Finite Element Model Tuning using Analytical Sensitivity Values

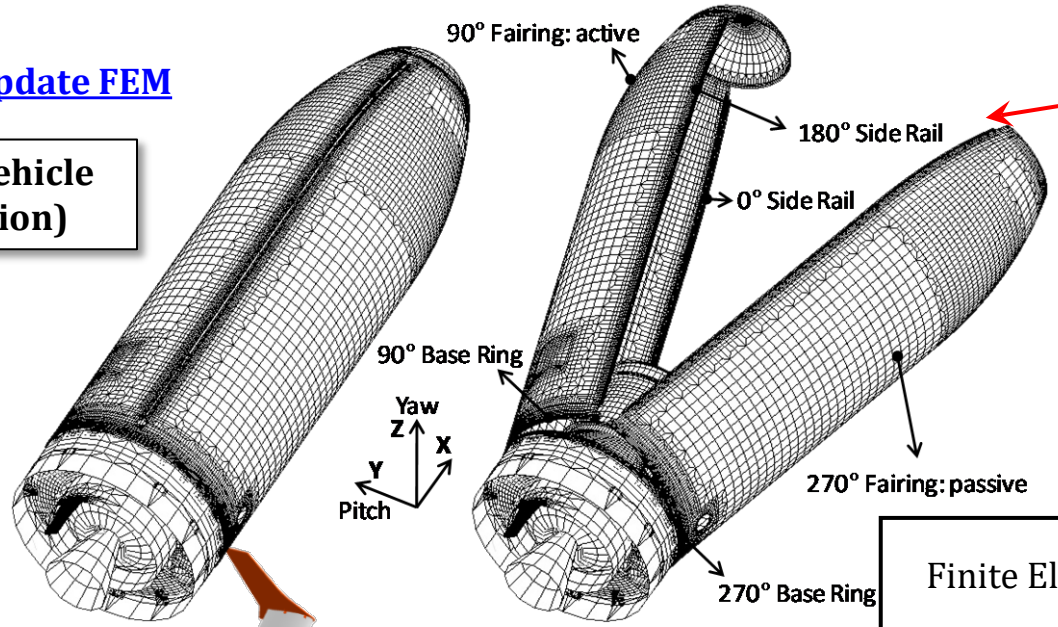


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Flutter Analysis Procedure @ NASA Armstrong

- Quality of Finite Element (FE) Model ??
  - Before FRR, validate FE Model wrt ground test data and **if needed update FEM using optimization**
- Tune FE Model using test data
  - From Industry
  - In-house development
- Flutter Analysis to guarantee the safety of flight
  - Based on validated FE Model

Taurus XL Launch Vehicle (Mishap investigation)





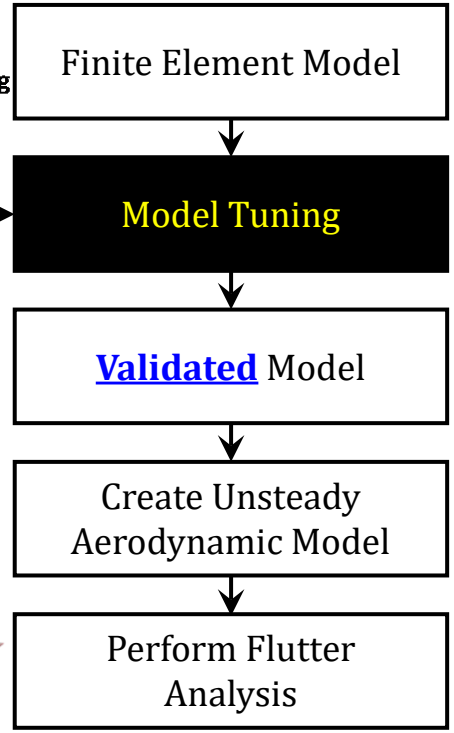








Weight, C.G., Moment of inertia, GVT data, & Static deformation



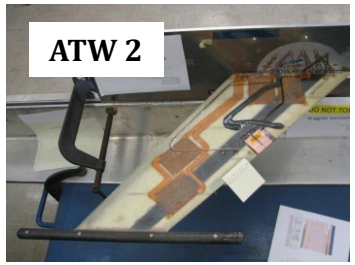
Model Correlation Requirements

- **References**
  - **MIL-STD-1540C** Section 6.2.10
  - **NASA-STD-5002** Section 4.2.6.d
  - AFFTC-TIH-90-001 (Structures Flight Test Handbook)
- **Frequency correlation**
  - **Primary modes**: within **3%** (MIL-STD) or **5%** (NASA-STD) of test frequencies
  - Secondary modes: no comments in standards (within **10%** of test frequencies)
- **Mass orthogonality: Auto Orthogonality Matrix (AOM)** $\boldsymbol{\Phi_G^T M \Phi_G}$
  - Use orthogonality matrix:
    - $\boldsymbol{\Phi_G}$ = mode shape from GVT
    - $\mathbf{M}$ = analytical mass matrix
  - **Primary modes**: Off-diagonal terms should be less than **10%** (0.1 when diagonal is 1.0).
  - Secondary modes: no comments in standards
- **Mode shape correlation: Cross Orthogonality Matrix (COM)** $\boldsymbol{\Phi_G^T M \Phi_A}$
  - Use cross-orthogonality matrix:
    - $\boldsymbol{\Phi_A}$ = mode shape from analysis
  - **Primary modes**: Off-diagonal terms should be less than **10%** & diagonal terms should be greater than **90%** (NASA-STD) or **95%** (MIL-STD).
  - Secondary modes: no comments in standards

Mathematical Backgrounds

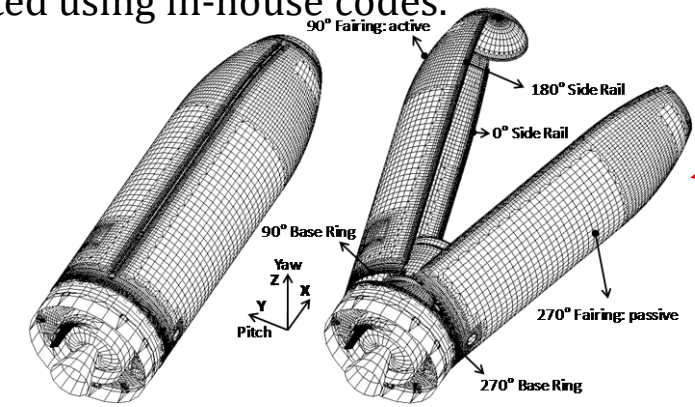
- Developed a model tuning tool for more than 13 years.
  - Improving FEM of the Quite Spike Boom, ATW2, X-56A, & others
  - Also used during the mishap investigation of the Taurus XL launch vehicle.
  - Major issue with the **previous** in-house **model tuning tool**
    - Sensitivity analyses of the performance indices were based on "a **finite difference method**."
    - Computation was long
      - ✓ can not use large number of design variables
      - ✓ Ex: **Twenty-four** design variables for **X-56A**







- Introduce an analytical sensitivity analysis for the model tuning procedure
  - Sensitivity values of the response functions are computed using the **MSC NASTRAN** sol 200 at a **single** design point
    - Performance indices for the model tuning procedure are computed using in-house codes.
    - **Response functions** in this study are as follows:
      - ✓ Total weight: W
      - ✓ Center of gravity location: $x_{CG}, y_{CG}, \& z_{CG}$
      - ✓ Mass moment of inertia: $I_{xxG}, I_{yyG}, I_{zzG}, I_{xyG}, I_{xzG}, \& I_{yzG}$
      - ✓ Frequencies: $f_j \quad j = 1, 2, \ldots, m$
      - ✓ Mode shapes: $\boldsymbol{\Phi_R}$, (order = $n_{RDOF} \times m$)
      - ✓ Static deformations: Δ_{ij} is the deformation at the j-th sensor under the i-th load condition





Taurus XL Launch Vehicle (Mishap investigation)

Analytical Sensitivity Analysis of Performance Indices

- **Type 1**: square of the relative error

  i-th performance index

$$J_i = \left(\frac{F_i(x_k) - F_{ir}}{F_{ir}}\right)^2 \quad (1)$$

  Sensitivity value of J_i

$$\frac{\partial J_i}{\partial x_k} = 2\left(\frac{F_i(x_k) - F_{ir}}{F_{ir}}\right)\left(\frac{1}{F_{ir}}\right)\frac{\partial F_i(x_k)}{\partial x_k} \quad (2)$$

  - Response functions except mode shapes and static deformations
    - $F_i(x_k) = W, x_{CG}, y_{CG}, z_{CG}, I_{xxG}, I_{yyG}, I_{zzG}, I_{xyG}, I_{xzG}, I_{yzG}$, & f_j $j = 1, 2, \dots, m$
    - F_{ir}: measured $F_i(x_k)$

- **Type 2**: off-diagonal terms of AOM & COM

$$J_{ij} \equiv {m_{ij}}^2 \quad (22)$$

$$\frac{\partial J_{ij}}{\partial x_k} = 2m_{ij}\frac{\partial m_{ij}}{\partial x_k} \quad (23)$$

  - An **off-diagonal term** m_{ij} is the i-th row and j-th column element of the **AOM or COM**.

- **Type 3**: diagonal terms of COM

$$J_{ii} \equiv 1 - {m_{ii}}^2 \quad (24)$$

$$\frac{\partial J_{ii}}{\partial x_k} = -2m_{ii}\frac{\partial m_{ii}}{\partial x_k} \quad (25)$$

  - The term m_{ii} is the i-th **diagonal element of the COM**.

- **Type 4**: static deformations

  Performance index for the i-th load case

$$J_{Li} = \frac{1}{m_p}\sum_{j=1}^{m_p}\left(\Delta_{ij} - \Delta_{ijT}\right)^2 \quad (26)$$

  Sensitivity value of J_{Li}

$$\frac{\partial J_{Li}}{\partial x_k} = \frac{1}{m_p}\sum_{j=1}^{m_p} 2\left(\Delta_{ij} - \Delta_{ijT}\right)\frac{\partial \Delta_{ij}}{\partial x_k} \quad (27)$$

  - Δ_{ij} is the **deformation** at the j-th sensor under the i-th load condition.
  - Δ_{ijT}: measured Δ_{ij}
  - m_p is the total number of deformation sensors during static test.

Obtained from NASTRAN sol 200

$$\boldsymbol{M_{AOM}} = \Phi_{GR}^{T}\mathbf{M_R}\Phi_{GR} \quad (\text{order} = m \times m)$$

$$\boldsymbol{M_{COM}} = \Phi_{GR}^{T}\mathbf{M_R}\Phi_{R} \quad (\text{order} = m \times m)$$

Obtained from NASTRAN sol 200 & in-house codes

The types 1 and 4 computations are straight-forward, however types 2 and 3 require a complicate derivation for the analytical sensitivity values.

$M_{AOM} = \Phi_{GR}^{T} M_R \Phi_{GR}$ & $M_{COM} = \Phi_{GR}^{T} M_R \Phi_R$

- To check the mass orthogonality of the measured, $\tilde{\Phi}_{GM}$, and computed, Φ_R, mode shapes, we should perform the matrix triple multiplication. Therefore, matrix sizes should be compatible.
  - $M_{AOM} = \Phi_G{}^{T} M \Phi_G = \tilde{\Phi}_{GM}{}^{T} M \tilde{\Phi}_{GM}$
  - $M_{COM} = \Phi_G{}^{T} M \Phi_A = \tilde{\Phi}_{GM}{}^{T} M \Phi_A$

???

