

Statistical Distribution of Gear Surface Fatigue Lives at High Reliability

Zachary A. Cameron¹

Timothy L. Krantz²

¹ NASA Goddard Space Flight Center

² NASA Glenn Research Center

Abstract

The reliability of gears is important for a wide variety of applications, and the gearing community has a desire for better prediction of fatigue life at very high reliability. To improve predictions, a set of 950 NASA-conducted gear surface fatigue test results were gathered, pooled together using the general standardized Weibull variate, and then fit to a Weibull three-parameter distribution. An iterative fitting method was developed and then applied to the gear fatigue data to determine the values of the distribution parameters. Additional analyses were done using bearing and gear data that demonstrate a trend of increasing minimum life coefficient as correlated to the timeframe of the fatigue testing, and material cleanliness is proposed as a primary explanation for such correlation. It is noted that this observed trend supports published findings from analytical modeling and experimental studies that improvements to material cleanliness improves minimum fatigue life. Further use and study of the three-parameter Weibull distribution is recommended as a tool to quantify fatigue life most accurately in the high reliability regime.

Keywords: Gear fatigue, reliability, Weibull distribution

Nomenclature

C_γ	minimum life coefficient; γ/n
$C_{\gamma,est}$	estimate for C_γ , current iteration
$C_{\gamma,k}$	adjustment to $C_{\gamma,est}$, iteration k
$F(L)$	probability of failure, cumulative distribution function
i	index, individual fatigue life datapoint
j	index, fatigue data group number
k	index, iteration of solution method
L	fatigue life
$S(L)$	probability of survival, cumulative distribution function
y	standardized Weibull variate
y^*	general standardized Weibull variate
β	shape parameter
γ	location parameter
η	scale parameter

1. Introduction

The reliability of gears is important for a wide variety of applications. Typically, gearing is designed using a safe-life approach for a required reliability appropriate for the application (Ref. 1). Usually, aerospace gearing provides a safety-critical function, sometimes with no redundancy of function should a gear fail to deliver torque. In such situations, the gearing must be designed for a very high reliability.

One failure mode for gears is surface fatigue. This work investigates the statistical description of the distribution of gear surface fatigue life using the Weibull distribution. The approach of this work was motivated and influenced by the investigations and findings of bearings. The fundamental phenomena responsible for the rolling contact fatigue of gears and bearings have much in common. However, the recommended approach for quantifying reliability

during the design process has differed for gears and bearings. For gears, the reliability consideration is introduced into standard design calculations as an adjustment to the allowable stress that will provide the required design lifetime at the required reliability for given loading and operating conditions (Ref. 1). The statistical distribution of the allowable stress is modeled as a Gaussian (normal) distribution. The standard deviation of the allowable stress is taken as the value 0.1 for spur and helical gears and 0.156 for bevel gears in standardized design practice. Another approach, the Lundberg-Palmgren approach to fatigue modeling, has been applied to gears in research papers. Coy, Townsend, and Zaretsky applied Lundberg-Palmgren theory to spur gears (Ref. 2). Vijayakar extended this work to spiral bevel gears (Ref. 3), and then using material factors derived from spur gear testing he found reasonable correlation in applying the theory to spiral bevel gear test data. A study of gear surface fatigue data to assess the goodness of fit to a statistical distribution in the high reliability regime has not been published. In this work, we explore the distribution of gear surface fatigue in the high reliability regime and bring forward the following contributions: (1) the concept of the standardized Weibull variate is generalized to include a non-zero value for minimum life, (2) an iterative method is developed to determine minimum life coefficient for a pooled dataset, (3) the method is applied to a large dataset of NASA gear fatigue data, and (4) minimum life coefficient values are determined that can be used to improve high-reliability gear design.

For bearings, reliability is introduced into design calculations using the two-parameter Weibull distribution (Refs. 4,5). Weibull assumed that the probability of a material fatigue was dependent upon the magnitude of stress and the stressed volume (Refs. 6,7) and derived a cumulative distribution function for survivability, the Weibull distribution. Lundberg and Palmgren (Ref. 8) applied Weibull's fatigue theory and developed equations for the load capacity of bearings that are widely adopted and accepted as the standardized design method (Ref. 4). The bearing reliability is introduced in the design calculation as a life adjustment factor. The Lundberg-Palmgren method models the bearing probability of survival as the two-parameter Weibull distribution. This approach has adequate accuracy for designing bearings for a 90-percent probability of survival. However, the bearing community has recognized that bearing failure data does not well fit the two-parameter Weibull distribution in the high reliability regime, as is evident from the discussions to follow.

Tallian (Ref. 9) investigated the goodness of fit of the Weibull distribution to bearing fatigue life data over a large range of survivability. To do such investigation, he needed a large set of data. His work pooled 93 groups of bearings comprising more than 2500 bearing fatigue data points using the Weibull variate concept. He examined trends at very high reliability values, and he showed a deviation of the pooled-data's distribution from that of the two-parameter Weibull distribution. Based on both the empirical pooled data and from theoretical arguments, Tallian discussed the concept of a lifetime having 100% reliability and developed design adjustment factors to account for the deviations from the two-parameter Weibull equation. These data supported the basis of high-reliability life factors published by the STLE (Ref. 4).

Other researchers studied bearing life for high reliability and in some cases have explored fitting the data to the three-parameter Weibull distribution. Findings have been published for several types of bearings, including deep groove ball by Snare (Ref. 10), tapered roller and ball by Takata et al. (Ref. 11), linear ball by Shimizu et al (Ref. 12), spherical roller by Ferreira (Ref. 13), and tapered roller by Kotzalas (Ref. 14). All of these works consistently find that bearing rolling contact fatigue data in the high reliability regime deviate from the two-parameter Weibull distribution, and most have adopted use of the three-parameter Weibull distribution to more accurately represent the data in the high reliability regime.

Engineering models based on principles of material fatigue, with no assumption made for the statistical distribution of the fatigue life, suggest the use of a three-parameter Weibull distribution. Tallian proposed an early unified model for rolling contact fatigue (Ref. 15) that included principles of crack growth, local plastic strain, ductility, competing failure modes, and defect population as defects per unit volume. The resulting rolling contact life distribution is a three-parameter Weibull distribution with a location parameter (minimum life) greater than zero, and the minimum life is influenced by the cleanliness of the material (and other factors). Sadeghi et al (Ref. 16) modeled rolling contact fatigue using nonhomogeneous microstructures including randomness of shape, size, and orientation of grains. The scatter of bearing fatigue lives resulting from the statistically generated microstructures were well described by three-parameter Weibull distribution. Walvekar and Sadeghi (Ref. 17) modeled rolling contact fatigue of case carburized steels with microstructure flaws. The flaws controlled the dispersion of the predicted fatigue lives, and the Weibull slopes correlated well to the slopes of experiments. The Weibull plots of the analysis results have the downwardly curved shapes of a three-parameter distribution in the high-reliability regime. These reviewed

works of bearing fatigue provided motivation, inspiration, and methodology for the present work investigating the statistical distribution of gear surface fatigue.

