Evaluation of Heat Transfer Coefficient from Velocity Distribution in Boundary Layer

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Abstract

This paper describes a method of evaluating heat transfer coefficient from a velocity distribution in the boundary layer. The power law velocity profile and universal velocity distribution in a smooth pipe have been used to evaluate the wall shear stress, and the heat transfer coefficient was evaluated from the wall shear stress using the Chilton-Colburn analogy. The predicted pressure drop was compared with the pressure drop computed by the Colebrook equation, and the predicted heat transfer coefficient was compared with the heat transfer coefficient calculated by the Dittus-Boelter equation. The accuracy of pressure drop prediction was within 1%, and the accuracy of heat transfer coefficient prediction was within 8%. This method has the potential to use a Navier-Stokes based CFD solution to evaluate the heat transfer coefficient in nodal or network flow codes.

1.Introduction

The knowledge of heat transfer coefficient is critical for an accurate numerical simulation of many Cryogenic Fluid Management (CFM) applications such as Tank Pressurization. Nodal or Network Flow Analysis tools are frequently used to estimate the pressurant requirement to pressurize a cryogenic tank which requires an accurate estimation of the heat transfer coefficient. The accuracy of prediction of Nodal Codes largely depends on the accuracy of empirical correlations used to estimate the heat transfer coefficients. For many common engineering applications such as flow in a circular tube, accurate correlations for evaluating heat transfer coefficients [1] are available. However, in non-conventional applications such as pressurization of a cryogenic tank and many CFM applications, there is a limited availability of accurate correlations for cryogenic fluid systems. The development of new correlations by performing a laboratory experiment is expensive and time-consuming. A possible alternative is to evaluate the heat transfer coefficient by performing Navier-Stoked based CFD calculations.

This paper describes a methodology of evaluating wall shear stress from the Universal Velocity Distribution [2] and determining the heat transfer coefficient from wall shear stress using the Chilton-Colburn analogy [3]. The methodology has been applied to evaluate wall shear stress and heat transfer coefficient in a pipe flow with smooth wall.

2. Friction and Heat Transfer Modeling in Nodal Codes

Nodal codes such as Generalized Fluid System Simulation Program (GFSSP) [4] use empirical correlations to evaluate friction and heat transfer in the boundary layer near the wall where viscous effects are predominant. In the nodal codes there is no attempt to discretize the region into a finer grid to accurately capture the gradient of temperature and velocity to estimate the transport of heat and momentum. Instead, pressure drop in a pipe is estimated from frictional loss which depends on friction factor and average velocity in the pipe. The friction factor is estimated from the Colebrook equation [5] and the heat transfer coefficient is determined from the Dittus-Boelter equation [1]. Both equations are empirically derived from experimental data and only valid in the ranges where experimental data were available.

