

Infrared Reflection Removal in Wind Tunnels using Polarization Theory

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Infrared radiometry with infrared cameras allows for high spatial resolution, noncontact measurements in wind tunnel experiments and has been increasingly used to visualize flow features. Reflections from test section walls present a major challenge in collecting these measurements as it is difficult or impossible to separate them from the true model emission signal. This paper outlines a method to quantitatively remove specular reflections using polarization theory for materials common in the Unitary Plan Wind Tunnel. The method was however ultimately ineffective due to unexpected diffuse reflection and was unable to remove the specular reflection. The experiment provided insight into ways that polarization could be utilized to improve data collection capabilities of optical measurement techniques in wind tunnel settings.

Nomenclature

c_p, c_s	=	p- and s-polarized diffuse reflection constant, respectively
E	=	energy
E_{diffuse}	=	diffuse reflection source energy
E_e, E_r	=	emitted and reflected energy, respectively
E_p, E_s	=	p- and s-polarized energy, respectively
E_{specular}	=	specular reflection source energy
$I_R, I_{R\text{ref}}$	=	image intensity of strong reflection source and reference reflection source, respectively
$I_S, I_{S\text{ref}}$	=	image intensity of sample with strong reflection source and reference reflection source, respectively
n	=	index of refraction
$\vec{n}$	=	local surface normal vector
p, s	=	image intensity with p- and s-polarization, respectively
$\vec{t}$	=	vector from camera to surface
v, h	=	image intensity with vertical and horizontal polarization, respectively
$\vec{w}$	=	plane of incidence normal vector
$\vec{w}_x, \vec{w}_y$	=	x and y component of the plane of incidence normal vector, respectively
α_p, α_s	=	p- and s-polarized specular reflectivity, respectively
ε	=	unpolarized emissivity
θ	=	angle of incidence
ρ	=	unpolarized reflectivity
ρ_p, ρ_s	=	p- and s-polarized reflectivity, respectively
τ	=	transmissivity
φ	=	polarization angle

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I. Introduction

Infrared (IR) radiometry is a nondestructive, noncontact technique that measures thermal radiation emanating from a surface and has been increasingly used in production-level wind tunnels such as the Unitary Plan Wind Tunnel (UPWT) at NASA Ames Research Center for thermal flow field visualizations [1]. Research-grade mid-wave IR cameras are utilized for IR radiometry in wind tunnels due to their high spatial resolution and high sensitivity. IR radiometric images of a model surface are useful both real-time and after the test to visualize aerodynamic features such as boundary layer transitions and shock impingements [2–4].

Without significant and unrealistic prior knowledge, a single image cannot be used to differentiate an IR signal due to model emission from that of reflection of the surrounding environment. In a wind tunnel test setting, the test-section walls act as a radiative source of infrared light that reflects off the wind tunnel model surface and into the camera [1]. For aluminum or steel models with high reflectivity, the background reflections will dominate the signal of the images [5].

One method of mitigation is to paint the model surface with a flat, black, polyurethane paint to reduce the reflectivity and increase the emissivity [1,6]. This has the additional benefit of increasing the temperature gradients on the model surface by decreasing the thermal conductivity. The polyurethane paint reduces the reflectivity of the metal surface from roughly 90% to 12% [1].

Unfortunately, not all production-level tests are able to paint the model surface in this manner. Some tests may desire preserving the bare metal surface while other measurement techniques may already require a surface coating such as fast-response Pressure-Sensitive Paint (PSP) [7] or ink speckles for model deformation measurements [8]. The ability to simultaneously utilize PSP for pressure measurement and IR radiometry for temperature measure was the original motivation to bring this specular reflection removal technique to the wind tunnel.

This paper presents a method to use polarization theory to quantitatively remove specular reflection from the IR signal. Only basic prior knowledge such as the camera internal calibration, camera external calibration, model geometry, and the index of refraction of the model surface are required. The method can be applied to any optical measurement technique where specular reflections are a major concern and during data collection only requires two cameras: one horizontally polarized and one vertically polarized.

