

Sidelobe Reduction and Mode-Purity Enhancement of Vortex Beams from a Programmable, Rectangular Phased Array Antenna

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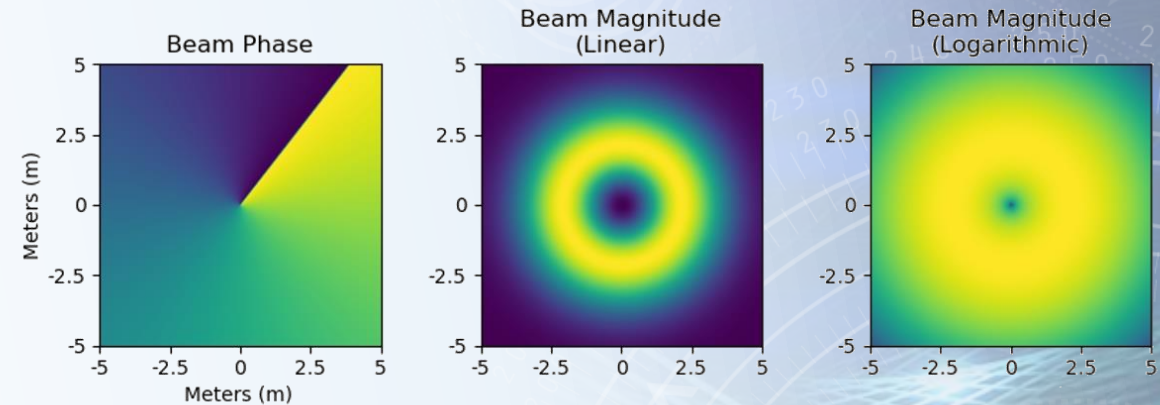
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What is Orbital Angular Momentum (OAM)?

A beam with OAM is also known as a vortex beam.

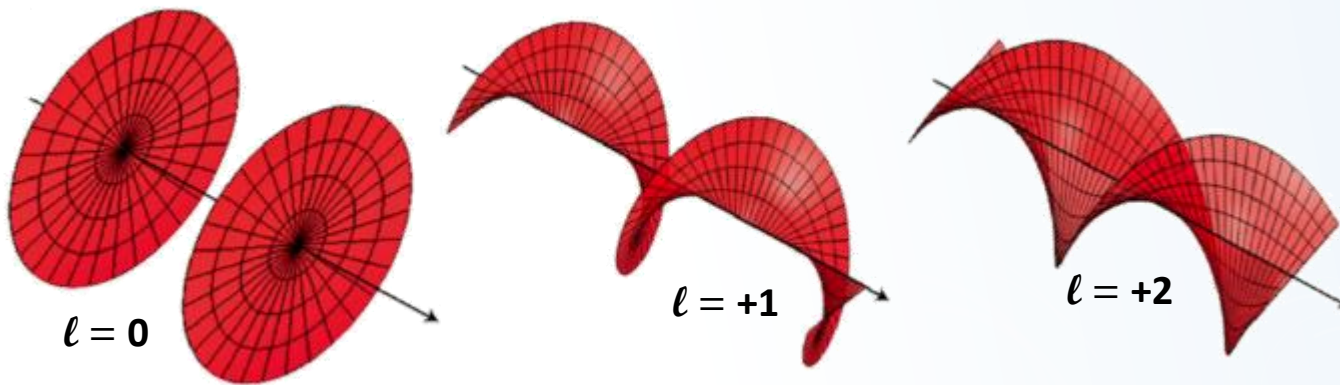
In a plane perpendicular to the beam axis, the phases of the electric and magnetic fields have $\ell\phi$ dependence where ℓ is an integer and ϕ is the azimuthal angle.

OAM L=1 Mode at 1m
26.5GHz 3m Beam Waist



There is a central intensity null.

For non-zero ℓ values the phase front is helical instead of planar.

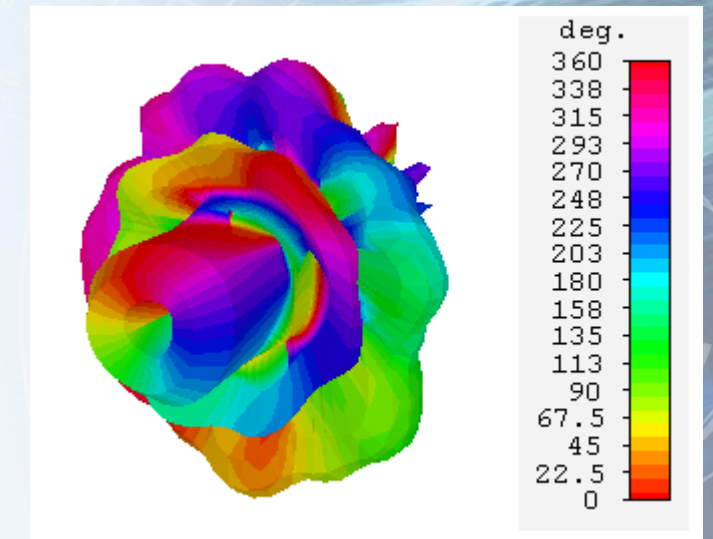
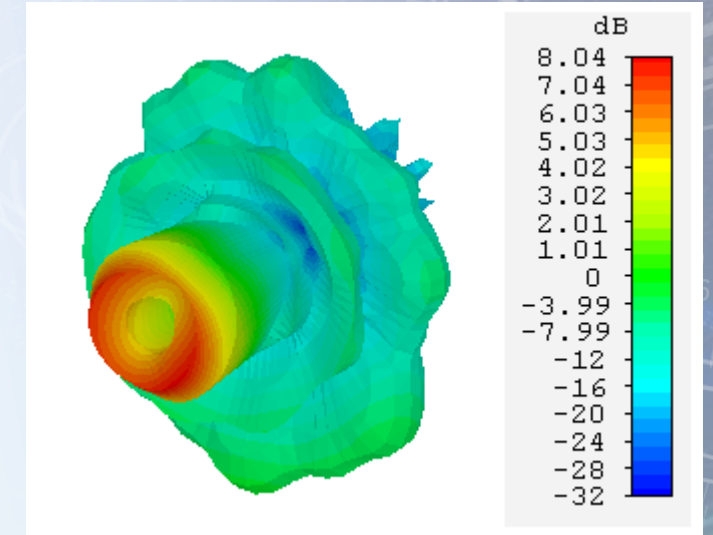
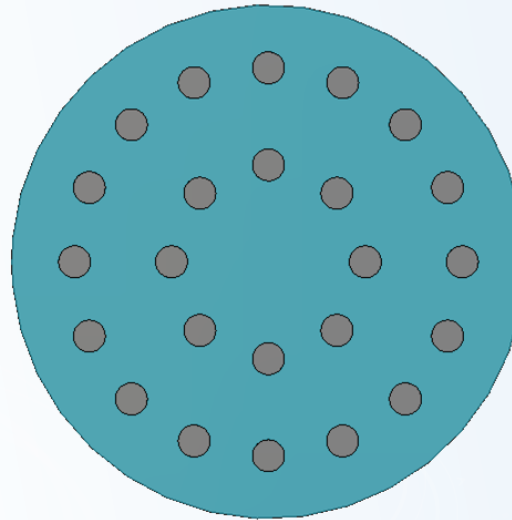
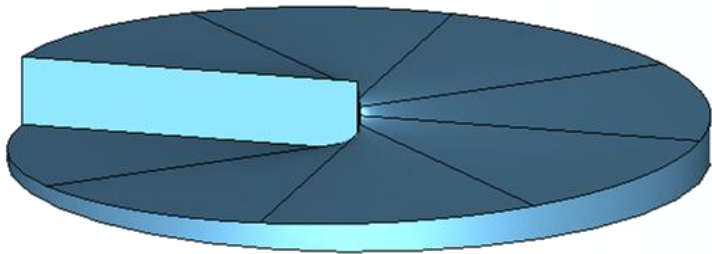


Why Study OAM?