The gearing community has a desire for better prediction of fatigue life at very high reliability values. This work explores the use of the three-parameter Weibull distribution to quantify surface fatigue of gears in the high reliability regime. As noted in a recent review article “large number of results and research have been carried out through the years to assess the mechanisms of crack initiation in the very high cycle fatigue region and to model the fatigue life behavior through the analysis of the experimental results” (Ref. 18). The appropriate method to be employed for the statistical description of fatigue depends on the nature of the material, the data available, and the goal of the analysis. Li et al. (Ref. 19) developed an improved backward statistical inference approach (ISIA) to fit probabilistic stress life curves to data for several materials, including railway axle steels, that exhibit distinct fatigue limits. The ISIA approach models life dispersion as lognormal, and stress-life curves are presented for probabilities of survival as high as 97.5% from datasets of limited size. Strzelecki (Ref. 20) recommends a different approach for designing to probabilities of survival higher than 95%. Strzelecki’s model, based on the random fatigue limit concept (Ref. 21), describes test results using the three-parameter Weibull distribution for the limited fatigue life range and the two-parameter Weibull distribution for the fatigue limit. Strzelecki demonstrated that the three-parameter Weibull distribution was an equal or better descriptor of the data than was the normal distribution for the four steels studied. Many gear applications require survival in the very high cycle or even gigacycle regime, and the high strength steels commonly used for gears and bearings do not exhibit the traditionally defined fatigue limit (Refs. 22–24). Murakami et al. (Ref. 25) explain that such exceptional behavior can be attributed to hydrogen trapped at nonmetallic inclusions that weaken plasticity induced crack closure mechanisms, and thereby distinct knee points of stress-life curves disappear. Guided by the essential structure of the stress-life relations of very high cycle gear fatigue having no traditional fatigue limit, the study herein adopts and extends the works of Tallian (Ref. 9) and Kotzalas (Ref. 14) for pooling of data to study the high reliability regime. We extend the concept of the standardized Weibull variate to include the use of a location parameter as part of the standardization equation. To implement such standardization, a new iterative method is developed to determine the minimum life coefficient. The iterative method is studied for convergence behavior and efficacy via Monte Carlo simulations. Finally, the method is applied to the NASA gear

surface fatigue database to produce an estimate of the minimum life coefficient for use in high reliability gear design.

2. Mathematics Background

The Weibull distribution is one belonging to a family of shape-scale-location distributions. The Weibull cumulative distribution function (CDF) to describe survivability can be defined as:

$$S(L) = 1 - F(L) = \exp\left(-\left(\frac{L - \gamma}{\eta}\right)^\beta\right) \quad [1]$$

where S is probability of survival, F is probability of failure, L is the fatigue life, β is the shape parameter, η is the scale parameter, and γ is the location parameter. Equation 1 is often termed the three-parameter Weibull distribution. The scale parameter scales the mathematical function to engineering units. The location parameter translates the distribution function along the abscissa. The shape parameter defines the shape of the function. If the shape factor is equal to one, then the failure rate is constant over time. A shape factor greater than one exhibits increasing failure rate with time as occurs for accumulative wear, corrosion, and fatigue. The shape parameter greater than one is typically observed for data of rolling contact fatigue, a fatigue mode of bearings and gears. The influence of the shape parameter on the Weibull CDF is shown in Figure 1.

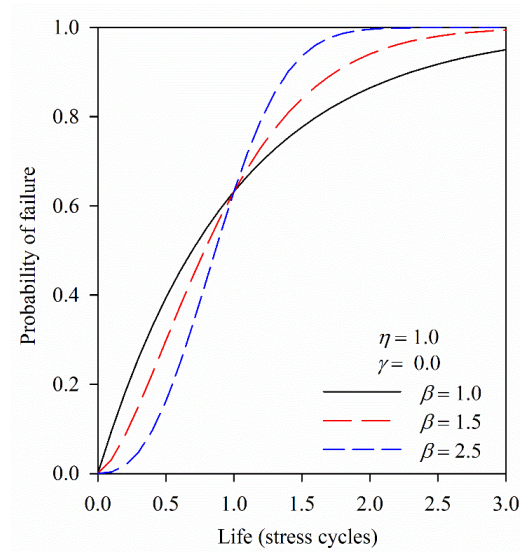


Figure 1.—Weibull CDF showing influence of shape parameter β on probability of failure with stress cycles.

Often the Weibull location parameter is assumed equal to zero, and then the distribution is termed the two-parameter Weibull distribution. This approximation is more than adequate for many engineering situations, but it is not always

accurate for quantifying high levels of reliability. In Figure 2 a two-parameter Weibull distribution is compared to a three-parameter distribution. The parameters were selected to minimize differences of failure probabilities over the mid-range failure values from 0.25 to 0.75. Over this mid-range of failure probability, the two functions are equal from an engineering perspective (Fig. 2(a)). However, there are differences in the high reliability regime that are of practical significance (Fig. 2(b)). Tallian (Ref. 9) termed such a difference as the “excess life” in his study of the distribution of bearing rolling contact fatigue life. When employed, the two-parameter Weibull approximation is a conservative design approach.

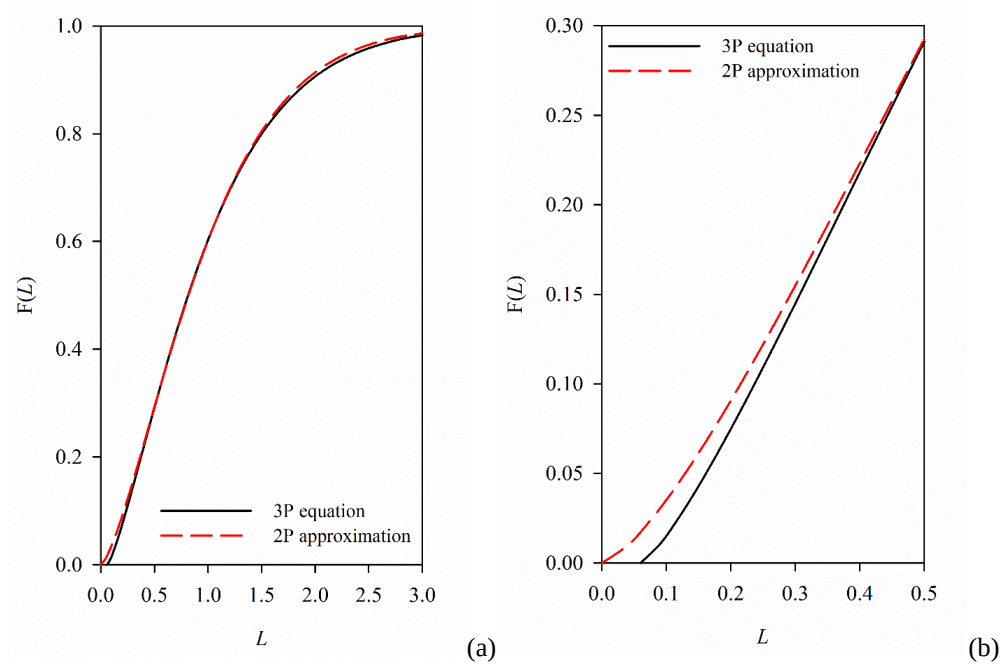


Figure 2.—Two Weibull distributions with similar values for mid-range failure probabilities. (a) Plot of the full range of failure probabilities. (b) Plot emphasizing only the high-reliability range of failure probabilities

Kotzalas explored using the three-parameter Weibull distribution for tapered roller bearing, and he defined a minimum life coefficient using the notation C_γ (Ref. 14). His definition for C_γ was appropriate for the sudden-death, best-of-four test censoring of his dataset. In this work we define the minimum life coefficient using a different equation that is appropriate for the available gearing data whereby the experiment used a fixed value, right-censoring test method. In this work C_γ is defined as the ratio of the location and scale parameters,

$$C_\gamma = \gamma/\eta \quad [2].$$

The Weibull cumulative distribution function, [eq. 1], can then be recast as

$$S(L) = 1 - F(L) = \exp\left(-\left(\frac{L}{\eta} - C_\gamma\right)^\beta\right) \quad [3]$$

and we note that for case of $C_\gamma \equiv 0$, the equation becomes the classical form for the two-parameter Weibull distribution

$$S(L) = 1 - F(L) = \exp\left(-\left(\frac{L}{\eta}\right)^\beta\right) \quad [4].$$

