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Debunking Stress Rupture Theories Using Weibull Regression Plots

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FALL TECHNICAL CONFERENCE 2022

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Outline

- Testing Overview
- Details on Stress Rupture Testing
- Weibull Regression Discussion
- Model Building
- Residual Analysis
- Team Dynamic
- Next Steps

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NASA Strand and Vessel Testing

- NASA's Engineering Safety Center (NESC) project to assess safety of Composite Overwrapped Pressure Vessels (COPVs)
- COPVs
  - Transport gasses under high pressure
  - Metal Liner
  - Wrapped by a Series of Carbon Strands
- Research Question: **Determine Reliability of COPVs at Use Conditions for the Expected Mission Life**
  - Primary Focus on Strands
  - Secondary Focus on Relationship to Vessels
  - Strands Less Expensive to Test
- https://www.nasa.gov/offices/nesc/home/Feature_COPVs_Jan-2012.html

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NASA Strand and Vessel Testing

- Analyses Use Classic Weibull Model

$$R(t_i) = e^{-\left(\frac{t_i}{t_{ref}} SR^{\rho}\right)^{\beta}}$$

- Observed Life Time: t_i
- SR: Stress Ratio, ratio of stress level to strength scale parameter
- Critical Parameters:
  - ρ: Sensitivity to Stress Ratio
  - β: Shape parameter for time to Failure
  - t_{ref}: Reference time to Failure when SR=1

NASA Strand Study

- Previous Strand Test
  - Relevant strand study conducted at a national lab
  - 57 strands at high loads for 10 years
  - Net information learned:
    - Strands either fail very early or
    - Last more than 10 years
  - Limited information based on 10 years of study!

- Estimates of Critical Parameters for Planning

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NASA Strand Study

- Team’s Initial Concept
  - Much larger study that the original 10 year study
  - Censor very early
    - Reduces time
    - Allows for the larger study in a practical amount of time

- Proceed in phases

- Have detailed data records to track any problems

NASA Strand Study

Experimental Phases

- Phase A – During “shake-out” of tests rigs
- Phase B – “Gold Standard” Experiment for Strands
- Phase C – “Proof” Study
- In Parallel: Vessel Studies (Opportunistic)

Phase A

- Conducted During Shake-Out of Equipment
  - Small study (although bigger than the national lab study!)
  - Statistical goal: Determine if the parameters from the national lab study are valid as the basis for planning the larger study!
  - Note: Phase A gave the team an opportunity to re-plan the larger experiment, if necessary!

Phase B

- "Gold Standard" Experiment
  - Planned time required: 1 year
  - Used 4 "blocks" of almost equal numbers of strands
    - Allowed the team to correct for time effects
    - Allowed the team to mitigate problems, especially early
  - Study assumed the "classic" Weibull model
  - Size of the experiment assured ability to assess model

Block	SR	Number	Proportion
1	0.80	176	0.718
	0.85	50	0.204
	0.90	19	0.078
	Sum	245	1.00
2	0.80	170	0.708
	0.85	50	0.208
	0.90	20	0.083
	Sum	240	1.00
3	0.80	174	0.710
	0.85	51	0.208
	0.90	20	0.082
	Sum	245	1.00
4	0.80	176	0.718
	0.85	49	0.200
	0.90	20	0.082
	Sum	245	1.00

Observations

- Phase A: Surprisingly Similar to Initial Study
- Phase B:
  - Serious problem occurred with the gripping in the first block
  - Serious conversations with possibility of replacing!
  - Other three blocks well behaved and by themselves produced better than the planned precision for the estimates
- Final Decision: Drop the First Block

NASA Strand Study: Benefits

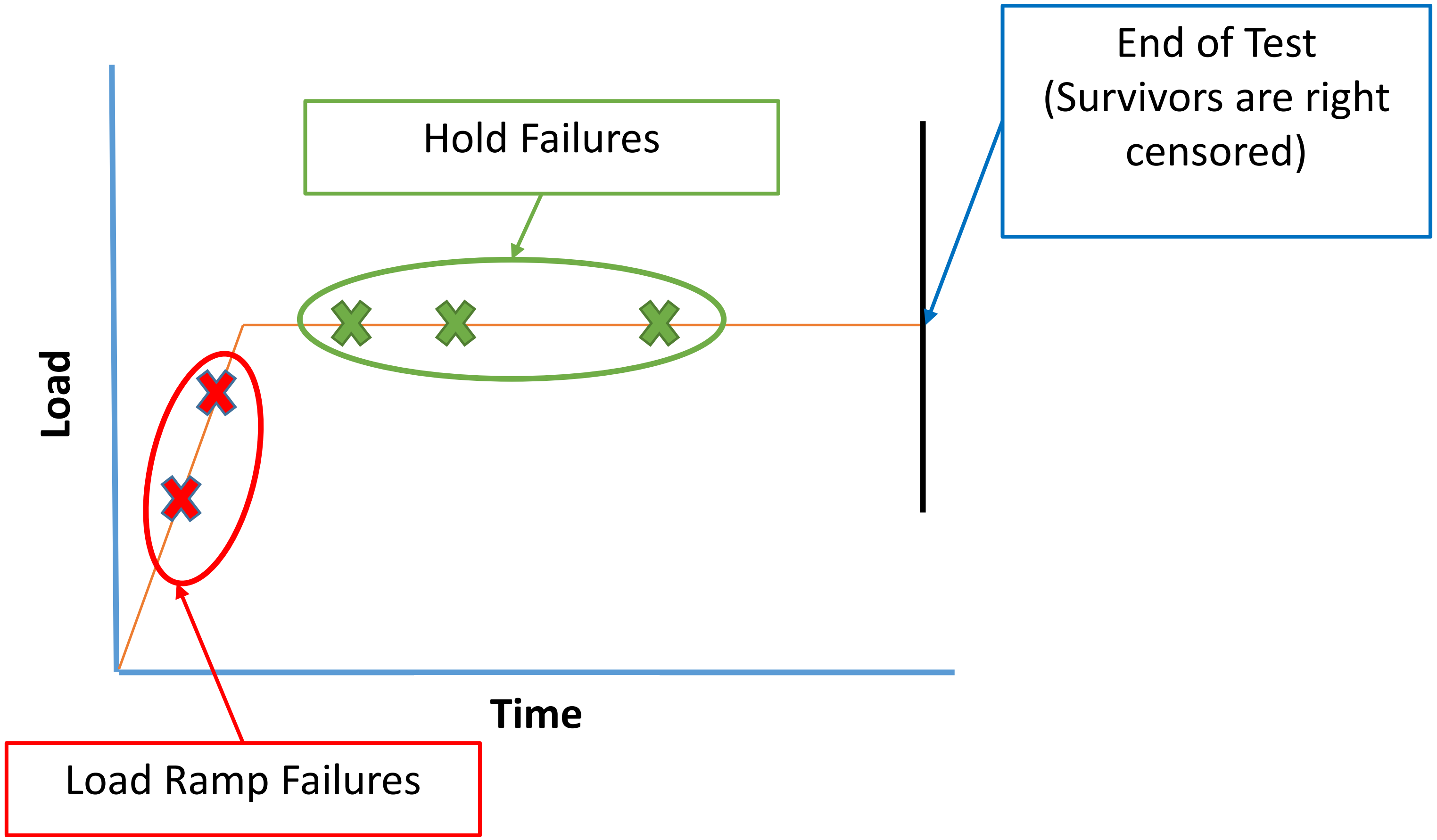
- Phase A:
  - Opportunity to Confirm Initial Study Parameter Estimates
  - Allowed opportunity to revise the experimental protocol if the estimates were significantly different

- Phase B:
  - Allowed opportunity to model changes in time over the year.
  - Mitigated the problem with the first block!
  - Provided simple mechanism for replacing the first block if needed!

