**Dynamical Core Damping of Thermal Tides in Martian Atmosphere**

Yuan Liana, Mark I. Richardsona, Claire E. Newmana, Chris Leea,b, Anthony Toigoc, Scott Guzewichd, Roger V. Yellee

a *Aeolis Research, Chandler, AZ 85224, USA*

b *University of Toronto, Toronto, Ontario, Canada*

c *Johns Hopkins University, Baltimore, MD 21218, USA*

d *Goddard Space Flight Center,* Greenbelt*, MD 20771, USA*

e *The University of Arizona, Tucson, AZ 85721*

*Corresponding author:* Yuan Lian, lian@aeolisresearch.com

**Abstract**

Atmospheric oscillations with daily periodicity are observed in *in-situ* near-surface pressure, temperature, and winds observations, and also in remotely sensed temperature and pressure observations of the Martian atmosphere. Such oscillations are interpreted as thermal tides driven by the diurnal cycle of solar radiation and occur at various frequencies, with the most prominent being the diurnal, semidiurnal, terdiurnal and quadiurnal tides. Mars global circulation models reproduce these tides with varying levels of success. Until recently, both the MarsWRF and newly developed MarsMPAS models were able to produce realistic diurnal and semidiurnal tide amplitudes but predicted higher-order mode amplitudes that were significantly weaker than observed. We use linear wave analysis to show that the divergence damping applied within both MarsWRF and MarsMPAS is responsible for suppressing the amplitude of thermal tides with frequency greater than 2 per sol, despite being designed to suppress only acoustic wave modes. Decreasing the strength of the divergence damping in MarsWRF and MarsMPAS allows for excellent prediction of the higher order tidal modes. This finding demonstrates that care must be taken when applying numerical dampers and filters that may eliminate some desired dynamical features in planetary atmospheres.

**1. Introduction**

Atmospheric waves provide important contributions to the energy, momentum, and tracer distributions within planetary atmospheres. Observations of atmospheric waves can serve as a probe of the atmospheric states due to the sensitivity of the waves to the thermal forcing and wind structures that generate, maintain, damp, and modify these waves. For example, measurement of thermal tides in the Martian atmosphere provides indirect insight into the global distribution of aerosol radiative heating that complements the direct measurement of dust and/or water ice cloud optical depths. In order to properly exploit this information, forward modeling of the waves is necessary using numerical modeling of the atmospheric state. Most often, the sensitivity of the model to variations in aerosol heating or other processes, such as boundary layer mixing, can be used to link the observed wave behavior to the dynamical behavior of the atmosphere. In regions of more complex interaction between the local and global circulation, such as at the Gale Crater landing site of the Curiosity rover, models are essential in order to unravel the relative roles of local, regional, and global atmospheric circulation. For example, models have been used to explain aspects of the observed daily variation of surface pressure in terms of global scale thermal tides, regional scale flows, and flows over the varied topography of the Gale Crater region [Rafkin et al., 2016; Richardson and Newman, 2018].

Implicit in the use of numerical models is the assumption that the dynamical core representation of wave propagation is accurate. If it is accurate, the model provides a direct linkage between the forcing physics and the observable state, and hence the model can be used to extract information about the atmosphere from the wave response. However, it is well known that numerical models do not provide perfect emulation of real atmospheres. This is due to the techniques required to discretize the fluid dynamical solutions both spatially and temporally. In the specific example of the thermal tide on Mars, which we use as our exemplar in this paper, it is known that aspects of the structure of the observed tide are better emulated by some models than others. Thermal tides are ubiquitous in the Martian atmosphere. Various Mars landers and rovers detected similar and repeatable daily surface pressure variations at different geological locations, suggesting that these temporal pressure variations were not localized events, but thermal tides [Leovy,1981; Schofield et al., 1997; Lewis et al., 1999; Guzewich et al., 2016; Banfield et al., 2020]. The Oxford Mars General Circulation Model (GCM) was able to simulate all of the daily structures in the Mars Pathfinder data [Lewis et al., 1999], while the Mars Weather Research and Forecasting (WRF) model [Richardson et al., 2007; Toigo et al., 2012] has struggled to capture some of the higher frequency, daily repeatable structures (specifically, the daily-repeatable surface pressure transgression around 8pm local time) [Guzewich et al., 2016; Fonseca et al., 2018; Richardson and Newman, 2018; Newman et al., 2017]. For MarsWRF, the cause was ultimately attributed to the dynamical core by testing several different physics parameterization schemes and dust distributions within the model and finding that no combination produced an improved match to observations. Significantly, we have found similar behavior in the Model for Prediction Across Scales (MPAS) model [Skamarock et al. 2012], whose Martian adaptation is described by Lian and Richardson [2022].

**2. Processes within the WRF and MPAS Dynamical Core**

While most GCMs solve the incompressible, hydrostatic primitive equations, the WRF and MPAS dynamical cores have the capability to solve fully-compressible, nonhydrostatic equations. This is useful when investigating regional motions in which the horizontal scale and vertical scale become comparable, or when investigating acoustic wave generation. However, acoustic waves produced by the compressibility of an atmosphere are usually unwanted in atmospheric models because they often lead to numerical instabilities. This is due to violation of the Courant-Friedrichs-Lewy (CFL) criterion: *i.e*., fluid motions associated with acoustic waves travel too fast to be resolved by model time step on the grid scale.

To mitigate the issues associated with acoustic wave modes, both WRF and MPAS utilize a divergence damping scheme that is intended to suppress the acoustic waves, without affecting gravity waves [Skamarock and Klemp, 1992; Klemp et al., 2018]. Skamarock and Klemp [1992] demonstrated that, when applied to wind fields in a domain where the effect of planetary rotation can be ignored, the divergence damping had little impact on the propagation properties of gravity waves in a Boussinesq fluid (*i.e*., divergence damping had little impact on gravity wave phase and amplitude). Gassmann and Herzog [2007] analyzed the divergence damping method in Skamarock and Klemp [1992] using fully compressible equations. They found that the damping must be applied to both horizontal and vertical momentum equations in order to avoid any impact on the gravity wave phases. Klemp et al. [2018] suggested that the proper formula of divergence damping in fully compressible equations required a divergence term different from that in Gassmann and Herzog [2007] and Skamarock and Klemp [1992]. With the modified divergence terms, they showed that the divergence damping in the horizontal momentum equations was sufficient to damp acoustic wave modes with negligible effect on gravity waves, whilst avoiding the complexity of dependence on vertical wavenumber.

The concept of divergence damping, as noted by Skamarock and Klemp [1992], was originally implemented to damp internal and inertial gravity waves in hydrostatic primitive equations. However, none of the aforementioned studies on divergence damping explored the impact of divergence damping on thermal tides, which have horizontal scales much larger than the depth of the atmosphere, and hence are generally considered to be hydrostatic in nature. In the remainder of this paper, we explore the impact on thermal tides and demonstrate that without proper tuning, divergence damping can significantly modify atmospheric thermal tides.

**3. Linear Analysis of Divergence Damping on Waves**

To evaluate the impact of divergence damping on thermal tides, we perform linear wave analysis that only includes a single source of atmospheric tides and a single wave dissipation mechanism via divergence damping as detailed below. This avoids the complication associated with various sources of wave generation and dissipation mechanisms in a 3D GCM.