3. Standardized Weibull Variate

In this work, we extend the efforts of Tallian and Kotzalas (Refs. 9,14) who pooled multiple groups of fatigue data via the standardized Weibull variate concept. Tallian assumed that the failures of bearings by rolling contact fatigue could be accurately described by the Weibull distribution. Using that principle, he pooled multiple fatigue datasets into a single composite dataset by standardizing each datapoint using

$$y_{i,j} = \left(\frac{L_{i,j}}{\eta_j}\right)^{\beta_j} \quad [5]$$

where $y_{i,j}$ is the *standardized Weibull variate* (also termed the standardized life). The i subscript denotes an individual life datapoint from a homogenous group of data, the group denoted by subscript j , and β_j and η_j are the estimated shape and scale factors for group “ j ”. This definition of the Weibull standardized variate is based on the two-parameter form of the Weibull distribution with $C_\gamma \equiv 0$. We propose to extend the concept of the Weibull variate to include the concept of a non-zero value for the minimum life coefficient C_γ . Introducing this concept, a *general standardized Weibull variate* is defined as

$$y_{i,j}^* = \left(\frac{L_{i,j}}{\eta_j} - C_\gamma\right)^{\beta_j} \quad [6].$$

As will be described later, C_γ is determined by an iterative solution method.

It is informative to plot the standardized Weibull variate ($y_{i,j}$ or $y_{i,j}^*$ as appropriate) as the abscissa and the natural log of the reciprocal of cumulative survivability as the ordinate. Plotted in this manner the general standardized variate y^* (equation 5) will plot as a straight line with a slope of one. However, when a three-parameter Weibull distribution with $C_\gamma \neq 0$ is standardized using only two parameters (per equation [5]), then the plotted data will deviate from a straight line at high probability of survival values, and the trend line will asymptotically approach the

abscissa at the value C_γ^β . The plot of Figure 3 illustrates the plotting of a three-parameter Weibull distribution for which the minimum life coefficient is not known “*a priori*”, but candidate values (i.e. estimates, $C_{\gamma est}$) can be used to calculate the general standardized Weibull variate y^* . In the plot of Figure 3, the four candidate values for $C_{\gamma est}$ include zero, the true value C_γ , and two intermediate values. The plotted lines deviate from straight lines with downward curvature in the high reliability region if the estimated value $C_{\gamma est}$ is less than the true value C_γ . This property is utilized to enable an iterative solution method for determining best estimate of C_γ from a set experimental data.

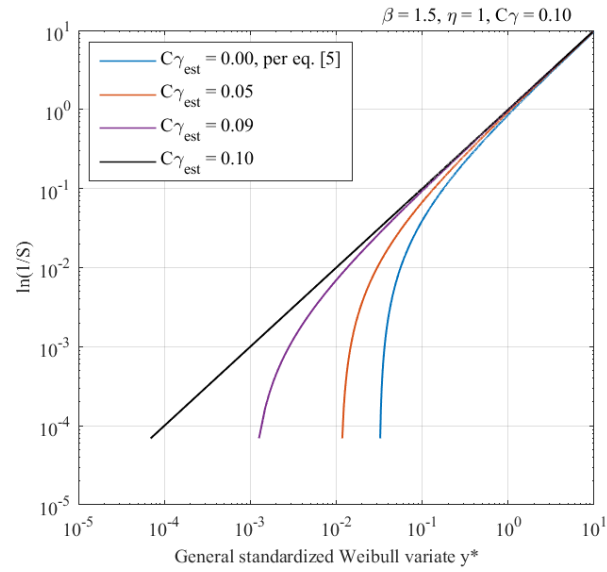


Figure 3.—General standardized Weibull variate plot using four estimates of the minimum life coefficient, $C_{\gamma est}$. By standardizing bearing fatigue data using equation 4, Tallian and Kotzalas respectively (Refs. 9,14) observed that plotted bearing fatigue data had trendlines with downward curvature, thereby indicating a minimum life coefficient $C_\gamma \neq 0$. Results from the bearings studies motivated the work herein examining if a non-zero minimum life coefficient accurately describes gear surface fatigue life. NASA gear fatigue datasets were gathered for such study, details to be described later. Initially, the gear data was standardized and pooled using [eq. 5] ($C_{\gamma est} = 0$) as follows. First, each gear data group was fit to a two-parameter Weibull distribution using the maximum likelihood method. Next, the shape parameter used for [eq. 5] was calculated as the median value of the shape estimates for all groups. The scale parameter for [eq. 5] was the maximum likelihood estimate for each group. The exploratory standardized gear data was then plotted using exact median-rank for the ordinate, see Figure 4. At high survivability values, the data significantly deviates from the $C_{\gamma est} \equiv C_\gamma$ line labeled as “ y_i^* theoretical” in Fig. 4 with a downward curvature

shape resembling trends of standardized bearing data (Refs. 9,14). This indicates that a three-parameter Weibull distribution is needed to quantify gear surface fatigue life at high reliability values. The section to follow will describe a method for estimating the minimum life coefficient C_γ of this pooled dataset.

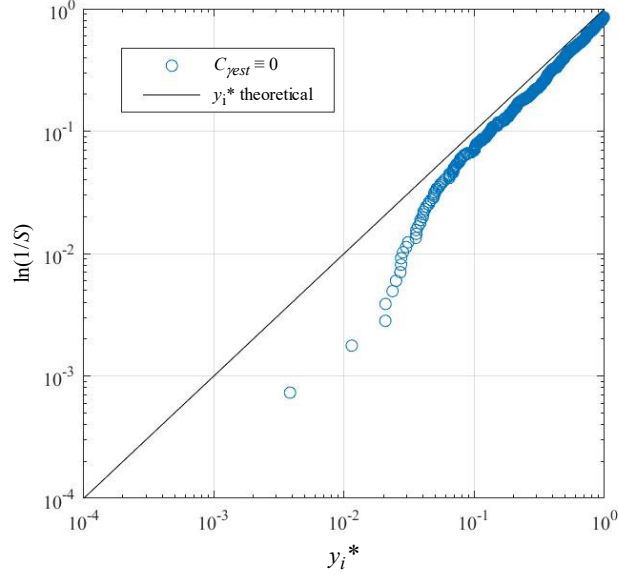


Figure 4.—Exploratory Weibull variate plot of pooled NASA gear fatigue data standardized assuming the minimum life coefficient equal to zero.

