mode mass with respect to the unknown exact TAM.

In general, the test modes can be written in terms of the FEM modes using:

$$\phi_T = \phi_F \gamma + R \quad (35)$$

where R is a matrix of residuals. Assuming that the residuals are small, it can be shown that the least-squares solution for γ is:

$$\gamma = \hat{O}_F^{-1} \phi_F^T M_{TAM} \phi_T. \quad (36)$$

Let $\tilde{\phi}_F$ be the sensor row partition of the FEM modes that were originally normalized to the FEM mass. Then for the i th mode, $\tilde{\phi}_{Fi}^T M_F \tilde{\phi}_{Fi} = 1$, and $\tilde{\phi}_{Fi}^T M_{TAM} \tilde{\phi}_{Fi} = \hat{m}_{Fi}$, which implies:

$$\phi_{Fi} = \frac{1}{\sqrt{\hat{m}_{Fi}}} \tilde{\phi}_{Fi}.$$

It can then be shown that generalized mass m_{Fi} in Eq. (34) is given by $m_{Fi} = \hat{m}_{Fi}^{-1}$. Neglecting the residuals and substituting Eq. (35) into (32) produces:

$$O_T \approx \hat{O}_T - \gamma^T \phi_F^T \Delta M_{TAM} \phi_F \gamma. \quad (37)$$

Substituting Eq. (34) into (37) produces the approximation for the corrected self-orthogonality matrix as:

$$O_T \approx \hat{O}_T - \gamma^T (\hat{O}_F - \hat{m}_F^{-1}) \gamma \quad (38)$$

where $\hat{m}_F = \text{diag}(\tilde{\phi}_F^T M_{TAM} \tilde{\phi}_F)$. As mentioned, O_T does not have a unit diagonal and must be normalized as:

$$O_{Tn} = (g^{-1/2})^T O_T (g^{-1/2}) \quad (39)$$

where g is a diagonal matrix equal to the O_T diagonal.

3.2 Dispersion of Eigenvalues

In past assessments [ref. 1], the component FI eigenvalues were considered as independent random variables. It was shown in Reference 2 that even though the variation of the FI eigenvalues is parametric with respect to the HCB representation, if they are varied independently, nonparametric variation of the stiffness matrix occurs in physical space. In contrast, if the FI eigenvalues are varied in unison (i.e., perfectly correlated), then it produces a purely parametric variation of the component stiffness matrix in physical space. As a component stiffness matrix varies, it is common to see the corresponding eigenvalues vary in a correlated manner to some extent, especially when they are closely spaced. Therefore, reality is somewhere between treating the FI eigenvalues as independent and treating them as perfectly correlated.

3.2.1 Randomization of Component FI Eigenvalues using Gaussian Process Models

In this discussion, Gaussian Process (GP) modeling [ref. 13] is used to represent the random space of component FI eigenvalues. This means that any finite ensemble of component FI eigenvalue realizations follow a multivariate normal distribution. The characteristics of the realizations are determined by the mean vector $\bar{\lambda}$ and covariance matrix Σ or covariance function $\Sigma(x, x')$. In general, the covariance matrix or function corresponding to the FI eigenvalues of a component is unknown. However, in this paper, it is assumed that the covariance function can be defined based on Euclidean distance. This covariance model is shown to be robust with respect to discrepancies between assumptions and reality in Reference 10. Therefore, if $Y(x)$ is a realization of the FI eigenvalues, the covariance function is defined as:

$$\text{Cov}(Y(x), Y(x')) = \Sigma(x, x') = \exp(-\|x - x'\|^2) \quad (40)$$

where x and x' are two points in FI eigenvalue space. The covariance between $Y(x)$ and $Y(x')$ decays exponentially as the distance between x and x' increases. The covariance matrix Σ_n is then generated by evaluating $\Sigma(x_i, x_j)$ in Eq. (40) at all pairs of the n component FI eigenvalues.

It is apparent that the covariance matrix derived based on Eq. (40) corresponds to unit scale or variance. In practice, it is desired to have the variance of the FI eigenvalues be based on the difference between the FEM and test eigenvalues, or frequencies from the component modal test. Let $\Delta\lambda$ be a vector of RMS uncertainties assigned to the FI eigenvalues based on the component modal test correlation results. In the case of a Gaussian distribution, the RMS uncertainty is equivalent to the standard deviation. The FI eigenvalue covariance matrix with the proper variance is:

$$\Sigma_{nv} = \text{diag}(\Delta\lambda) * \Sigma_n * \text{diag}(\Delta\lambda) \quad (41)$$

where $\text{diag}(\Delta\lambda)$ is a diagonal matrix. If the j th eigenvalue λ_j is not uncertain, then $\Delta\lambda_j = 0$ and the j th row and column of Σ_{nv} are null (i.e., Σ_{nv} is positive semi-definite). Within MATLAB®, the command:

$$Y = \text{mvnrnd}(\bar{\lambda}, \Sigma_{nv}, 1) \quad (42)$$

produces a finite realization of the random FI eigenvalues under a GP prior with a specific mean and covariance, which can be easily implemented within a MC analysis.

Assuming that the difference between the component test and analysis frequencies is set as the standard deviation of the FI frequencies, and assuming an approximately normal distribution, implies that approximately 68% of the time the frequency uncertainty is less than the test-based value, while approximately 32% of the time the frequency uncertainty is greater than the test value. This tends to underestimate the uncertainty in the component FI eigenvalues during a MC analysis. It may be more appropriate to have the test/analysis frequency errors from the component tests be set at the halfway point in the corresponding normal probability distributions. Therefore, during the MC analysis, 50% of the time the FI frequency uncertainty will be below the test-based error, and 50% of the time it will be above. This can be accomplished by dividing the eigenvalue uncertainties based on test/analysis frequency errors by 0.6745, which increases the overall uncertainty in the FI eigenvalues.

3.2.2 Mixed Boundary Approach for Assigning HCB Eigenvalue Dispersions

As discussed, test/analysis frequency error is used to identify the HCB FI eigenvalue uncertainties. However, one of the biggest challenges in the propagation of component test-analysis frequency error into uncertainty in the HCB flight-configuration FI eigenvalues is that the component test and flight-configuration boundary conditions and/or hardware are almost never the same. Because of this, it is difficult to match test-configuration modes with flight-configuration FI modes. Boundary condition mismatch can be alleviated using a mixed-boundary approach. In general, the HCB flight-configuration FI modes will be over-constrained when compared to the test-configuration modes. The HCB component stiffness matrix can be written as:

$$K_{CB} = \begin{bmatrix} K_S & 0 \\ 0 & \lambda \end{bmatrix} = \begin{bmatrix} K_{cc} & K_{cb} & 0 \\ K_{bc} & K_{bb} & 0 \\ 0 & 0 & \lambda \end{bmatrix} \quad (43)$$

where K_S is the component physical stiffness matrix statically reduced to the interface or boundary degrees of freedom (DOF), and the boundary DOF have been divided into two subsets: the c -set contains all DOF that are free in the component test-configuration, and the b -set contains the DOF that are constrained in the component test-configuration. When the HCB flight-configuration is constrained at the test-configuration interface DOF (b -set), it produces the mass and stiffness matrices:

$$M_C = \begin{bmatrix} M_{cc} & M_{cq} \\ M_{qc} & M_{qq} \end{bmatrix} \quad K_C = \begin{bmatrix} K_{cc} & 0 \\ 0 & \lambda \end{bmatrix} \quad (44)$$

with corresponding eigenvalues λ_C and mass normalized eigenvectors $\phi_C = [\phi_{cc}^T \quad \phi_{cq}^T]^T$. These eigenvalues and eigenvectors are consistent with the boundary conditions of the test-configuration modes used in the component test-analysis correlation. Error or uncertainty in the analytical test-configuration eigenvalues can be readily mapped into uncertainty $\Delta\lambda_C$ in the component eigenvalues in Eq. (44). The HCB representation of the component using λ_C and ϕ_C as FI modal properties has the stiffness matrix and corresponding displacement vector given by:

$$K_B = \begin{bmatrix} K_{Sb} & 0 \\ 0 & \lambda_C \end{bmatrix} \quad u_B = \{x_b^T \quad q_C^T\}^T \quad (45)$$

where K_{Sb} is K_S statically reduced to the b -set, x_b is the physical displacement of the b -set, and q_c are the modal coordinates of the FI modes with the c -set free. The transformation between displacement vector u_B and the original HCB displacement vector u_{CB} is:

$$u_{CB} = \begin{Bmatrix} x_c \\ x_b \\ q \end{Bmatrix} = \begin{bmatrix} \psi & \phi_{cc} \\ I & 0 \\ 0 & \phi_{cq} \end{bmatrix} \begin{Bmatrix} x_b \\ q_c \end{Bmatrix} = T u_B \quad (46)$$

