Multihierarchy Gaussian Process Models for Probabilistic Aerodynamic Databases using Uncertain Nominal and Off-Nominal Configuration Data

2023 AIAA SciTech Forum

January 23-27, 2023 - National Harbor, Maryland

<u>James B. Scoggins</u>, Thomas J. Wignall, Tenavi Nakamura-Zimmerer, Karen Bibb

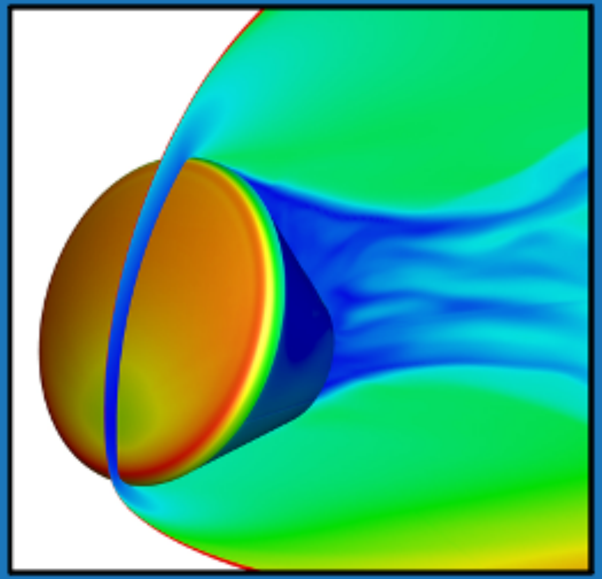
NASA Aerofusion-MLUQ Early Career Initiative, NASA Langley Research Center

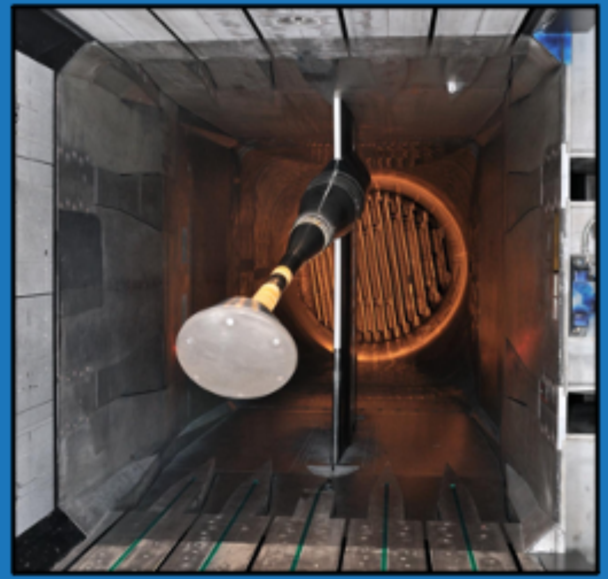
Outline

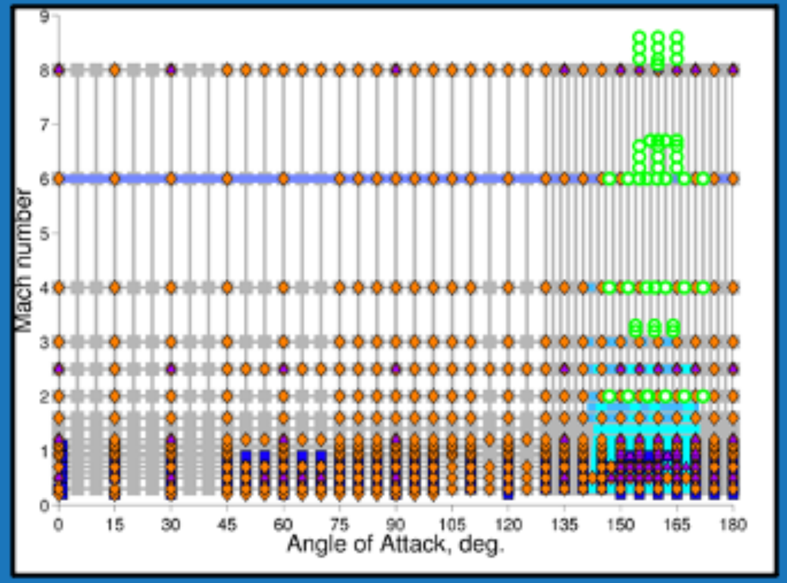
- Data fusion problem for aerodynamic databases
- Introduction to Gaussian Processes (GPs)
- Multihierarchy Gaussian Process Regression
- Example dataset based on Orion wind-tunnel data
- Results

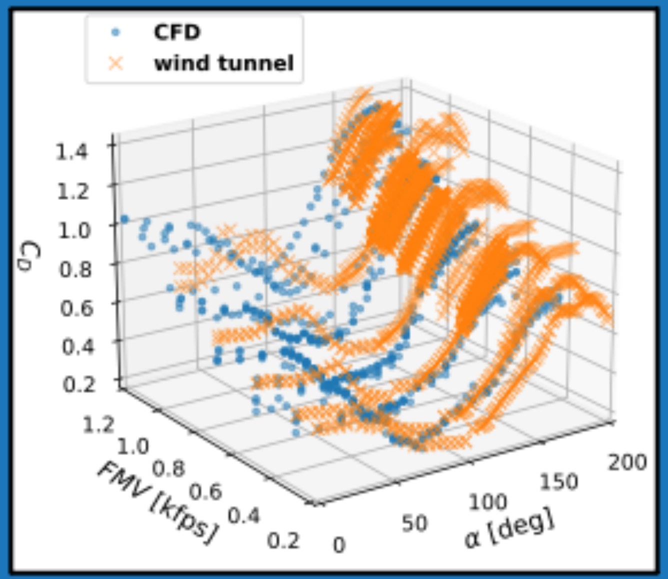
Aerodynamic databases

CFD Solutions and Wind Tunnel Tests



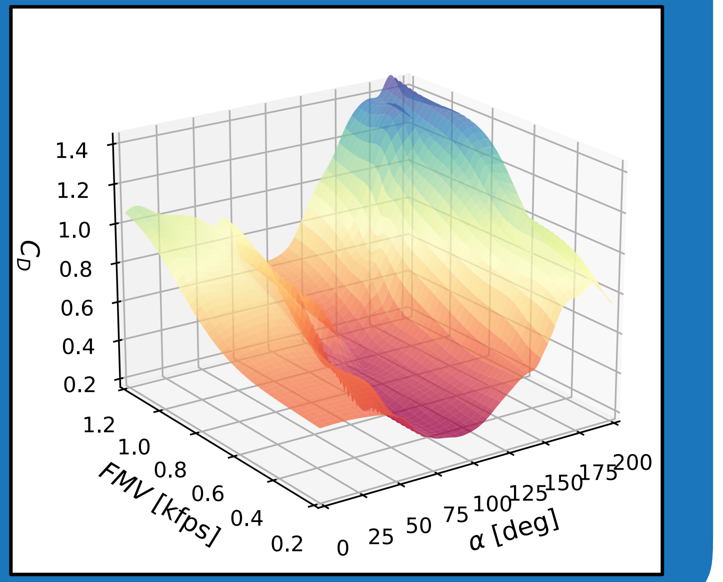






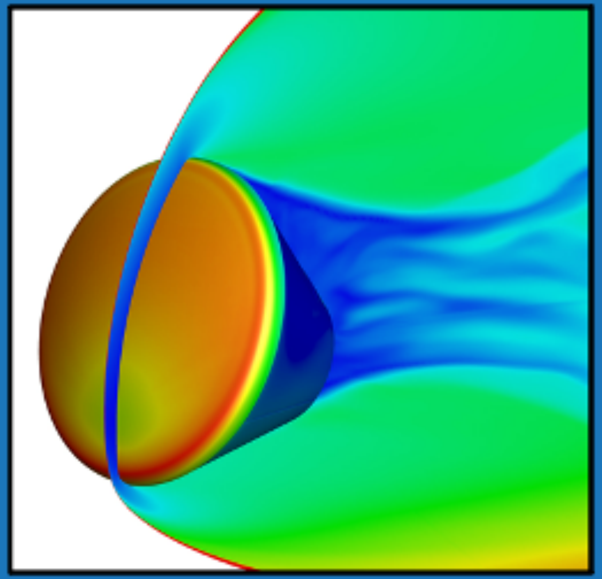
Aerodynamic Database Construction

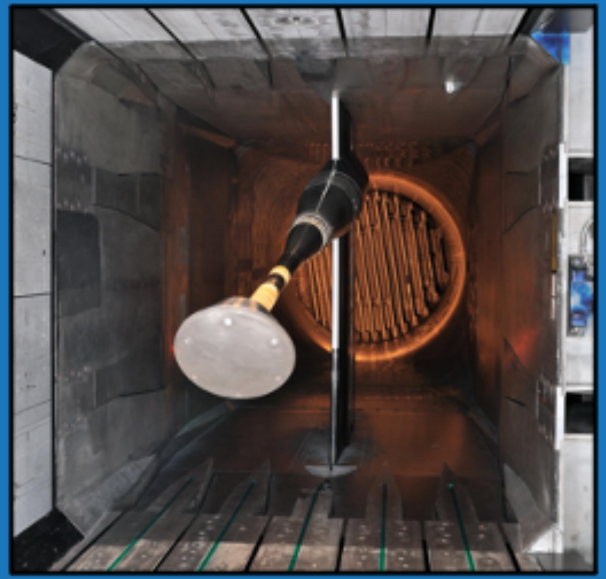
$$\begin{bmatrix} \text{FMV} & \text{Mach or Velocity} \\ \alpha & \text{Angle of Attack} \\ \beta & \text{Side-slip Angle} \\ \text{Re} & \text{Reynolds Number} \\ \vdots & \end{bmatrix} \Longrightarrow \begin{bmatrix} C_L & \text{Lift Force} \\ C_D & \text{Drag Force} \\ C_m & \text{Pitching Moment} \\ \vdots & \end{bmatrix}$$

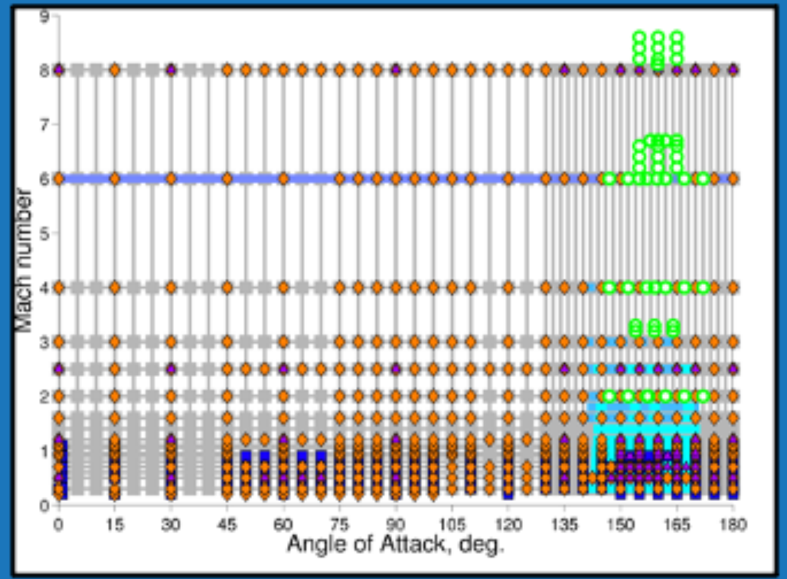


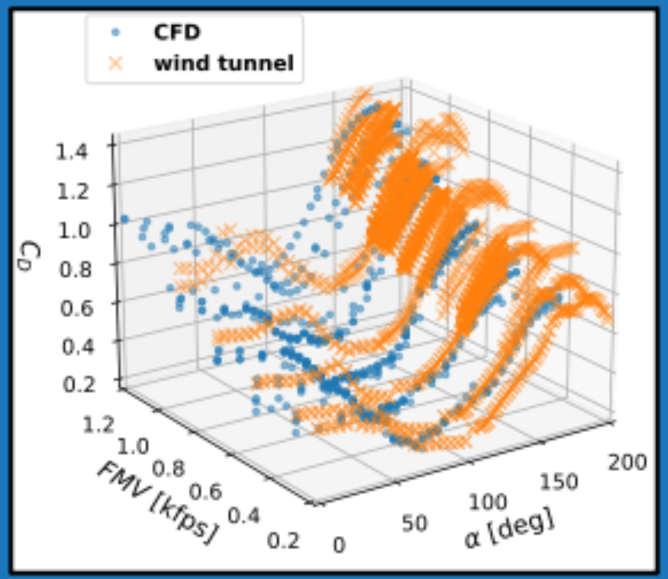
Aerodynamic databases

CFD Solutions and Wind Tunnel Tests



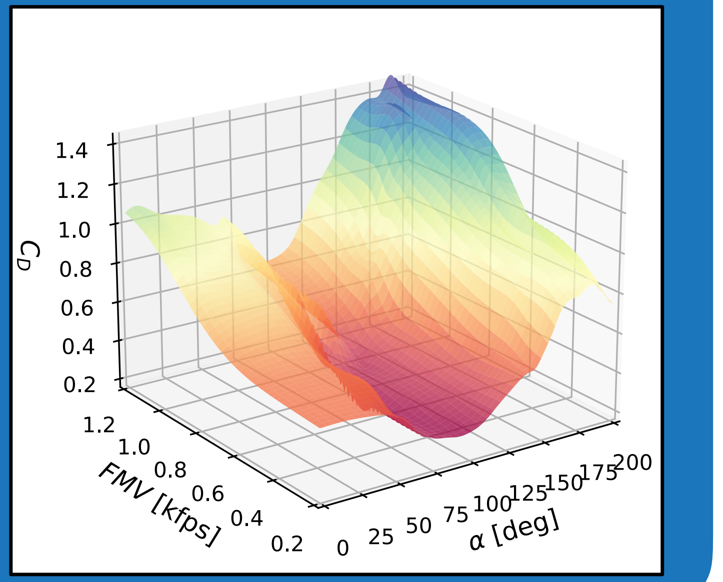




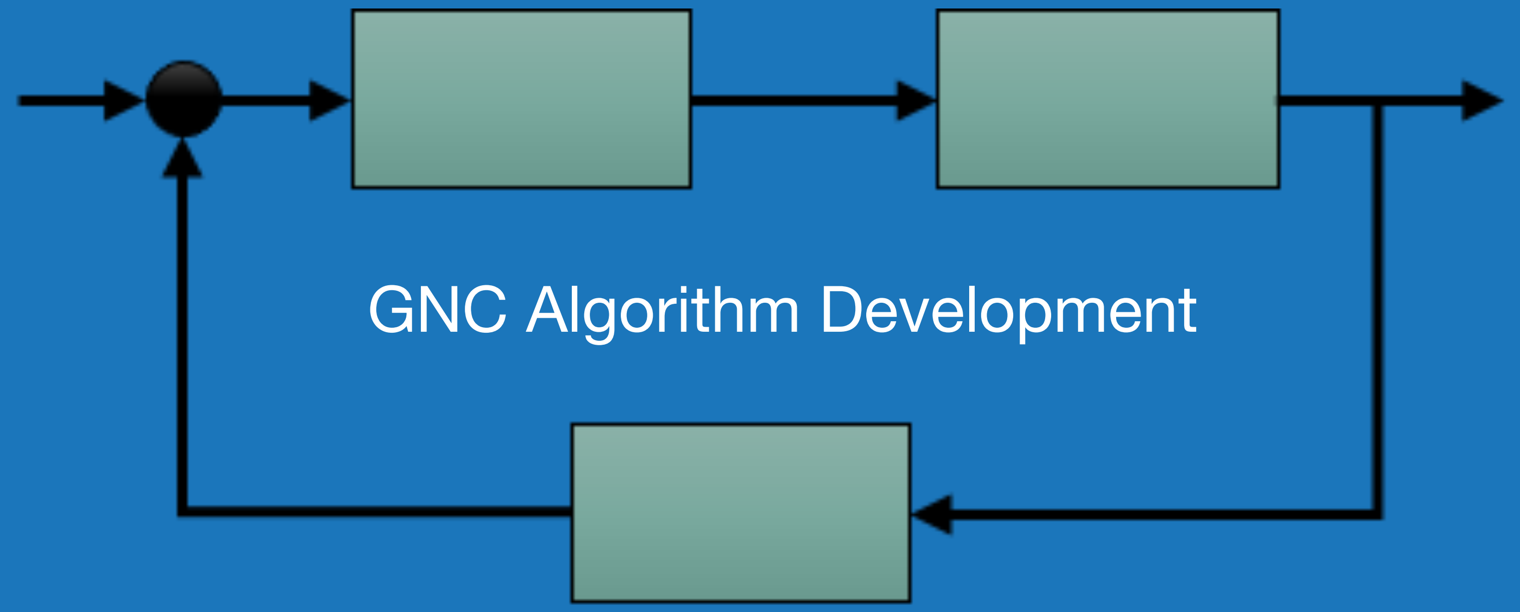


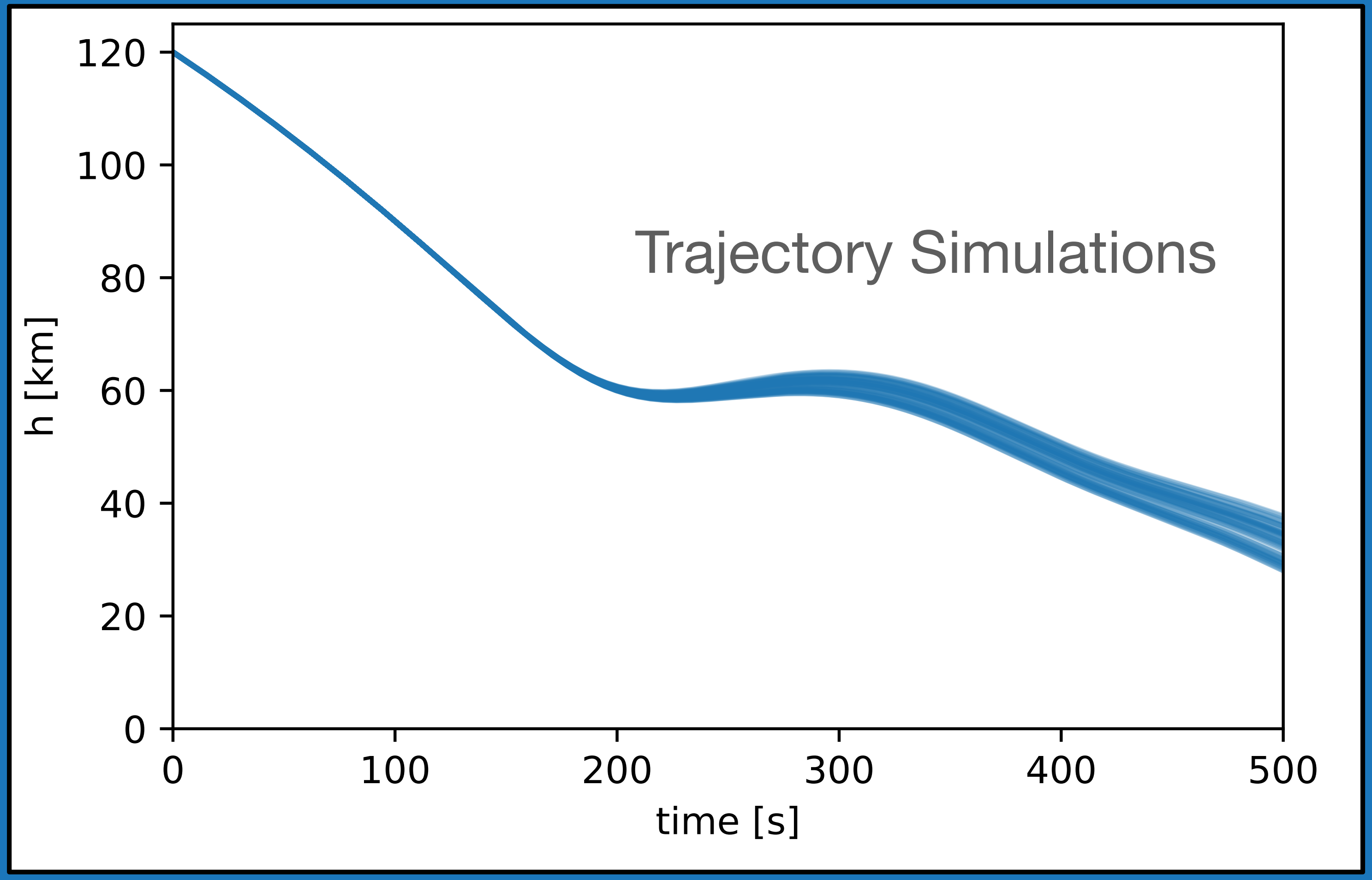
Aerodynamic Database Construction

$$\begin{bmatrix} \text{FMV} & \text{Mach or Velocity} \\ \alpha & \text{Angle of Attack} \\ \beta & \text{Side-slip Angle} \\ \text{Re} & \text{Reynolds Number} \\ \vdots & \end{bmatrix} \Longrightarrow \begin{bmatrix} C_L & \text{Lift Force} \\ C_D & \text{Drag Force} \\ C_m & \text{Pitching Moment} \\ & \vdots \end{bmatrix}$$



