

# High-precision predictions of properties of chemically disordered crystals

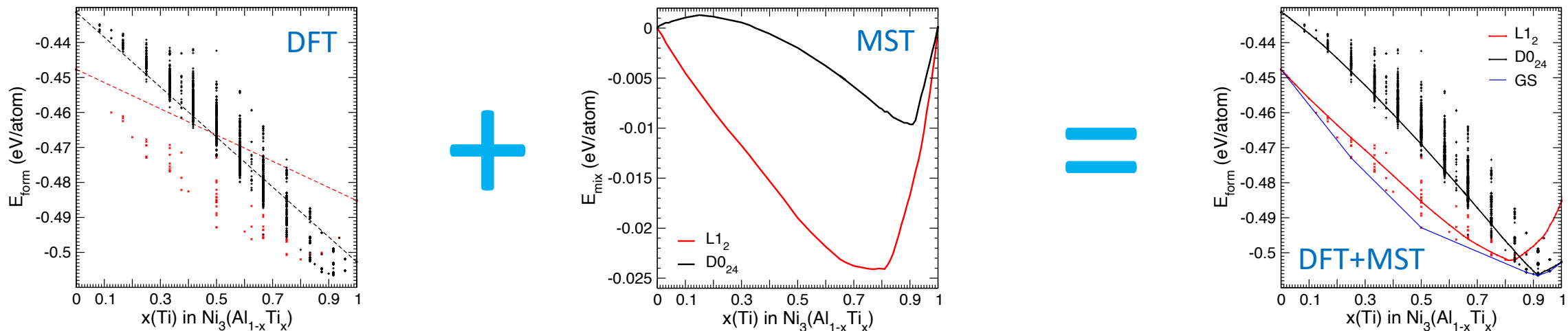


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# Outline

- Multiple scattering theory (MST) combined with density functional theory (DFT) allows to predict properties of chemically disordered materials from the first principles.
- Such predictions suffer from the systematic errors, which depend on crystal geometry. Each computed property of a particular crystal structure typically has a relatively small random error and a larger systematic error.
- Cancellation of systematic errors allows more accurate predictions. We propose a computational methodology based on the subtraction of the systematic errors in MST.
- To exemplify it, we apply it to the precipitated alloys. Considering precipitation strengthening in Ni superalloys, we compute the relative enthalpies of the competing  $\text{Ni}_3(\text{Al,Ti})_1$  crystal structures with a chemical disorder on the Al+Ti sublattice.
- Such predicted composition-structure-property dependencies are useful for the guided design of the next-generation alloys with improved strength. Our predictions are validated by comparison with the results of other DFT methods and with experiment.

# Subtraction of systematic errors

- Approximation = Exact + Error

$$X_i = X_0 + \epsilon_i$$

- $X_0$  is the exact value;  $X_i$  are approximations;  $\epsilon_i$  are errors;  $\epsilon_1 < \epsilon_2$ .

$$X_1 = X_0 + \epsilon_1 = (X_0 + \epsilon_2) + (\epsilon_1 - \epsilon_2) = X_2 + (X_1 - X_2).$$

- Assume slow (rapid) variation of systematic (random) error; random errors can be small compared to systematic errors.
- If 2 approximations differ in accuracy, then systematic error can be reduced to that of the higher-accuracy approximation.

# Density Functional Theory & Multiple Scattering Theory

## Density functional theory (DFT)

$$\left[ -\frac{1}{2} \frac{\hbar^2}{m_e} \nabla^2 + \frac{e^2}{4\pi\epsilon_0} \left( \frac{1}{2} \int \frac{\rho(r')}{|r-r'|} dr' + \sum_I \frac{Z_I}{|r-R_I|} \right) + \frac{\delta E_{XC}[\rho(r)]}{\delta \rho(r)} \right] \varphi_i(r) = \epsilon_i \varphi_i(r).$$

$$\rho(r) = \sum_i |\varphi_i(r)|^2$$

- Formulated in terms of stationary electronic wave functions and charge densities.
- Applicable to ordered structures only.