2.1 Moody's Diagram (Colebrook Equation)

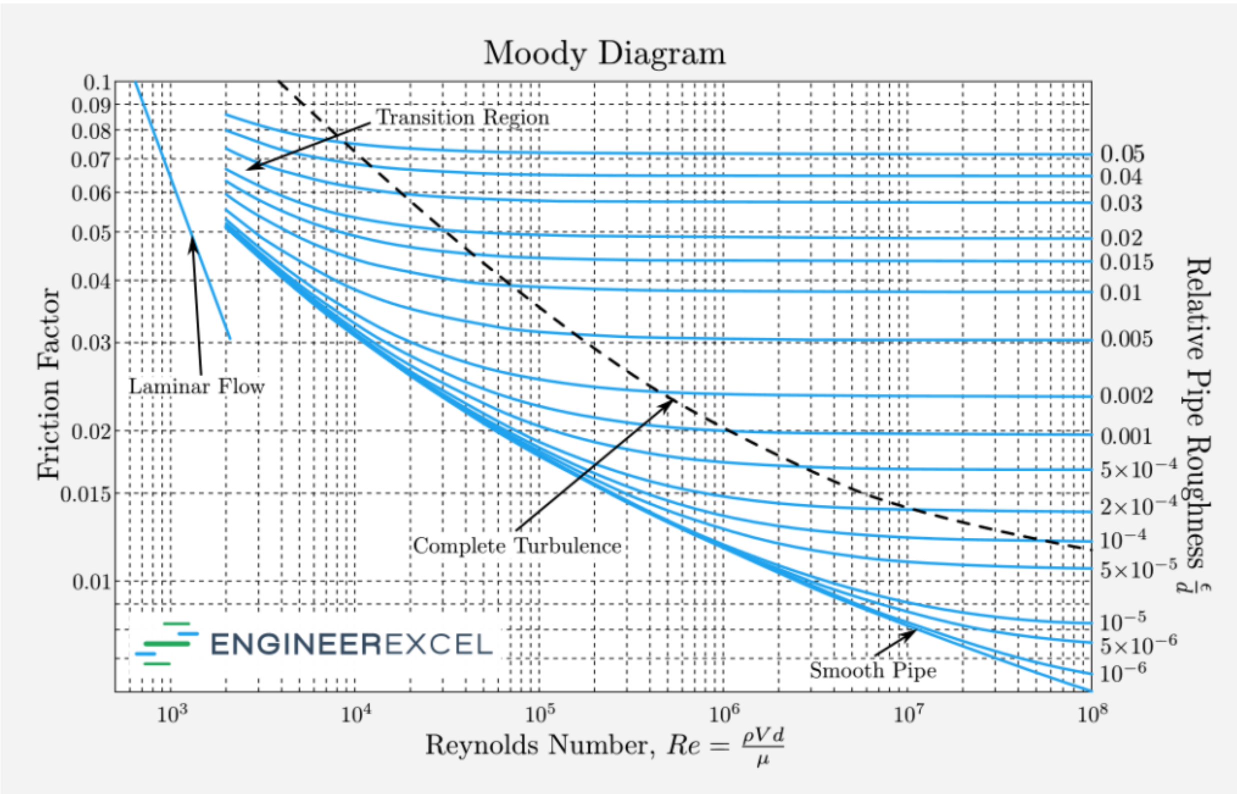

Figure 1. The relationship of Friction Factor with Reynolds Number in a circular pipe

The frictional pressure drop in a pipe can be expressed as

$$h_f = \frac{fLV^2}{2Dg} \qquad (1)$$

f is the friction factor to be determined from Moody's diagram shown in Figure 1. Figure 1 and Equation (1) are used for engineering calculation. However, in GFSSP, pressure drop is computed by using the following equation:

The frictional pressure drop in a pipe is expressed as

$$\Delta \mathrm{P} = K_f \dot{m}^2 \qquad (2)$$

Frictional resistance factor, K_f is expressed as

$$K_f = \frac{8fL}{\rho_u \pi^2 D^5 g_c} \qquad (3)$$

For laminar flow, the friction factor is expressed as

$$f = \frac{64}{Re_D} \qquad (4)$$

For turbulent flow the friction factor is calculated from the Colebrook equation

$$\frac{1}{\sqrt{f}} = -2\log\left[\frac{\varepsilon}{3.7D} + \frac{2.51}{Re\sqrt{f}}\right] \qquad (5)$$

2.2 Dittus-Boelter Equation

The heat transfer between solid wall and fluid is expressed as:

$$\dot{Q} = hA\left(T_w - T_f\right) \qquad (6)$$

The heat transfer coefficient, *h* is calculated from the Dittus Boelter Equation given in Eqn (7)

$$Nu = 0.023\, Re^{0.8} Pr^{0.33} \qquad (7)$$

The Dittus Boelter Equation is experimentally derived and expressed in terms of Nusselt Number, Nu,Reynolds Number, Re, and Prandtl Number, Pr, which are defined as:

