

Background

Generative adversarial networks (GANs) can learn complex data distributions and generate new realizations of what are effectively synthetic observations. This attractive ability motivates application to certification of materials by analysis, whereby statistical models replace or augment physical testing. GANs consist of two basic components (see Figure 1):

- Generator: maps values from a latent space (typically high-dimensional noise) to output.
- Discriminator: a simple binary classifier.

These two components of a GAN compete during training: the discriminator attempts to correctly classify generated and observed data, while the generator attempts to fool the discriminator. The unique nature of this loss function results in generators that can produce samples from a distribution targeting the observed data. By generating new outputs from values in the latent space, the generator produces realistic, sharp virtual observations.

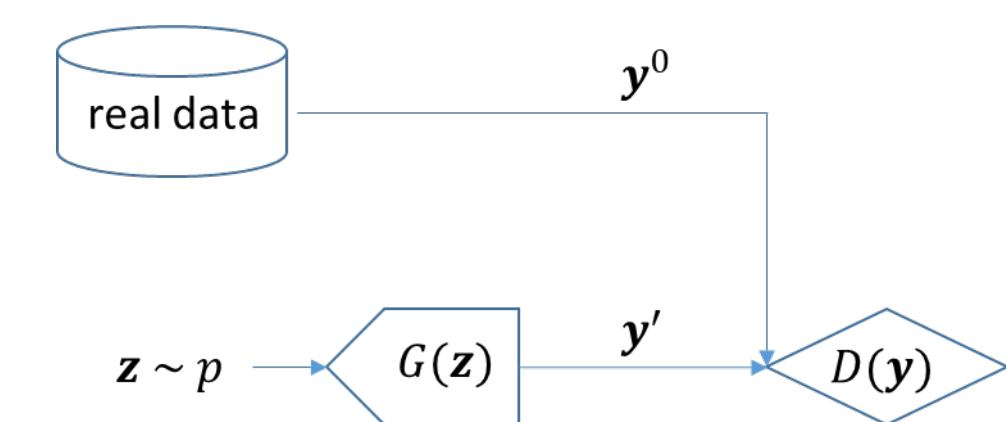


Figure 1: Basic anatomy of a GAN.

Physics-informed GANs (PIGANs) are of particular interest, leveraging partial differential equations (PDEs) to incorporate physical constraints and domain knowledge. Thus, **PIGANs can produce probabilistic and physically admissible solutions to PDEs without relying on more computationally expensive methods**, such as finite element analysis. By relating observed data to unobserved quantities of interest through the PDEs, PIGANs can then be used to solve non-deterministic inverse problems.

Contribution

Previous work has demonstrated this capability [1, 2], but required that every observed sample must have identical sensor configurations (i.e. number and arrangement), which may be impossible for real-world cases possessing heterogeneity and missingness.

The present work relaxes the assumption that samples must share sensor locations. A PIGAN is introduced that discriminates samples conditional on location information, termed location-aware discrimination. This discriminator also operates on a single (location, response) pair whereas GANs typically discriminate on the entire material sample (i.e. unconditional). The latter is costly as the number of points grows exponentially with dimension, and a corresponding increase in data and network complexity may be required. However, discriminating on a single point may hamper the model's ability to make inference on spatially-varying quantities, as explored in the final section.

Experimentation

The goal is to make inference on unobserved functions related to the observed data through PDEs. Equations 1 and 2 provide tangible examples with analytic solutions; a practical example is making inference on a material's elastic modulus having observed only its deformation.

Spatially-Constant Inference

In light of the possible shortcomings of location-aware PIGANs, it is productive to first confirm their ability to learn fixed unobserved functions. Equation 1 describes a scenario where the observed data $U(\cdot)$ behaves like a sine curve, and the unobserved $E(\cdot)$ is constant across its domain at a Beta-distributed value.

$$\begin{aligned} E(x) \cdot U(x) + U_{xx}(x) &= 0 \\ E \mid \beta &\sim \pi^2(4 + 2\beta - 1) \\ \beta &\sim \text{Beta}(2, 2) \end{aligned} \quad (1)$$

Location-aware PIGANs are able to correctly represent solutions $U(\cdot)$ and make accurate inference hidden on functions $E(\cdot)$ when $E(\cdot)$ is constant across its domain (see Figure 2).