- Use full-order model (FOM)
  - Size of the mass matrix is too large (ex 438321 by 438321)
  - Matrix multiplication itself requires a long computation time. ($M_R \Phi_R$)
    - At every design optimization points, we should perform matrix triple multiplication. (smaller order model should be used.)
- Use test model
  - The matrix M should be reduced to the test DOF or the measured mode shape, $\tilde{\Phi}_{GM}$, should be expanded to the full order. This can cause an accuracy issue due to oversimplification.
- Use reduced order model (ROM)
  - ROM with an intermediate matrix size which is created using the system equivalent reduction and expansion process (SEREP).

$$M_{AOM} = \Phi_{GR}^{T} M_R \Phi_{GR}$$
$$M_{COM} = \Phi_{GR}^{T} M_R \Phi_R$$

$$\Phi_{GR} = \begin{bmatrix} \Phi_M \Psi_M \\ \Phi_S \Psi_M \end{bmatrix} \tilde{\Phi}_{GM} \quad \text{: Expansion}$$

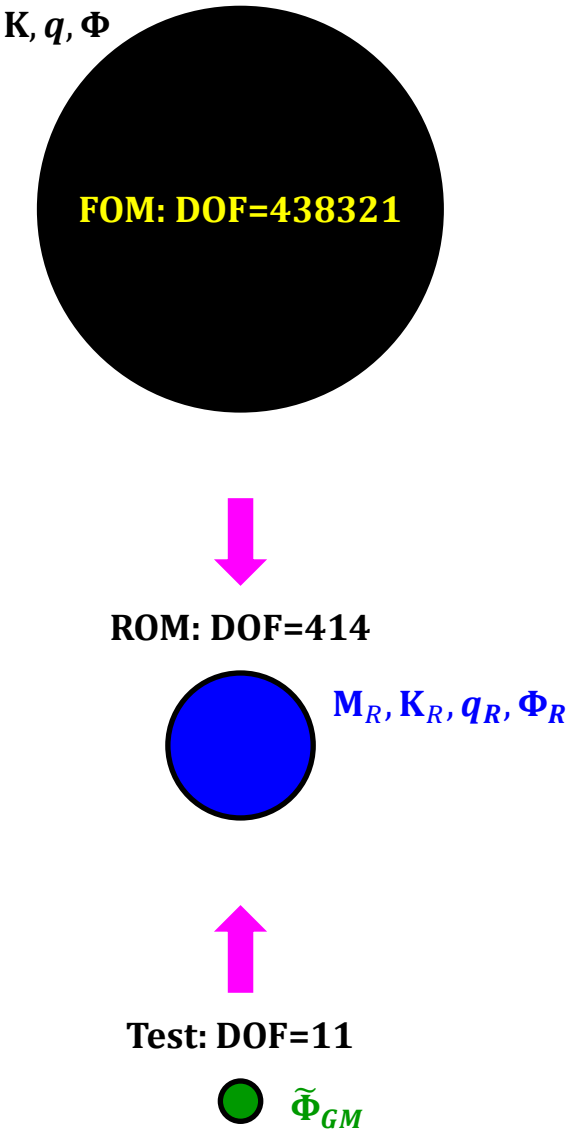
$$q = \begin{Bmatrix} q_R \\ q_O \end{Bmatrix} = \begin{Bmatrix} q_M \\ q_S \\ q_O \end{Bmatrix}$$

$$M_R = \Psi_R^T \Psi_R$$
$$\Psi_R = \left(\Phi_R^T \Phi_R\right)^{-1} \Phi_R^T$$

$$q = \Phi \Psi_R q_R = \begin{bmatrix} \Phi_R \Psi_R \\ \Phi_O \Psi_R \end{bmatrix} q_R = T q_R \quad \text{: Reduction}$$

$$\Phi = \begin{bmatrix} \Phi_R \\ \Phi_O \end{bmatrix} = \begin{bmatrix} \Phi_M \\ \Phi_S \\ \Phi_O \end{bmatrix}$$

  - The frequencies and mode shapes using ROM are identical with modal results from the FOM.
  - The number of DOF for the ROM is determined using the MSC NASTRAN with ASET cards.
    - The Guyan reduction is used for the MSC NASTRAN with ASET cards.



Analytical Sensitivity of Matrices $\frac{\partial M_{AOM}}{\partial x_k}$ and $\frac{\partial M_{COM}}{\partial x_k}$

- The expanded GVT mode shape matrix $\boldsymbol{\Phi_{GR}}$ is orthonomalized once at the starting design configuration during each optimization cycle. Therefore, $\boldsymbol{\Phi_{GR}}$ will be assumed as a constant matrix during optimization.

Normalize with starting configuration
Assume constant during optimization

$$\boldsymbol{\Phi_{GR}} = \begin{bmatrix} \boldsymbol{\Phi_M \Psi_M} \\ \boldsymbol{\Phi_S \Psi_M} \end{bmatrix} \boldsymbol{\tilde{\Phi}_{GM}} \qquad \text{(order} = n_{RDOF} \times m\text{)}$$

Order expansion using **SEREP**

$$\boldsymbol{\Psi_M} = \left(\boldsymbol{\Phi_M^T \Phi_M}\right)^{-1} \boldsymbol{\Phi_M^T} \qquad \text{(order} = m \times n_{MDOF}\text{)}$$

Expansion GVT to ROM

Constant during optimization (measured data)

- The analytical sensitivity of mass matrix, $\frac{\partial \mathbf{M}_R}{\partial x_k}$, is needed, and you can refer my paper for details.
  - In this derivation, $\boldsymbol{\Phi_R}$ and the corresponding sensitivity value, $\frac{\partial \boldsymbol{\Phi_R}}{\partial x_k}$, are the response function values and will be computed using the MSC NASTRAN code in this study.

NASTRAN Sol 200: $\boldsymbol{\Phi}_R$ and $\frac{\partial \boldsymbol{\Phi}_R}{\partial x_k}$

$$\frac{\partial \boldsymbol{M_{AOM}}}{\partial x_k} = \frac{\partial \boldsymbol{\Phi_{GR}^T M_R \Phi_{GR}}}{\partial x_k} = \frac{\partial \boldsymbol{\Phi_{GR}^T}}{\partial x_k}\mathbf{M}_R\boldsymbol{\Phi_{GR}} + \boldsymbol{\Phi_{GR}^T}\frac{\partial \mathbf{M}_R}{\partial x_k}\boldsymbol{\Phi_{GR}} + \boldsymbol{\Phi_{GR}^T}\mathbf{M}_R\frac{\partial \boldsymbol{\Phi_{GR}}}{\partial x_k}$$

$$\frac{\partial \boldsymbol{M_{COM}}}{\partial x_k} = \frac{\partial \boldsymbol{\Phi_{GR}^T M_R \Phi_R}}{\partial x_k} = \frac{\partial \boldsymbol{\Phi_{GR}^T}}{\partial x_k}\mathbf{M}_R\boldsymbol{\Phi_R} + \boldsymbol{\Phi_{GR}^T}\frac{\partial \mathbf{M}_R}{\partial x_k}\boldsymbol{\Phi_R} + \boldsymbol{\Phi_{GR}^T}\mathbf{M}_R\frac{\partial \boldsymbol{\Phi_R}}{\partial x_k}$$

$$\frac{\partial \mathbf{M}_R}{\partial x_k} = \frac{\partial \boldsymbol{\Psi}_R^T}{\partial x_k}\left(\boldsymbol{\Phi}_R^T\boldsymbol{\Phi_R}\right)^{-1}\boldsymbol{\Phi}_R^T + \boldsymbol{\Phi_R}\left(\boldsymbol{\Phi}_R^T\boldsymbol{\Phi_R}\right)^{-T}\frac{\partial \boldsymbol{\Psi_R}}{\partial x_k}$$

$$\frac{\partial \boldsymbol{\Psi_R}}{\partial x_k} = \left(\boldsymbol{\Phi}_R^T\boldsymbol{\Phi_R}\right)^{-1}\left[\frac{\partial \boldsymbol{\Phi}_R^T}{\partial x_k} - \left(\frac{\partial \boldsymbol{\Phi}_R^T}{\partial x_k}\boldsymbol{\Phi_R} + \boldsymbol{\Phi}_R^T\frac{\partial \boldsymbol{\Phi_R}}{\partial x_k}\right)\left(\boldsymbol{\Phi}_R^T\boldsymbol{\Phi_R}\right)^{-1}\boldsymbol{\Phi}_R^T\right]$$

Derived in Appendix

$$\mathbf{M}_R = \boldsymbol{\Psi}_R^T\boldsymbol{\Psi_R} \qquad \text{(order} = n_{RDOF} \times n_{RDOF}\text{)}$$

$$\boldsymbol{\Psi_R} = \left(\boldsymbol{\Phi}_R^T\boldsymbol{\Phi_R}\right)^{-1}\boldsymbol{\Phi}_R^T \qquad \text{(order} = m \times n_{RDOF}\text{)}$$

$$\boldsymbol{\Phi_R} \equiv \begin{bmatrix} \boldsymbol{\Phi_M} \\ \boldsymbol{\Phi_S} \end{bmatrix} \qquad \text{(order} = n_{RDOF} \times m\text{)}$$

$$\boldsymbol{\Phi} = \begin{bmatrix} \boldsymbol{\Phi_R} \\ \boldsymbol{\Phi_O} \end{bmatrix} = \begin{bmatrix} \boldsymbol{\Phi_M} \\ \boldsymbol{\Phi_S} \\ \boldsymbol{\Phi_O} \end{bmatrix}$$

$$I = \boldsymbol{\Phi}_R^T\mathbf{M}_R\boldsymbol{\Phi_R} \qquad \text{(order} = m \times m\text{)}$$

$$\boldsymbol{M_{AOM}} = \boldsymbol{\Phi_{GR}^T}\mathbf{M}_R\boldsymbol{\Phi_{GR}} \qquad \text{(order} = m \times m\text{)}$$

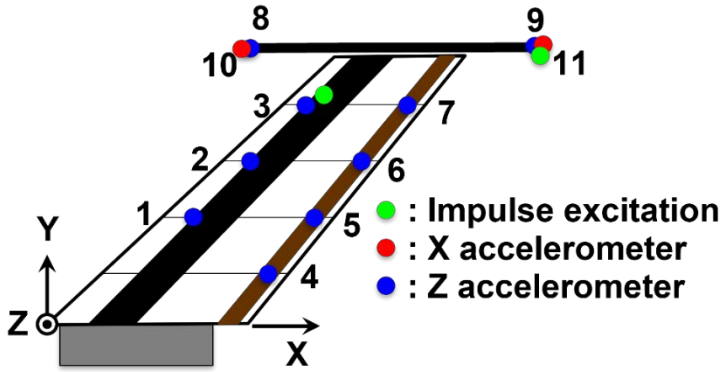
$$\boldsymbol{M_{COM}} = \boldsymbol{\Phi_{GR}^T}\mathbf{M}_R\boldsymbol{\Phi_R} \qquad \text{(order} = m \times m\text{)}$$

m : number of modes
n_{RDOF} : DOF in ROM
n_{MDOF} : DOF in GVT

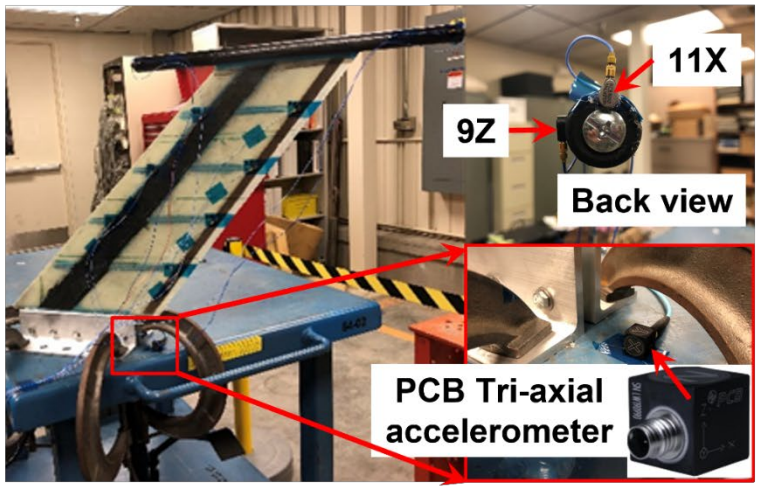
Ground Vibration Test of Baseline Aerostructures Test Wing 4

- Sample case: Baseline ATW4.
- Select 11 PCB teardrop accelerometers.
  - 9 in z direction
  - 2 in x direction
- Use two PCB tri-axial accelerometers to detect the mounting table motion.
- For out-of-plane motion, the accelerometer position 3 was excited using the impulse hammer.
- In case of in-plane motion, the accelerometer position 11, trailing-edge of the wing tip boom, was excited in x direction.