$Nu = \frac{hD}{k}$; $Re = \frac{\rho u D}{\mu}$; $Pr = \frac{C_p \mu}{k}$

3. Power Law and Universal Velocity Distribution

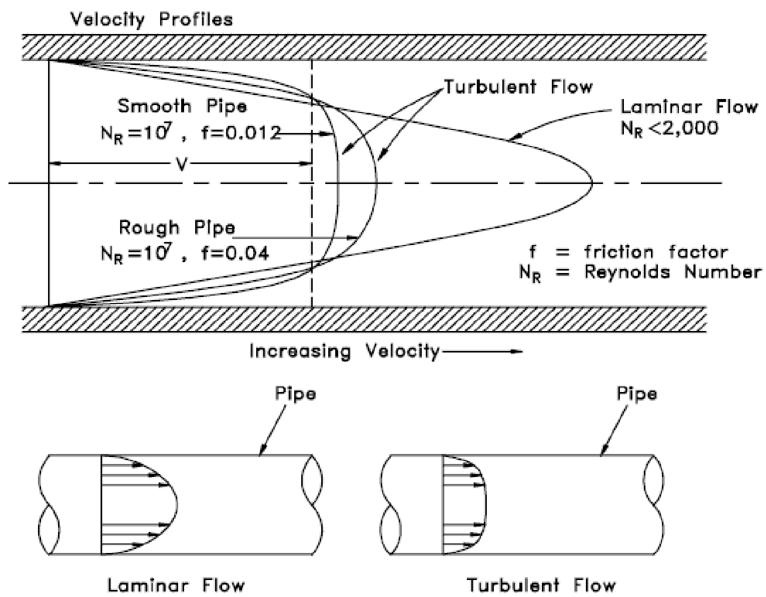

Figure 2. Velocity Distribution in a Pipe

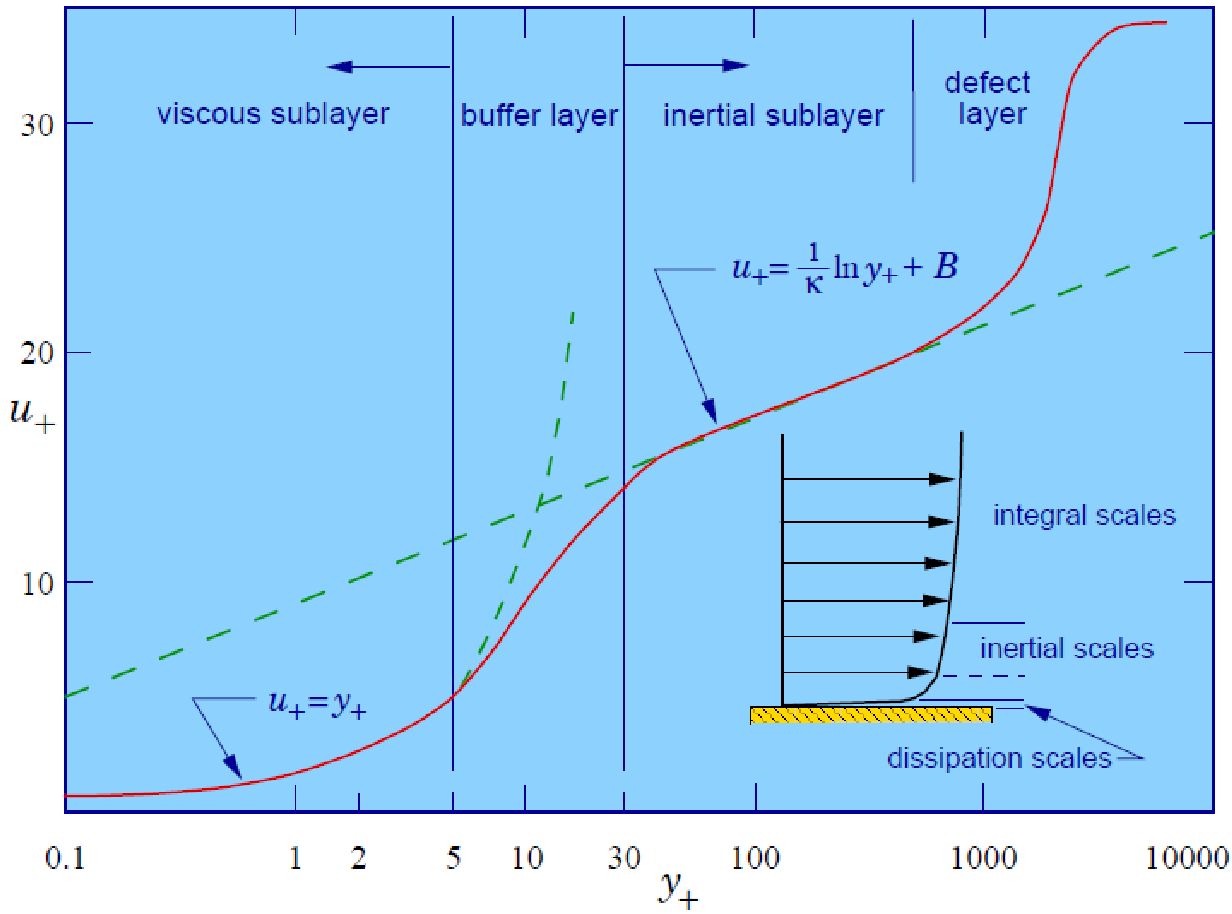

Figure 3. Universal Velocity Distribution in Turbulent Boundary Layer

Figure 2 shows the velocity distribution in a pipe flow for both laminar and turbulent flow. For laminar flow, the velocity distribution is parabolic and can be determined analytically. However, for turbulent flow the velocity distribution is determined experimentally and can be expressed as a power law [2] as shown in Equation 8.

$$\frac{u}{U} = \left(\frac{y}{R}\right)^{1/n} \qquad (8)$$

The exponent of the power law, n is a function of Reynolds number. Equation 8 can also be expressed in terms of average velocity, $\bar{u}$ instead of center line velocity, U as shown in Equation 9.

$$\frac{u}{\bar{u}} = \frac{(n+1)(2n+1)}{2n^2}\left(\frac{y}{R}\right)^{1/n} \qquad (9)$$

Where the value of n actually depends on Reynolds number and it increases with Reynolds number. The Table 1 below shows the range of the parameter n as a function of Reynolds number.

Table 1. Power Law Index as a function of Reynolds number [2]

Reynolds number	n
4000	6
23,000	6.6
110,000	7
1.1e6	8.8
2.e6	10
3.2e6	10