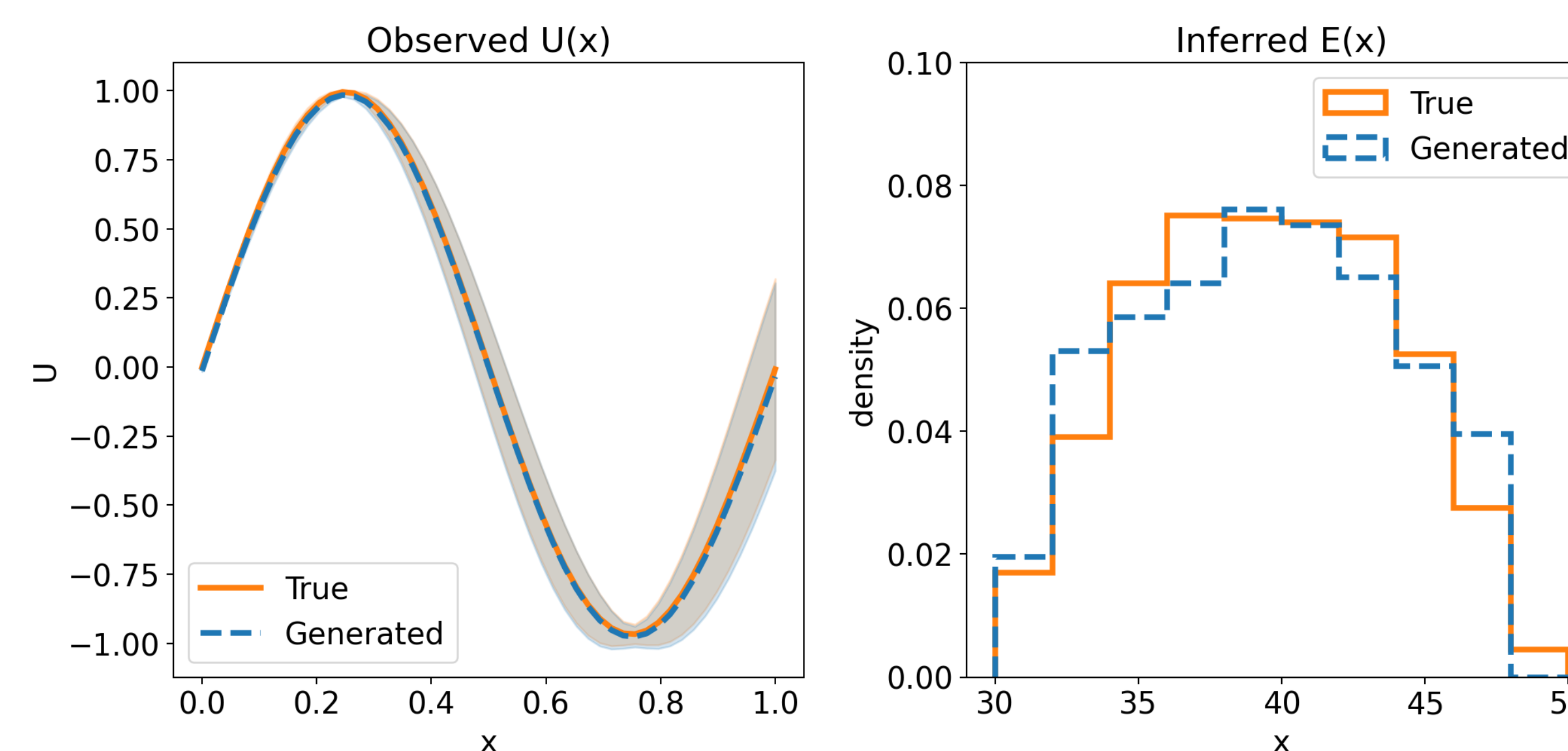


Figure 2: Summary output from a location-aware PIGAN trained on Eq. 1. Left: mean and variance of observed and generated samples of $U(\cdot)$. Right: histogram of real and generated values of $E(\cdot)$.