## Multiple scattering theory (MST)

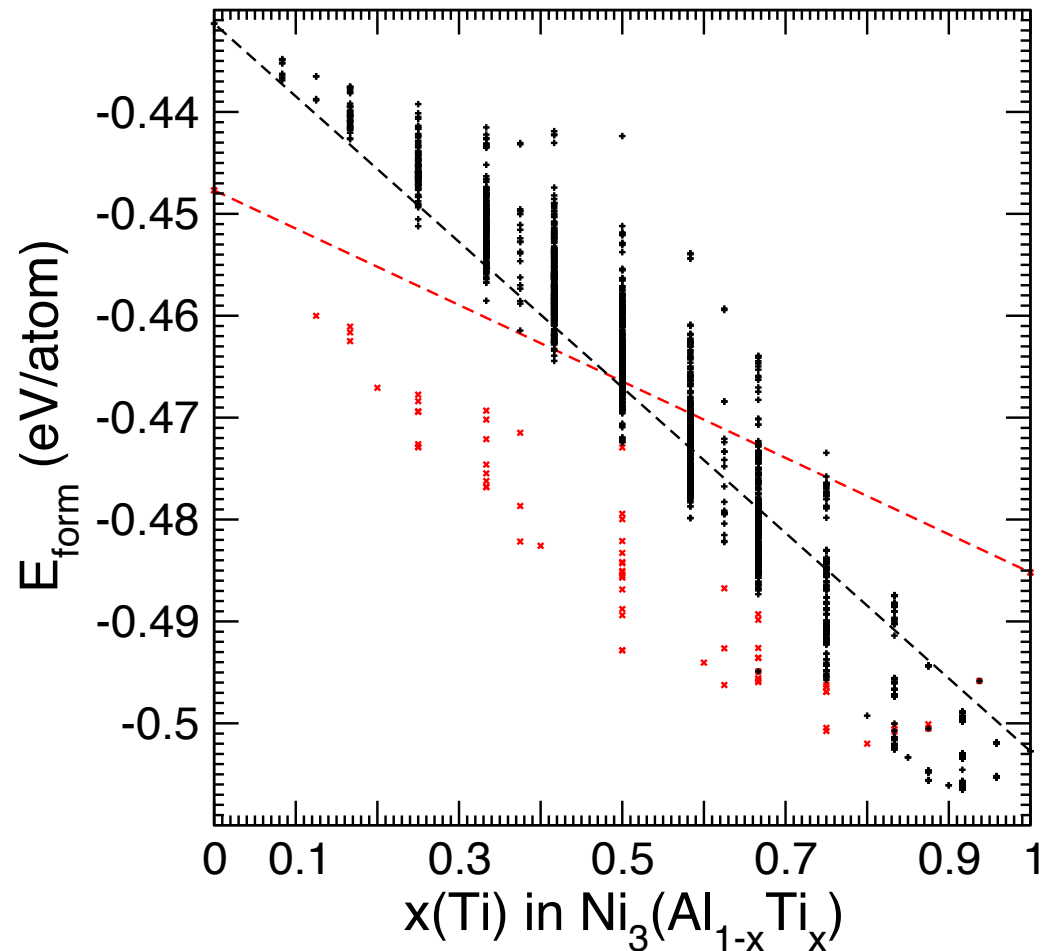
$$\det |t^{-1}(E) - G^0(q, E)| = 0,$$

$$\rho(r) = -\pi^{-1} \operatorname{Im} \oint G_{II}(r, E) dE$$

- Formulated in terms of Green's functions  $G$  and scattering matrices  $t$ .
  - Can be combined with CPA to address chemical disorder in small unit cell.
- 
- Both methods are applicable to ordered structures.
  - Disorder can be addressed using large supercells.
  - Computational cost increases with number of electrons.

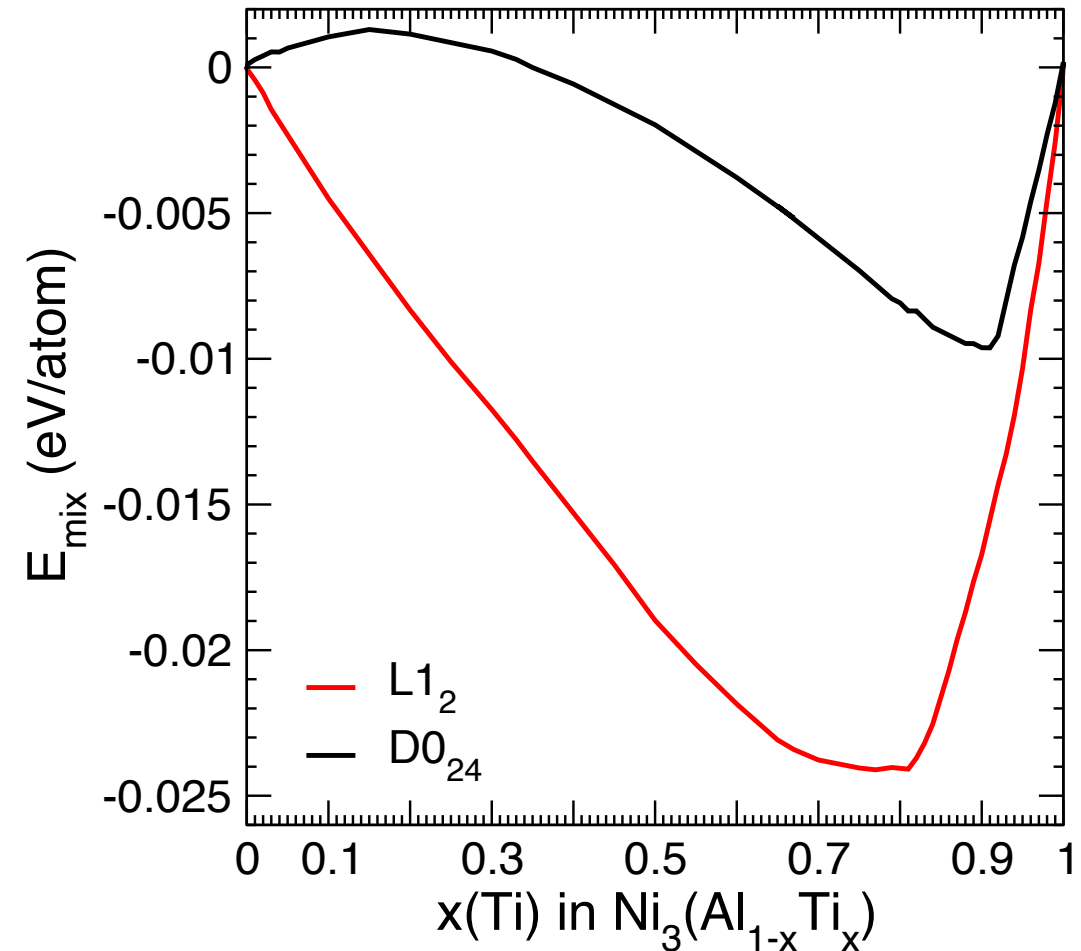
# Combining formation and mixing energies

Formation energy from DFT



$E_{\text{form}}$  is relative to energies of elemental solids.