The universal distribution in turbulent boundary layer is shown in Figure 3. The dimensionless velocity is plotted against non-dimensional distance. The dimensionless velocity (u_+) is normalized with the frictional velocity (u_τ), and non-dimensional distance (y_+) is a Reynolds number defined with the frictional velocity. The boundary layer can be divided into four regions: viscous sublayer, buffer layer, inertial sublayer and defect layer. The velocity profiles can be expressed as [6]:

Viscous sublayer

$$u_+ = y_+ \qquad (10)$$

Buffer Layer

$$u_+ = -3.05 + 5.0\, ln y_+ \qquad (11)$$

Inertial sublayer

$$u_+ = 5.5 + 2.5\, ln y_+ \qquad (12)$$

$$u_+ = \frac{u}{u_\tau} \;\; ; u_\tau = \sqrt{\frac{\tau_w}{\rho;}} \;\; ; y_+ = \frac{\rho u_\tau y}{\mu}$$

Note that the wall shear stress can be evaluated from Universal Velocity Distribution and will be described in the following section.

4. A Methodology to compute wall shear stress and heat transfer coefficient

GFSSP uses the method described in section 2 to calculate pressure drop in a pipe. In fluid mechanics, this method is known as Darcy's method. In this section an alternate method was used to evaluate pressure drop. In this method, the power law and universal velocity distribution were used to calculate pressure drop and compared with the conventional Darcy's method.