References

- [1] James E Warner, Julian Cuevas, Geoffrey F Bomarito, Patrick E Leser, and William P Leser. Inverse estimation of elastic modulus using physics-informed generative adversarial networks. 2020.
- [2] Yibo Yang and Paris Perdikaris. Adversarial uncertainty quantification in physics-informed neural networks. *Journal of Computational Physics*,

Spatially-Varying Inference

Now consider the same differential equation when $E(\cdot)$ varies with x according to Equation 2.

$$\begin{aligned} E \mid x, \beta_1, \beta_2 &\sim \beta_1 + \beta_2 x^2 \\ \beta_1 &\sim \pi^2(8 + 2\text{Beta}(2, 2) - 1); \quad \beta_2 \sim -5\pi^2 \text{Beta}(2, 2) \end{aligned} \quad (2)$$

Location-aware PIGANs are still able to learn $U(\cdot)$ and $E(\cdot)$ when $E(\cdot)$ is spatially-varying (see Figure 3). The spatial correlation of real and generated $E(\cdot)$ samples is shown in Figure 4; the location-aware PIGAN exhibits some inconsistencies, a subject of future investigation.

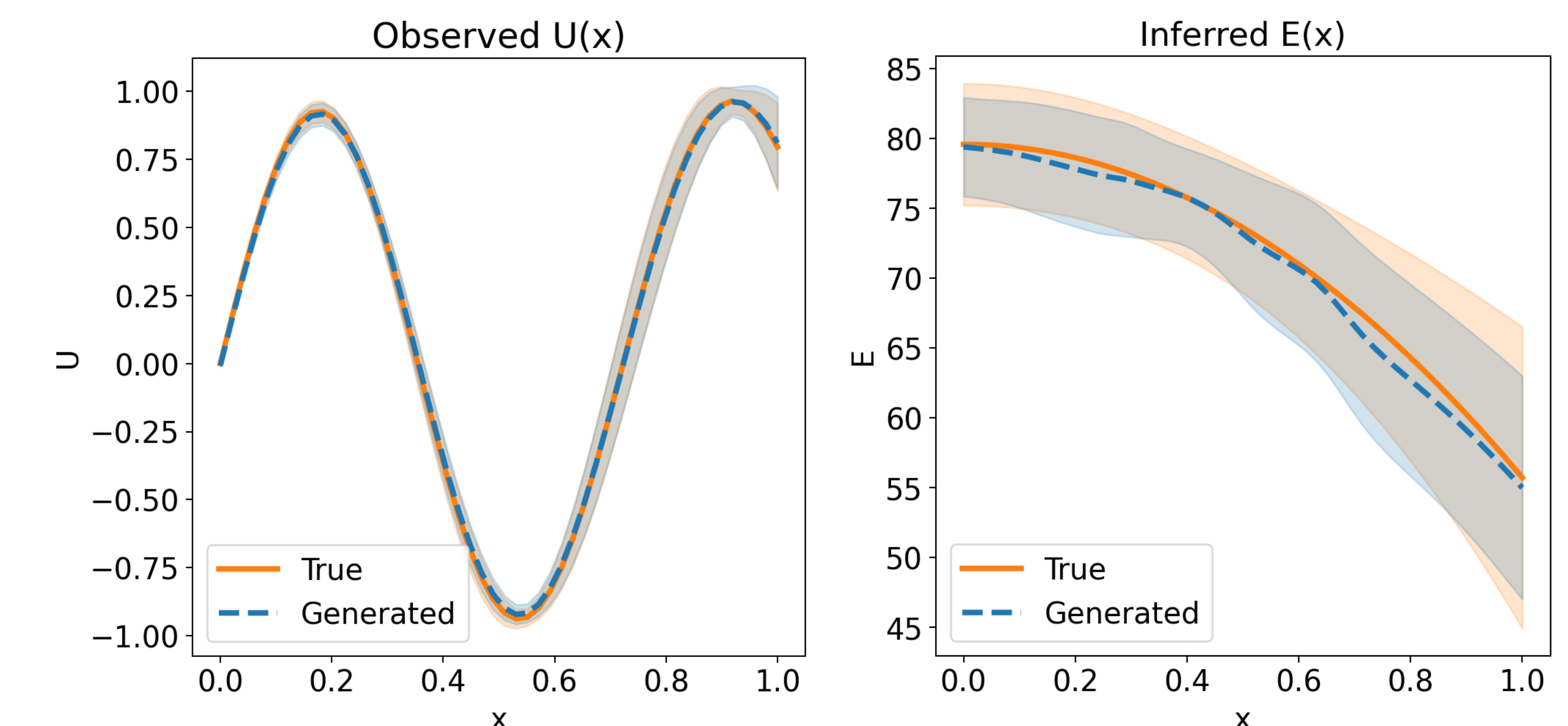


Figure 3: Summary output from a location-aware PIGAN trained on Eq. 2. Left: mean and variance of real and generated $U(\cdot)$. Right: mean and variance of real and generated $E(\cdot)$.