Analysis: Stress Rupture Data

Description of Stress Rupture Test

- Stress Rupture
  - Failures occur after a period of time where there is no increase in load
- Failures are needed to determine reliability
- Must extrapolate from where test is performed versus where reliability predictions are made
- Test strands at higher loads and then extrapolate
- Need a model to make predictions



Basic Weibull Distribution

- Probability Density Function

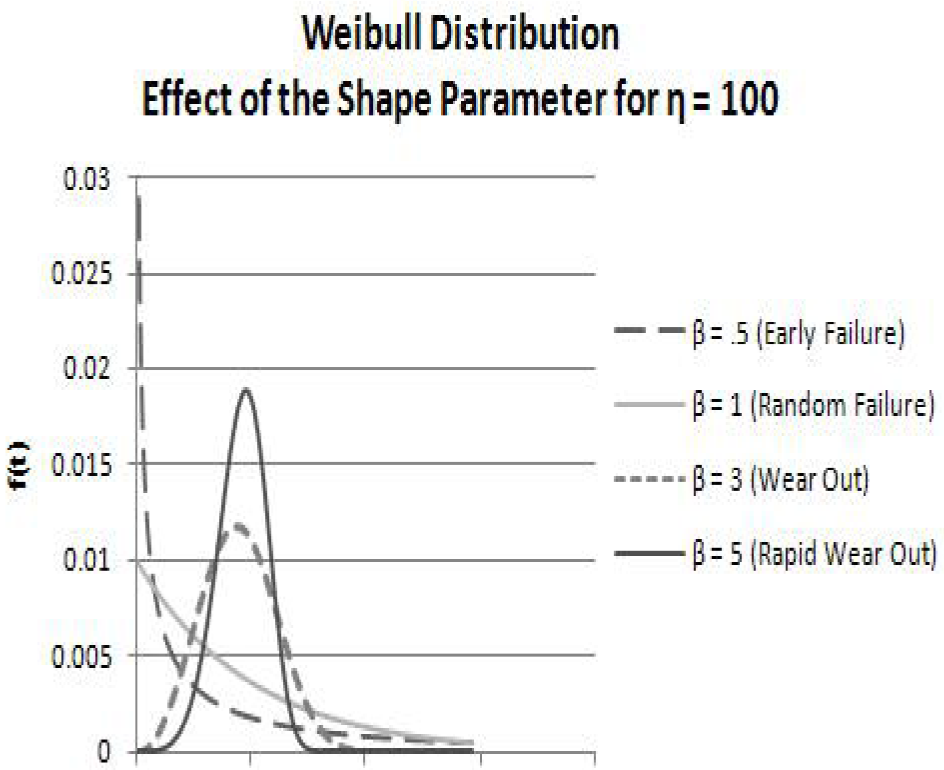
$$f(t,\beta,\eta) = \frac{\beta}{\eta}\left(\frac{t}{\eta}\right)^{\beta-1} \mathrm{e}^{-\left(\frac{\mathrm{t}}{\eta}\right)^{\beta}}$$

- Survivor Function = 1 – F(t)

$$S(t) = e^{-\left(\frac{t}{\eta}\right)^{\beta}}$$

β: shape parameter

η: scale parameter (characteristic life, time where 63.2% of units will fail)



Smallest Extreme Value Distribution

- Smallest extreme value (SEV) distribution is an alternate parameterization of the Weibull distribution
- SEV represents Weibull as a log-location-scale distribution
  - If t_i is Weibull, then $\log(t_i)$ is SEV
- Weibull/SEV relationship mimics the Normal/Lognormal relationship
- Parameters
  - Log-location: $\mu = \log(\eta)$ Scale: $\sigma = \frac{1}{\beta}$
- Residuals
  - $e_i = \log t_i - \mu$
  - Scaled: $z_i = \beta e_i = \beta(\log t_i - \mu) = \frac{\log t_i - \mu}{\sigma}$
- Survivor Function
  - $S(t_i) = P(T > t_i) = e^{-e^{z_i}}$

Classic Stress Rupture Model: Weibull

- Classic Weibull Survival Function

$$S(t_i) = P(T > t_i) = e^{-\left(\frac{t_i}{t_{ref}} SR^{\rho}\right)^{\beta}}$$

Note: $\eta = t_{ref} SR^{-\rho}$

- Observed Life Time: t_i
- SR: Stress Ratio, ratio of stress level to strength scale parameter
- Critical Parameters:
  - ρ: controls the relationship between the failure time and stress ratio (SR)
  - β: Shape parameter for time to Failure
  - t_{ref}: Reference time to Failure

Classic Stress Rupture Model: SEV

- SEV Survival Function

$$S(t_i) = e^{-\left(\frac{t_i}{t_{ref}} SR^{\rho}\right)^{\beta}} = e^{-e^{\beta\left(\log t_i - \theta + \rho \ln(SR)\right)}}$$