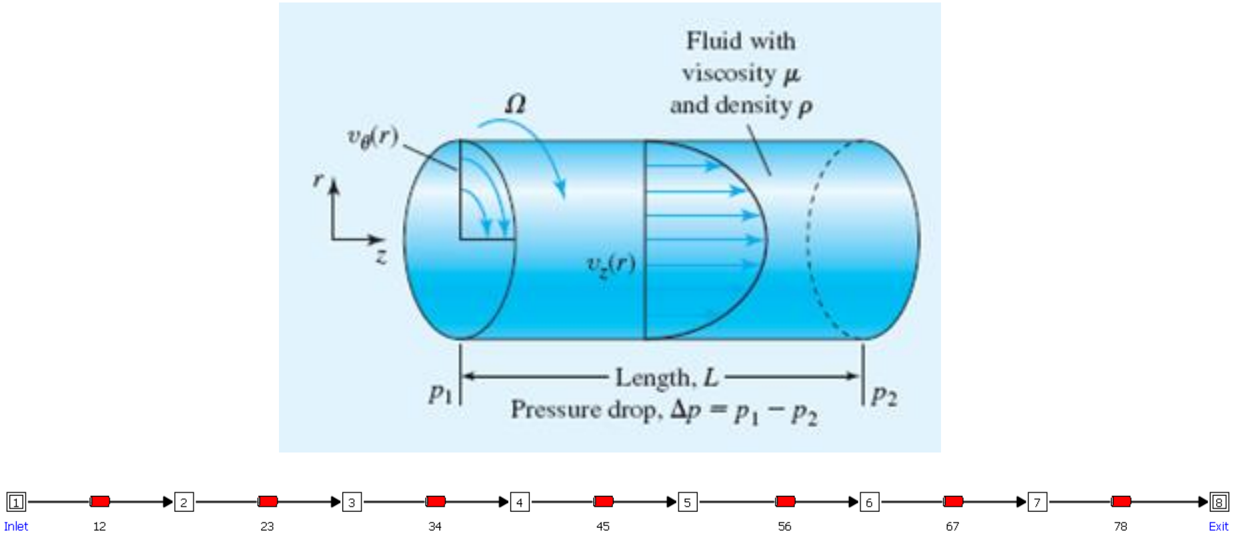

Figure 4. GFSSP Model of Pipe Flow

Figure 4 shows a GFSSP model of pipe flow. This model has been used to calculate pressure drop by conventional methods as well as the alternate method as described below.

4.1 <u>Iterative Algorithm to evaluate Wall Shear Stress from Velocity Profile</u>

To calculate shear stress from the universal velocity distribution, one needs to find the correct y_+ where the computed shear stress matches with experimental data. The Colebrook equation was considered as the benchmark because it was derived from experimental data. Following are the steps of the iterative algorithm

1. Calculate Pressure Loss Coefficient using Colebrook Equation (Equation 5)

$$C_p = \frac{p_{inlet} - p_{exit}}{0.5\, \rho_{inlet} u_{inlet}^2}$$

2. Guess a distance from the wall, y, and obtain velocity, u, from the power law (Equation 9)

3. Obtain Wall Shear Stress, τ_w by iteratively solving Universal Velocity Distribution (Equations 10 to 12)

4. Calculate C_p using the computed wall shear stress

5. Check if $C_{p,UVD}$ matches $C_{p,Colebrook}$

6. If not, adjust y and repeat steps 2 to 4 until both values of C_p match

7. Determine y+ when if $C_{p,UVD} \sim C_{p,Colebrook}$

To validate this algorithm several fluids are considered: Liquid water, liquid Oxygen, Gaseous Nitrogen, and Helium. The intent was to establish if there is a unique y_+ for all fluids where the computed shear stress matches with the Colebrook equation. The operating conditions for various case runs are given in Table 2.

Table 2. Operating Conditions for Parametric Study

Case Number	P_{inlet} (psia)	P_{exit} (psia)	T_{inlet} (Deg F)	Fluid	L/D
LIQUID					
1	15	14.7	60	**Water**	84
2	20	14.7	60	**Water**	84
3	25	14.7	60	**Water**	84
4	15	14.7	60	**Water**	168
5	20	14.7	60	**Water**	168
6	25	14.7	60	**Water**	168
7	50	49	-280	**LO2**	168
8	50	49.5	-280	**LO2**	168
9	50	49.75	-280	**LO2**	168
GAS					
10	15	14.7	60	**GN2**	168
11	16	14.7	60	**GN2**	168
12	18	14.7	60	**GN2**	168
13	50	14.7	60	**GHE**	168
14	30	14.7	60	**GHE**	168
15	20	14.7	60	**GHE**	168

4.2 Evaluation of Heat Transfer Coefficient from Wall Shear Stress

Once the wall shear stress was calculated, heat transfer coefficient can be evaluated from a boundary layer analogy. Momentum & Heat Transfer in Boundary Layer is related by Chilton-Colburn Analogy [3]:

$$\frac{C_f}{2} = St\,Pr^{2/3} \qquad 0.6 < Pr < 60;\ 10{,}000 < Re < 300{,}000 \qquad (13)$$

$$C_f = \frac{\tau_w}{\rho V^2/2}; \; St = \frac{h}{\rho V C_p}\; ; \; \mathrm{Pr} = \frac{C_p \mu}{k}$$

C_f is a non dimensional shear stress which has been evaluated from Universal Velocity Distribution (UVD) (Equations 10-12) discussed in the previous section.