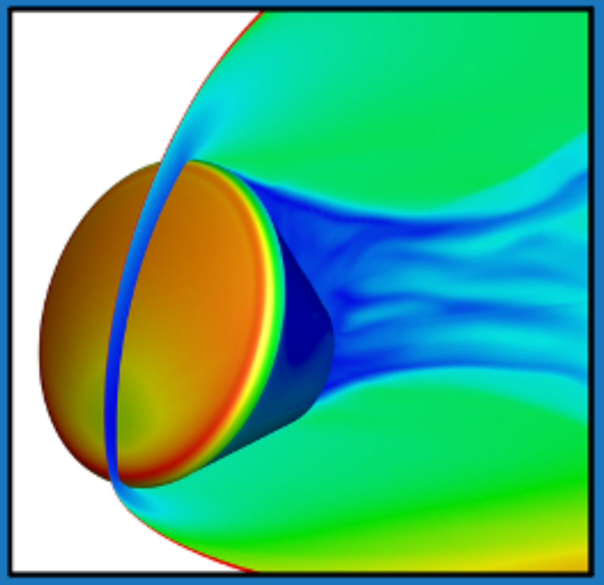
Engineering and Mission Design

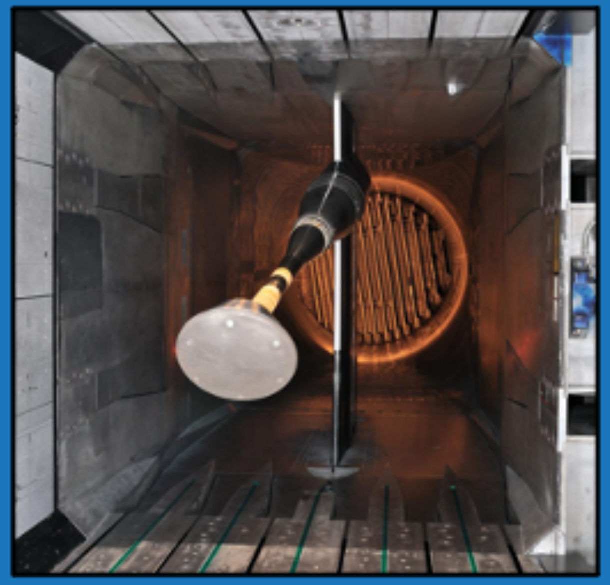


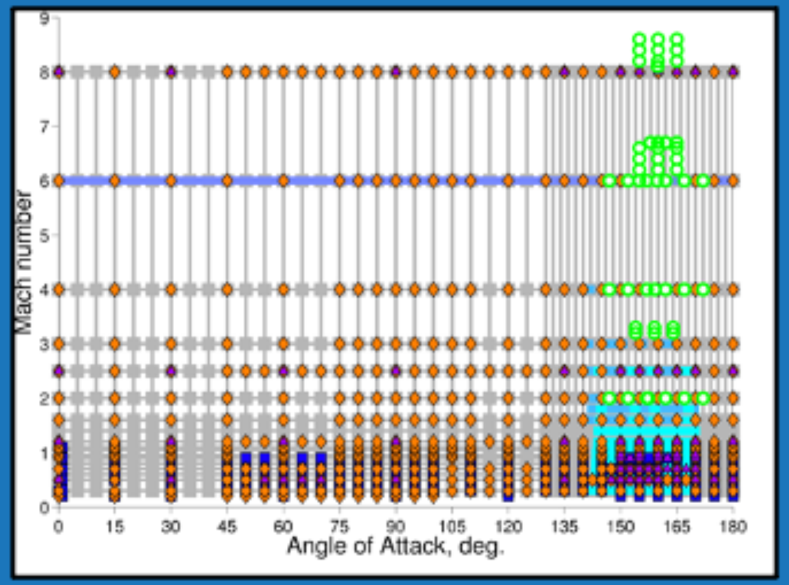


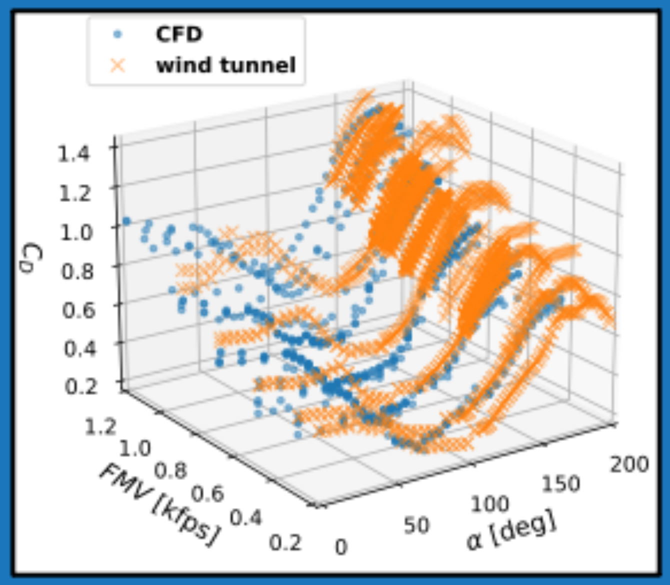
Aerodynamic databases

CFD Solutions and Wind Tunnel Tests

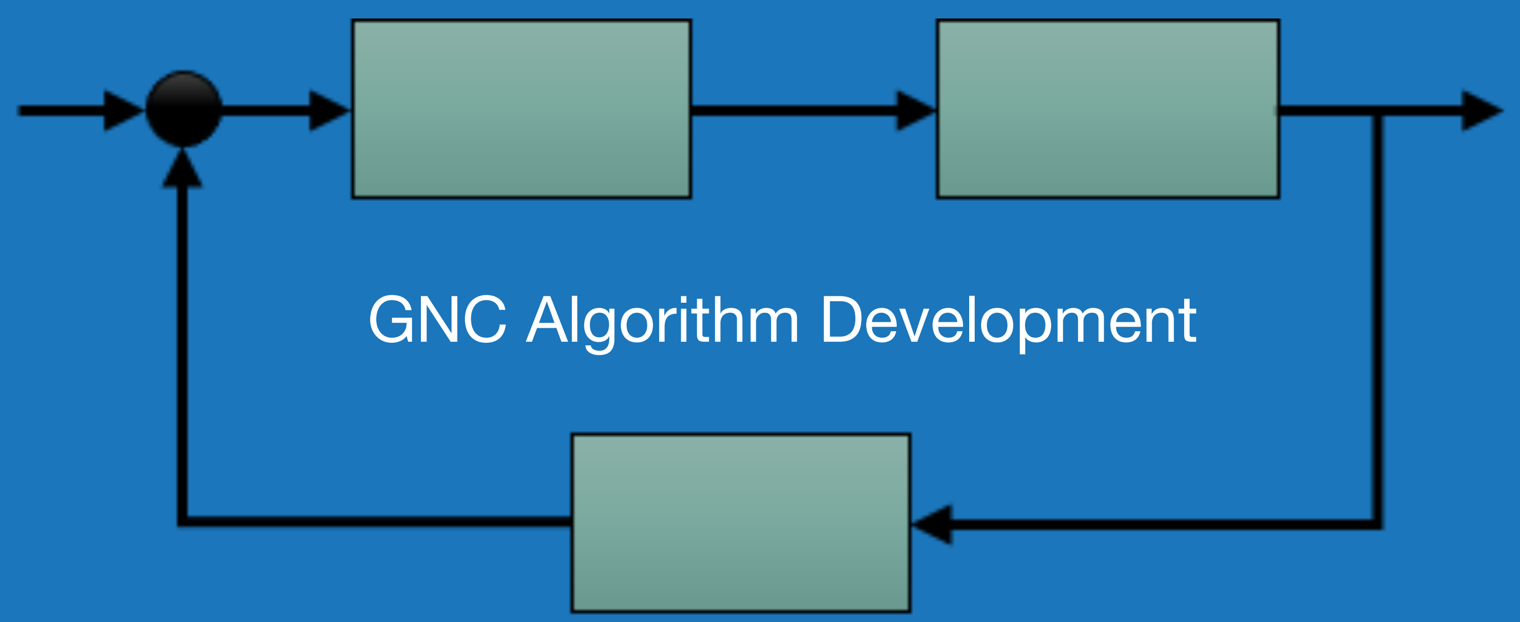


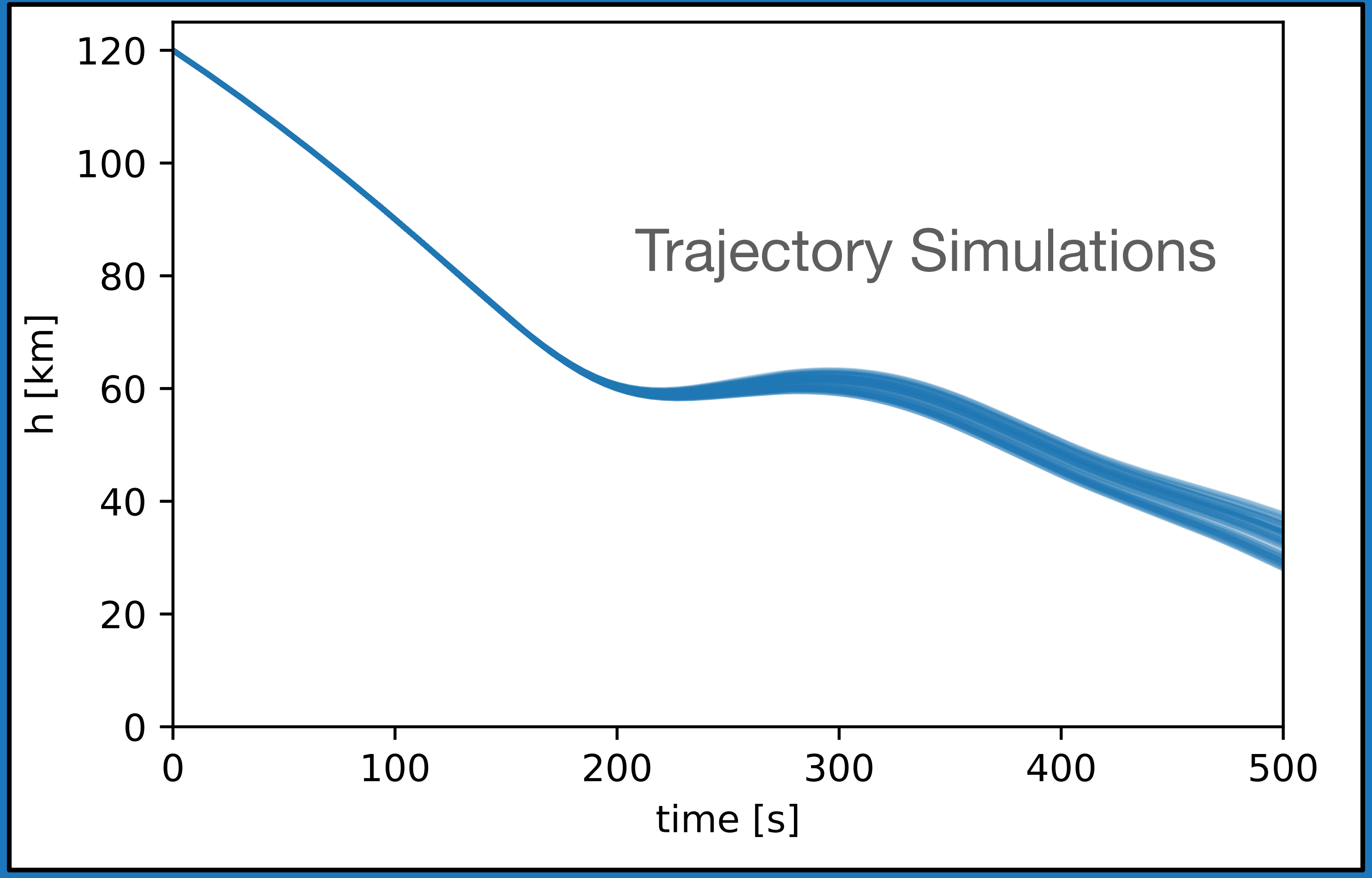






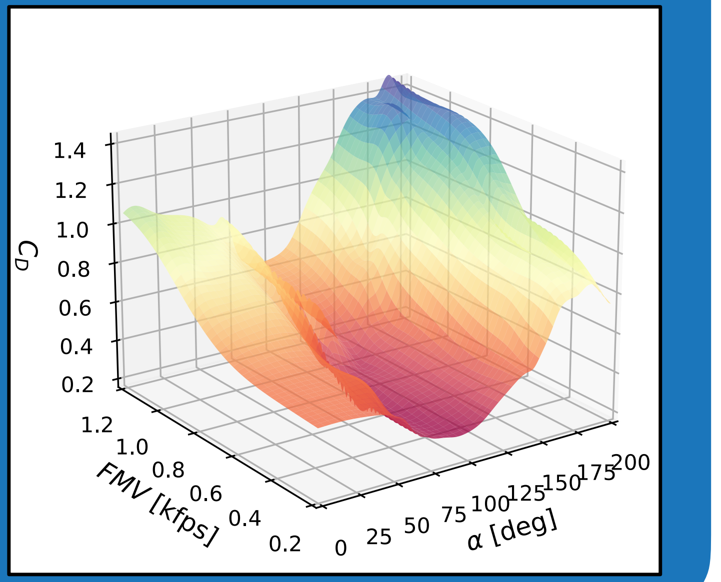
Engineering and Mission Design





Aerodynamic Database Construction

$$\begin{bmatrix} \text{FMV} & \text{Mach or Velocity} \\ \alpha & \text{Angle of Attack} \\ \beta & \text{Side-slip Angle} \\ \text{Re} & \text{Reynolds Number} \\ & \vdots \end{bmatrix} \Longrightarrow \begin{bmatrix} C_L & \text{Lift Force} \\ C_D & \text{Drag Force} \\ C_m & \text{Pitching Moment} \\ & \vdots \end{bmatrix}$$

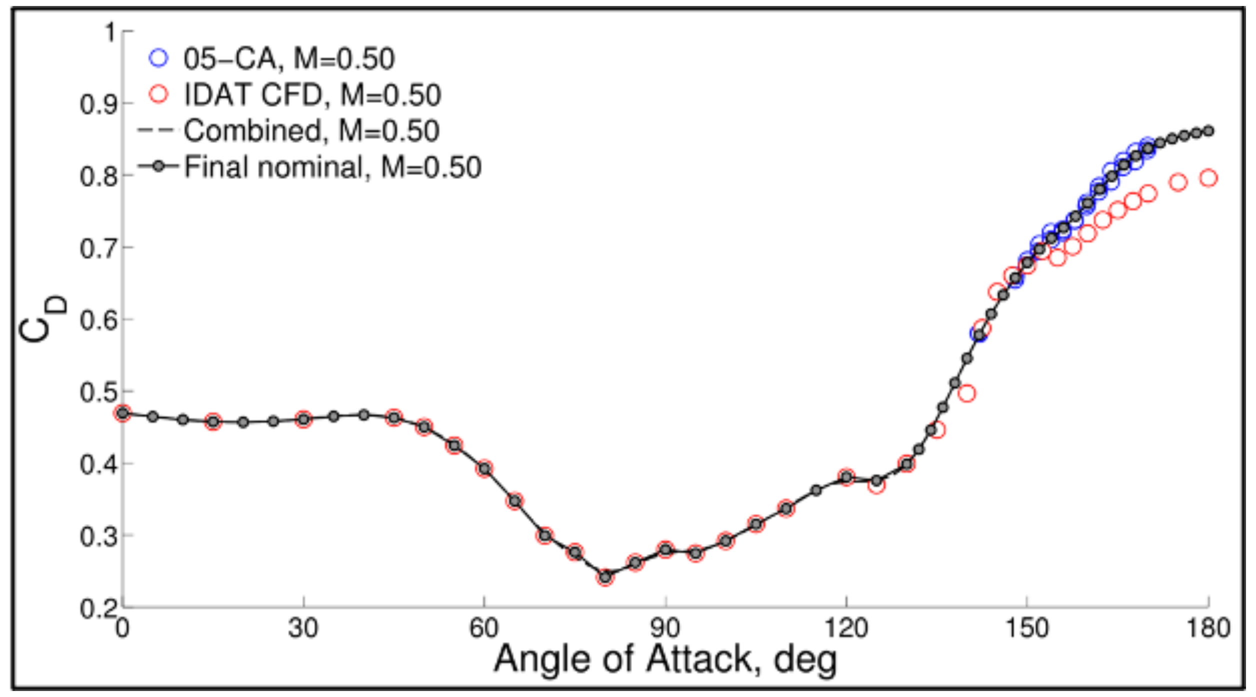


Data fusion with uncertainties

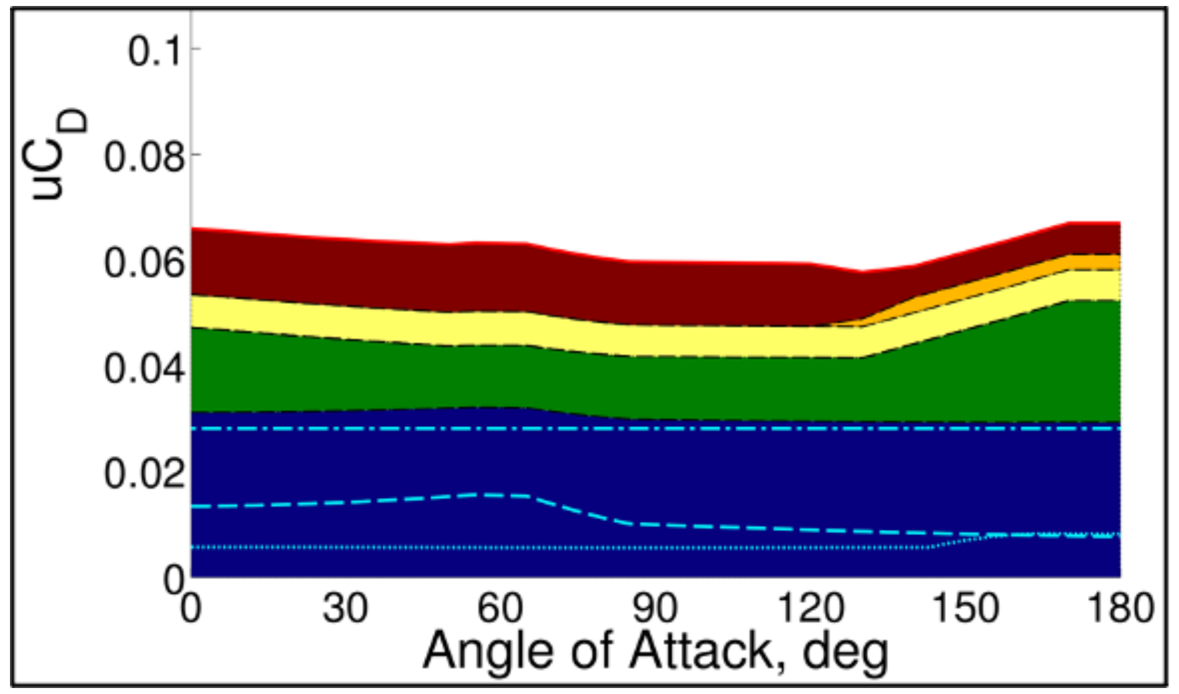
Typical data is noisy, with varying degrees of fidelity to flight vehicle

- Data continuously updated as design matures
- Different levels of fidelity in computational tools
- Wind tunnel models approximate vehicle geometry and roughness
- Wind tunnels cannot always reproduce flight conditions

Current state of the practice: “UQ by Inspection”



Nominal aerocoefficients constructed using expert judgment, given multiple sources of data.



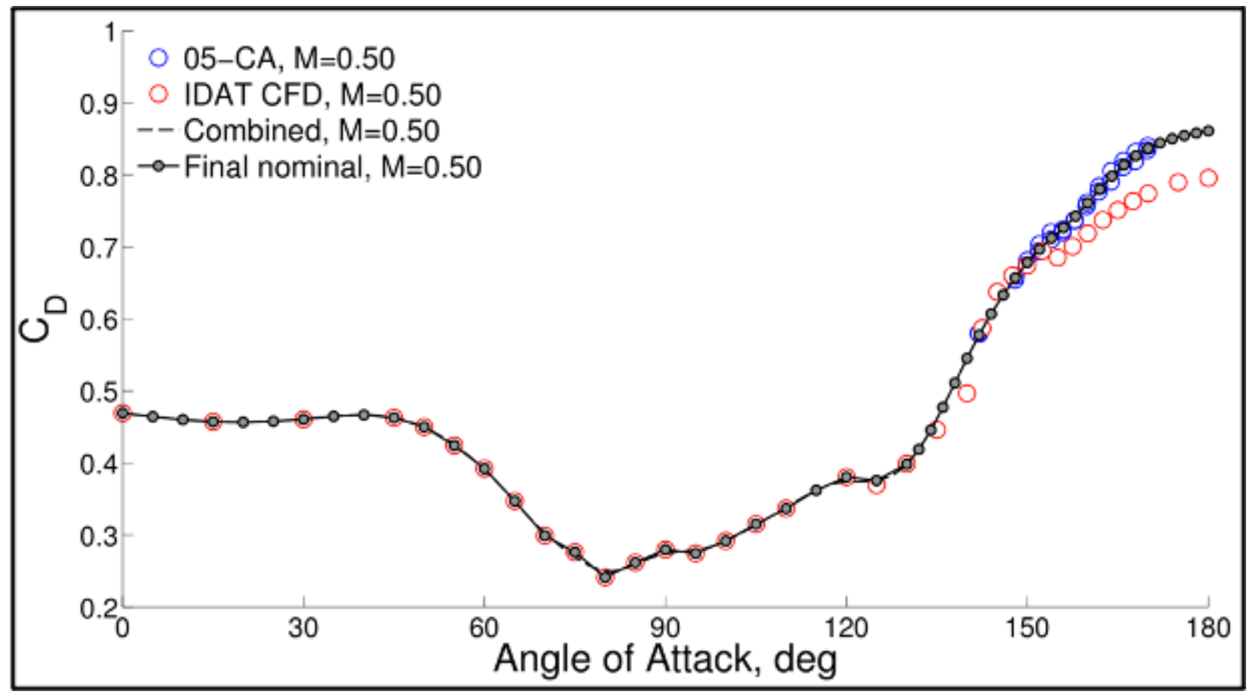
Uncertainty buildup based on dispersion factors, tuned to cover varying data sources.