4. Method to Estimate Minimum Life Coefficient

The method for estimating the minimum life coefficient is described in detail below. The approach employed herein was based on previous studies by Tallian (Ref. 9) and Kotzalas (Ref. 14), but the present method differs in some details. Before describing the method step by step, we first highlight the impactful differences from previous work. As to the first difference, as was discussed above, we make use of the generalized standard variate (eq. [6]) rather than the standard variate (eq. [5]). A second difference regards the choice of the parameter β_j used to standardize each datapoint (such a choice is required regardless if using eq. [5] or eq. [6]). In the previous works, a different value of β_j was determined for each group using a two-parameter Weibull, best-fit criteria. In this work, incorporating the idea that the underlying standardized dataset comprises a homogenous Weibull distribution, we first postulated and then studied the idea that a common value for β_j would provide a more accurate standardization. After a study using Monte Carlo simulation, we found that a common value for β_j indeed provided a more accurate standardization. The common value of β_j used in applying eq. [6] to our dataset was the median value for all group estimates, and such value was found iteratively as will be detailed below. The third difference regards what portion

of the full dataset is used for determining the minimum life coefficient after standardized pooling of all the data. Both Tallian and Kotzalas observed that the standardized bearing data deviated from two-parameter Weibull distributions in both the very-low and very-high reliability regions. Only the high reliability region (lower tail of the distribution) is of interest for design, and so only data from the lower tail of the pooled is used for the data fit. Kotzalas used pooled data for 80% and greater probability of survival. We chose to use a larger proportion of our dataset. Guided by Monte Carlo studies, pooled data above 67% and greater probability of survival was used in this study. Our available dataset numbered 950 datapoints. We note that Kotzalas's approach was reasonable and appropriate given that his dataset numbered 9702 datapoints.

The detailed iterative method is now described with the aid of a flow chart, see Figure 5. To initiate the iterative method, first each group is fit to a two-parameter Weibull distribution using the maximum likelihood method. Next, the median of all of the group's shape parameters was determined, β_k , where k is the current iteration number. Then, β_k was used as the current iteration's shape parameter common for all groups. The two-parameter maximum likelihood fit was then reapplied to each group but with the shape parameter fixed to the previously calculated β_k . This provided adjusted scale parameter for each group designated as $\eta_{j,k}$. A range of location parameter estimates, C_{yest} , were then applied to the individual lives to produce "modified standardized Weibull variates" using the median shape parameter and recalculated scale parameters. For each C_{yest} value, the sum squared error (SSE) between the lower half of the total number of standardized Weibull variates and the twice natural logged of one over survivability was computed. The location parameter estimate corresponding to the lowest SSE was selected as the estimate for the current iteration, and its value stored. For the next iteration, the previous loops' location parameter estimate was multiplied with the group's scale parameter estimates to remove the impact of the location parameter estimate from individual lives within a given group. These lives were then applied to the fitting process again, finding new shape and location parameters, and fitting a new location parameter to remove from the data. This was repeated for M number of iterations until the procedure converged to within engineering precision. The final three parameter estimates of this algorithm was the M^{th} loop's common shape parameter estimate β_M , the individual group scale parameter estimates $\eta_{j,M}$, and the summation of all the loops' location parameter estimates.

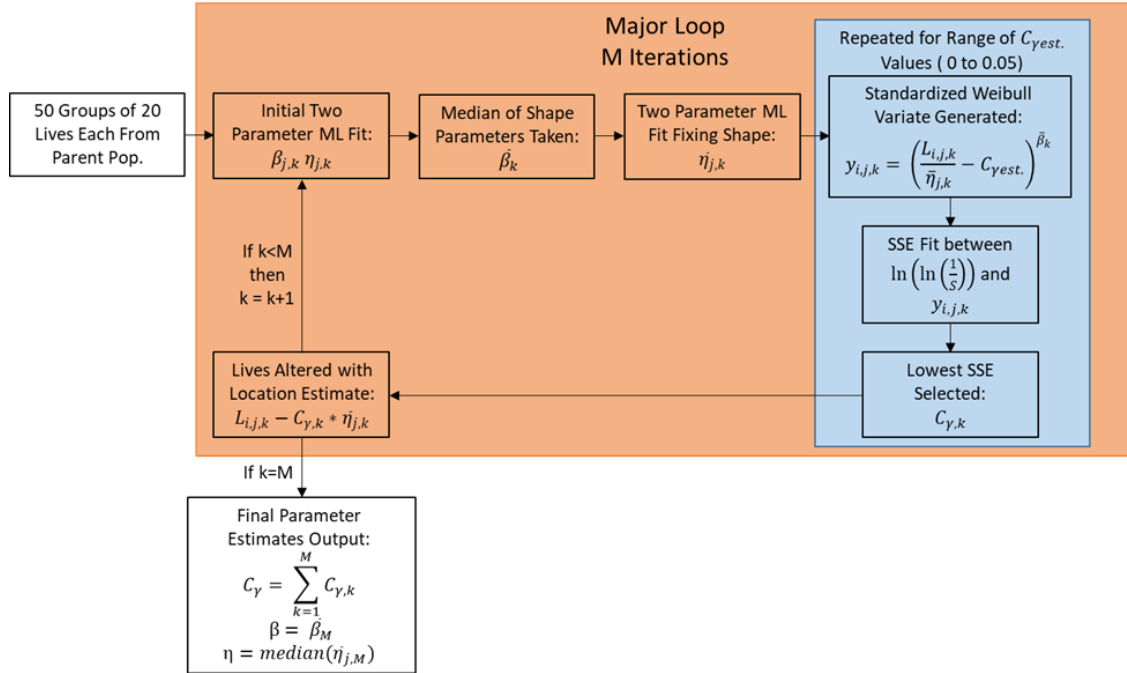


Figure 5.— Flow of iterative process to estimate minimum life coefficient using the general standardized Weibull variate.

Monte Carlo simulations were conducted to assess the efficacy of this fitting method. The fitting method makes use of estimates of population parameters to pool data via the standardized Weibull variate [eq. 6], and then the estimated minimum life coefficient C_γ is determined iteratively from the pooled data. The resulting C_γ is only an estimate of the true value, the deviation depending on the total number of datapoints available, the group sizes, and the random sampling effect. Exact mathematical treatment of the sampling distribution for C_γ is not available, and so Monte Carlo technique was used to study the convergence behavior of the iterative method and to quantify the median of the sampling distribution for C_γ .

The Monte Carlo studies were done using a group size (20) and number of groups (50) to imitate the available NASA gear fatigue database. To generate Monte Carlo data, a parent population of 10,000 virtual gears was defined. Each ordered member of the parent population was assigned a failure probability using median rank. Shape, scale and location parameters were selected, and thereby the life of each virtual gear was calculated corresponding to the failure probability. For each Monte Carlo simulation, 1,000 random selections were made from the parent population creating 50 groups of 20 samples each. The characteristics of the fitting method will now be described using one example Monte Carlo simulation. This example used a parent population with a shape factor of 1.30, a

scale factor of 100 hours, and a minimum life coefficient of 0.030. To highlight the random sampling effect, each group created by Monte Carlo was sorted according to the fatigue life values. Figure 6 plots the lifetimes of the sorted samples for all 50 groups, and the trends illustrate the wide range of results that should be expected for a group size is 20 even when all samples come from one population. To effectively explore the high reliability regime using such a database having modest group sizes, pooling of multiple groups is required.

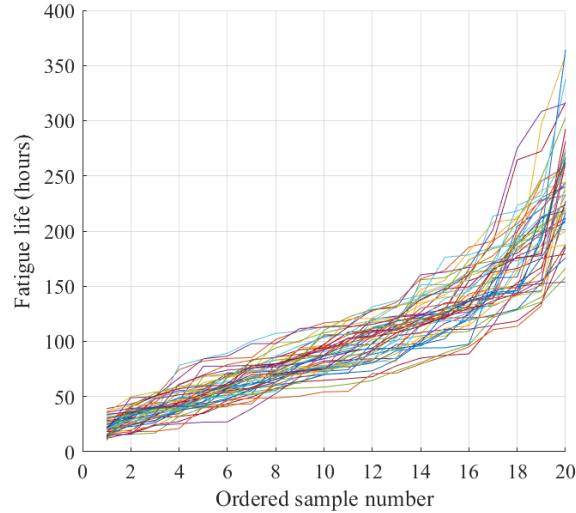


Figure 6.—Fatigue lives of 50 groups of 20 Monte Carlo random samples, samples ordered by fatigue life values within each group.