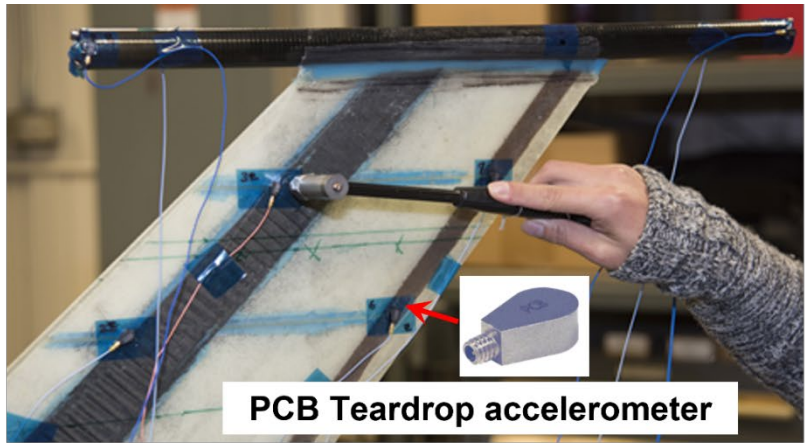
Accel #	Grid number (DOF)
1	1847 (Z)
2	2315 (Z)
3	1484 (Z)
4	1370 (Z)
5	2232 (Z)
6	2606 (Z)
7	1754 (Z)
8	2 (Z)
9	31 (Z)
10	2 (X)
11	31 (X)



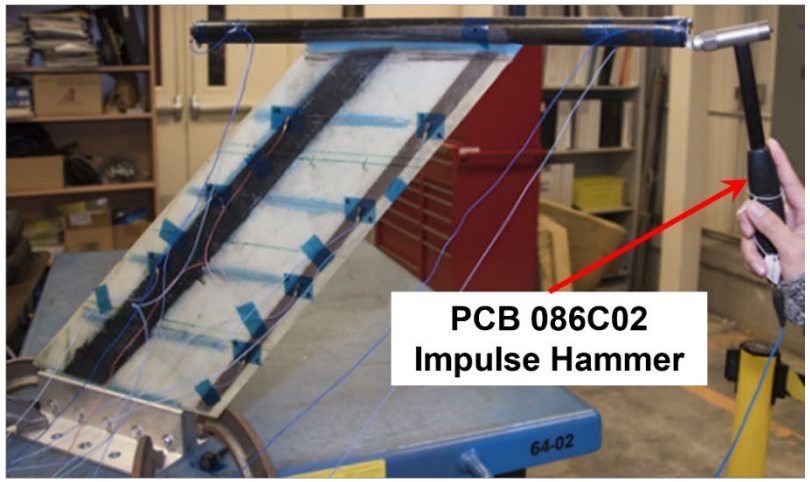
(a) Accelerometer DOF and impact locations



(b) Accelerometer locations

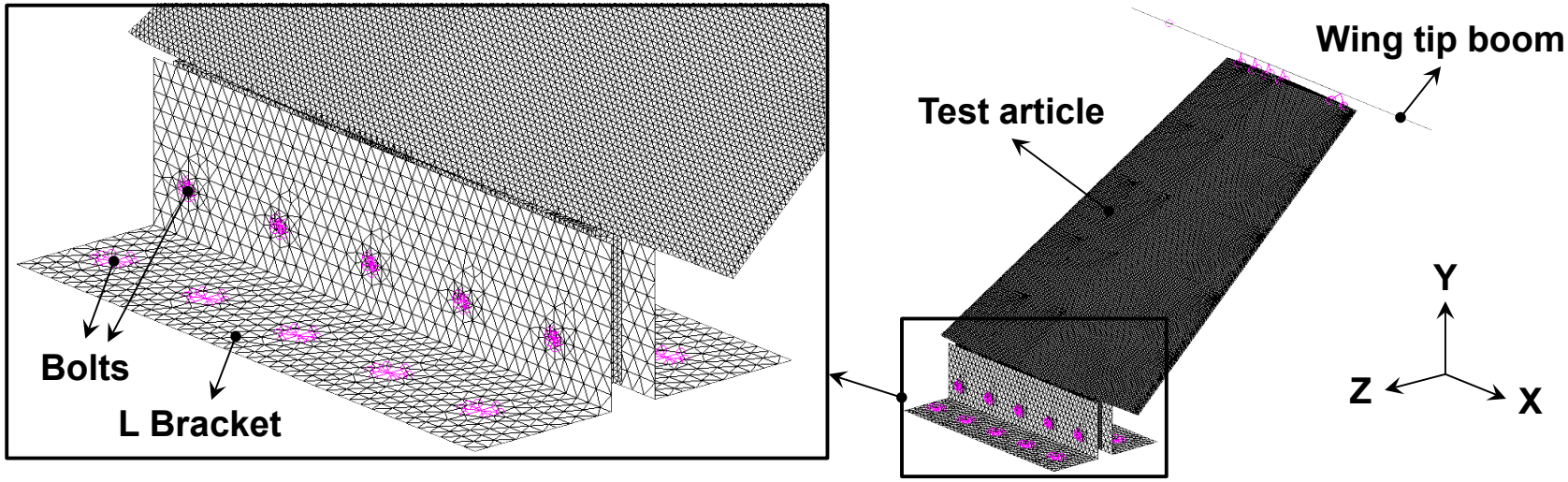


(c) Impact at node 3 in Z- direction



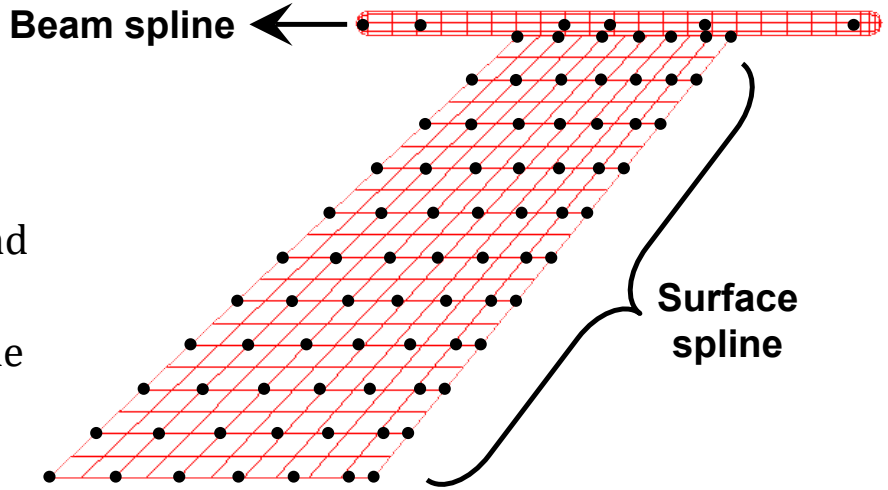
(d) Impact at node 11 in X- direction

Structural and Aerodynamic Models



(a) Finite element structural dynamic model

- The finite element model of ATW4 with the L-bracket for mounting the test article to the foundation is given in this figure.
- The ZAERO based unsteady aerodynamic model with spline points is also provided.
- The wing is modeled using the flat aerodynamic trapezoids and the wing-tip boom is modeled using the body element.
- A surface spline is used for the wing portion, and a beam spline is used for the wing-tip boom.



(b) Aerodynamic model

V-g and V-f Curves using Baseline ATW 4 Model

	Modal Participation	Measured	Baseline ATW4	
			FEM	% Error
Weight	N/A	1.686	1.742	3.33
X-C.G.		13.37	13.67	2.24
Y-C.G.		7.763	7.944z	2.33
Mode 1	**5.37%**	**14.65**	**17.26**	**17.8**
Mode 2	**93.5%**	**29.55**	**33.54**	**13.5**
Mode 3	**0.87%**	**84.92**	**91.18**	**7.38**
Mode 5	**0.27%**	**181.8**	**184.5**	**1.47**
Mode 6	0.00%	210.7	228.0	8.21

3~5% requirements

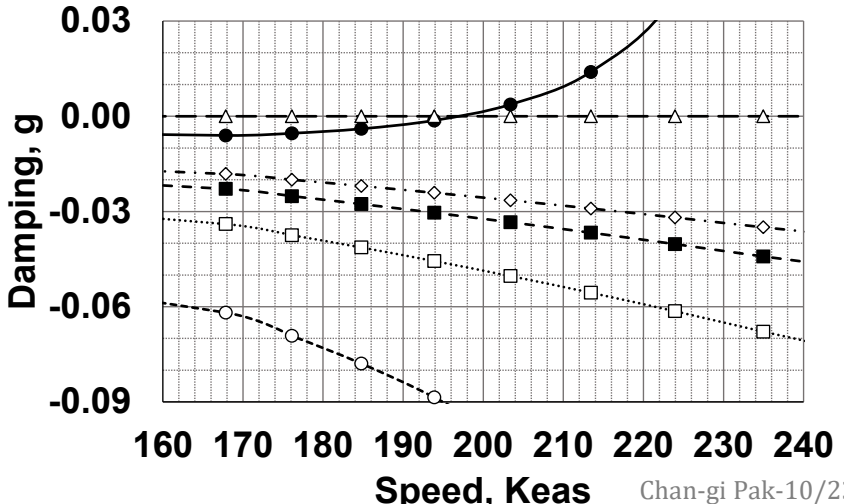
Flutter boundary	Baseline ATW4
Speed (Keas)	196.40
Frequency (Hz)	29.92

- Measured frequencies are compared with analytical frequencies using the baseline model.
- Auto and cross orthogonality matrices between measured and analytical mode shapes obtained from the baseline model are also provided in this table.
- Modal participation factors from flutter analysis are in this table.
- Modes 1, 2, 3, and 5 are the primary modes in this model tuning.
- Frequency errors correspond to the first three primary modes violation NASA as well as military standards.
- Off-diagonal terms of AOM and COM, as well as diagonal terms of COM, violate military and NASA standards.
- The V-g diagram using the baseline model is in this figure.
- The flutter speed was 196.4 keas and the flutter frequency was 29.92 Hz.