Data fusion with uncertainties

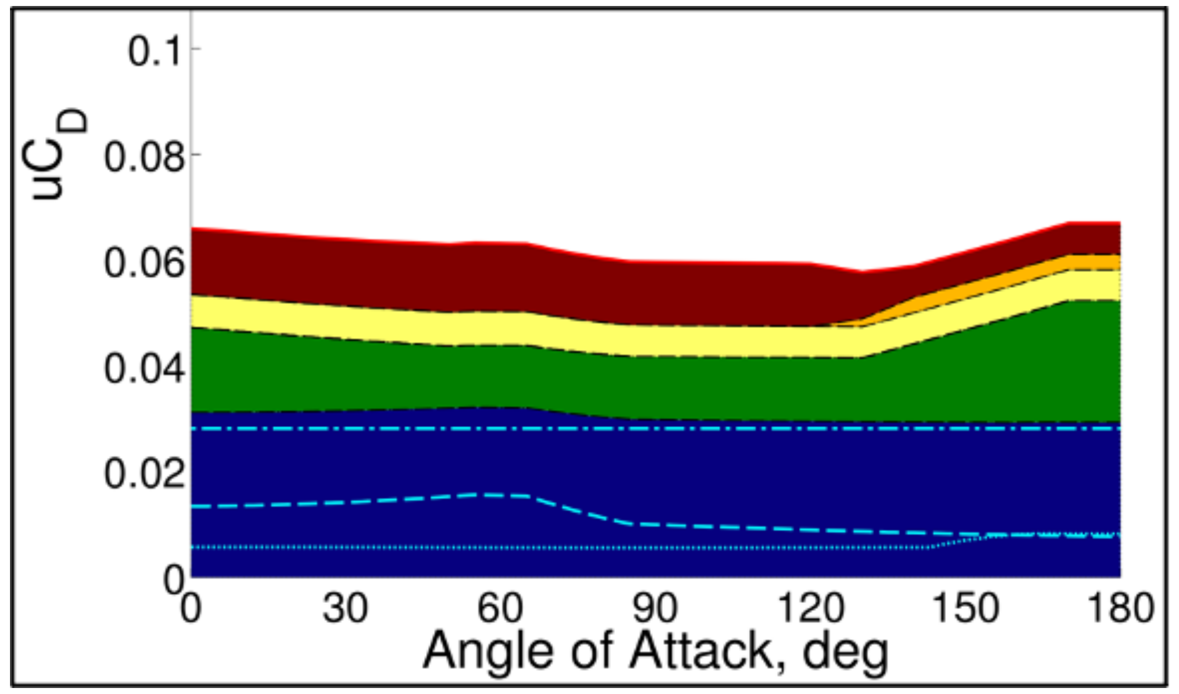
Typical data is noisy, with varying degrees of fidelity to flight vehicle

- Data continuously updated as design matures
- Different levels of fidelity in computational tools
- Wind tunnel models approximate vehicle geometry and roughness
- Wind tunnels cannot always reproduce flight conditions

Current state of the practice: "UQ by Inspection"

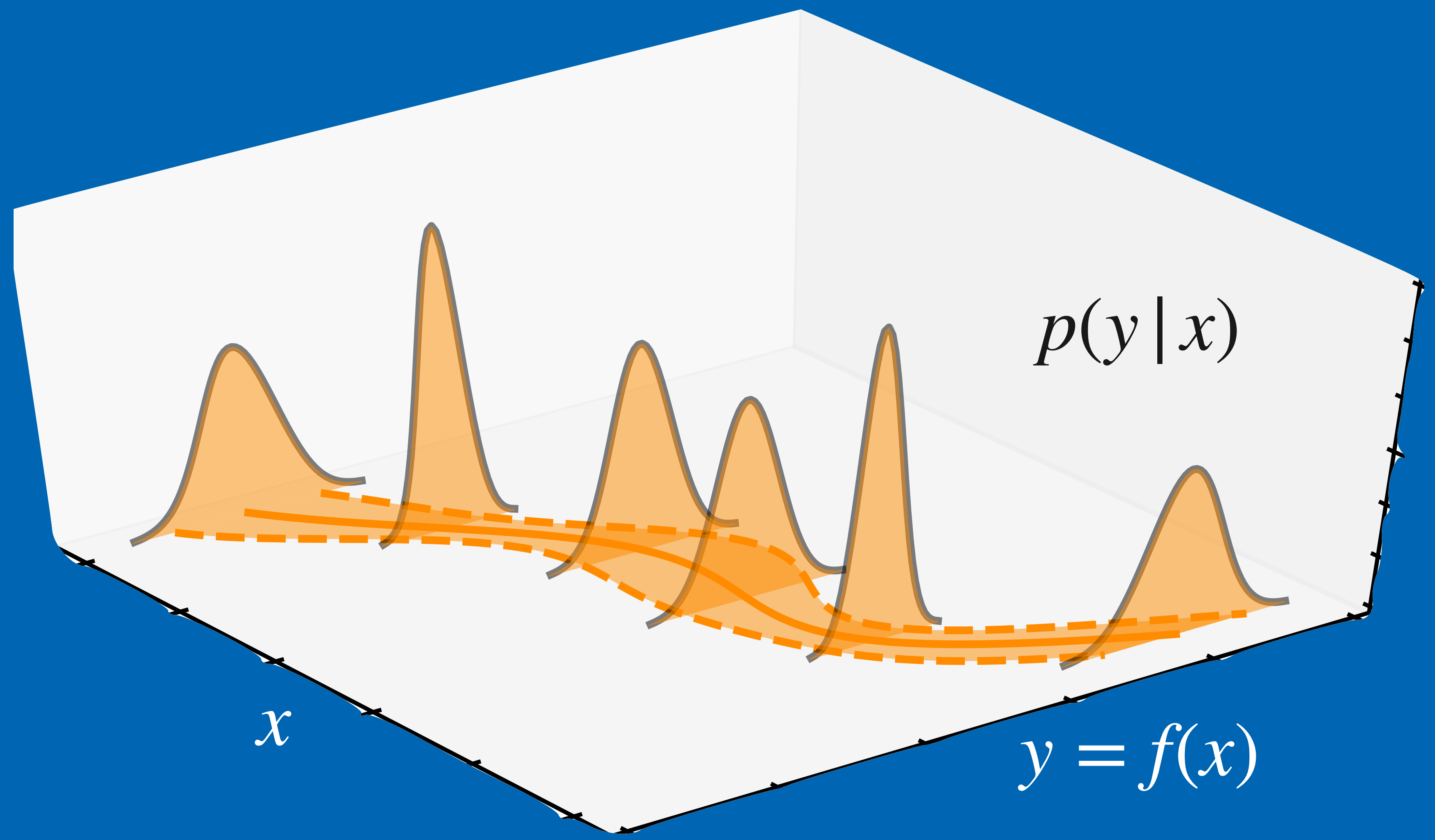


Nominal aerocoefficients constructed using expert judgment, given multiple sources of data.



Uncertainty buildup based on dispersion factors, tuned to cover varying data sources.

Want to "learn" a surrogate conditional probability distribution, given all data sources

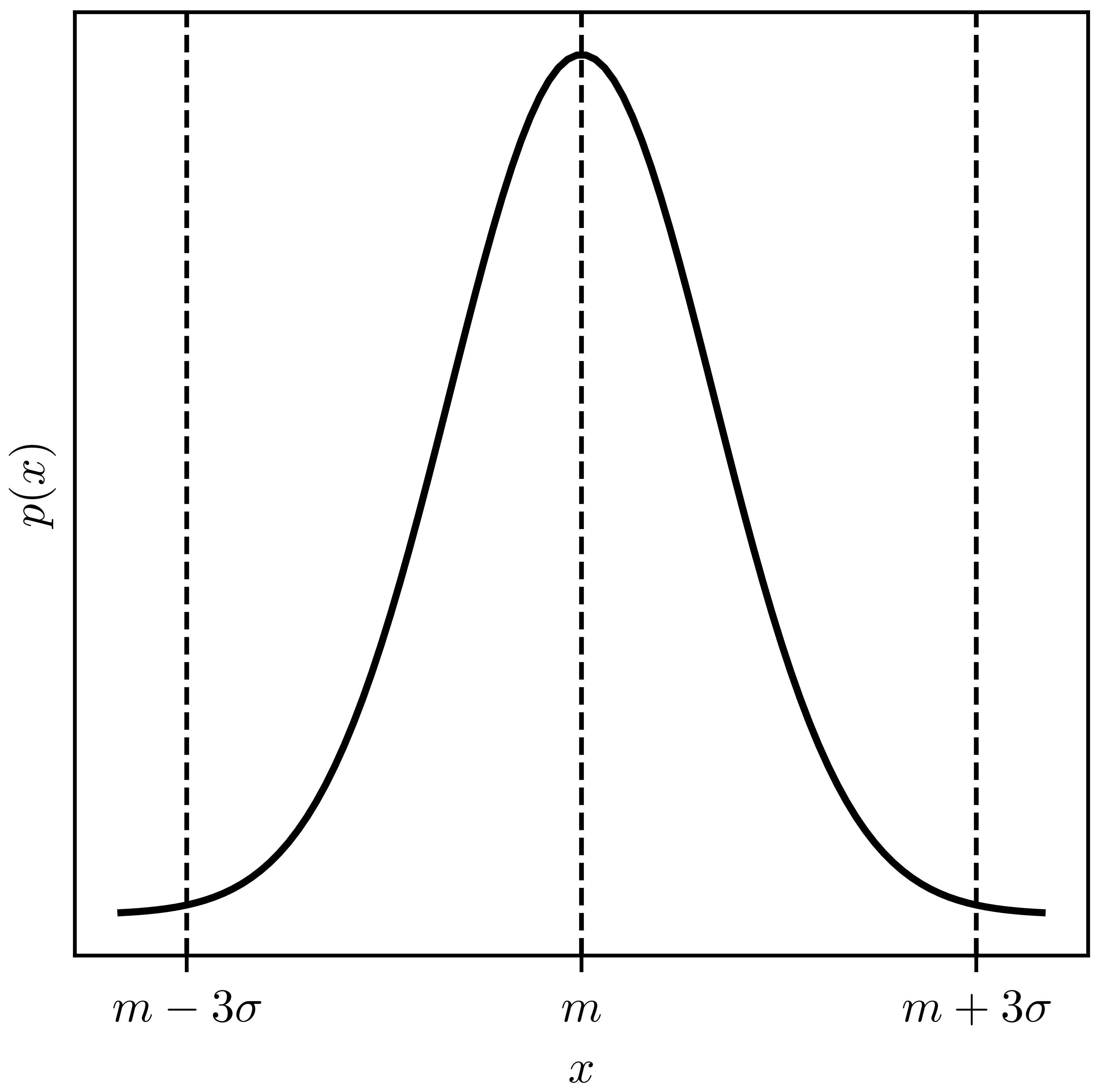


- $p(y|x)$ defines the "probability of outcome y given x"
- surrogate model is "stochastic" but not "random"

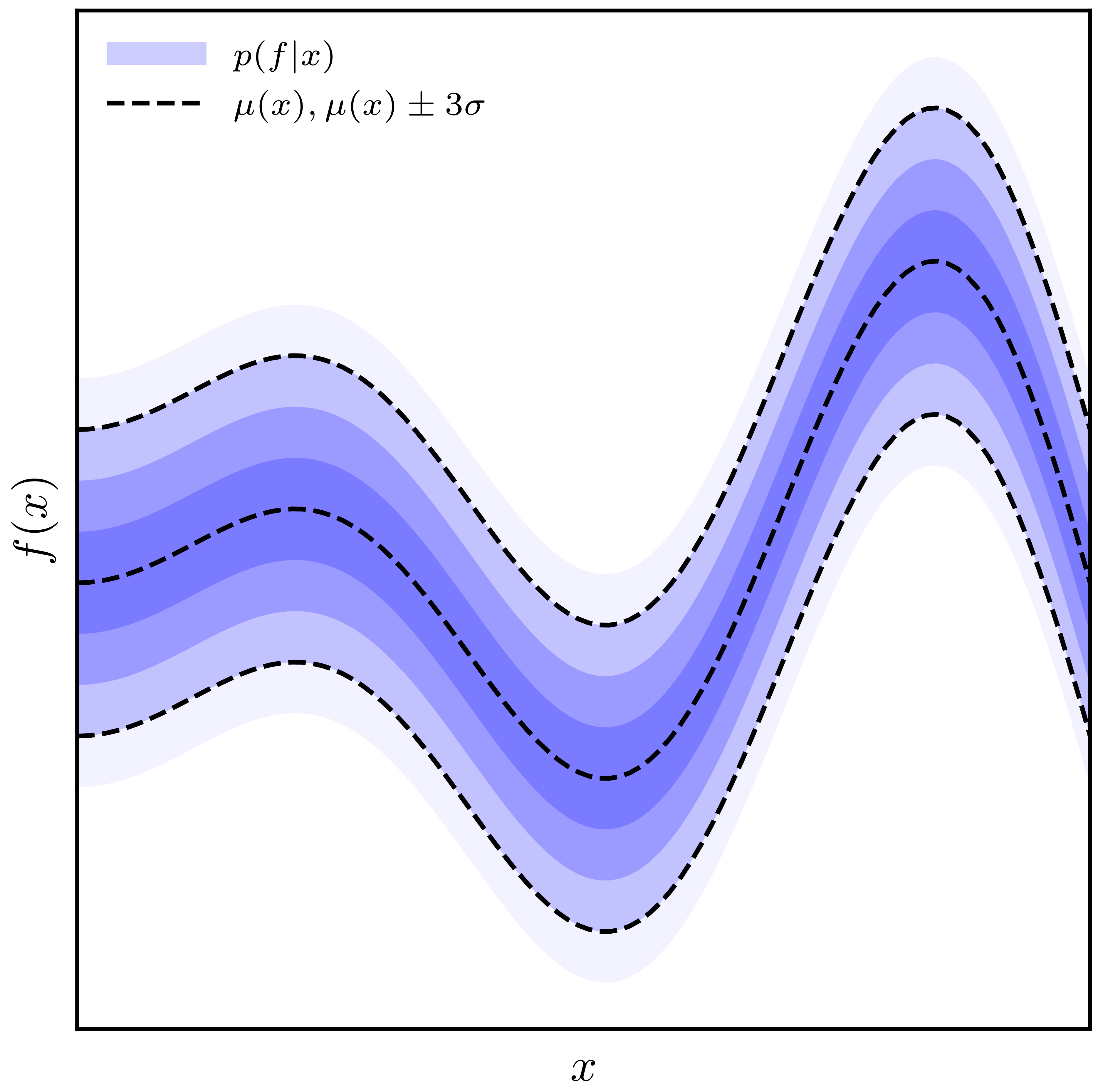
Gaussian Processes (GPs)

Generalization of the Gaussian distribution for random scalars to random functions

$x \sim \mathcal{N}(m, \sigma^2)$

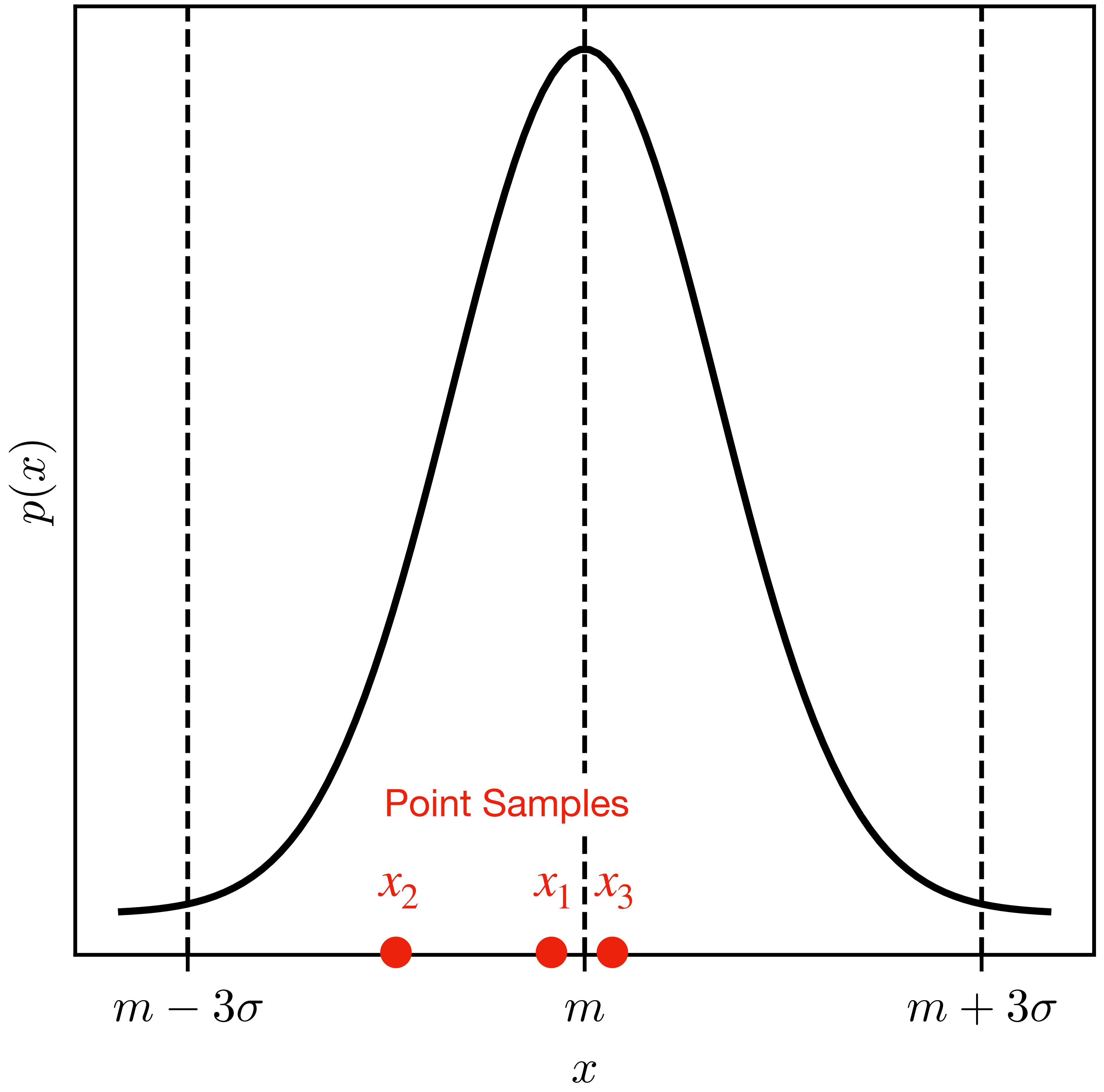


$f(x) \sim \mathcal{GP}(\mu(x), k(x, x'))$

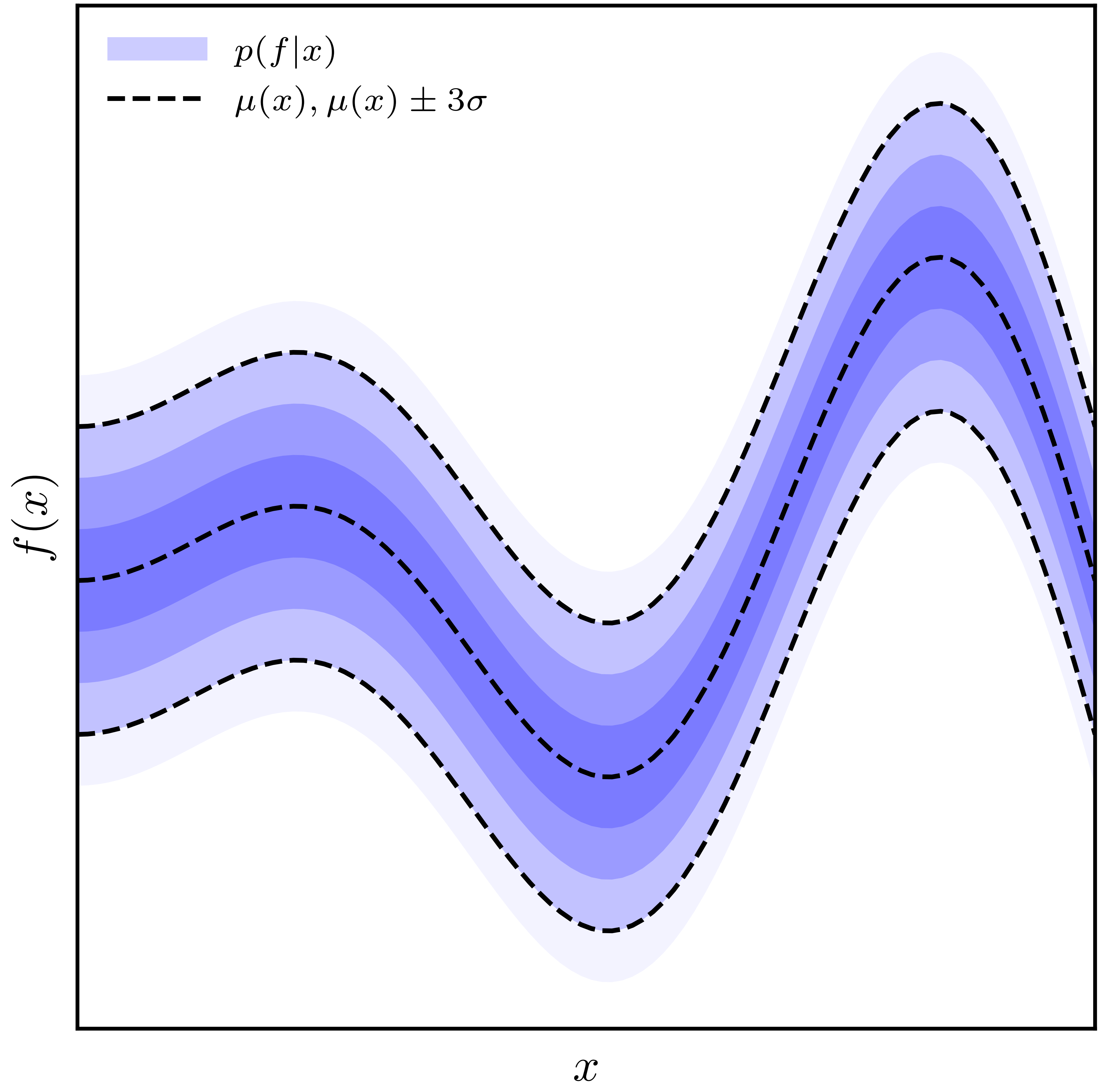


Generalization of the Gaussian distribution for random scalars to random functions

