BLUE RIDGE RESEARCH AND CONSULTING, LLC

Estimating sonic boom metrics across a community using a Kalman filter

Shane V. Lympany & Juliet A. Page

183rd Meeting of the
Acoustical Society of America

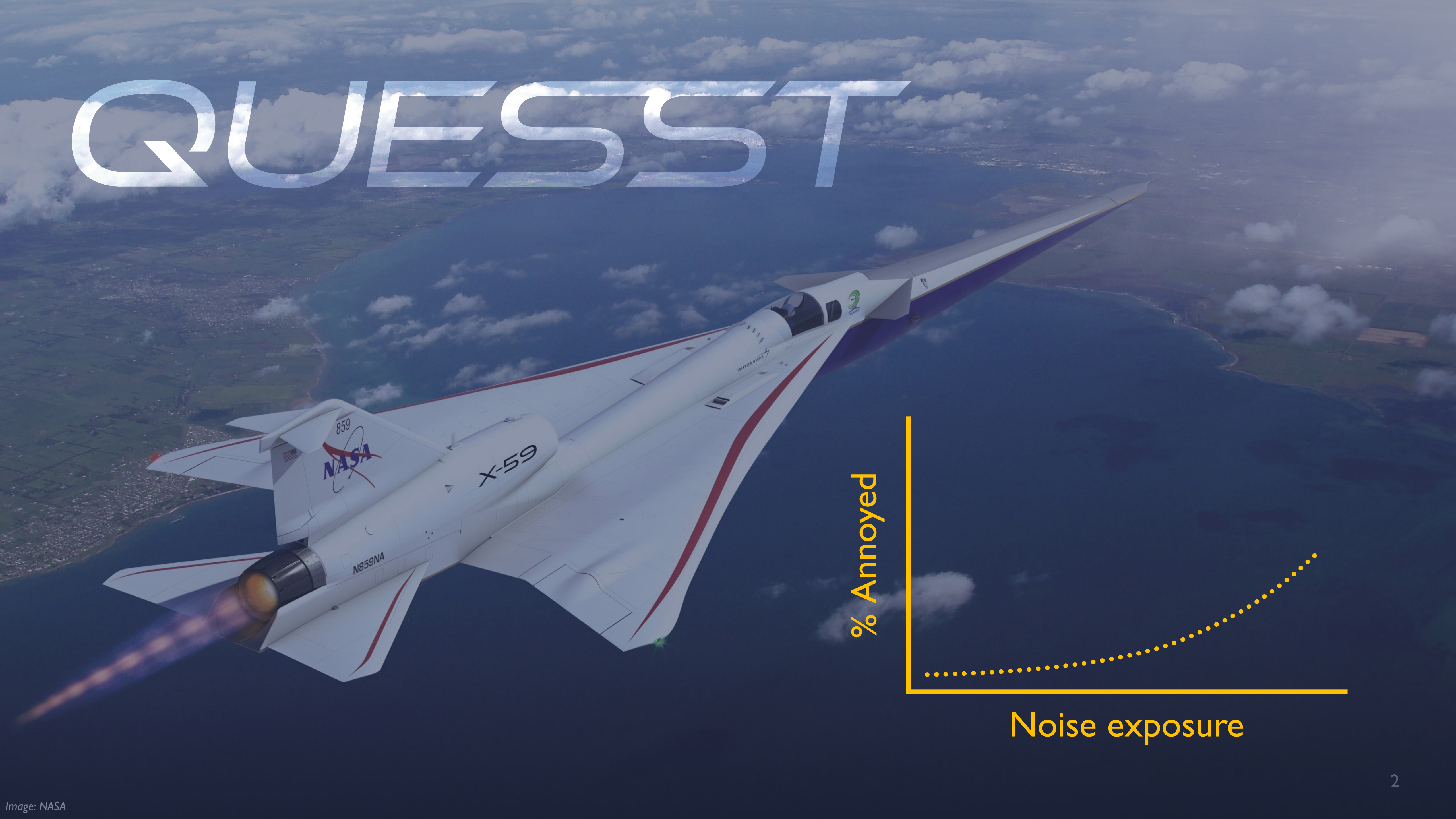
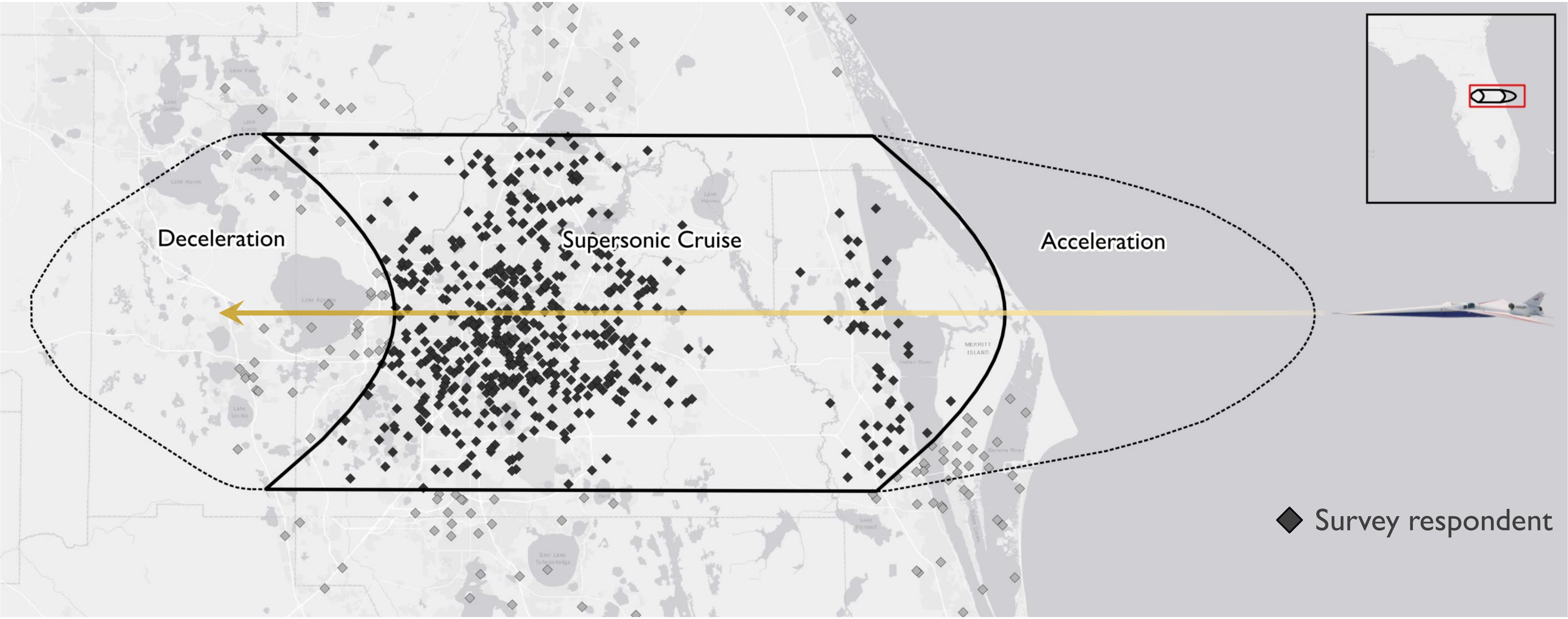


Image: NASA

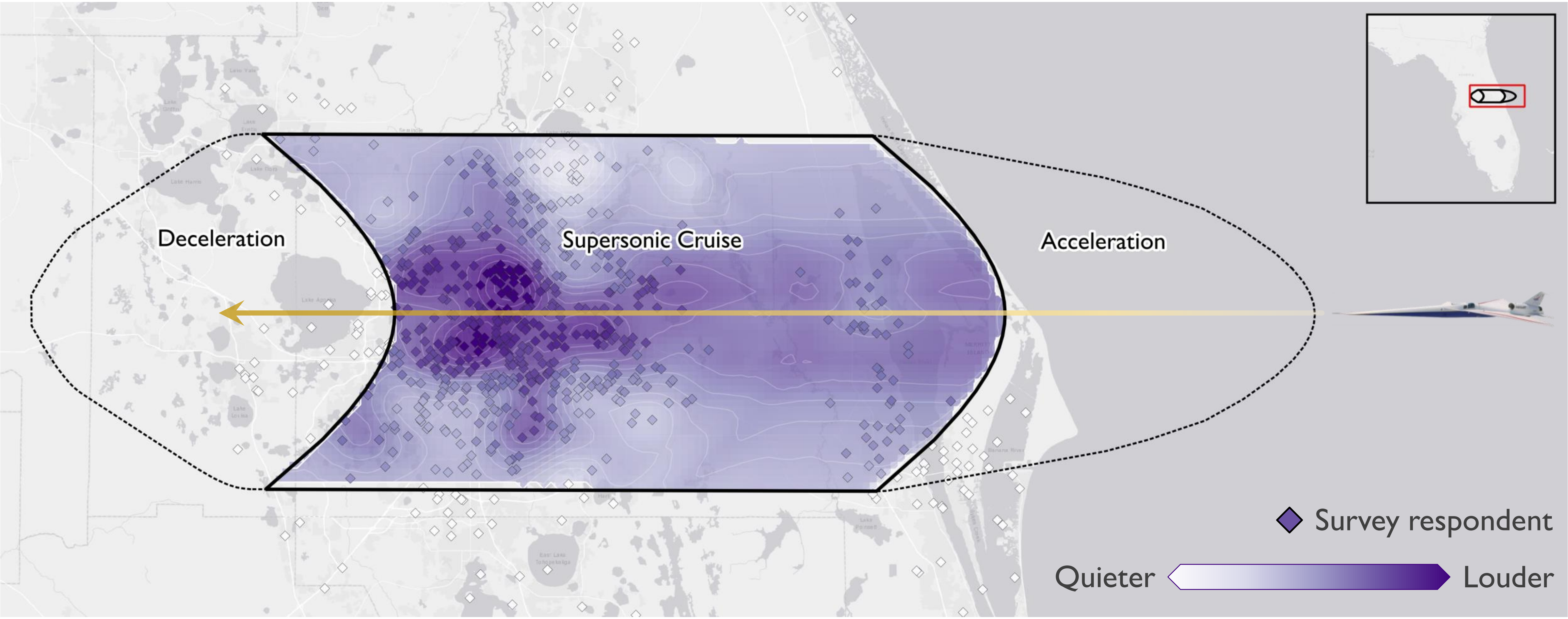
The Challenge: Estimate the Noise Exposure

Question: how to estimate noise exposure and uncertainty across sonic boom footprint?



The Solution: Kalman Filter

Answer: combine calculated and measured noise exposure data using a Kalman filter



Outline

1. Sources of noise exposure data
2. Prior noise estimation methods
3. Kalman filter method
4. Simulations and results
5. Conclusions and future work

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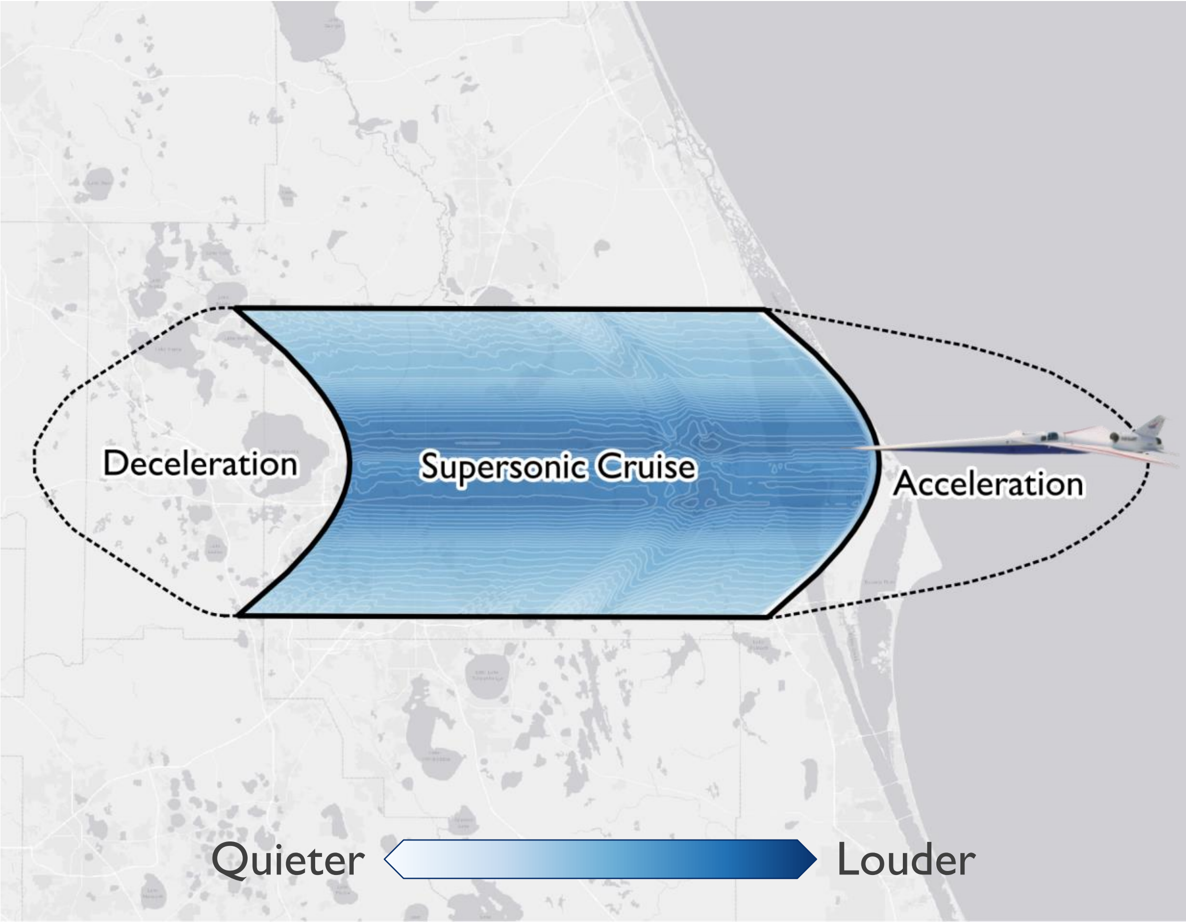
Calculated Noise Exposure