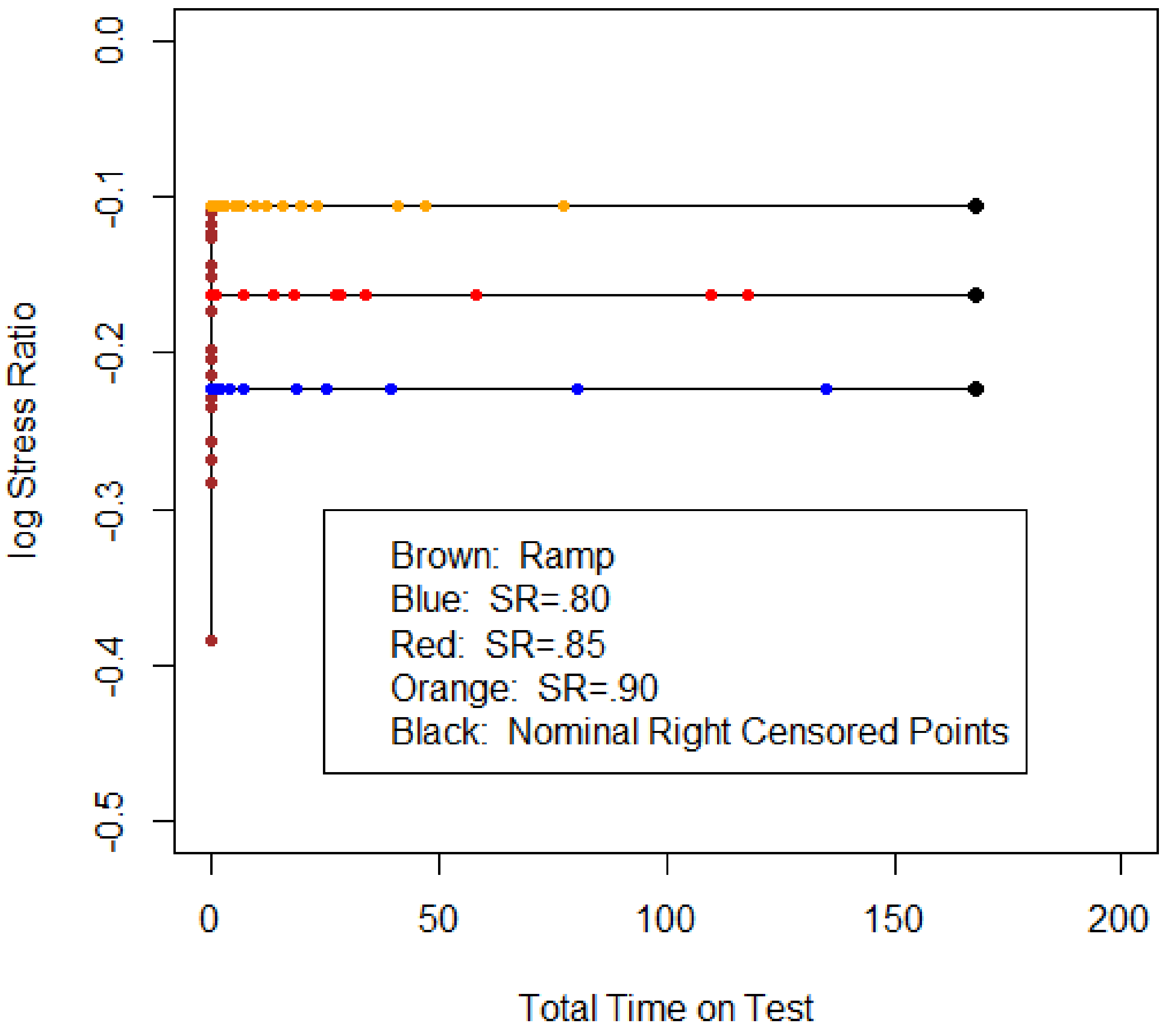
where $\theta = \log(t_{ref})$ and $\mu = \log(\eta) = \theta - \rho \ln(SR)$

Now working with a linear model, similar to simple linear regression

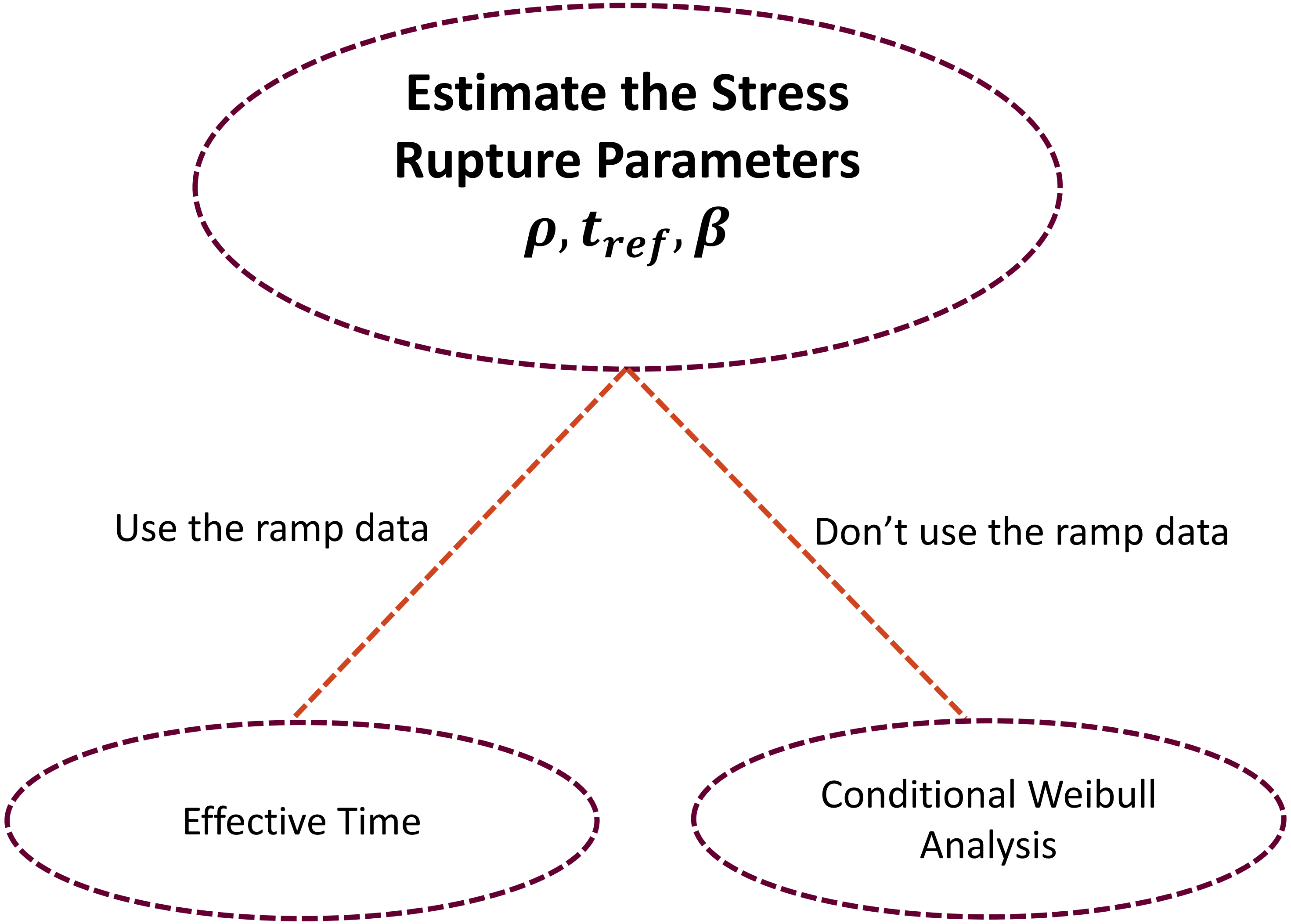
- Scaled Residuals
  - $z_i = \beta e_i = \beta(\log t_i - \mu) = \beta(\log t_i - \theta + \rho \ln(SR))$
  - Used for predictions of the log probability for specific observations

Weibull Regression: How do we fit this model?