$$x \sim \mathcal{N}(m, \sigma^2)$$



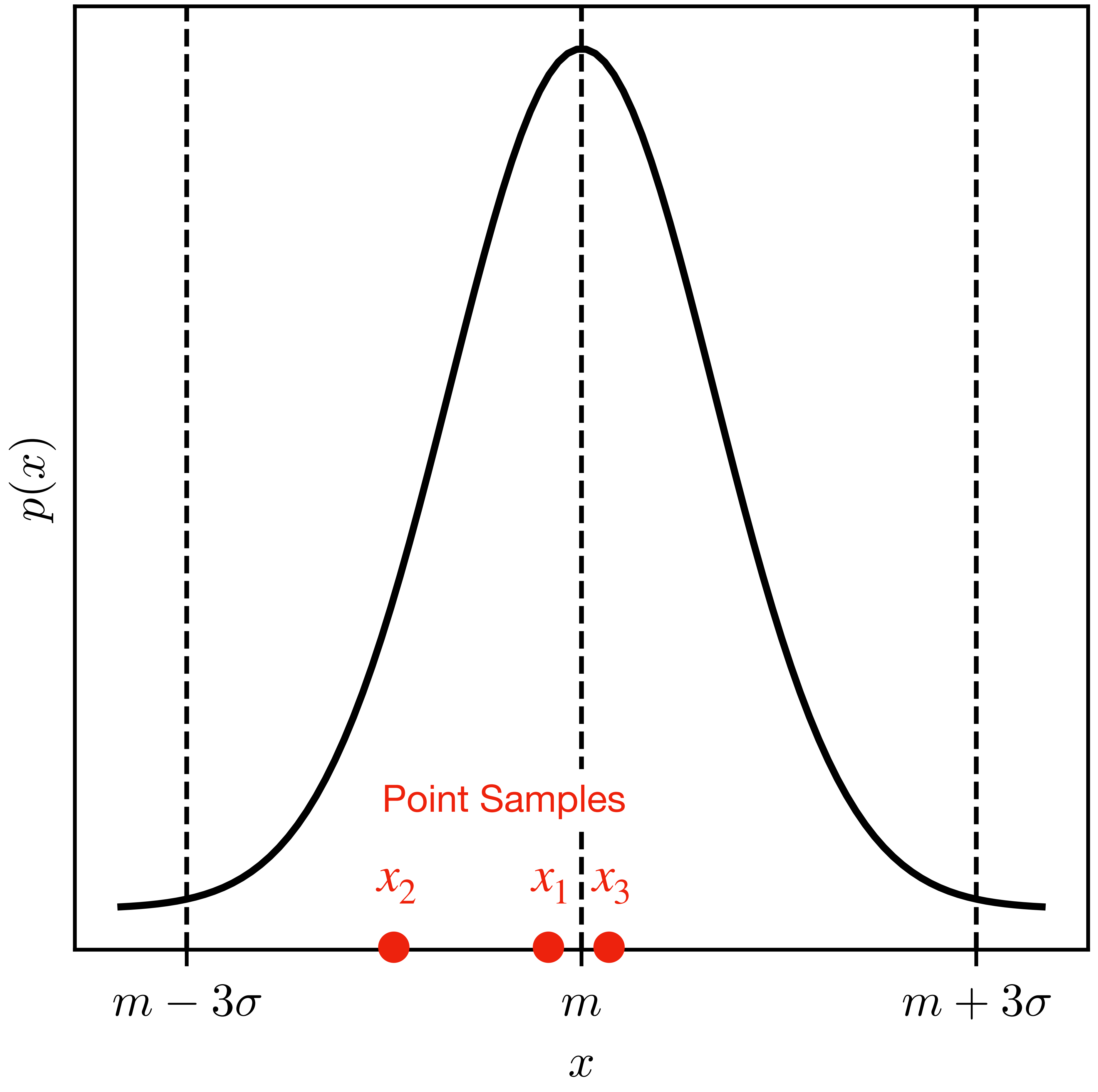
$$f(x) \sim \mathcal{GP}(\mu(x), k(x, x'))$$



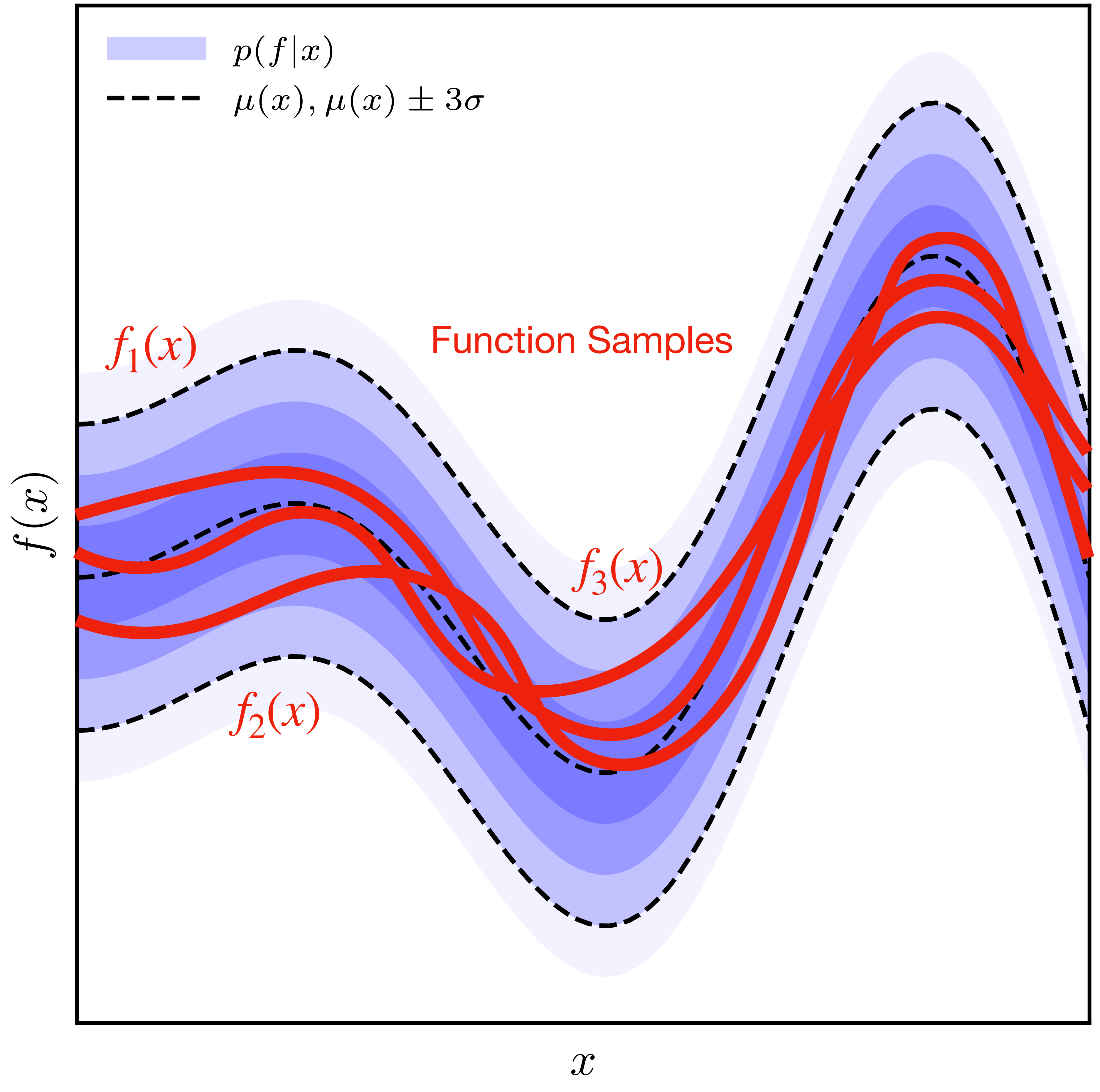
Gaussian Processes (GPs)

Generalization of the Gaussian distribution for random scalars to random functions

$x \sim \mathcal{N}(m, \sigma^2)$



$f(x) \sim \mathcal{GP}(\mu(x), k(x, x'))$

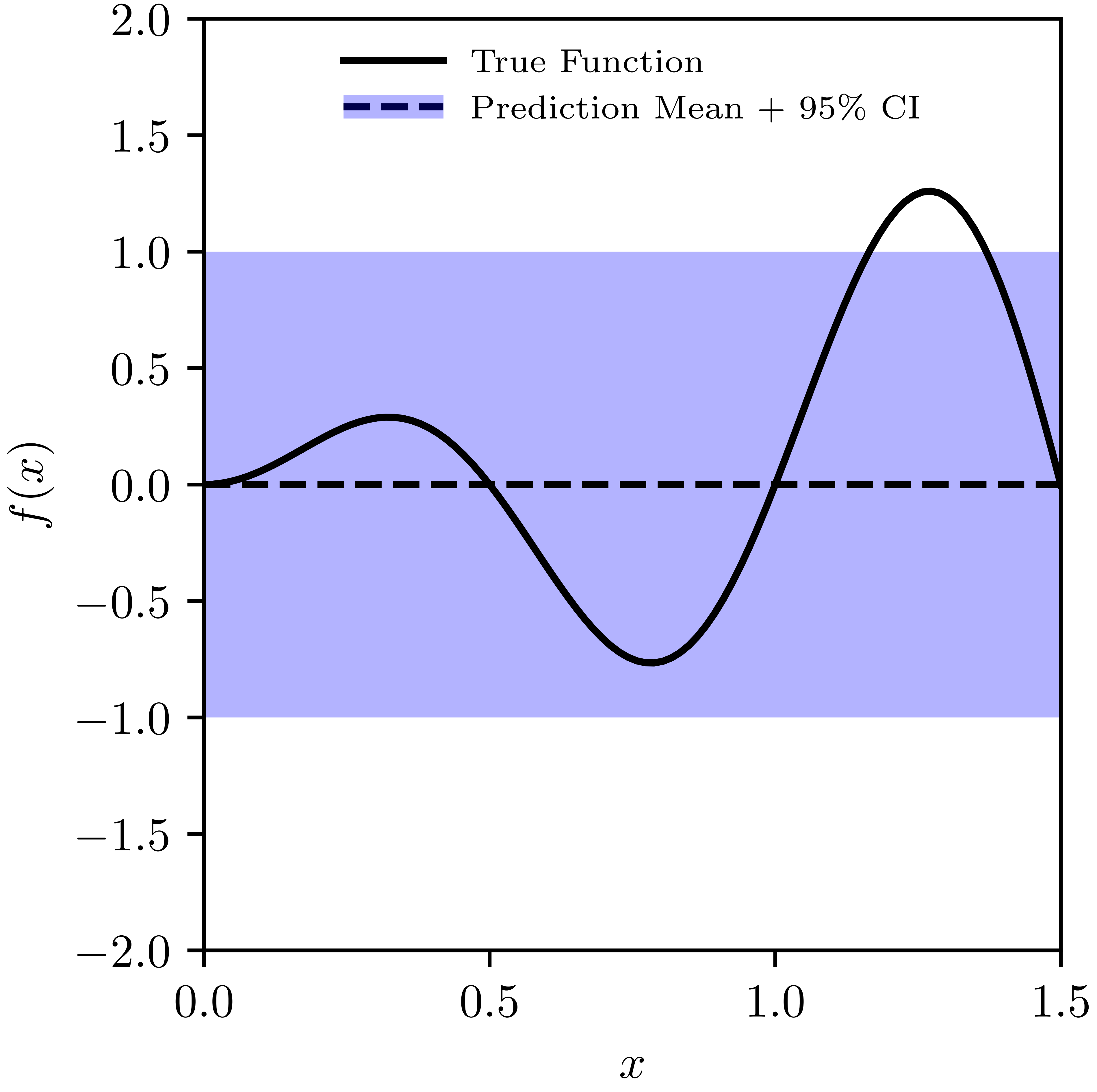


Gaussian Process Regression

Any finite set of index points $\{x_1, \ldots, x_n\}$ represents a multivariate Gaussian distribution for function values.

$$\underbrace{f(x) \sim p(f\,|\,x)}_{\text{prior}} = \mathcal{GP}(\mu, k)$$

$$\Longrightarrow \begin{bmatrix} f(x_1) \\ f(x_2) \\ f(x_3) \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} \mu(x_1) \\ \mu(x_2) \\ \mu(x_3) \end{bmatrix}, \begin{bmatrix} k(x_1,x_1) & k(x_1,x_2) & k(x_1,x_3) \\ k(x_2,x_1) & k(x_2,x_2) & k(x_2,x_3) \\ k(x_3,x_1) & k(x_3,x_2) & k(x_3,x_3) \end{bmatrix} \right)$$



Gaussian Process Regression

Any finite set of index points $\{x_1, \ldots, x_n\}$ represents a multivariate Gaussian distribution for function values.

prior

$$f(x) \sim p(f\,|\,x) = \mathcal{GP}(\mu, k)$$

$$\implies \begin{bmatrix} f(x_1) \\ f(x_2) \\ f(x_3) \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} \mu(x_1) \\ \mu(x_2) \\ \mu(x_3) \end{bmatrix}, \begin{bmatrix} k(x_1, x_1) & k(x_1, x_2) & k(x_1, x_3) \\ k(x_2, x_1) & k(x_2, x_2) & k(x_2, x_3) \\ k(x_3, x_1) & k(x_3, x_2) & k(x_3, x_3) \end{bmatrix} \right)$$

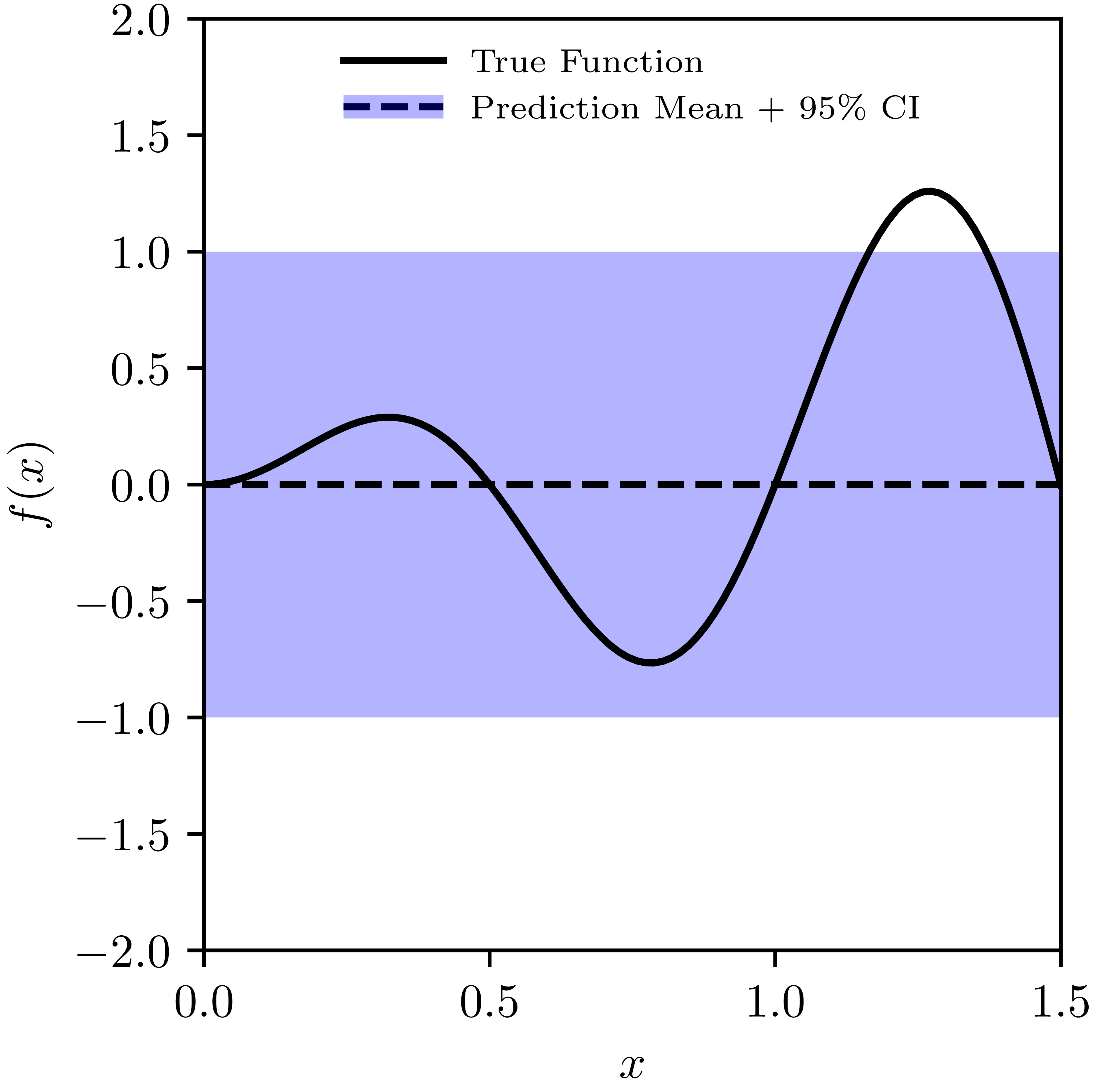
Regression performed by conditioning the distribution on data $\mathcal{D} = (X, y)$

posterior

$$\hat{f}(x) \sim p(f\,|\,\mathcal{D}, x) = \mathcal{GP}(\hat{\mu}, \hat{k})$$

$$\hat{\mu}(x) = \mu(x) + k(x, X)\; k(X, X)^{-1}\; (y - \mu(X))$$

$$\hat{k}(x, x') = k(x, x') + k(x, X)\; k(X, X)^{-1}\; k(X, x')$$



Gaussian Process Regression

Any finite set of index points $\{x_1, \ldots, x_n\}$ represents a multivariate Gaussian distribution for function values.

prior

$$f(x) \sim p(f \,|\, x) = \mathcal{GP}(\mu, k)$$

$$\implies \begin{bmatrix} f(x_1) \\ f(x_2) \\ f(x_3) \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} \mu(x_1) \\ \mu(x_2) \\ \mu(x_3) \end{bmatrix}, \begin{bmatrix} k(x_1,x_1) & k(x_1,x_2) & k(x_1,x_3) \\ k(x_2,x_1) & k(x_2,x_2) & k(x_2,x_3) \\ k(x_3,x_1) & k(x_3,x_2) & k(x_3,x_3) \end{bmatrix} \right)$$

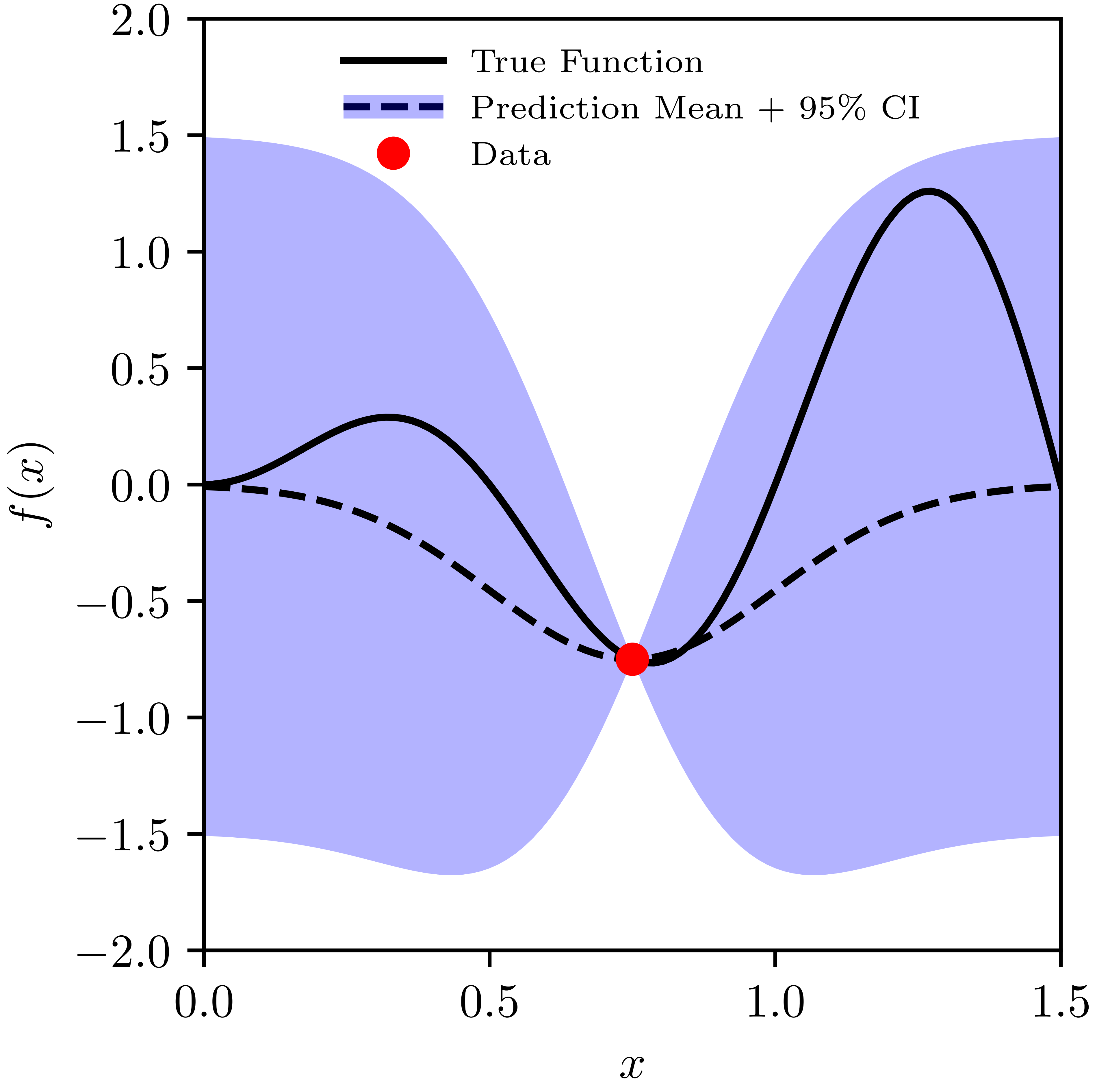
Regression performed by conditioning the distribution on data $\mathcal{D} = (X, y)$

posterior

$$\hat{f}(x) \sim p(f \,|\, \mathcal{D}, x) = \mathcal{GP}(\hat{\mu}, \hat{k})$$

$$\hat{\mu}(x) = \mu(x) + k(x, X)\; k(X, X)^{-1}\; (y - \mu(X))$$

$$\hat{k}(x, x') = k(x, x') + k(x, X)\; k(X, X)^{-1}\; k(X, x')$$



Gaussian Process Regression

Any finite set of index points $\{x_1, \ldots, x_n\}$ represents a multivariate Gaussian distribution for function values.