**3.1 Laplace’s Tidal Equations**

The perturbation equation sets describing atmospheric motions on synoptic scales in a motionless, isothermal, hydrostatic and compressible atmosphere are (see Appendix for detailed derivation):

$$\begin{array}{c}-iω\frac{ρ'}{ρ\_{∘}}+\frac{w'}{H}=χ\#\left(1\right)\end{array}$$

$$\begin{array}{c}-iωP'+w'g=γgHχ-\left(γ-1\right)J\#\left(2\right)\end{array}$$

$$\begin{array}{c}\frac{∂P'}{∂z}-\frac{P'}{H}=-\frac{ρ'}{ρ\_{∘}}g\#\left(3\right)\end{array}$$

$$\begin{array}{c}χ-\frac{∂w'}{∂z}=i\frac{ω}{4a^{2}Ω^{2}}\left[F\left(P'\right)-α\_{d}F\left(χ\right)\right]\#\left(4\right)\end{array}$$

In the above equations, an arbitrary dynamical quantity $X$ is expressed as $X=X\_{∘}+X^{'}$, where $X\_{∘}$ is the mean background state. The perturbation $X^{'}$ can be described in wave form:

$$X^{'}= \overline{X}\left(θ,z\right) e^{i\left(ωt +sλ\right)}$$

where: $\overline{X}\left(θ,z\right)$ and $\left(ωt+sλ\right)$ are, respectively, the amplitude and phase of thermal tides respectively; $θ$ and $λ$ are, respectively, the latitude and longitude in radians; $ω$ is the wave frequency; $s$ is the zonal wavenumber (positive value for westward wave propagation); $ρ$ is atmospheric density; $w$ is vertical velocity; $P=\frac{p^{'}}{ρ\_{∘}}$ where $p$ is pressure; $g$ is acceleration due to gravity; $γ=\frac{c\_{p}}{c\_{v}}$ where $c\_{p}$ and $c\_{v}=c\_{p}-R$ are heat capacities for constant pressure and volume, respectively; $R$ is the specific gas constant; $a$ is the radius of the planet; $Ω$ is the rotation rate of the planet; $J$ is the external heating source that drives the thermal tides; and the divergence in spherical coordinate is expressed as (following Skamarock and Klemp, [1992]):

$$χ'=∇⋅\vec{v^{'}}=\frac{1}{acosθ}\frac{∂u^{'}}{∂λ}+\frac{1}{acosθ}\frac{∂v^{'}cosθ}{∂θ}+\frac{∂w^{'}}{∂z}$$

Note that in WRF and MPAS models, a revised definition of divergence damping term

$$χ\_{D}=\frac{1}{ρ\tilde{θ}}∇⋅ρ\tilde{θ}\vec{v}$$

is used to represent gravity waves more appropriately (Klemp et al., [2018]; Skamarock et al., [2021]), where $\tilde{θ}$ is the potential temperature. This modification weakens the damping effect on the thermal tides but its effect is still not negligible (see Section 3 in Appendix). Nonetheless, we choose the current definition $χ $here since it was widely used by previous analysis on divergence damping (*e.g.*, Sadourny [1975]; Skamarock and Klemp, [1992]; Gassmann and Herzog [2007]; Whitehead et al., [2011]). Further, $α\_{d}$ in Eq. 4 is the divergence damping coefficient, formulated in WRF and MPAS as $α\_{d}=2α\_{h}\frac{L\_{d}^{2}}{Δt}$ (Skamarock and Klemp, [1992]; Klemp et al., [2018]), where $α\_{h}$ is a dimensionless coefficient (typically 0.1 in WRF and MPAS, which is suitable for lower-atmosphere applications on Earth). $L\_{d}$ is the dissipation length scale (e.g., mean horizontal grid spacing $Δx$), and $Δt$ is the Runge-Kutta sub-timestep in both GCMs. It can be seen that, besides changing $α\_{h}$, modifying $L\_{d}$ and/or $Δt$ can both change $α\_{d}$. Here we choose to change $α\_{h}$ only while keeping $L\_{d}=220 km$ and $Δt = 40  s$ (typical values for global Mars simulations) for a more controlled investigation on the effect of divergence damping. The Runge-Kutta sub-timestep $Δt = 40  s$ (corresponding to a model time step of 120 *s*) is significantly shorter than the typical value used for Earth simulations with comparable grid resolution. The reason is that Mars has a very thin atmosphere that responds to solar radiation rapidly and very steep terrain variations that leads to strong vertical motions, which prone to the violation of vertical CFL. Finally, the linear differential operator $F$ in Eq. 4 is defined as:

$$F=\frac{∂}{∂μ}\left(\frac{1-μ^{2}}{ν^{2}-μ^{2}}\frac{∂}{∂μ}\right)-\frac{1}{ν^{2}-μ^{2}}\left[\left(\frac{s}{ν}\right)\frac{ν^{2}+μ^{2}}{ν^{2}-μ^{2}}+\frac{s^{2}}{1-μ^{2}}\right]$$

where $ν=\frac{ω}{2Ω}$ is the scaled wave frequency and $μ=sinθ$.

Equations (1)-(4) can be further reduced to a Laplace’s Tidal Equation (LTE) and a vertical structure equation (VSE) using separation of variables for solutions at each wave frequency $ω$ and longitudinal wavenumber $s$ [e.g., Chapman and Lindzen, 1970] assuming:

$$\begin{array}{c}P^{'}\&=\sum\_{n}^{}L\_{n}\left(z\right)Θ\_{n}\left(θ\right)\\J\&=\sum\_{n}^{}G\_{n}\left(z\right)Θ\_{n}\left(θ\right)\end{array}$$

where the subscription $n$ represents multiple possible solutions (Hough modes [Hough, 1897]) for the following equations:

$\begin{array}{c}F\left(Θ\_{n}\right)=-\frac{4a^{2}Ω^{2}}{gh\_{n}}Θ\_{n}\#\left(5\right)\end{array}$ $\begin{array}{c}H\frac{∂^{2}L\_{n}}{∂z^{2}}-\left(1-i\frac{α\_{d}ω}{γgh\_{n}}\right)\frac{∂L\_{n}}{∂z}-\frac{1-γ}{γ}\frac{L\_{n}}{h\_{n}}=\frac{i}{ω}\frac{1-γ}{γ}\frac{∂G\_{n}}{∂z}-\left(\frac{i}{ω}+\frac{α\_{d}}{gh\_{n}}\right)\frac{1-γ}{γ}\frac{G\_{n}}{H}\#\left(6\right)\end{array}$

In the LTE (Eq. 5) and VSE (Eq. 6), $h\_{n}$ is termed the *equivalent depth*, which results from the separation of variables and can be determined by eigenvalues $ζ\_{n}=\frac{4a^{2}Ω^{2}}{gh\_{n}}$ of Eq. 5 as a boundary value problem [Wang et al., 2016].

The LTE has two boundary conditions, $Θ\_{n}=0$ at both $θ=-\frac{π}{2}$ and $θ=\frac{π}{2}$. The VSE is also subject to two boundary conditions. First, the perturbation vertical velocity $w^{'}=0$ at the lower boundary $z=0$; second, there is no downward propagating wave at the top boundary $z=z\_{top}$. The latter boundary condition implies that, for a homogenous second order ordinary differential equation, $c\_{1}\frac{∂^{2}X}{∂z^{2}}+c\_{2}\frac{∂X}{∂z}+c\_{3}X=0$ where $c\_{1}$, $c\_{2}$ and $c\_{3}$ are arbitrary coefficients as functions of $z$, only the solution representing the upward propagating wave will be retained, *i.e.*, the solution $X=Ce^{ik\_{z}z}$ with vertical wavenumber $k\_{z}>0$. This is also called the *radiation boundary condition* (see Appendix for details).

For simplicity, we assume that the atmosphere is radiatively transparent and the surface absorbs all solar insolation. This is because the Martian atmosphere is very thin and there is a strong convective/radiative coupling between diurnal ground temperature variations and atmospheric dynamics [Gierasch and Goody, 1968]. The thermal tides are therefore primarily excited by the diffusive heat exchange between the surface and the atmosphere in the planetary boundary layer (PBL) [Chapman and Lindzen, 1970]:

$$G\_{n}=iωc\_{p}ΔT\_{s}e^{-k\_{d}z}e^{i\left(ωt+sλ\right)}$$

where $ΔT\_{s}$ is the near surface temperature anomaly at the subsolar point and $k\_{d}$ is the decaying factor that is correlated to eddy diffusivity, $κ\_{e}$, in the PBL as $k\_{d}=\sqrt{\frac{ω}{κ\_{e}}}e^{i\frac{λ}{4}}$. For Mars, we assume $ΔT\_{s}≈40K$ (roughly the maximum amplitude of diurnal temperature variations measured by multiple Mars landers). The eddy diffusivity on Mars is small due to the thin atmosphere (*i.e.*, roughly 100 times lower surface pressure than on Earth). Assuming $κ\_{e}=0.1m^{2}s^{-1}$, a typical value in the Martian surface layer [Martinez et al., 2009], and considering the solutions at $λ=0$ such that $k\_{d}=\sqrt{\frac{ω}{κ\_{e}}}$, which is equivalent to the properties of migrating tides that are independent of longitude, we can estimate the e-folding scale for $G\_{n}$ as $k\_{d}Δz≈1$. For diurnal, semidiurnal and terdiurnal tides of interest, $k\_{d}$ is estimated to be 0.03 to 0.05 $m^{-1}$, corresponding to $Δz ≈$ 33 to 20 *m*. This depth scale is similar to the finest vertical resolution used in the PBL scheme employed by MarsWRF and MarsMPAS.