Data Structure for the Ramp and Hold Phase B Data

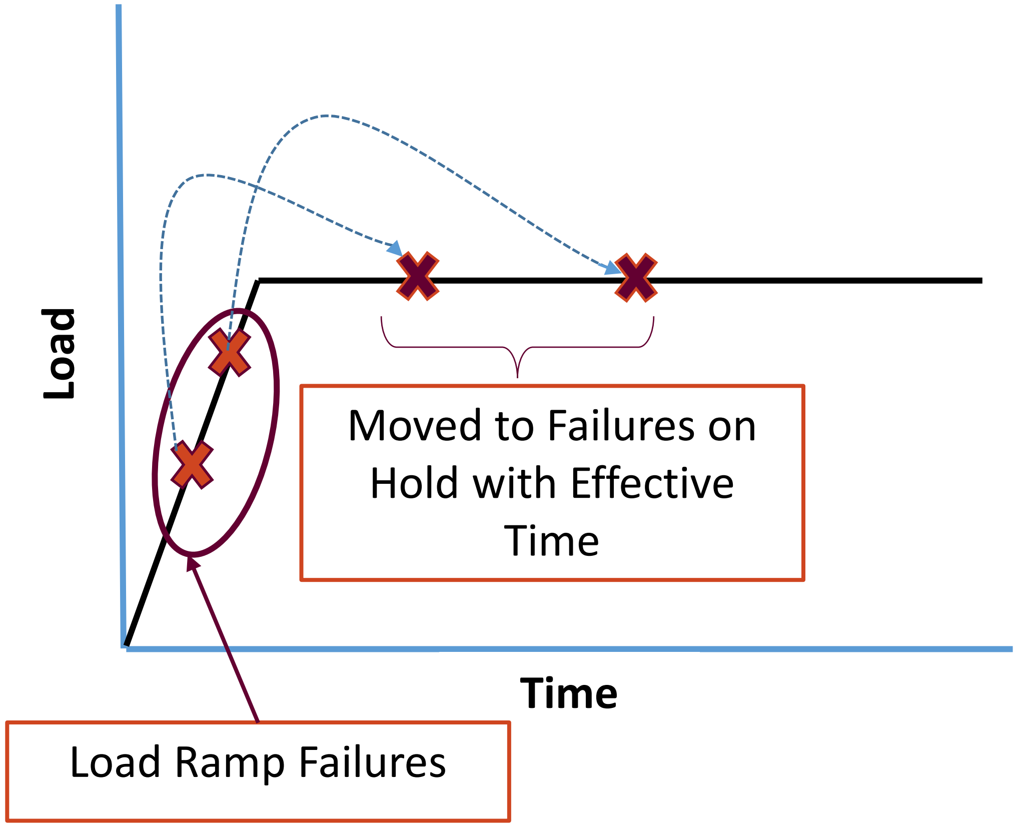


Right Censored Data


Estimate the Stress Rupture Parameters
ρ, t_{ref}, β
Use the ramp data
Don't use the ramp data
Effective Time
Conditional Weibull Analysis

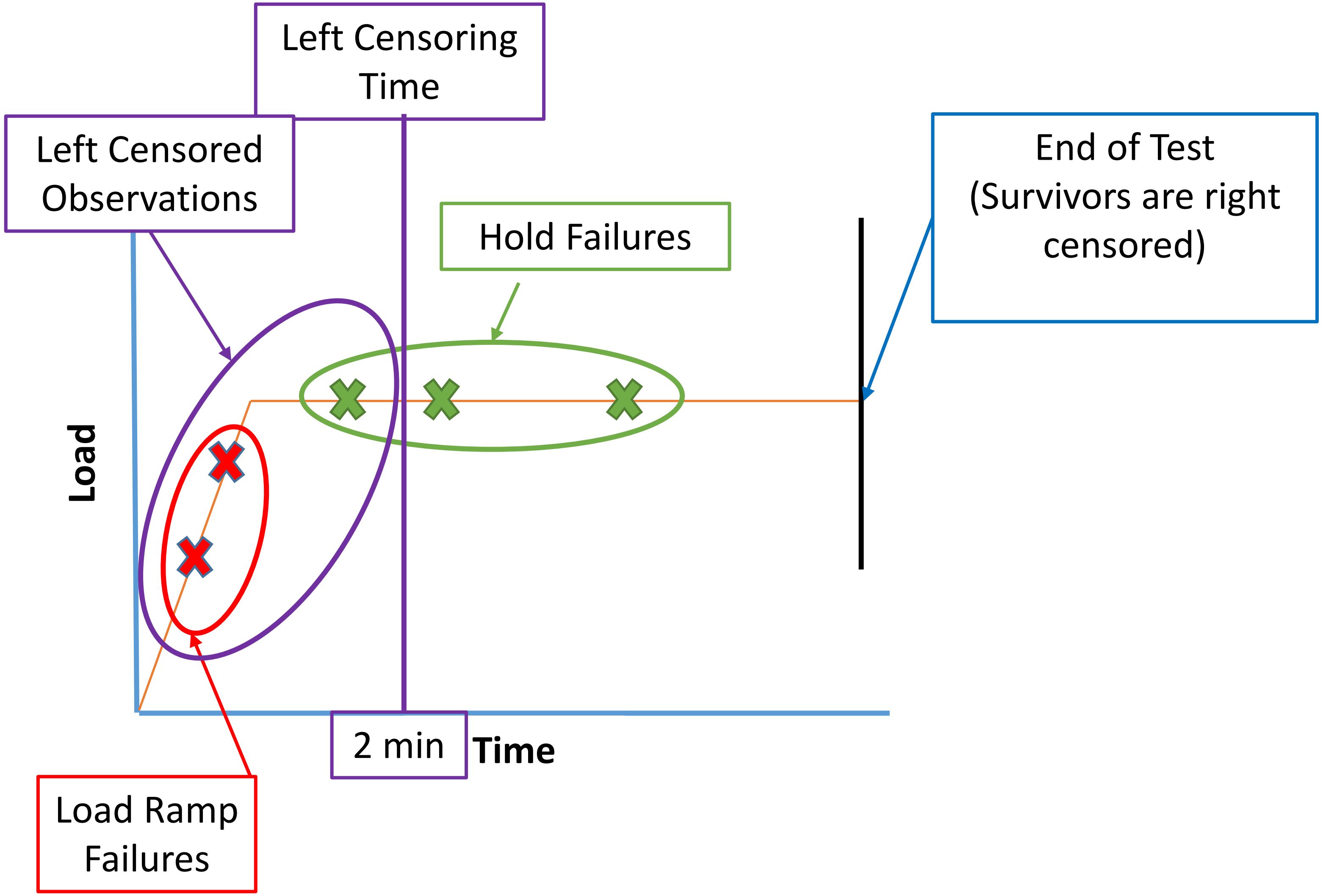
What is Effective Time?

- Basic Idea:
  - "Correct" the actually observed ramp time for what the time would have been at a specified target load
- Effective Time on Load
  - $t_{eff} = \frac{t}{\rho+1}$
- Yin and Sheng (1987)
  - Theoretical development of the concept
  - Parameters
    - $\beta_{ramp} = \beta(\rho + 1)$
    - $\eta_{ramp} = \left[\frac{(\rho+1)}{R^{\rho+1}}\right]^{1/\rho+1}$
- Ramp behavior is a two-dimensional problem and stress rupture is really a three-dimensional problem



Effective Time→Left Censored

- Left Censors specimen that fail on load and those that fail within the first two minutes of stress rupture testing
- Left censoring assumes the data from the ramp can be modeled by the same model parameters as the hold data