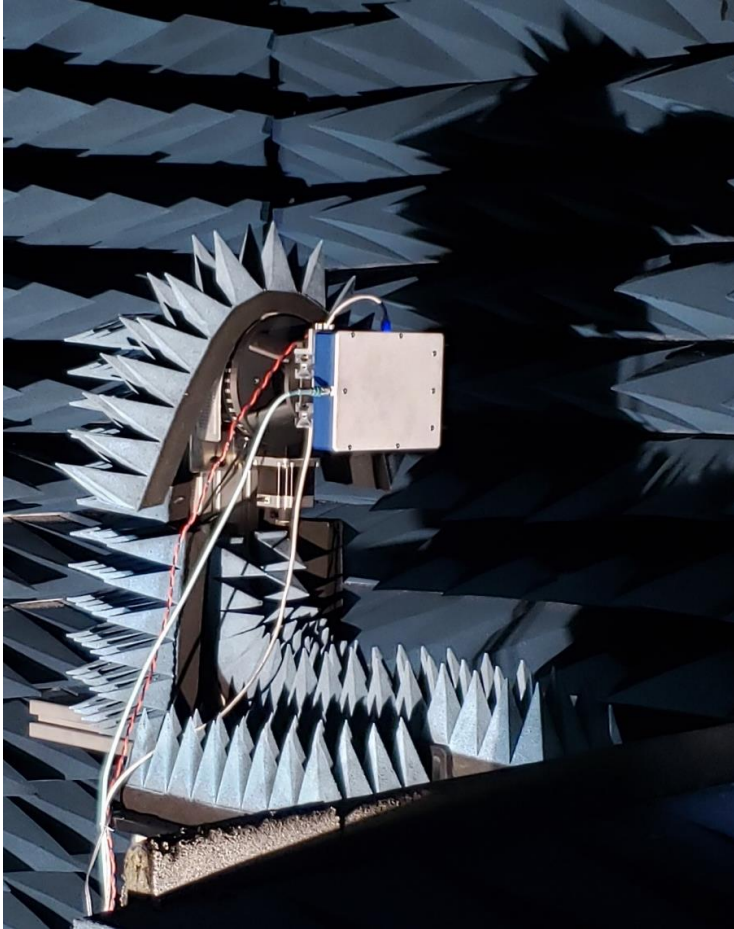
Increase data rate.

Multiple orthogonal modes can be transmitted at the same frequency.

Research in RF continues, with meta surfaces and uniform circular arrays (UCAs).



OAM with Rectangular Arrays

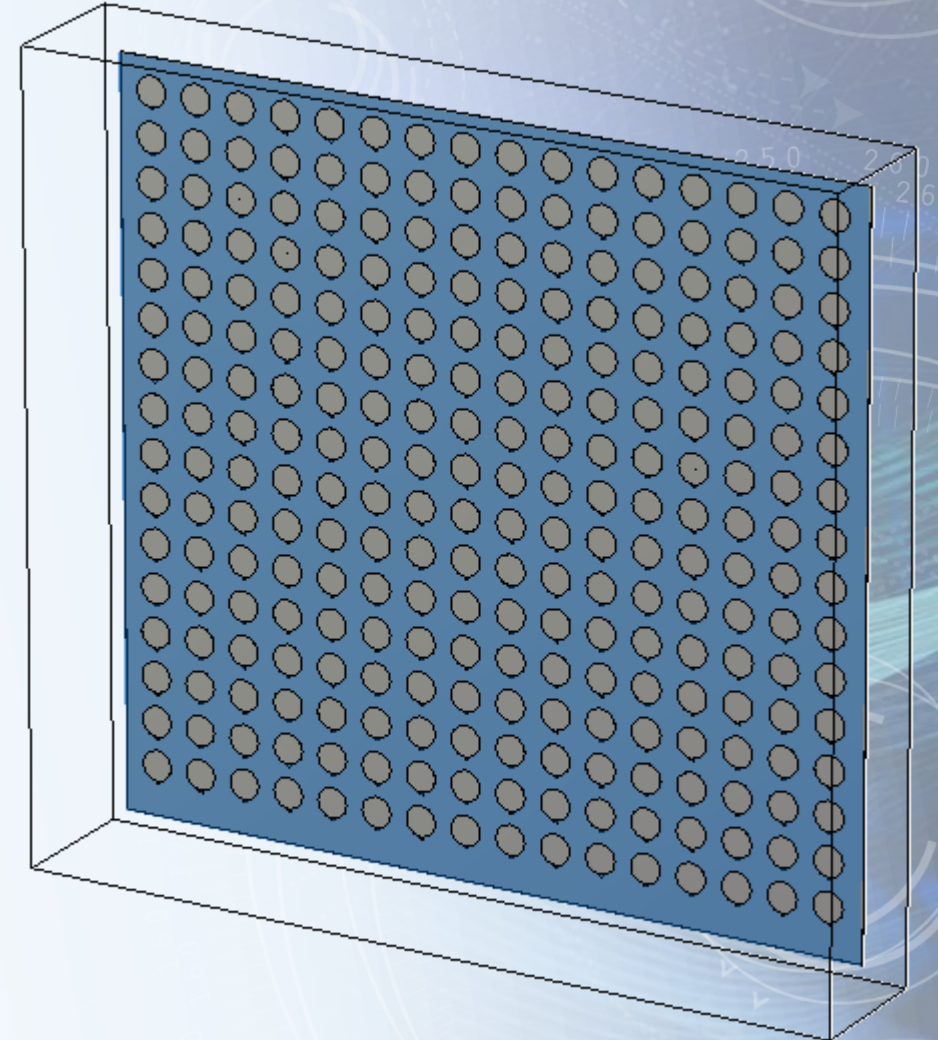


Rectangular arrays are widely available

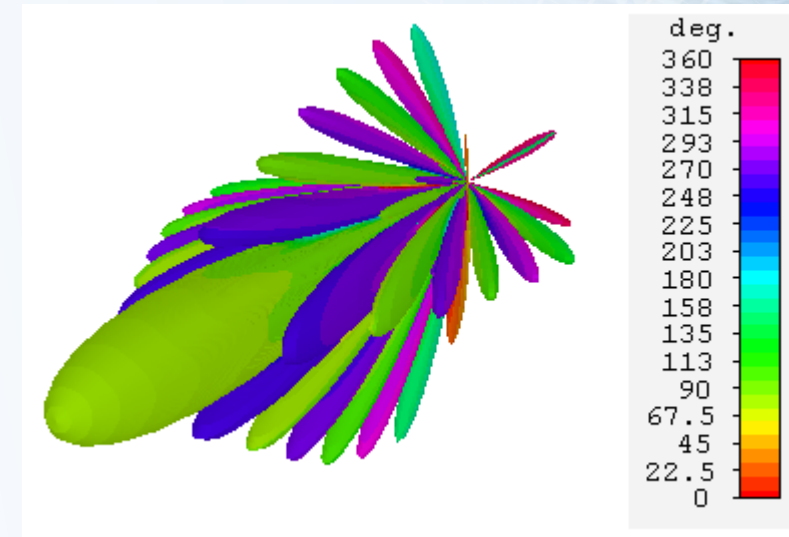
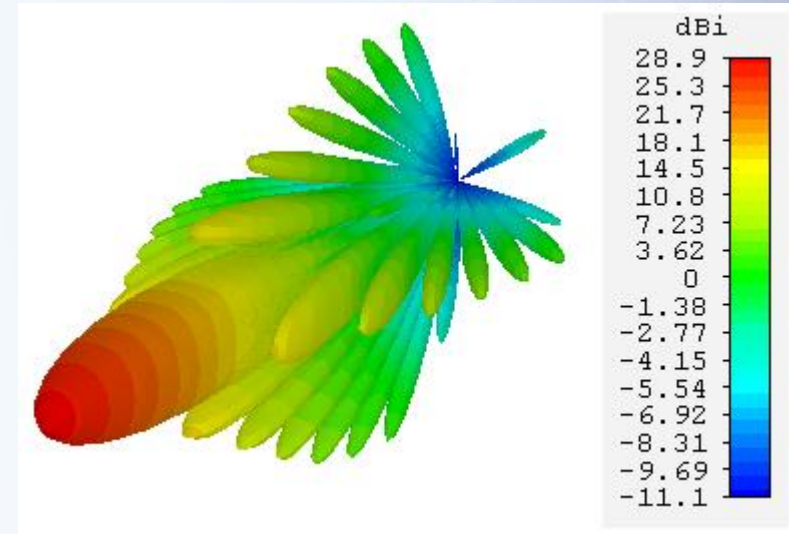
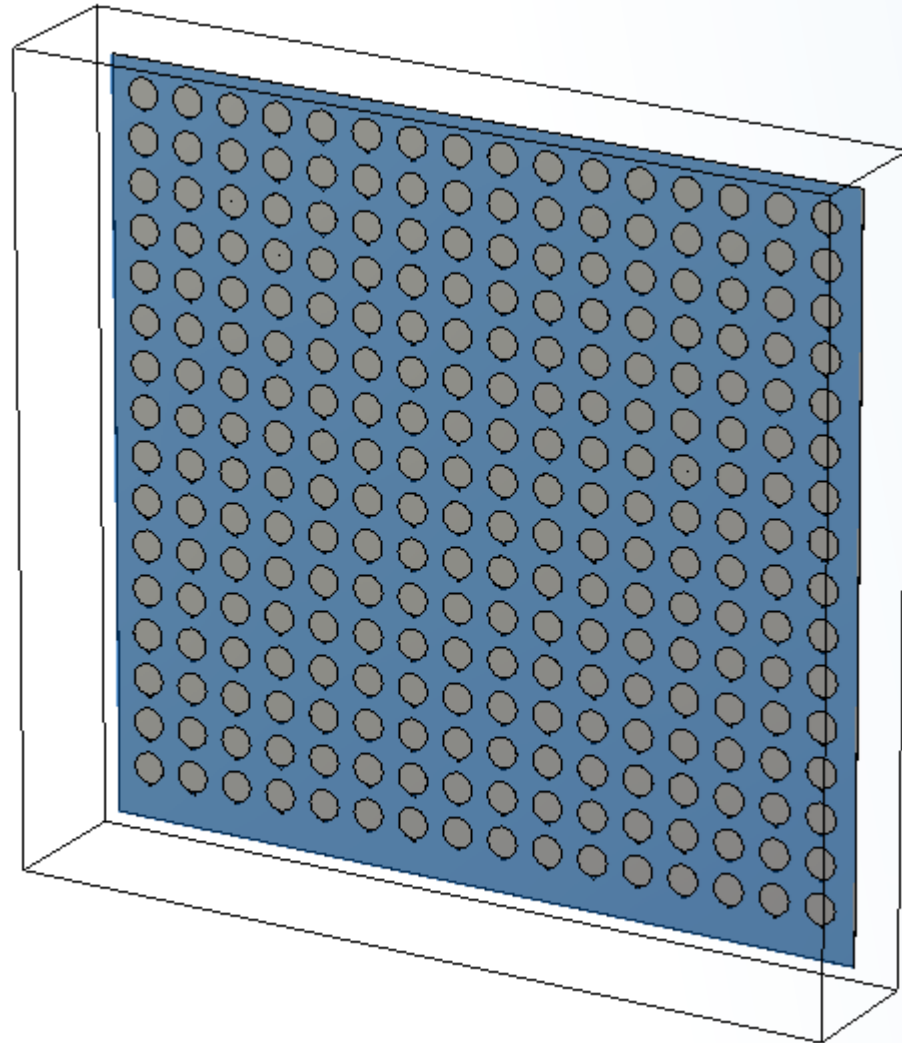
Extreme Waves