Baseline ATW4											
AOM						COM					
Mode	**1**	**2**	**3**	**5**	**6**	**Mode**	**1**	**2**	**3**	**5**	**6**
1	**1.000**	-0.431	-0.019	-0.110	0.143	**1**	**-0.867**	-0.287	0.007	0.038	0.016
2	-0.431	**1.000**	-0.491	0.775	0.382	**2**	-0.046	**0.798**	0.046	-0.080	0.003
3	-0.019	-0.491	**1.000**	-0.565	-0.405	**3**	0.218	-0.480	**-0.842**	0.055	-0.041
5	-0.110	0.775	-0.565	**1.000**	0.477	**5**	-0.226	0.773	0.111	**-0.534**	0.158
6	0.143	0.382	-0.405	0.477	**1.000**	**6**	-0.300	0.453	0.114	0.161	**0.816**

10% requirements

90% requirements



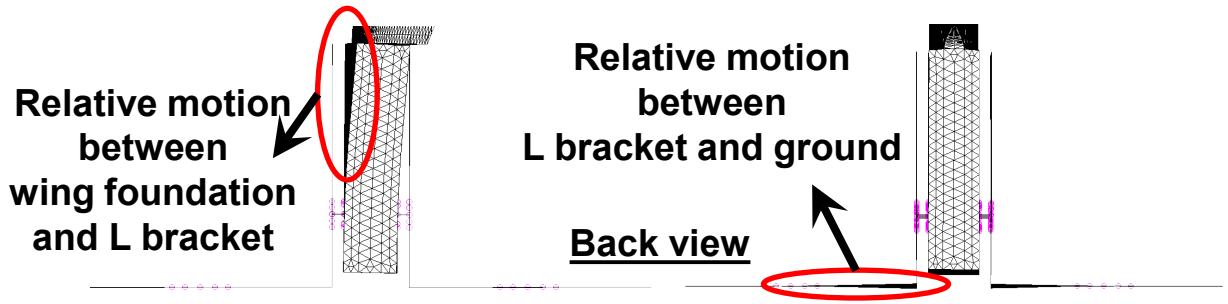
Delivered Model

	Modal Participation	Measured	Baseline ATW4		Delivered model	
			FEM	% Error	FEM	% Error
Weight	N/A	1.686	1.742	3.33	1.742	3.33
X-C.G.		13.37	13.67	2.24	13.67	2.24
Y-C.G.		7.763	7.944	2.33	7.944	2.33
Mode 1	**5.37%**	**14.65**	**17.26**	**17.8**	**14.62**	**-0.17**
Mode 2	**93.5%**	**29.55**	**33.54**	**13.5**	**29.15**	**-1.36**
Mode 3	**0.87%**	**84.92**	**91.18**	**7.38**	**77.37**	**-8.89**
Mode 5	**0.27%**	**181.8**	**184.5**	**1.47**	**158.9**	**-12.6**
Mode 6	0.00%	210.7	228.0	8.21	194.0	-7.92

Flutter boundary	**Baseline ATW4**	**Delivered model**	
		Value	% difference
Speed (Keas)	196.40	172.63	-12.1
Frequency (Hz)	29.92	25.85	-13.6

Delivered model											
AOM						COM					
Mode	**1**	**2**	**3**	**5**	**6**	**Mode**	**1**	**2**	**3**	**5**	**6**
1	**1.000**	0.042	0.049	0.072	0.004	**1**	**0.998**	0.064	-0.024	-0.014	-0.003
2	0.042	**1.000**	-0.036	0.008	0.039	**2**	0.107	**-0.993**	0.034	0.030	-0.001
3	0.049	-0.036	**1.000**	0.093	0.007	**3**	0.024	0.004	**-0.999**	-0.022	0.005
5	0.072	0.008	0.093	**1.000**	0.031	**5**	0.061	-0.032	-0.070	**-0.933**	0.347
6	0.004	0.039	0.007	0.031	**1.000**	**6**	0.013	-0.029	-0.009	0.321	**0.946**

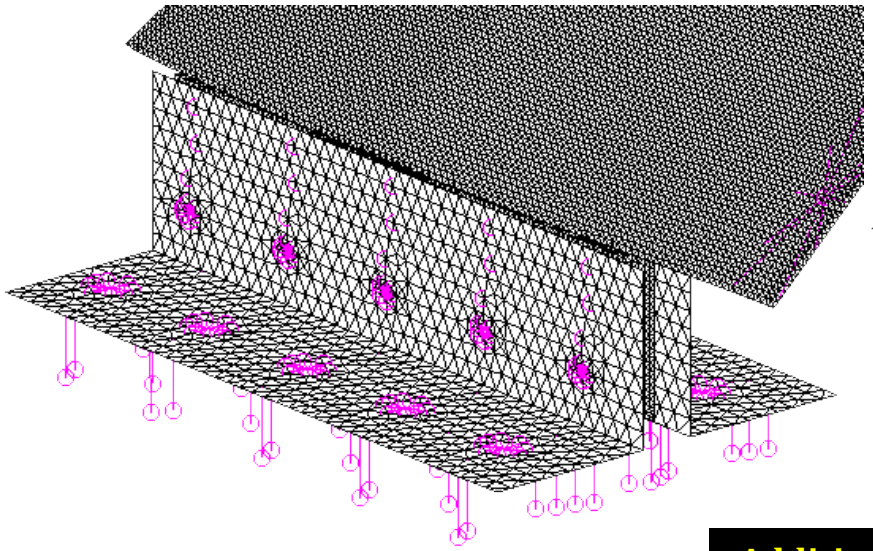
- The small business company delivered the updated finite element model. We perform frequency and mode shape correlations using delivered model.
- The frequency errors for the first two primary modes satisfy the military and NASA standards. Frequency errors in modes 3, 5, and the secondary mode, mode 6, however, violate frequency correlation requirements.
- Off-diagonal terms of AOM satisfy requirements. One of the off-diagonal terms of COM, which corresponds to the primary mode, violates the 10% requirement slightly.
- The flutter boundary has changed more than 12% with the delivered model.



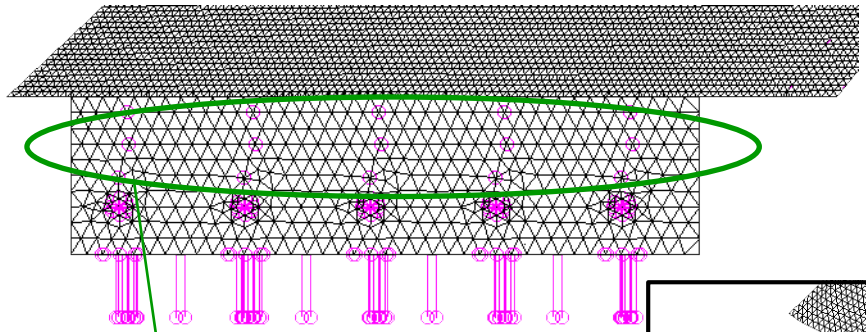
- It should be noted that due to the idealization error associated with the foundation area, we have relative motion between the L-bracket and test article, and between the L-bracket and foundation. These two figures are back views of the foundation area. Small relative motions are observed here.

Idealization Error: Wing Root Boundary Condition

- To fix the idealization error, some additional constraints are defined between the L-bracket and test article and between the L-bracket and foundation.