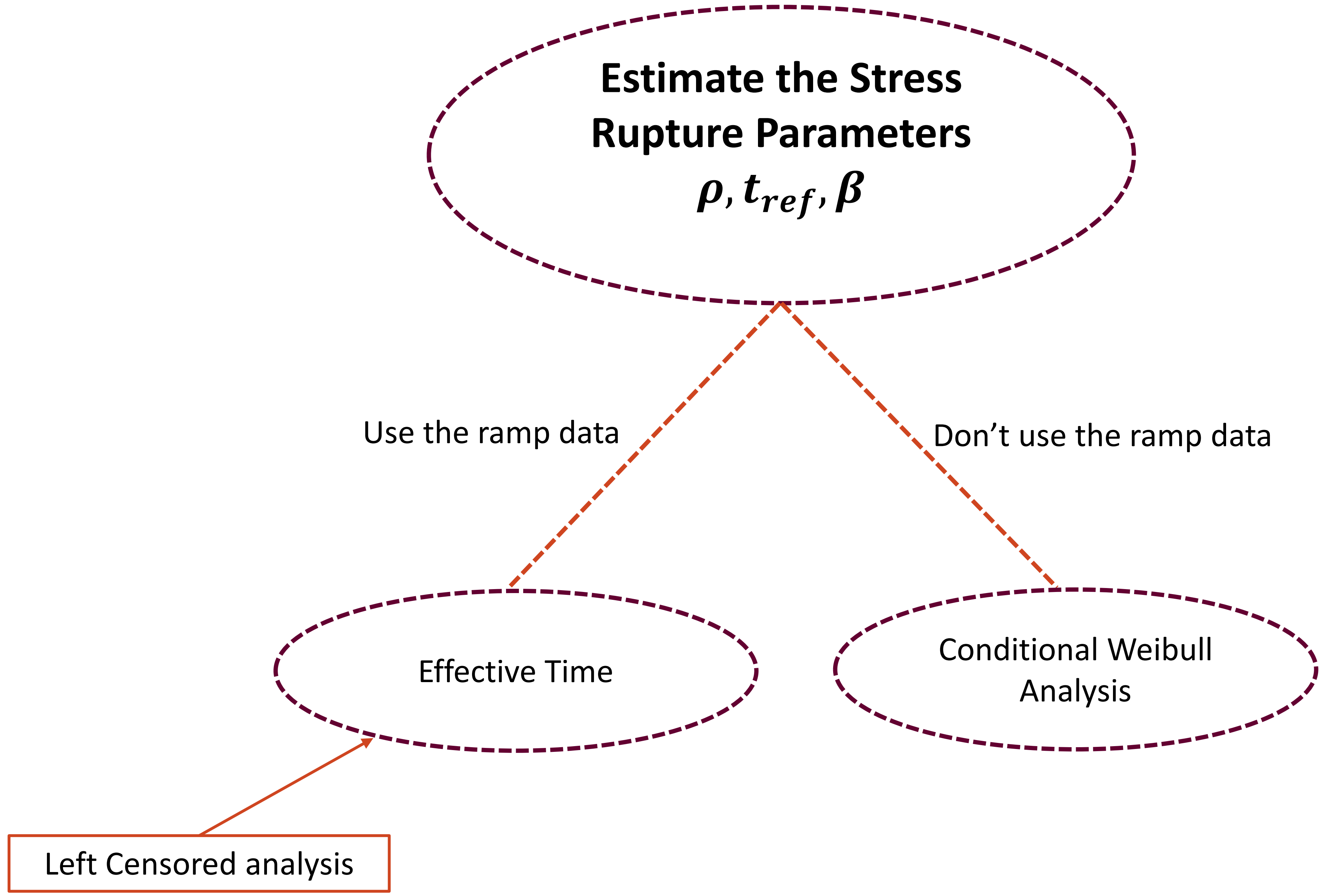


Deeper Dive into Left Censoring

- Left censor when failure times are missing
  - Only have information that specimen failed before a specified time
- It is not appropriate to left censor outliers
- Left censoring assumes that the failure time is missing at random
  - Left censoring ramp failures and failures within a certain window of time violates this assumption
- Left censoring assumes the missing failure times follow the exactly same distribution as the other data being estimated
  - Does this make sense for the ramp and hold

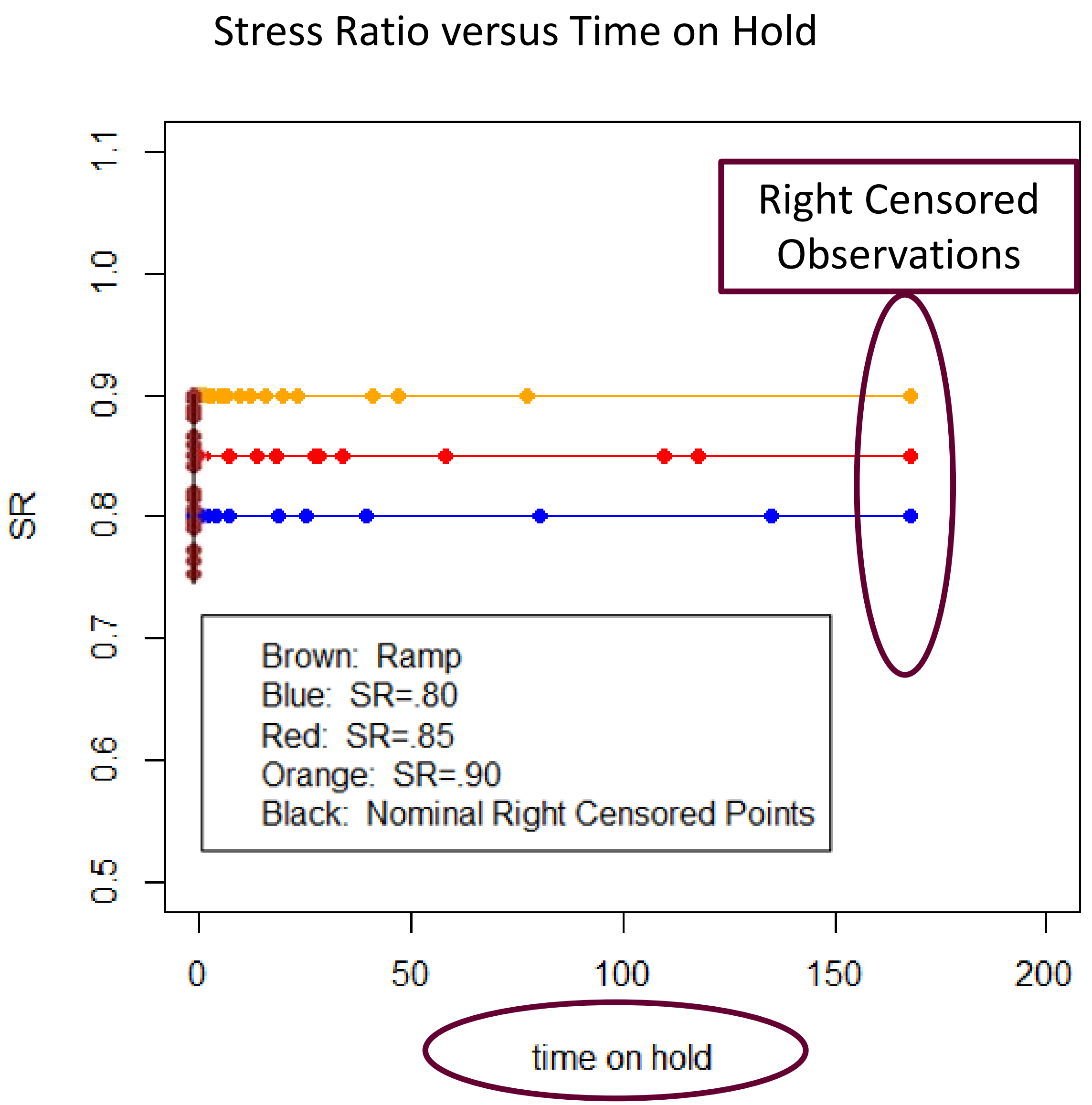
"Left-censored observations occur in life test applications when a unit has failed at the time of its first inspection; all that is known is that the unit failed before the inspection time. In other situations, left-censored observations arise when the exact value of a response has not been observed and we have, instead, an upper bound on that response.."