II. Theory

A. General Theory

All energy that reaches an object is absorbed, reflected, or transmitted through that object, and all energy emanating from an object is from emission, reflection, or transmission. It is assumed that the model is opaque in the infrared spectrum, at steady state or at least quasi-steady state, and all reflectivity is specular. With those assumptions we can ignore the absorption and transmission terms which implies emissivity (ϵ) and reflectivity (ρ) sum to unity, as given in Eq. (1).

$$1 = \epsilon + \rho \quad (1)$$

According to the Fresnel formulas and Snell's law, if the index of refraction of air is assumed to be that of free space, then reflectivity and emissivity are functions of the angle of incidence (AOI, θ) which is purely geometric, and the index of refraction (n) of the material which is assumed to be constant both spatially and with temperature [9,10]. AOI refers to the angle between the local surface normal vector and the vector from the camera to the surface at the point of interest. For simplicity, the materials are typically assumed to be dielectric, and so n is used to denote real values for index of refraction. This method is still valid for materials that are not dielectrics (i.e., conductive materials) but for simplicity the equations given will be for real values.

Reflectivity for p- and s-polarizations (ρ_p and ρ_s , respectively) as a function of AOI and index of refraction are given Eqs. (2) and (3). Equations (1), (2), and (3) were experimentally confirmed in [11]. Fig 1 shows a representative plot of reflectivity for the p- and s-polarizations.

$$\rho_p(\theta) = \left| \frac{n^2 \cos \theta - \sqrt{n^2 - \sin^2 \theta}}{n^2 \cos \theta + \sqrt{n^2 - \sin^2 \theta}} \right|^2 \quad (2)$$

$$\rho_s(\theta) = \left| \frac{\cos \theta - \sqrt{n^2 - \sin^2 \theta}}{\cos \theta + \sqrt{n^2 - \sin^2 \theta}} \right|^2 \quad (3)$$

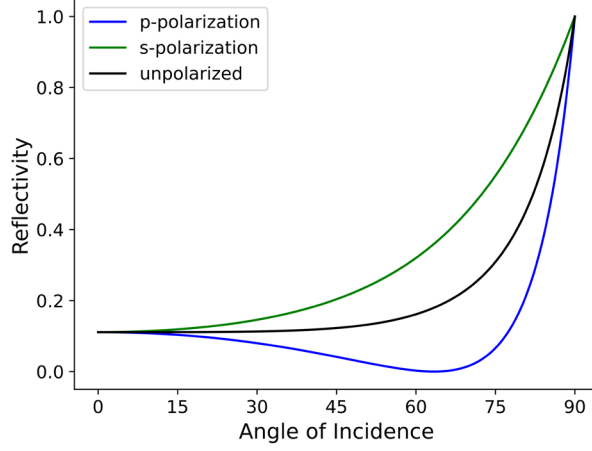


Fig 1: Reflectivities of Material with Index of Refraction of 2.0

The energy received by the camera can be defined as the sum of the emission and reflection energy that reach the camera. This is defined rigorously in Eq. (4) where E is the radiant energy that reaches the camera, E_e is the black body radiant energy emitted from the model, E_r is the radiant energy from a reflection source, and τ is the total transmissivity of the mediums between the camera and model such as the polarizer and camera lens. Equation (5) simply replaces ε with $1 - \rho$ from Eq. (1).

$$E = \varepsilon \tau E_e + \rho \tau E_r \quad (4)$$

$$E = (1 - \rho) \tau E_e + \rho \tau E_r \quad (5)$$

Because the emitted and reflected energy are transmitted through a polarizer, they are each reduced by half according to Malus' law [12]. From this fact Eq. (6) can be derived for the energy observed by camera with a p-polarizer (E_p) and Eq. (7) for an s-polarizer (E_s).