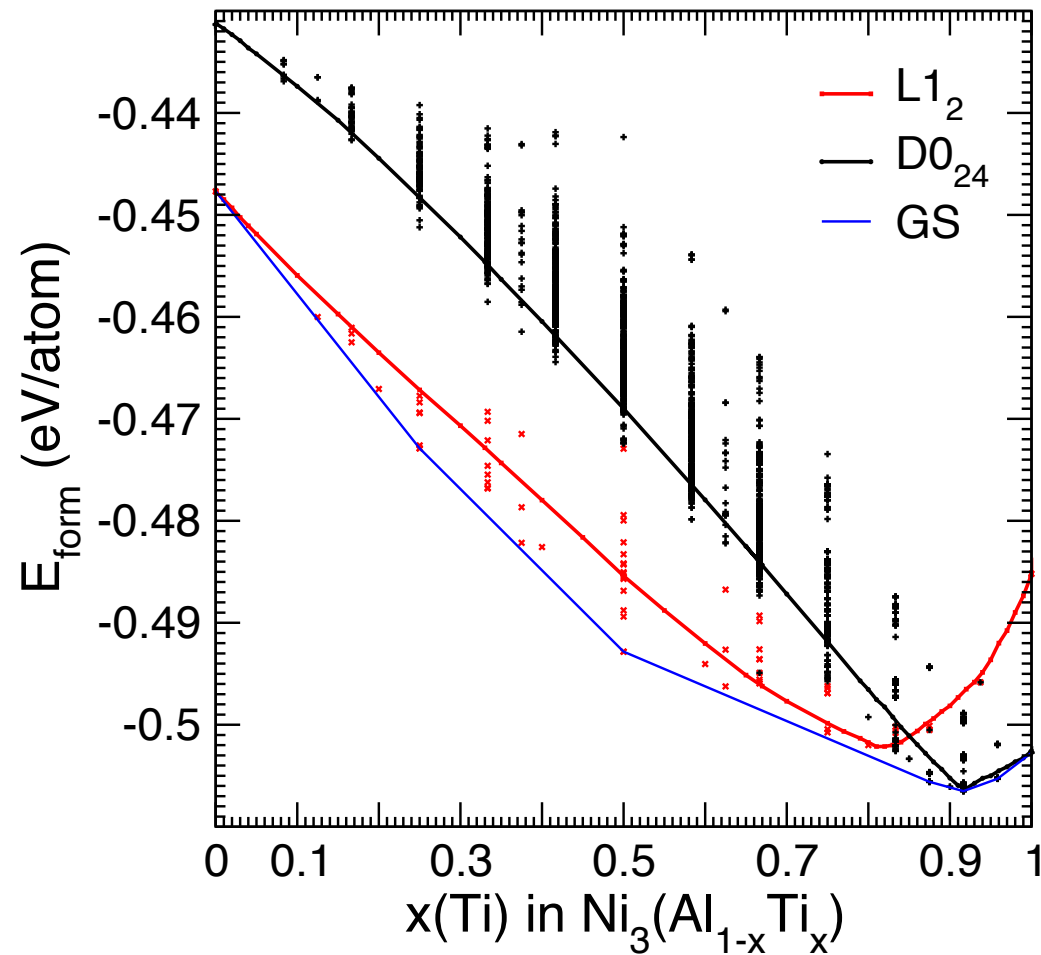
Mixing energy from MST+CPA



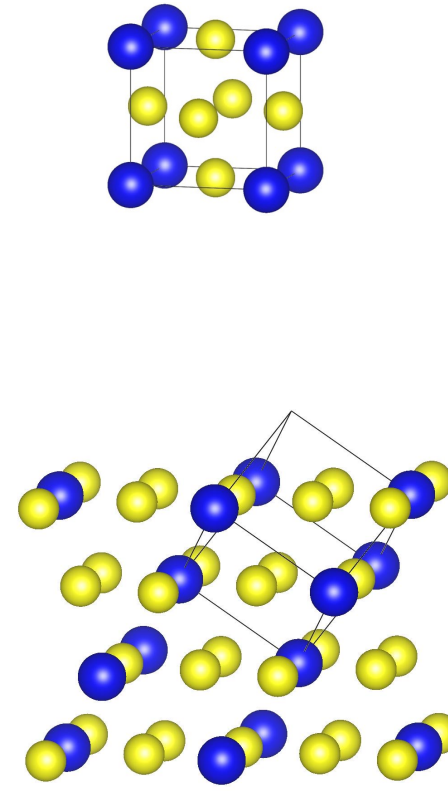
$E_{\text{mix}}$  is relative to energies of terminal compositions.

# Formation energies in $\text{Ni}_3(\text{Al,Ti})_1$ alloys

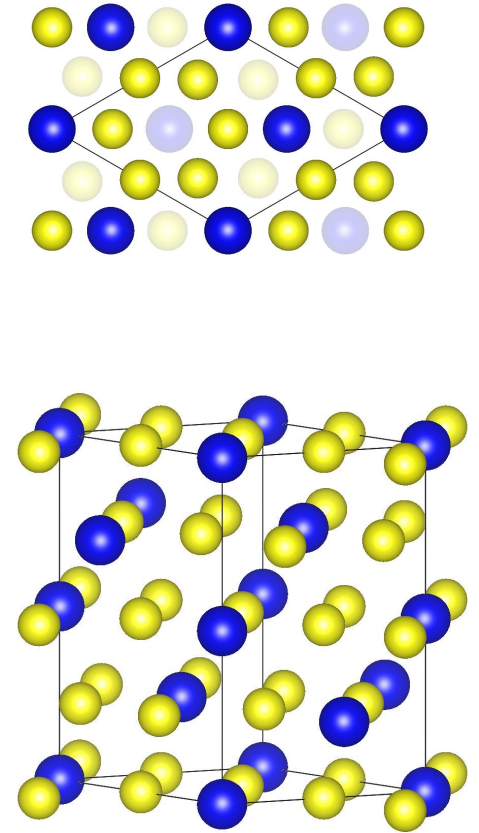
DFT + MST on the same energy scale.