The iterative process was then applied to the sample groups of Figure 6 to test the capability to determine the minimum life coefficient. The procedure converged to stable values for the common shape parameter β_j and minimum life coefficient C_γ after 10 iterations. The trending for the minimum life coefficient with successive iterations is provided in Figure 7. This stable convergence trend was typical of all the Monte Carlo simulations regardless of the values used for the parent population. The final fit of the data for the example Monte Carlo simulation after the ten iterations is provided in Figure 8. From this plot, one can surmise that a dataset of size approximately 1000 datapoints are of sufficient number to reliably assess if a non-zero minimum life coefficient exists for experimental data, and the method appears to be of practical accuracy.

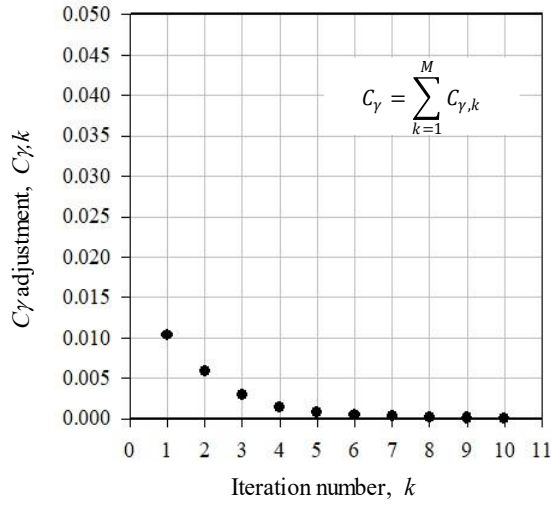


Figure 7.— Successive adjustments to the minimum life coefficient as a function of iteration number k .

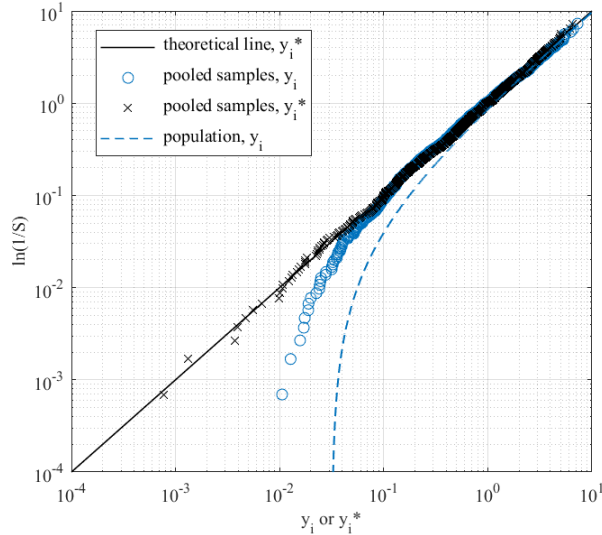


Figure 8. Final data fit of the Monte Carlo example simulation.

To further assess the efficacy of the iterative method to estimate the minimum life coefficient from pooled data, a matrix of Monte Carlo simulations was completed. For this study, the parent populations covered a full matrix of shape and minimum life parameters. The shape parameter of the parent populations ranged from 1.2 to 1.6 in increments of 0.1 and minimum life coefficients varied from 0.02 to 0.06 in increments of 0.01. The scale parameter of the parent population was 1.0 for each case. For each combination of shape and minimum life coefficient, the Monte Carlo simulation were completed 2000 times with a different random sampling for each simulation. With all simulations completed, the median value of the final solutions were collected, assessed, and plotted per Figure 9. From this plot, we see that regardless of the parent population, the minimum life coefficient tends to be

underestimated. Kotzalas (Ref. 14) also noted such a characteristic of the pooling-and-fitting methods of his study. From a practical standpoint, one can consider that fitting the data to the three-parameter distribution provides, in general, conservative estimates of guaranteed minimum lifetime for high probabilities of survival. The larger the value of the parent population's minimum life coefficient, the more capable is this methodology of detecting any difference from zero. The method is more accurate for smaller values of the shape parameter. From these Monte Carlo studies we judge the method as capable and valuable for the study of experimental data.

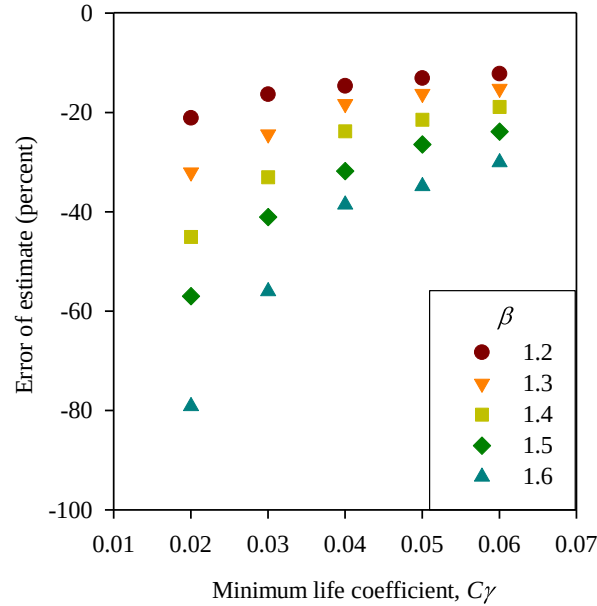


Figure 9— Trends of error of results in estimating the minimum life coefficient $C\gamma$.

5. Gear Surface Fatigue Database - Test Facility, Procedure, Test Gears, and Results

Laboratory data from prior research studies were collected for the present study. The data were produced using the NASA Glenn Research Center's gear test apparatus during a 27-year period (1973-2000). The test rig (Figure 10), first described in Ref. (26), is belt driven by an electric motor. The rig uses the four-square principle, and thus the motor only needs to overcome the frictional losses in the system. A depiction of the apparatus is shown in Figure 10(b). Oil pressure and leakage replacement flow is supplied to load vanes through a shaft seal. As the oil pressure is increased to the cavity inside one of the reaction gears increasing torque is applied to its shaft. This torque is transmitted through the test gears and back to the reaction gears, and thereby with motion power is circulated. The desired torque and corresponding stress levels on the test gear teeth are obtained by adjusting the hydraulic pressure. The test gear design parameters are given in Table I. Tolerances for the gear geometries were specified to meet

American Gear Manufacturers Association (AGMA) 2000-A88 quality level class 12 (Ref. 27). The two identical test gears are started under no load, and then load is applied gradually with gears rotating. The gears were tested with the faces offset, as shown in Figure 10(b), by 3.3 mm (0.130 in.) to give an active face width of 3.0 mm (0.120 in.). The gears had no lead crowning but do have a nominal 0.13-mm- (0.005-in.-) radius edge break at the tips and sides of the teeth. Tests began using a running-in torque of 14 Nm (130 in-lb) for at least 1 hour. After run-in, torque was then increased to the test torque for the remainder of the fatigue test. Most of the tests used a testing torque of 72 Nm (640 in-lb). For such torque, the peak of the Hertz pressure distribution for line contact condition at the pitch-line and static torque equilibrium is 1.7 GPa (250 ksi). Two other tests torque values were used for 19 tests at each of these two torques producing peak Hertz pressures of 1.5 GPa (220 ksi) or 1.9 GPa (280 ksi), respectively.