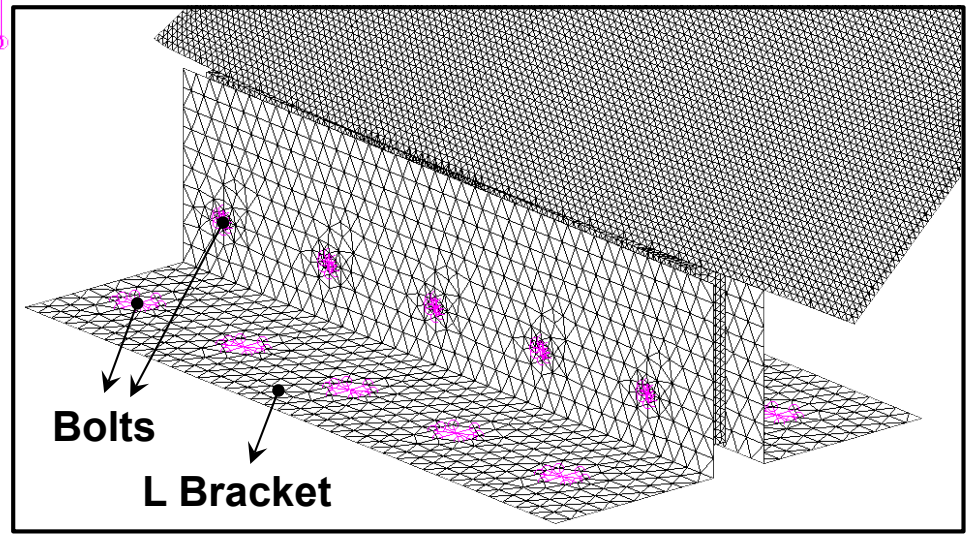


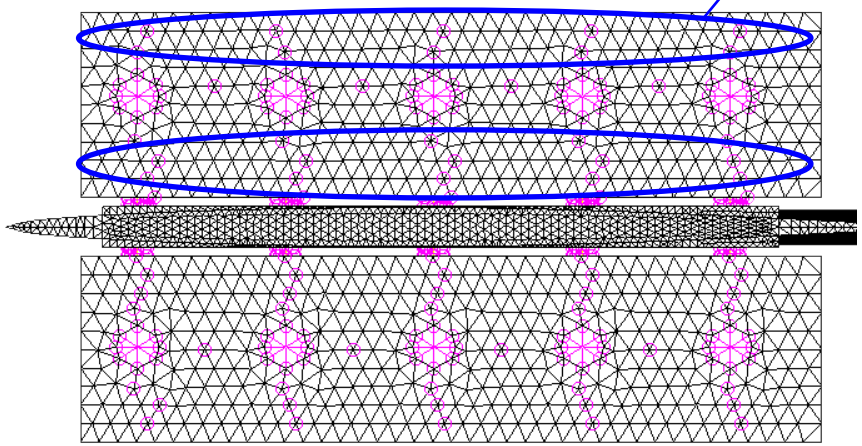
(a)Tri view



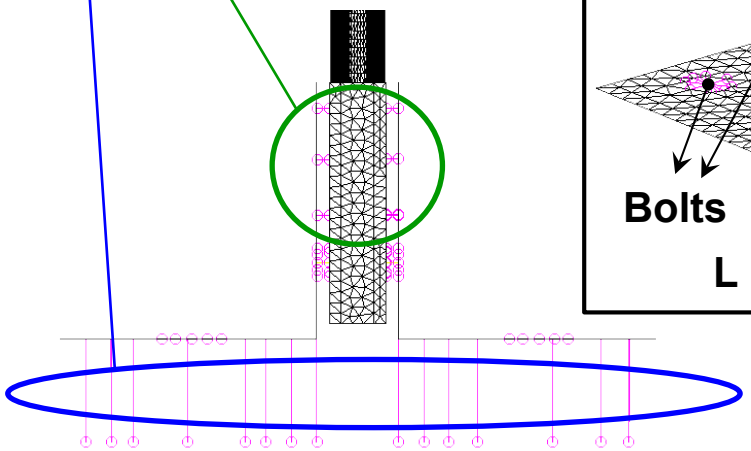
(c) Side view

Additional constraints





(b) Bottom view



(d) Back view

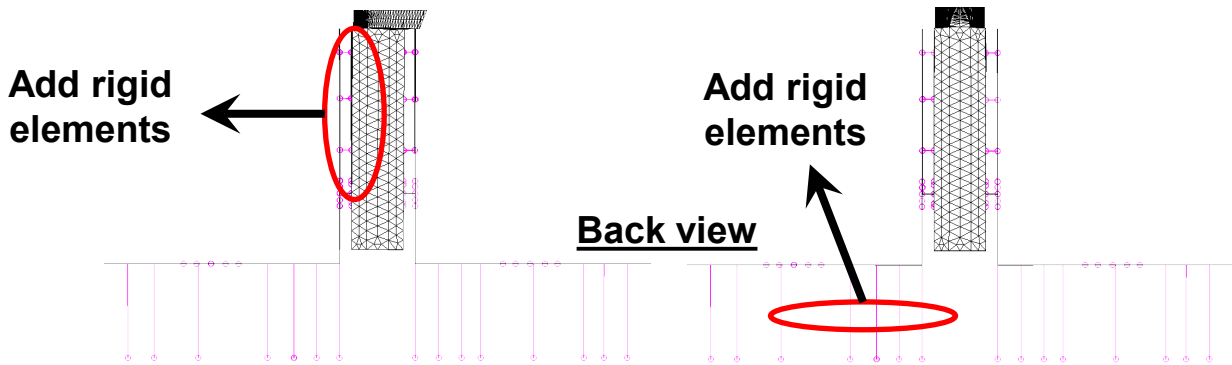
Fixed Model

	Modal Participation	Measured	Delivered model		Fixed model	
			FEM	% Error	FEM	% Error
Weight	N/A	1.686	1.742	3.33	1.744	3.43
X-C.G.		13.37	13.67	2.24	13.64	2.01
Y-C.G.		7.763	7.944	2.33	7.934	2.20
Mode 1	**5.37%**	**14.65**	**14.62**	**-0.17**	**15.17**	**3.55**
Mode 2	**93.5%**	**29.55**	**29.15**	**-1.36**	**30.08**	**1.79**
Mode 3	**0.87%**	**84.92**	**77.37**	**-8.89**	**81.01**	**-4.60**
Mode 5	**0.27%**	**181.8**	**158.9**	**-12.6**	**175.3**	**-3.58**
Mode 6	0.00%	210.7	194.0	-7.92	198.8	-5.65

Flutter boundary	Baseline ATW4	Delivered model		Fixed model	
		Value	% difference	Value	% difference
Speed (Keas)	196.40	172.63	-12.1	177.43	-9.66
Frequency (Hz)	29.92	25.85	-13.6	26.85	-10.3

Fixed Model											
AOM						COM					
Mode	1	2	3	5	6	Mode	1	2	3	5	6
1	**1.000**	0.042	0.049	0.075	0.004	1	**0.999**	0.026	0.039	0.004	0.000
2	0.042	**1.000**	-0.037	0.020	0.032	2	0.069	**-0.997**	-0.011	-0.045	0.000
3	0.049	-0.037	**1.000**	0.110	-0.014	3	0.009	0.028	**0.999**	-0.028	0.023
5	0.075	0.020	0.110	**1.000**	0.080	5	0.067	-0.059	0.130	**0.932**	0.328
6	0.004	0.032	-0.014	0.080	**1.000**	6	0.007	-0.020	-0.042	-0.250	**0.967**

- Frequencies from the fixed model are compared with the GVT frequencies. Mode 2 satisfies the military standard. Modes 1, 3, & 5 satisfy the NASA standard. Frequency error for the mode 6 is less than 6%.
- One off-diagonal term of AOM violates 10% requirement. One off-diagonal term of COM, corresponds to the primary mode, also violate 10 % requirement.
- Flutter speed improved more than 2.4 percent by correcting boundary condition problems with the delivered model.



- It should be noted that the idealization error can't be improved with a model tuning procedure.
- The fixed model already gives pretty good frequencies and mode shapes.
- Total weight error of 3.43 % is larger than 3%, and We'll further improve the model using the model tuning tool developed in this study.

Model Tuning Iteration #1

	Modal Participation	GVT	Fixed model		After iteration 1	
			FEM	% Error	FEM	% Error
Weight	**N/A**	1.686	1.744	3.43	**1.686**	0.00
X-C.G.		13.37	13.64	2.01	13.55	1.32
Y-C.G.		7.763	7.934	2.20	7.825	0.80
Mode 1	**5.37%**	**14.65**	**15.17**	**3.55**	**15.57**	**6.34**
Mode 2	**93.5%**	**29.55**	**30.08**	**1.79**	**30.68**	**3.79**
Mode 3	**0.87%**	**84.92**	**81.01**	**-4.60**	**82.44**	**-2.93**
Mode 5	**0.27%**	**181.8**	**175.3**	**-3.58**	**178.3**	**-1.90**
Mode 6	0.00%	210.7	198.8	-5.65	203.0	-3.69

Candidate DV #	Property	Iteration 1
1 - 15	MAT1	3
16 - 31	MAT8	2
32 - 34	PBARL	2
35	PSHELL	
36 - 120	CONM2	
121 - 153	PELAS	
154 - 475	PCOMP (thickness)	18
476 - 797	PCOMP (ply angle)	
Total number of design variables		25

Fixed model

AOM						COM					
Mode	**1**	**2**	**3**	**5**	**6**	**Mode**	**1**	**2**	**3**	**5**	**6**
1	**1.000**	0.042	0.049	0.075	0.004	**1**	**0.999**	0.026	0.039	0.004	0.000
2	0.042	**1.000**	-0.037	0.020	0.032	**2**	0.069	**-0.997**	-0.011	-0.045	0.000
3	0.049	-0.037	**1.000**	0.110	-0.014	**3**	0.009	0.028	**0.999**	-0.028	0.023
5	0.075	0.020	0.110	**1.000**	0.080	**5**	0.067	-0.059	0.130	**0.932**	0.328
6	0.004	0.032	-0.014	0.080	**1.000**	**6**	0.007	-0.020	-0.042	-0.250	**0.967**

After Iteration #1

AOM						COM					
Mode	**1**	**2**	**3**	**5**	**6**	**Mode**	**1**	**2**	**3**	**5**	**6**
1	**0.949**	0.035	0.058	0.072	0.002	**1**	**0.973**	0.024	-0.038	0.004	0.000
2	0.035	**0.960**	-0.033	0.018	0.030	**2**	0.06	**-0.977**	0.012	-0.045	0.000
3	0.058	-0.033	**0.968**	0.110	-0.015	**3**	0.021	0.024	**-0.982**	-0.029	-0.022
5	0.072	0.018	0.110	**0.975**	0.079	**5**	0.066	-0.058	-0.132	**0.92**	-0.323
6	0.002	0.030	-0.015	0.079	**0.977**	**6**	0.006	-0.019	0.044	-0.245	**-0.956**

- A total of 797 candidate design variables, as listed in the above table, are prepared in this study.
- Once the sensitivity values of the response functions are obtained, the sensitivity values of the performance indices are computed using the in-house post-processing codes.
- Absolute values of the objective sensitivity are sorted to select the most efficient design variables for optimizing the objective function value selected among performance indices.
- The first objective function was the total weight error without any constraints. Twenty-five design variables are selected for the first optimization run.
- After the first optimization run, total weight and x and y cg locations are improved, however the first two frequency correlations and the off-diagonal term of the cross-orthogonality matrix become worse than the fixed model.

Model Tuning Iteration #2

	Modal Participation	GVT	After iteration 1		After iteration 2	
			FEM	% Error	FEM	% Error
Weight	N/A	1.686	**1.686**	0.00	1.721	2.05
X-C.G.		13.37	13.55	1.32	13.71	2.51
Y-C.G.		7.763	7.825	0.80	8.008	3.16
Mode 1	**5.37%**	**14.65**	**15.57**	**6.34**	**15.07**	**2.91**
Mode 2	**93.5%**	**29.55**	**30.68**	**3.79**	**29.91**	**1.21**
Mode 3	**0.87%**	**84.92**	**82.44**	**-2.93**	**81.71**	**-3.79**
Mode 5	**0.27%**	**181.8**	**178.3**	**-1.90**	**177.8**	**-2.16**
Mode 6	0.00%	210.7	203.0	-3.69	202.4	-3.95

Candidate DV #	Property	Iteration 1	Iteration 2
1 - 15	MAT1	3	3
16 - 31	MAT8	2	2
32 - 34	PBARL	2	2
35	PSHELL		
36 - 120	CONM2		7
121 - 153	PELAS		
154 - 475	PCOMP (thickness)	18	50
476 - 797	PCOMP (ply angle)		
Total number of design variables		25	64

After Iteration #1 (after mass normalize)

	AOM						COM					
Mode	**1**	**2**	**3**	**5**	**6**	**Mode**	**1**	**2**	**3**	**5**	**6**
1	**1.000**	0.037	0.061	0.075	0.002	**1**	**0.999**	0.025	-0.039	0.004	0.000
2	0.037	**1.000**	-0.034	0.019	0.031	**2**	0.061	**-0.997**	0.012	-0.046	0.000
3	0.061	-0.034	**1.000**	0.113	-0.015	**3**	0.021	0.024	**-0.998**	-0.029	-0.022
5	0.075	0.019	0.113	**1.000**	0.081	**5**	0.067	-0.059	-0.134	**0.932**	-0.327
6	0.002	0.031	-0.015	0.081	**1.000**	**6**	0.006	-0.019	0.045	-0.248	**-0.967**

After Iteration #2

	AOM						COM					
Mode	**1**	**2**	**3**	**5**	**6**	**Mode**	**1**	**2**	**3**	**5**	**6**
1	**1.062**	0.042	0.039	0.074	-0.004	**1**	**1.030**	0.027	0.040	-0.004	0.000
2	0.042	**1.051**	-0.043	0.025	0.029	**2**	0.067	**-1.022**	-0.011	0.046	0.000
3	0.039	-0.043	**1.016**	0.113	-0.013	**3**	-0.002	0.033	**1.006**	0.029	0.024
5	0.074	0.025	0.113	**1.005**	0.079	**5**	0.065	-0.063	0.133	**-0.934**	0.326
6	-0.004	0.029	-0.013	0.079	**1.002**	**6**	0.000	-0.017	-0.043	0.249	**0.969**

- The first frequency error is selected for the objective function of the second optimization run. In this case, total weight, X and Y-CG locations, and frequency errors of the Modes (2, 3, 5, and 6) are chosen as the inequality constraint functions.
  - Frequencies f_j $j = 2, 3, 5, \& 6$ ($\leq 5\%$) and Weight, $\boldsymbol{X_{CG}}$, & $\boldsymbol{Y_{CG}}$($\leq 3\%$)
- Before starting the second optimization run, sensitivity analysis with 797 design variables is again performed to decide the most efficient design variables. An additional 39 design variables, for a total of 64 (= 25 + 39) are selected.
- This table show the first frequency error of 6.34 percent reduces to 2.91 percent after the second optimization run.
- It should be noted that the Y-CG location becomes an active constraint at 3.16 percent.
- The frequency errors for Modes (1, 2, and 5) are less than three percent. Similarly, frequency errors for Modes (3 and 6) are less than five percent.
- The off-diagonal terms of the AOM and COM are similar values with the previous optimization results.