- Meeker and Escobar (p.38)


Estimate the Stress Rupture Parameters
ρ, t_{ref}, β
Use the ramp data
Don't use the ramp data
Effective Time
Conditional Weibull Analysis
Left Censored analysis

Fit-to-Hold Analysis

- Focus on Time on Hold not Time on Test.
- Fit the Classic Weibull Model using Maximum Likelihood Estimation with Right Censoring.
- Failures on load are not used because they have no time on hold!

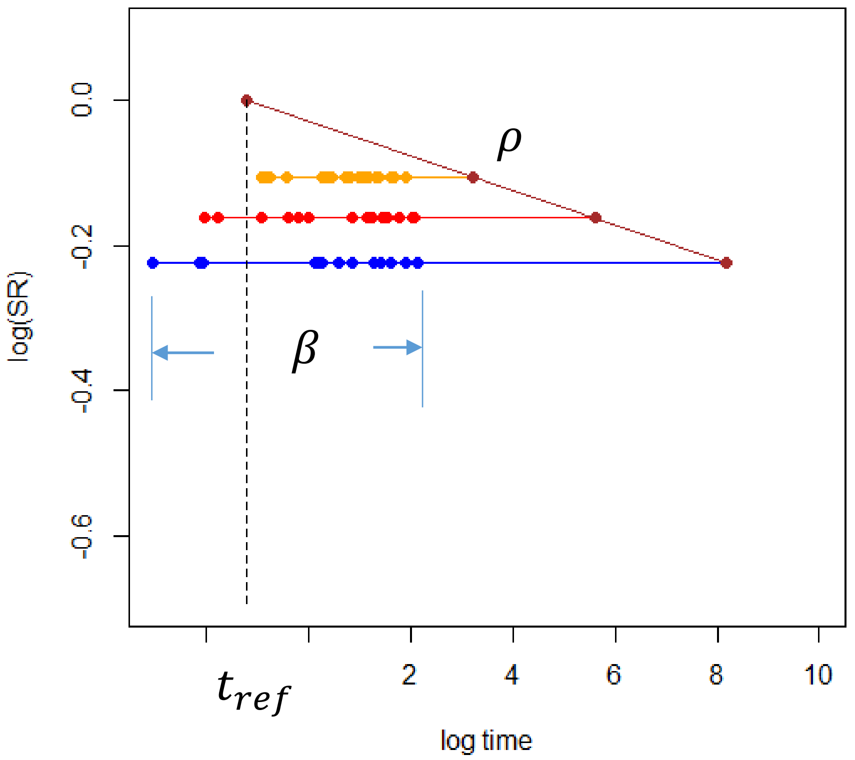


True Structure of the Stress Rupture Model

$$\mu = \log(\eta) = \theta - \rho \ln(SR)$$

- ρ: controls the relationship between the failure time and stress ratio (SR)
- β: Shape parameter for time to Failure
- t_{ref}: Reference time to Failure

Log Stress Ratio versus Log Time



Stress Rupture model explains the behavior of the items *on hold*.

- Weibull regression gives us estimates for ρ, β and t_{ref}

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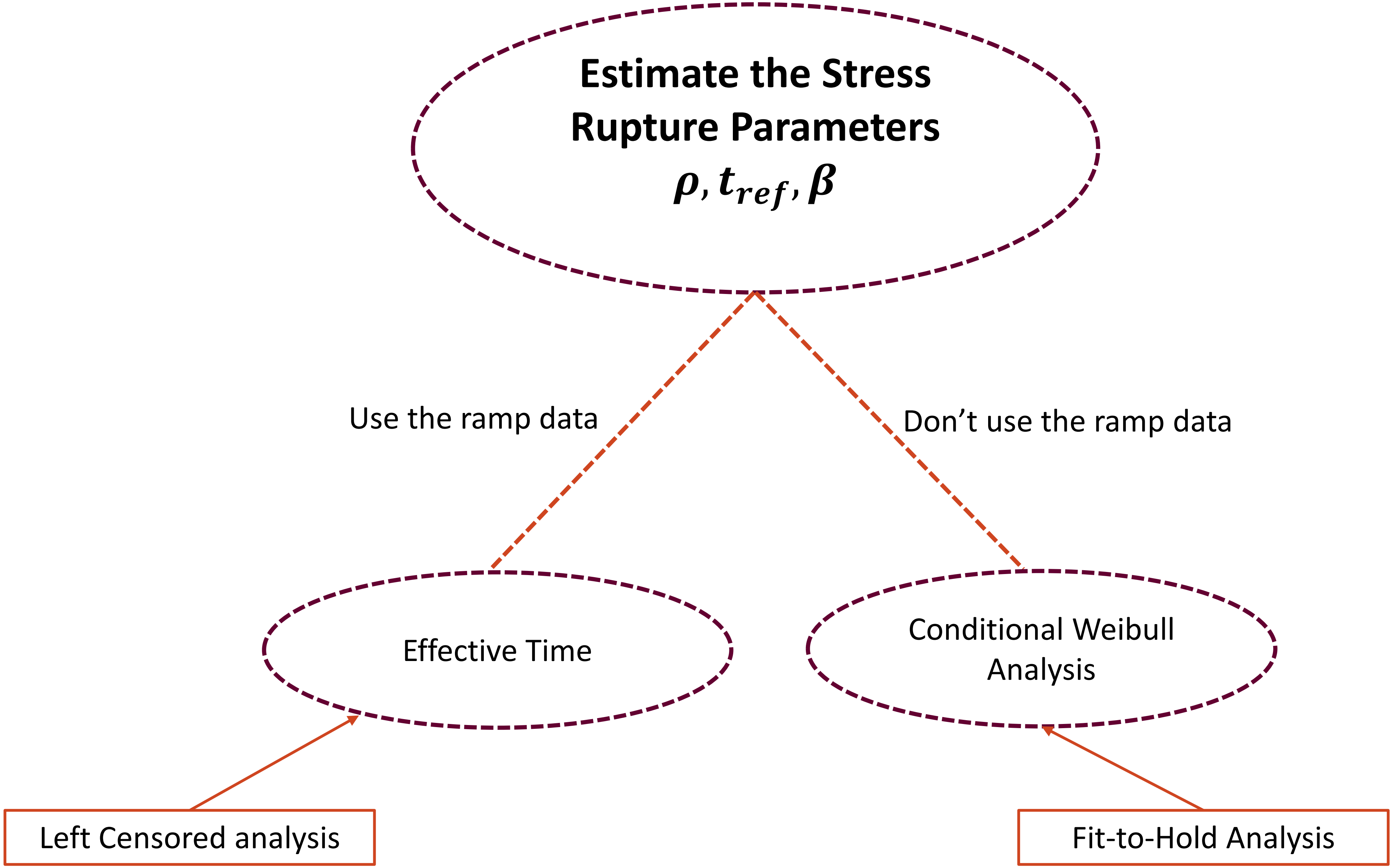
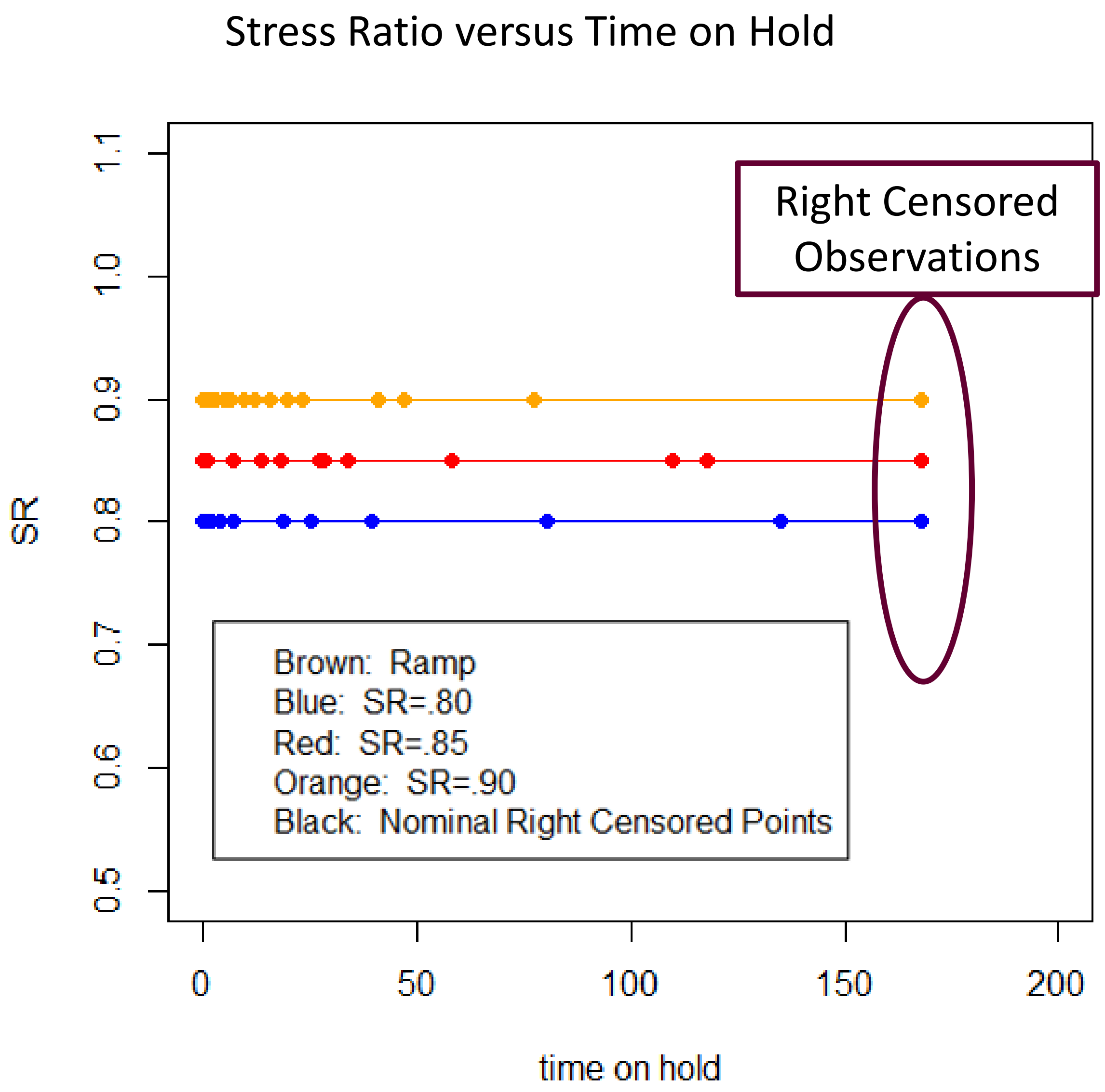