$$E_p = (1 - \rho_p) \tau \left(\frac{E_e}{2} \right) + \rho_p \tau \left(\frac{E_r}{2} \right) \quad (6)$$

$$E_s = (1 - \rho_s) \tau \left(\frac{E_e}{2} \right) + \rho_s \tau \left(\frac{E_r}{2} \right) \quad (7)$$

Algebraic manipulation of Eqs. (6) and (7) yields Eqs. (8) and (9) to express E_e and E_r in terms of the base quantities.

$$E_e = \frac{2(\rho_p E_s - \rho_s E_p)}{\tau(\rho_p - \rho_s)} \quad (8)$$

$$E_r = \frac{2((\rho_p - 1)E_s - (\rho_s - 1)E_p)}{\tau(\rho_p - \rho_s)} \quad (9)$$

III. Experiment

A. Tested Samples

To test the feasibility of the method, three samples were used with 3 different surface coatings: a bare aluminum sample (no coating), an aluminum sample coated with a flat, black, polyurethane paint, and an aluminum sample coated with fast-response PSP and its associated epoxy undercoat [13]. The bare aluminum sample represented the worst-case since the aluminum has not only low emissivity, roughly 10% based on previous experiments [1], but is also not a dielectric and so the index of refraction has a large imaginary term for the extinction coefficient. The polyurethane paint sample represented the best-case as it has high emissivity, estimated to be 88% based on previous experiments [1], and will act as a dielectric. The polyurethane paint is commonly employed at the UPWT for similar IR visualization tests due to its excellent material properties [1,6]. The PSP sample represented an intermediate case since it has an emissivity between the bare aluminum and polyurethane paint and acts as a dielectric.

B. Calibrations

1. Internal Calibration

An internal camera calibration was performed to measure the camera's focal length, principal point, and radial lens distortion. Additional terms such as pixel skew, pixel aspect ratio, and tangential lens distortion were ignored since they are negligible for modern cameras. Calib.io Camera Calibrator software [14] was used to measure internal calibration parameters from images of Calib.io's 14x19 coded checkerboard calibration board. The board was made with Calib.io's Lambertian substrate to reduce IR reflections and was chilled in a freezer to increase the contrast between the black and white squares.

2. External Calibration

An external camera calibration was performed to measure the position and orientation of the sample relative to the camera to estimate the AOI across the sample. The samples were placed on an optics table with a regular grid of 1/4"-20 tapped holes with 1" spacing. The holes surrounding the samples and associated image locations were used as 3D-to-2D correspondence points to solve the associated Perspective-n-Point problem [15] and obtain the external calibration. This is a similar method to that employed in other wind tunnel experiments [7,13].

3. Non-Uniformity Correction

Since the polarizer was placed directly in front of the lens and parallel to the cooled camera sensor, the center of the image appeared significantly darker than the rest of the image. This is a narcissism effect and is due to the reflection of the camera's thermal emission (or relative lack thereof from the cooled sensor). The magnitude of this effect is shown in Fig 2. Since reflection strength varies with polarization angle, the narcissism effect required an external non-uniformity correction (NUC) every time the polarizer orientation was changed. The correction was performed by holding a diffuse, flat plate at room temperature in front of the camera to entirely fill the field of view. The camera software then used an image of this to apply the correction. The external NUC accounted for the narcissism and for any changes in the temperature of the camera sensor. As a precaution, the samples were positioned such that they were outside of the center of the image to avoid the region where the narcissism effect was strongest. The positioning is shown in Fig 3.