prior

$$f(x) \sim p(f\,|\,x) = \mathcal{GP}(\mu, k)$$

$$\implies \begin{bmatrix} f(x_1) \\ f(x_2) \\ f(x_3) \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} \mu(x_1) \\ \mu(x_2) \\ \mu(x_3) \end{bmatrix}, \begin{bmatrix} k(x_1, x_1) & k(x_1, x_2) & k(x_1, x_3) \\ k(x_2, x_1) & k(x_2, x_2) & k(x_2, x_3) \\ k(x_3, x_1) & k(x_3, x_2) & k(x_3, x_3) \end{bmatrix} \right)$$

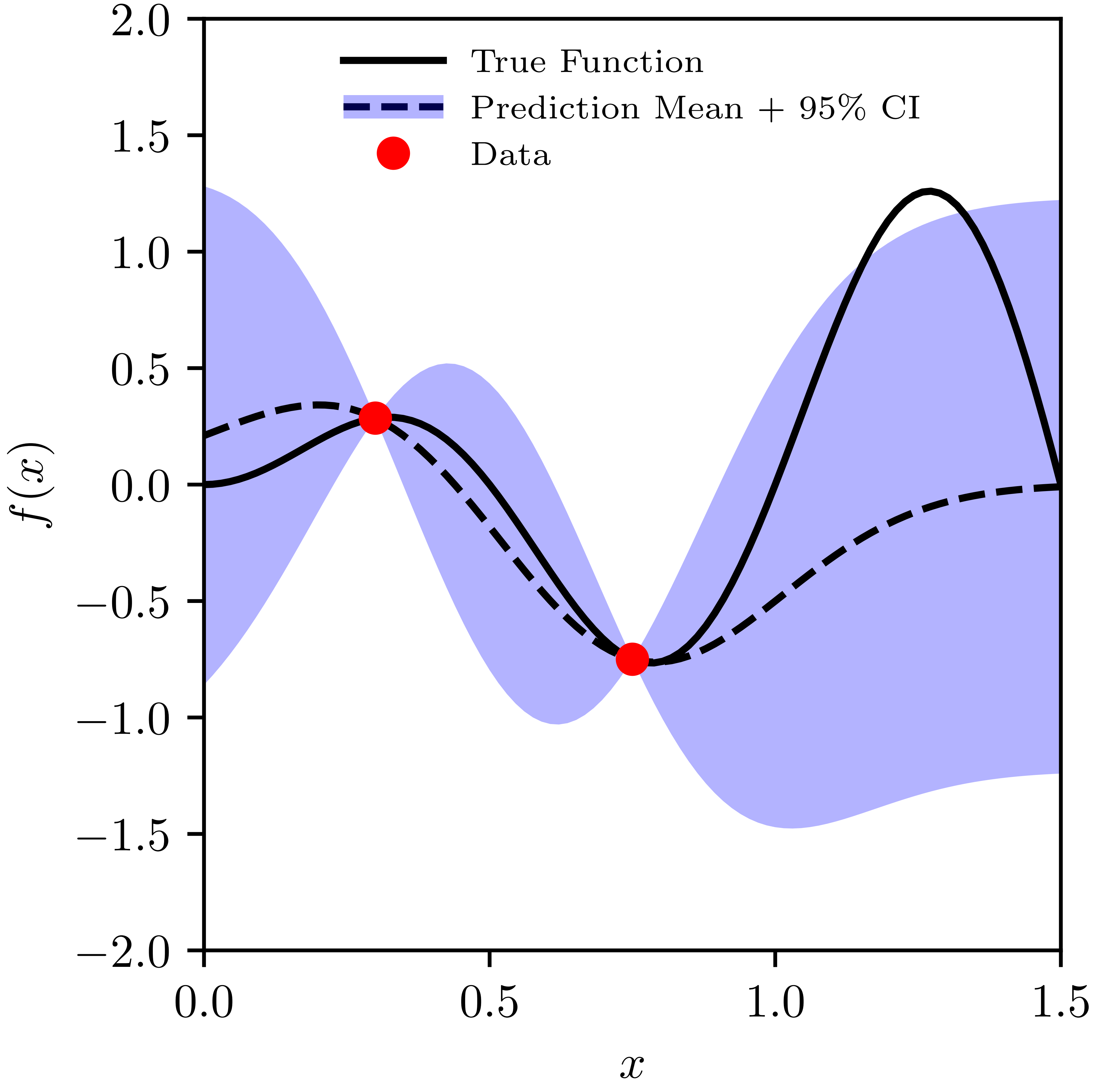
Regression performed by conditioning the distribution on data $\mathcal{D} = (X, y)$

posterior

$$\hat{f}(x) \sim p(f\,|\,\mathcal{D}, x) = \mathcal{GP}(\hat{\mu}, \hat{k})$$

$$\hat{\mu}(x) = \mu(x) + k(x, X)\; k(X, X)^{-1}\; (y - \mu(X))$$

$$\hat{k}(x, x') = k(x, x') + k(x, X)\; k(X, X)^{-1}\; k(X, x')$$



Gaussian Process Regression

Any finite set of index points $\{x_1, \ldots, x_n\}$ represents a multivariate Gaussian distribution for function values.

prior

$$f(x) \sim p(f\,|\,x) = \mathcal{GP}(\mu, k)$$

$$\implies \begin{bmatrix} f(x_1) \\ f(x_2) \\ f(x_3) \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} \mu(x_1) \\ \mu(x_2) \\ \mu(x_3) \end{bmatrix}, \begin{bmatrix} k(x_1, x_1) & k(x_1, x_2) & k(x_1, x_3) \\ k(x_2, x_1) & k(x_2, x_2) & k(x_2, x_3) \\ k(x_3, x_1) & k(x_3, x_2) & k(x_3, x_3) \end{bmatrix} \right)$$

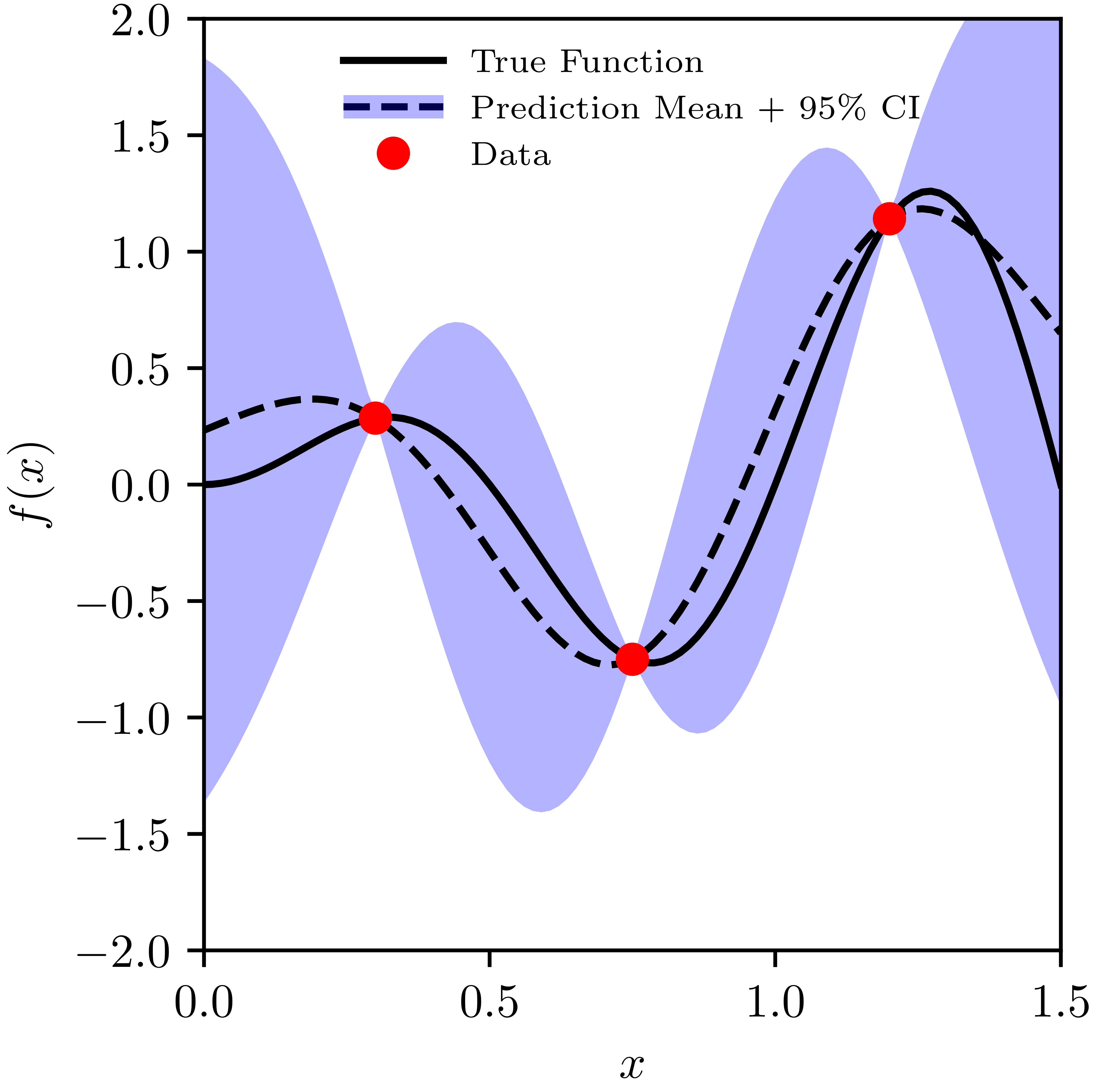
Regression performed by conditioning the distribution on data $\mathcal{D} = (X, y)$

posterior

$$\hat{f}(x) \sim p(f\,|\,\mathcal{D}, x) = \mathcal{GP}(\hat{\mu}, \hat{k})$$

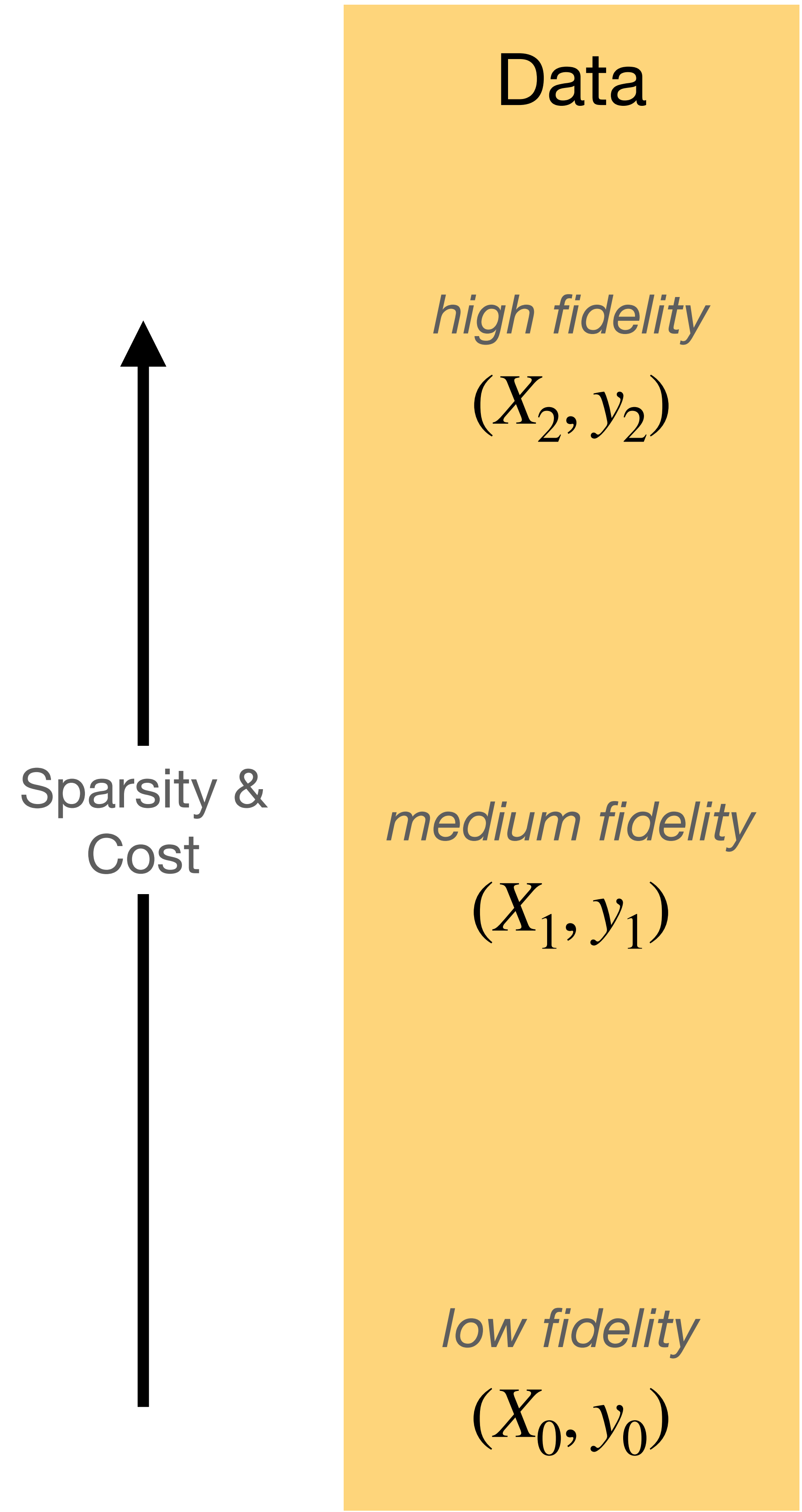
$$\hat{\mu}(x) = \mu(x) + k(x, X)\; k(X, X)^{-1}\; (y - \mu(X))$$

$$\hat{k}(x, x') = k(x, x') + k(x, X)\; k(X, X)^{-1}\; k(X, x')$$



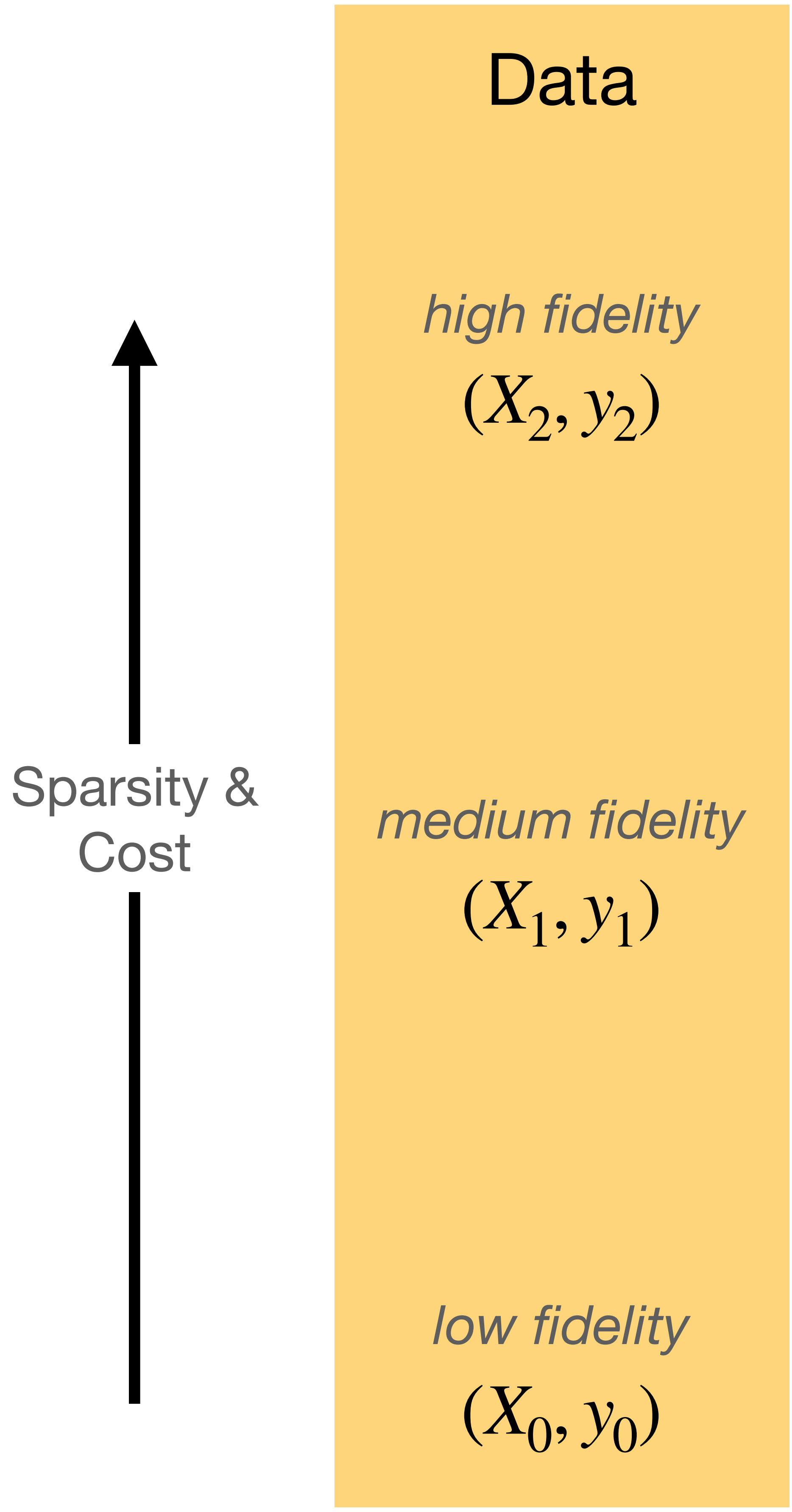
Multifidelity Gaussian Process Regression

- Multiple sources of data of increasing fidelity
  - Increasing CFD mesh resolutions
  - Heat flux correlations and 3D CFD solutions
  - CFD solutions and wind tunnel data



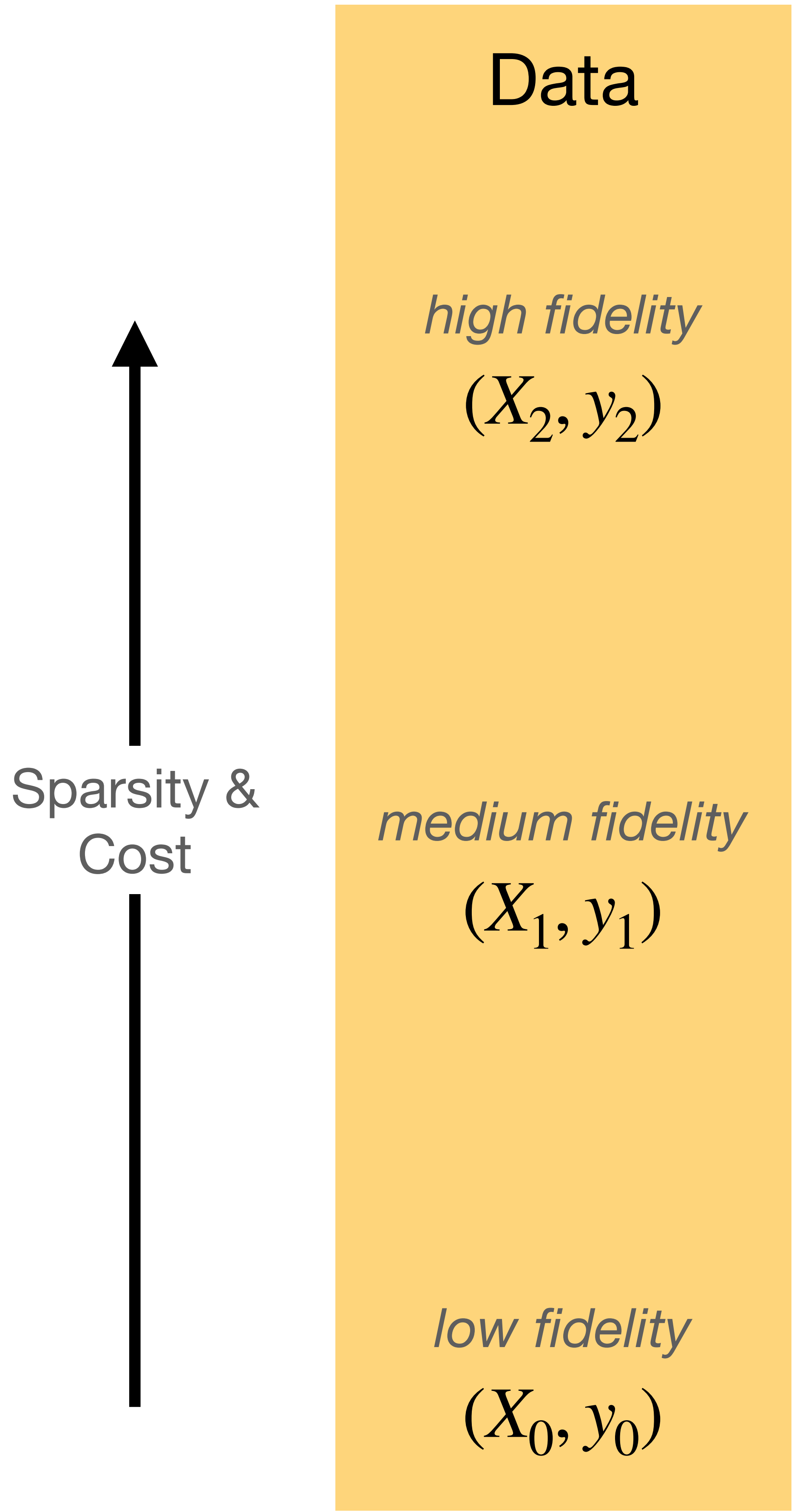
Multifidelity Gaussian Process Regression

- Multiple sources of data of increasing fidelity
  - Increasing CFD mesh resolutions
  - Heat flux correlations and 3D CFD solutions
  - CFD solutions and wind tunnel data
- Low fidelity is dense and cheap to obtain, high fidelity is sparse and expensive