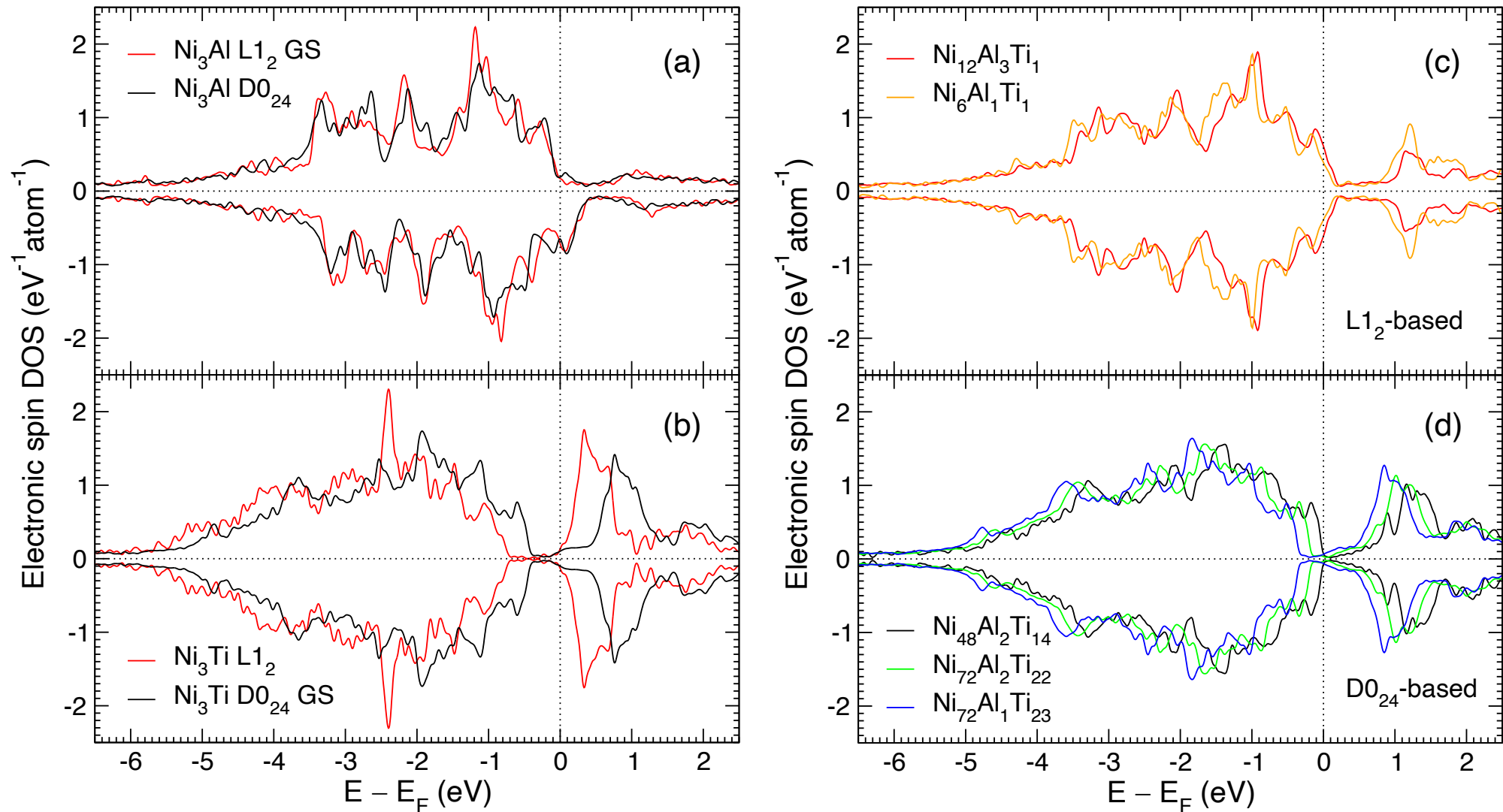
$L1_2$



$\text{D0}_{24}$

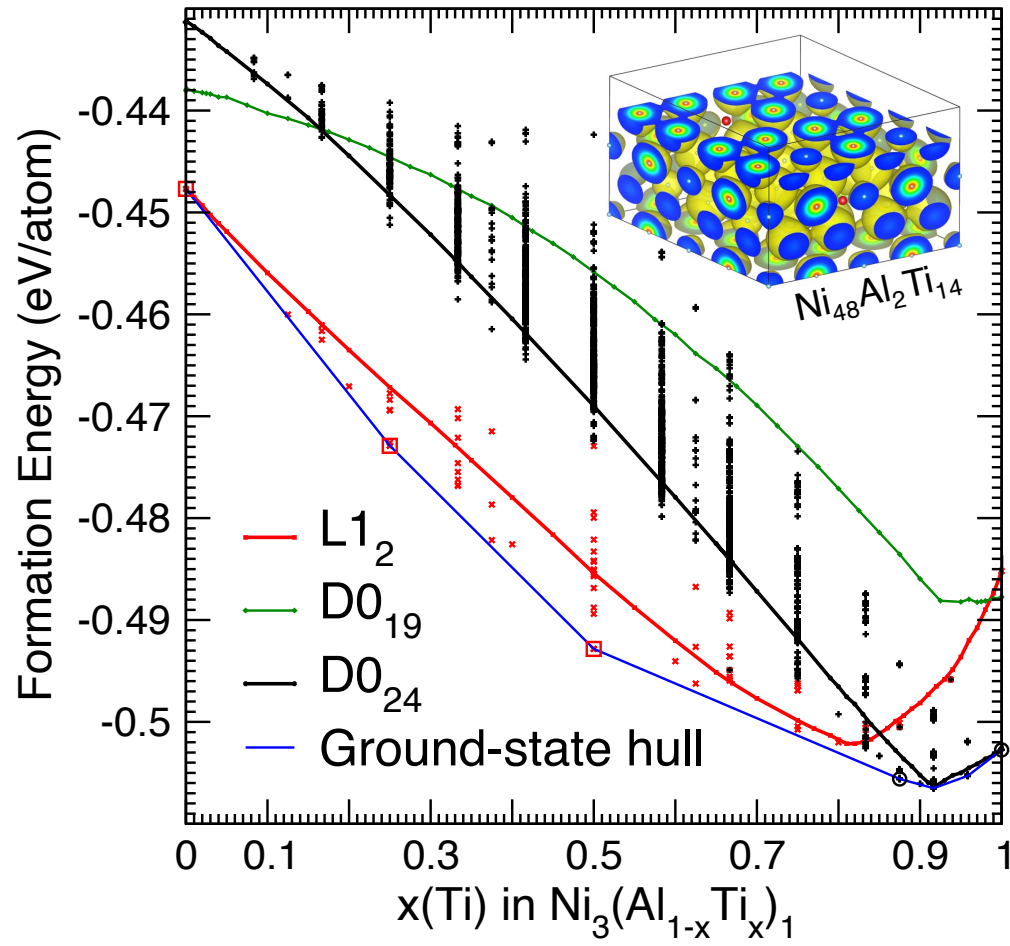


# Electronic density of states (DOS) in $\text{Ni}_3(\text{Al}_{1-x}\text{Ti}_x)$