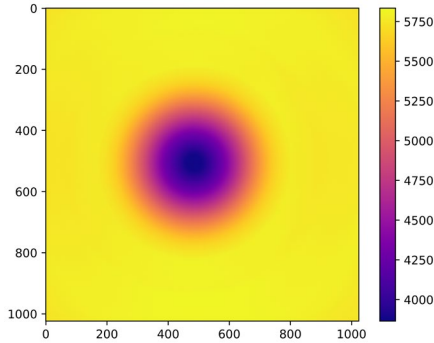


Fig 2: Polarizer Non-Uniformity

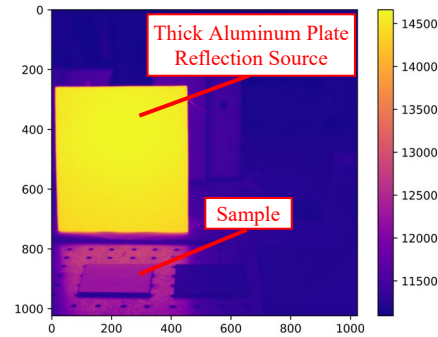


Fig 3: Sample Positioned Outside Non-Uniformity

C. Experimental Setup

Shown in Fig 4, the experimental setup consisted of a camera viewing one sample at a time with a reflection source acting on the sample. The camera was a FLIR SC8200 Research Grade Mid-Wave (3-5 μm) IR camera with a MoxTek ProFlux SIR polarizer (3-5 μm). The samples were 1/8" thick, 3" squares of aluminum with one of the aforementioned surface coatings. The samples were thermally isolated from the table with a piece of glass. The reflection source was thermally insulated from the table with several sheets of A4 paper. The AOI of the sample was roughly 65° to maximize the difference between the p- and s-polarization reflectivity.

Two reflection sources were used during the test. The first was an aluminum plate with a black coating from FLIR, which acted as a black reference. This source was at room temperature to produce minimal reflection on the samples. The second consisted of a thick aluminum plate in contact with a heat lamp controlled by a variac. After allowing the thick aluminum plate to reach steady state, the plate

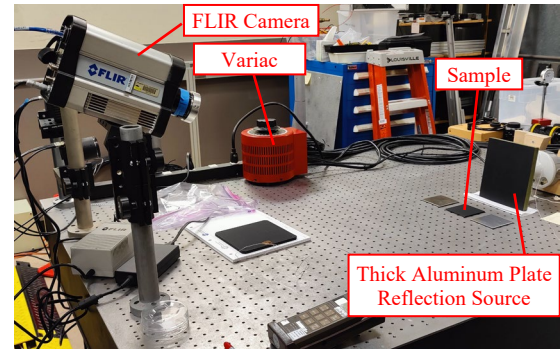


Fig 4: Experimental Setup

thermal emission was relatively uniform both temporally and spatially. Data was collected with the variac set to 40 volts with the resulting surface temperature of the thick aluminum plate at roughly 110° F (measured by a type-PTJ thermocouple). The thick aluminum plate was intended to act as a strong reflection source.

For data collection, the FLIR ResearchIR software was used to save batches of 100 images with an exposure time of 6 milliseconds and a frame rate of 10 frames per second. The batches of images were averaged into a single frame before data processing to mitigate the effect of camera noise. The thick plate reflection source and sample were given ample time to reach thermal equilibrium before the data collection began and every time new sample was imaged.

D. Reflection Measurement

The data collection attempted to measure the magnitude of reflection of the samples. This was done by collecting a batch of polarized images with the thick plate acting as a reflection source, then inserting the black reference sample between the thick plate and the samples and immediately collecting a batch of images with the same polarization. After collecting data with both polarizations, Eq. (10) was used to determine the polarized reflectivities. Equation (10) is based on a method similar to that of Appendix A in [1].

In Eq. (10), I_S is the image intensity of the sample with the thick plate acting as the reflection source and I_{Sref} is the image intensity of the sample with the black reference acting as the reflection source. Similarly, I_R is the image intensity of the thick plate and I_{Rref} is the image intensity of the black reference.

$$\rho = \frac{I_S - I_{Sref}}{I_R - I_{Rref}} \quad (10)$$

Equation (10) was applied on a pixel-wise basis to the samples, and the results are shown in Fig 5. The AOI for the experiment extends over a relatively small region, which is why the reflectivities appear to be nearly constant. As expected, the s-polarized reflectivity was higher than the p-polarized reflectivity, and the unpolarized reflectivity was the average of the two. Additionally, as expected the flat, polyurethane paint sample had the lowest reflectivity, the bare aluminum sample had the highest, and the PSP sample was between them.