Multifidelity Gaussian Process Regression

- Multiple sources of data of increasing fidelity
  - Increasing CFD mesh resolutions
  - Heat flux correlations and 3D CFD solutions
  - CFD solutions and wind tunnel data
- Low fidelity is dense and cheap to obtain, high fidelity is sparse and expensive
- **Goal:** use low fidelity data to inform high fidelity model (with uncertainties)



Multifidelity Gaussian Process Regression

- Multiple sources of data of increasing fidelity
  - Increasing CFD mesh resolutions
  - Heat flux correlations and 3D CFD solutions
  - CFD solutions and wind tunnel data
- Low fidelity is dense and cheap to obtain, high fidelity is sparse and expensive
- **Goal:** use low fidelity data to inform high fidelity model (with uncertainties)
- Autoregressive (AR1) model [1] linearly combines GP models for increasing fidelity levels

$$f_k(x) = \rho_k f_{k-1}(x) + \delta_k(x)$$

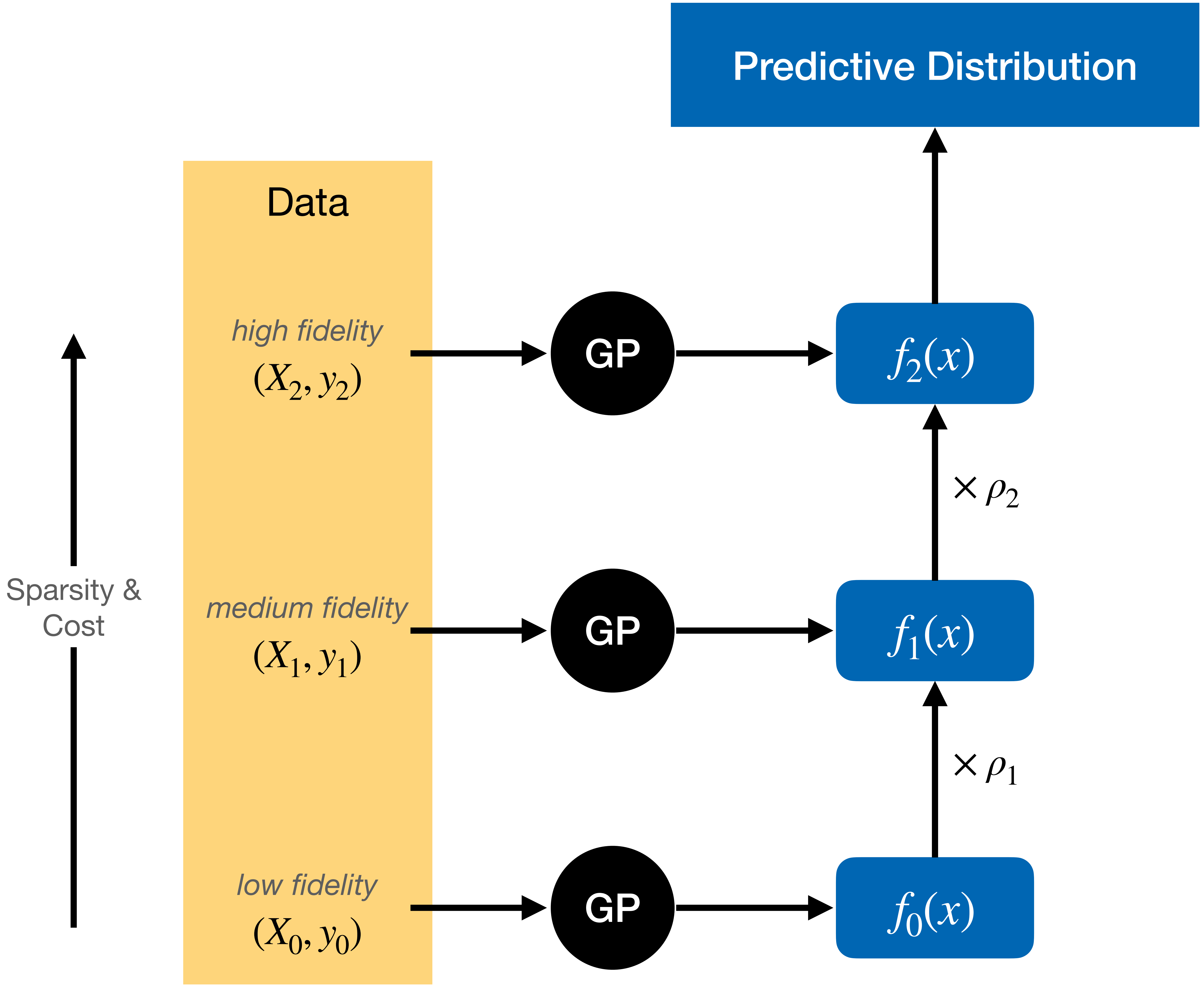


Diagram of the AR1 multifidelity GP model.

[1] Kennedy and O'Hagan. *Biometrika* 87:1-13, 2000.

Multifidelity Gaussian Process Regression

- Multiple sources of data of increasing fidelity
  - Increasing CFD mesh resolutions
  - Heat flux correlations and 3D CFD solutions
  - CFD solutions and wind tunnel data
- Low fidelity is dense and cheap to obtain, high fidelity is sparse and expensive
- **Goal:** use low fidelity data to inform high fidelity model (with uncertainties)
- Autoregressive (AR1) model [1] linearly combines GP models for increasing fidelity levels

$$f_k(x) = \rho_k f_{k-1}(x) + \delta_k(x)$$

- **Requires an obvious hierarchy of fidelity levels!**

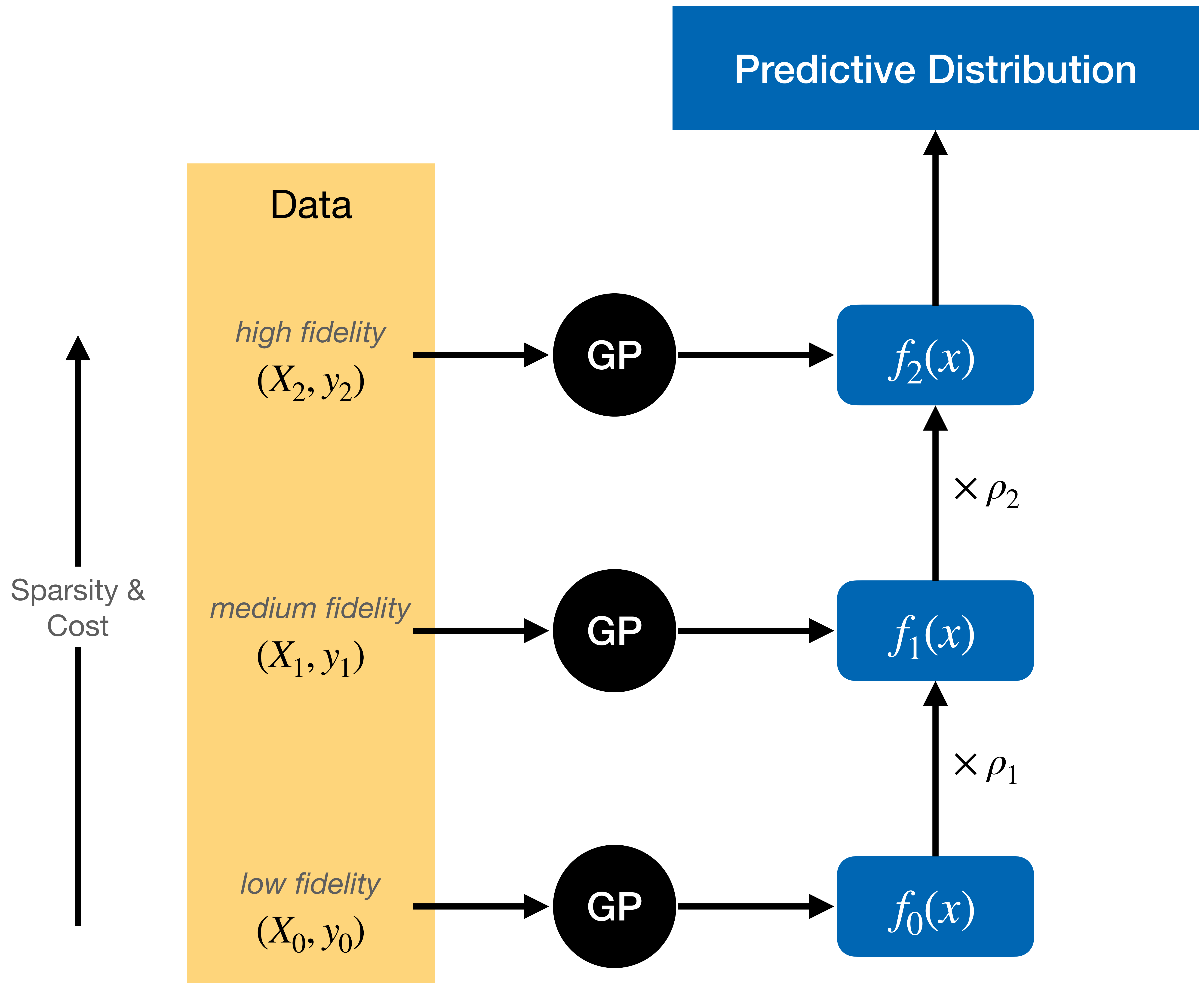


Diagram of the AR1 multifidelity GP model.

[1] Kennedy and O'Hagan. *Biometrika* 87:1-13, 2000.

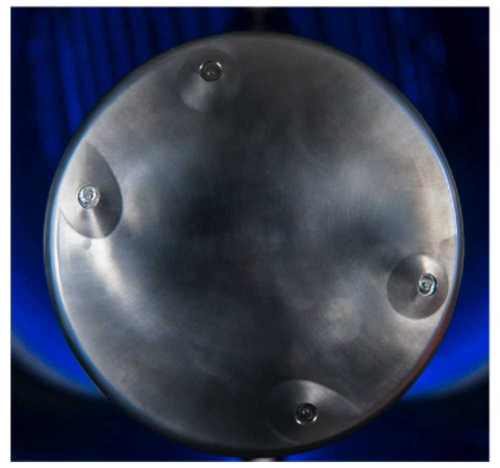
Multihierarchy GP Model

- Real world data typically cannot be organized into hierarchy of fidelity levels with single "truth"

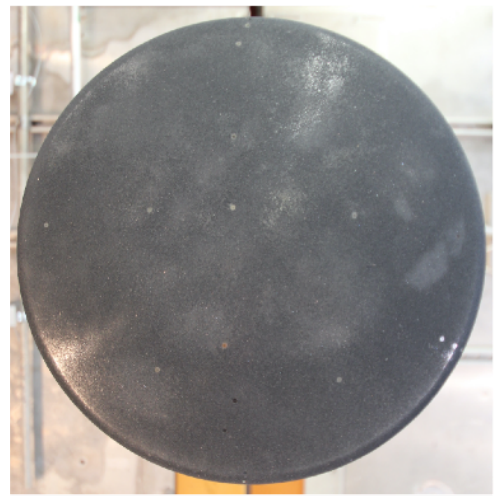
- Easier to categorize "nominal" and "off-nominal" data

Orion heatshield models used in 133-CA test campaign in the National Transonic Facility

Asymmetric, Smooth



Symmetric, Rough



Symmetric, Smooth

Data

+ second effect

(X_2, y_2)

+ first effect

(X_1, y_1)

nominal

(X_0, y_0)

Multihierarchy GP Model

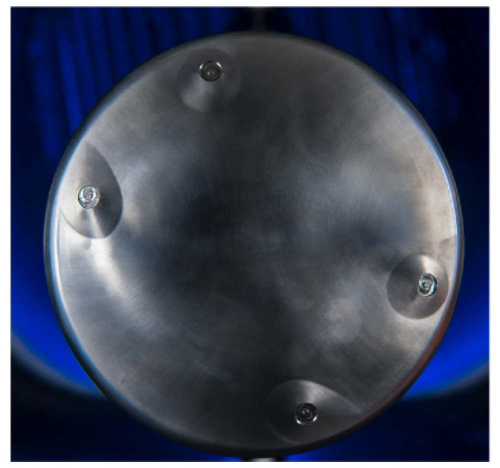
- Real world data typically cannot be organized into hierarchy of fidelity levels with single "truth"
- Easier to categorize "nominal" and "off-nominal" data

Predictive Distribution

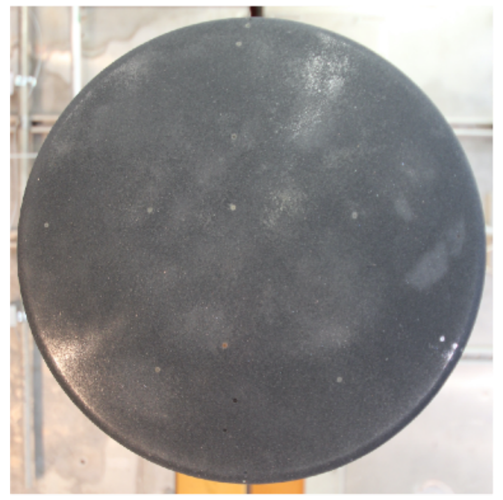
$$f(x) = f_0(x) + w_1 \Delta f_1(x) + w_2 \Delta f_2(x)$$

Orion heatshield models used in 133-CA test campaign in the National Transonic Facility

Asymmetric, Smooth



Symmetric, Rough



Symmetric, Smooth

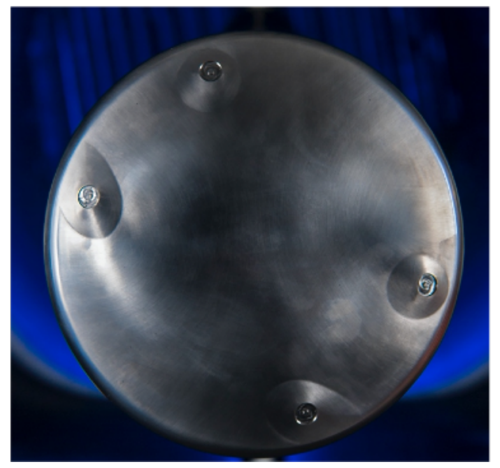
Data
+ second effect (X_2, y_2)
+ first effect (X_1, y_1)
nominal (X_0, y_0)

Multihierarchy GP Model

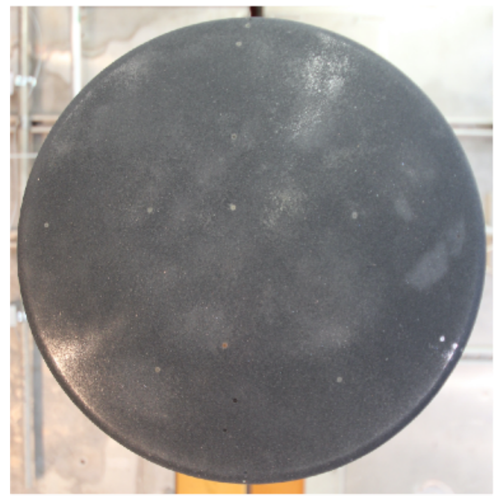
- Real world data typically cannot be organized into hierarchy of fidelity levels with single “truth”
- Easier to categorize “nominal” and “off-nominal” data

Orion heatshield models used in 133-CA test campaign in the National Transonic Facility

Asymmetric, Smooth



Symmetric, Rough



Symmetric, Smooth

Data

+ second effect

(X_2, y_2)

+ first effect

(X_1, y_1)

nominal

(X_0, y_0)

GP

$f_0(x)$

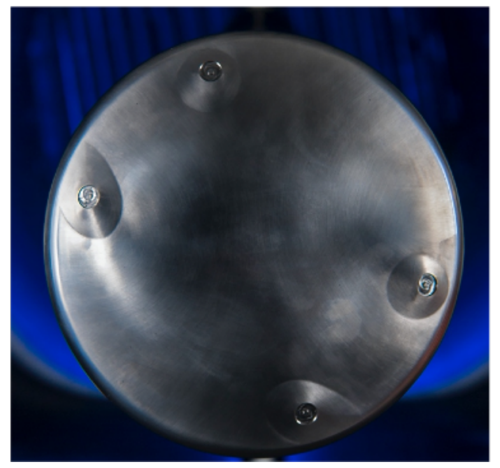
Predictive Distribution

$f(x) = f_0(x) + w_1 \Delta f_1(x) + w_2 \Delta f_2(x)$

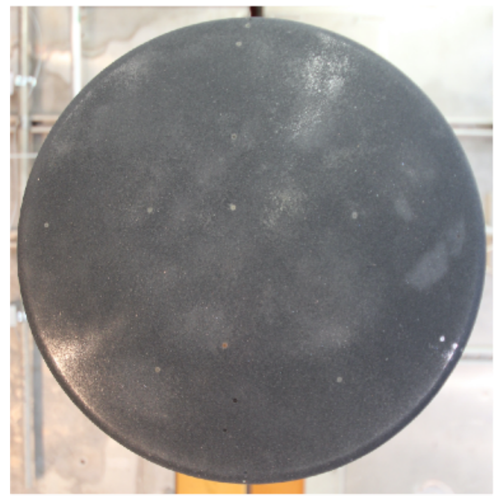
Multihierarchy GP Model

- Real world data typically cannot be organized into hierarchy of fidelity levels with single “truth”
- Easier to categorize “nominal” and “off-nominal” data

Asymmetric, Smooth

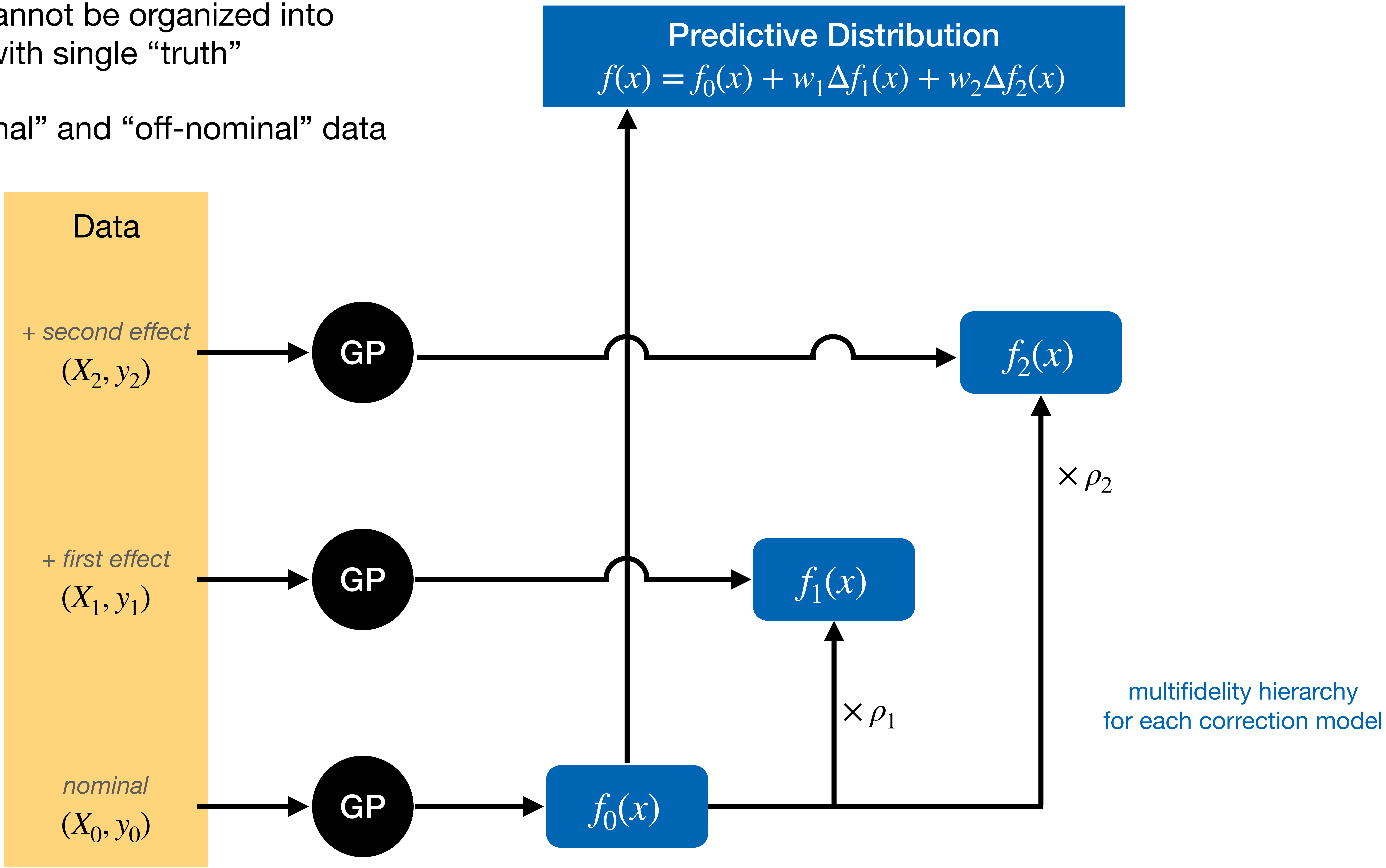


Symmetric, Rough



Symmetric, Smooth

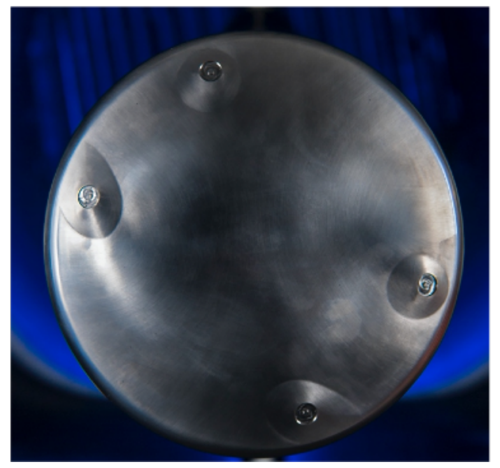
Orion heatshield models used in 133-CA test campaign in the National Transonic Facility