Ka-band

16x16 rectangular array



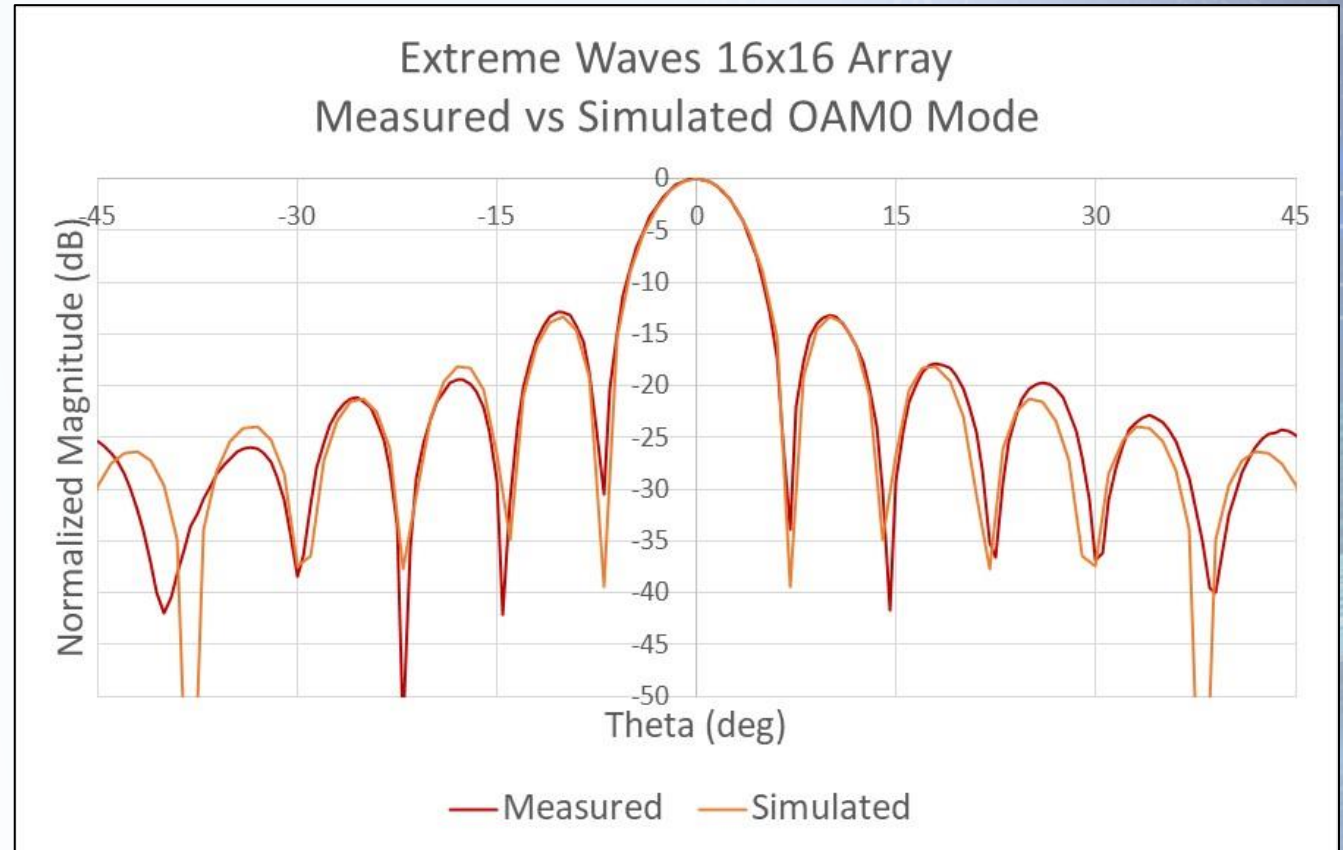
Rectangular Array Simulation $\ell = 0$



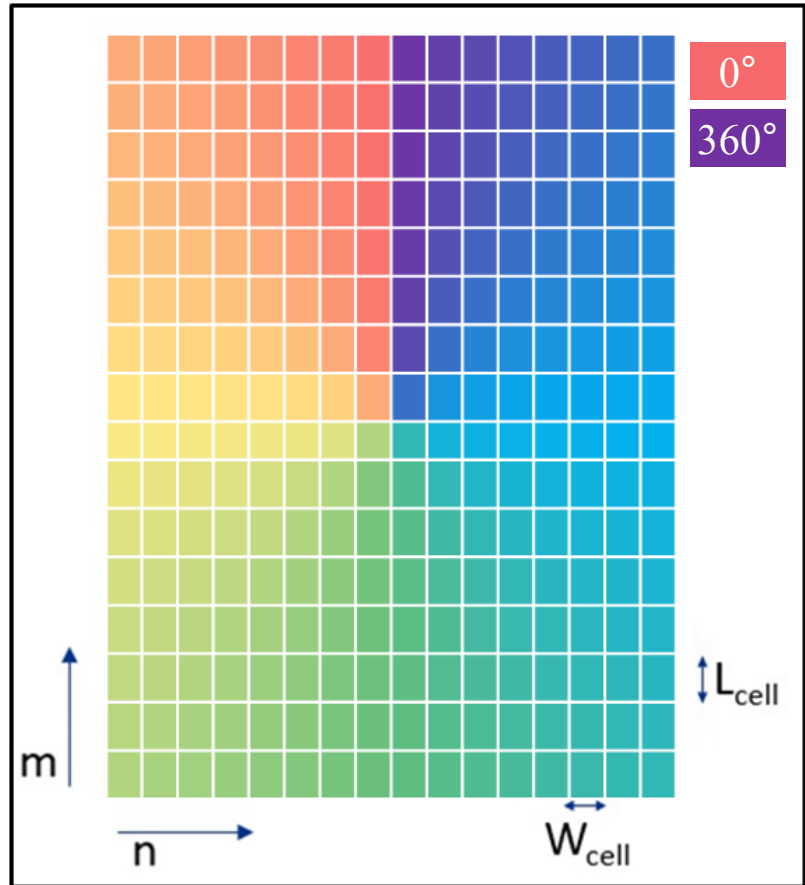
Rectangular Array Measurements $\ell = 0$