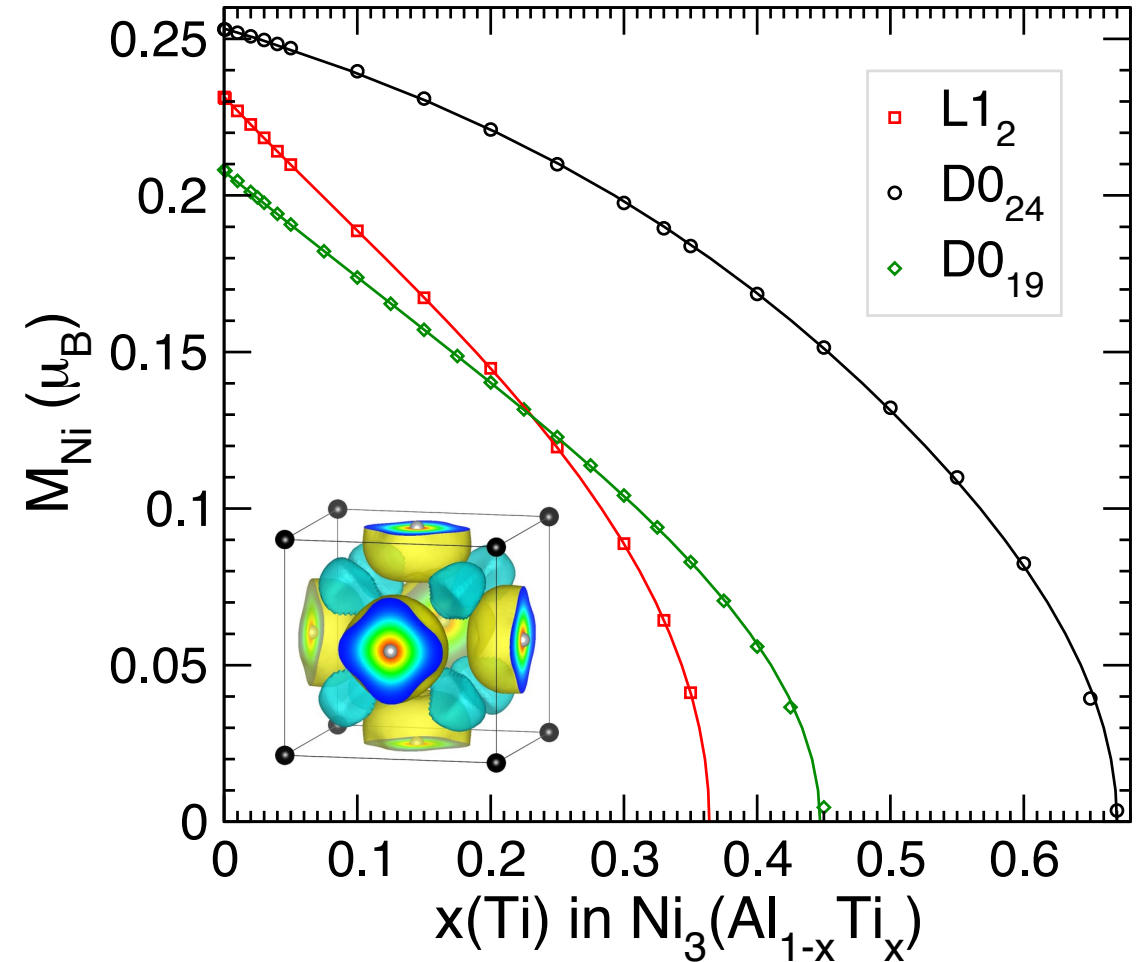


# Phase transitions in $\text{Ni}_3(\text{Al,Ti})_1$ alloys

## Structural stability

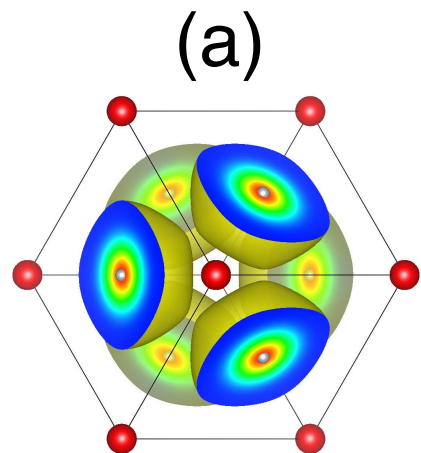