PCBoom calculations



Description:

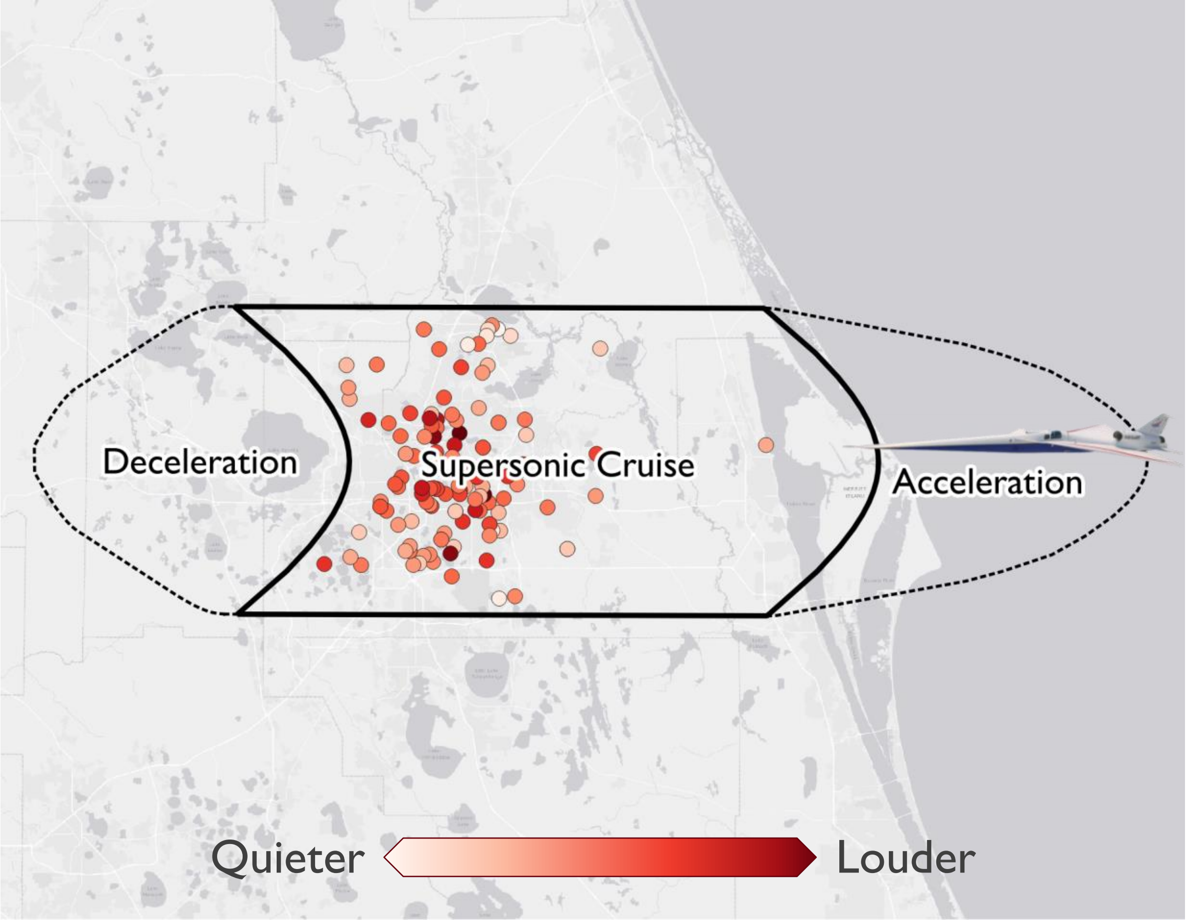
Calculations on a fine-resolution grid across the sonic boom footprint

Sources of uncertainty:

- Near-field pressure predictions
- Trajectory data
- Meteorological measurements
- Modeling uncertainty

Measured Noise Exposure

Acoustical measurements



Description:

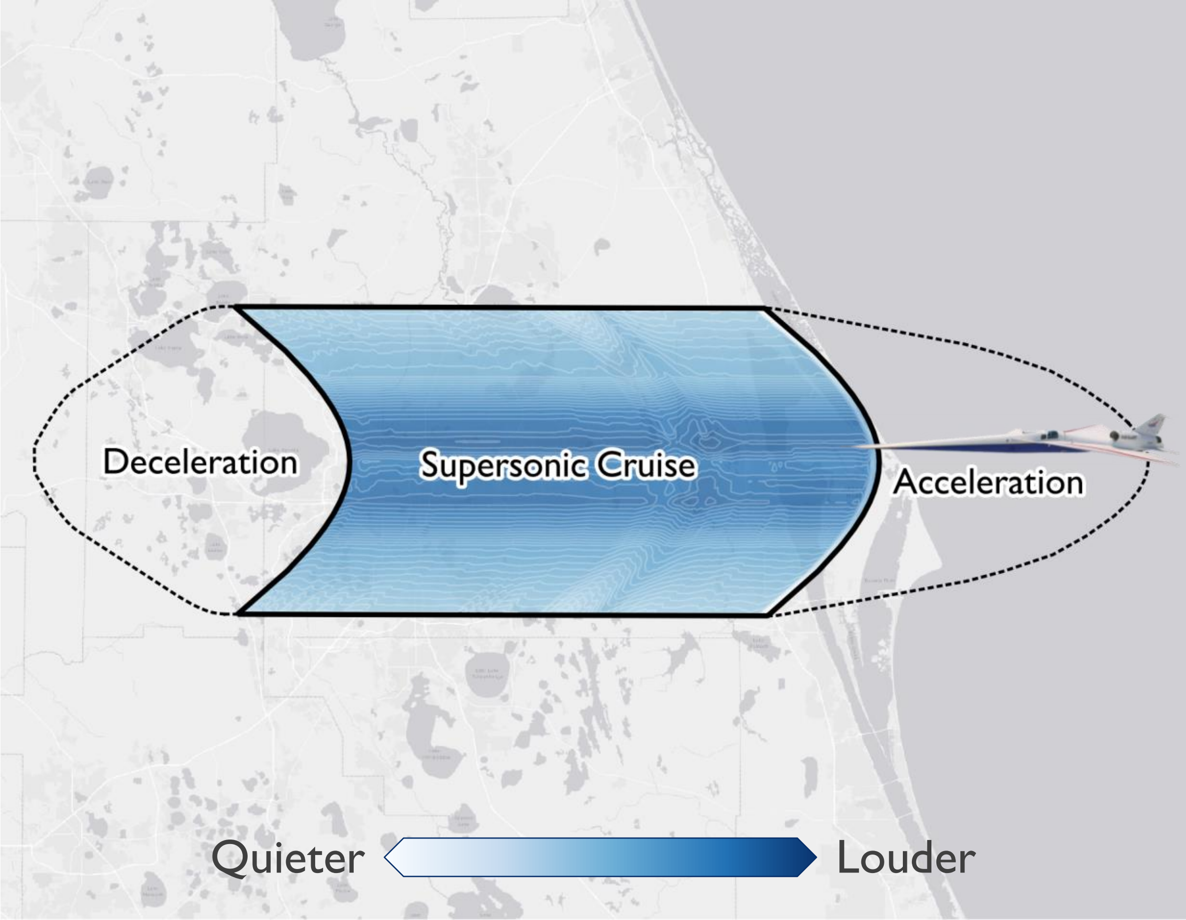
Measurements at sparse locations scattered throughout the sonic boom footprint

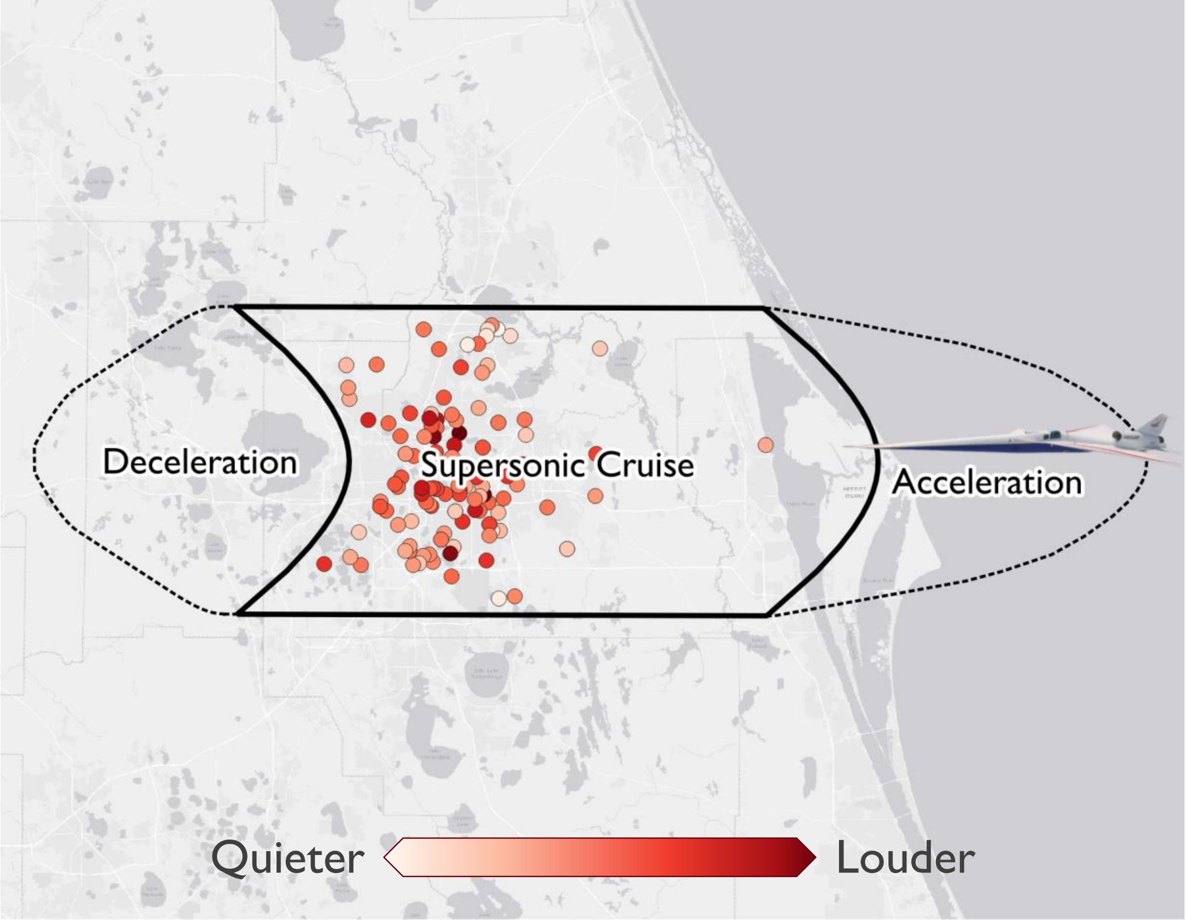
Sources of uncertainty:

- Instrumentation error
- Ambient noise contamination
- Localized turbulence

Estimated Noise Exposure

Question: how to combine the calculated and measured noise exposure?