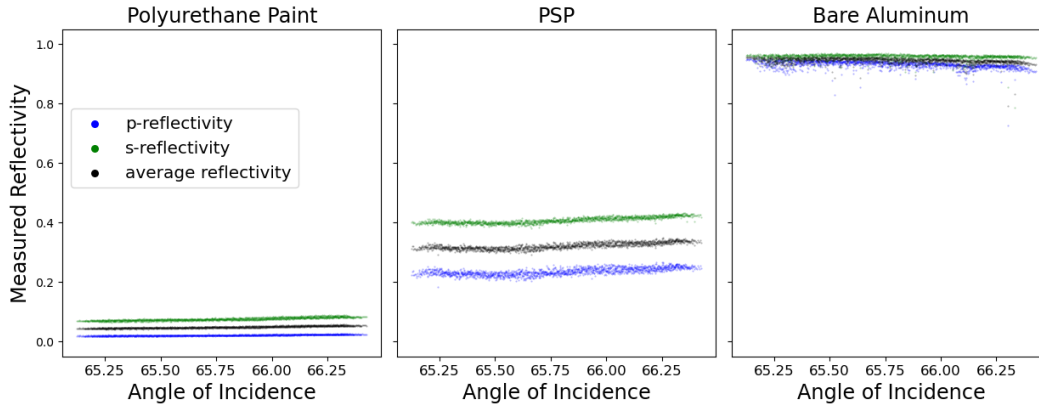


Fig 5: Measured Polarized Reflectivities

Unexpected was the fact that the polyurethane paint reflectivity was less than 10%, and that the p-reflectivity and s-reflectivity were so similar in magnitude for all the samples. An attempt was made to estimate each the samples' indices of refraction using the measured polarized reflectivities. Unfortunately, none of the measured values correspond to real valued indices of refraction, and the complex valued indices of refraction that match the measurements were non-physical. This was likely from a violation of the assumption that there is no diffuse reflection.

Diffuse reflection likely occurred since the materials did not have a mirror-like or even polished surface finish. The diffuse reflection can be added to the model, which updates Eqs. (6) and (7) to be Eqs. (11) and (12). These new equations have the additional terms α_p and α_s , which represent the proportion of reflection that is specular. $E_{specular}$ and $E_{diffuse}$ represent the specular and diffuse reflection source energies, respectively. The final new terms, c_p and c_s , are catch all terms that represent the proportion of diffuse reflection that is directed toward the camera. Diffuse reflection is significantly more complex than specular reflection since it is the summation of all non-specular reflection from all AOIs other than the that of the specular reflection.

$$E_p = (1 - \rho_p)\tau \left(\frac{E_e}{2}\right) + \rho_p\alpha_p\tau \left(\frac{E_{specular}}{2}\right) + c_p\tau E_{diffuse} \quad (11)$$

$$E_s = (1 - \rho_s)\tau \left(\frac{E_e}{2}\right) + \rho_s\alpha_s\tau \left(\frac{E_{specular}}{2}\right) + c_s\tau E_{diffuse} \quad (12)$$

With the addition of the diffuse reflection terms, the system of equations becomes underdetermined and impossible to solve without additional information.

IV. Discussion

A. Reflection Removal

Due to the relatively large diffuse reflection of the samples, the method was unable to remove specular reflection from the data. Other, more carefully constrained experiments that utilized this method were not hindered by this effect [10–12], but this effect would need to be addressed for viability in a tunnel such as the UPWT.

Since the system of equations becomes underdetermined with the addition of the diffuse terms, it is impossible to solve without additional information. Currently, it is unclear how to effectively address this issue, but there are several potential avenues.