## Magnetism

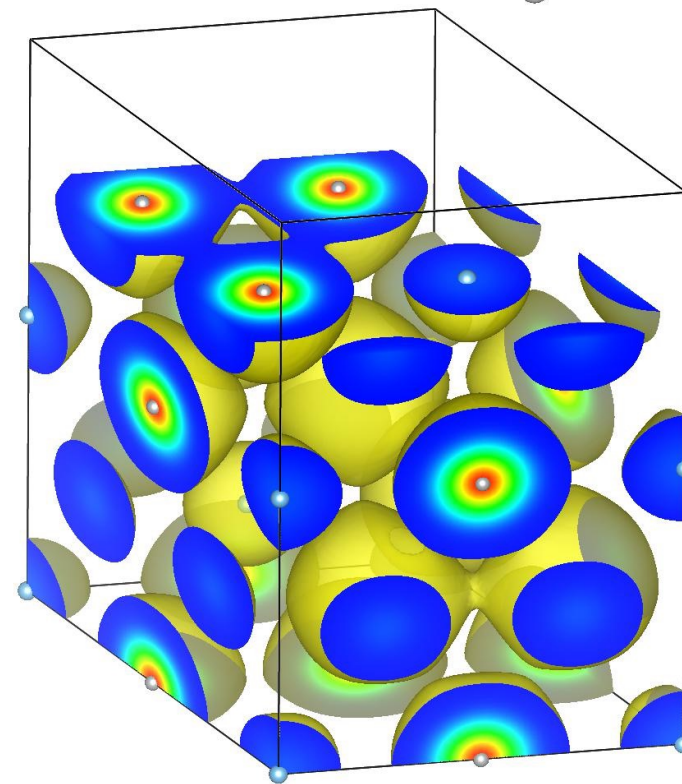
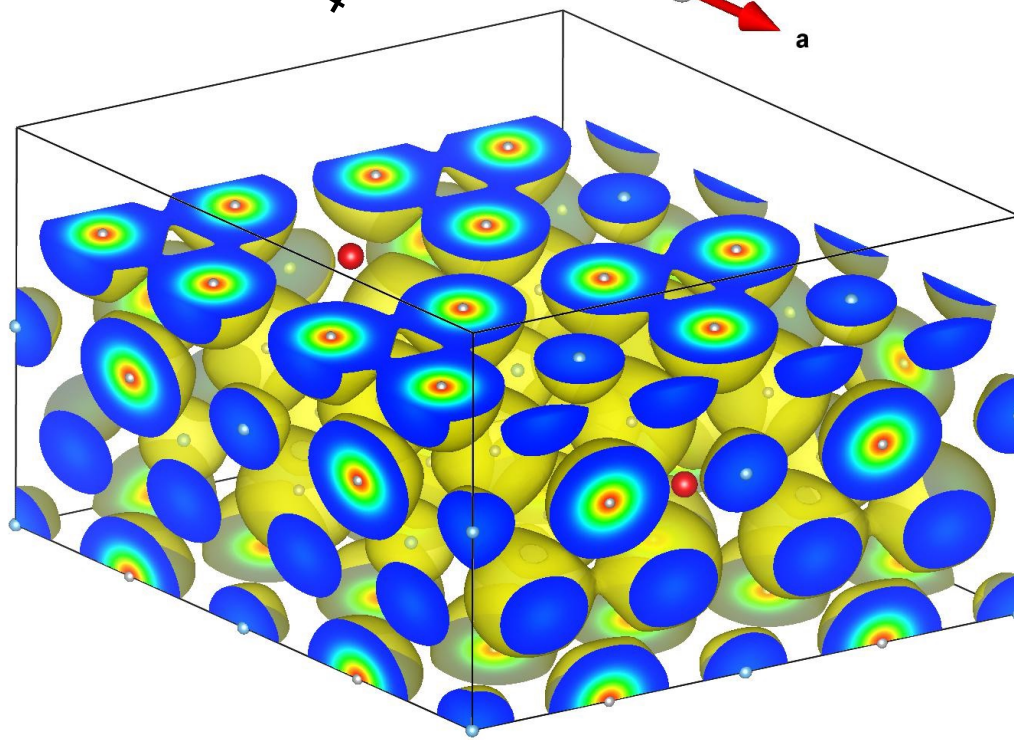
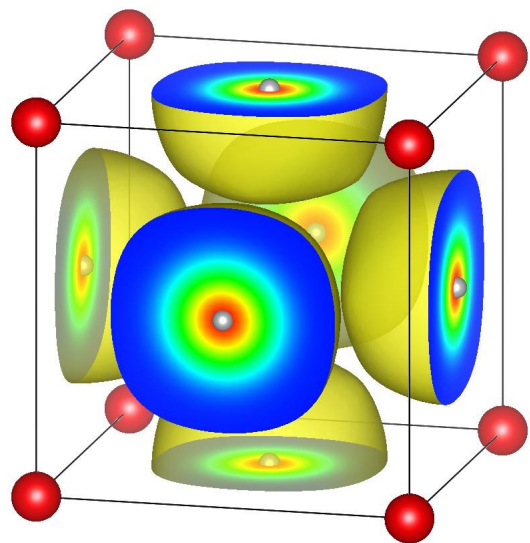
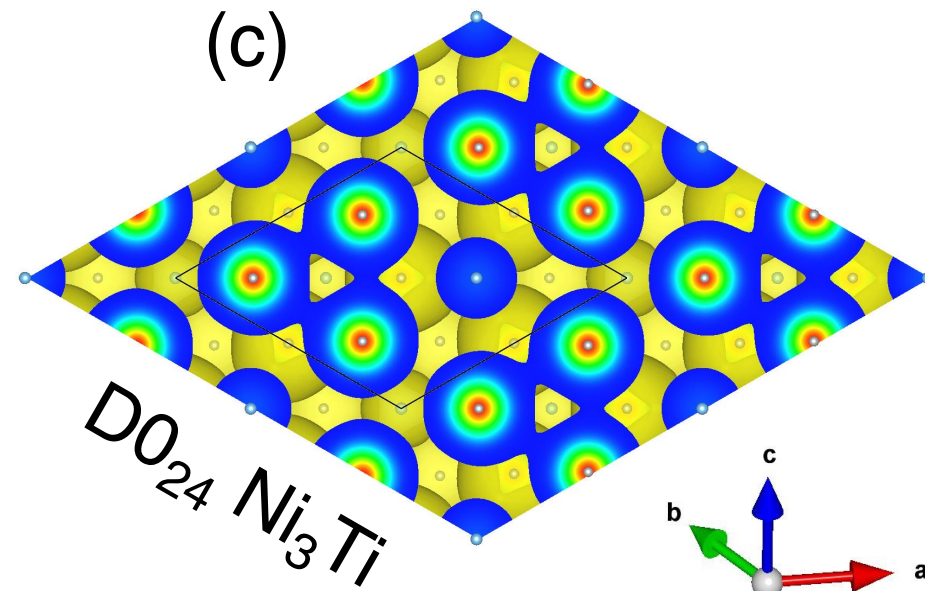
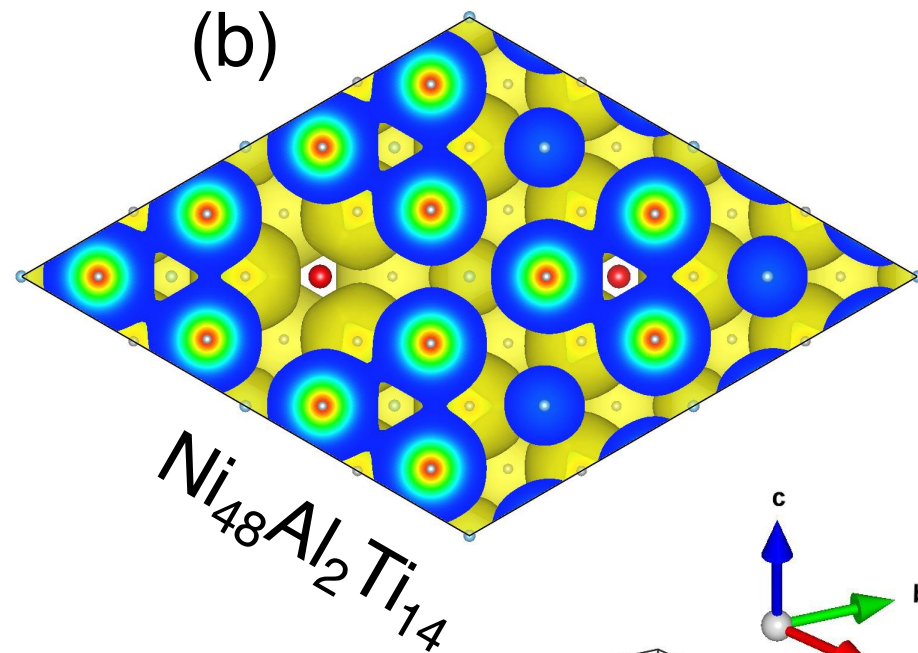