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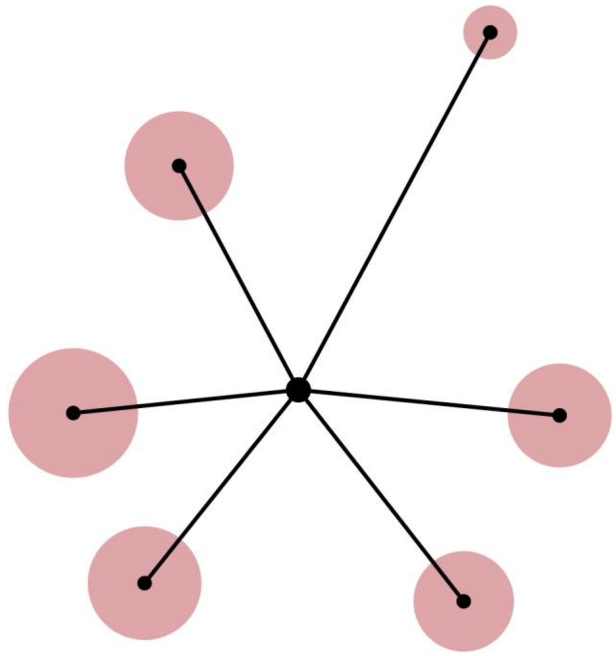
Inverse Distance Weighting

Summary:

Weighted average based on distance between measurements and grid points

Observations:

- Method does not account for measurement uncertainty
- Inverse distance may not describe a measurement's region of influence



J. Page, et al. (2018), "Quiet Supersonic Flights 2018 (QSF18) Test: Galveston, Texas Risk Reduction for Future Community Testing with a Low-Boom Flight Demonstration Vehicle," NASA/CR-2020-220589, Volume 1.

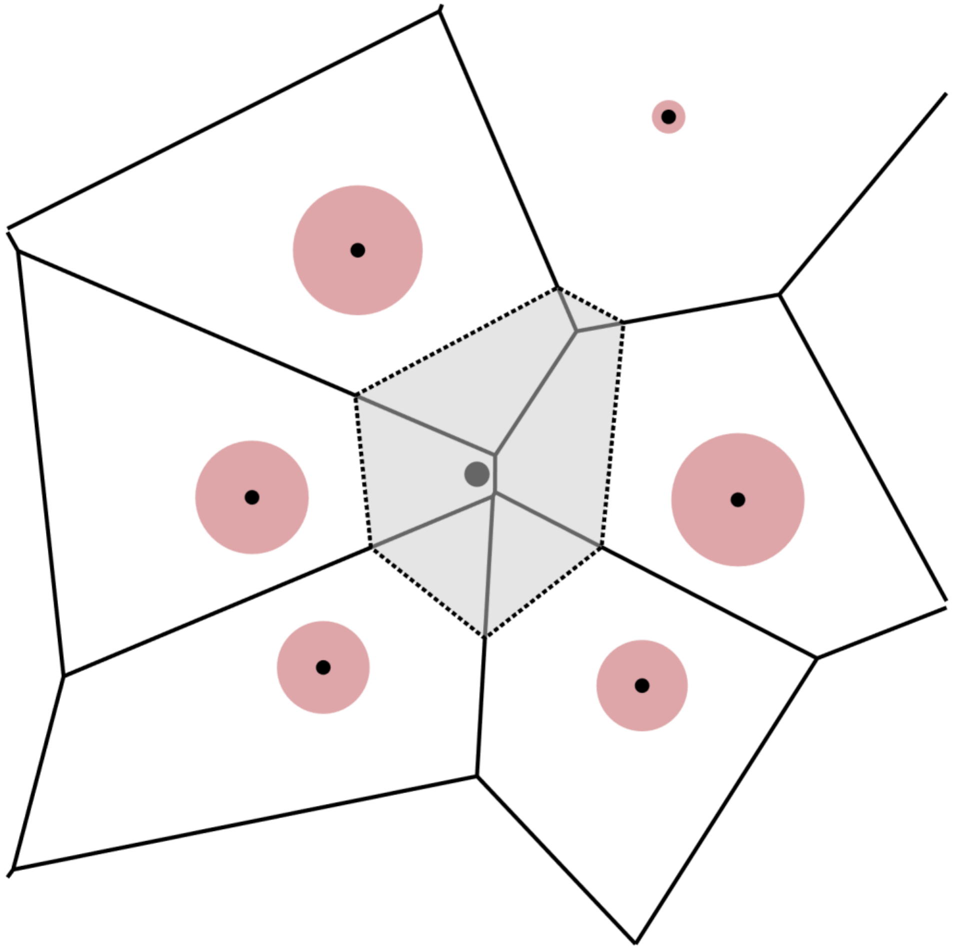
Natural Neighbor Interpolation

Summary:

Weighted average based on interpolation of measurements to grid points

Observations:

- Method extrapolates poorly beyond measurement locations
- Interpolation neglects physics knowledge between measurement locations



J. Klos (2020), "Recommendations for Using Noise Monitors to Estimate Noise Exposure During X-59 Community Tests," NASA/TM-2020-5007926.

Data Assimilation in Meteorology

Meteorology:
Estimate the atmospheric state from gridded forecasts and sparse observations

Quesst:
Estimate the noise exposure from gridded calculations and sparse measurements

Mathematics:
Derived from a Kalman filter

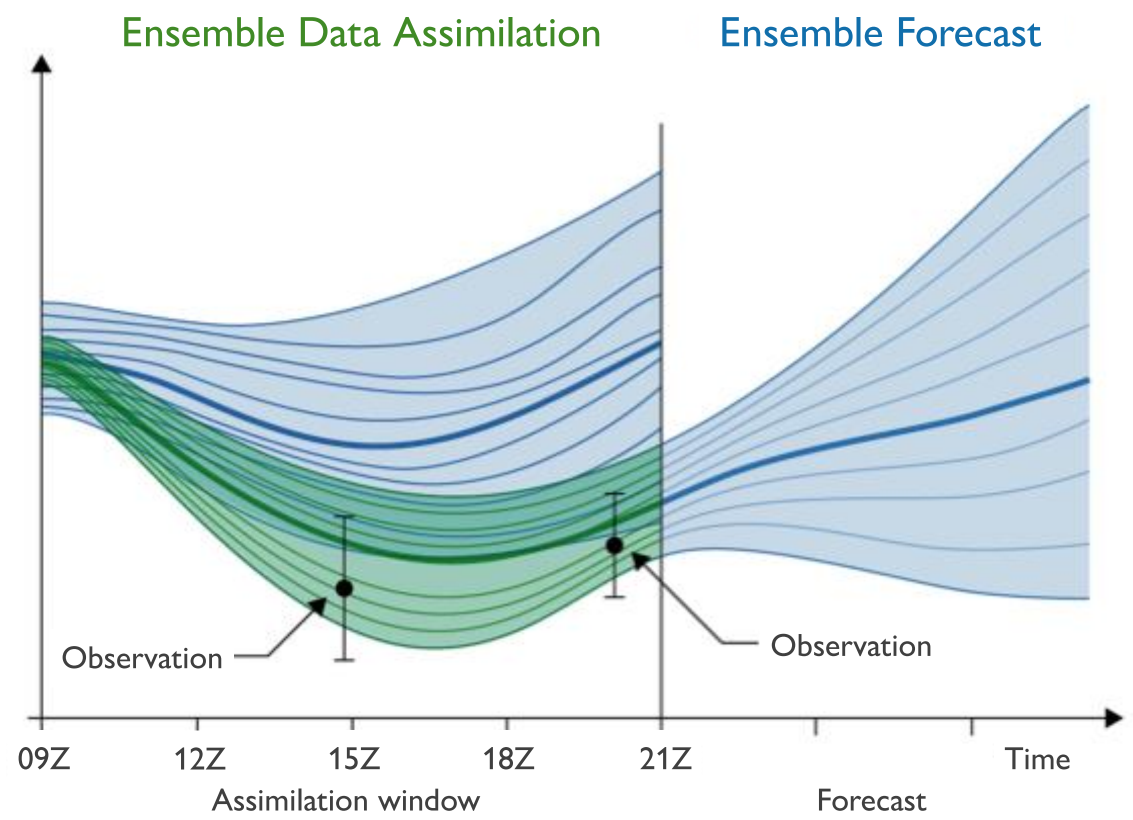


Image: European Center for Medium-Range Weather Forecasts (ECMWF)

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What is a Kalman Filter?

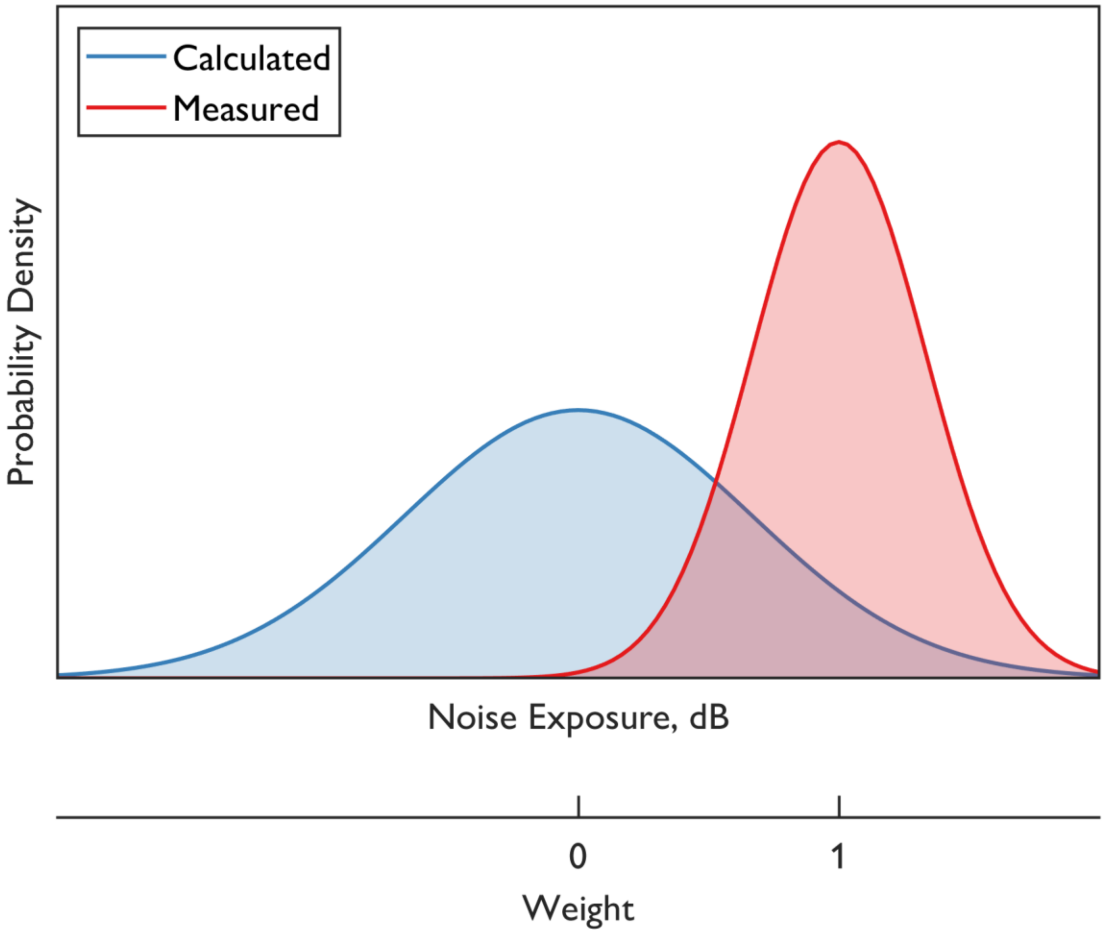
Summary: weighted average based on uncertainty of calculations and measurements

Weights:

Kalman filter applies more weight to more certain data

Result:

Estimate has lower uncertainty than calculations or measurements alone



What are the Kalman Filter Weights?