The relation between K_B and K_{CB} is:

$$K_B = T^T K_{CB} T. \quad (47)$$

The test-configuration HCB FI eigenvalues λ_c can be randomized (λ_{Cr}) based upon the component test-analysis correlation results, and the uncertainty can be propagated into the random flight-configuration HCB component stiffness (K_{CBr}) using the expression

$$K_{CBr} = T^{-T} K_{Br} T^{-1} = T^{-T} \begin{bmatrix} K_{Sb} & 0 \\ 0 & \lambda_{Cr} \end{bmatrix} T^{-1} \quad (48)$$

3.3 Dispersion of Stiffness

In the case of a flight component that has rigid-body modes and a positive semidefinite stiffness matrix, additional steps must be taken to decompose the nominal HCB stiffness matrix (Eq. (7)) for subsequent randomization using method 3 and Eq. (12). As in the case of component mass randomization, the slosh mode stiffness K_s must be preserved. In addition, the rigid-body stiffness must be preserved as the null matrix. The procedure follows a modified mass matrix decomposition. The nominal component HCB stiffness matrix can be written as the direct sum:

$$K_{CB} = K_s + K_e. \quad (49)$$

The slosh and elastic mode stiffness matrix components are:

$$K_s = P_s^T K_{CB} P_s \text{ and} \quad (50)$$

$$K_e = P_e^T K_{CB} P_e. \quad (51)$$

Substituting Eq. (16) into Eq. (51) yields:

$$K_e = M_{CB} \phi_e \phi_e^T K_{CB} \phi_e \phi_e^T M_{CB} = M_{CB} \phi_e \lambda_e \phi_e^T M_{CB} \quad (52)$$

where λ_e is the diagonal matrix containing the component elastic mode eigenvalues. The elastic stiffness can then be decomposed into the form of Eq. (7) as $K_e = L_{ke} L_{ke}^T$ where:

$$L_{ke} = M_{CB} \phi_e \lambda_e^{1/2}. \quad (53)$$

The shape parameter p_k is now:

$$p_k = \text{fix} \left\{ \frac{1 + \gamma_{ke}}{\delta_k^2} \right\} \quad (54)$$

where $\text{fix}(x)$ denotes rounding the value down to the nearest integer and:

$$\gamma_{ke} = \frac{(\text{tr}(K_e))^2}{\text{tr}(K_e^T K_e)} \quad (55)$$

which produces parameter $\theta_k = p_k - (n_e + 1)$. The randomized version of K_e is:

$$K_{er} = \frac{1}{\theta_k} L_{ke} X X^T L_{ke}^T. \quad (56)$$

Note that random matrix X is $(n_e \times p_k)$, and n_e must be greater than p_k . Adding the unperturbed slosh stiffness produces the randomized HCB stiffness matrix:

$$K_{CBr} = K_s + K_{er}. \quad (57)$$

In contrast with the random mass matrix ensemble, the stiffness matrix ensemble preserves the inverse of the nominal stiffness as the mean of the random stiffness matrix inverses, $E(K_{CBr}^{-1}) = K_{CB}^{-1}$. Method 3 gives the mean of the random stiffness matrix ensemble as:

$$E(K_{CBr}) = \frac{p_k}{\theta_k} K_{CB}. \quad (58)$$

In the case in which parametric uncertainty is included by randomizing the HCB component FI eigenvalues, a different approach must be taken. During each iteration within the MC analysis, the HCB FI eigenvalues are randomized as discussed in the previous section. The resulting stiffness matrix in Eq. (48) maintains the model form associated with the original HCB representation. Method 3 is then applied to the randomized matrix as though that matrix were the nominal HCB stiffness matrix. In this manner, the maximum-entropy-based nonparametric model-form uncertainty is layered on top of the parametric uncertainty associated with the random HCB eigenvalues. In this case, the elastic partition of the random HCB stiffness matrix is:

$$K_e = \phi_{ke} \lambda_{ke} \phi_{ke}^T \quad (59)$$

where ϕ_{ke} are the elastic orthonormal modes, and λ_{ke} are the associated elastic eigenvalues of $K_{\lambda r}$. The decomposition in Eq. (53) is then replaced by:

$$L_{ke} = \phi_{ke} \lambda_{ke}^{1/2}. \quad (60)$$

The remaining procedure used in Eqs. (54) to (58) remains the same. Note that this alternative stiffness decomposition can be used instead of Eq. (53) in the case where the HCB FI eigenvalues are not randomized.

Once the eigenvalue and mass dispersions have been specified, test-analysis XO can be used to identify the dispersion of the component stiffness matrix. In this case, the RMS diagonal value of the component test-analysis XO matrix is used as a diagonal cross-generalized mass (*DCGM*) metric to identify the stiffness dispersion. The same MC-based process used to identify the mass dispersion is used to identify the HCB stiffness matrix dispersion value. A series of MC analyses is performed in which the HCB stiffness matrix dispersion value is swept over a range. The most probable value of *DCGM* is computed for each MC analysis in the series. The goal is to select the stiffness dispersion value that gives a most probable *DCGM* value equal to the test value.

4.0 Applications

The HPV method for UQ, along with the procedure for the development of component uncertainty models outlined in the previous sections, are applied to two examples to demonstrate the efficacy of the proposed process. The first application concerns a simple generic spacecraft (GSC), where the test data are simulated and the exact solutions are known. The second application is on the SLS Interim Cryogenic Propulsion Stage (ICPS).

4.1 GSC

The GSC is illustrated in Figure 1. The corresponding unconstrained FEM possesses 588 translational DOF. The corresponding HCB component representation includes 12 physical translational DOF at the four base interfaces, and 41 FI modal DOF to 100 Hz. The first 16 component elastic modes to 60.0 Hz are considered to be the example target modes. To simulate test data, the GSC FEM was altered to represent the test-configuration. Parametric variations were applied by changing several of the bar and shell physical properties in the nominal FEM. In addition, model-form variation was added by applying 5% dispersion to the parametrically varied mass matrix using the methods described in Section 3.1, and 20% dispersion to the parametrically varied stiffness matrix using the approach outlined in Section 3.3. These dispersion values were selected purely as an example based on past experience.

The test-configuration corresponds to the GSC constrained at the four base interface locations. In the case of the test-configuration, there are 19 FEM modes with frequencies less than 60.0 Hz that are designated as target modes. The test/analysis correlation results for the test-configuration are listed in Table 1. Note that the frequency error is computed with respect to the FEM values, and the XO is computed as the root-sum-square (RSS) for a linear combination of random modes within 3% of the frequency of the unique-best-fit mode [ref. 14].