Model Tuning Iteration #3

	M.P.	GVT	After iteration 1		After iteration 2		After iteration 3	
			FEM	% Error	FEM	% Error	FEM	% Error
Weight	N/A	1.686	**1.686**	0.00	1.721	2.05	1.726	2.39
X-C.G.		13.37	13.55	1.32	13.71	2.51	13.79	3.17
Y-C.G.		7.763	7.825	0.80	8.008	3.16	8.159	5.10
Mode 1	**5.37%**	**14.65**	**15.57**	**6.34**	**15.07**	**2.91**	**14.73**	**0.57**
Mode 2	**93.5%**	**29.55**	**30.68**	**3.79**	**29.91**	**1.21**	**29.20**	**-1.21**
Mode 3	**0.87%**	**84.92**	**82.44**	**-2.93**	**81.71**	**-3.79**	**82.63**	**-2.69**
Mode 5	**0.27%**	**181.8**	**178.3**	**-1.90**	**177.8**	**-2.16**	**179.1**	**-1.50**
Mode 6	0.00%	210.7	203.0	-3.69	202.4	-3.95	205.4	-2.52

Candidate DV #	Property	Iter 1	Iter 2	Iter 3
1 - 15	MAT1	3	3	3
16 - 31	MAT8	2	2	2
32 - 34	PBARL	2	2	2
35	PSHELL			
36 - 120	CONM2		7	47
121 - 153	PELAS			
154 - 475	PCOMP (thickness)	18	50	80
476 - 797	PCOMP (ply angle)			
Total number of design variables		25	64	134

After Iteration #2 (after mass normalize)

AOM						COM					
Mode	1	2	3	5	6	Mode	1	2	3	5	6
1	**1.000**	0.040	0.038	0.072	-0.004	1	**0.999**	0.026	0.039	-0.004	0.000
2	0.040	**1.000**	-0.042	0.024	0.028	2	0.065	**-0.997**	-0.011	0.045	0.000
3	0.038	-0.042	**1.000**	0.112	-0.013	3	-0.002	0.033	**0.998**	0.029	0.024
5	0.072	0.024	0.112	**1.000**	0.079	5	0.065	-0.063	0.133	**-0.932**	0.325
6	-0.004	0.028	-0.013	0.079	**1.000**	6	0.000	-0.017	-0.043	0.249	**0.968**

After Iteration #3

AOM						COM					
Mode	1	2	3	5	6	Mode	1	2	3	5	6
1	**1.038**	0.045	-0.001	0.068	0.029	1	**1.018**	0.026	0.040	-0.004	0.000
2	0.045	**1.044**	-0.054	0.040	0.041	2	0.071	**-1.018**	-0.008	0.044	0.000
3	-0.001	-0.054	**0.970**	**0.076**	-0.033	3	-0.041	0.044	**0.983**	0.024	0.026
5	0.068	0.040	**0.076**	**0.975**	0.075	5	0.061	-0.076	**0.098**	**-0.926**	0.315
6	0.029	0.041	-0.033	0.075	**0.957**	6	0.032	-0.027	-0.062	0.239	**0.946**

- The off-diagonal term of the AOM, fourth row (Mode 5) and third column, is selected as the objective function for the third optimization run.
- Total weight, x and y cg locations, all the frequencies, most of off-diagonal terms of the AOM and COM except 4-3, 5-4, and 4-5 terms, and all the diagonal terms of the COM are constraints.
- Maximum allowable for the weight and the x-y cg locations are 5% during the third optimization run.
- Maximum allowable for modes 1, 2, mode 3, 5, and mode 6 are 3, 5, and 10 % respectively.
- Ten percent limit is used for off-diagonal terms and 90 % limit is use for diagonal terms.
- After the third optimization run, objective function value becomes 7.6%. The 4-3 term of the COM becomes 9.8%.

Model Tuning Iteration #4

	M.P.	GVT	After iteration 1		After iteration 2		After iteration 3		After iteration 4	
			FEM	% Error	FEM	% Error	FEM	% Error	FEM	% Error
Weight	N/A	1.686	**1.686**	0.00	1.721	2.05	1.726	2.39	1.706	1.17
X-C.G.		13.37	13.55	1.32	13.71	2.51	13.79	3.17	13.64	2.04
Y-C.G.		7.763	7.825	0.80	8.008	3.16	8.159	5.10	**7.986**	2.87
Mode 1	**5.37%**	**14.65**	**15.57**	**6.34**	**15.07**	**2.91**	**14.73**	**0.57**	**15.11**	**3.16**
Mode 2	**93.5%**	**29.55**	**30.68**	**3.79**	**29.91**	**1.21**	**29.20**	**-1.21**	**29.82**	**0.88**
Mode 3	**0.87%**	**84.92**	**82.44**	**-2.93**	**81.71**	**-3.79**	**82.63**	**-2.69**	**82.94**	**-2.34**
Mode 5	**0.27%**	**181.8**	**178.3**	**-1.90**	**177.8**	**-2.16**	**179.1**	**-1.50**	**178.8**	**-1.64**
Mode 6	0.00%	210.7	203.0	-3.69	202.4	-3.95	205.4	-2.52	205.2	-2.62

Candidate DV #	Property	Iteration 4
1 - 15	MAT1	3
16 - 31	MAT8	2
32 - 34	PBARL	2
35	PSHELL	
36 - 120	CONM2	47
121 - 153	PELAS	
154 - 475	PCOMP (thickness)	80
476 - 797	PCOMP (ply angle)	
Total number of design variables		134

After Iteration #3 (after mass normalize)											
AOM						COM					
Mode	1	2	3	5	6	Mode	1	2	3	5	6
1	**1.000**	0.043	-0.001	0.068	0.029	1	**0.999**	0.026	0.039	-0.004	0.000
2	0.043	**1.000**	-0.054	0.040	0.041	2	0.069	**-0.996**	-0.008	0.043	0.000
3	-0.001	-0.054	**1.000**	0.078	-0.034	3	-0.042	0.045	**0.998**	0.024	0.026
5	0.068	0.040	0.078	**1.000**	0.078	5	0.062	-0.077	0.099	**-0.938**	0.319
6	0.029	0.041	-0.034	0.078	**1.000**	6	0.033	-0.028	-0.063	0.244	**0.967**

After Iteration #4 (after mass normalize)											
AOM						COM					
Mode	1	2	3	5	6	Mode	1	2	3	5	6
1	**1.000**	0.041	0.023	0.067	0.018	1	**0.999**	0.024	0.040	0.004	0.000
2	0.041	**1.000**	-0.044	0.030	0.036	2	0.066	**-0.997**	-0.009	-0.044	0.000
3	0.023	-0.044	**1.000**	0.081	-0.029	3	-0.017	0.035	**0.999**	-0.025	0.025
5	0.067	0.030	0.081	**1.000**	0.075	5	0.061	-0.069	0.099	**0.938**	0.320
6	0.018	0.036	-0.029	0.075	**1.000**	6	0.022	-0.023	-0.058	-0.247	**0.967**

- After normalize, the 4-3 term of the AOM and COM become 7.8% and 9.9% respectively.
- After the third optimization run, the constraint for the y-cg location become active constraint.
- Therefore, the y-cg location is selected as the objective function for the final optimization run.
  - Weight & $\boldsymbol{X_{CG}}$ (≤ 5%); Frequencies f_j $j = 1$ & 2 (≤ 3%); Frequencies f_j $j = 3$ & 5 (≤ 5%); Frequencies f_j $j = 6$ (≤ 10%); $\boldsymbol{AOM_{ij}}$, $\boldsymbol{COM_{ij}}$, (≤ 10%) except $\boldsymbol{COM_{65}}$ & $\boldsymbol{COM_{56}}$; & $\boldsymbol{COM_{ii}}$ (≥ 90%)
- After the last optimization run, mode 1 related constraint becomes an active constraint.
- Total weight and x and y cg location errors are less than 3%. The maximum frequency error is 3.16%.
- All the off-diagonal terms of the AOM satisfies the 10 % requirements. All the off-diagonal terms of the COM corresponds to the primary modes satisfies the 10% requirements. All the diagonal terms of the COM are greater than 90%.

Baseline ATW4 vs. After Iteration #4

	M.P.	GVT	Baseline ATW4		After iteration 4	
			FEM	% Error	FEM	% Error
Weight	N/A	1.686	1.742	3.33	1.706	1.17
X-C.G.		13.37	13.67	2.24	13.64	2.04
Y-C.G.		7.763	7.944	2.33	**7.986**	2.87
Mode 1	**5.37%**	**14.65**	**17.26**	**17.8**	**15.11**	**3.16**
Mode 2	**93.5%**	**29.55**	**33.54**	**13.5**	**29.82**	**0.88**
Mode 3	**0.87%**	**84.92**	**91.18**	**7.38**	**82.94**	**-2.34**
Mode 5	**0.27%**	**181.8**	**184.5**	**1.47**	**178.8**	**-1.64**
Mode 6	0.00%	210.7	228.0	8.21	205.2	-2.62

Design variable changes more than five percent

DV#	Property	% change	DV#	Property	% change	DV#	Property	% change
34	diameter	-12.7	115	comn2	-6.33	371	pcomp	-6.16
60	conm2	-7.27	303	pcomp	-6.17	372	pcomp	-6.20
65	conm2	6.22	304	pcomp	-6.20	373	pcomp	-6.16
108	conm2	-5.16	305	pcomp	-6.17	374	pcomp	-7.24
109	conm2	-6.89	306	pcomp	-7.24	375	pcomp	-7.23
110	conm2	-7.98	307	pcomp	-7.23	376	pcomp	-7.24
111	conm2	-8.83	308	pcomp	-7.24	377	pcomp	-5.20
112	conm2	-8.78	309	pcomp	-5.19	378	pcomp	-5.10
113	conm2	-8.08	310	pcomp	-5.09	379	pcomp	-5.21
114	conm2	-7.57	311	pcomp	-5.20			

Baseline ATW4

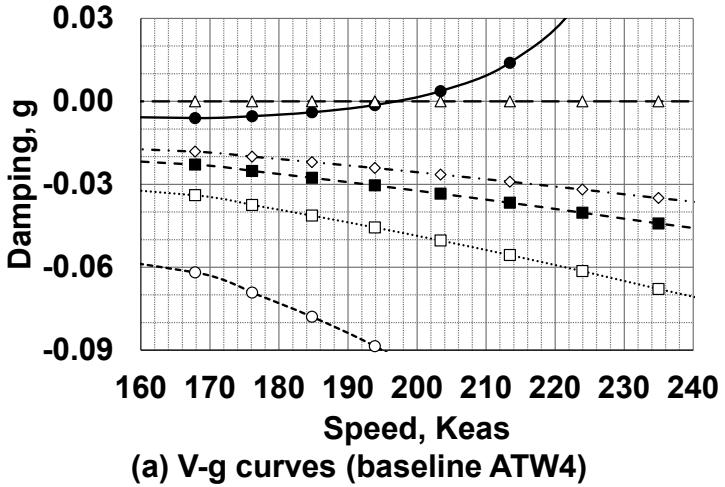
AOM						COM					
Mode	1	2	3	5	6	Mode	1	2	3	5	6
1	**1.000**	-0.431	-0.019	-0.110	0.143	1	**-0.867**	-0.287	0.007	0.038	0.016
2	-0.431	**1.000**	-0.491	0.775	0.382	2	-0.046	**0.798**	0.046	-0.080	0.003
3	-0.019	-0.491	**1.000**	-0.565	-0.405	3	0.218	-0.480	**-0.842**	0.055	-0.041
5	-0.110	0.775	-0.565	**1.000**	0.477	5	-0.226	0.773	0.111	**-0.534**	0.158
6	0.143	0.382	-0.405	0.477	**1.000**	6	-0.300	0.453	0.114	0.161	**0.816**

After Iteration #4

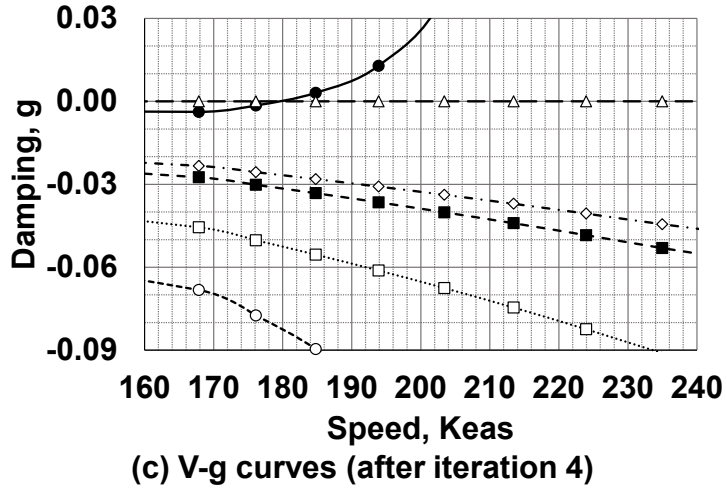
AOM						COM					
Mode	1	2	3	5	6	Mode	1	2	3	5	6
1	**1.000**	0.041	0.023	0.067	0.018	1	**0.999**	0.024	0.040	0.004	0.000
2	0.041	**1.000**	-0.044	0.030	0.036	2	0.066	**-0.997**	-0.009	-0.044	0.000
3	0.023	-0.044	**1.000**	0.081	-0.029	3	-0.017	0.035	**0.999**	-0.025	0.025
5	0.067	0.030	0.081	**1.000**	0.075	5	0.061	-0.069	0.099	**0.938**	0.320
6	0.018	0.036	-0.029	0.075	**1.000**	6	0.022	-0.023	-0.058	-0.247	**0.967**

- All the design variables changed more than 5% are summarized in the above table.
- The maximum change is inner diameter of the circular wing tip boom.
- After model tuning, frequencies, mode shapes, total weight, and cg locations correlate excellent with target values.