**3.2 Model Parameters**

We solve LTE and VSE using the physical parameters for Mars shown in Table 1.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | $$a \left[km\right]$$ | $$Ω\left[s^{-1}\right]$$ | $$g\left[ms^{-2}\right]$$ | $$c\_{p}\left[Jkg^{-1}K^{-1}\right]$$ | $$R \left[Jkg^{-1}K^{-1}\right]$$ | $$T\_{s}\left[K\right]$$ | $$κ\_{e}\left[m^{2}s^{-1}\right]$$ |
| value | 3389.9 | $$7.088×10^{-5}$$ | 3.727 | 767.3 | 191.8 | 300 | 0.1 |

Table 1. Physical parameters used by the linear wave analysis. Note that $T\_{s}=300K$ is chosen for demonstration purposes. The results are relatively insensitive to the typical surface temperature at the subsolar point near the equator over a Martian year.

For the present analysis, we consider the diurnal tide, semidiurnal tide, terdiurnal tide and quadiurnal tide. These are the frequencies of the dominant modes seen in the daily cycles of near-surface pressure observed by Mars landers and rovers (e.g., Insight lander and Curiosity rover). The mode of thermal tide is defined by the combination of scaled wave frequency, $ν$, and zonal wave number, $s$ (Table 2).

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | Diurnal (DW1) | Semidiurnal(SW2) | Terdiurnal(TW3) | Quadiurnal(QW4) |
| $$ν$$ | 0.5 | 1 | 1.5 | 2 |
| $$s$$ | 1 | 2 | 3 | 4 |

Table 2. Parameters for various modes of the thermal tides. DW1 stands for diurnal tide, westward propagating with zonal wavenumber of 1. Similar notations apply to SW2, TW3 and QW4.

**3.3 Results**

The solutions to the LTE and VSE shown in the following are obtained from the associated Legendre polynomials (ALP) for the LTE [Wang et al., 2016] and the second-order central differential scheme for VSE. There are 90 grid points evenly spaced from $-\frac{π}{2}$ and $\frac{π}{2}$ (in latitude) for LTE. In the vertical, the model top is chosen to be high enough (*i.e.*, higher than the PBL height) that the tidal forcing term $G\_{n}$ vanishes.



Figure 1. Hough functions $Θ$ for DW1 (a), SW2 (b) and TW3 (c). Only the first three symmetric modes for each type of tides are shown.

Figure 1 demonstrates the Hough functions (i.e., the eigenvectors $Θ\_{n}$, where $n$=0,1,2,3, …) for the first three symmetric Hough modes of the DW1, SW2 and TW3 tides. The order of the Hough modes is sorted by the magnitude of the eigenvalues from the largest to the smallest. The Hough functions depict the latitudinal structures of perturbations to physical quantities (*e.g.*, pressure, temperature or vertical velocity) associated with the different types of thermal tides, as shown by the gravest Hough modes of the diurnal and semidiurnal tides in the MarsWRF simulations (Guzewich et al. [2016]). For each mode (each eigenvalue), an equivalent depth $h\_{n}$ can be obtained (Table 3).

|  |  |  |  |
| --- | --- | --- | --- |
|  | DW1 | SW2 | TW3 |
| $h\_{n}$(mode) | 516.76 (1,1) | 5854.08 (2,2) | 9591.76 (3,3) |
| $h\_{n}$(mode)  | 89.98 (1,3) | 1576.22 (2,4) | 3785.63 (3,5) |
| $h\_{n}$(mode)  | 36.19 (1,5) | 715.46 (2,6) | 2020.85 (3,7) |

Table 3. Equivalent depth $h\_{n}$[m] corresponding to different Hough modes for DW1, SW2 and TW3 tides. The subscripts (X,Y) mean the tide X and its dominant symmetric Hough modes Y.



Figure 2. The ratios between pressure perturbations with $α\_{h}=0.1$ and $α\_{h}=0$ for the first three symmetric Hough modes *n*=(1,1),(1,3),(1,5) for diurnal tides.

The divergence damping impacts all three types of tides and various modes associated with each tide. Figure 2 shows the effect of divergence damping ($α\_{h}=0.1$) on the first three symmetric modes of the DW1 tide. The effect of divergence damping, measured by the ratio of amplitudes between pressure perturbations with or without it activated ($r\_{div}=\frac{δp\left.​\right|\_{α\_{h}>0}}{δp\left.​\right|\_{α\_{h}=0}}$ ) shows about a factor of four variation near the surface from Hough modes (1,1) to (1,5). Note that $r\_{div}$ is greater than 1 for all Hough modes at lower altitudes in Fig. 2. This appears to be counterintuitive because divergence damping shall suppress the growth of wave amplitude vertically. However, a non-zero divergence damping coefficient $α\_{d}$ affects both wave amplitude and phase (see the dispersion relations given by Eq. A33 and Eq. A34 in Appendix), and the solution of the surface pressure perturbation (*i.e.*, the initial wave amplitude at the lower boundary) can be larger than that of $α\_{d}=0$ for some wave modes (characterized by $ω$ and $h\_{n}$) under the lower boundary condition $w^{'}=0$ at $z=0$.



Figure 3. Same as Figure 2 except that the lower boundary condition is changed to $p^{'}$=constant at $z=0$.

Given proper boundary conditions, the divergence damping would show familiar behavior of other wave damping mechanisms (*e.g.*, viscous damping), which suppress the growth of wave amplitudes with increasing altitude. For instance, the ratio $r\_{div}$ would be smaller than 1 away from the surface and decrease with increasing altitude if the lower boundary condition were $p^{'} $= constant at $z=0$ (Fig. 3). With this lower boundary condition, $r\_{div}$ would also decrease with increasing order of Hough modes because of the decreasing equivalent depth (decreasing vertical wavelength). This behavior is again similar to that of viscous damping, which exhibits a stronger damping effect on shorter vertical wavelengths (e.g., Lian and Yelle, [2019]; Vadas and Fritts, [2005]).



Figure 4. The ratios between pressure perturbations with and without divergence damping at the lowest model level ($z≈0$). (a) the ratio $r\_{div}=\frac{δp\left.​\right|\_{α\_{h}=0.1}}{δp\left.​\right|\_{α\_{h}=0}}$ as a function of tide frequency; (b) the ratio $r\_{div}=\frac{δp\left.​\right|\_{α\_{h}>0}}{δp\left.​\right|\_{α\_{h}=0}}$ as a function of dimensionless divergence damping coefficient $α\_{h}$ for the dominant Hough modes in QW4.

Figure 4a shows the effect of the divergence damping ($α\_{h}=0.1$) as a function of tide frequency at the lowest model level. $r\_{div}$ becomes smaller when the frequency of tides increases. For example, $r\_{div}$ for QW4 is more than a factor of three smaller than that for DW1. This behavior is consistent with the MarsWRF and MarsMPAS model results, which show that simulations using the typical divergence damping coefficient $α\_{h}=0.1$ overly suppress the high-frequency oscillations associated with daily pressure variations, essentially making DW1 and SW2 the only recognizable thermal tides in these models (see section 4.2).

The strength of the divergence damping is controlled by the non-dimensional damping coefficient $α\_{h}$ in both WRF and MPAS. Figure 4b shows $r\_{div}$ as a function of $α\_{h}$ at the lowest model level for the QW4 tide (4,4). The QW4 tide experiences very little damping for $α\_{h}$ ranging from $10^{-4}$ to $10^{-3}$, but it shows slight amplification when $α\_{h}\~10^{-2}$ and becomes noticeably damped when $α\_{h}>0.1$.