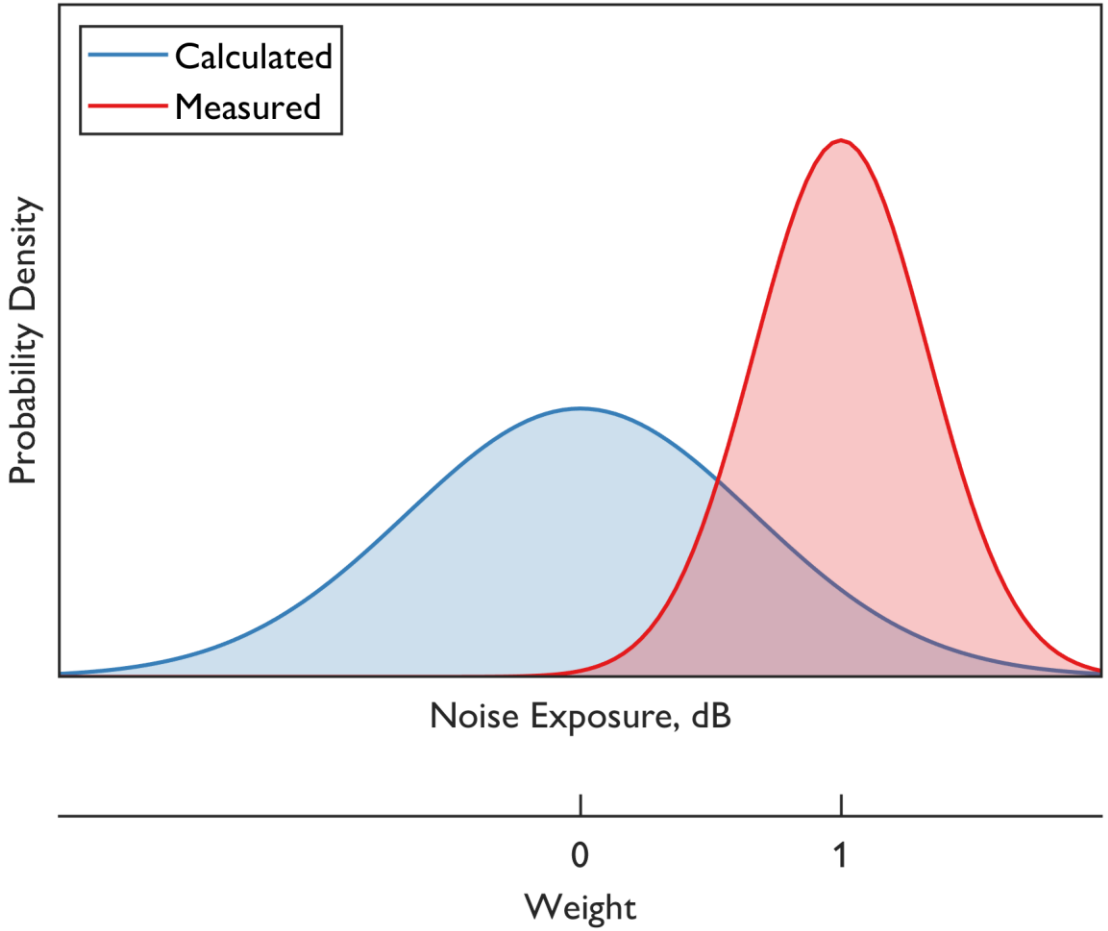
Optimal weights: maximize the accuracy and minimize the uncertainty of the estimate

Calculated Measured

Weight = 0

Calculated Measured

Weight = 1



What are the Kalman Filter Weights?

Optimal weights: maximize the accuracy and minimize the uncertainty of the estimate

Maximize the accuracy:

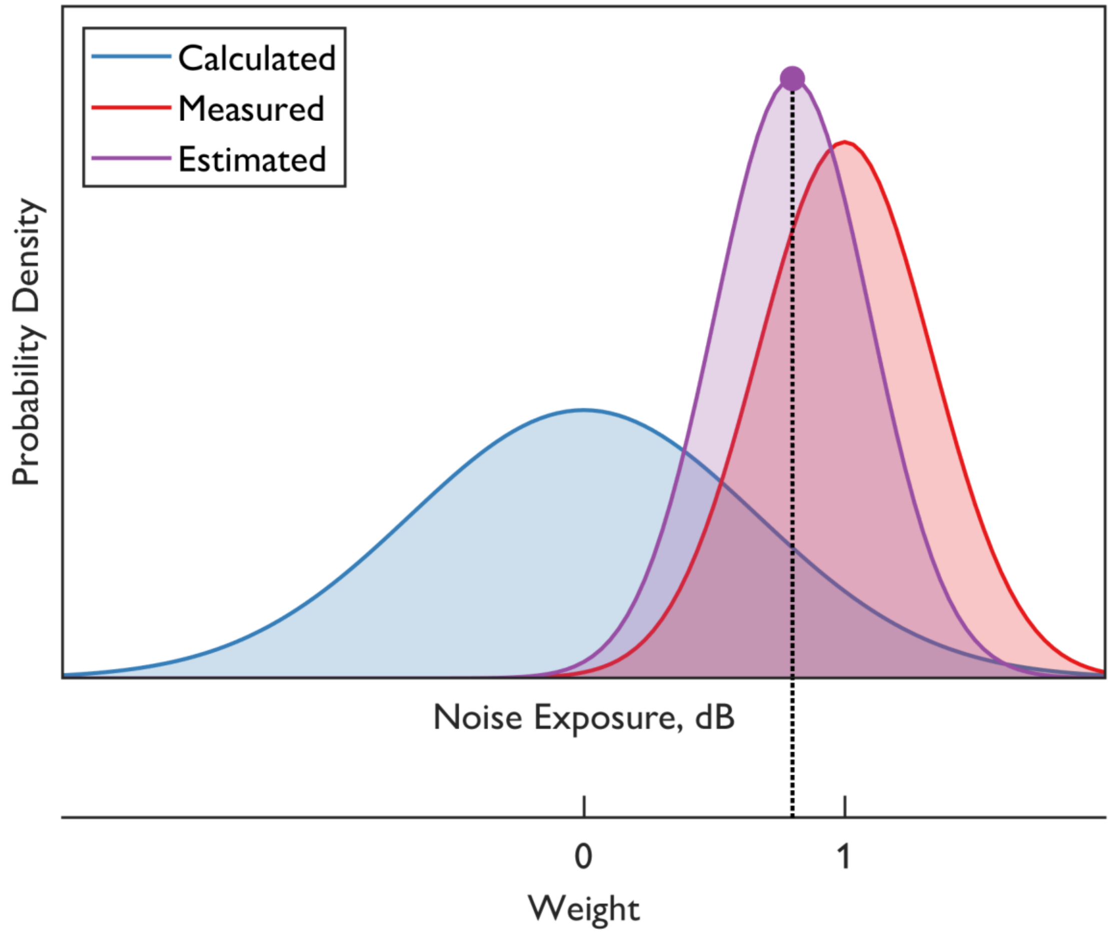
Maximize the probability of the estimate using Bayesian estimation

Calculated Measured

Weight = 0

Calculated Measured

Weight = 1

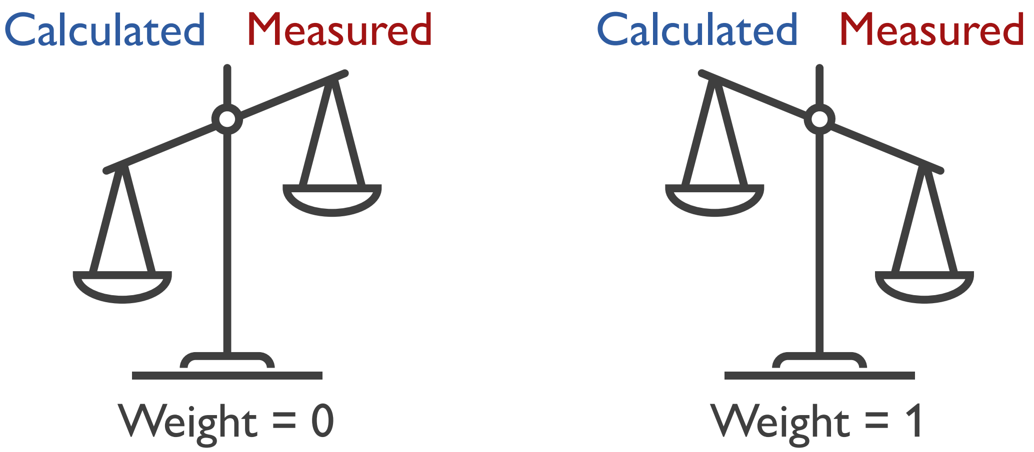


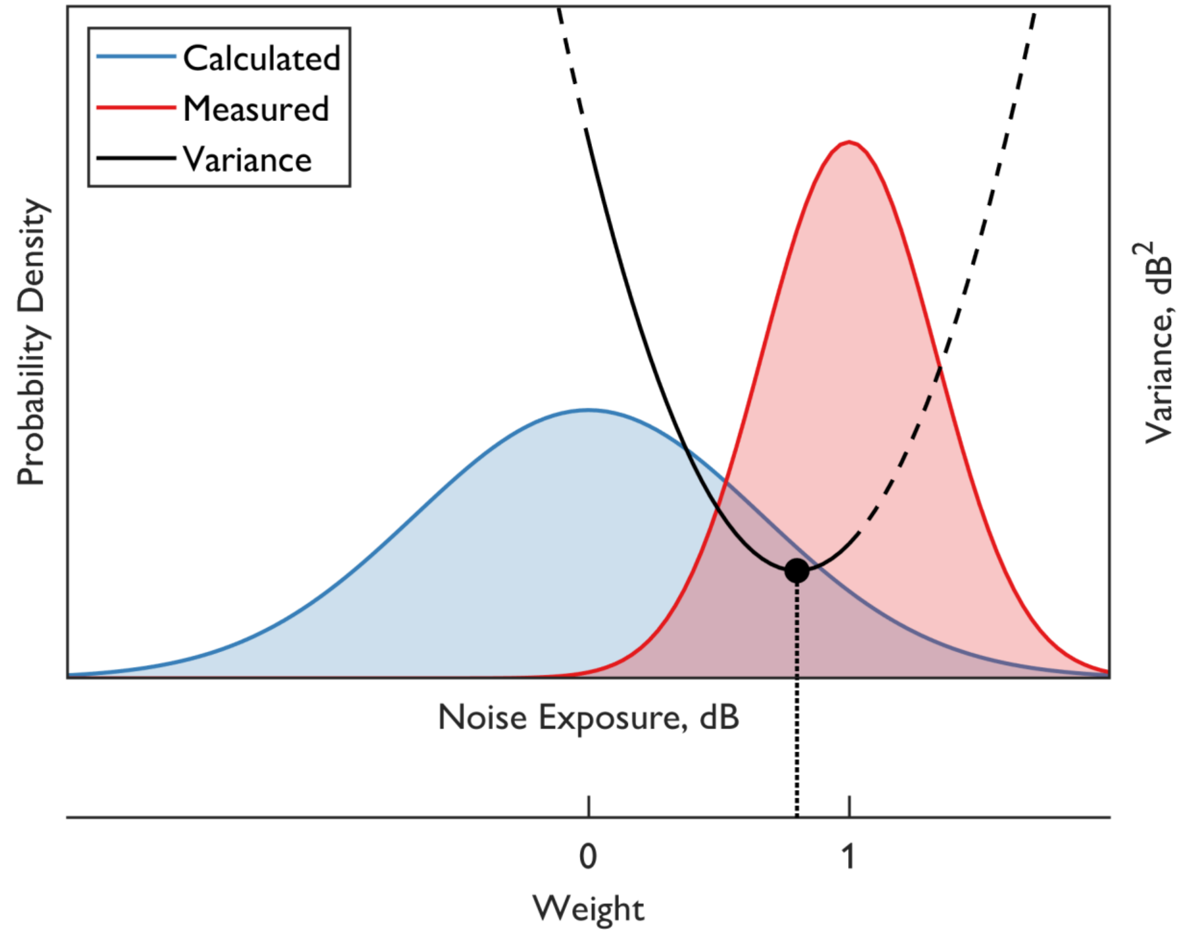
What are the Kalman Filter Weights?

Optimal weights: maximize the accuracy and minimize the uncertainty of the estimate

Minimize the uncertainty:

Minimize the variance of the estimate using weighted least squares



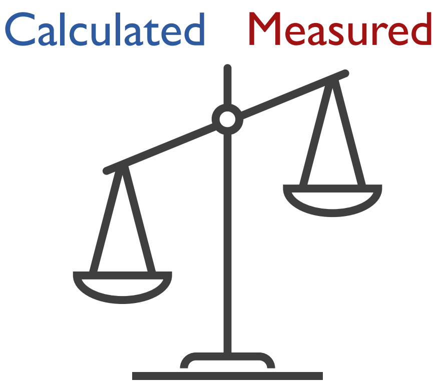


What are the Kalman Filter Weights?