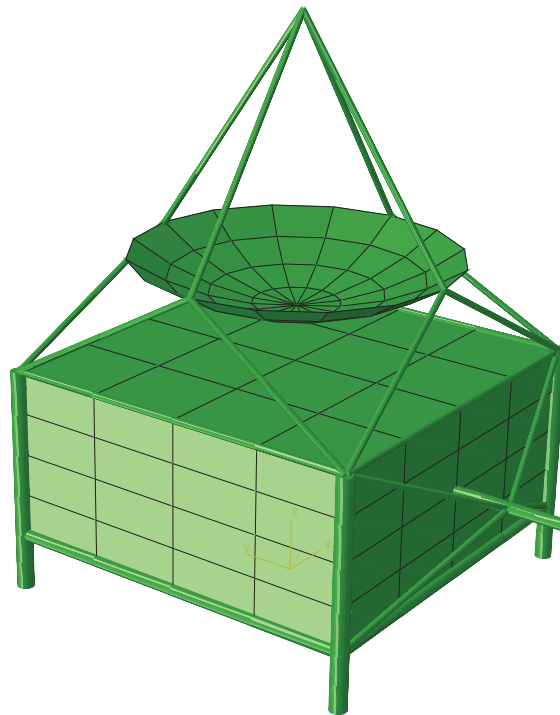


Figure 1. Overall GSC Configuration

Table 1. Test/Analysis Correlation Results for GSC in Test-configuration

FEM		Test				
Mode	Freq	Mode	Freq	% Error	XO	Description
1	14.70	1	11.90	-19.09	100	1st symmetric panel breathing
2	17.90	3	17.37	-2.93	72	1st vertical probe bending
3	18.98	2	16.75	-11.71	70	1st antenna bending about Y
4	20.65	4	20.93	1.37	100	1st antenna bending about X
5	22.18	5	21.86	-1.46	96	2nd antenna bending about Y
6	25.76	6	25.82	0.25	98	2nd antenna bending about X
7	26.21	7	26.64	1.65	98	3rd antenna bending about Y
8	29.04	10	29.78	2.56	99	Antenna vertical bounce
9	29.56	11	31.05	5.05	99	1st asymmetric panel breathing
10	35.74	8	29.13	-18.49	99	1st/2nd order panel bending – X/Y
11	35.96	9	29.22	-18.73	96	1st/2nd order panel bending – Y/X
12	36.60	12	36.48	-0.31	99	1st torsion
13	44.58	13	41.51	-6.89	85	Antenna/top panel bending
14	45.31	14	43.07	-4.94	73	Higher order antenna bending
15	46.71	16	44.58	-4.55	75	Probe panel displacement along Y
16	53.52	15	43.37	-18.97	66	2nd order panel bending
17	56.60	20	63.26	11.78	98	2nd order antenna /support bending about X
18	56.60	19	61.97	9.50	76	2nd order antenna /support bending about Y
19	58.58	22	65.61	12.00	63	1st antenna /support torsion

Based on the 19 test-configuration target modes, the test/analysis correlation XO metric is given by $DCGM_{test} = 88.55$, while the self-orthogonality metrics are $ODM_{test} = 2.66$ and $MOD_{test} = 1.17$. A static TAM possessing 210 translational DOF was used in the computations. Note that the ODM metric is greater than twice (i.e., 2.3 times) the MOD metric value. The absolute value of the test mode self-orthogonality matrix with the diagonal unit values removed is shown in Figure 2. The 342 off-diagonal terms are dominated by four modes. This results in the large discrepancy between the two self-orthogonality metrics. As the off-diagonal terms become evenly distributed, the two metrics approach the same value.

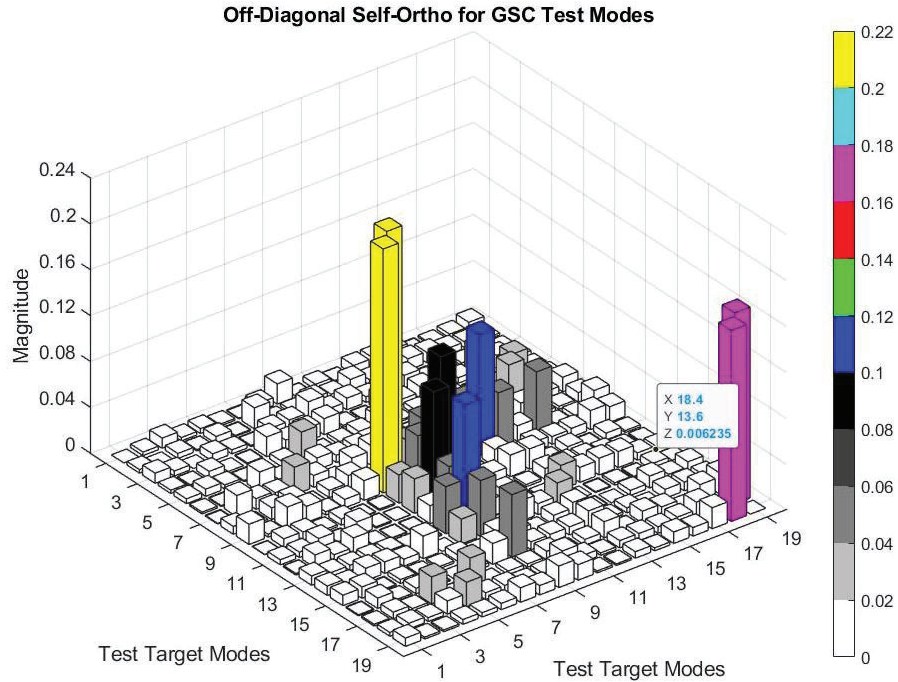


Figure 2. Absolute Value of GSC Test Mode Self-Orthogonality Matrix with Diagonal Terms Removed

The HCB mass matrix dispersion is initially determined as discussed in Section 3. A series of MC analyses was performed in which the value of the mass dispersion is swept over an interval until the most probable value of the self-orthogonality metrics from the MC ensemble correlates with the test values. In this analysis 3,000 iterations were used in the MC analysis, 30 free-free elastic modes were used in the random and FEM mode matching, 3% RSS was used in the corresponding XO computations, and metrics were computed for the 16 FEM target modes. Figure 3 illustrates the variation of *ODM* and *MOD* self-orthogonality metrics versus the dispersion of the HCB mass matrix. Both metrics are linear with respect to the mass matrix dispersion. A mass dispersion of 13.5% yields a most probable *ODM* value of 2.62, which is similar with the test value of 2.66. The corresponding most probable *DCGM* value is 92.89, compared with the test value of 88.55. A mass matrix dispersion of 7.25% produces a most probable *MOD* metric value of 1.14, compared to the test value of 1.17, and a corresponding most probable *DCGM* value of 97.86. Recall these MC-based *DCGM* values are due to mass dispersion. FI eigenvalue and stiffness dispersions affect the *DCGM* XO metric, but have a limited effect on the *ODM* or *MOD* self-orthogonality metrics.