The gears were tested at 10,000 rpm, and therefore the pitch-line velocity was 46.5 m/s (9154 ft/min). Inlet and outlet oil temperatures were monitored. Cooled lubricant was supplied to the inlet of the gear mesh at 0.8 liter/min (49 in.³/min) and 320 ± 7 K (116 ± 13 °F). The lubricant outlet temperature was recorded and observed to have been 348 ± 4.5 K (166 ± 8 °F). The lubricant was circulated through a 5- μ m- (200- μ in.-) rated fiberglass filter to remove wear particles. The reaction gears were lubricated by a separate lubricating system isolated from the test gear system with shaft seals. The tests ran continuously until a vibration detection transducer automatically stopped the rig, and then the gears were inspected for surface fatigue defined as one or more spalls or pits covering at least 50 percent of the width of the line contact on any one tooth. The fatigue was either spalling or pitting. The database does not include any instances of failure by micropitting. If the gear pairs operated for more than 500 hours (300 million stress cycles) without failure, the test was usually suspended. A small number of tests were operated for up to 1000 hours (600 million cycles).

Data for this study came largely from previously published NASA Technical Notes, Memorandum and Reports having open, unrestricted distribution. Some data were from laboratory notes available to the authors. Data from as many tests as possible were gathered, but some previous NASA-conducted studies were excluded if the gears tested were not representative of aerospace quality. For example, data for experimental steels for which heat treatments were not yet satisfactory were judged not representative and thereby excluded from this study. Fatigue data for 50 unique “groups” of gears were utilized. Herein a “group” was a set of gears manufactured in one lot and then tested using one lubricant formulation. The gear fatigue database of this study is summarized in Table II. The steel alloys

and gear technologies represented in the database are listed in Table III. The tests were conducted in from year 1973 to 2000. The total dataset comprised 950 data points. The iterative method discussed in Section 4 was applied to this data, with a slight adjustment to the method. In fitting the gear fatigue data, for each C_{test} value the sum squared error (SSE) between the lower one-third of the total number of standardized Weibull variates and the twice natural logged of one over survivability was computed. In the Monte Carlo studies described above, the lower one-half of the total number of standardized Weibull variates was utilized for this step of the iterative process. This ad hoc adjustment was based on a visual assessment of the final fit, favoring a good fit in the lower tail of the distribution while still utilizing a substantial number of datapoints. Per inspection of the result as a Weibull variate plot (Figure 11), the data is effectively described using the three-parameter Weibull distribution. The resulting converged solution was a common shape parameter of 1.3 and a minimum life coefficient of 0.041.

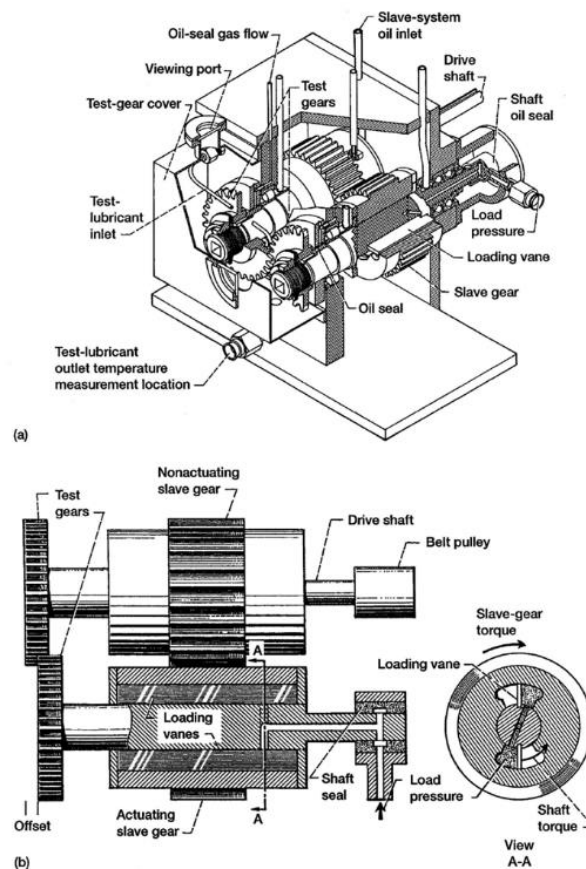


Figure 10.—NASA Glenn Research Center gear fatigue test apparatus. (a) Cutaway view. (b) Schematic view.

Table I.—Spur test gear design parameters.

Number of teeth	28
Module, mm	3.175
Diametral pitch	8
Circular pitch, mm (in.)	9.975 (0.3927)
Whole depth, mm (in.)	7.62 (0.300)
Addendum, mm (in.)	3.18 (.125)
Chordal tooth thickness reference, mm (in.)	4.85 (0.191)
Pressure angle, deg.	20
Pitch diameter, mm (in.)	88.90 (3.500)
Outside diameter, mm (in.)	95.25 (3.750)
Root fillet, mm (in.)	1.02 to 1.52 (0.04 to 0.06)
Measurement over pins, mm (in.)	96.03 to 96.30 (3.7807 to 3.7915)
Pin diameter, mm (in.)	5.49 (0.216)
Backlash reference, mm (in.)	0.254 (0.010)
Tip relief, mm (in.)	0.010 to 0.015 (0.0004 to 0.0006)

Table II.—Summary of Gear Fatigue Database

Total number of fatigue tests	950
Number of groups	50
Tests resulting in fatigue	770
Tests suspended prior to fatigue	180
Average number tests per group	19
Average number failed per group	15

Table III.—Alloys and Technologies Tested

<u>Alloys</u>	<u>Melting Methods</u>
AISI 1552	VAR
AISI 4340	CVM
AISI 4620	VIM-VAR
AISI 4640	<u>Hardening Methods</u>
AISI 9310	Case Carburize
CBS600	Induction Harden
CBS1000M	Through Harden
Ferrium C69	Nitride
M50	<u>Other Technologies</u>
M50NiL	(15) Lubricants
Nitralloy N	(5) Lube Additives
Pyrowear53	Standard Forged
Super Nitralloy	Ausforged
Vasco Matrix II	Superfinishing
Vasco MAX 350	DLC Coatings
	Shot Peening

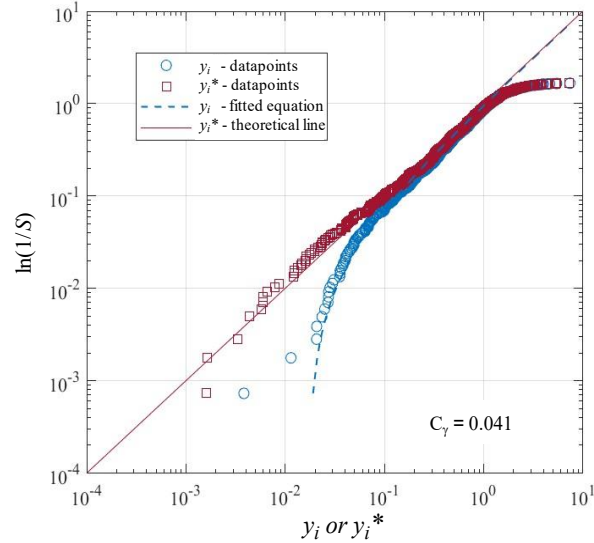


Figure 11— Standardized Weibull variate plot of NASA gear fatigue data, full dataset result.

To compare these results for gears to previous studies for bearings, one can compare the values for the minimum life coefficient C_γ in a consistent manner using eq. [2]. Tallian (Ref. 9) did not explicitly state a value for C_γ , but he plotted the pooled variate datapoints with an “interpolated line” and noted the line’s asymptote value on the abscissa was 0.004. Denoting the asymptote of the y_i trend as “Z”, the relation to the minimum life coefficient is

$$C_\gamma = Z^{(1/\beta)}. [7]$$

The mean value of the group's shape parameters of Tallian's dataset was 1.3 {per author's closure (Ref. 9)}.