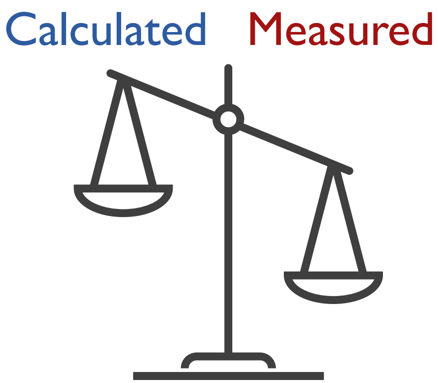
Optimal weights: maximize the accuracy and minimize the uncertainty of the estimate

Comparison:

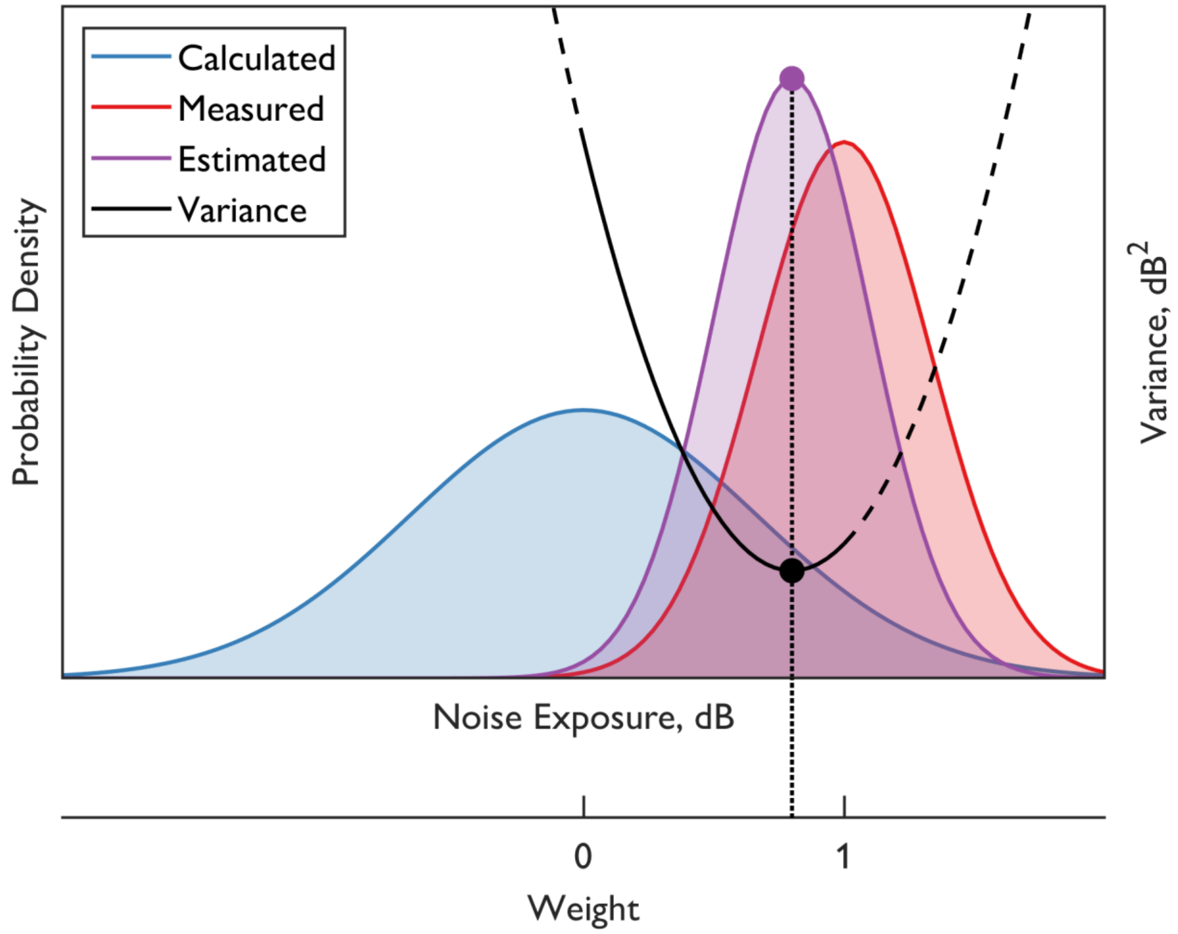
Bayesian estimation and weighted least squares are identical for Gaussian errors



Weight = 0



Weight = 1



Noise Estimation using the Kalman Filter

Calculated on a fine-resolution grid:

Calculated noise exposure
from PCBoom

★ **Calculated uncertainty**
covariance matrix

Measured at sparse locations:

Measured noise exposure
from ground recording systems

★ **Measured uncertainty**
standard deviation or variance

→ **Kalman filter** →

Estimated on a fine-resolution grid:

Estimated noise exposure

Estimated uncertainty

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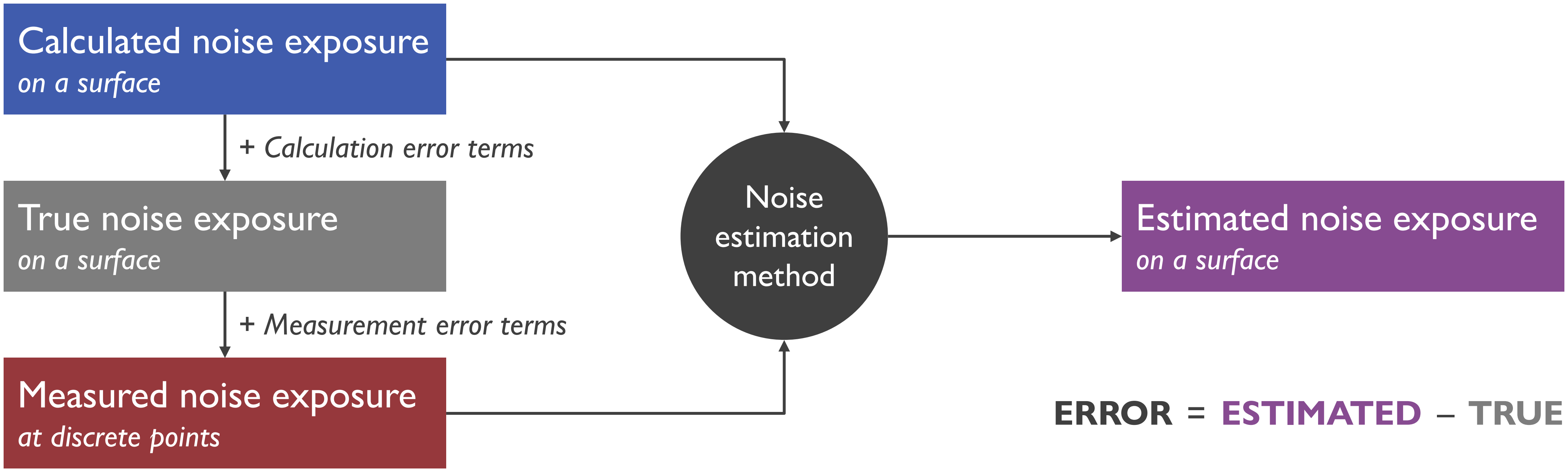
Simulation Process