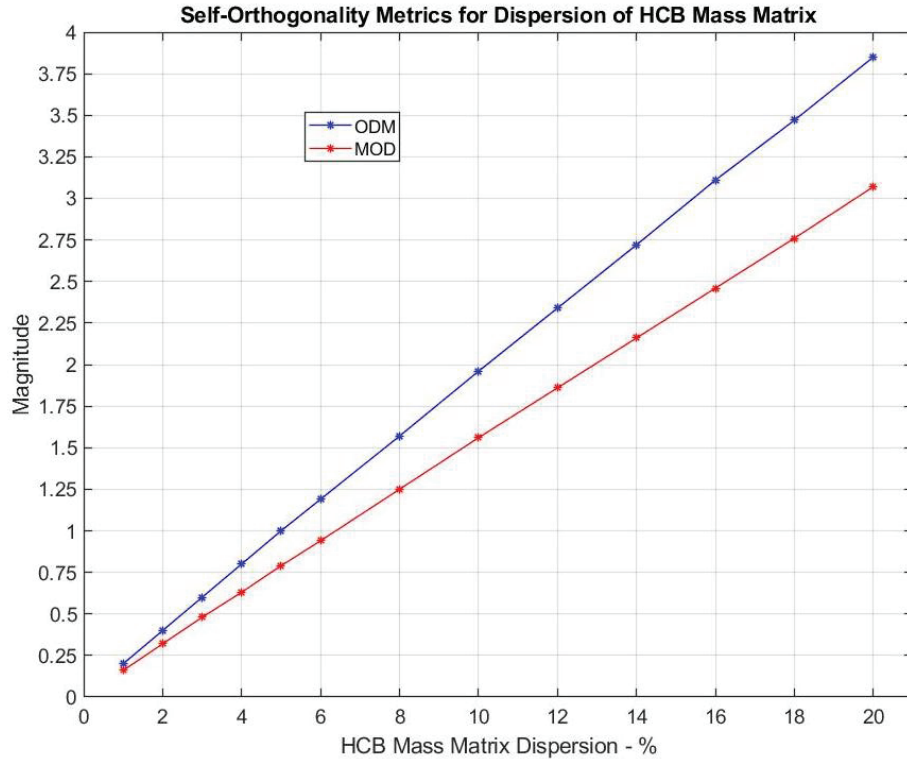


Figure 3. Self-Orthogonality Metrics vs. Dispersion of GSC HCB Mass Matrix

In this application, the test-configuration corresponds to the FI configuration in the HCB component. Therefore, the FEM/test frequency errors listed in Table 1 can be used to determine the HCB FI eigenvalue uncertainties. The frequency error magnitudes were converted to the corresponding absolute eigenvalue errors and assigned to the first 19 HCB FI modes. The remaining 22 FI modes were assigned an eigenvalue uncertainty of 10.35%, which corresponds to the median value from the first 19 FI modes. These test-based eigenvalue errors were then divided by 0.6745 and assigned as the standard deviation of the HCB FI eigenvalues during MC analysis. This produces FI eigenvalue probability distributions in which the test value is at the 50/50 probability point, where half the time the difference between the random and FEM frequency will be greater than the test value and half the time it will be less, as discussed in Section 3.2.1. The resulting eigenvalue uncertainties are shown in Figure 4.

The impact of FI eigenvalue uncertainty on the determination of the HCB mass matrix dispersion can be examined by applying the eigenvalue uncertainty and mass dispersion at the same time during the MC analysis. In this case, the mass dispersion of 13.5% produces a most probable *ODM* value of 2.73, compared with the test value of 2.66, and the most probable *ODM* value of 2.62 when no eigenvalue uncertainty was applied. A mass dispersion of 7.25% in conjunction with the FI eigenvalue uncertainty produces a most probable *MOD* value of 1.14, compared with the test value of 1.17, and the most probable *MOD* value of 1.14 when no eigenvalue uncertainty was applied. Therefore, if the *ODM* metric is used to select the mass matrix dispersion, the dispersion value would be approximately 13.5%, while using the *MOD* metric results in a dispersion value of approximately 7.25%. As discussed, the application of FI eigenvalue uncertainty has little impact on either of the self-orthogonality metrics.

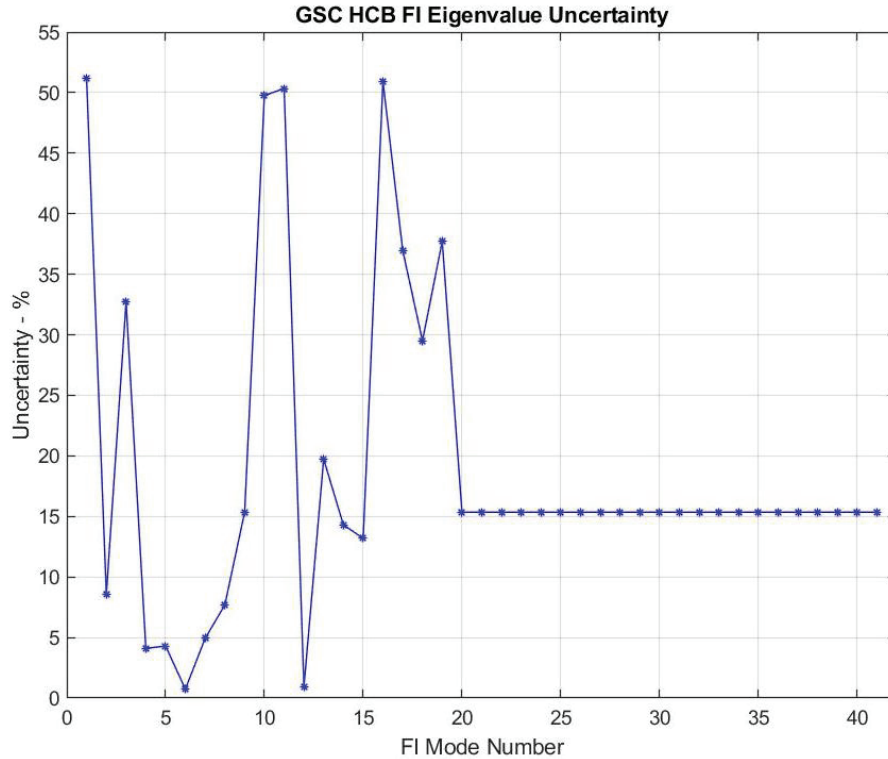


Figure 4. GSC HCB FI Eigenvalue Uncertainty Based on Component Test

In contrast, eigenvalue uncertainty has a significant impact on the *DCGM* XO metric. When no eigenvalue uncertainty is applied, using the *ODM* metric produced 13.5% mass dispersion, which resulted in a most probable *DCGM* metric value of 92.89, compared with the test value of 88.55. If eigenvalue uncertainty is applied, the 13.5% mass dispersion gives a most probable *DCGM* value of 87.03, which is significantly below the test value without any stiffness matrix dispersion. Using the *ODM* self-orthogonality metric overestimates the uncertainty in the analytical mass matrix. Conversely, using the *MOD* self-orthogonality metric when no eigenvalue uncertainty is introduced resulted in a mass dispersion of 7.25%, which produces a most probable *DCGM* value of 97.86. When the eigenvalue uncertainty is applied, the 7.25% mass dispersion results in a most probable *DCGM* metric value of 90.98, compared to the 88.55 test value. Therefore, using the *MOD* self-orthogonality metric to determine the HCB mass matrix dispersion produces a *DCGM* XO metric that is less than the test value. This indicates the HCB stiffness matrix dispersion can be applied to account for uncertainty in stiffness, which is known to exist in this academic example. In contrast, using the *ODM* metric produces a mass dispersion that coupled with the eigenvalue uncertainty overwhelms the *DCGM* metric and does not allow for stiffness uncertainty. Therefore, the *MOD* metric was used to select the mass dispersion for the GSC example.

Once the mass dispersion was determined to be 7.25%, a second series of MC analyses was performed in which the mass and designated eigenvalue uncertainty was applied and the HCB stiffness matrix dispersion was swept over a range of values until the most probable *DCGM* metric corresponded to the test value. A stiffness dispersion of 8.25% produces a most probable *DCGM* metric value of 88.51, compared with the test value of 88.55. The corresponding most probable *MOD* value is 1.17, compared to the value of 1.14 when only 7.25% mass dispersion and eigenvalue uncertainty were applied, and the same value of 1.14 when only 7.25% mass

dispersion was applied during MC analysis. The eigenvalue and stiffness matrix uncertainty have little effect on the *MOD* self-orthogonality metric. In many cases, it has been observed that the mass matrix dispersion can be determined independently from the stiffness dispersion using the *MOD* and *DCGM* metrics. Within a single MC analysis, where the mass and stiffness matrix dispersions are fixed, the *MOD* and *DCGM* metrics are uncorrelated as shown in Figure 5 for the case of 7.25% mass and 8.25% stiffness dispersions.