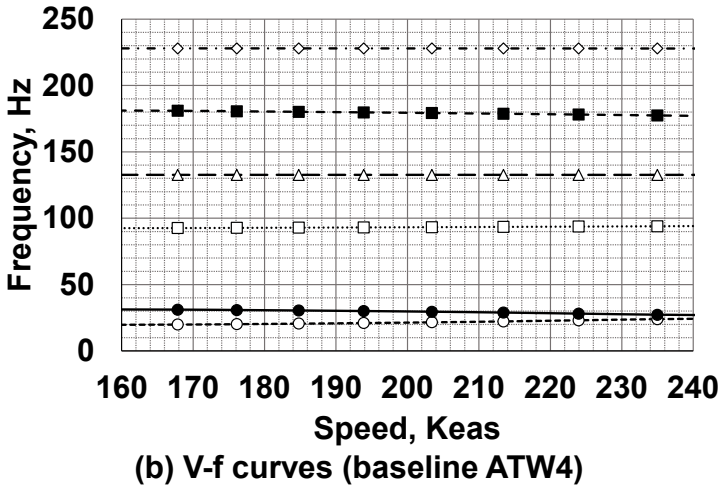
Flutter Analysis After Iteration #4



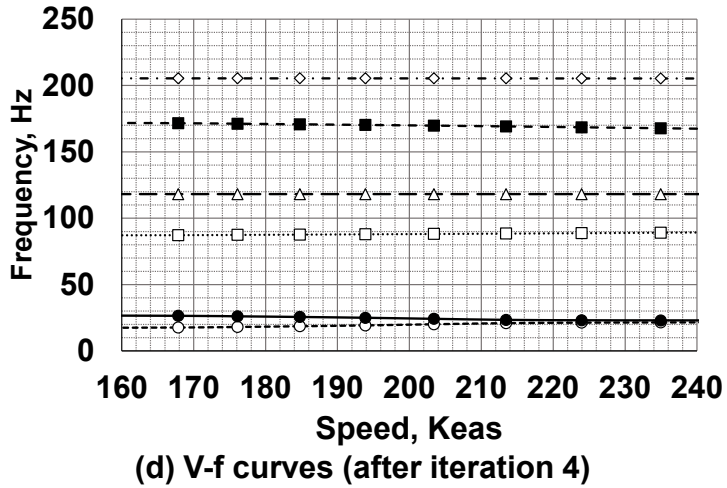
(a) V-g curves (baseline ATW4)



(c) V-g curves (after iteration 4)



(b) V-f curves (baseline ATW4)



(d) V-f curves (after iteration 4)

- After the finite element model tuning, flutter analysis is performed again.
- Roughly, 2.5% (= 12.1% - 9.66%) of this flutter speed improvement is due to fixing the idealization error in the wing root boundary condition.

Flutter boundary	Baseline ATW4	Delivered model		Fixed model		After iteration 4	
		Value	% difference	Value	% difference	Value	% difference
Speed (Keas)	196.40	172.63	-12.1	177.43	-9.66	178.89	-8.91
Frequency (Hz)	29.92	25.85	-13.6	26.85	-10.3	25.93	-13.3

Flutter Analysis After Iteration #4 (continue)

	M.P.	GVT	Fixed model		After iteration 4	
			FEM	% Error	FEM	% Error
Weight	**N/A**	1.686	1.744	3.43	1.706	1.17
X-C.G.		13.37	13.64	2.01	13.64	2.04
Y-C.G.		7.763	7.934	2.20	**7.986**	2.87
Mode 1	**5.37%**	**14.65**	**15.17**	**3.55**	**15.11**	**3.16**
Mode 2	**93.5%**	**29.55**	**30.08**	**1.79**	**29.82**	**0.88**
Mode 3	**0.87%**	**84.92**	**81.01**	**-4.60**	**82.94**	**-2.34**
Mode 5	**0.27%**	**181.8**	**175.3**	**-3.58**	**178.8**	**-1.64**
Mode 6	0.00%	210.7	198.8	-5.65	205.2	-2.62

Average 2.01%

Average 3.25%

Flutter boundary	Baseline ATW4	Fixed model		After iteration 4	
		Value	% difference	Value	% difference
Speed (Keas)	196.40	177.43	-9.66	178.89	-8.91
Frequency (Hz)	29.92	26.85	-10.3	25.93	-13.3

Fixed Model

AOM						COM					
Mode	**1**	**2**	**3**	**5**	**6**	**Mode**	**1**	**2**	**3**	**5**	**6**
1	**1.000**	0.042	0.049	0.075	0.004	**1**	**0.999**	0.026	0.039	0.004	0.000
2	0.042	**1.000**	-0.037	0.020	0.032	**2**	0.069	**-0.997**	-0.011	-0.045	0.000
3	0.049	-0.037	**1.000**	0.110	-0.014	**3**	0.009	0.028	**0.999**	-0.028	0.023
5	0.075	0.020	0.110	**1.000**	0.080	**5**	0.067	-0.059	0.130	**0.932**	0.328
6	0.004	0.032	-0.014	0.080	**1.000**	**6**	0.007	-0.020	-0.042	-0.250	**0.967**

After Iteration #4

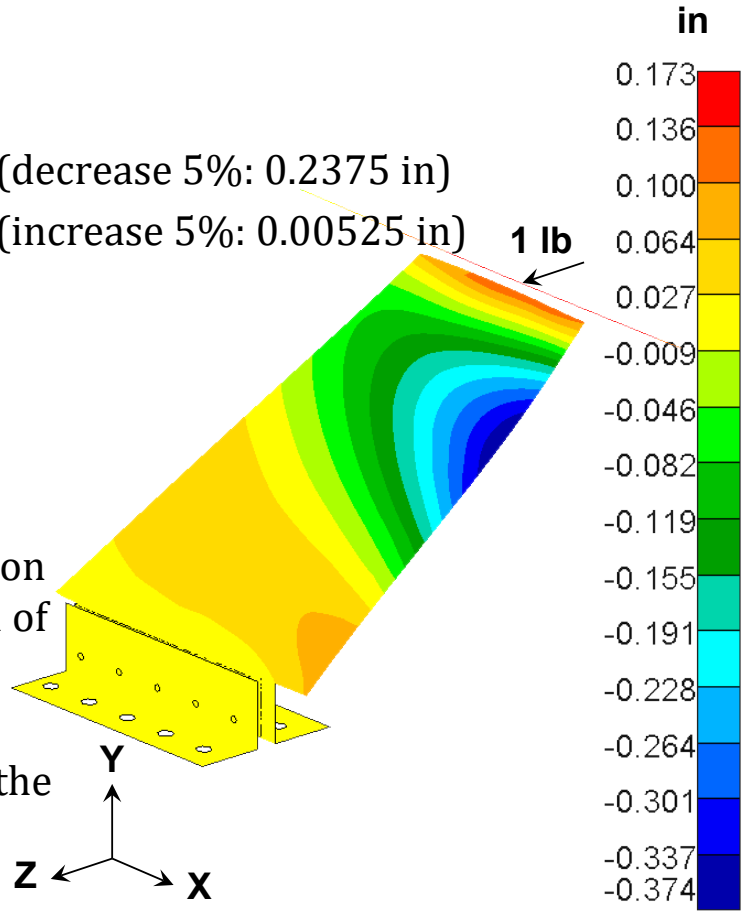
AOM						COM					
Mode	**1**	**2**	**3**	**5**	**6**	**Mode**	**1**	**2**	**3**	**5**	**6**
1	**1.000**	0.041	0.023	0.067	0.018	**1**	**0.999**	0.024	0.040	0.004	0.000
2	0.041	**1.000**	-0.044	0.030	0.036	**2**	0.066	**-0.997**	-0.009	-0.044	0.000
3	0.023	-0.044	**1.000**	0.081	-0.029	**3**	-0.017	0.035	**0.999**	-0.025	0.025
5	0.067	0.030	0.081	**1.000**	0.075	**5**	0.061	-0.069	0.099	**0.938**	0.320
6	0.018	0.036	-0.029	0.075	**1.000**	**6**	0.022	-0.023	-0.058	-0.247	**0.967**

- The average frequency error of the fixed model and the tuned model are 3.25% and 2.01%, respectively.
  - Frequency improvement: 1.24%
  - Total weight improvement: 2.26%
- One off-diagonal term of the AOM and COM corresponds to the primary modes slightly violate 10% requirement, 11% and 13%.
- However, the flutter speed difference is less than 0.75%. (9.66-8.91=0.75)
- Therefore, the **3%** requirement (military standard) for the frequency correlation of the primary modes with **10%** (11% and 13%) requirements for the off-diagonal terms of the AOM and COM is **accurate enough to have good flutter speed**.