**4. Effect of Divergence Damping in WRF/MPAS**

The linear wave analysis above shows that divergence damping can impact the amplitude and phase of pressure perturbations associated with thermal tides. Similar impacts exist for wind fields and other physical quantities via polarization relationships derived from the perturbation equations (see Eq. A13-Eq. A16 in Appendix). Here we examine the effect of divergence damping on the diurnal pressure cycles under Martian conditions in WRF/MPAS simulations. Only the daily surface pressure cycles are analyzed because the vertical profiles of thermal tides can be affected by thermal structures, and damped by various mechanisms such as eddy viscosity and other numerical filters such as Smagorinsky viscosity in WRF/MPAS, in addition to divergence damping. These factors make it hard to directly compare the vertical structures between the idealized linear wave model and 3D GCM.

**4.1 Observations**

Multiple Mars landers and spacecraft have detected atmospheric tides via measurements of daily surface pressure variations (e.g., Zurek and Leovy [1981]; Haberle et al., [2014]; Guzewich et al., [2016]; Newman et al., [2018]) and daily atmospheric thermal variations (e.g., Conrath [1975]; Lee et al., [2009]). These variations, with various frequencies, persist throughout the Martian year. Using Curiosity Rover’s Rover Environmental Monitoring Station (REMS) and Insight Lander measurements from Planetary Data System (PDS) data over a Martian year as examples, the near-surface pressure shows wave-like variations over a Martian day, with the largest-amplitude wave being the diurnal tide (Figure 5). Other higher-frequency oscillations are superposed on top of the diurnal tide.



Figure 5. Daily surface pressure perturbations $Δp=p\_{s}-\overline{p\_{s}}$ with 1-sigma error bars as a function of local time from measurements by REMS (a) and by Insight lander (b) during the typical seasons of a Martian year, where $\overline{p\_{s}}$ is the diurnally averaged surface pressure. The 1-sigma error bars are estimated from the standard deviation of surface pressure perturbations over 10 consecutive sols for every solar longitude $L\_{s}$. The pressure perturbations are separated by 40 Pa apart in (a) and 20 Pa apart in (b) for better illustration. The error bars are relatively small, suggesting that these pressure perturbations are representative of diurnal surface pressure variations on Mars.

The detailed properties of these diurnal variations can be obtained via spectral analysis. The surface pressure perturbation can be represented by superposition of wave solutions $p^{'}\left(t\right)=\sum\_{}^{}A\_{σ}cos\left[σt+δ\_{σ}\right]$, where $A\_{σ}$ is the amplitude of a wave with frequency per sol $σ$, $t$ is the local time, and $δ\_{σ}$ is the wave phase [Guzewich et al., 2013]. For simplicity, we ignore the migrating and non-migrating nature of tides and their phases. Performing Fourier transformation (FFT) on the measured pressure perturbation $p^{'}=p-\overline{p}$ (where $\overline{p}$ is the diurnal averaged pressure) over 10 sols (sol 373- sol 383), we obtain the amplitude of pressure perturbation in frequency space. Figure 7 shows the wave amplitude as a function of frequency with unit of $sol^{-1}$. The largest wave amplitude of 12 Pa corresponds to the diurnal tide ($σ=1 sol^{-1}$), followed by several distinct peaks correspond to higher order thermal tides such as the semidiurnal tide ($σ=2 sol^{-1}$), terdiurnal tide ($σ=3 sol^{-1}$) and quadiurnal tide ($σ=4 sol^{-1}$) etc.

**4.2 Model Predictions**

We run the Mars GCM for 10 sols starting from $L\_{s}=120^{∘}$ as a direct comparison to the observations. The choice of this particular $L\_{s}$ is somewhat random but the Insight measurements near this $L\_{s}$ showed relatively larger amplitudes of high-order oscillations with smaller error bars over 10 sols compared to other $L\_{s}$. The simulations are very similar whether performed using MarsWRF or MarsMPAS, but the specific results shown in this paper use MarsMPAS, which includes a uniform horizontal mesh with roughly 240 km spacing (equivalent to roughly 4° MarsWRF spacing) and a 45-layer terrain-following vertical coordinate with the top of the uppermost layer at roughly 120 km. Higher horizontal grid resolution may introduce some topographic effect to the modeled thermal tides but the effect is not large enough to invalidate our comparisons. The suite of available physics parameterizations is standard between MarsWRF and MarsMPAS (Richardson et al., [2007]; Lian and Richardson, [2021]), and we use the K-distribution method (KDM) radiative transfer (RT) scheme, YSU PBL scheme, surface/subsurface scheme, and a simple microphysics model of the CO2 condensation-sublimation cycle. To represent aerosol forcing in the Martian atmosphere, we use a fully interactive two-moment dust scheme that generates aerosol radiative properties employed in the KDM RT scheme [Lee et al., 2018]. Both the cases with and without divergence damping are compared against observations. Since the purpose of the GCM simulations is to illustrate the impact of divergence damping in otherwise identical models, and not to provide and optimal fit to surface pressure observations, the model distribution of aerosol heating has not been specifically tuned to maximize the best fit match, and for simplicity water ice cloud opacity is not treated.



Figure 6. Modeled surface pressure perturbations $Δp=p\_{s}-\overline{p\_{s}}$ as a function of local time at Insight lander site near solar longitude $L\_{s}=120^{∘}$. The black dashed line and solid line show the cases with and without divergence damping (*i.e.*, $α\_{h}=0.1$ and $α\_{h}=0$). The red line shows the Insight measurement (Sol 380) as a reference.

Figure 6 shows the model-predicted daily pressure cycles compared to the Insight measurements. Both the modeled and observed pressure variations show periodic oscillations with comparable magnitude over a sol. Without divergence damping ($α\_{h}=0$), the GCM is able to capture the distinct peaks and valleys in the observed pressure curves during various time of the sol, *i.e.*, the peaks and valleys at midnight, in the early morning and later afternoon/early evening. The exact timing of the predicted pressure perturbation shows some difference with those of the observations, such as the peak near 7am and the valley near 5pm. With the divergence damping ($α\_{h}=0.1$), the modeled pressure curve becomes overly smoothed and only exhibits diurnal variation with a peak near 8am and a valley near 5pm. Moreover, divergence damping reduces the amplitudes and changes the phases of the thermal tides compared to the case without divergence damping.



Figure 7. Similar to Fig. 6 but it shows the amplitude of modeled pressure perturbations as a function of frequency of thermal tides using Fourier transformation (FFT) method. The frequencies $σ=$1, 2, 3, 4 … [sol-1] mean diurnal, semidiurnal, terdiurnal, quadiurnal … tides respectively. The red line shows the FFT of Insight measurement (Sol 373-383) as a reference.

The FFT analysis of the modeled pressure variations suggest that divergence damping can significantly impact the higher-order thermal tides. Figure 7 shows the amplitude-frequency relation for the model-predicted and the observed pressure perturbations. Without divergence damping, the GCM is able to predict almost all dominant modes of the observed thermal tides with comparable amplitudes. However, only diurnal and semidiurnal tides are recognizable once we switch on divergence damping. The amplitudes of higher-order modes become an order of magnitude smaller than those in the case without divergence damping. Additional case studies (not shown in the figure) with various strength of the divergence damping suggest that, a damping coefficient of $α\_{h}∼0.001$ is able to capture most of the high-order thermal tides without impacting numerical stability (particularly during the dusty southern summer on Mars), compared to $α\_{h}=0$.

**5. Conclusions**

We performed wave analysis to show how divergence damping can impact the daily variations of atmospheric thermal tides. A linear wave study suggests that divergence damping can affect the wave amplitudes and phases in the entire atmosphere. The specific impact, *e.g.*, either damping or amplifying the wave amplitude near the surface, depends on the wave modes and the boundary conditions. Consistent with this linear wave analysis, spectral analysis of GCM-predicted diurnal pressure perturbations shows that strong divergence damping can suppress the thermal tides with order higher than the diurnal and semidiurnal tides. Thus, the strength of the divergence damping must remain reasonably low to properly represent the observed pressure cycles in numerical models.