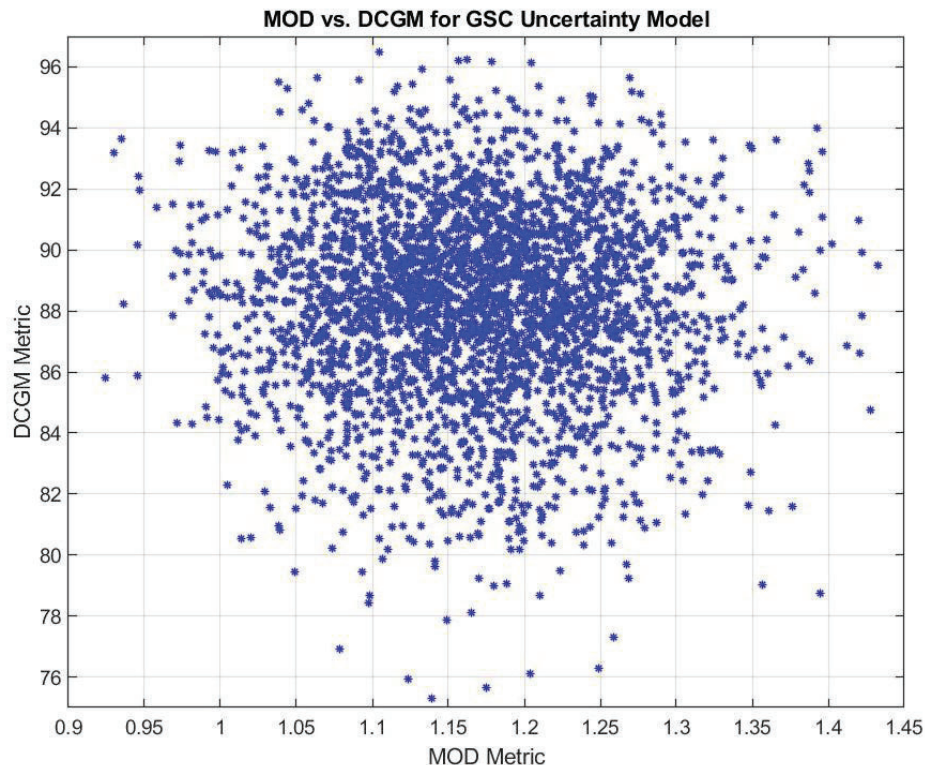


Figure 5. *MOD* versus *DCGM* Metric for 7.25% Mass and 8.25% Stiffness Dispersions

However, while the *MOD* and *DCGM* metrics for each random GSC model are uncorrelated over the ensemble, the most probable values of each metric are related from ensemble to ensemble as the uncertainty is varied.

Using the GSC uncertainty model consisting of: 7.25% HCB mass dispersion, HCB FI eigenvalue 50/50 uncertainty based on test/analysis frequency errors, and 8.25% HCB stiffness matrix dispersion, a MC modal analysis was performed with 10,015 ensemble members. The first 30 flight configuration elastic modes were used in mode matching, and 3% RSS was used in the computation of XO values. Figure 6 shows the GSC flight-configuration target mode frequency statistics. The P99/90 and P01/90 values are computed using order statistics [ref. 15]. The P98/90 enclosure covers all of the GSC flight-configuration test frequencies using the uncertainty model based on the test-configuration correlation results. The flight-configuration FEM frequencies are included in Figure 6 for comparison.

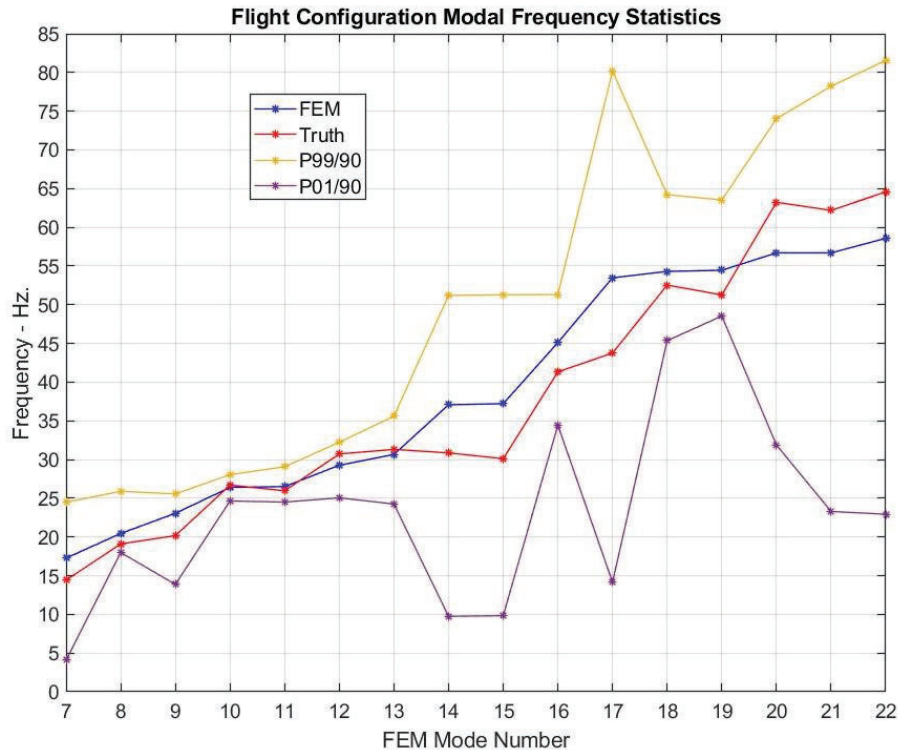


Figure 6. GSC Flight-configuration Target Mode Frequency Statistics

A MC analysis was applied to the GSC flight-configuration frequency response using the derived uncertainty model with 10,015 ensemble members. Modal damping was set to 0.5% and the number of HCB FI modes was increased to include frequencies to 120 Hz. Figure 7 illustrates the input location at one of the base interfaces, and the response locations at the antenna and stinger tips. Figure 8 shows the acceleration response along X at the antenna tip due to an input along X at the base. Figure 9 illustrates the same input and output location acceleration response, but along the Y direction. Figures 10 and 11 present the acceleration response at the stinger tip in the X and Z directions due to input at the base in the X and Z directions, respectively.

For the most part, the uncertainty model developed for the GSC HCB component based on the test-configuration modal correlation results covers the true flight-configuration frequency response with the P98/90 probability enclosure. The main exception is the acceleration response along X at the antenna tip and the stinger tip due to X input at 20.20 Hz where the true response is greater than the P99/90 prediction. The true flight-configuration mode at 20.20 Hz corresponds to the FEM flight-configuration mode 9 at 23.05 Hz representing a 12.52% frequency error. Note the uncertainty model covers the true flight-configuration mode frequency at 20.2 Hz as seen in Figure 6, even though the acceleration frequency response in the X direction at that frequency is not covered. The reason for this shortcoming is not known.