Measured array in Far-field range

Elements calibrated to have ~ same magnitude and phase



Rectangular Array Simulations



Phase mapping of each phased array element for an OAM $\ell = 1$ pattern.

$$\Phi_{\ell}[\phi] = -\ell \phi$$

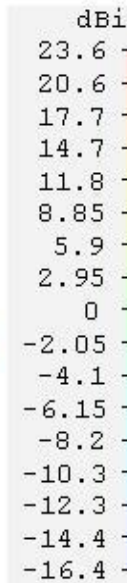
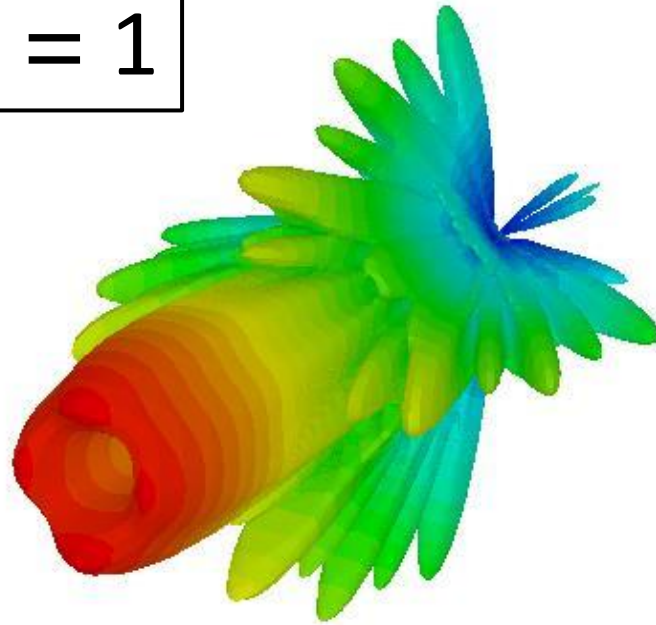
$$\varphi_{m,n}^{(1)} = \text{atan2} \left[\left(\frac{M+1}{2} - m \right) \cdot L_{\text{cell}}, \left(\frac{N+1}{2} - n \right) \cdot W_{\text{cell}} \right]$$

Rectangular Array Simulations

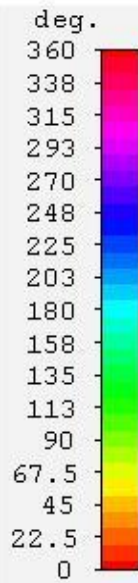
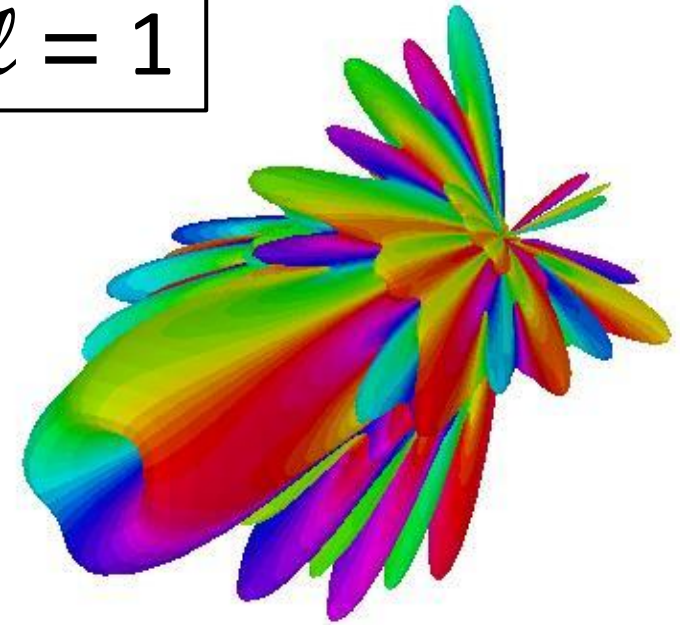
Far-field
magnitude and
phase patterns
produced in CST
Microwave Studio.

All 256 elements
have uniform
amplitude.

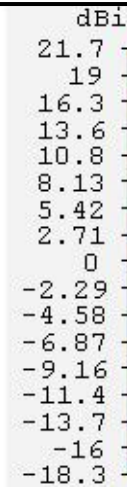
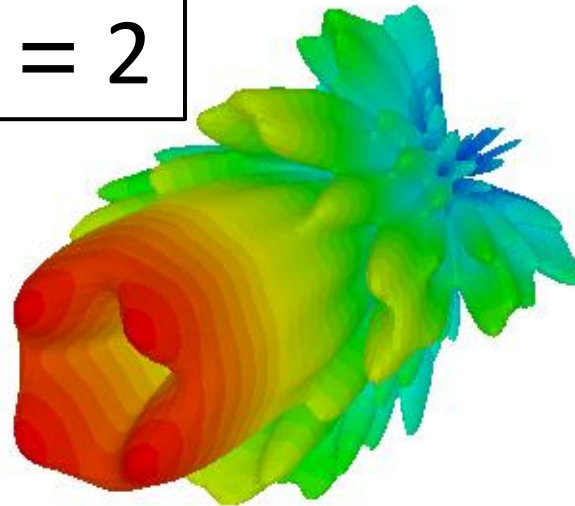
$\ell = 1$



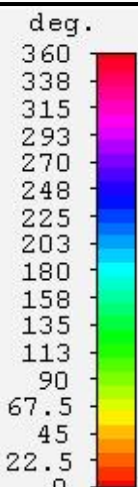
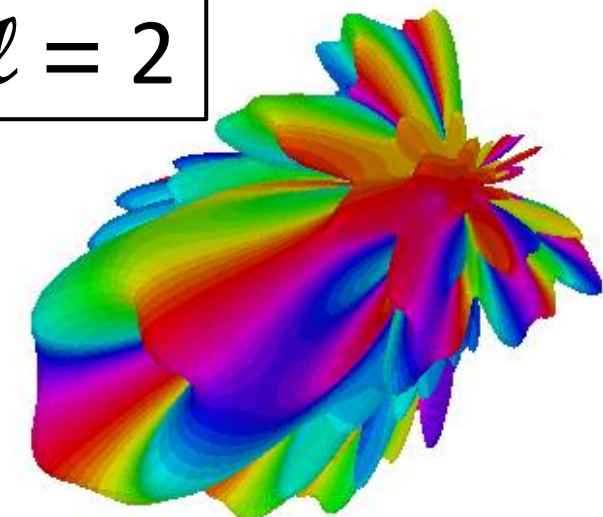
$\ell = 1$