First, the diffuse reflection could be reduced if the material is given a mirror-like or even polished surface finish. In a future experiment the aluminum could be polished, and the polyurethane paint could be wet sanded to achieve this effect. There is currently no procedure performed at NASA Ames to achieve a mirror-like surface finish for PSP coated samples. Due to the delicate nature of the PSP coating, any such procedure would be difficult to implement without severely reducing the PSP pressure sensing performance. Other surface coatings may better lend themselves to this method such as ink speckles for model deformation measurements since the surface is polished after the application of the ink.

Furthermore, if diffuse reflection cannot be reduced, it may be possible to measure or estimate the amount of diffuse reflection energy reaching the camera. With the assumption that the diffuse reflection is Lambertian, it can be inferred that c_p equals c_s and they are constant with respect to AOI. If α_p and α_s can be modeled based on a function of AOI, then an additional pair of polarized cameras would balance the systems of equations and the emission could once again be separated from the reflection.

B. Reflection Mitigation

One viable way to capitalize on the difference in reflectivity based on polarization is to collect p-polarized data rather than unpolarized data during the test. For all AOIs, the p-polarized reflectivity is less than the unpolarized reflectivity and the p-polarized emissivity is greater than the unpolarized emissivity. The magnitude of this effect is based on the index of refraction of the material and the AOI, but it can be considerable.

Fig 6 depicts the magnitude of the effect for the materials used in this study based on estimated values for the index of refraction from available literature [1,16]. The plots are representative of the sample materials and can be used for qualitative analysis but should not be used for quantitative methods. The y-axis is representative of ‘performance’ where higher values mean better performance. The y-axes are clipped since they approach infinity at Brewster’s angle where the p-polarized reflectivity is zero and the performance is ideal.

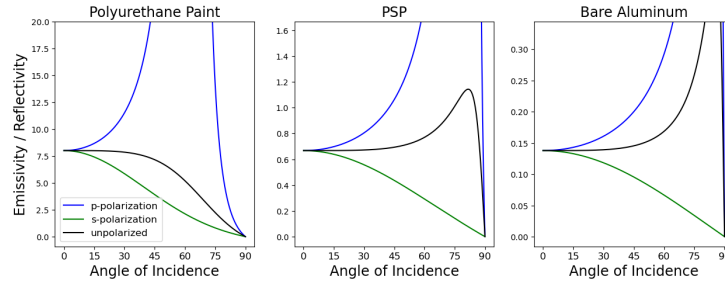


Fig 6: Representative Emissivity / Reflectivity for Selected Materials

One caveat is that the polarizations are horizontal and vertical with respect to the camera sensor. Our experiment aligned these with the plane of incidence and therefore with the p- and s-polarizations, but for cylindrical or spherical models, alignment is not possible over the entire model surface. Equations (15) and (16) set up a system of equations to calculate the p-polarization signal, p , and s-polarization signal, s , from the horizontally polarized, h , and vertically polarized signal, v . This is derived from Malus’ law via the polarization angle, ϕ , which can be calculated for points on the model’s surface.

Each point of the model geometry can be mapped to a pixel using projection to acquire the h and v for that point. Then φ can be calculated using Eqs. (13) and (14) for that point using the vector from the camera to that point, $\vec{t}$, and the point's local normal vector with respect to the camera coordinate frame, $\vec{n}$. The plane that contains $\vec{t}$ and $\vec{n}$ is referred to as the plane of incidence, and $\vec{w}$ is the normal vector of this plane. The solution to the system of equations formed from Eqs. (15) and (16) can then be found to determine the p- and s-polarized signals.

$$\vec{w} = \vec{t} \times \vec{n} \quad (13)$$

$$\varphi = \arctan2(\vec{w}_x, \vec{w}_y) \quad (14)$$

$$v = p(\sin^2\varphi) + s(\cos^2\varphi) \quad (15)$$

$$h = p(\cos^2\varphi) + s(\sin^2\varphi) \quad (16)$$

C. Other Limitations

Even ignoring the diffuse reflection issue, the method suffers from several other limitations, which taken together make it difficult to scale this method into a production-level wind tunnel such as the UPWT. Some limitations are due to the nature of the underlying physics that this method leverages and cannot be avoided, while others can be addressed but are difficult to implement.