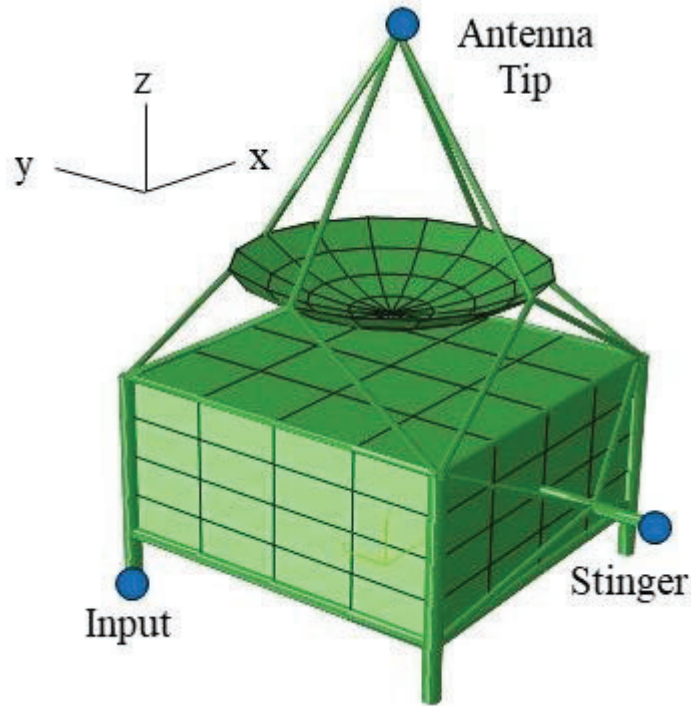


Figure 7. GSC Frequency Response Input and Output Locations

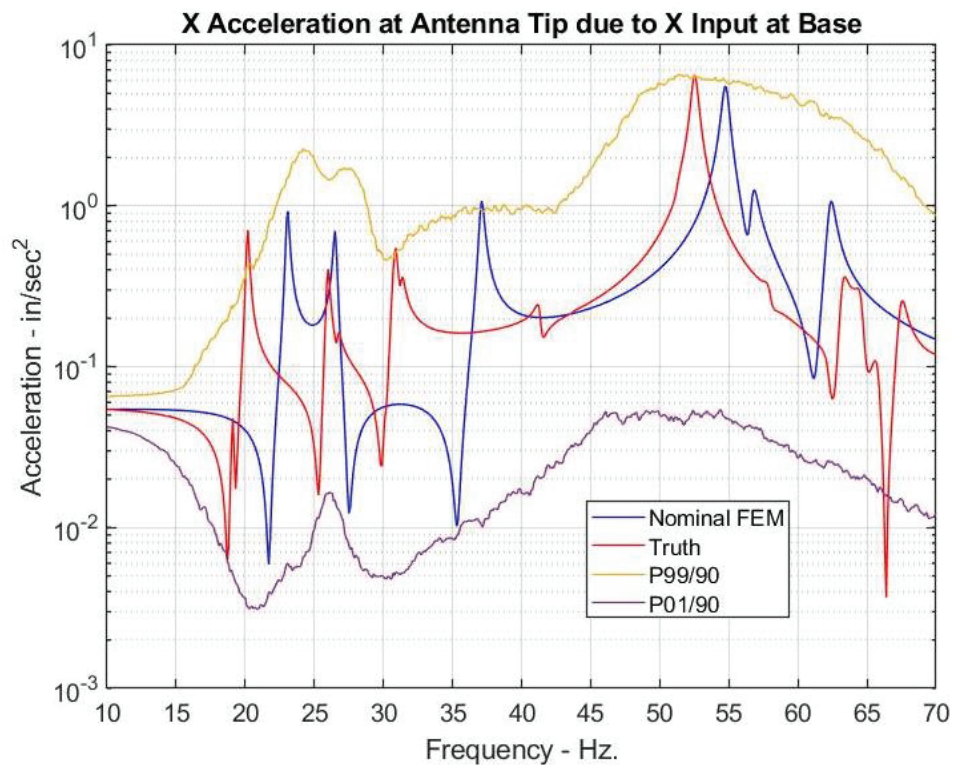


Figure 8. GSC Flight-Configuration Acceleration at Antenna Tip Along X due to Input at Base Along X

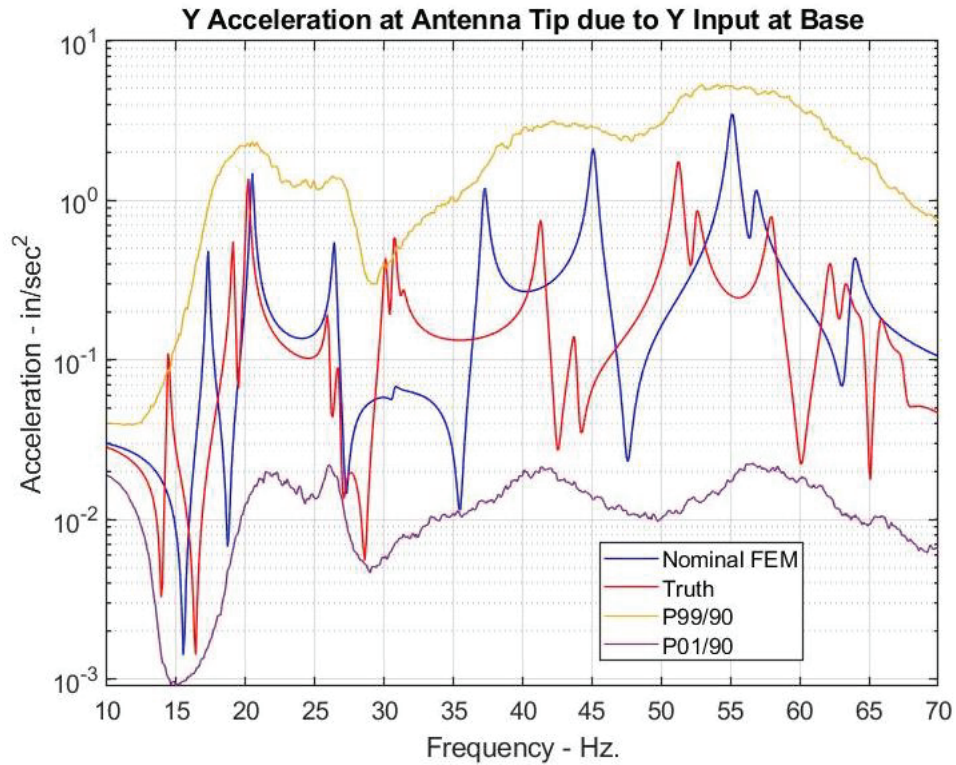


Figure 9. GSC Flight-Configuration Acceleration at Antenna Tip Along Y due to Input at Base Along Y

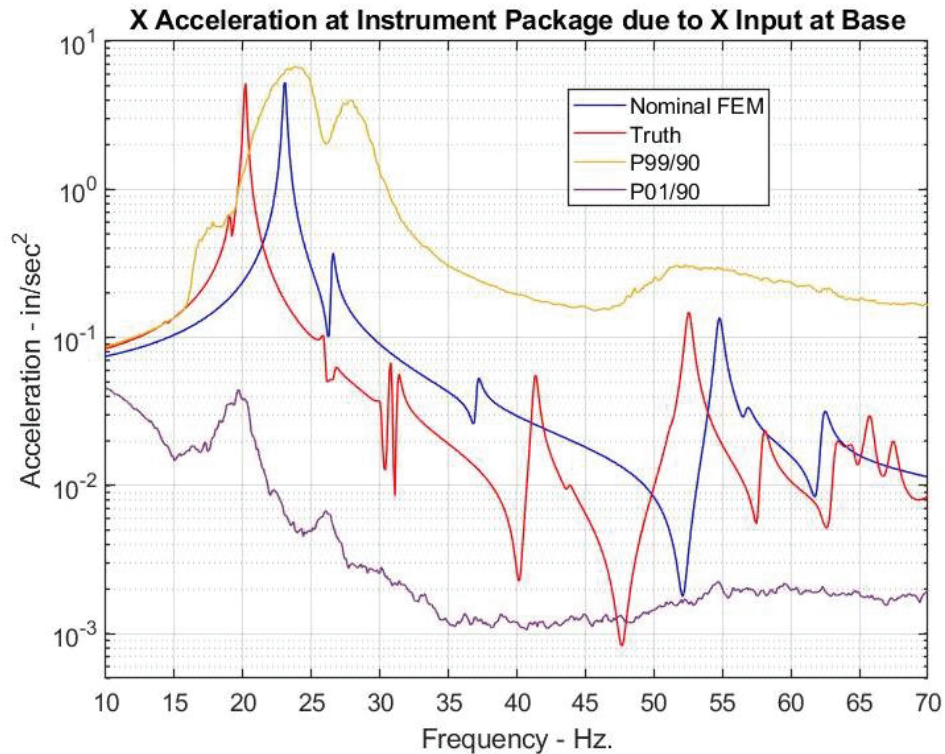


Figure 10. GSC Flight-Configuration Acceleration at Stinger Tip Along X due to Input at Base Along X