$\ell = 2$



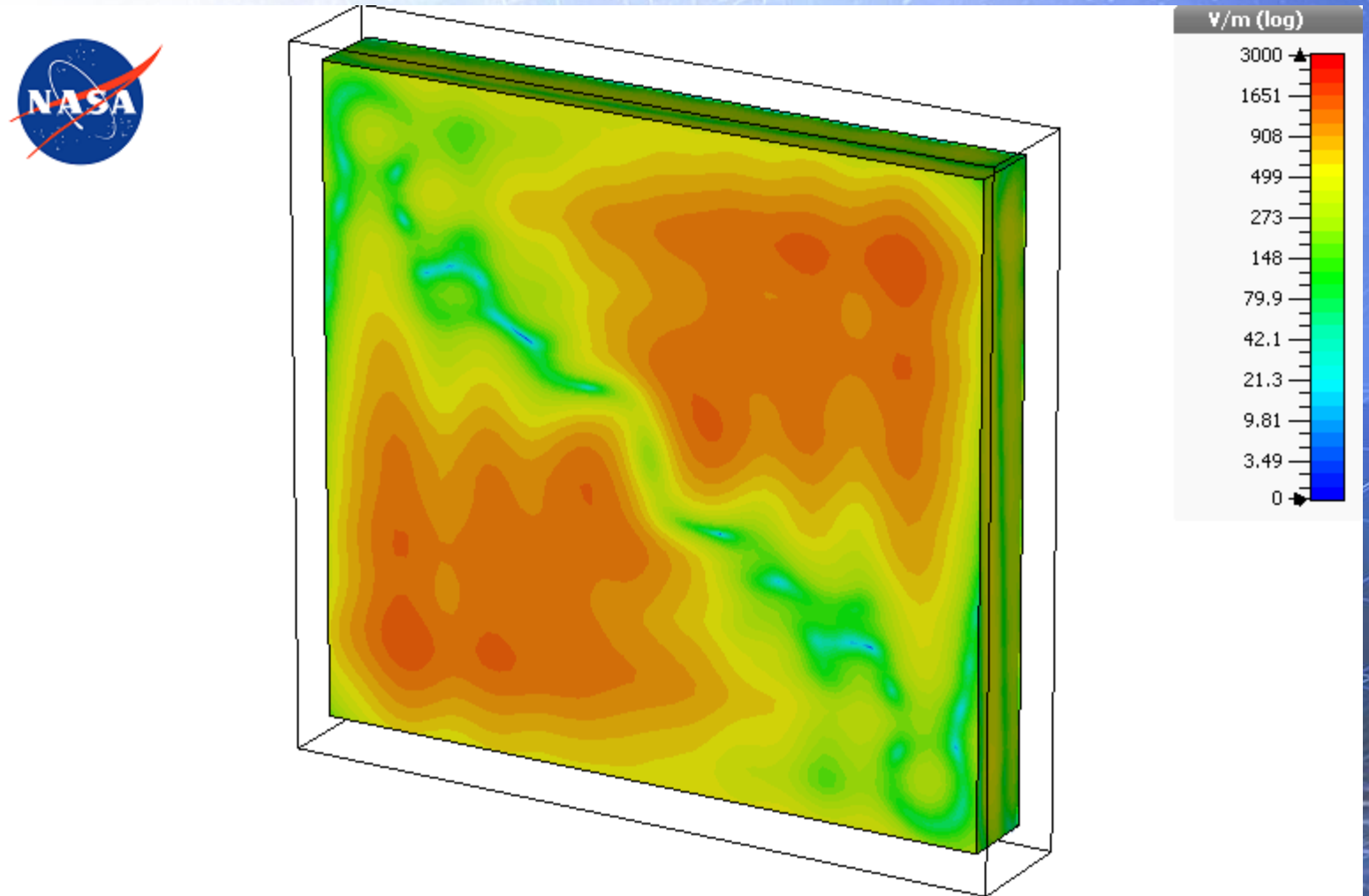
$\ell = 2$



Rectangular Array Simulations

Absolute value of
Electric Field
produced in CST
Microwave Studio.

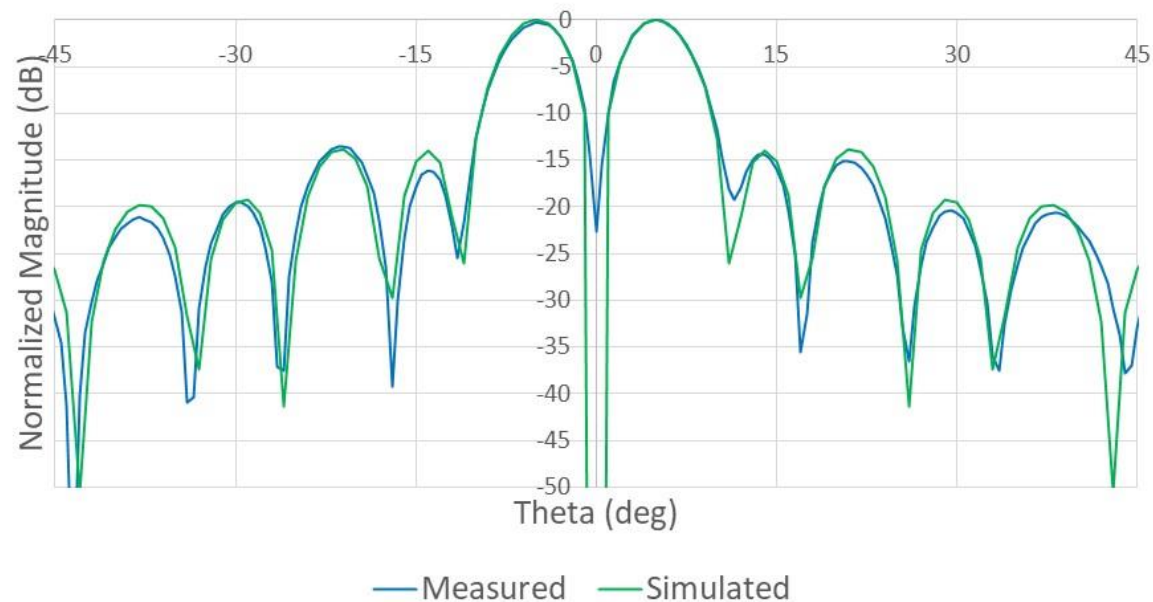
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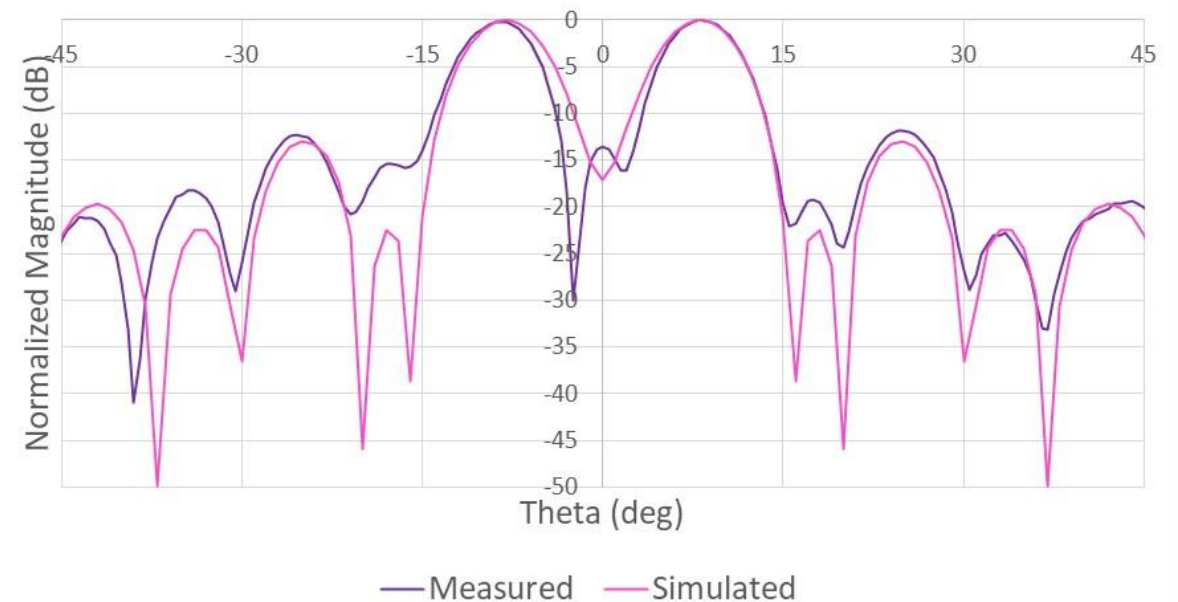
Rectangular Array Measurements

Measured array with OAM phases
applied for modes $\ell=1$ and $\ell=2$

Extreme Waves 16x16 Array
Measured vs Simulated OAM1 Mode



Extreme Waves 16x16 Array
Measured vs Simulated OAM2 Mode



Construction of LG Taper on Rectangular Array

Amplitude taper to approximate the profile of a focused Laguerre-Gaussian beam

- $f_{p,\ell}[\rho] = A_{p,\ell} \frac{\rho^{|\ell|}}{w_0} L_n^{|\ell|}[\rho^2] e^{-\frac{\rho^2}{2}}$
- $A_{p,\ell} = \sqrt{\frac{2}{\pi} \cdot \frac{p!}{(p+|\ell|)!}}, \quad \rho \equiv \sqrt{2} r/w_0$

ρ = scaled radius
 p = radial mode index
 ℓ = azimuthal mode index

w_0 = beam waist at focus
 $L_{nk}[x]$ = generalized Laguerre polynomial
 $A_{p,l}$ = normalization amplitude

Assume maximum programmable attenuation level of array ($\alpha_{max} \approx 17$ dB) and numerically solve for the ideal beamwaist, using:

- $\alpha_{max}^{(p,\ell)} = -20 \log_{10} \left| \frac{f_{p,\ell}[\rho_{bnd}]}{f_{p,\ell}[\rho_{pk}]} \right| \text{ dB}$
- $\rho_{bnd} = (M - 1) L_{cell} / (\sqrt{2} w_0).$

ρ_{bnd} = scaled array boundary
 ρ_{pk} = scaled radial position of beam intensity peak
 M = number of elements