Goal: evaluate the accuracy of the noise estimation methods

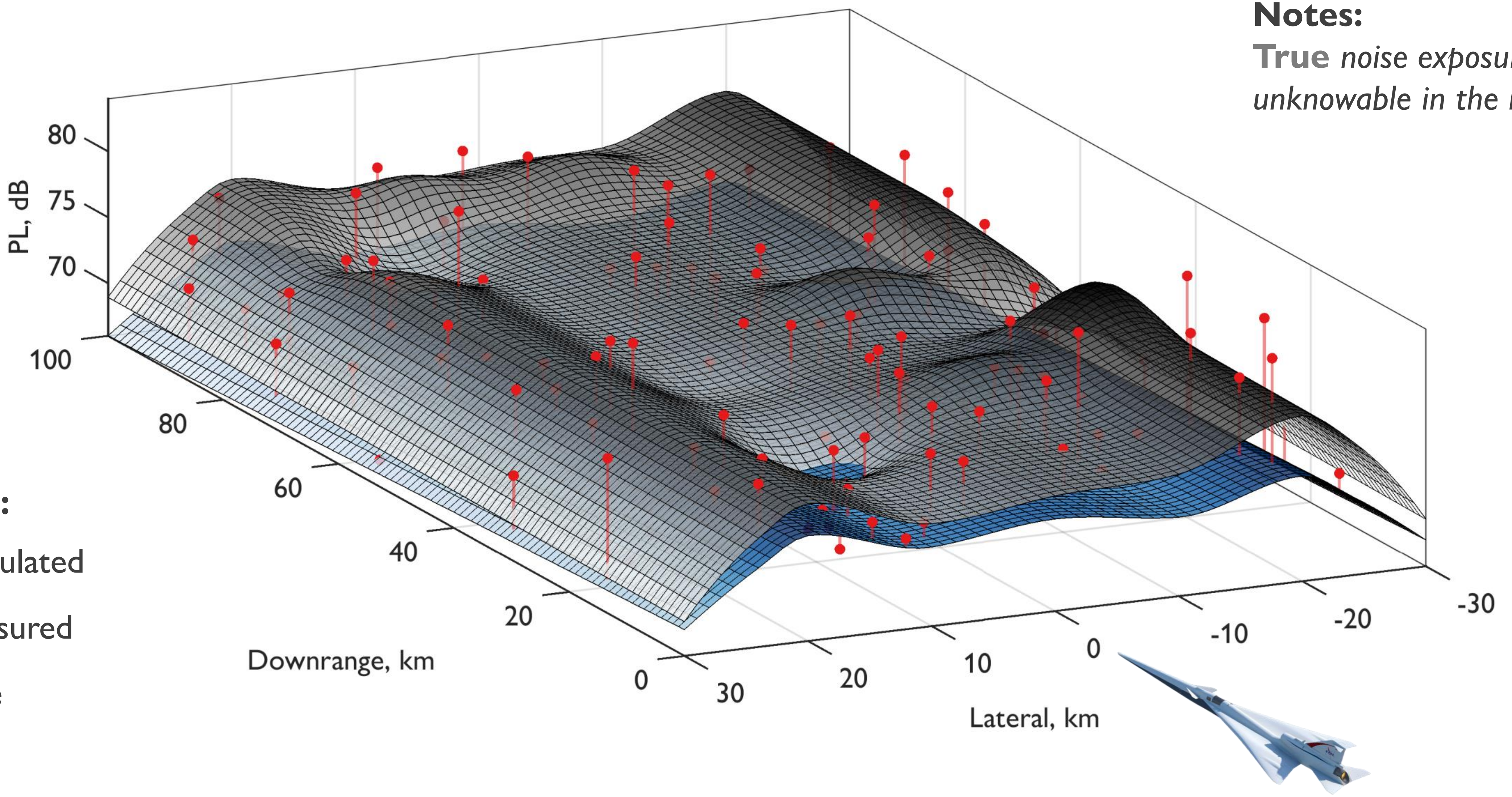


ERROR = ESTIMATED – TRUE

× 2,500 simulations

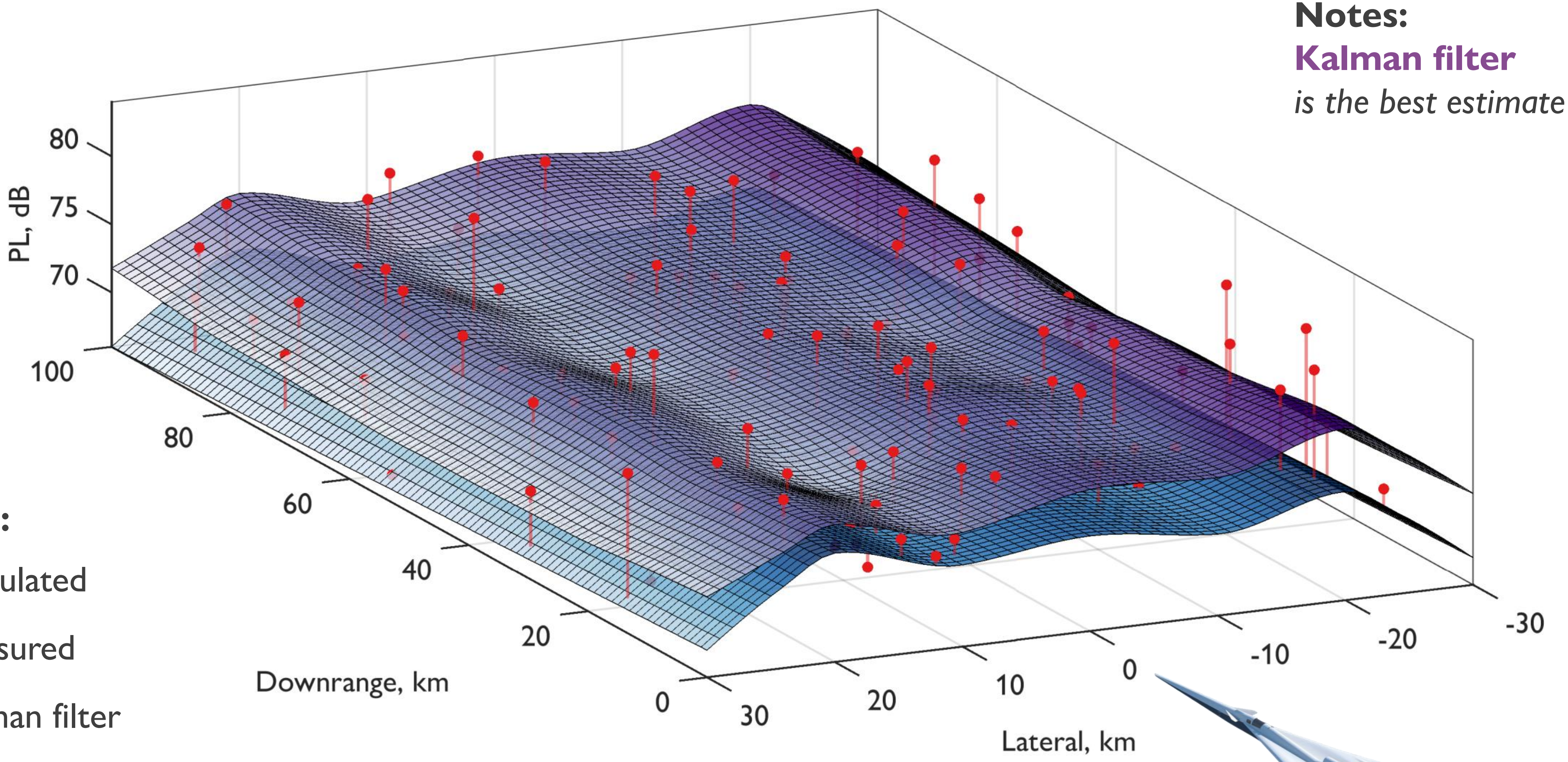
Example: Simulated Noise Exposure

Notes:
True *noise exposure is unknowable in the real world*



Legend:
- Calculated
- Measured
- True

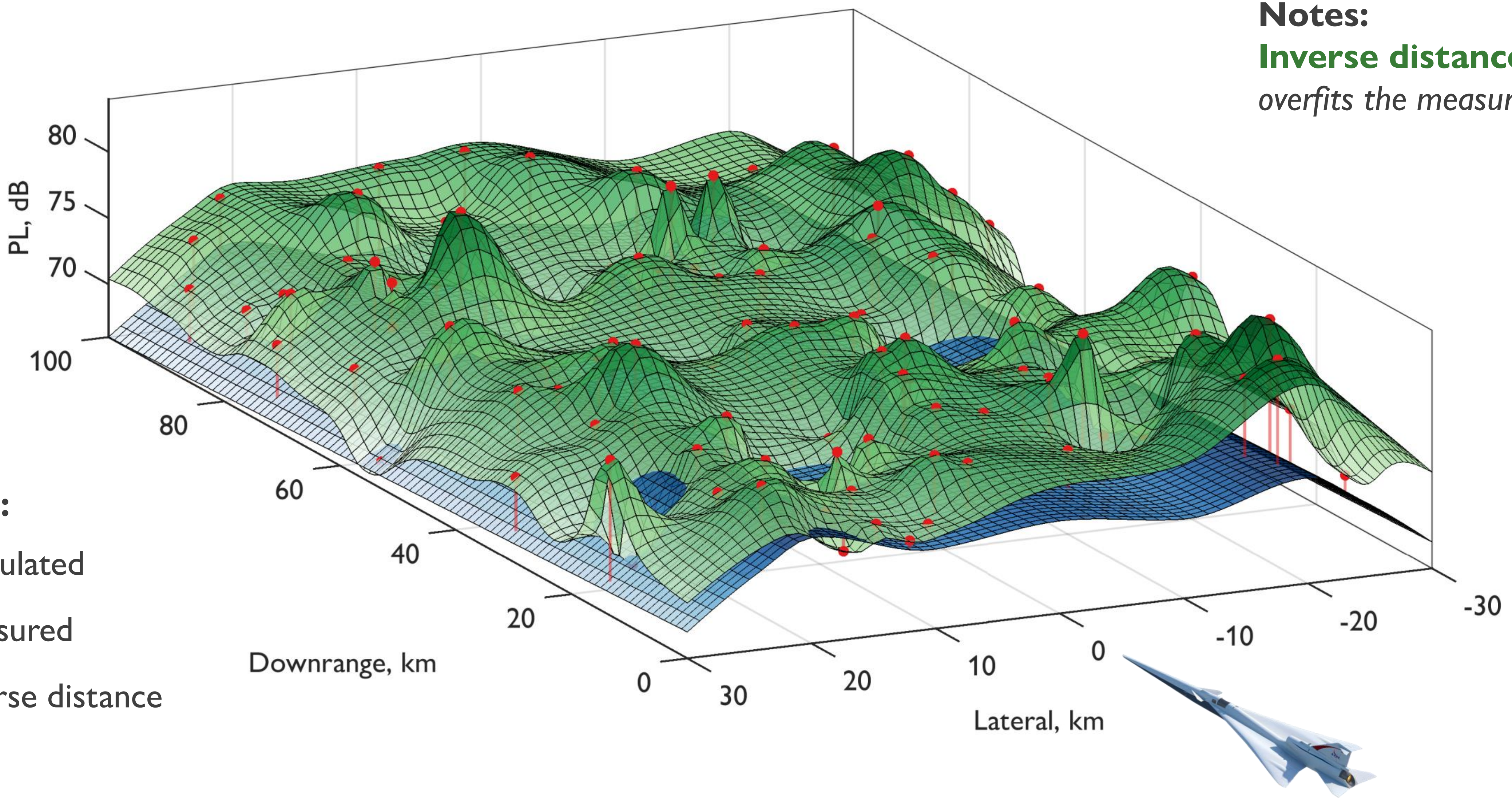
Example: Kalman Filter



Notes:
Kalman filter
is the best estimate

Legend:
- Calculated
- Measured
- Kalman filter

Example: Inverse Distance Weighting



Notes:

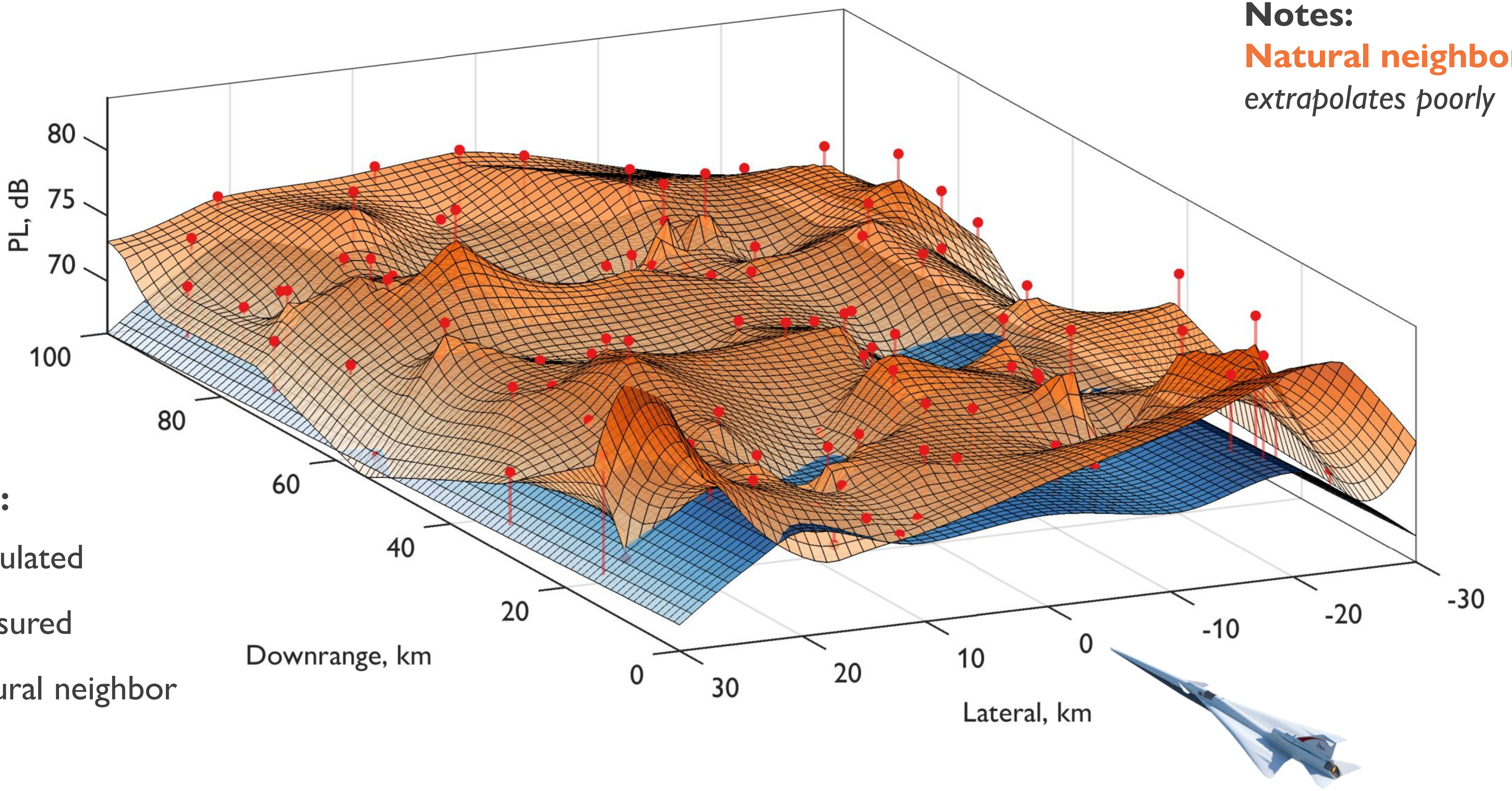
Inverse distance

overfits the measurements

Legend:

- Calculated
- Measured
- Inverse distance

Example: Natural Neighbor Interpolation



Notes:
Natural neighbor
extrapolates poorly

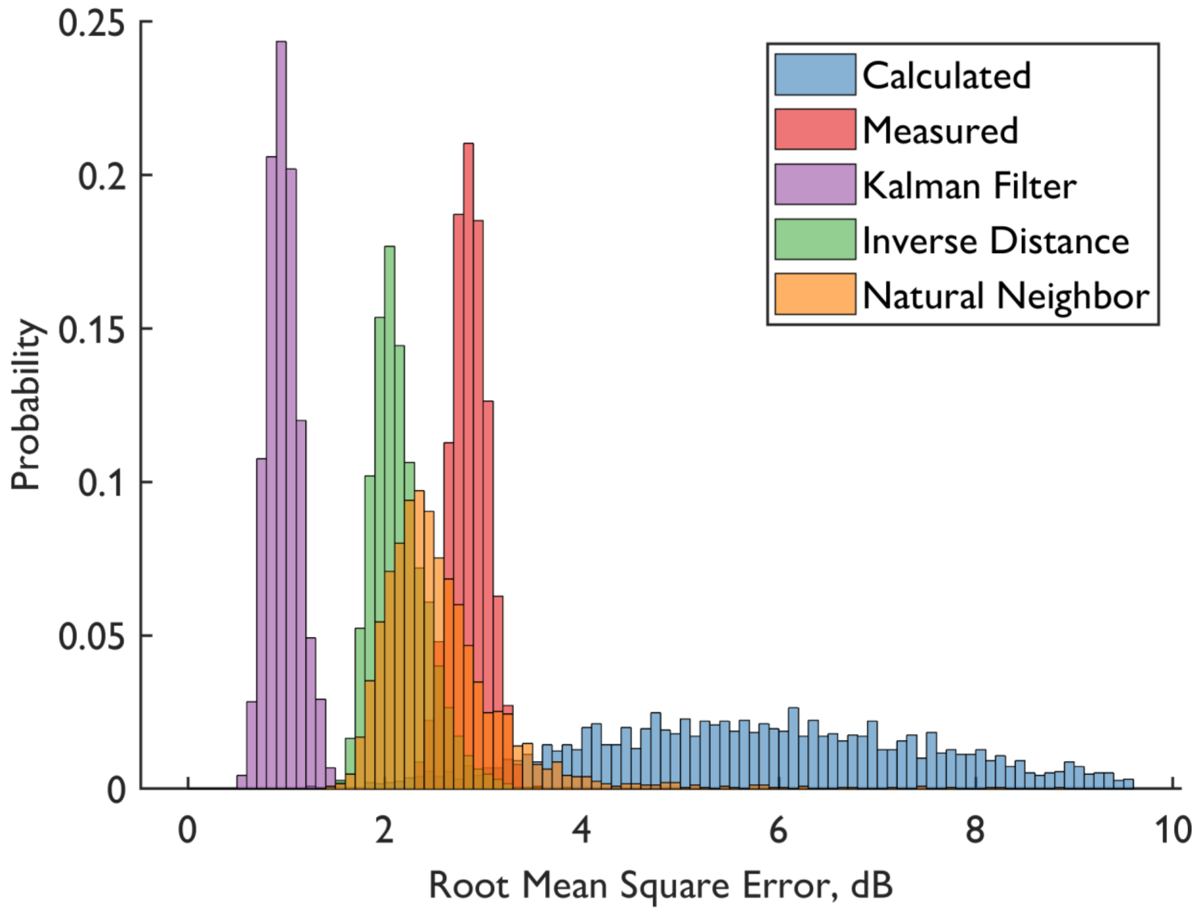
Legend:
- Calculated
- Measured
- Natural neighbor