Applying eq. [7] using $Z = 0.004$ and $\beta = 1.3$, Tallian's data result is $C_\gamma = 0.014$. Executing the same methodology to Kotzalas' plot (Ref. 14), using an asymptote value of $Z = 0.02$ and his stated value for $\beta = 1.5$, then Kotzalas's data result is $C_\gamma = 0.074$. These values for bearings are of similar magnitude as for the gear data of this study.

Note that the minimum life coefficients from the two prior bearing studies and this study for gears, when ordered from smallest to largest, correlate to the timeframes when the tests were conducted from oldest to most recent. We propose that a primary explanation for the correlation is material cleanliness, which in general improved during such timeframe. According to analytical modeling results, the minimum life increases with improvements to cleanliness of the material (Refs. 15–17). Empirical data also suggest that minimum life coefficient may be strongly influenced by the material cleanliness. Consider results from Morrison, et al. from bearings tests (Ref. 28) whereby starting with a single ingot of air melted AISI 52100 steel, some of the material was re-melted progressively using consumable electrode vacuum melting. Bearing inner races were machined from the first, second, and fifth remelts, and then bearing fatigue tests were completed. With each re-melt a progressive reduction of nonmetallic content occurred, and the L10 fatigue lifetimes increased accordingly. The L10 fatigue lifetime of the fifth-remelted material was four times greater than that of the air-melted material. Another study with corroborating evidence comes from a study by Bamberger and Clark to evaluate influence bearing steel processing methods on fatigue using rolling contact fatigue tester (Ref. 29). Cleaner material as was made by vacuum-induction melt followed by vacuum-arc-remelt (VIM-VAR) had a L10 fatigue lifetime 1.9 times greater than baseline material produced using air-melt followed by vacuum-arc remelt (AM-VAR) process. More recent studies have quantified steel cleanliness and have linked the relationships of inclusion size to gear bending fatigue performance (Ref. 30) and rolling contact fatigue performance (Ref. 31).

To further explore if better material cleanliness correlates with larger values for minimum life coefficient, the gear surface fatigue data from more recent timeframe (1988 to 2000) was segregated from the total dataset, and the data was fit by the iterative method. The dataset size was roughly one-half of the total of 950 gear datapoints. The resulting fitted statistical distribution has a minimum life coefficient C_γ equal 0.062 and shape parameter β equal 1.0. The fitted distribution is presented graphically in Figure 12. The gear and bearing results are compared and

summarized in Table IV. Assuming that the cleanliness of the material was improved for the more recently made test articles, the evidence is compelling that better material cleanliness increases the fatigue life capability and increases the minimum life coefficient. Of course, other factors will influence the statistical distribution and the value of the minimum life. Echoing statements of Kotzalas (Ref. 14), more work should be undertaken to understand the influence of component type, design technologies, and material cleanliness levels on the distribution of fatigue life.

Table IV.— Minimum life coefficients of surface fatigue data for bearings and gears.

Data source	Test article	Years data was produced	Minimum life coefficient C_γ
Tallian (Ref. 9)	Roller bearings	Prior to 1960	0.014
Cameron - Krantz	Spur gears	1973-2000	0.041
	Spur gears	1988-2000	0.062
Kotzalas (ref. 14)	Tapered roller bearings	1985-2003	0.074

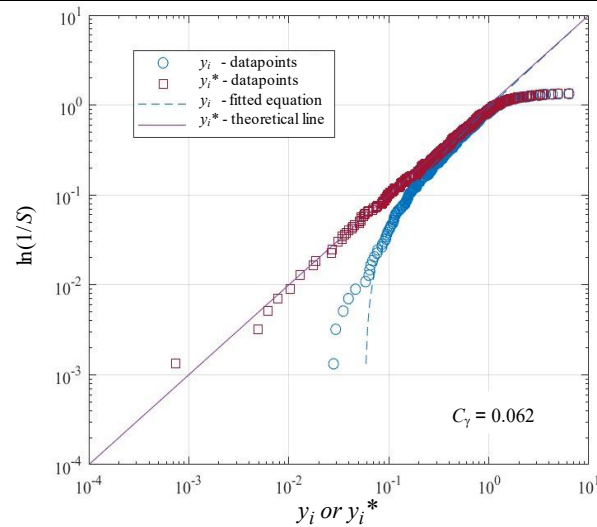


Figure 12.— Standardized Weibull variate plot of NASA gear fatigue data, more recent data result.

This study demonstrates that describing gear surface fatigue using a three-parameter rather than a two-parameter Weibull distribution more accurately describes gear surface fatigue in the high reliability regime. Such result is consistent with prior published studies of bearings. The three-parameter equation form given by eq. [3] could be used with the results of Table IV to quantify fatigue in the high reliability regime for data sets with a modest number of data points after fitting of the remaining two free parameters. To illustrate, an example dataset for alloy EX53 (Ref. 32) was analyzed fitting the distribution parameters to both eq. [3] and eq. [4] by least squares regression of median ranks. The number of tests was 21, with 15 tests resulting in fatigue while 6 tests were suspended after 300

million cycles with no fatigue. Results of the regression are provided in Table V. The total sum-squared error of regression (SSE) was smaller for the three-parameter solution (SSE=1.23) compared to the two-parameter solution (SSE=1.74) validating that the three-parameter solution models the data with improved accuracy. From the fitted equations, the estimated fatigue lives for 99% cumulative probability of survival are 14.8×10^6 cycles for the three-parameter solution and 8.7×10^6 cycles for the two-parameter solution. Such a difference is of practical significance. The improved accuracy using the three-parameter equation will allow for more reliable utilization of gear fatigue experiments for research, development, and design purposes. Some may be reluctant to abandon use of the two-parameter Weibull distribution that has well-served the bearing and gear designers and technologists. However, in applying the Weibull distribution to fatigue data we suggest a viewpoint that all Weibull distributions are defined by three parameters, and the two-parameter distribution is the specialized case whereby the minimum life coefficient is assumed as equal to zero. For purposes of quantifying fatigue in the high-reliability regime, the results summarized in Table IV provide values for the minimum life coefficient that are based on large numbers of fatigue tests.

Table V.— Results of fitting fatigue data for alloy EX53 (Ref. 32) to the three-parameter (eq. [3]) and two-parameter (eq. [4]) Weibull distributions.

Weibull distribution	Parameters			Fatigue life (cycles 10^6)	
	β	η	C_γ	$S = 0.95$	$S = 0.99$
Three-parameter; eq. [3]	1.49	170	0.041	30.2	14.8
Two-parameter; eq. [4]	1.56	167	—	24.8	8.7

6. Summary and Conclusions

NASA gear fatigue test results were gathered and pooled together to study the statistical distribution of gear surface fatigue lives. Former studies of bearing data employed the standard Weibull variate for pooling of data. This work defined and employed a generalized standard Weibull variate to apply the concept of a non-zero minimum life coefficient to the standardization of fatigue data. A method was developed for determining the minimum life coefficient and Weibull shape parameter for pooled datasets comprising multiple fatigue studies using an iterative solution method. The convergence behavior and efficacy of the newly proposed iterative solution method was

established using results from Monte Carlo simulations. Using results from multiple NASA gear test projects numbering 950 datapoints in total, the pooled dataset was fit to a three-parameter Weibull distribution and the minimum life coefficient was determined. Additional analyses were done using bearing and gear data that demonstrate a trend of increasing minimum life coefficient as correlated to the timeframe of the fatigue testing, and material cleanliness is proposed as a primary explanation for such correlation. It is noted that this observed trend supports published findings from analytical modeling (Refs. 15-17) and experimental studies (Refs. 28-31) that improvements to material cleanliness improves minimum fatigue life. Further use and study of the three-parameter Weibull distribution is recommended as a tool to quantify fatigue life most accurately in the high reliability regime.