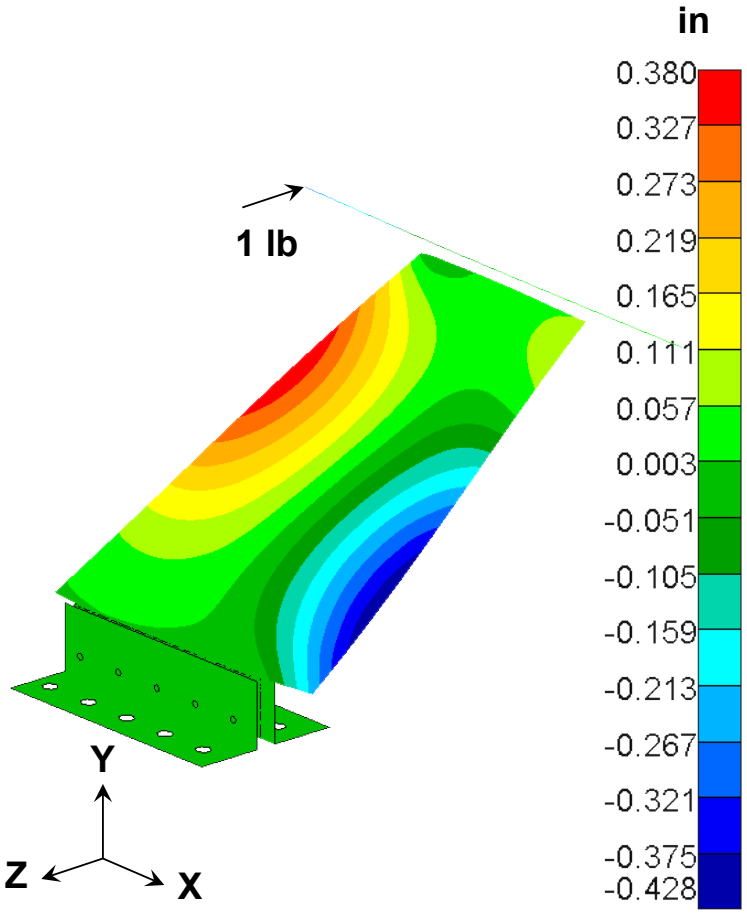
Model Tuning using Static Deformation

- The thicknesses of the aluminum L bracket, and a single layer of a composite plate are changed by 5 percent to create a sample numerical test case.
  - Use two design variables
  - Create target deformation shapes
    - Thickness of Aluminum L bracket, 0.25 in (decrease 5%: 0.2375 in)
    - Single layer of a composite plate, 0.005 in (increase 5%: 0.00525 in)

- Two load cases are considered in this model tuning with static deformation data.
  - In the first case, one pound of positive Z direction load is applied at one inch, aft of the mid-chord of the wing-tip boom.
  - On the other hand, one pound of negative Z direction load is applied at the leading edge of the wing-tip boom in the second load case.
  - Structural deformations under these two load cases are shown in these figures.

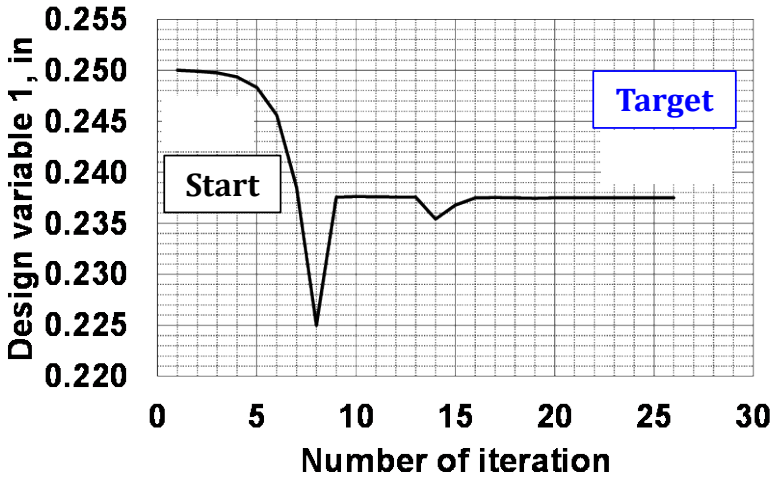


(a) Z deformation under load case 1

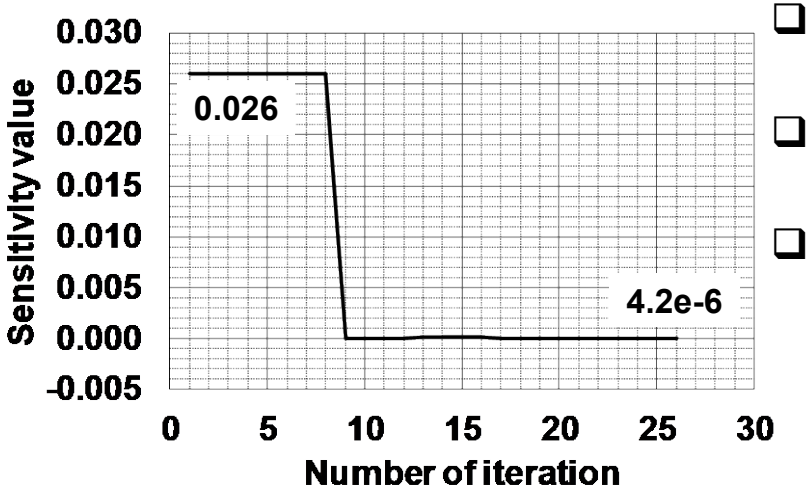


(b) Z deformation under load case 2

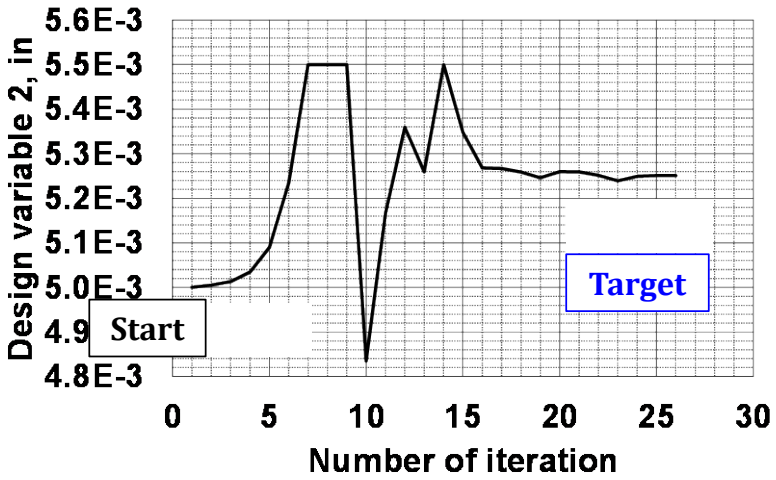
Histories of design variables and design variable sensitivities during optimization run



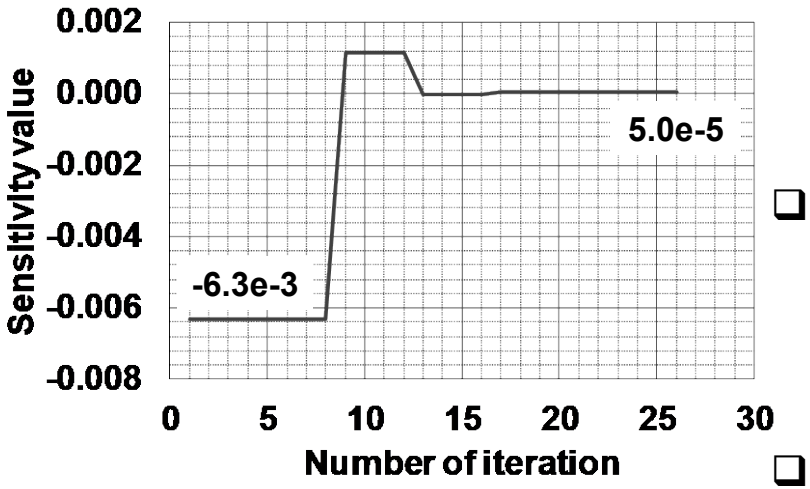
(a) Aluminum thickness



(c) Sensitivity values (aluminum)



(b) Composite layer thickness



(d) Sensitivity values (composite layer)

- The histories of two design variables during the optimization run are shown in these figures.
- The initial starting values of design variables one and two are 0.25 and 0.005 in, respectively.
- Figures (c) and (d) show the sensitivity values of the objective function with respect to design variables one and two at the start of the optimization run are 0.026 and -0.0063, respectively.
- At the end of the optimization run, these sensitivity values become 4.2E-06 and 5.0E-05; therefore, the Kuhn-Tucker condition is satisfied with an unconstrained optimization problem.
- After the optimization run, the design variables one and two become 0.2375 and 0.00525 in, respectively, as shown in Figs. (a) and (b), which are target values used to generate target wing deformations in the previous figures.
- The performance of the static deformation module is demonstrated in this section.

Conclusion

- A structural model tuning tool based on analytical sensitivity values has been developed.
  - **Sensitivity values of performance indices** are computed using the **Nastran-generated sensitivity values** together with the **in-house computer codes**.

- All the **analysis modules for the new structural model tuning tool are tested using the aerostructures test wing (ATW) 4**.
  - This wing was developed at the National Aeronautics and Space Administration (NASA) Amstrong Flight Research Center (Edwards, California) using **NASA Small Business Technology Transfer Phase II**, NNX15CD08C, for demonstrating flutter and advanced aeroelastic test techniques.

- Structural dynamic and static model tunings are performed separately using measured ground vibration test data and computer simulated static test data, respectively.
  - After the tuning, all **the weight-related errors**, including total weight as well as x and y center of gravity locations, are **less than 3%.**
  - The **frequency errors** of the first five out-of-plane modes are **all less than 3.2%.**
  - All the **off-diagonal terms of the auto-orthogonality matrix (AOM)** are **less than 10%,** and the diagonal terms are equal to 1.0.
  - Similarly, all the **off-diagonal terms of the cross-orthogonality matrix (COM) for the primary modes** are **less than 10%.** Off-diagonal terms of the COM for the secondary modes become 24.7% and 32.0%.
  - All the **diagonal terms of the COM** are greater than 93.8%. The final FE model correlates well with the test data.
  - **The flutter speed decreases by 8.91% compared with the baseline ATW4.**
    - **Fixing idealization error improves 2.5% of flutter speed.**
    - The **3% requirement for the frequency correlation** of the primary modes with **10% requirements for the off-diagonal terms of the AOM and COM** is accurate enough to have good flutter speed.
  - The performance of the static deformation module has been demonstrated using the ATW4 model.

Questions?

$$\widehat{\Phi}_{GR} = \left[\frac{1}{\sqrt{M_{AOMii}}}\right] \Phi_{GR} \quad \text{diagonal matrix}$$

$$\frac{\partial \widehat{\Phi}_{GR}}{\partial x_k} = \left[\frac{1}{\sqrt{M_{AOMii}}}\right] \frac{\partial \Phi_{GR}}{\partial x_k} + \frac{\partial \left[\frac{1}{\sqrt{M_{AOMii}}}\right]}{\partial x_k} \Phi_{GR}$$

$$\frac{\partial M_{AOM}}{\partial x_k}$$

$$\frac{\partial \Phi_{GR}}{\partial x_k} = \begin{bmatrix} \frac{\partial \Phi_M}{\partial x_k} \left(\Phi_M^{\mathrm{T}} \Phi_M\right)^{-1} \Phi_M^{\mathrm{T}} + \Phi_M \frac{\partial \Psi_M}{\partial x_k} \\ \frac{\partial \Phi_S}{\partial x_k} \left(\Phi_M^{\mathrm{T}} \Phi_M\right)^{-1} \Phi_M^{\mathrm{T}} + \Phi_S \frac{\partial \Psi_M}{\partial x_k} \end{bmatrix} \widetilde{\Phi}_{GM}$$

$$\frac{\partial \Psi_M}{\partial x_k} = \left(\Phi_M^{\mathrm{T}} \Phi_M\right)^{-1} \left[\frac{\partial \Phi_M^{\mathrm{T}}}{\partial x_k} - \left(\frac{\partial \Phi_M^{\mathrm{T}}}{\partial x_k} \Phi_M + \Phi_M^{\mathrm{T}} \frac{\partial \Phi_M}{\partial x_k}\right) \left(\Phi_M^{\mathrm{T}} \Phi_M\right)^{-1} \Phi_M^{\mathrm{T}}\right]$$

$$\frac{\partial \Phi_R}{\partial x_k} \equiv \begin{bmatrix} \frac{\partial \Phi_M}{\partial x_k} \\ \frac{\partial \Phi_S}{\partial x_k} \end{bmatrix}$$

$$\frac{\partial \left[\frac{1}{\sqrt{M_{AOMii}}}\right]}{\partial x_k} = ??$$