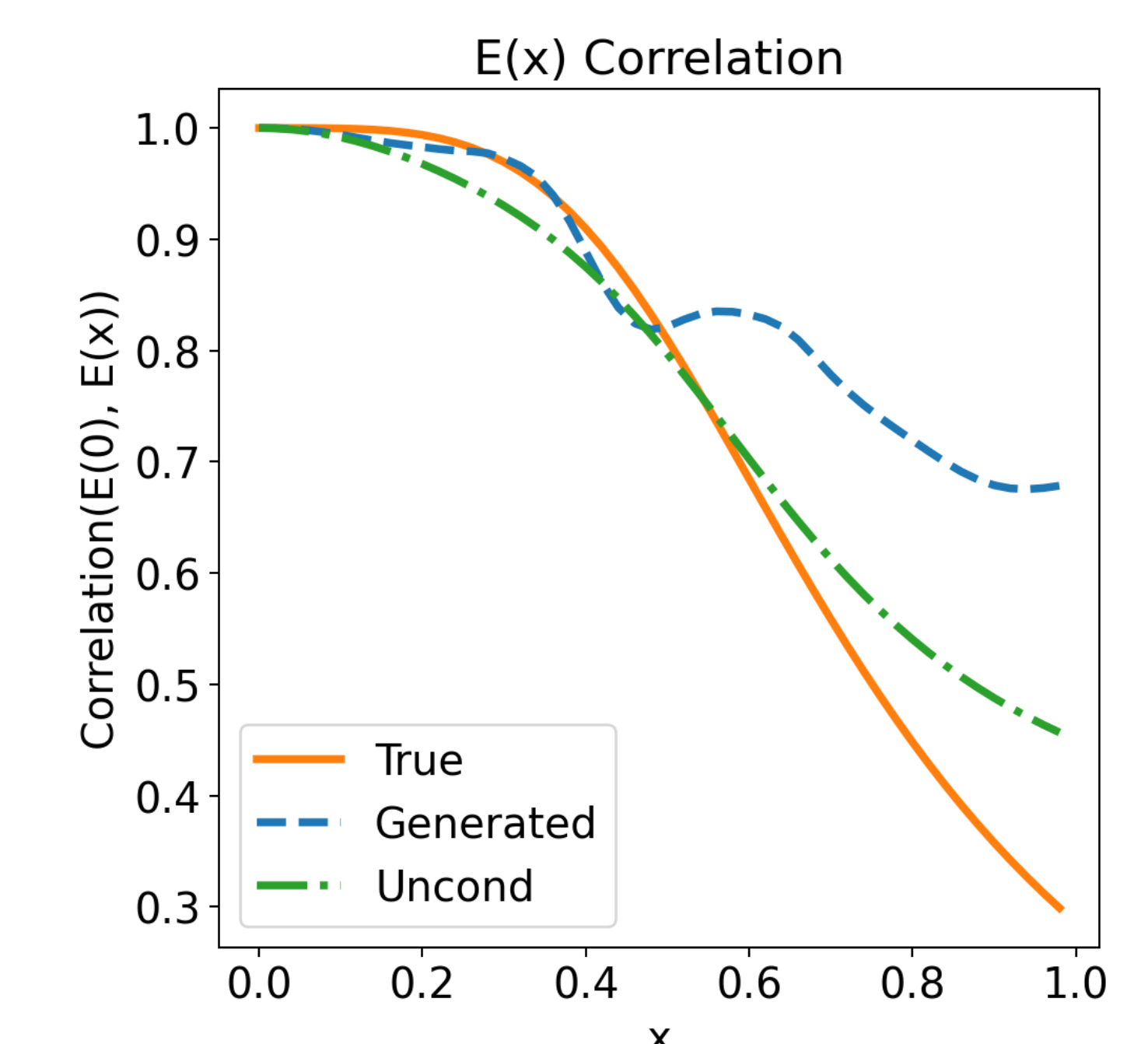


Figure 4: Correlation between $E(0)$ and $E(\cdot)$ for the true data (orange), location-aware PIGAN (blue), and a standard PIGAN (green).

Conclusion

Location-aware PIGANs show promise in making meaningful inference under less-stringent data collection requirements.