The heat transfer coefficient can be evaluated from Equation 13.

$$h = \frac{C_p g_c}{V Pr^{2/3}} \tau_w \qquad (14)$$

5. Results and Discussion

The result of parametric study is shown in Table 3. The GFSSP model (Figure 4) was run with different fluids and different pressures at inlet and exit boundaries (Table 2) to obtain solutions at different Reynolds number and Prandtl number. Pressure coefficients computed by Colebrook equation and UVD method are compared. Heat transfer coefficients computed by Dittus-Boelter equation and Chilton-Colburn analogy are compared. The value of y_+ where pressure coefficients between the two methods match are also shown in the table. It was found that the pressure coefficients match when $y_+ \sim 28.54$. In the universal velocity distribution (Figure 3), this point is at the edge of the buffer layer and the inertial sublayer.

Table 3. Result of Parametric Study

Case Number	P_{inlet} (psia)	P_{exit} (psia)	T_{inlet} (Deg F)	Fluid	L/D	Reynolds Number	Prandtl Number	Pressure Coefficient (Colebrook)	Pressure Coefficient (UVD)	Fractional Difference in Cp	HT Coefficient (Dittus-Boelter)	HT Coefficient (Chilton-Colburn)	Fractional Difference in HTC	y^+	u^+
LIQUID															
1	15	14.7	60	**Water**	84	**33136**	7.94	1.9273	1.9272	**4.12E-05**	0.215	0.2149	**2.47E-04**	**29.1944**	13.82
2	20	14.7	60	**Water**	84	**165610**	7.94	1.3634	1.3615	**1.39E-03**	0.7787	0.7593	**2.49E-02**	**28.107**	13.6302
3	25	14.7	60	**Water**	84	**239391**	7.94	1.2684	1.2683	**7.88E-05**	1.0457	1.0218	**2.29E-02**	**27.9549**	13.6031
4	15	14.7	60	**Water**	168	**10113**	7.94	5.1729	5.1726	**5.80E-05**	0.1664	0.176	**5.77E-02**	**28.8799**	13.7659
5	20	14.7	60	**Water**	168	**51815**	7.94	3.482	3.4797	**6.61E-04**	0.6148	0.607	**1.27E-02**	**28.6727**	13.7298
6	25	14.7	60	**Water**	168	**75245**	7.94	3.2098	3.2076	**6.85E-04**	0.8285	0.8125	**1.93E-02**	**28.4152**	13.6847
7	50	49	-280	**LO2**	168	**193255**	2.28	2.6448	2.6424	**9.07E-04**	0.2375	0.2317	**2.44E-02**	**27.9813**	13.6078
8	50	49.5	-280	**LO2**	168	**131486**	2.28	2.86E+00	2.8515	**1.75E-03**	0.1745	0.1702	**2.46E-02**	**28.3684**	13.6765
9	50	49.75	-280	**LO2**	168	**89324**	2.28	3.0947	3.0918	**9.37E-04**	0.1281	0.1253	**2.19E-02**	**28.3915**	13.6806
GAS															
10	15	14.7	60	**GN2**	168	**25154**	0.72	4.1465	4.1452	**3.14E-04**	0.0066	0.0067	**1.52E-02**	**29.5469**	13.8631
11	16	14.7	60	**GN2**	168	**58274**	0.72	3.5061	3.5025	**1.03E-03**	0.013	0.0129	**7.69E-03**	**28.5526**	13.6264
12	18	14.7	60	**GN2**	168	**100504**	0.72	3.2572	3.256	**3.52E-04**	0.0201	0.0201	**0.00E+00**	**28.499**	13.5053
13	50	14.7	60	**GHE**	168	**151422**	0.67	3.858	3.8539	**1.06E-03**	1.66E-01	0.1794	**8.01E-02**	**28.1418**	12.793
14	30	14.7	60	**GHE**	168	**78984**	0.67	3.9826	3.9809	**4.27E-04**	0.0987	0.1037	**5.07E-02**	**28.4401**	13.0931
15	20	14.7	60	**GHE**	168	**38970**	0.67	4.1477	4.1447	**7.23E-04**	0.0561	0.0576	**2.67E-02**	**28.9479**	13.4865
					L = 84 inch D = 1 inch or 0.5 inch									**28.54**	