Multihierarchy GP Model

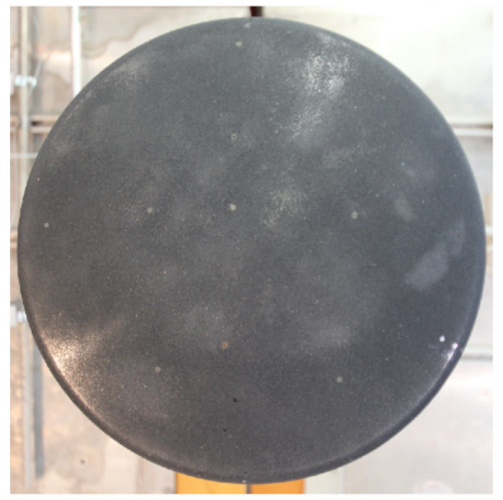
- Real world data typically cannot be organized into hierarchy of fidelity levels with single “truth”
- Easier to categorize “nominal” and “off-nominal” data

Asymmetric, Smooth

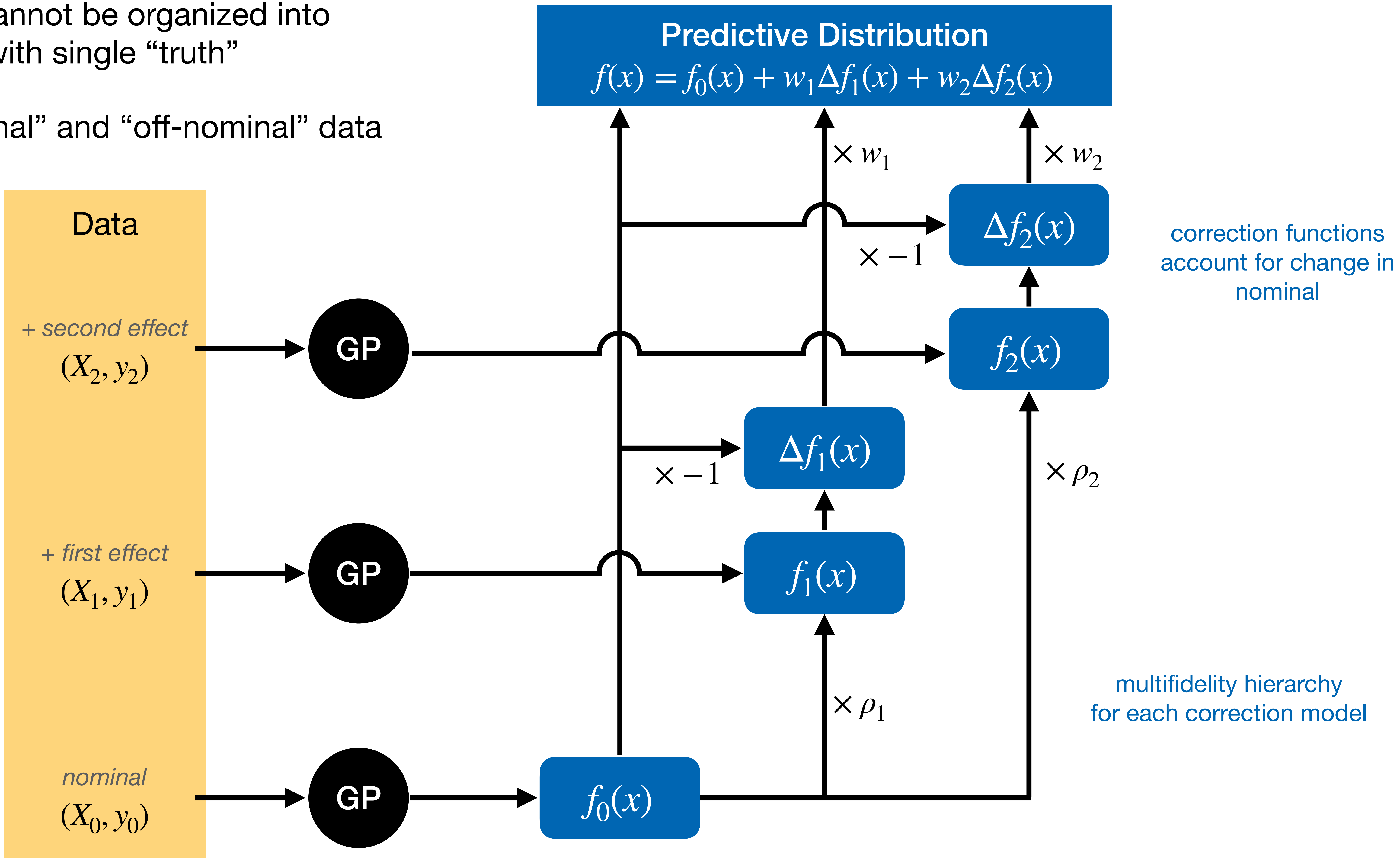


Orion heatshield models used in 133-CA test campaign in the National Transonic Facility

Symmetric, Rough



Symmetric, Smooth

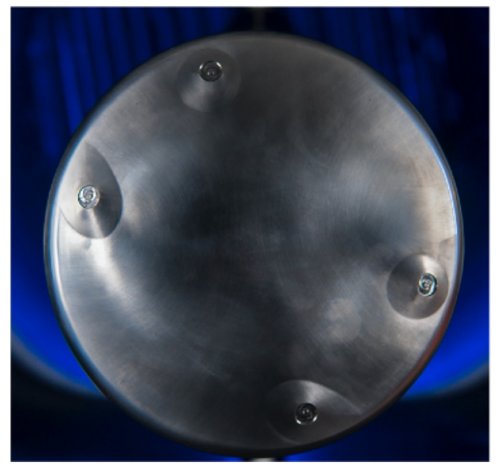


Multihierarchy GP Model

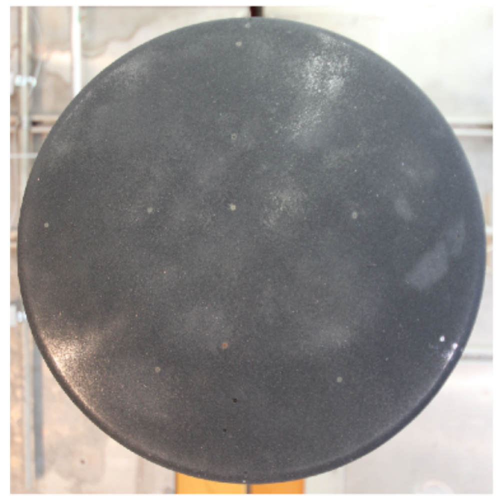
- Real world data typically cannot be organized into hierarchy of fidelity levels with single "truth"
- Easier to categorize "nominal" and "off-nominal" data

Orion heatshield models used in 133-CA test campaign in the National Transonic Facility

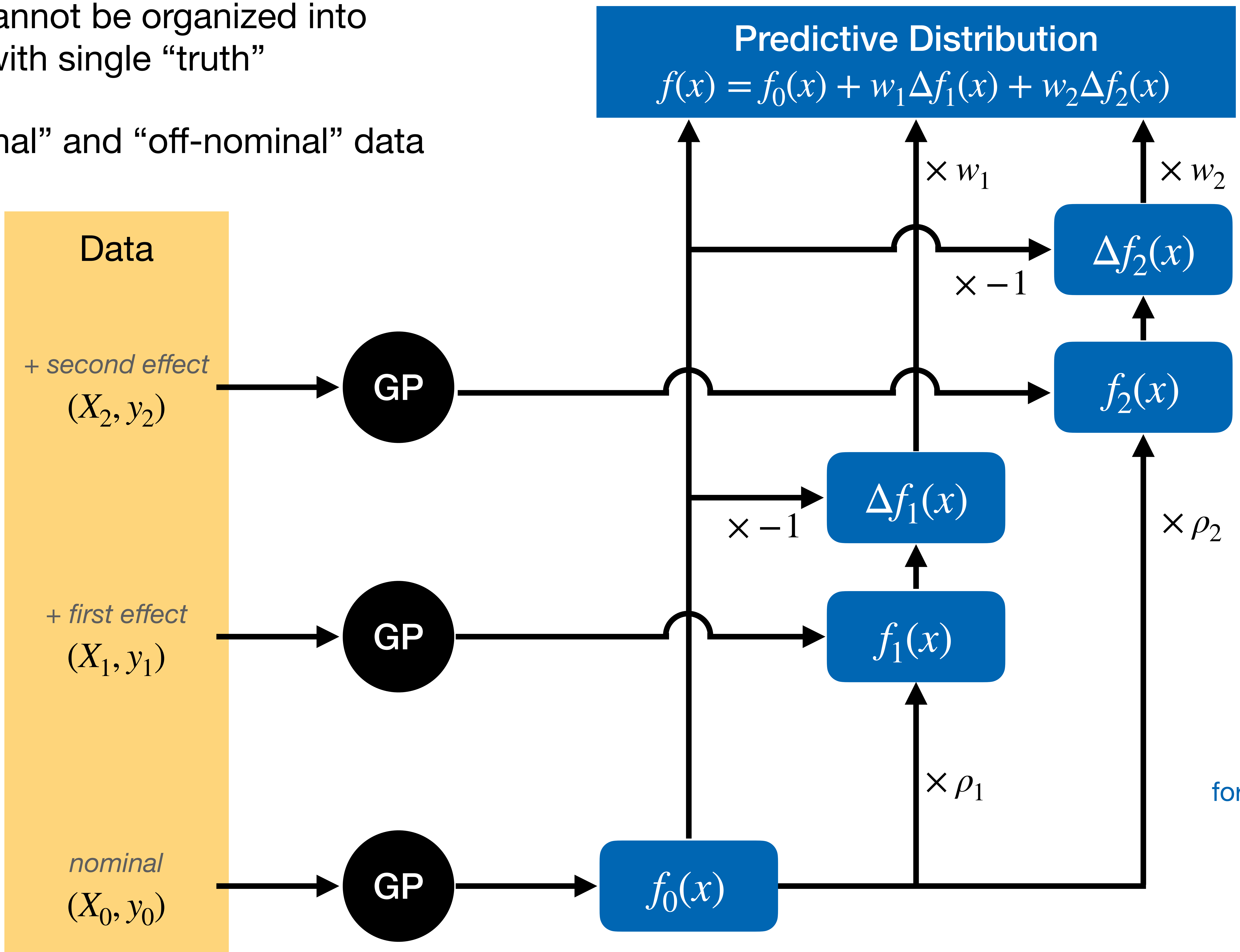
Asymmetric, Smooth



Symmetric, Rough



Symmetric, Smooth



no assumed truth! weight parameters learned from flight data or estimated from previous vehicles

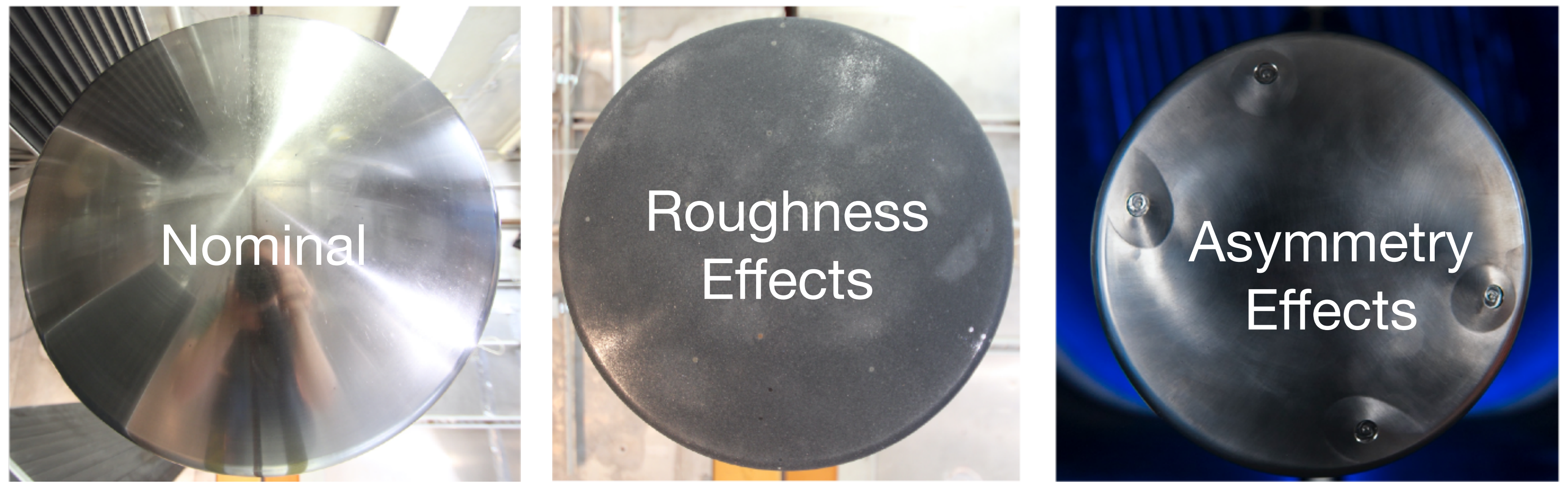
correction functions account for change in nominal

multifidelity hierarchy for each correction model

Example datasets taken from Orion NTF testing

- All reported results based on Orion 133-CA test campaign performed in the National Transonic Facility at NASA LaRC [1]

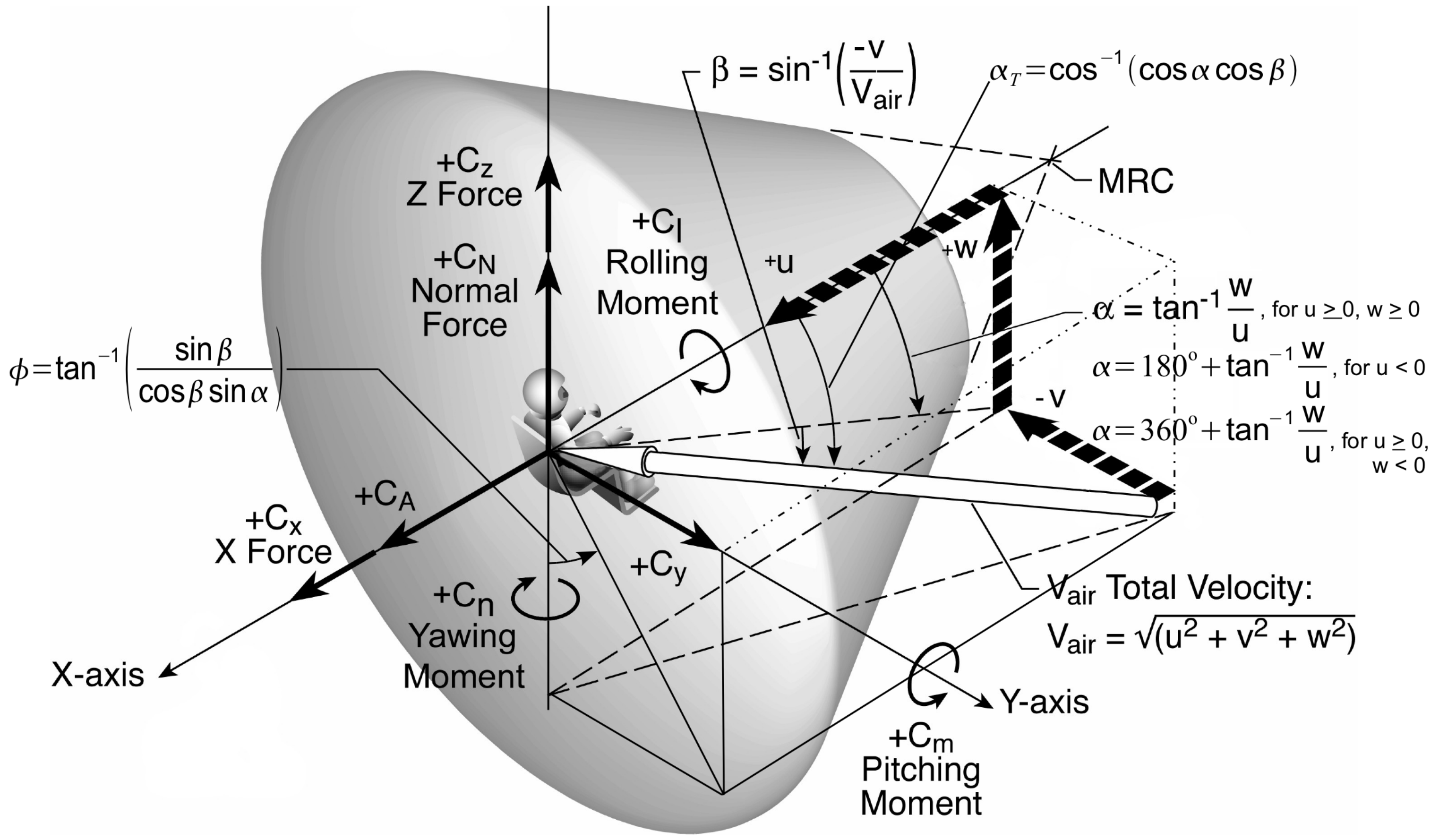
Train



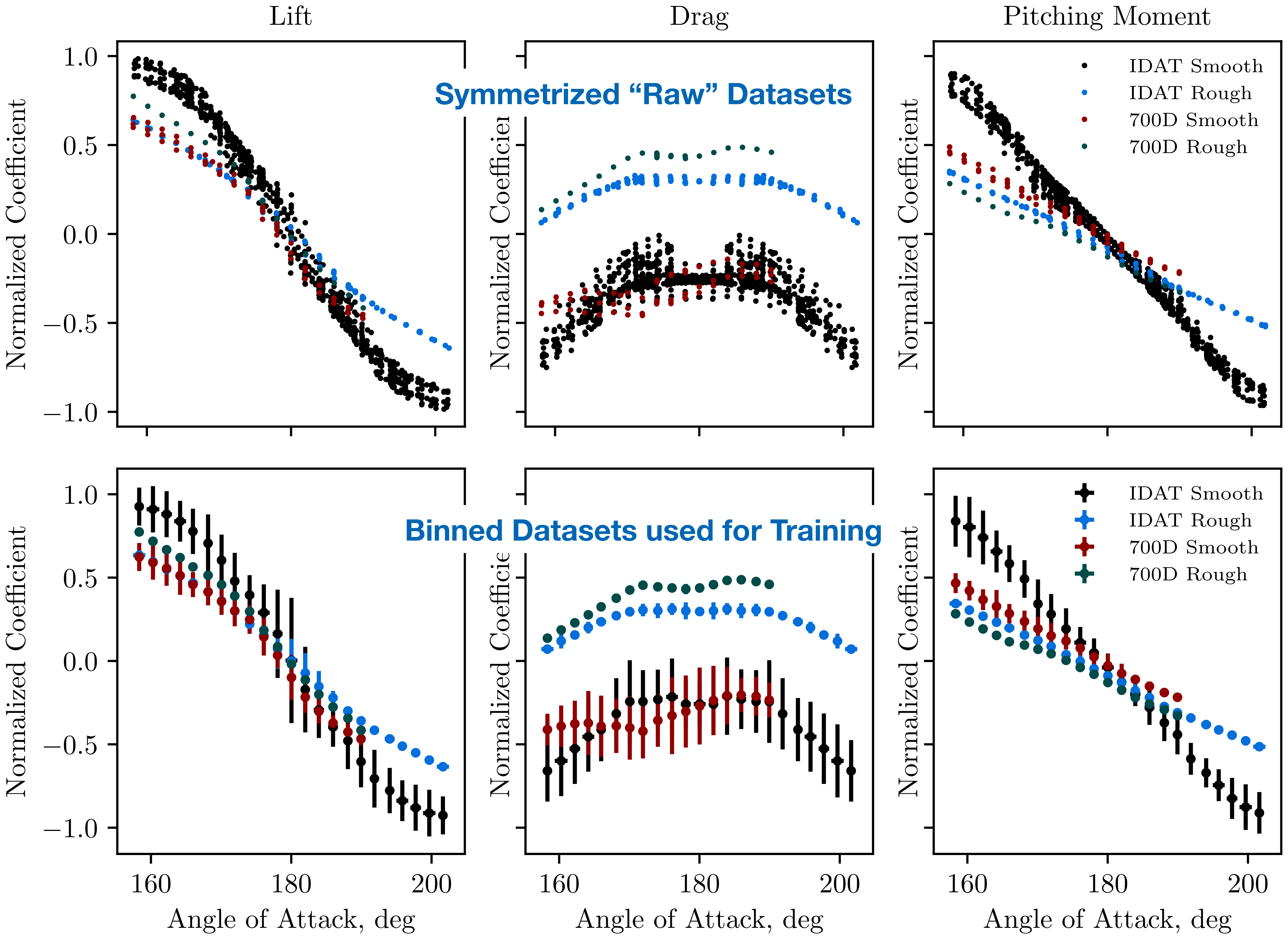
IDAT Smooth **IDAT 240 Grit** **700D Smooth**

700D Fenced + 240 Grit

Test



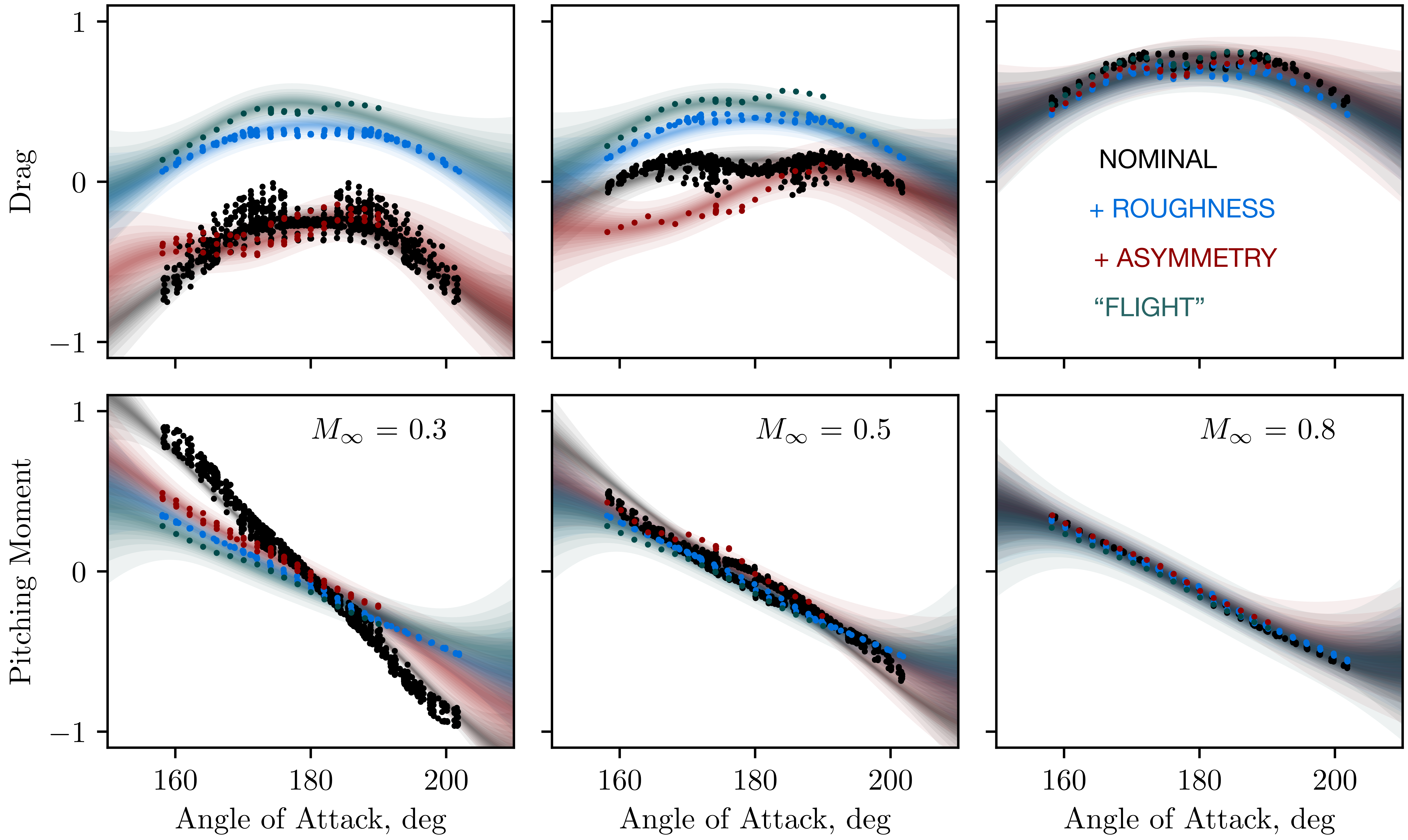
Orion "IDAT" Geometry with coordinates, forces, and moments.