Structural, electronic, and magnetic transitions.

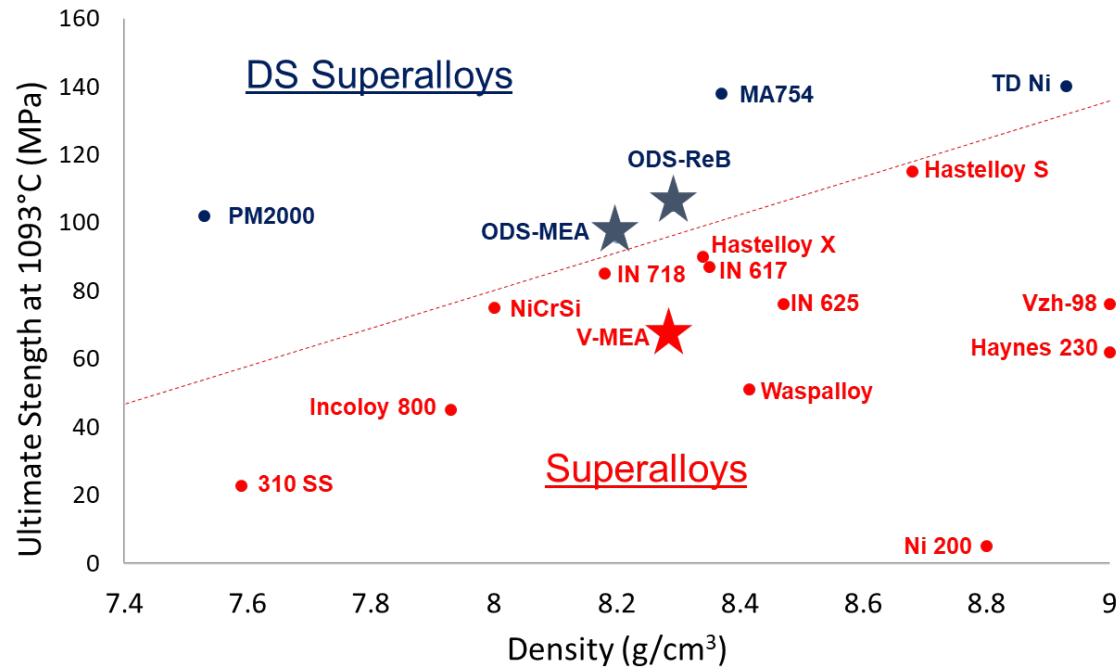




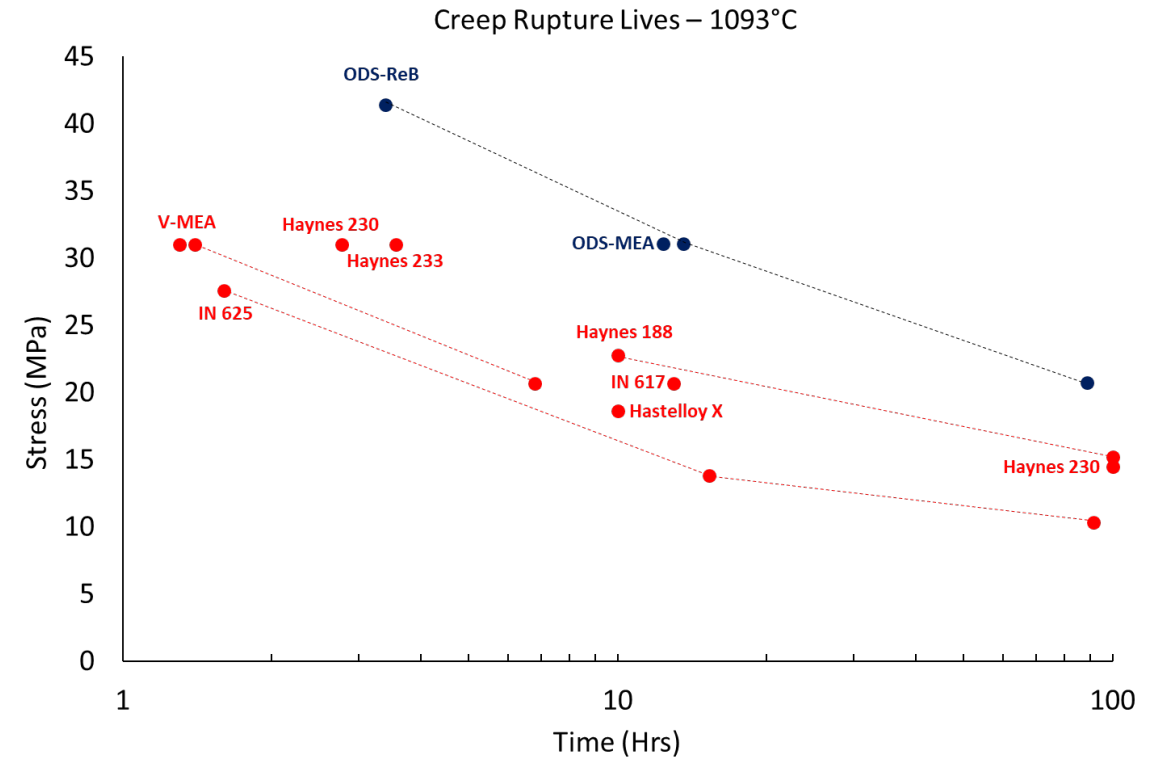
$L1_2 \text{ Ni}_3\text{Al}$



# Experimental strength of alloys



Scatter plot of alloy density and ultimate tensile strength at 1093°C for conventional and dispersion strengthened alloys. V-MEA, ODS-MEA and ODS-ReB are denoted by star points.



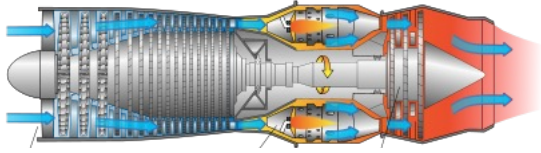
Scatter plot of creep rupture life (hours) and stress (MPa) at 1093°C for conventional superalloys. ODS-MEA and ODS-ReB are denoted by blue points.

✓ Best-in-class alloys are designed.

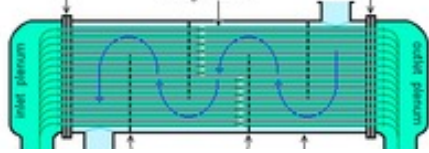


# Applications of superalloys

Combustion engines



Heat exchangers



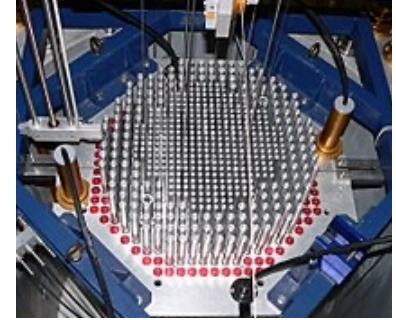
Turbines



Generators



Nuclear reactors



Power plants



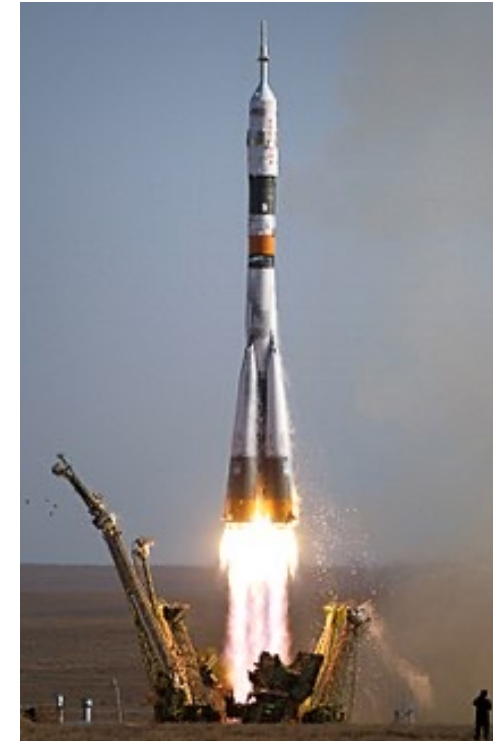
Planes, trains, automobiles



Satellites



Rockets



Ships, Submarines



Superalloys retain **strength** at high temperature, stress, **radiation**.  
Electric and **thermal conductivity** of a metal, corrosion resistance.  
Superalloys are used in **engines**, **nuclear reactors**, **heat exchangers**.

- Considered subtraction of systematic errors.
- Combined full-potential DFT and multiple-scattering theory with CPA;
- Applied to precipitation in Ni superalloys.
- Formation of  $\chi$  and  $\eta$  phases along the  $\gamma'$  stacking faults correlates with improved creep strength of Ni superalloys.
- Composition-structure-property relations allow to design an alloy with superior strength and creep properties at high operating temperature.