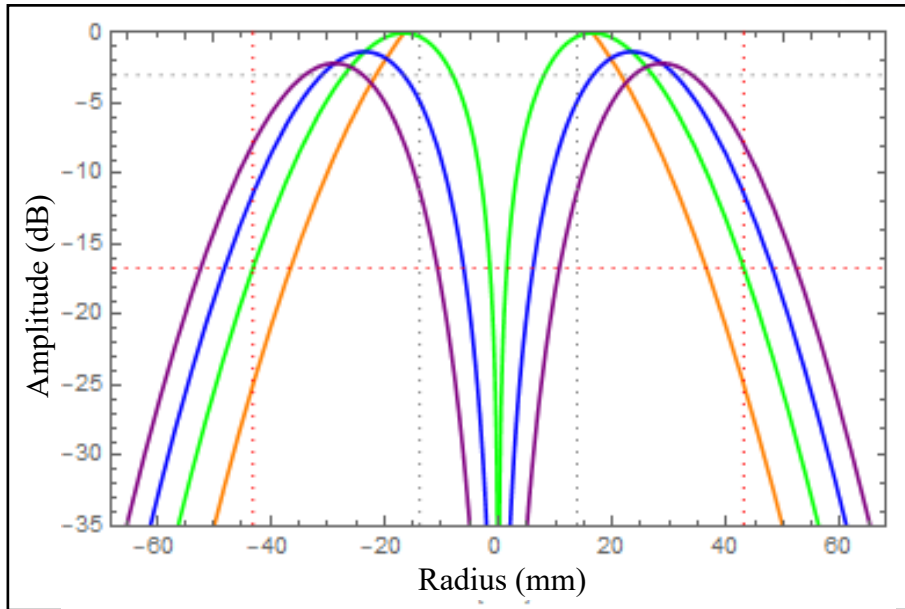
Determine attenuation for mean of relative field magnitude at each radiating element.

- $b_{m,n}^{(p,\ell)} = \frac{1}{L_{cell} W_{cell}} \int_{y_{min}[m]}^{y_{max}[m]} \int_{x_{min}[n]}^{x_{max}[n]} \left| \frac{f_{p,\ell}[\sqrt{x^2+y^2}]}{f_{p,\ell}[\rho_{pk}]} \right|^2 dx dy$

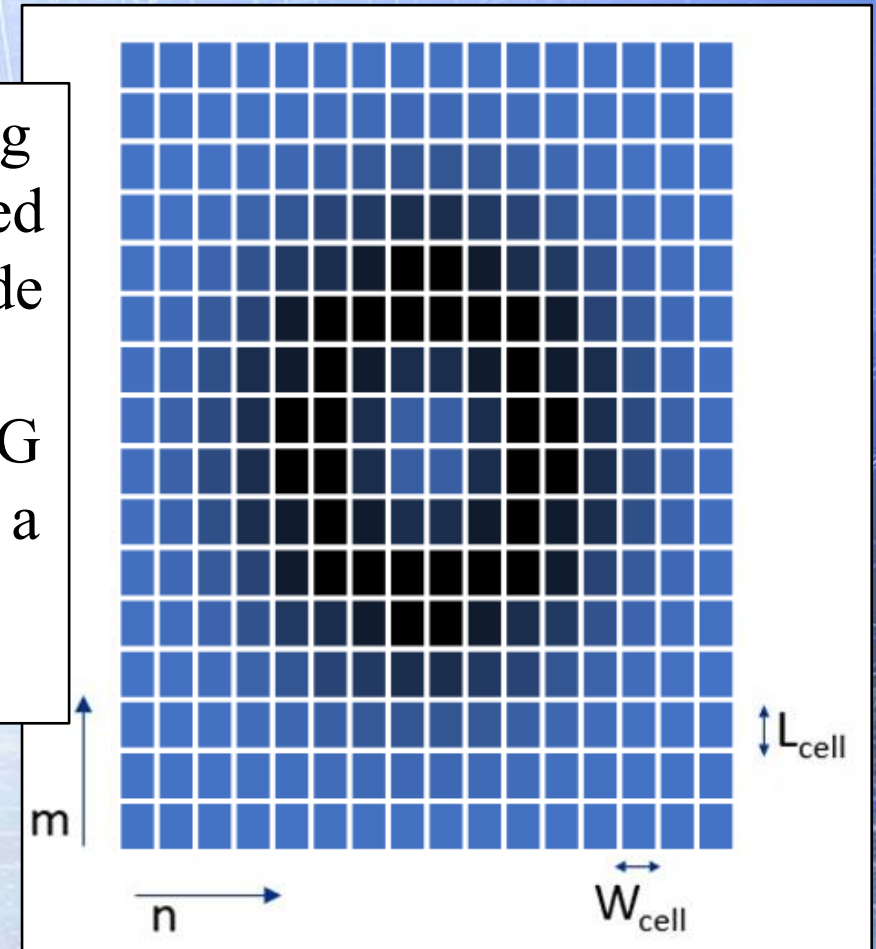
Rectangular Array with Tapered Amplitude

Amplitude taper to approximate the profile of a focused Laguerre-Gaussian beam

Example tapering profiles for various LG modes.



Mapping of tapered amplitudes for a $\ell = 1$ LG beam to a 16x16 array

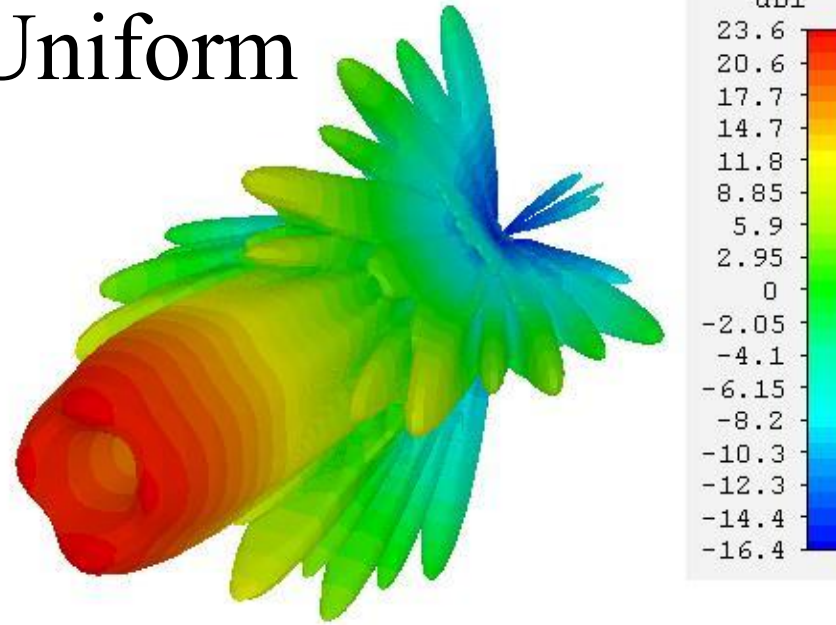


Amplitude Tapered Array Simulations

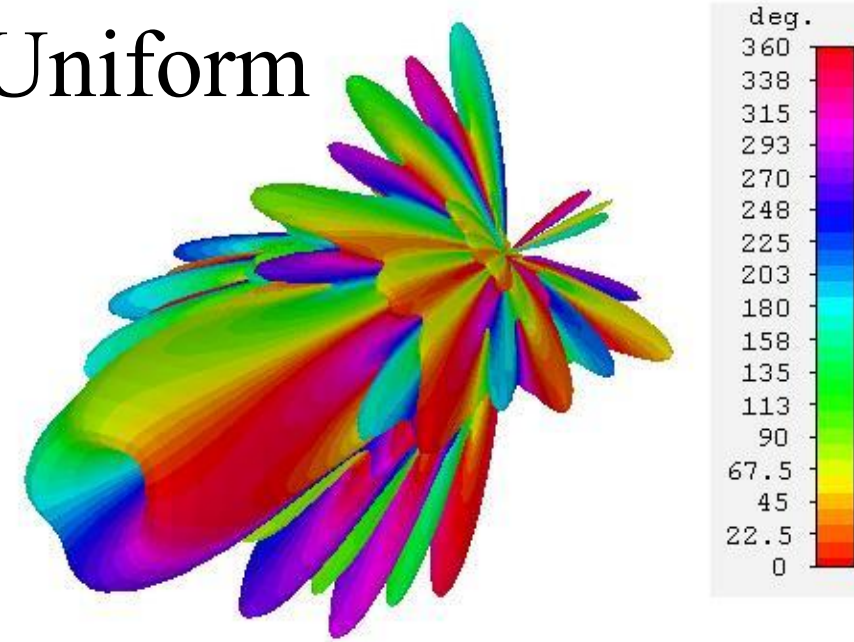
Far-field
magnitude and
phase patterns
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The amplitude
taper drastically
reduces sidelobes,
shown here for
the $\ell = +1$ case.