We emphasize that the study presented here is to demonstrate the impact of divergence damping on physical quantities in general. The exact behavior of the divergence damping in a 3D GCM may be complicated. For example, MPAS implements a rigid lid approximation, therefore the reflection of upward propagating waves near the top of the atmosphere is permitted. Wave-absorbing layers (*e.g.*, Rayleigh damping of vertical velocity, horizontal velocities and temperature, see Klemp et al., [2008]) are introduced to suppress the numerical instabilities introduced by this wave reflection, but these layers also introduce complexity to the linear wave model analysis and change the behavior of the solutions. Further, a more rigorous linear wave analysis should include both the eddy viscosity and various other wave damping mechanisms used in the GCM.

Other mechanisms affecting our linear wave analysis include the excitation sources of the atmospheric tides. We assume that all solar radiation is absorbed by the ground, which in turn exchanges heat with the atmosphere via diffusive mixing. This approach is overly simplified because the radiative heating and cooling of the Martian atmosphere can greatly impact the thermal structures of the atmosphere, *e.g.*, dust aerosols, water ice and the major component of the atmosphere $CO\_{2}$ are all radiatively active in both solar and infrared (IR) wavelengths. These atmospheric sources of tidal excitations, like various wave damping mechanisms, need to be considered in the linear wave model in order to establish a better comparison to Mars GCMs.

Numerical diffusion in WRF and MPAS is well-designed to suppress numerical noise and instabilities for climate simulations in general. However, the parameters controlling the numerical diffusion (such as divergence damping, off-centering in the vertically implicit time step, external mode filter and other viscous dissipation) need to be assessed thoroughly for specific planetary atmosphere applications. For instance, prior work on Titan showed that excessive horizontal diffusion reduced the magnitude of the stratospheric superrotation on Titan [Newman et al., 2011]. Likewise, we speculate that the default divergence damping coefficient excessively damps short-period thermal tides on Mars. Thankfully, the damping is cleanly “broken out” in the code and its effects are readily tested. While implicit damping is also unavoidable in the numerical solvers of differential equations due to truncation errors (Lauritzen et al., [2011]), this damping does not seem to have a deleterious effect. We recommend that other Mars GCMs, especially if they are unable to match the full spectrum of waves in Martian pressure data, should also be examined in terms of detailed numerical damping and dissipation mechanisms in their dynamical cores.

*Acknowledgement*.

This work is supported by NASA Solar System Works (SSW) grant NNH18ZDA001N-SSW. The submission of manuscript has no conflict of interest with this SSW grant.

*Data Availability Statement.*

The Mars Insight lander and REMS pressure data used in this study is publicly available from PDS nodes:

<https://atmos.nmsu.edu/data_and_services/atmospheres_data/INSIGHT/insight.html>

[https://atmos.nmsu.edu/data\_and\_services/atmospheres\_data/MARS/curiosity/rems.html](https://atmos.nmsu.edu/data_and_services/atmospheres_data/INSIGHT/insight.html)

The official WRF and MPAS GCMs are available via https://github.com/wrf-model/WRF and https://github.com/MPAS-Dev/MPAS-Model. The codes solving LTE/VSE and performing FFT analysis of both observed and simulated Martian thermal tides in this study can be obtained from Aeolis Research public repository at https://github.com/AeolisResearch/divergence\_damping. The Mars version of WRF or MPAS GCMs and other codes used in this study are available from the corresponding author, Yuan Lian, upon reasonable request.

APPENDIX

**1. Thermal Tide Equations**

We derive the set of wave equations that describe thermal tides in a motionless, isothermal, hydrostatic and compressible atmosphere using linear wave theory. The derivation is similar to that in Chapman and Lindzen [1970] except that divergence damping terms are applied to the horizontal momentum equations. The governing equations of hydrostatic flow in spherical coordinates are:

$$\begin{array}{c}\frac{Dρ}{Dt}+ρχ=0\#\left(A1\right)\end{array}$$

$$\begin{array}{c}\frac{Du}{Dt}-fv=-\frac{1}{ρ}\frac{1}{acosθ}\frac{∂p}{∂λ}+α\_{d}\frac{1}{acosθ}\frac{∂χ}{∂λ}\#\left(A2\right)\end{array}$$

$$\begin{array}{c}\frac{Dv}{Dt}+fu=-\frac{1}{ρ}\frac{∂p}{a∂θ}+α\_{d}\frac{1}{a}\frac{∂χ}{∂θ}\#\left(A3\right)\end{array}$$

$$\begin{array}{c}\frac{∂p}{∂z}=-ρg\#\left(A4\right)\end{array}$$

$$\begin{array}{c}\frac{R}{γ-1}\frac{DT}{Dt}=\frac{gH}{ρ}\frac{Dρ}{Dt}+J\#\left(A5\right)\end{array}$$

In the above equations, $ρ$ is the density, $u$, $v$, $w$ are the zonal, meridional, and vertical velocities respectively, $p$ is the pressure, $a$ is the radius of the planet, $g$ is the gravity, $θ$ and $λ$ are the latitude and longitude in radians respectively, $f=2Ωsinθ$ is the Coriolis parameter, $R$ is the gas constant, $γ=\frac{c\_{p}}{c\_{v}}$ is the ratio between specific heat at constant pressure ($c\_{p}$) and constant volume ($c\_{v}$), $H$ is the pressure scale height (same as density scale height in an isothermal atmosphere), $α\_{d}$ is the divergence damping coefficient, formulated in MPAS as $α\_{d}=2α\_{h}\frac{L\_{d}^{2}}{Δt}$, where $α\_{h}$ is a dimensionless coefficient (typically 0.1 in WRF and MPAS), $L\_{d}$ is the dissipation length scale, which is typically the smallest distance between adjacent grid cells ($L\_{d}≈220km$ for $4^{∘}$ grid resolution), $Δt$ is the Runge-Kutta split time step (typically three sub-timesteps for MarsMPAS, therefore $Δt = 40s$ for model time step of 120*s*), $J$ is the source of thermal tides (which will be explained later). The total derivative $\frac{D}{Dt}=\frac{∂}{∂t}+\vec{v}⋅∇$. The divergence damping terms with damping coefficients $α\_{d}$ are applied to Eq. A2 and A3 in a way similar to that in Skamarock and Klemp [1992]. The equation of state for an ideal atmosphere is $p=ρRT$, and the divergence

$$\begin{array}{c}χ=\frac{1}{acosθ}\frac{∂u}{∂λ}+\frac{1}{acosθ}\frac{∂vcosθ}{∂θ}+\frac{∂w}{∂z}\#\left(A6\right)\end{array}$$

Defining an arbitrary dynamics quantity $X$ as a sum of mean and perturbation parts as $X=X\_{∘}+X^{'}$, recognizing $u\_{∘}=0$, $v\_{∘}=0$ t, $w\_{∘}=0$ and $\frac{∂p\_{∘}}{∂z}=-ρ\_{∘}g$ for the background atmosphere and ignoring $O^{2}\left(X^{'}\right)$, we can obtain the set of perturbation equations as:

$$\begin{array}{c}\frac{∂ρ^{'}}{∂t}+w^{'}\frac{∂ρ\_{∘}}{∂z}=-ρ\_{∘}χ^{'}\#\left(A7\right)\end{array}$$

$$\begin{array}{c}\frac{∂u^{'}}{∂t}-fv^{'}=-\frac{1}{ρ\_{∘}}\frac{1}{acosθ}\frac{∂p^{'}}{∂λ}+α\_{d}\frac{1}{acosθ}\frac{∂χ^{'}}{∂λ}\#\left(A8\right)\end{array}$$

$$\begin{array}{c}\frac{∂v^{'}}{∂t}+fu^{'}=-\frac{1}{ρ\_{∘}}\frac{∂p^{'}}{a∂θ}+α\_{d}\frac{∂χ^{'}}{a∂θ}\#\left(A9\right)\end{array}$$

$$\begin{array}{c}\frac{∂p^{'}}{∂z}=-ρ^{'}g\#\left(A10\right)\end{array}$$

$$\begin{array}{c}\frac{R}{γ-1}\left(\frac{∂T^{'}}{∂t}+w^{'}\frac{∂T\_{∘}}{∂z}\right)=\frac{gH}{ρ\_{∘}}\left(\frac{∂ρ^{'}}{∂t}+w^{'}\frac{∂ρ\_{∘}}{∂z}\right)+J\#\left(A11\right)\end{array}$$

$$\begin{array}{c}\frac{p^{'}}{p\_{∘}}=\frac{ρ^{'}}{ρ\_{∘}}+\frac{T^{'}}{T\_{∘}}\#\left(A12\right)\end{array}$$