The most severe limitation is that the method is inaccurate at both low and high AOIs. Low and high are relative and depend on the index of refraction of the material and the accuracy of the AOI estimation for each pixel. As shown in Fig 7, below roughly 20° the p- and s-polarized reflectivities are similar in magnitude which causes the method to become sensitive to small input errors. As shown in Fig 8, above 70°, the reflectivities have a relatively large rate of change with respect to the AOI and again become sensitive to small input errors. Additionally, at high AOI, more of the model surface area is imaged by each pixel which reduces the spatial resolution of the method.

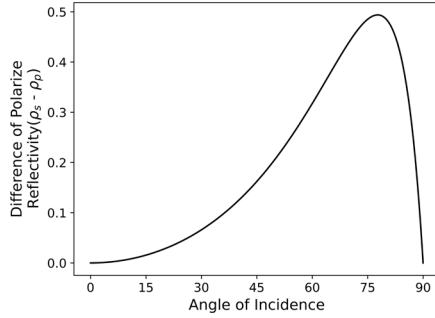


Fig 7: Difference of Polarized Reflectivities for Material with Index of Refraction of 2.0 (lower means more sensitive to input errors)

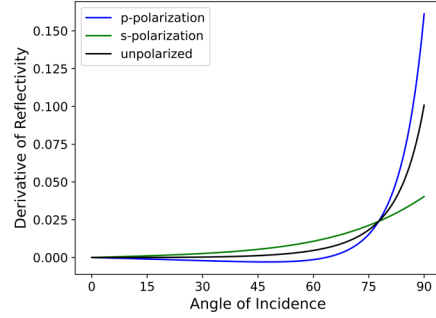


Fig 8: Derivative of Reflectivity for Material with Index of Refraction of 2.0 (higher means more sensitive to input errors)

Another limitation is that the signal into the camera is reduced by roughly half by the polarizer, and so the images will have roughly half the intensity. This decreases the signal to noise ratio of the system since read noise is constant and shot noise is proportional to the square root of the intensity. However, this issue can be sidestepped by doubling the exposure time of the camera. While not ideal, a high frame rate is not typically required since the thermal gradients do not change very quickly in time.

Another limitation is in collecting the p- and s-polarized data simultaneously. In the lab, the samples are at steady state which allowed the images to be separated in time and it was possible to rotate the polarizer between data collections. In the wind tunnel, the cameras are isolated to the operator and cannot be accessed. This would then require two cameras, one vertically polarized and one horizontally polarized. The cameras would need to have the same field of view so that the same reflection source is visible to both cameras. A polarizing beam splitter could be utilized to this end. While feasible, it complicates the setup and would be difficult to implement and properly align the cameras.

A final limitation is frequently performing external NUCs. The FLIR SC8200 cameras have an internal paddle that can move in front of the camera sensor to act as a black reference. This allows for an operator to perform a NUC during the test to correct for thermal drift of the camera sensor due to extended runtime or changes in the tunnel temperature. With the polarizer, an external rather than internal NUC was required to additionally correct for the polarizer non-uniformity. An external, motorized black reference could be used, but this would again complicate the tunnel setup and would be difficult to implement.

V. Conclusion

The results of the experiment seem to have potential and are frustratingly close to separating the reflection signal from the emission signal. While the experiment did not accomplish the goal of removing reflection from IR images, it did provide insight which we hope will be useful in utilizing polarization in a wind tunnel setting. The greatest limitation to bring this method to the UPWT at NASA Ames Research Center was that the IR reflections could not be modeled with a simple specular model and likely require the addition of more complex diffuse terms. However, there is hope in use polarization to improve performance since, relative to unpolarized data, p-polarized data has higher emissivity and lower reflectivity. We are optimistic that an improvement to this method will be found and that the method could make its way into use at the UPWT.