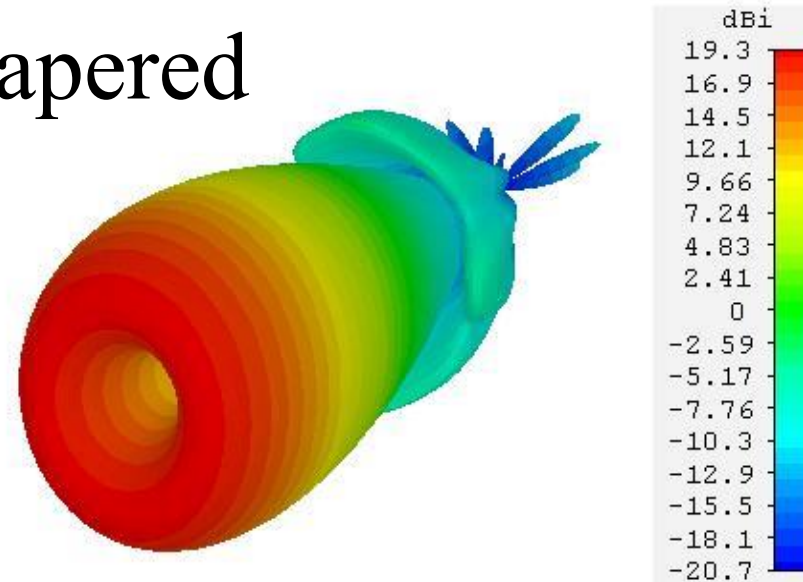
Uniform



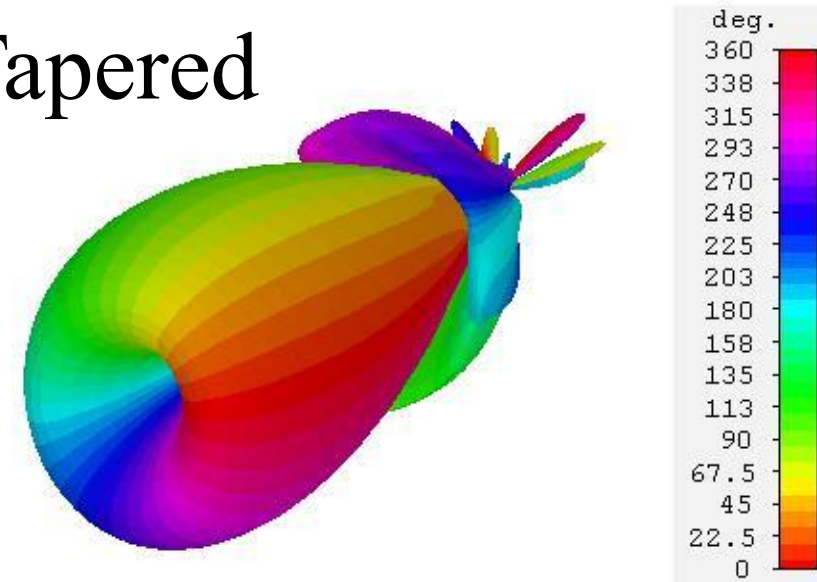
Uniform



Tapered



Tapered



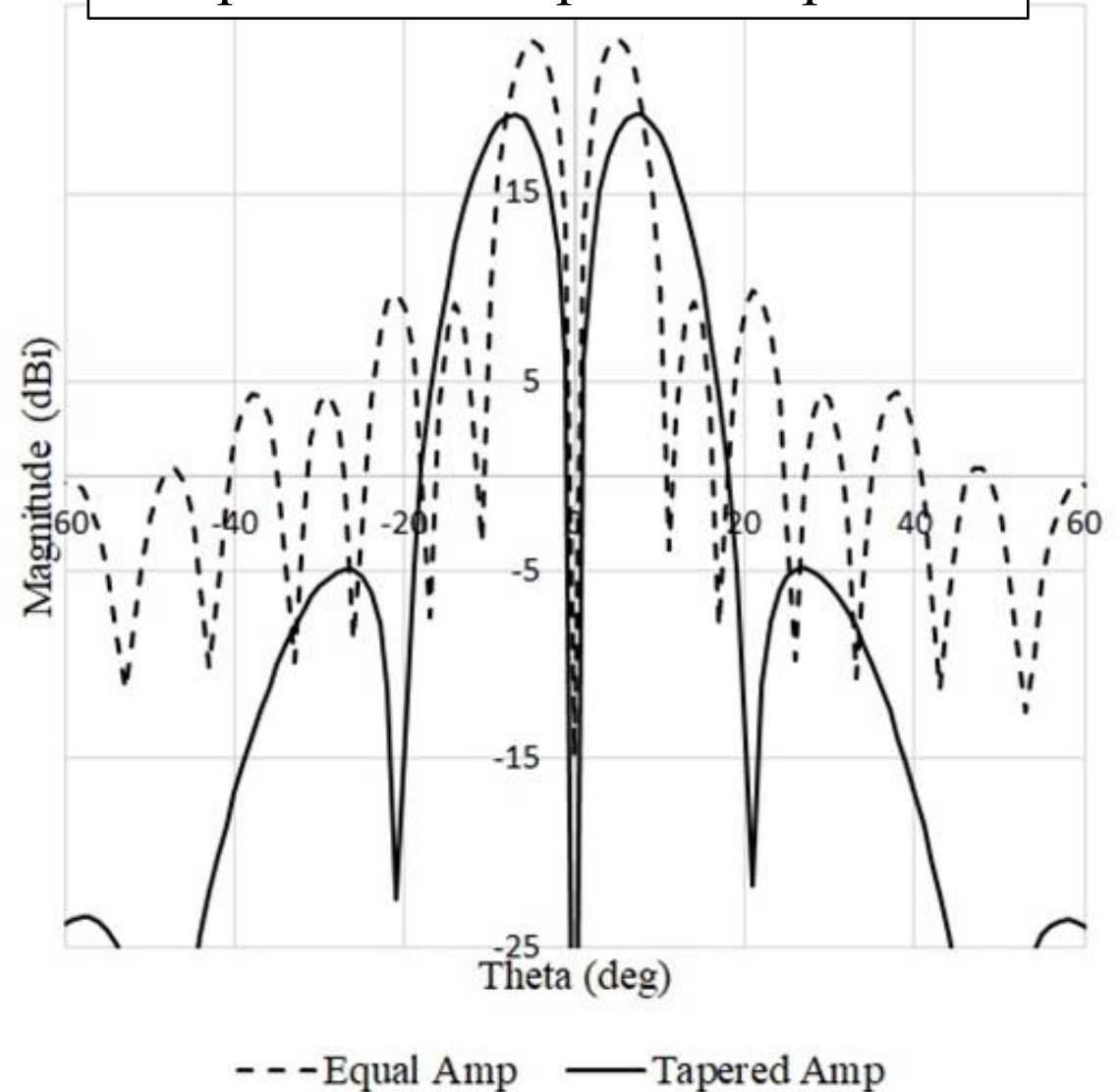
Amplitude Tapered Array Simulations

2.4 dB reduction in main lobe magnitude for Tapered vs. Uniform

10.7 dB reduction of sidelobe levels

Number of sidelobes drastically reduced.

Far-field Magnitude for 16x16 Array Excited with Uniform Amplitude vs. Tapered Amplitude.