The perturbations can be expressed in wave form $A^{'}=\overline{A}\left(θ,z\right)e^{i\left(ωt+sλ\right)}$, where $\overline{A}$ is the wave amplitude, $ω$ is the wave frequency that depicts phase directions by positive value (westward moving) or negative value (eastward moving), and $s$ is the zonal wavenumber. A few useful properties of the wave perturbations are:

$$\begin{array}{c}\frac{∂A^{'}}{∂t}\&=iωA'\\\frac{∂A'}{∂λ}\&=isA'\end{array}$$

Equations A8 and A9 can be used to eliminate $u^{'}$ and $v^{'}$. Similarly, Eq. A11 and Eq. A12 can be used to eliminate $T^{'}$. After some mathematical manipulations, we obtain a set of wave equations in terms of $w^{'}$, $ρ^{'}$, $P^{'}=\frac{p^{'}}{ρ\_{∘}}$ and $χ^{'}$:

$$\begin{array}{c}χ^{'}=-iω\frac{ρ^{'}}{ρ\_{∘}}+\frac{w^{'}}{H}\#\left(A13\right)\end{array}$$

$$\begin{array}{c}-iωP^{'}+w^{'}g=γgHχ^{'}-\left(γ-1\right)J\#\left(A14\right)\end{array}$$

$$\begin{array}{c}\frac{∂P^{'}}{∂z}-\frac{P^{'}}{H}=-\frac{ρ^{'}}{ρ\_{∘}}g\#\left(A15\right)\end{array}$$

$$\begin{array}{c}χ^{'}-\frac{∂w^{'}}{∂z}=\frac{iω}{4a^{2}Ω^{2}}\left[F\left(P^{'}\right)-α\_{d}F\left(χ'\right)\right]\#\left(A16\right)\end{array}$$

In Eq. A16, the linear differential operator $F$ is defined as:

$$\begin{array}{c}F=\frac{∂}{∂μ}\left(\frac{1-μ^{2}}{ν^{2}-μ^{2}}\frac{∂}{∂μ}\right)-\frac{1}{ν^{2}-μ^{2}}\left[\left(\frac{s}{ν}\right)\frac{ν^{2}+μ^{2}}{ν^{2}-μ^{2}}+\frac{s^{2}}{1-μ^{2}}\right]\#\left(A17\right)\end{array}$$

Where $ν=\frac{ω}{2Ω}$ is the scaled wave frequency, and $μ=sinθ$. Equations A13 – A16 can be further reduced to a single equation by eliminating $w^{'}$ and $ρ^{'}$:

$$\begin{array}{c}\begin{array}{c}-\frac{γH}{1-γ}\frac{∂^{2}P^{'}}{∂z^{2}}+\frac{γ}{1-γ}\frac{∂P^{'}}{∂z}+\frac{i}{ω}\left(\frac{∂J}{∂z}-\frac{J}{H}\right)\&=\frac{g}{4a^{2}Ω^{2}}\left[F\left(P^{'}\right)-α\_{d}F\left(χ^{'}\right)\right]\\\&=\frac{g}{4a^{2}Ω^{2}}F\left(P^{'}-α\_{d}χ^{'}\right)\end{array}\#\left(A18\right)\end{array}$$

Equation A18 can be converted to expressions similar to the classical tidal equations using the separation of variables technique (*e.g.*, Chapman and Lindzen [1970]). We define $P^{'}$, $χ^{'}$ and $J$ as sum of all possible solutions (denoted by subscription $n$):

$$\begin{array}{c}P^{'}\&=\sum\_{n}^{}L\_{n}\left(z\right)Θ\_{n}\left(θ\right)\\χ^{'}\&=\sum\_{n}^{}M\_{n}\left(z\right)Θ\_{n}\left(θ\right)\\J\&=\sum\_{n}^{}G\_{n}\left(z\right)Θ\_{n}\left(θ\right)\end{array}$$

Note that the term $e^{i\left(ωt+sλ\right)}$ is implied in $L\_{n}$, $M\_{n}$, and $G\_{n}$ since it does not affect the solutions to Eq. A18. Applying separation of variables to Eq. A18, we have:

$$\begin{array}{c}\frac{1}{L\_{n}-α\_{d}M\_{n}}\left[ηH\frac{∂^{2}L\_{n}}{∂z^{2}}-η\frac{∂L\_{n}}{∂z}-\frac{i}{ω}\left(\frac{∂G\_{n}}{∂z}-\frac{G\_{n}}{H}\right)\right]=-\frac{g}{4a^{2}Ω^{2}}\frac{F\left(Θ\_{n}\right)}{Θ\_{n}}\#\left(A19\right)\end{array}$$

Where $η=\frac{γ}{1-γ}$. Equation A19 states that the left-hand side (LHS) and right-hand side (RHS) of the equation are functions of $z$ and $θ$ respectively. Therefore, both sides must be equal to a constant, *e.g.*, $\frac{1}{h\_{n}}$, where $h\_{n}$ is called the *equivalent depth*. This leads to two equations:

$$\begin{array}{c}F\left(Θ\_{n}\right)=-\frac{4a^{2}Ω^{2}}{gh\_{n}}Θ\_{n}\#\left(A20\right)\end{array}$$

$$\begin{array}{c}ηH\frac{∂^{2}L\_{n}}{∂z^{2}}-η\frac{∂L\_{n}}{∂z}-\frac{i}{ω}\left(\frac{∂G\_{n}}{∂z}-\frac{G\_{n}}{H}\right)=\frac{L\_{n}-α\_{d}M\_{n}}{h\_{h}}\#\left(A21\right)\end{array}$$

In the case when $α\_{d}=0$, Eq. A20 and Eq. A21 simply become the classical Laplace’s Tidal Equation and the vertical structure equation seen in many literatures.

In order to solve Eq. A21, $M\_{n}$ must be eliminated. This can be achieved by reducing Eq. A13, Eq. A14 and Eq. A15 to:

$$\begin{array}{c}χ'=i\frac{ω}{g}\frac{1}{1-γ}\frac{∂P'}{∂z}+\frac{J}{gH}\#\left(A22\right)\end{array}$$

Applying separation of variables again, Eq. A22 becomes:

$$M\_{n}Θ\_{n}=i\frac{ω}{g}\frac{1}{1-γ}\frac{∂L\_{n}}{∂z}Θ\_{n}+\frac{G\_{n}Θ\_{n}}{gH}$$

Now we have:

$$\begin{array}{c}M\_{n}=i\frac{ω}{g}\frac{1}{1-γ}\frac{∂L\_{n}}{∂z}+\frac{G\_{n}}{gH}\#\left(A23\right)\end{array}$$

Substituting $M\_{n}$ to Eq. A21, we can obtain the vertical structure equation as:

$$\begin{array}{c}H\frac{∂^{2}L\_{n}}{∂z^{2}}-\left(1-i\frac{α\_{d}ω}{γgh\_{n}}\right)\frac{∂L\_{n}}{∂z}-\frac{L\_{n}}{ηh\_{n}}=\frac{i}{ηω}\frac{∂G\_{n}}{∂z}-\left(\frac{i}{ω}+\frac{α\_{d}}{gh\_{n}}\right)\frac{G\_{n}}{ηH}\#\left(A24\right)\end{array}$$

The Laplace’s Tidal Equation Eq. A20 and the vertical structure equation Eq. A24 can be solved numerically with properly defined boundary conditions. We follow the same procedure described in Wang et al. [2016] to solve Eq. A20 using the normalized associated Legendre polynomials. The solutions to Eq. A20 provide a set of eigenvalues $ζ\_{n}=\frac{4a^{2}Ω^{2}}{gh\_{n}}$ and eigenfunctions $Θ\_{n}$ for each pair of normalized wave frequency $ν$ and zonal wavenumber $s$, which define the modes of thermal tides such as diurnal tide ($ν=0.5$, $s=1)$, semidiurnal tide ($ν=1$, $s=2$ ) and terdiurnal tide ($ν=1.5$, $s=3$). The equivalent depth thus can be obtained as:

$\begin{array}{c}h\_{n}=\frac{4a^{2}Ω^{2}}{gζ\_{n}}\#\left(A25\right)\end{array}$

Subsequently, the vertical structure equation Eq. A24 can be solved using the finite difference method using the equivalent depth in Eq. A25.