Acknowledgements

The authors acknowledge the support of the NASA Revolutionary Vertical Lift Technology Project which sponsored this work and the many researchers, engineers, and technicians whose efforts and dedication provided the gear fatigue data.

References

1. American Gear Manufacturers Association. Design Guidelines for Aerospace Gearing. AGMA 911-A94, Alexandria, Va.: American Gear Manufacturers Association, (1994).
2. Coy, John J. Townsend Dennis P. and Ervin V. Zaretsky. "Analysis of Dynamic Capacity of Low-contact-ratio Spur Gears Using Lundberg-Palmgren Theory". TN D-8029, NASA, Lewis Research Center, (1975).
3. Vijayakar, S., "Contact and Bending Durability Calculation for Spiral-Bevel Gears", Project Report: for NASA/Glenn Research Center PO: NNC15QA19P, Advanced Numerical Solutions, Inc., Hilliard, Ohio (2016).
4. Zaretsky, E. V., Ed., STLE Life Factors for Rolling Bearings, STLE Publication SP-34, Society of Tribologists and Lubrication Engineers, Park Ridge, IL, 2nd edition (1999).
5. Vlcek, B., Hendricks, R. and Zaretsky, E., "Determination of Rolling-Element Fatigue Life from Computer Generated Bearing Tests," Annual Meeting of Society of Tribologists and Lubrication Engineers, (2003).
6. Weibull, W., "A Statistical Representation of Fatigue in Solids," Transactions of the Royal Institute Technology Stockholm, n. 27 (1949).
7. Weibull, W., "A Statistical Distribution Function of Wide Applicability." Journal of Applied Mechanics 103.730 (1951): 293-297.
8. Lundberg, G., and Palmgren, A., "Dynamic Capacity of Rolling Bearings," Acta Polytech., Mech. Eng. Ser 1 (3) (1947).
9. Tallian, T., "Weibull Distribution of Rolling Contact Fatigue Life and Deviations Therefrom," ASLE Trans., 5, pp.183-196 (1962).
10. Snare, B., "How Reliable are Bearings?," Ball Bearing J., 162, pp. 3-5 (1970).
11. Takata, H., Suzuki, S., and Maeda, E., "Experimental Study of the Life Adjustment Factor for Reliability of Rolling Element Bearings," Proc. JSLE Int. Trib. Conf., pp. 603-608. (1985).
12. Shimizu, S., Sharma, C., and Takeki, S., "Life Prediction of Linear Rolling Element Bearings: A New Approach to Reliable Life Assessment," ASME J. Tribol., 124(1), pp. 121-128. (2002).
13. Ferreira, J., Balthazar, J., and Araujo, A., "An Investigation of Rail Bearing Reliability Under Real Conditions of Use," Eng. Failure Anal., 10, pp. 745-758. (2003).
14. Kotzalas, M., "Statistical Distribution of tapered Roller Bearing Fatigue Lives at High Levels of Reliability," ASME Journal of Tribology, vol. 127, (2005).
15. Tallian, T., "A Unified Model for Rolling Contact Life Prediction", ASME J. of Tribology, Vol. 104, pp. 336-356 (1982).

16. Sadeghi, F., Jalalahmadi, B., Slack, T.S., Raje, N. and Arakere, N.K., "A Review of Rolling Contact Fatigue." *Journal of Tribology*, Vol 131, (2009).
17. Walvekar, A.A. and Sadeghi, F., "Rolling Contact Fatigue of Case carburized Steels," *International Journal of Fatigue*, 95, (2017).
18. Tridello, A., Boursier Niutta, C., Rossetto, M., Berto, F. and Paolino, D.S., "Statistical Models for Estimating the Fatigue Life, the Stress–Life Relation, and the P–S–N Curves of Metallic Materials in Very High Cycle Fatigue: A Review." *Fatigue & Fracture of Engineering Materials & Structures*, 45(2), (2022).
19. Li, C., Wu, S., Zhang, J., Xie, L., & Zhang, Y., "Determination of the Fatigue PSN curves–A critical Review and Improved Backward Statistical Inference Method." *International Journal of Fatigue*, 139, (2020).
20. Strzelecki, P., "Determination of Fatigue Life for Low Probability of Failure for Different Stress Levels Using 3-parameter Weibull Distribution." *International Journal of Fatigue*, 145 (2021).
21. Pascual, F. and Meeker, W., "Estimating Fatigue Curves with the Random Fatigue-Limit Model." *Technometrics* 41, no. 4 (1999).
22. Bathias, C., "Gigacycle Fatigue of Bearing Steels." *Materials Science and Technology* 28, no. 1 (2012).
23. Gabelli, A., Lai, J., Lund, T., Rydén, K., Strandell, I., & Morales-Espejel, G. E., "The Fatigue Limit of Bearing Steels–Part II: Characterization for Life Rating Standards". *International Journal of Fatigue*, 38, (2012).
24. Furuya, Y., Hirukawa, H., and Takeuchi, E., "Gigacycle Fatigue in High Strength Steels." *Science and Technology of Advanced Materials*, 20, no. 1, (2019).
25. Murakami, Y., Takagi, T., Wada, K., & Matsunaga, H. "Essential Structure of SN Curve: Prediction of Fatigue Life and Fatigue Limit of Defective Materials and Nature of Scatter." *International Journal of Fatigue*, 146, (2021).
26. Townsend, D. P., Chevalier, J.L., and Zaretsky, E.V., "Pitting Fatigue Characteristics of AISI M–50 and Super Nitralloy Spur Gears," NASA TN D–7261, <https://ntrs.nasa.gov/api/citations/19730012684/downloads/19730012684.pdf>, (1973).
27. AGMA 2000–A88. *Gear Classification and Inspection Handbook: Tolerances and Measuring Methods for Unassembled Spur and Helical Gears (Including Metric Equivalents)*.
28. Morrison, T., et al. "The Effect of Material Variables on the Fatigue Life of AISI 52100 Steel Ball Bearings," *ASLE Transactions*, 5, No.2 (1962).
29. Bamberger, E., and Clark, J., "Development and Application of the Rolling Contact Fatigue Test Rig", in *Rolling Contact Fatigue Testing of Bearing Steels*, ASTM STP 771, J. Hoo, Ed. (1982).
30. Roy, T., De, N., Smith, C. and Wink, C., "Evaluation of Steel Cleanliness by Extreme Value Statistics and its Correlation with Fatigue Performance.", AGMA 19FTM12, American Gear Manufacturers Association, Washington, D.C., (2019).
31. Cao, Z., Shi, Z., Yu, F., Wu, G., Cao, W., & Weng, Y., "A New Proposed Weibull Distribution of Inclusion Size and its Correlation with Rolling Contact Fatigue Life of an Extra Clean Bearing Steel." *International journal of fatigue* 126 (2019).
32. Townsend, D., "Surface Fatigue Life and Failure Characteristics of EX-53, CBS 1000M, and AISI 9310 Gear Materials," NASA Technical Paper 2513, <https://ntrs.nasa.gov/api/citations/19860003141/downloads/19860003141.pdf> (1985).