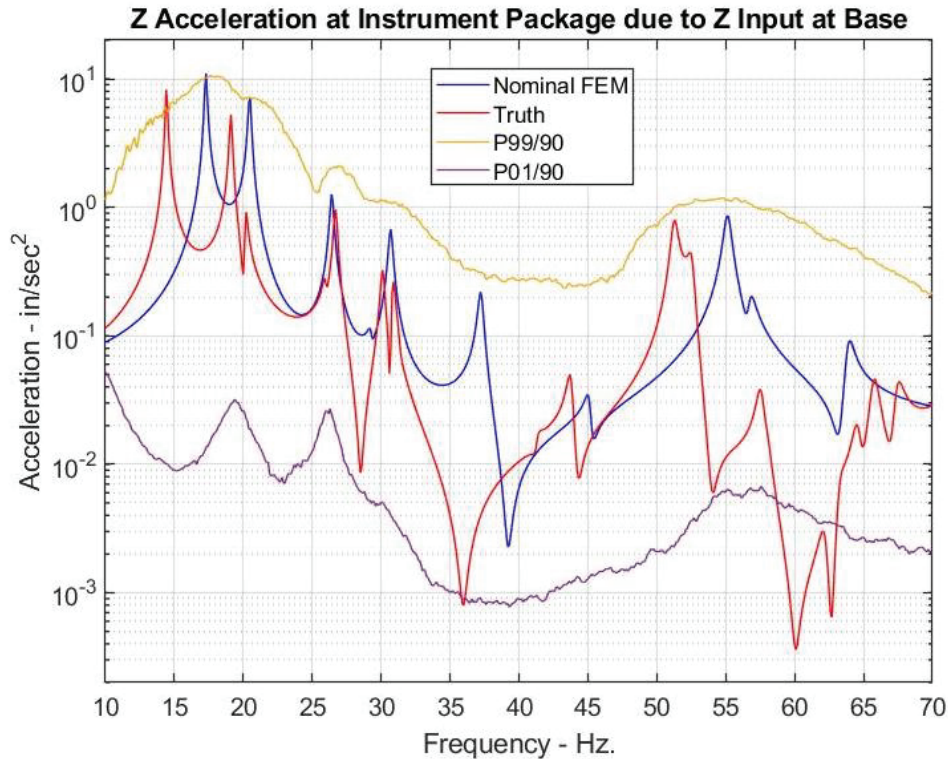


Figure 11. GSC Flight-Configuration Acceleration at Stinger Tip Along Z due to Input at Base Along Z

In this particular example, the TAM has little effect on the off-diagonal self-orthogonality terms. However, if the GSC test-configuration is simulated by dispersing the constrained FEM mass matrix by 10% instead of 5% as described in the first paragraph of Section 4.1, the TAM does have an impact upon the test self-orthogonality. The effect of the TAM error is examined in this case by considering the first 20 constrained FEM modes as the target modes. The test self-orthogonality matrix is computed using the full FEM mass matrix and the test-configuration modes that are matched to the FEM target modes. The corresponding test self-orthogonality metrics are $ODM = 1.03$ and $MOD = 0.78$. Repeating the process, but instead computing the test self-orthogonality matrix using a 210 DOF static TAM results in self-orthogonality metrics $ODM = 1.11$ and $MOD = 0.83$, representing a 7.8% and 6.4% error, respectively. Applying the approach outlined in Section 3.1.3 to correct for the error in the test self-orthogonality due to the TAM, produces an adjusted test self-orthogonality matrix with metrics $ODM = 1.03$ and $MOD = 0.78$. These values correspond to the test self-orthogonality metric values when the full FEM mass matrix and test modes are used in the self-orthogonality computation, which are of course never available in reality.

4.2 ICPS

During the course of the SLS UQ analysis, the mass matrix dispersion procedure outlined in Section 3.1 was applied to the ICPS pretest HCB component. The ICPS HCB component contained 325 DOF, including 288 physical interface DOF and 37 FI modes. There are six rigid-body modes and four slosh modes. There was assumed to be no uncertainty in the slosh modes. The mass dispersion analysis was based upon the Integrated Spacecraft Payload Element (ISPE) configuration 3 modal test-analysis correlation results [ref. 1]. There were 11 ISPE FEM target

modes matched to 11 test modes. Dispersion of the ICPS HCB mass matrix was based upon the self-orthogonality of the 11 test modes with respect to a static TAM with 263 DOF. The resulting test self-orthogonality metrics were $ODM_{test} = 4.08$ and $MOD_{test} = 2.84$. These two metrics are numerically similar because the off-diagonal terms in the test self-orthogonality matrix are more uniformly distributed over all of the modes as seen in Figure 12. This is in contrast with the GSC example in Section 4.1, where the off-diagonal test self-orthogonality terms are dominated by a smaller number of modes. The ISPE XO metric corresponding to the 11 matched test modes and pretest FEM target modes with respect to the static TAM was $DCGM_{test} = 90.44$.

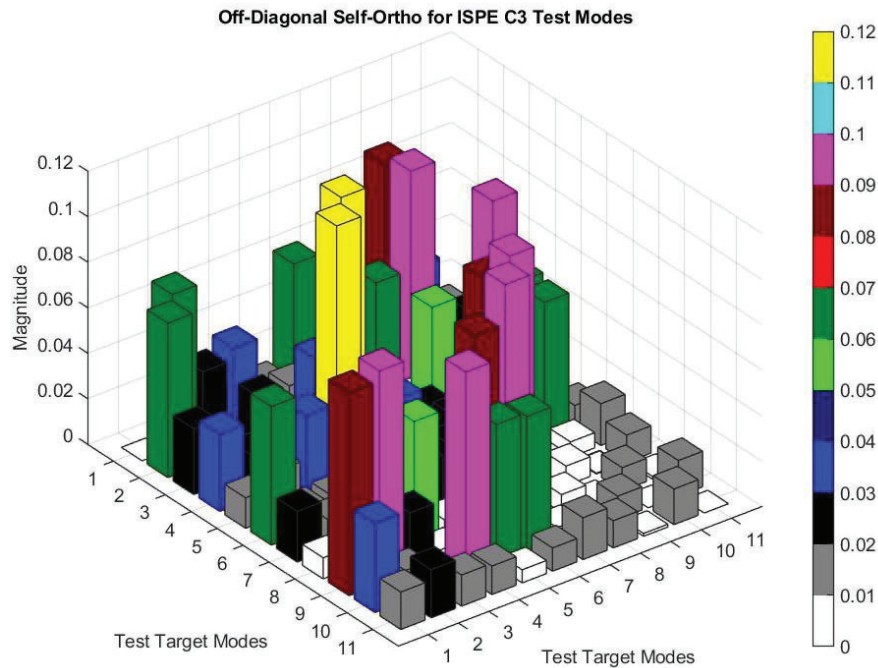


Figure 12. Absolute Value of ISPE C3 Test Mode Self-Orthogonality Matrix Off-Diagonal Terms

Dispersion of the ICPS HCB mass matrix was determined using a 3,000 ensemble member MC analysis where nominal and random modes were matched using 1% RSS XO over the first 60 elastic modes. Note that 1% RSS was used in this case because it was equivalent to what NASA employed in the generation of the ISPE test modes. The XO and self-orthogonality metrics were computed for the first 37 non-slosh elastic modes having frequencies less than 20 Hz. Using the ODM self-orthogonality metric, an HCB mass matrix dispersion of 21% produces a most probable ODM value of 4.09, which is similar to the test value of 4.08. The corresponding most probable $DCGM$ value for this mass dispersion is 70.42, which is significantly less than the test value of 90.44. As in the GSC example, the use of the ODM metric to set the mass matrix dispersion indicates a level of mass uncertainty that overwhelms the XO $DCGM$ metric that is used to set the stiffness matrix dispersion. A 21% mass dispersion within the ICPS uncertainty model precludes the application of FI eigenvalue or stiffness matrix uncertainty, which is clearly not realistic.