Mode Purity Beam Analysis

Decomposition

- Gaussian Beam Mode Analysis can be used to analyze the mode spectrum of simulations and measurements, as all OAM beams can be described as superpositions of pure modes.
- Multiplying a pure mode's electric field by the complex conjugate of a simulated or measured beam and then integrating over the plane perpendicular to the propagation axis gives mode coefficient (a_p^ℓ) and normalized coupling coefficients (C_p^ℓ):

$$\begin{aligned} \bullet \quad a_p^\ell &= \iint U_s(x,y) u_p^{\ell*}(x,y) dx dy & U_s &= \text{simulated beam field} \\ \bullet \quad |C_p^\ell|^2 &= \frac{|\iint U_s(x,y) u_p^{\ell*}(x,y) dx dy|^2}{\iint U_s(x,y) U_s^*(x,y) dx dy} & u_p^\ell &= \text{Normalized LG field} \\ & & p &= \text{radial mode index} \\ & & \ell &= \text{azimuthal mode index} \end{aligned}$$

Optimization

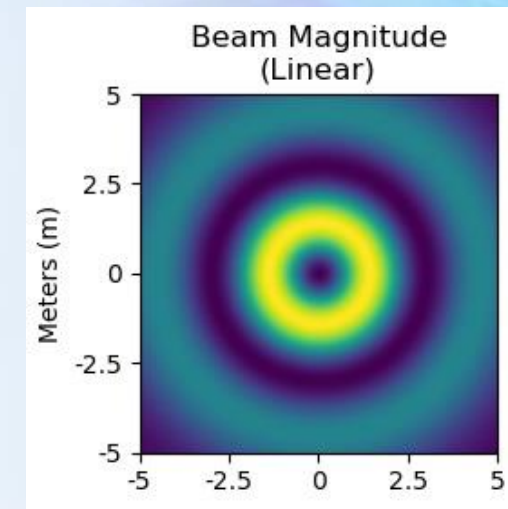
- Accurate description of radial mode requires an “optimal” beam waist to be calculated – one where the desired LG mode is maximized.
- Accomplished by iterating over all modes, and optimizing the beam waist for the desired mode

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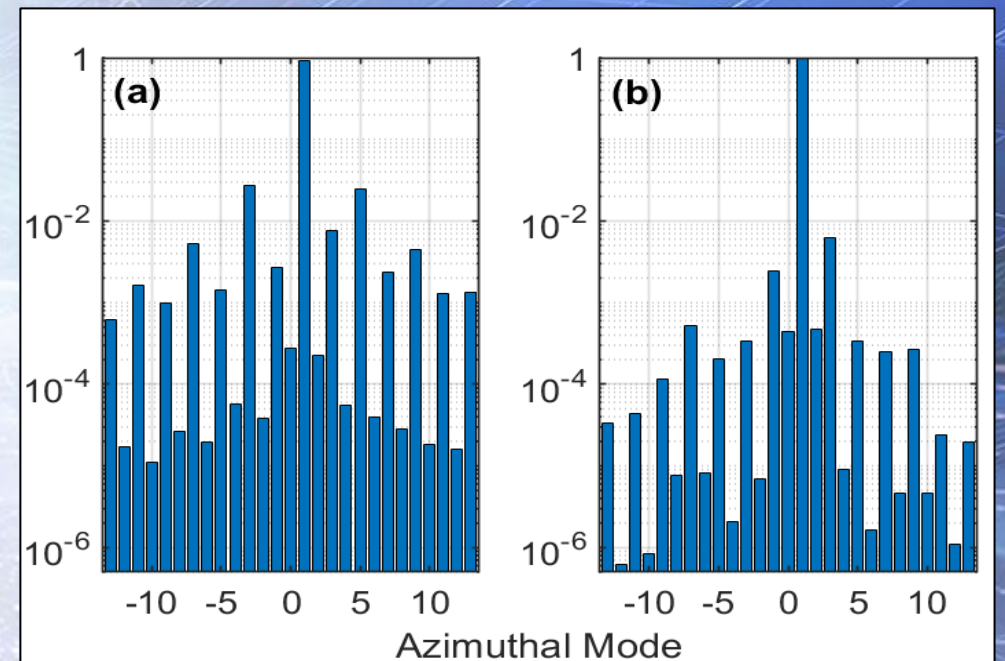
Mode Purity Calc.

Radial and azimuthal decomposition of both uniform and tapered excitation simulations over three radial and seven azimuthal modes.

- Note that the $\ell = 1, p = 0$ modes are the most significant values
- Crosstalk between azimuthal modes in the tapered excitation is greatly reduced (highest crosstalk in bold.)

Comparison of summed azimuthal modal content of (a) uniform and (b) tapered excitation simulations.

<u>Uniform</u>	$\ell = -3$	$\ell = -2$	$\ell = -1$	$\ell = 0$	$\ell = 1$	$\ell = 2$	$\ell = 3$
p = 0	0.0136	0.0000	0.0001	0.0001	0.8720	0.0001	0.0051
p = 1	0.0017	0.0000	0.0009	0.0001	0.0000	0.0000	0.0002
p = 2	0.0055	0.0000	0.0001	0.0000	0.0093	0.0000	0.0004
Tot	0.0270	0.0000	0.0022	0.0002	0.9115	0.0001	0.0072
<u>Taper</u>	$\ell = -3$	$\ell = -2$	$\ell = -1$	$\ell = 0$	$\ell = 1$	$\ell = 2$	$\ell = 3$
p = 0	0.0000	0.0000	0.0000	0.0002	0.9694	0.0004	0.0061
p = 1	0.0000	0.0000	0.0023	0.0002	0.0001	0.0000	0.0000
p = 2	0.0001	0.0000	0.0000	0.0000	0.0097	0.0000	0.0000
Tot	0.0003	0.0000	0.0023	0.0004	0.9881	0.0004	0.0061



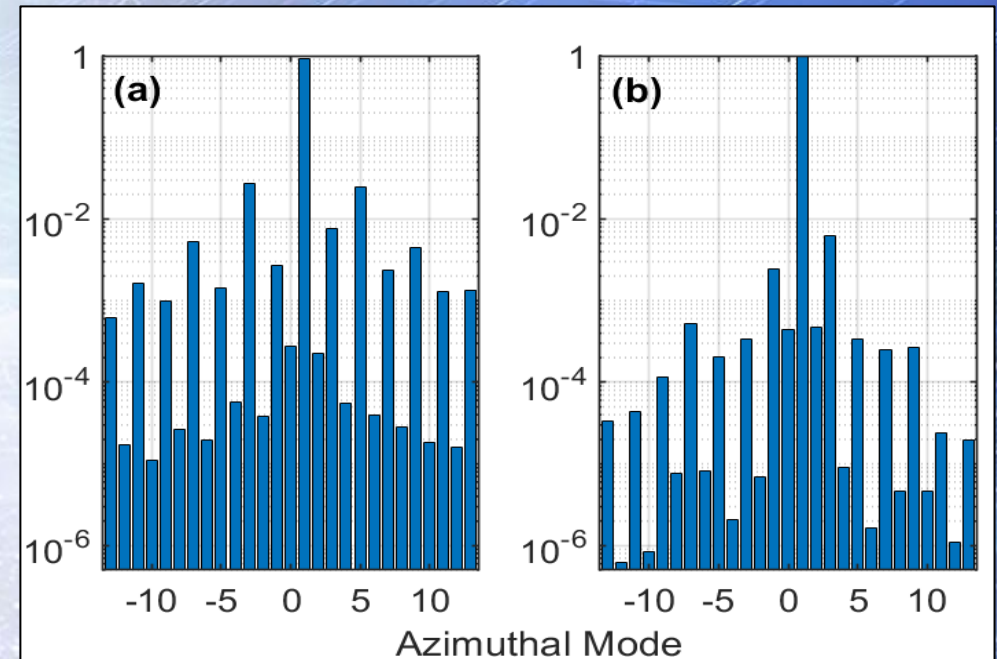
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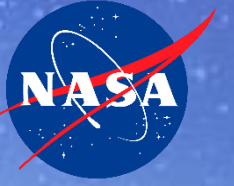
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Uniform Excitation							
dB	$\ell = -3$	$\ell = -2$	$\ell = -1$	$\ell = 0$	$\ell = 1$	$\ell = 2$	$\ell = 3$
$p = 0$	-37.37	-103.56	-79.41	-85.01	-1.19	-77.12	-45.76
$p = 1$	-55.10	-147.96	-61.19	-79.23	-86.15	-104.24	-76.00
$p = 2$	-45.19	-105.26	-84.25	-97.44	-40.71	-107.60	-68.24
$p = 3$	-62.78	-111.15	-76.88	-98.76	-64.16	-108.43	-69.26
$p = 5$	-65.41	-131.37	-71.67	-104.29	-40.77	-97.02	-96.23
Sum	-31.3	-88.4	-51.3	-71.1	-0.8	-73.0	-42.3
Tapered Excitation							
dB	$\ell = -3$	$\ell = -2$	$\ell = -1$	$\ell = 0$	$\ell = 1$	$\ell = 2$	$\ell = 3$
$p = 0$	-93.82	-120.54	-115.04	-75.02	-0.27	-67.43	-44.30
$p = 1$	-89.16	-117.99	-52.63	-73.59	-78.79	-137.72	-88.13
$p = 2$	-79.75	-121.72	-94.23	-118.49	-40.29	-94.34	-86.63
$p = 3$	-80.52	-116.54	-86.75	-94.47	-45.94	-97.92	-102.96
$p = 5$	-85.85	-119.25	-97.04	-97.22	-52.70	-103.22	-122.16
Sum	-69.4	-103.3	-52.3	-67.2	-0.1	-66.6	-44.2





Thank you!