- Subtraction of systematic errors resulted in higher-accuracy predictions, useful for materials design.

We acknowledge funding by NASA Aeronautics Research Mission Directorate via Transformational Tools and Technologies (TTT) Project.

# Publications

- N.A. Zarkevich. Theoretical and computational methods for accelerated materials discovery (Brief Review). **Mod. Phys. Lett. B** **35** (12), 2130003 (2021). <https://doi.org/10.1142/S0217984921300039>
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- T.M. Smith, B.D. Esser, N. Antolin et al. Segregation and  $\eta$  phase formation along stacking faults during creep at intermediate temperatures in a Ni-based superalloy. **Acta Materialia** **100**, 19-31 (2015). <https://doi.org/10.1016/j.actamat.2015.08.053>
- T.M. Smith, B. Esser, N. Antolin et al. Phase transformation strengthening of high-temperature superalloys. **Nat. Commun.** **7**, 13434 (2016). <https://doi.org/10.1038/ncomms13434>
- T.M. Smith, A.C. Thompson, T.P. Gabb et al. Efficient production of a high-performance dispersion strengthened, multi-principal element alloy. **Sci. Rep.** **10**, 9663 (2020). <https://doi.org/10.1038/s41598-020-66436-5>
- N.A. Zarkevich, D.D. Johnson. Between Harmonic Crystal and Glass: Solids with Dimpled Potential-Energy Surfaces Having Multiple Local Energy Minima (Feature Paper). **Crystals** **12** (1), 84 (2022). <https://doi.org/10.3390/cryst12010084>
- N.A. Zarkevich, T.M. Smith, E.N. Baum, J.W. Lawson. Compositional glass: a state with inherent chemical disorder, exemplified by Ti-rich  $\text{Ni}_3(\text{Al,Ti})_1\text{DO}_{24}$  phase (Feature Paper). **Crystals** **12** (8), 1049 (2022). <https://doi.org/10.3390/cryst12081049>