Results: Error Metrics

Root mean square error: quantify the accuracy of the noise estimation methods

1. Calculate error between estimated and true noise exposure at each grid point
2. Calculate RMS value of errors across the grid for each simulation
3. Calculate mean value of RMS errors across 2,500 simulations

Method	RMS Error, dB
Calculated noise exposure	5.9
Measured noise exposure	2.9
Kalman filter	1.0 ✓
Inverse distance weighting	2.1
Natural neighbor interpolation	3.3



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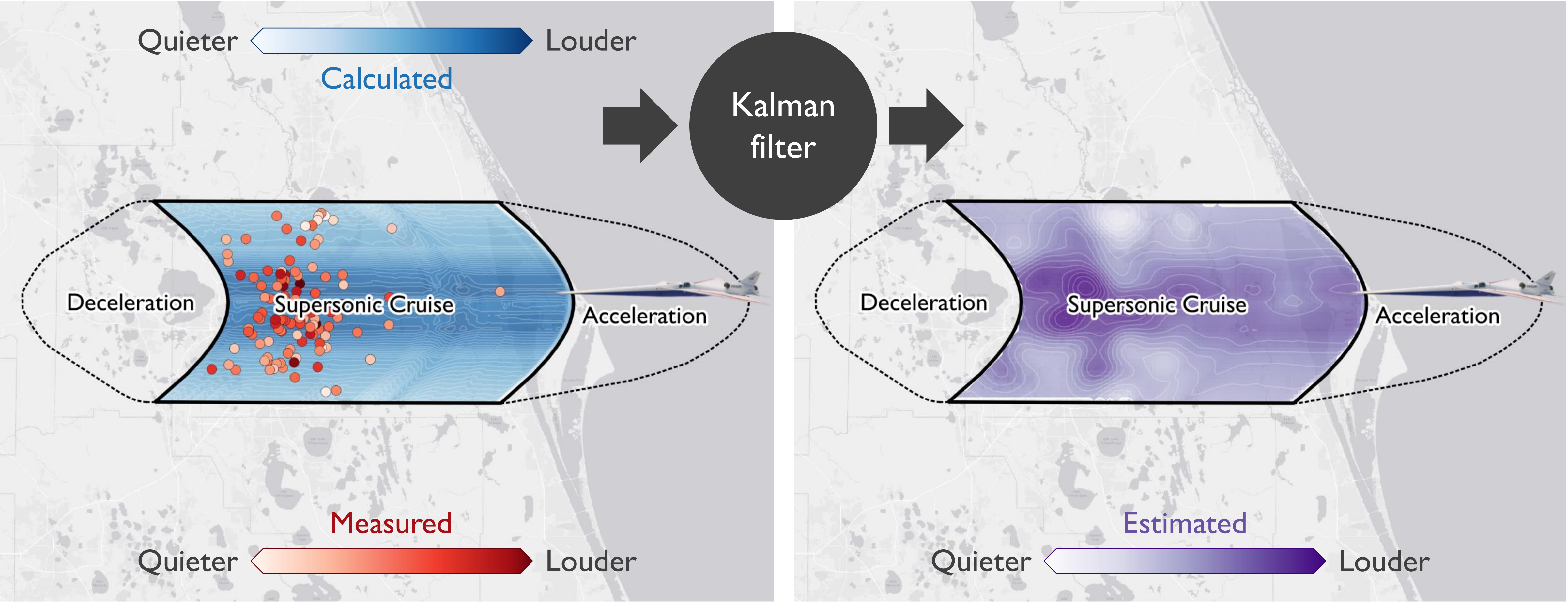
Summary

Key idea: combine calculated and measured noise exposure data using a Kalman filter



Summary

Key idea: combine calculated and measured noise exposure data using a Kalman filter

Conclusions:

The Kalman filter produces the best estimate of the true noise exposure

Future work:

Quantify uncertainty in the calculated and measured noise exposure

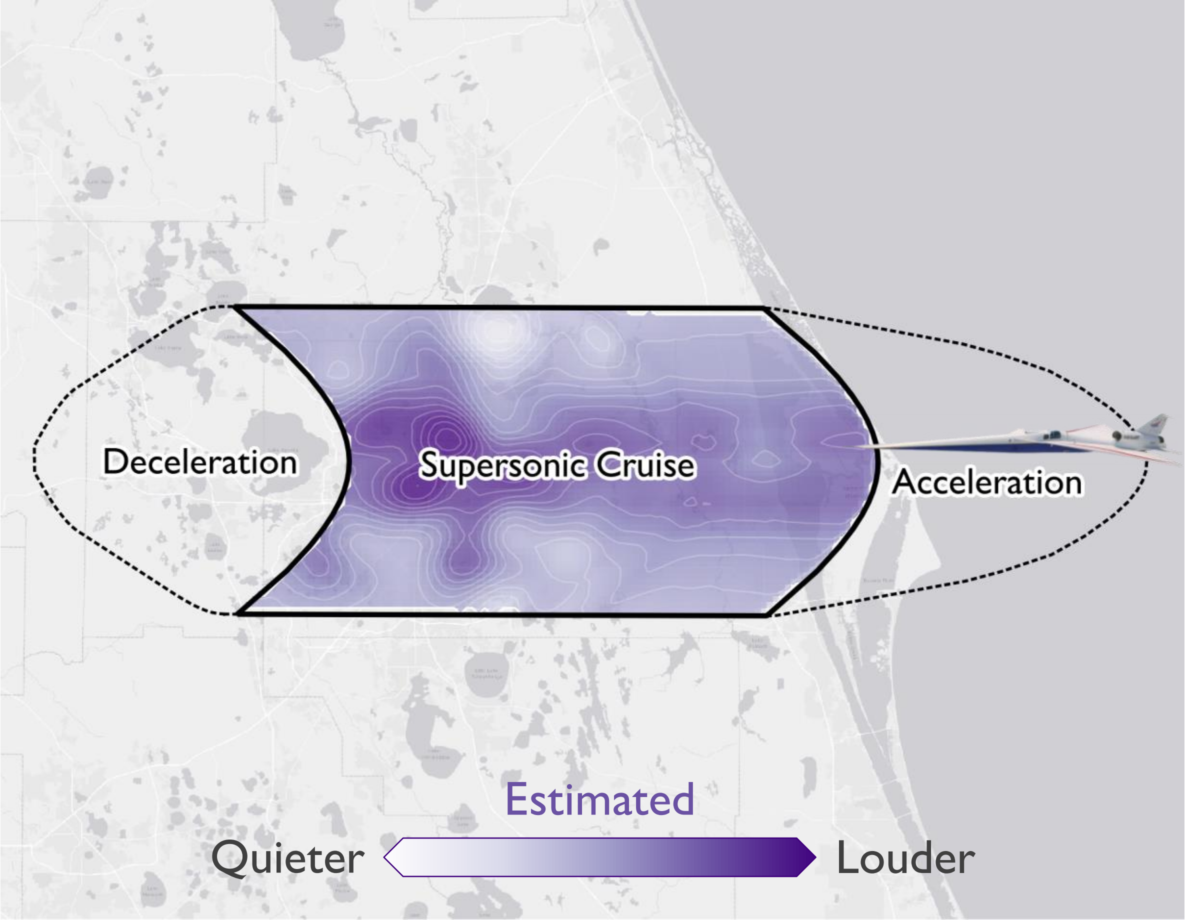


Image: NASA

Backup

Kalman Filter

Summary: weighted average based on uncertainty of calculations and measurements

Method:

1. **RESIDUAL** = **MEASURED** – **CALCULATED**
at measurement locations

2. **WEIGHT** = *f* (**CALCULATED UNCERTAINTY** , **MEASURED UNCERTAINTY**)

3. **ESTIMATE** = **CALCULATED** + **WEIGHT** × **RESIDUAL**
on calculated grid

Inverse Distance Weighting

Summary: weighted average based on distance between measurements and grid points

Method:

1. **RESIDUAL = MEASURED – CALCULATED**
   at measurement locations

2. **WEIGHT** = $\frac{1}{\text{DISTANCE}^3}$

3. **ESTIMATE = CALCULATED + WEIGHT × RESIDUAL**
   on calculated grid

J. Page, et al. (2018), "Quiet Supersonic Flights 2018 (QSF18) Test: Galveston, Texas Risk Reduction for Future Community Testing with a Low-Boom Flight Demonstration Vehicle," NASA/CR-2020-220589, Volume 1.

Natural Neighbor Interpolation

Summary: weighted average based on interpolation of measurements to grid points

Method:

1. INTERPOLATED = *f* (MEASURED)
   on calculated grid

2. WEIGHT = *f* (CALCULATED UNCERTAINTY , INTERPOLATED UNCERTAINTY)

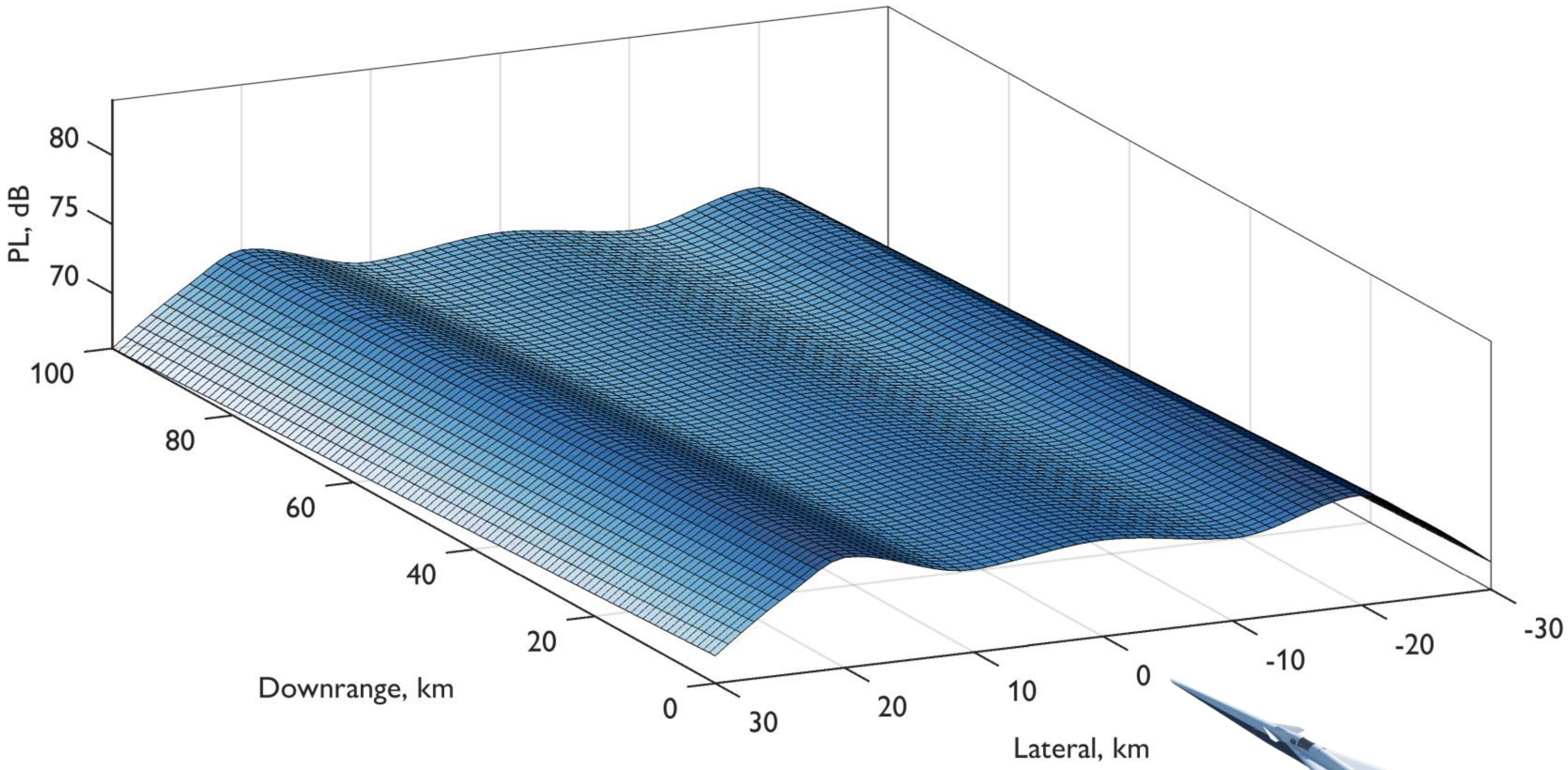
3. ESTIMATE = (1 – WEIGHT) × CALCULATED + WEIGHT × INTERPOLATED
   on calculated grid

J. Klos (2020), "Recommendations for Using Noise Monitors to Estimate Noise Exposure During X-59 Community Tests," NASA/TM-2020-5007926.