Defining:

$$y\_{n}=L\_{n}\sqrt{ρ\_{∘}}$$

$$ϵ\_{n}=G\_{n}\sqrt{ρ\_{∘}}$$

Eq. A24 can be further rewritten to an alternative form:

$$\begin{array}{c}\frac{∂^{2}y\_{n}}{∂z^{2}}+i\frac{α\_{d}ω}{γgh\_{n}H}\frac{∂y\_{n}}{∂z}+\left(i\frac{α\_{d}ω}{γgh\_{n}}\frac{1}{2H^{2}}-\frac{1}{4H^{2}}-\frac{1}{ηh\_{n}H}\right)y\_{n}=\frac{i}{ηωH}\left(\frac{∂ϵ\_{n}}{∂z}+\frac{ϵ\_{n}}{2H}\right)-\left(\frac{i}{ω}+\frac{α\_{d}}{gh\_{n}}\right)\frac{ϵ\_{n}}{ηH^{2}}\#\left(A26\right)\end{array}$$

In the case where $α\_{d}=0$ and $ϵ\_{n}=0$, Eq. A26 describes undamped waves without external sources.

**2. Boundary Conditions**

Both Eq. A20 and Eq. A24 are second order linear differential equations. Solving these equations requires two boundary conditions. For the Laplace’s Tidal Equation Eq. A20, two boundary conditions are

$$Θ\_{n}=0,   θ=-\frac{π}{2}$$

$$Θ\_{n}=0,   θ=    \frac{π}{2}$$

For the vertical structure equation Eq. A24, we require that the vertical velocity at the surface is zero and there is no downward propagating wave at the upper boundary. At the lower boundary

$$w^{'}=0,   z=0$$

Again, using Eq. A13, Eq. A14 and Eq. A15, we can establish the polarization relation between $w^{'}$ and $P^{'}$

$$\begin{array}{c}w'=iωη\frac{H}{g}\frac{∂P'}{∂z}+i\frac{ω}{g}P'+\frac{J}{g}\#\left(A27\right)\end{array}$$

Applying $w^{'}=0$ at $z=0$ and the separation of variables, we obtain the lower boundary condition in terms of $L\_{n}$

$$\begin{array}{c}\frac{∂L\_{n}}{∂z}+\frac{L\_{n}}{ηH}-\frac{i}{ωηH}G\_{n}=0   at  z=0\#(A28)\end{array}$$

Again, Eq. A28 can be expressed by $y\_{n}$ and $ϵ\_{n}$ as

$$\begin{array}{c}\frac{∂y\_{n}}{∂z}+\frac{1}{H}\left(\frac{1}{2}+\frac{1}{η}\right)y\_{n}=i\frac{ϵ\_{n}}{ωηH}  at  z=0\#\left(A29\right)\end{array}$$

Equation A29 can be solved analytically using variation of constants.

The determination of the upper boundary condition relies on the excitation mechanism of the thermal tides. For simplicity, we assume that the atmosphere is radiatively transparent and the surface absorbs all solar insolation. The thermal tides are therefore primarily excited by the diffusive heat exchange between the surface and the atmosphere in the planetary boundary layer (PBL) [Chapman and Lindzen, 1970]

$$G\_{n}=iωc\_{p}ΔT\_{s}e^{-k\_{d}z}e^{i\left(ωt+sλ\right)}$$

Where $ΔT\_{s}$ is the surface temperature anomaly at the subsolar point, $k\_{d}$ is the decaying factor that is correlated to eddy diffusivity $κ\_{e}$ in the PBL as $k\_{d}=\sqrt{\frac{ω}{κ\_{e}}}e^{i\frac{λ}{4}}$. It can be seen that $G\_{n}$ decays exponentially when altitude increases. At altitude high enough, $G\_{n}≈0$, and Eq. A26 becomes

$$\begin{array}{c}\frac{∂^{2}y\_{n}}{∂z^{2}}+i\frac{α\_{d}ω}{γgh\_{n}H}\frac{∂y\_{n}}{∂z}+\left(i\frac{α\_{d}ω}{γgh\_{n}}\frac{1}{2H^{2}}-\frac{1}{4H^{2}}-\frac{1}{ηh\_{n}H}\right)y\_{n}=0  at  z=z\_{top}\#\left(A30\right)\end{array}$$

Assuming solution to Eq. A30 is in form of

$$y\_{n}=Ae^{ik\_{z}z}$$

Where $A$ is a constant and $k\_{z}$ is the vertical wavenumber, we can solve Eq. A30 analytically using the second-order polynomial

$$\begin{array}{c}k\_{z}^{2}+c\_{1}k\_{z}+c\_{2}=0\#\left(A31\right)\end{array}$$

where $c\_{1}=\frac{α\_{d}ω}{γgh\_{n}H}$ and $c\_{2}=\frac{1}{4H^{2}}+\frac{1}{ηh\_{n}H}-i\frac{α\_{d}ω}{γgh\_{n}}\frac{1}{2H^{2}}$.

The solution to Eq. A31 is

$$\begin{array}{c}k\_{z}=\frac{-c\_{1}\pm \sqrt{c\_{1}^{2}-4c\_{2}}}{2}\#\left(A32\right)\end{array}$$

Two roots in Eq. A31 represents a pair of upward (excited by surface source, $k\_{z}^{u}$) and downward (reflected, $k\_{z}^{d}$) propagating waves

$$\begin{array}{c}k\_{z}^{u}=\frac{-\frac{α\_{d}ω}{γgh\_{n}H}+\sqrt{\left(\frac{α\_{d}ω}{γgh\_{n}H}\right)^{2}-\frac{1}{H^{2}}-\frac{4}{ηh\_{n}H}+i\frac{α\_{d}ω}{γgh\_{n}}\frac{2}{H^{2}}}}{2}\#\left(A33\right)\end{array}$$

$$\begin{array}{c}k\_{z}^{d}=\frac{-\frac{α\_{d}ω}{γgh\_{n}H}-\sqrt{\left(\frac{α\_{d}ω}{γgh\_{n}H}\right)^{2}-\frac{1}{H^{2}}-\frac{4}{ηh\_{n}H}+i\frac{α\_{d}ω}{γgh\_{n}}\frac{2}{H^{2}}}}{2}\#\left(A34\right)\end{array}$$

The real and imaginary parts of $k\_{z}$ represent the vertical wavenumber and damping rate respectively

$$\begin{array}{c}real\left(k\_{z}\right)=-\frac{1}{2}\left[\frac{α\_{d}ω}{γgh\_{n}H}\pm cos\left(\frac{ϕ}{2}\right)\left(a^{2}+b^{2}\right)^{\frac{1}{4}}\right]\#\left(A35\right)\end{array}$$

$$\begin{array}{c}imag\left(k\_{z}\right)=\mp \frac{1}{2}sin\left(\frac{ϕ}{2}\right)\left(a^{2}+b^{2}\right)^{\frac{1}{4}}\#\left(A36\right)\end{array}$$

Where $ϕ=arcsin\left(b/\sqrt{a^{2}+b^{2}}\right)$, $a=\left(\frac{α\_{d}ω}{γgh\_{n}H}\right)^{2}-\frac{1}{H^{2}}-\frac{4}{ηh\_{n}H}$ and $b=\frac{2α\_{d}ω}{γgh\_{n}H^{2}}$. For positive $a$ and *b*, $ϕ=\frac{π}{4}$ if $a=b$, $0\leq ϕ<\frac{π}{4}$ if $a>b$ and $\frac{π}{2}\geq ϕ>\frac{π}{4}$ if $a<b$. Using the typical values for $α\_{h}=0.1$, $h\_{n}∼516m, ω=7.27×10^{-5}s^{-1}$, $L\_{d}$=220 km, $Δt $=40 s, $g=3.727ms^{-2}$, $H∼15$km, $\left|η\right|=3.5$ and $γ=1.4$, the dominant terms within the square root in $k\_{z}$ for diurnal tide mode (1,1) can be estimated as $\left(\frac{α\_{d}ω}{γgh\_{n}H}\right)^{2}\~\frac{1.47}{h\_{n}H}$, $\left|\frac{4}{ηh\_{n}H}\right|∼\frac{1.14}{h\_{n}H}$ and $i\frac{α\_{d}ω}{γgh\_{n}}\frac{2}{H^{2}}∼i\frac{0.45}{h\_{n}H}$. These terms are comparable and equally important in determining the vertical wavenumber $k\_{z}$.