Using the MOD self-orthogonality metric, an HCB mass matrix dispersion of 18.25% produces a most probable MOD value of 2.84, which corresponds to the test value. The equivalent most probable $DCGM$ value for this mass dispersion is 73.03, which is also significantly less than the test value of 90.44. In contrast with the GSC example, the use of the MOD metric to set the mass

matrix dispersion also indicates a level of mass uncertainty that overwhelms the XO *DCGM* metric. This result indicates that either the self-orthogonality metrics *ODM* and *MOD* are not valid as indicators of mass matrix uncertainty, or there is error or inconsistency in the ISPE test self-orthogonality and XO matrices. Correction for the use of a static TAM in the self-orthogonality computation provided minimal differences in metric values.

5.0 Conclusion

There is some level of uncertainty in every FEM that propagates into uncertainty in predicted results. The purpose of UQ analysis is to provide statistical bounds on prediction accuracy based on model uncertainty. Uncertainty quantification does not improve the accuracy of models but accepts that the models are inaccurate and attempts to quantify the impact of that inaccuracy on predicted results. This report summarizes HPV method for UQ, which is based on uncertainty models for HCB components. This method combines a parametric variation of the HCB FI modal frequencies with a NPV that randomly varies the HCB mass and stiffness matrices using random matrix theory. The major advantage of the NPV method is that it covers errors in model form.

The HPV method uses uncertainty models for HCB components based on component modal test/analysis correlation results. The NPV based dispersion of the HCB mass matrix is derived from the test self-orthogonality matrix. Two different test self-orthogonality metrics were considered, the RMS value of the off-diagonal terms (*ODM*), and the mean absolute value of the off-diagonal terms (*MOD*). Fixed-interface mode eigenvalue uncertainty within the HCB stiffness matrix is based on frequency error between matching HCB FI modes and test modes. The NPV method is then applied to the HCB stiffness matrix by layering it on top of the FI eigenvalue variation. The stiffness matrix dispersion level is based upon the FEM/test XO matrix using the *DCGM* metric, which is the RMS value of the diagonal terms.

The validity and efficacy of the HPV method and corresponding component uncertainty model development procedure were examined by applying the approach to two examples. The GSC example was a representation where the test model was known. It was found that the *ODM* and *MOD* self-orthogonality metrics were linear with respect to the HCB mass matrix dispersion value, and neither were correlated to the resulting *DCGM* XO metric. It was found that in this example, using the *ODM* metric for test self-orthogonality to determine mass dispersion resulted in a high level of mass uncertainty that produced a *DCGM* XO value that was much less than the test value. This precludes the application of uncertainty in the HCB stiffness matrix, which is known to exist in the example, and would further reduce the *DCGM* value. In contrast, using the *MOD* metric resulted in a lower level of mass uncertainty and a value of *DCGM* that exceeded the test value, so that eigenvalue uncertainty and stiffness matrix dispersion could be applied and meet the test metric values for *MOD* and *DCGM*. For the most part, the uncertainty model developed for the GSC HCB component based on the test-configuration modal correlation results using the *MOD* and *DCGM* metrics covered the flight-configuration test frequencies and frequency response with the P98/90 probability enclosure. The second example considered the application of the HPV method to the SLS ICPS component based on the ISPE configuration 3 test results. In this application, the use of the proposed test self-orthogonality metrics produced a corresponding *DCGM* XO metric that overwhelmed the test value so that no eigenvalue uncertainty or stiffness matrix dispersion could be applied.

Based on this work and other assessments [refs. 1, 2, 5, and 6], the HPV method provides another tool to the toolset used for complex system UQ analysis that accounts for both parametric and model-form uncertainty, and is based on test data. From experience gathered to date using the HPV method, additional design specific applications must be investigated to determine which self-orthogonality metric provides the best mass matrix dispersion results, and to provide further confidence in the validity of the HPV method of UQ analysis.

6.0 Acronyms

DCGM	Diagonal Cross-Generalized Mass
DOF	Degree of Freedom
FEM	Finite Element Model
FI	Fixed Interface
GP	Gaussian Process
GSC	Generic Spacecraft
HCB	Hurty/Craig-Bampton
HPV	Hybrid Parametric Variation
Hz	Hertz
ICPS	Interim Cryogenic Propulsion Stage
ISPE	Integrated Spacecraft Payload Element
MC	Monte Carlo
MOD	Mean Off-Diagonal
NPV	Nonparametric Variation
ODM	Off-Diagonal Mass
RMS	Root Mean Square
RMT	Random Matrix Theory
RSS	Root Sum Square
SLS	Space Launch System
TAM	Test Analysis Model
UQ	Uncertainty Quantification
XO	Cross-Orthogonality

7.0 References

1. Kammer, D.; Blueloch, P.; and Sills, J.: "Test-Based Uncertainty Quantification and Propagation Using Hurty/Craig-Bampton Substructure Representations," in *IMAC*, Orlando, FL, 2019.
2. Kammer, D.; Blueloch, P.; and Sills, J.: "Variational Coupled Loads Analysis using the Hybrid Parametric Variation Method," in *IMAC*, Houston, TX, 2020.
3. Wishart, J.: "Generalized Product Moment Distribution in Samples," *Biometrika*, vol. 20A, no. 1-2, pp. 32-52, 1928.
4. Craig, R. R.; and Bampton, M. C. C.: "Coupling of Substructures for Dynamic Analysis," *AIAA Journal*, vol. 6, pp. 1313-1319, 1968.
5. Kammer, D.; Blueloch, P.; and Sills, J.: "SLS Integrated Modal Test Uncertainty Quantification using the Hybrid Parametric Variation Method," in *IMAC*, 2021.
6. Kammer, D.; Blueloch, P.; and Sills, J.: "Probability Bounds Analysis Applied to

- Multi-Purpose Crew Vehicle Nonlinearity,” in *IMAC*, Orlando, FL, 2022.
7. Soize, C.: “A Nonparametric Model of Random Uncertainties for Reduced Matrix Models in Structural Dynamics,” *Probabilistic Engineering Mechanics*, vol. 15, no. 3, pp. 277-294, 2000.
 8. Soize, C.: “Maximum Entropy Approach for Modeling Random Uncertainties in Transient Elastodynamics,” *J. Acoust. Soc. Am.*, vol. 109, no. 5, pp. 1979-1996, 2001.
 9. Adhikari, S.: “Wishart Random Matrices in Probabilistic Structural Mechanics,” *J. Eng. Mech.*, vol. 134, no. 12, pp. 1029-1044, 2008.
 10. Dereniowski, D.; and Kubale M.: “Cholesky Factorization of Matrices in Parallel and Ranking of Graphs,” *5th International Conference on Parallel Processing and Applied Mathematics*, 2003.
 11. Capiez-Lernout, E.; Pellissetti, M.; Pradlwarter, H.; Schueller, G. I.; and Soize, C.: “Data and model uncertainties in complex aerospace engineering systems,” *J. Sound Vib.*, vol. 295, no. 3-5, p. 923–938, 2006.
 12. Ben-Israel, Adi; and Greville, T.: *Generalized Inverses - Theory and Applications*, 2nd ed., New York: Springer, 2003.
 13. Gramacy, R.: *Surrogates, Gaussian Process Modeling, Design, and Optimization for the Applied Sciences*, CRC Press, Boca Raton, FL, 2020.
 14. Blelloch, P.: “Cross-Orthogonality of Closely Spaced Modes,” in *IMAC*, 2006.
 15. Kabe, A.; and Sako, B.: *Structural Dynamics Fundamentals and Advanced Applications – Volume 2*, Elsevier Science, June 2020.

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