Estimate the Stress
Rupture Parameters
ρ, t_{ref}, β
Use the ramp data
Don't use the ramp data
Effective Time
Conditional Weibull
Analysis
Left Censored analysis
Fit-to-Hold Analysis

Weibull Regression: Model Building

Full Model

- Separate individual models to each stress ratio
  - Two parameters for the SR=1 data: $\eta_{.80}$ and $\alpha_{.80}$
  - Two parameters for the SR=2 data: $\eta_{.85}$ and $\alpha_{.85}$
  - Two parameters for the SR=3 data: $\eta_{.90}$ and $\alpha_{.90}$
- Largest possible Weibull model for the data
  - Has the largest log-likelihood
- Will compare to the Full Model to subset models to determine whether the improvement in log-likelihood justifies the extra parameters



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Fit-to-Hold (subset model)

- Model the data that have achieved the target load as defined by the experimental protocol (no ramp data)

- Defines that the time at the sustained constant load begins the moment the test item achieves the target load

- Assumes a Weibull distribution to describe the time to failure under the sustained constant load

- Experimental protocol uses right-censoring at a nominal time

Fit-to-Hold (subset model)

- Ramp and Hold data are modeled separately
  - Three parameters to explain the hold data: ρ, β, and $\theta = \log t_{ref}$
- Model assumes

$$\alpha_{.80} = \alpha_{.85} = \alpha_{.90} = \beta$$

Comparisons

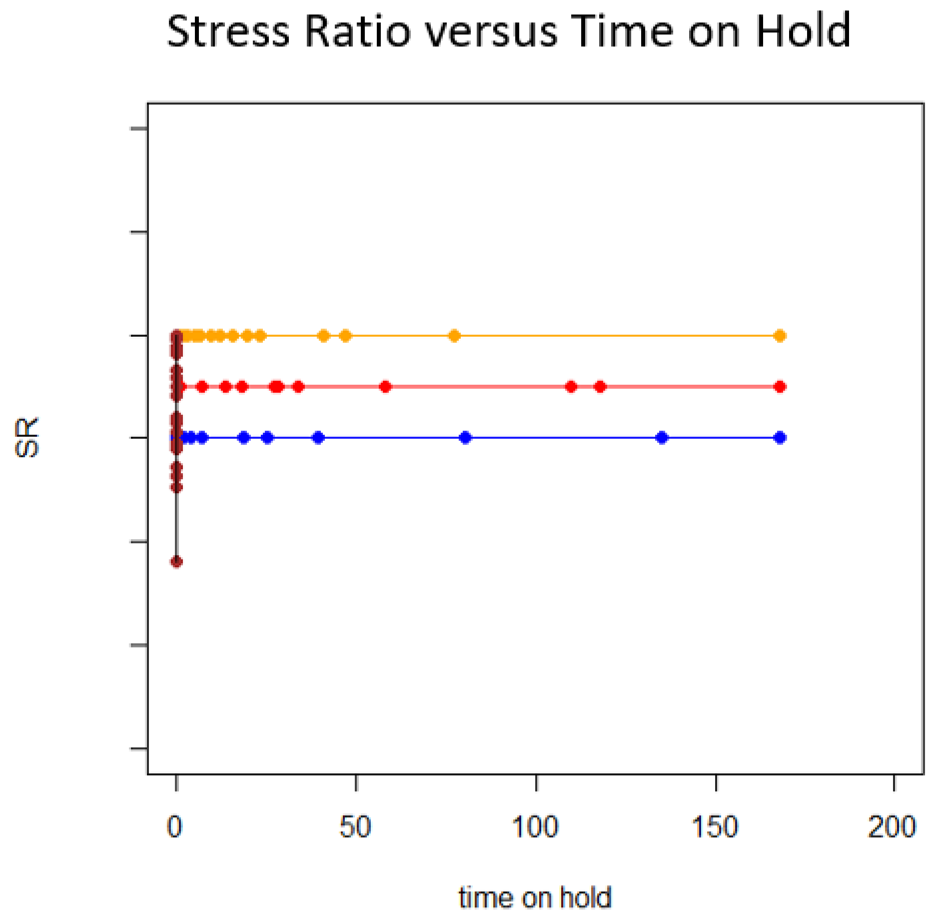
Model:	Fit-to-Hold	Full Model
Number of Observations:	708	708
True Log-Likelihood:	−306.411	−305.900
Log-Likelihood Statistic:	612.822	611.800
AIC:	618.822	623.800