In the case where divergence damping is absent, the solution for the vertical wavenumber simply becomes

$$k\_{z}=\frac{\pm \sqrt{-\frac{1}{H^{2}}-\frac{4}{ηh\_{n}H}}}{2}$$

In order to obtain meaningful wave solutions, $-\frac{1}{H^{2}}-\frac{4}{ηh\_{n}H}>0$ is required. For typical value of $γ≡\frac{c\_{p}}{c\_{v}}=1.4$ ($η=-3.5$), this means $\frac{4}{3.5h\_{n}}>\frac{1}{H}$. This is easily achievable since the typical equivalent depth $h\_{n}<H$. For undamped waves, the wave energy flux (*e.g.*, the heat flux $ρ\_{∘}\left.<w^{'}T^{'}\right.>$, where $\left.<w^{'}T^{'}\right.>=w^{'}T^{'\*}$ and $T^{'\*}$ is the conjugate of $T^{'}$) in the region where $G\_{n}≈0$ remains constant.

The radiation boundary condition requires that there is no downward propagating wave energy (or alternatively the solution needs to be bounded at $z=\infty $, *i.e.*, $imag\left(k\_{z}\right)>0$), therefore the solution to Eq. A30 becomes

$$y\_{n}=Ae^{ik\_{z}^{u}z}$$

Differentiating $y\_{n}$ with respect to $z$, we have

$$\begin{array}{c}\frac{∂y\_{n}}{∂z}-ik\_{z}^{u}y\_{n}=0   at   z=z\_{top}\#\left(A37\right)\end{array}$$

Equations A29 and A37 are the lower and upper boundary conditions that can be used to solve Eq. A26.

**3. Impact of divergence damping definitions**

Klemp et al., [2018] suggested that a more appropriate divergence damping term should be used when damping the acoustic wave modes

$$χ\_{D}^{'}=\frac{1}{\overline{ρ}\overline{\tilde{θ}}}∇⋅\overline{ρ}\overline{\tilde{θ}}\vec{v}^{'}=χ^{'}-\frac{g}{c\_{s}^{2}}w^{'}$$

where $\tilde{θ}$ is the potential temperature. This revised definition leads to a slight modification to Eq. A18

$$\begin{array}{c}\begin{array}{c}-\frac{γH}{1-γ}\frac{∂^{2}P^{'}}{∂z^{2}}+\frac{γ}{1-γ}\frac{∂P^{'}}{∂z}+\frac{i}{ω}\left(\frac{∂J}{∂z}-\frac{J}{H}\right)\&=\frac{g}{4a^{2}Ω^{2}}\left[F\left(P^{'}\right)-α\_{d}F\left(χ\_{D}^{'}\right)\right]\\\&=\frac{g}{4a^{2}Ω^{2}}F\left(P^{'}-α\_{d}χ\_{D}^{'}\right)\end{array}\#\left(A38\right)\end{array}$$

The detailed derivation is not shown here but it can be trivially done by replacing $χ'$ with $χ\_{D}^{'}$ in Eq. A8 and A9.

Replacing divergence damping $χ'$ to $χ\_{D}^{'}$ changes the vertical structure equation. Using $χ\_{D}^{'}$, Eq. A23, A24 and A26 become

$$\begin{array}{c}M\_{n}=- i\frac{ω}{γgH}L\_{n}-\frac{G\_{n}}{ηgH}\#\left(A39\right)\end{array}$$

$$\begin{array}{c}H\frac{∂^{2}L\_{n}}{∂z^{2}}-\frac{∂L\_{n}}{∂z}-\left(1+i\frac{α\_{d}ω}{γgh\_{n}}\right)\frac{L\_{n}}{ηh\_{n}}=\frac{i}{ηω}\frac{∂G\_{n}}{∂z}-\left(\frac{i}{ω}-\frac{α\_{d}}{ηgh\_{n}}\right)\frac{G\_{n}}{ηH}\#\left(A40\right)\end{array}$$

$$\begin{array}{c}\frac{∂^{2}y\_{n}}{∂z^{2}}-\left(i\frac{α\_{d}ω}{γgh\_{n}}\frac{1}{ηH^{2}}+\frac{1}{4H^{2}}+\frac{1}{ηh\_{n}H}\right)y\_{n}=\frac{i}{ηωH}\left(\frac{∂ϵ\_{n}}{∂z}+\frac{ϵ\_{n}}{2H}\right)-\left(\frac{i}{ω}-\frac{α\_{d}}{ηgh\_{n}}\right)\frac{ϵ\_{n}}{ηH^{2}}\#\left(A41\right)\end{array}$$

In order to evaluate the impact of the revised divergence damping term on the vertical wave structure, we perform order of magnitude analysis by ignoring the source term $ϵ\_{n}$. The dispersion relation for Eq. A41 is given by

$$\begin{array}{c}k\_{z}^{2}=-\frac{1}{4H^{2}}-\frac{1}{ηh\_{n}H}-i\frac{α\_{d}ω}{γgh\_{n}}\frac{1}{ηH^{2}}\#\left(A42\right)\end{array}$$

Using the typical values for $h\_{n}∼516m, ω=7.27×10^{-5}s^{-1}$, $L\_{d}$=220 km, $Δt $=40 s, $g=3.727ms^{-2}$, $H∼15$km, $\left|η\right|=3.5$ and $γ=1.4$ again, we calculate damping rates (*i.e.*, the imaginary part of $k\_{z}$ in Eq. A31 and Eq. A42) for a range of $α\_{h}$. Figure A1 shows the ratio of damping rates between the case with the original definition of divergence damping $χ' $and the case with the revised definition of divergence damping $χ\_{D}^{'}$. The latter has a weaker damping effect on the atmospheric tides than the former when the divergence damping coefficient $α\_{h}<0.6$.



Figure A1. The ratio of divergence damping rates on typical diurnal tide between the case with the original definition of divergence damping $χ' $and the case with the revised definition of divergence damping $χ\_{D}^{'}$.

The dispersion relations in Eq. A31 and Eq. A42 also provide a rule of thumb on the choices of the divergence damping coefficient $α\_{h}$. Using Eq. A42 as an example, the dispersion relation can be rewritten as

$$\begin{array}{c}k\_{z}^{2}=-\frac{1}{4H^{2}}-\frac{1}{ηh\_{n}H}(1+i\frac{α\_{d}ω}{γgH})\#\left(A43\right)\end{array}$$

The effect of divergence damping can be ignored if $\frac{α\_{d}ω}{γgH}\ll 1$. Recall that $α\_{d}=2α\_{h}\frac{L\_{d}^{2}}{Δt}$, this means

$$α\_{h}\ll \frac{1}{2}\frac{Δt}{L\_{d}^{2}}\frac{γgH}{nΩ}$$

Using the typical values above, we have $α\_{h}\ll \frac{0.44}{n}$, where $n=1,2,3 …$ is the tide mode such as diurnal, semidiurnal and terdiurnal tides. It is apparent that $α\_{h}$ needs to be smaller for higher order tides to avoid excessive damping in MarsWRF and MarsMPAS.

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