Heat Transfer Coefficient – Btu/sec-ft^2-R

The discrepancy in Pressure Coefficients computed with Colebrook Equation and UVD Method is shown in Figure 5 The maximum discrepancy is less than 0.7%. Discrepancy in Heat Transfer Coefficient computed by Dittus-Boelter Equation and Chilton-Colburn Analogy is shown in Figure 6. The maximum discrepancy is less than 8%.

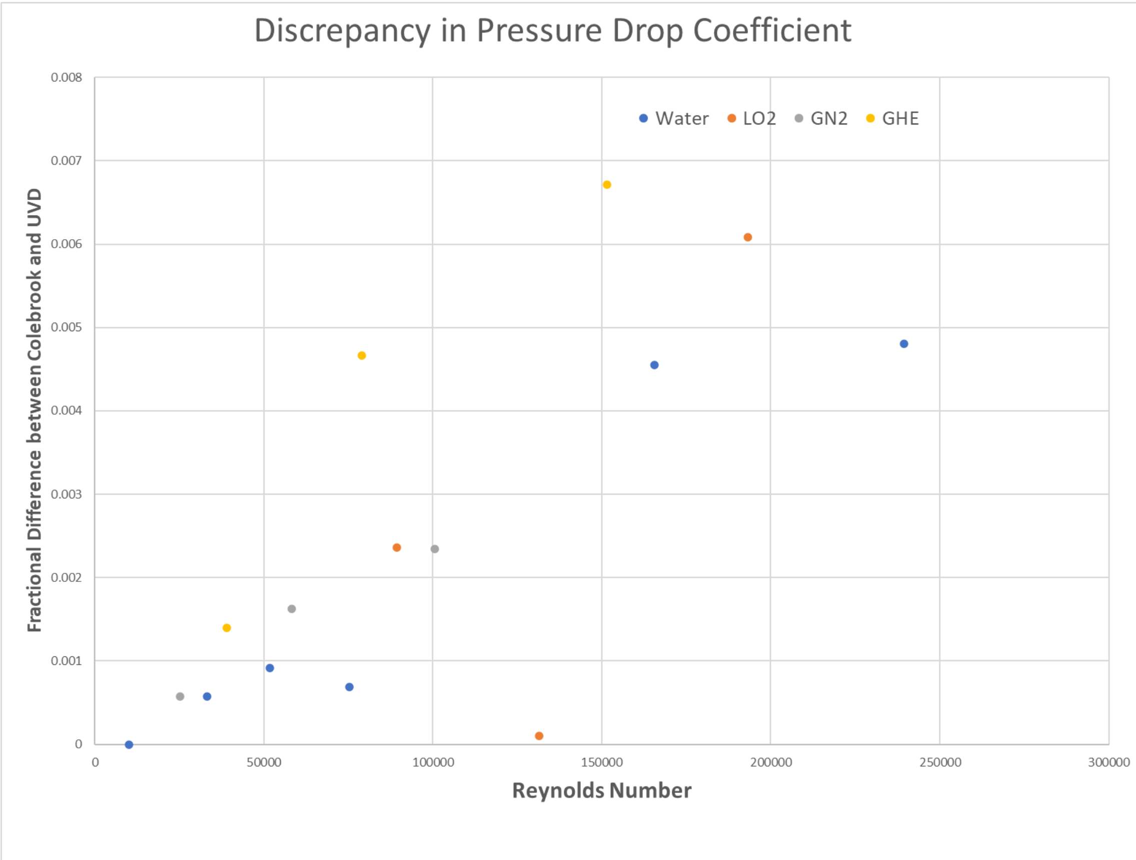

Figure 5. Discrepancy in Pressure Coefficients computed with Colebrook Equation and UVD Method