Simulation Process

Goal: evaluate the accuracy of the noise estimation methods

Calculated noise exposure
on a surface



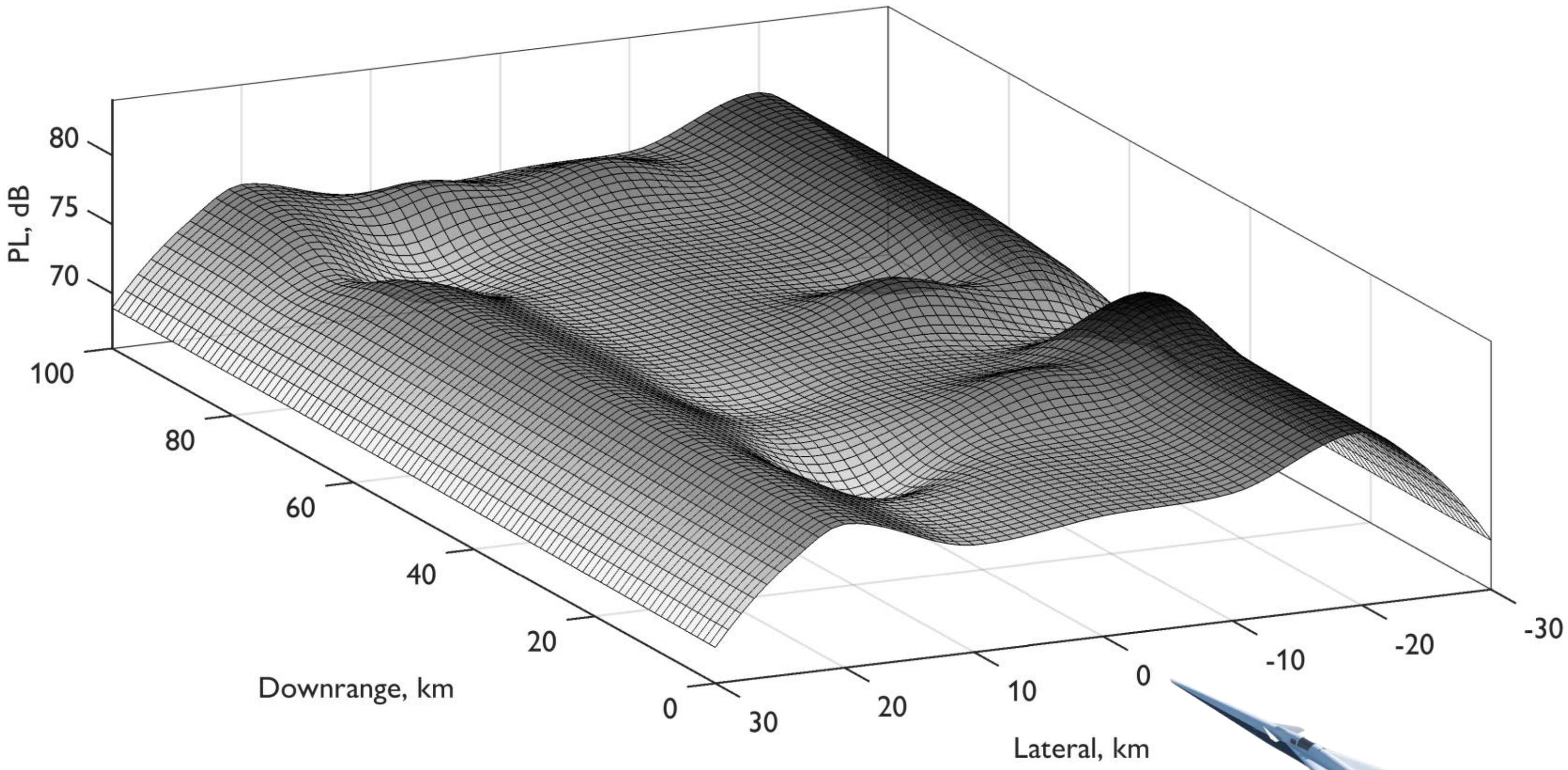
Simulation Process

Goal: evaluate the accuracy of the noise estimation methods

Calculated noise exposure
on a surface

+ Calculation error terms

True noise exposure
on a surface



Simulation Process

Goal: evaluate the accuracy of the noise estimation methods

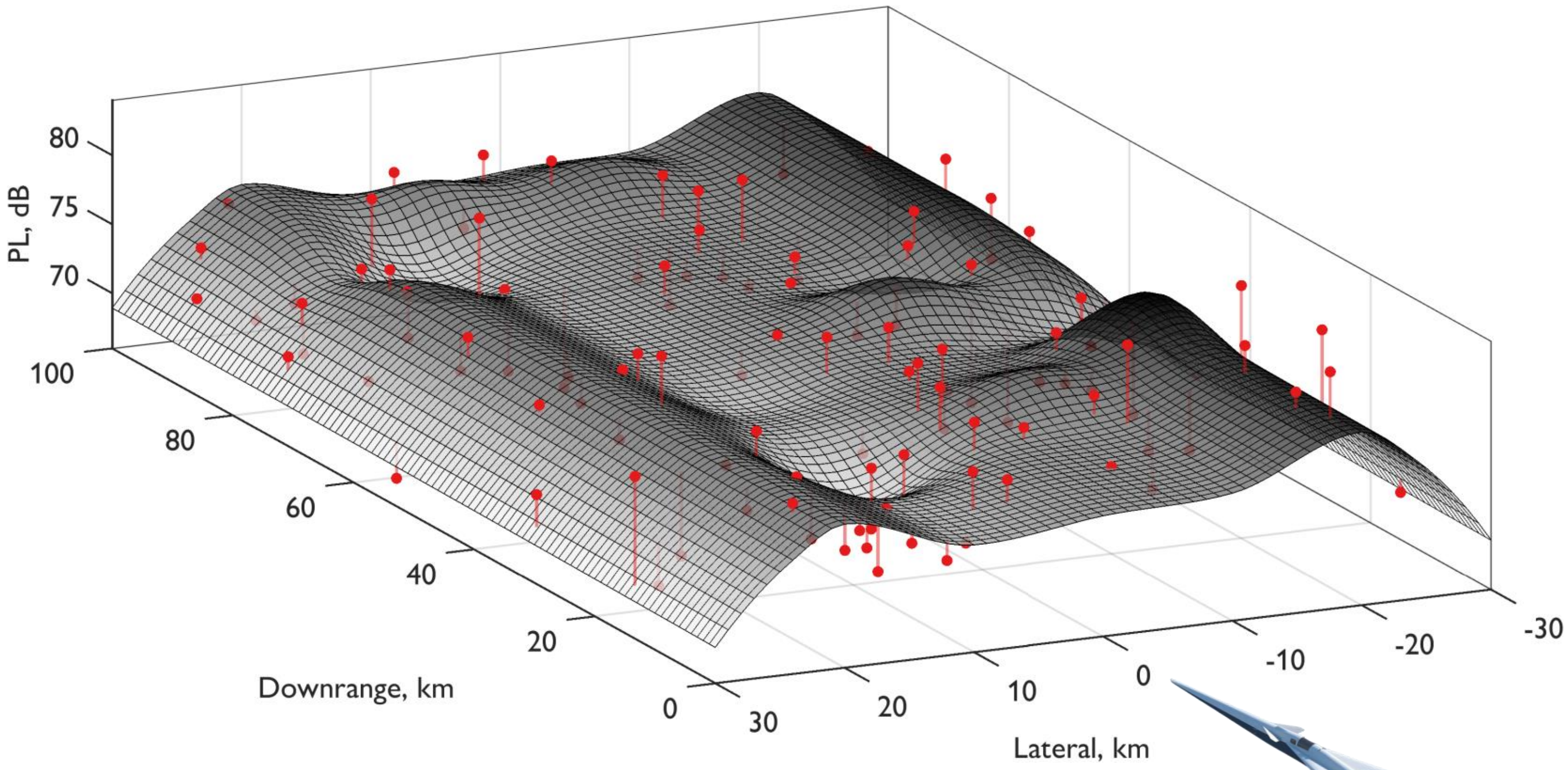
Calculated noise exposure
on a surface

+ *Calculation error terms*

True noise exposure
on a surface

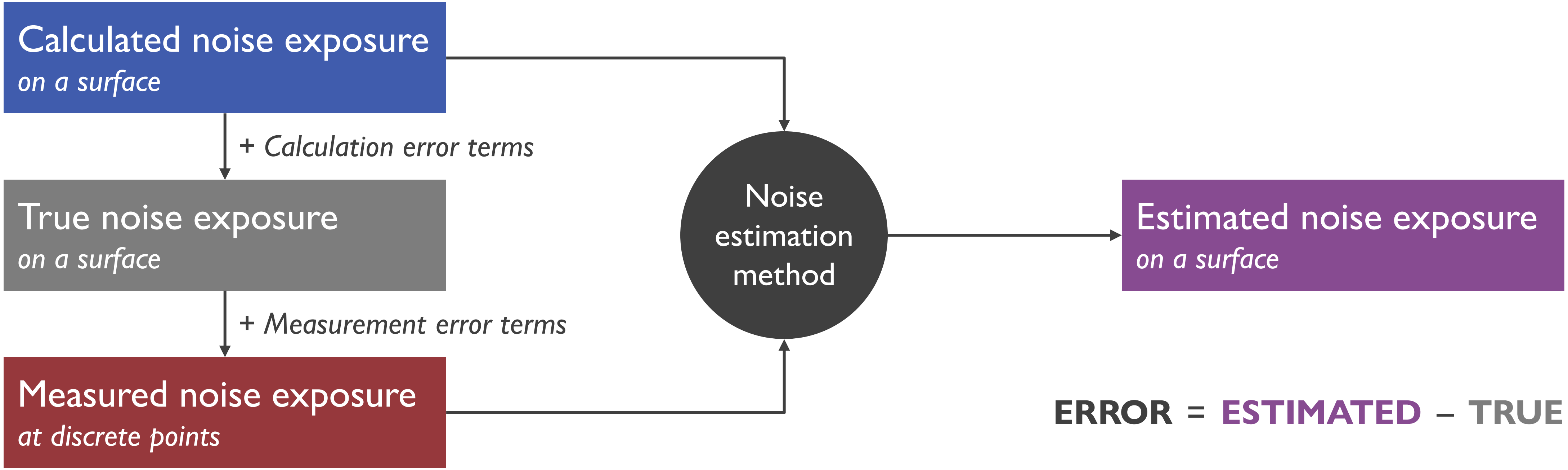
+ *Measurement error terms*

Measured noise exposure
at discrete points



Simulation Process

Goal: evaluate the accuracy of the noise estimation methods



× 2,500 simulations

QUESST

Image: NASA