References

- [1] Garbeff, T. J., and Baerny, J. K. Recent Advancements in the Infrared Flow Visualization System for the NASA Ames Unitary Plan Wind Tunnels. 2017.
- [2] Garbeff, T. J., Panda, J., Ross, J. C., and Smith, N. T. Experimental Visualizations of a Generic Launch Vehicle Flow Field: Time-Resolved Shadowgraph and Infrared Imaging. 2017.
- [3] Le Sant, Y., Marchand, M., Millan, P., and Fontaine, J. “An Overview of Infrared Thermography Techniques Used in Large Wind Tunnels.” *Aerospace science and technology*, Vol. 6, No. 5, 2002, pp. 355–366.
- [4] Garbeff, T. J., Baerny, J., and Ross, J. C. Wind Tunnel Flow Field Visualizations of the Space Launch System Vehicle Ascent. 2019.
- [5] Möllmann, K.-P., and Vollmer, M. *Infrared Thermal Imaging: Fundamentals, Research and Applications*. John Wiley & Sons, 2017.
- [6] Garbeff, T. J., and Baerny, J. A Qualitative Investigation of Selected Infrared Flow Visualization Image Processing Techniques. 2019.
- [7] Roozeboom, N. H., Powell, J., Baerny, J. K., Murakami, D. D., Ngo, C. L., Garbeff, T. J., Ross, J. C., and Flach, R. “Development of Unsteady Pressure-Sensitive Paint Application on NASA Space Launch System.” *AIAA Scitech 2019 Forum*, 2019, pp. 1–16. <https://doi.org/10.2514/6.2019-3502>.
- [8] Schairer, E. T., Kushner, L. K., Garbeff, T. J., and Heineck, J. T. “Model Deformation Measurements of Sonic Boom Models in the NASA Ames 9- by 7-Ft Supersonic Wind Tunnel.” *53rd AIAA Aerospace Sciences Meeting*, No. January, 2015, pp. 1–17. <https://doi.org/10.2514/6.2015-1913>.
- [9] Li, N., Zhao, Y., Pan, Q., and Kong, S. G. “Removal of Reflections in LWIR Image with Polarization Characteristics.” *Optics express*, Vol. 26, No. 13, 2018, pp. 16488–16504.
- [10] Ogasawara, N., Kobayashi, C., and Yamada, H. Infrared Image Correction for the Reduction of Background Reflection. No. 10214, 2017, pp. 351–357.
- [11] SUZUKI, S., OGASAWARA, N., and Ogasawara, N. “Quantitative Evaluation of Polarized Emissivity and Polarized Reflectivity Using Infrared Thermographic Instrument.” *Advanced Experimental Mechanics*, Vol. 3, 2018, pp. 167–172.
- [12] Suzuki, S., and Ogasawara, N. “Infrared Thermographic Test for Removing Background Reflection Based on Polarization Theory.” *NDT & E International*, Vol. 103, 2019, pp. 19–25.
- [13] Gregory, J. W., Asai, K., Kameda, M., Liu, T., and Sullivan, J. P. “A Review of Pressure-Sensitive Paint for High-Speed and Unsteady Aerodynamics.” *Proceedings of the Institution of Mechanical Engineers, Part G: Journal of Aerospace Engineering*, Vol. 222, No. 2, 2008, pp. 249–290.
- [14] calib.io. Camera Calibration Boards, Targets and Software by Calib.Io. <https://calib.io/>.
- [15] NVIDIA Corporation. Isaac SDK — ISAAC 2020.1 Documentation. <https://docs.nvidia.com/isaac/isaac/packages/pnp/doc/pnp.html>.
- [16] Rakić, A. D. “Algorithm for the Determination of Intrinsic Optical Constants of Metal Films: Application to Aluminum.” *Applied optics*, Vol. 34, No. 22, 1995, pp. 4755–4767.