- The p-value associated with the χ^2 based on the difference in the log-likelihood statistics is 0.7959
  - The three extra parameters in the full model are not significant
- Smallest AIC value for Fit-to-Hold (adjustment for parameters)

Weibull Regression: Including Ramp Fit-to-Hold versus Effective Time

Adaptations to Include Ramp Failures

- Rigorous Approach
  - Add two additional parameters for a Weibull Distribution fit to only the ramp data along with the Fit-to-Hold Analysis
    - Two parameters for the ramp data: η and α
    - Three parameters to explain the hold data: ρ, β, and $\theta = \log t_{ref}$
- Left Censored Analysis
  - Assume that ALL data follow the same failure mechanism
  - Left censor all ramp failures and some early stress rupture failures
    - Three parameters to explain the ramp and hold data: ρ, β, and $\theta = \log t_{ref}$



Comparisons

	Rigorous	Left-Censored	Full Model
Overall log $\mathcal{L}$:	-251.103	-390.779	-250.592
Log-Like Stat:	502.206	781.558	501.184
AIC:	512.206	789.558	517.184
Ramp log $\mathcal{L}$:	55.308		55.308
Hold log $\mathcal{L}$:	-306.4114		-305.900

- All fit statistics indicate that the rigorous model is the superior fit to the data compared to the Left-Censored approach
  - Smallest AIC value (512.206) and log-likelihood statistic (502.206) and the largest overall log-likelihood value (-251.103)The three extra parameters in the full model are not significant
- The probability that the left censored analysis explains the data at least as well as the rigorous model is 1.03946 E-62
- Counter-intuitive to penalize the maximum likelihood fit to the data with left censoring especially when we know the precise time these items failed on the ramp

Weibull Regression: Residuals

Proper Basis for Constructing Probability Plots

- Estimate the Model

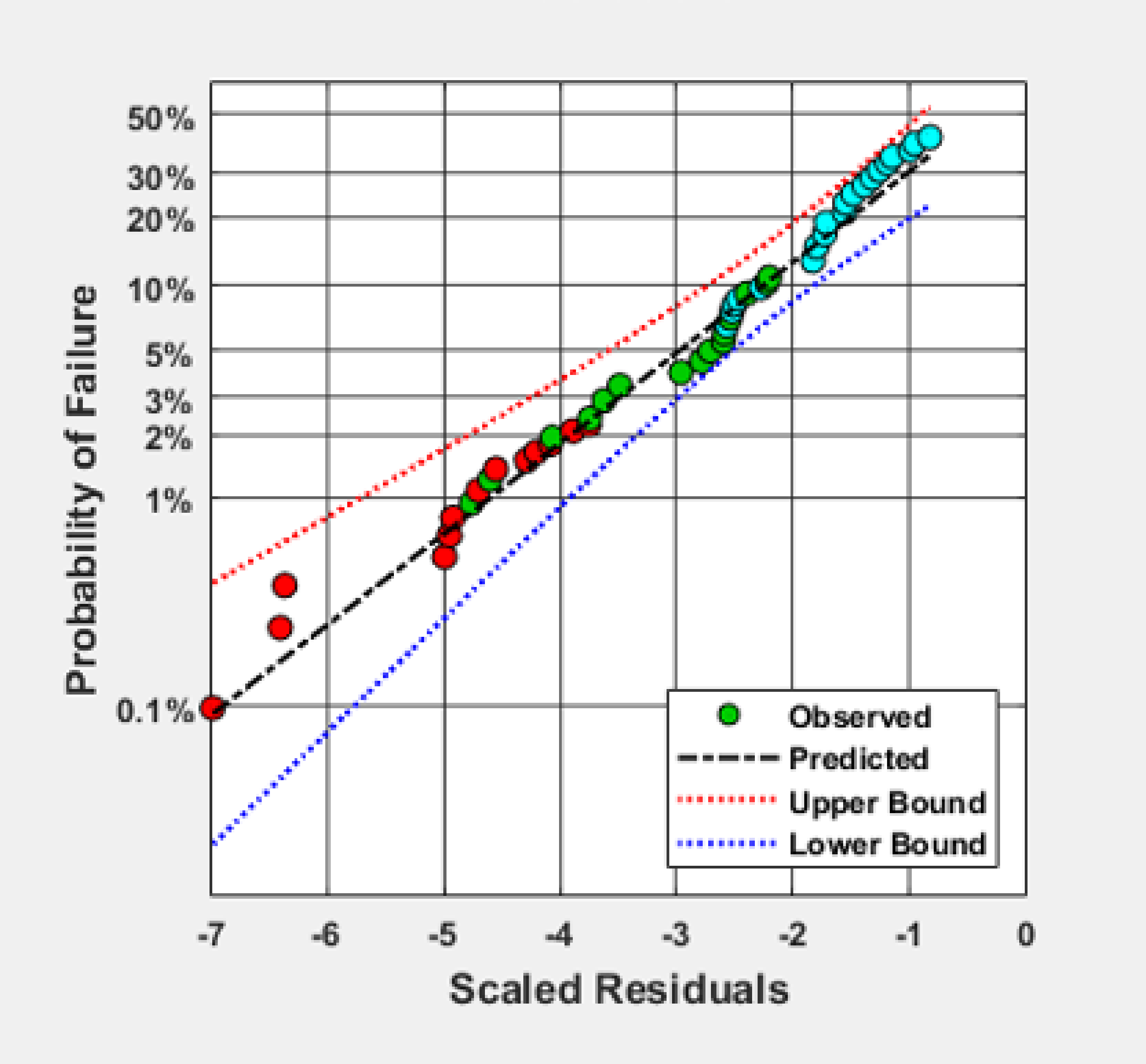
- Construct the Scaled Residuals

- Calculate the Median Ranks for these Residuals (Overall not by SR!)

- Plot $\ln[-\ln(1 - mr_i)]$ versus the βe_i

- Method Extends Easily to More Complicated Models

Weibull Probability Plot



Team Dynamic Discussion Questions

- Have you ever been part of a team where members have very strong feelings for very different analyses?
- Best practices for being a trail blazer?
- How do you make people focus on the data, not their preconceived beliefs?

Where to Next…

- Working on Tutorials that cover best practices for analyzing Stress Rupture Data

- Advocating for the Use of the Fit-to-Hold Method for analysis
  - Model Building
  - Residual Analysis

Team

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