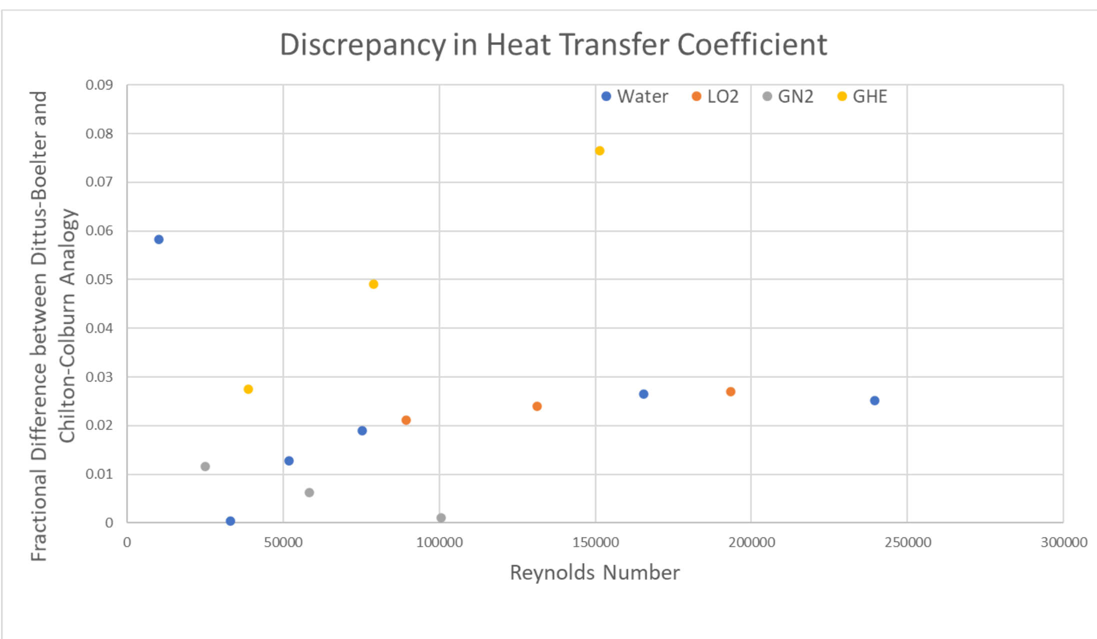

Figure 6. Discrepancy in Heat Transfer Coefficient computed by Dittus Boelter Equation and Chilton-Colburn Analogy

6. Conclusions

A GFSSP model was developed to demonstrate that the power law velocity profile and universal velocity distribution can be used to evaluate the wall shear stress and the heat transfer coefficient can be computed from the wall shear stress using the Chilton-Colburn analogy. The accuracy of pressure drop prediction was within 1% and the accuracy of heat transfer coefficient prediction was within 8%. This method has the potential to use a Navier-Stokes based CFD solution to evaluate heat transfer coefficient in nodal or network flow codes. There are two options: 1. Nodal code can compute wall shear stress from CFD computed velocity profiles and compute heat transfer coefficients from Chilton-Coburn or any other boundary layer analogy. 2. Nodal code can compute heat transfer coefficients from Chilton-Coburn or any other boundary layer analogy from CFD computed shear stress. Option 2 appears to be simpler in developing CFD-Nodal tool integration.

7. Acknowledgement

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8. Nomenclature

A	Area
B	Constant in Log Law of Wall
C_p	Pressure Coefficient, or Specific Heat in Calculation of Pr
D	Pipe Diameter
g	Gravitational Acceleration
g_c	Conversion Constant (= 32.174 $\frac{lb_m ft}{lb_f sec^2}$)
h	Heat Transfer Coefficient
h_f	Frictional Head
k	Thermal Conductivity
K_f	Friction Coefficient
L	Pipe Length
$\dot{m}$	Flow Rate
Nu	Nusselt Number
P	Pressure
$\dot{Q}$	Heat Transfer Rate
R	Pipe Radius

T Temperature

U Center Line Velocity in Pipe

u, V Velocity

$\bar{u}$ Average Velocity in Pipe

u_+ Non-Dimensional Velocity in Log Law of the Wall

y Distance from the Wall

y_+ Non-Dimensional Distance in Log Law of the Wall

Greek

ϵ Wall Roughness

μ Viscosity

κ Constant in Log Law of the Wall

τ_w Wall Shear Stress

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