Slices of data around Mach 0.3 and Reynolds 7.5x10⁶.

[1] Brauckmann. *CAP WTT Report EG-CAP-12-65*, NASA LaRC, 2022 (under preparation).

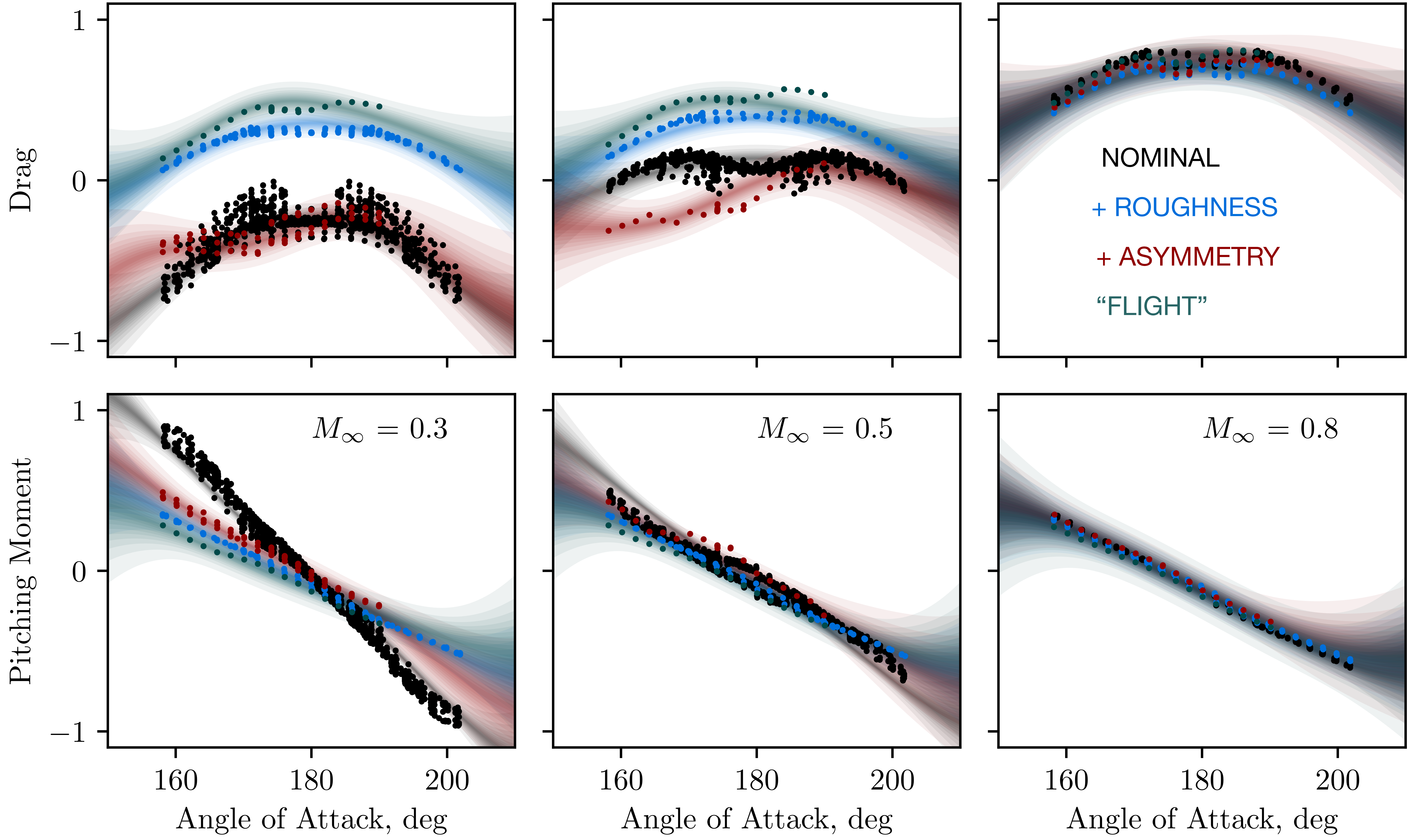
Model distributions compared to data



Aerodynamic coefficient function distributions at 3 Mach numbers and Reynolds 7.5x10[6].

Model distributions compared to data

excellent agreement
with “noisy” data

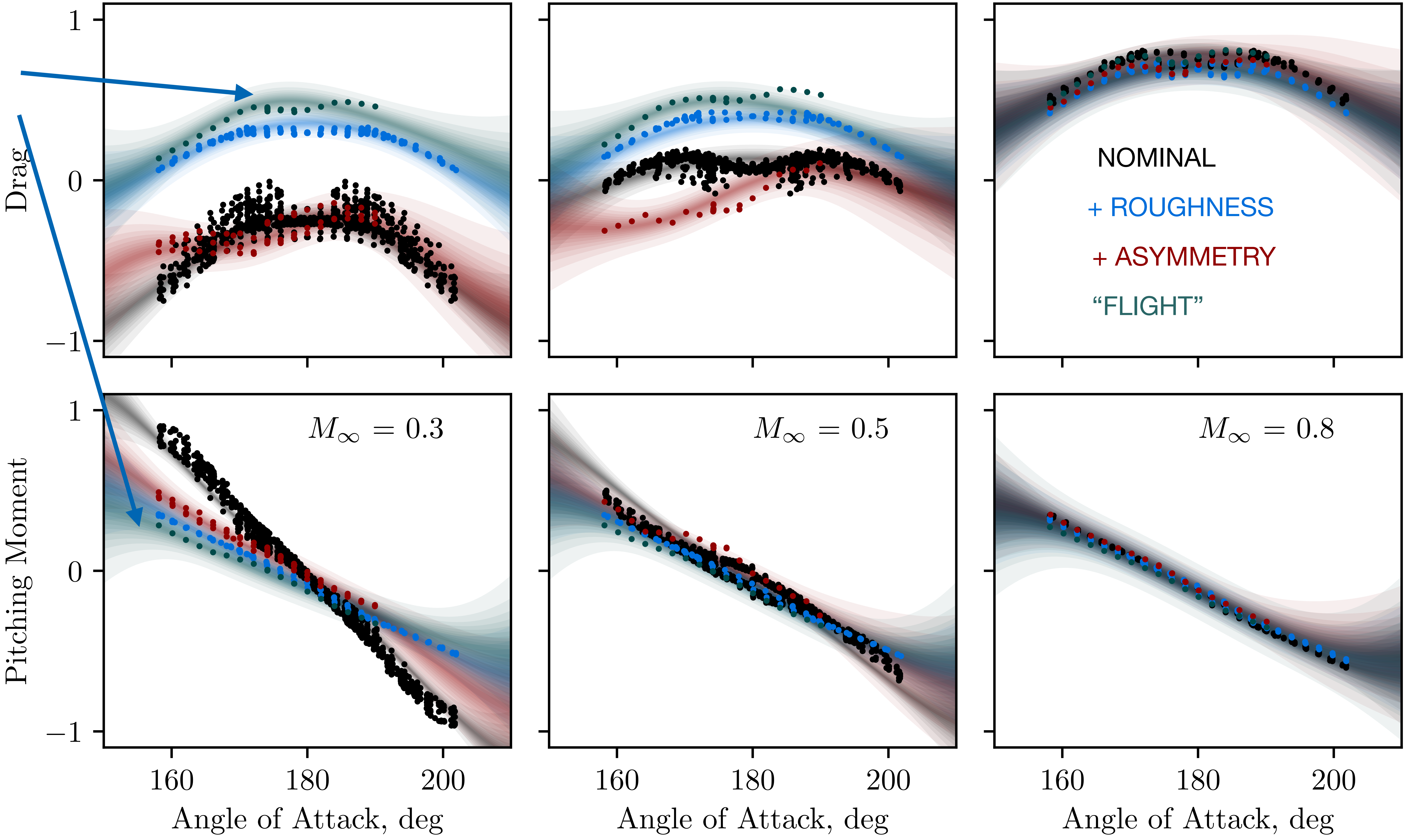


Aerodynamic coefficient function distributions at 3 Mach numbers and Reynolds 7.5x10⁶.

Model distributions compared to data

simulated “flight" data is reasonably reproduced, only correction weights

excellent agreement with “noisy” data

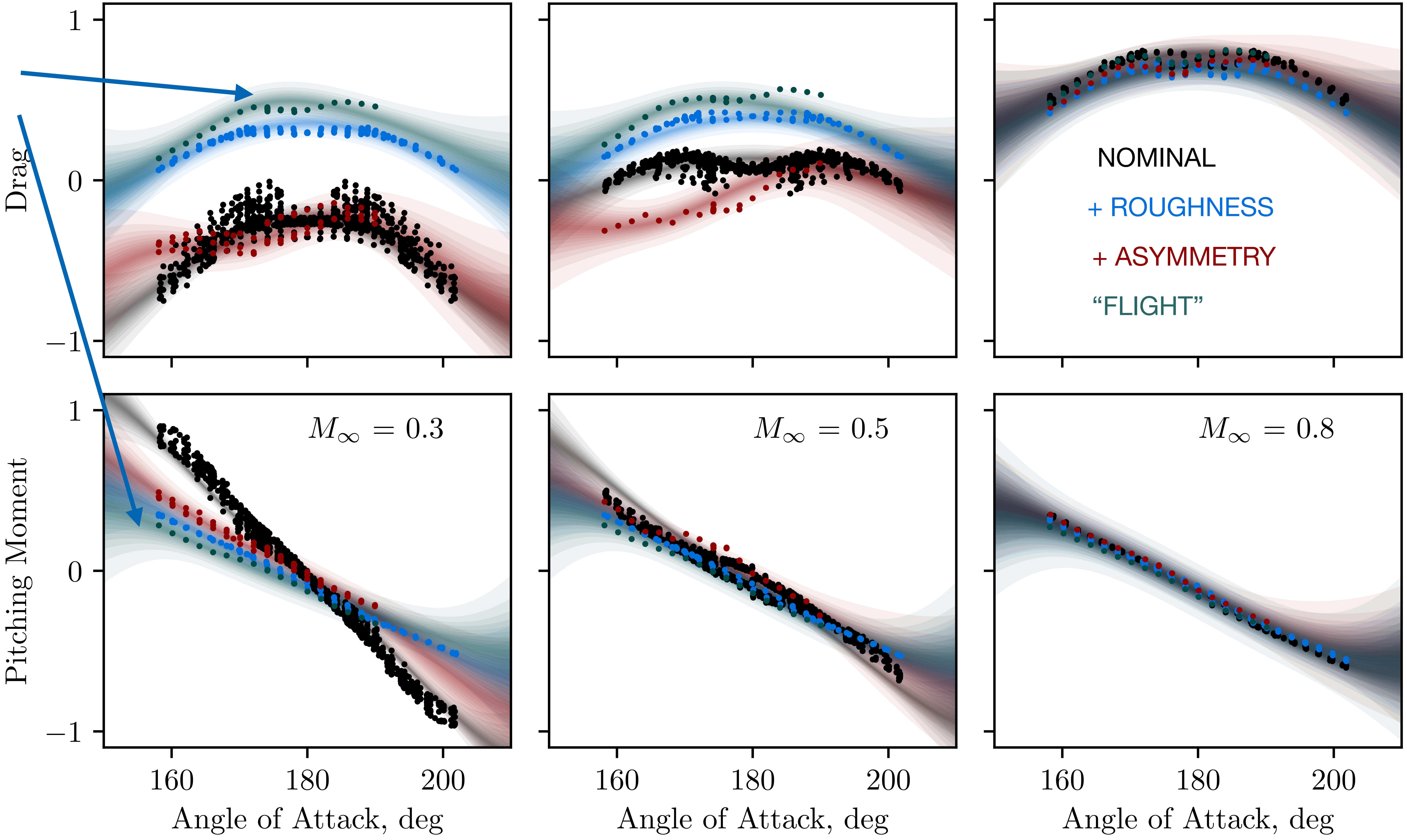


Aerodynamic coefficient function distributions at 3 Mach numbers and Reynolds 7.5x10⁶.

Model distributions compared to data

simulated “flight" data is reasonably reproduced, only correction weights

excellent agreement with “noisy” data

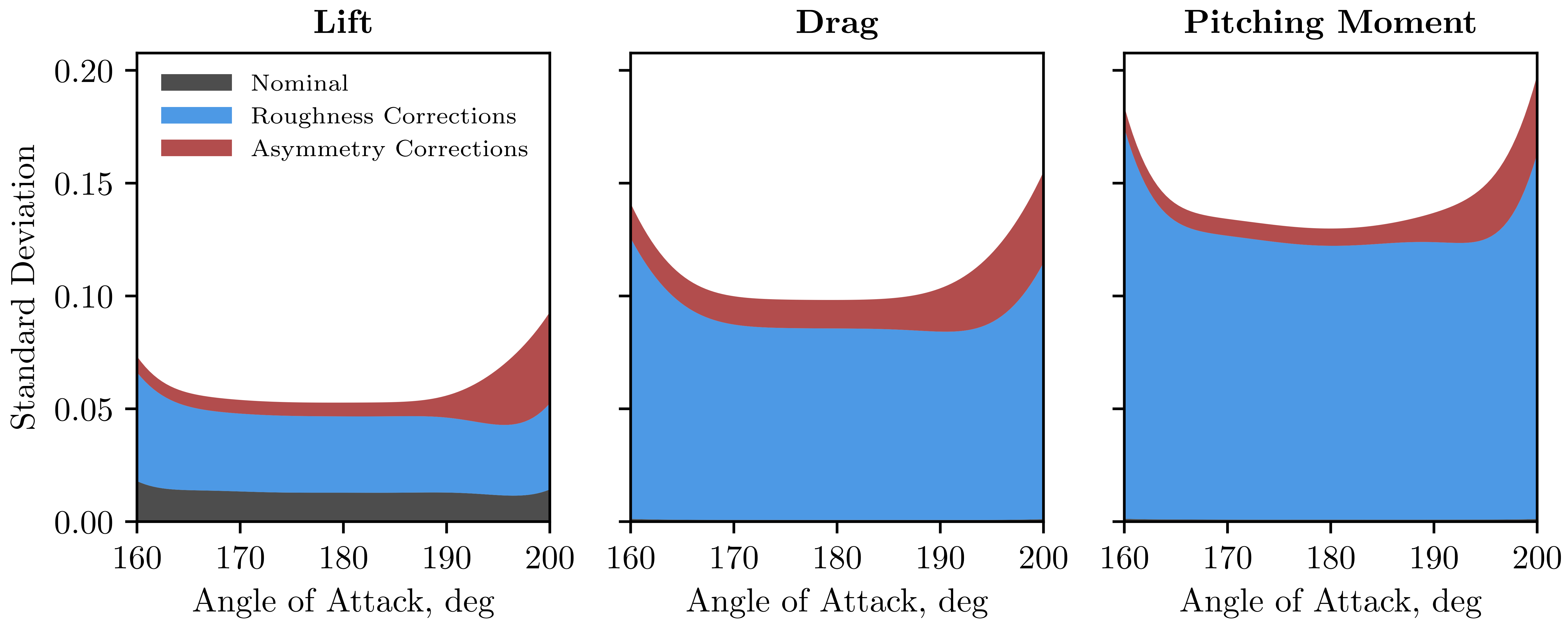


“out-of-distribution” uncertainty in regions of no data

Aerodynamic coefficient function distributions at 3 Mach numbers and Reynolds 7.5x10^6.

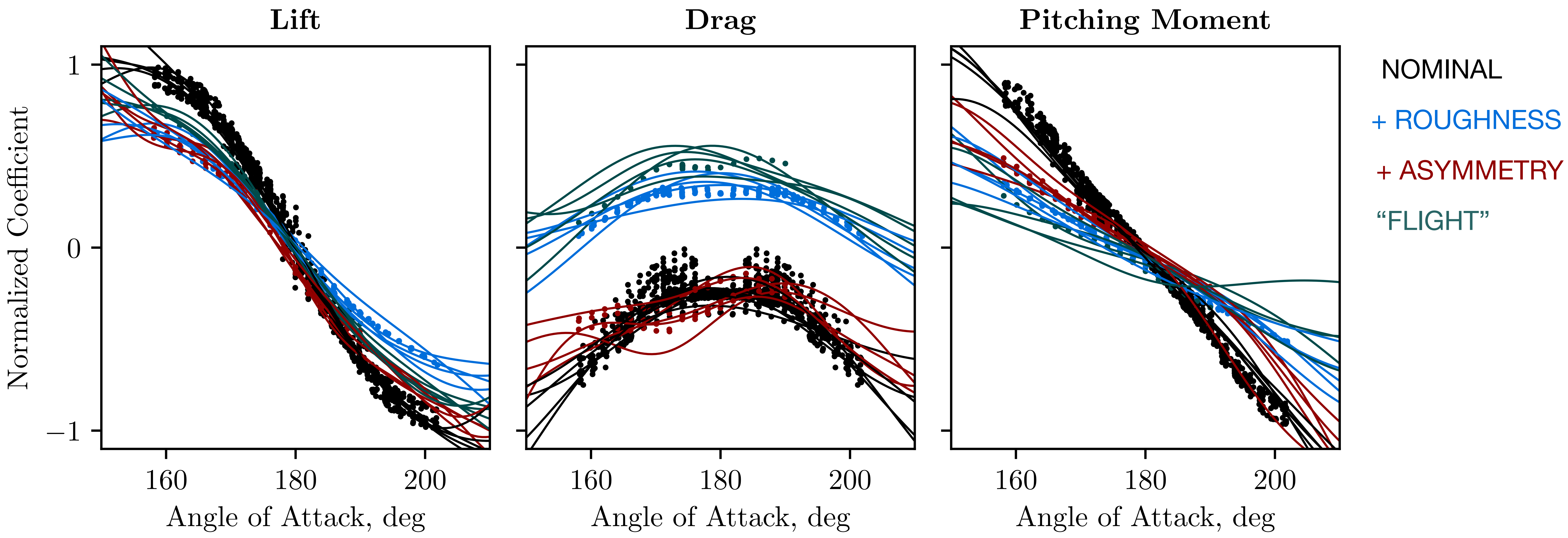
Understanding uncertainty sources

- Can attribute uncertainty from different sources to overall uncertainty in predictive distribution
- Roughness corrections account for significant uncertainty in drag and pitching moment
- Only uncertainties present in data, does not include those associated with factors not considered here



Uncertainty buildup from different sources at Mach 0.5 and Reynolds 7.5×10^6.

Function sampling from model distributions



Aerodynamic coefficient function samples at Mach 0.5 and Reynolds 7.5x10[6].

- State-of-the-practice uses random dispersion offsets from the nominal for Monte Carlo trajectory simulations
- Proposed approach allows for function sampling that is more consistent with underlying conditional distribution
- **Each function is a plausible explanation of the data, reproduces conditional distribution in aggregate**

Probability distributions for derived quantities

Model distributions can used to obtain conditional distributions on derived quantities of interest

Probability distributions for derived quantities

Model distributions can used to obtain conditional distributions on derived quantities of interest

- **Example:** trim angle of attack found using *Bayes' Theorem*

$$p(\alpha = \alpha_{\text{trim}}) = p(\alpha \,|\, C_{\text{m,cg}} = 0) \propto p(C_{\text{m,cg}} = 0 \,|\, \alpha)\; p(\alpha)$$

Probability distributions for derived quantities

Model distributions can used to obtain conditional distributions on derived quantities of interest

- **Example:** trim angle of attack found using *Bayes' Theorem*

$$p(\alpha = \alpha_{\text{trim}}) = p(\alpha \,|\, C_{\text{m,cg}} = 0) \propto \boxed{p(C_{\text{m,cg}} = 0 \,|\, \alpha)}\, p(\alpha)$$

- Probability of pitching moment being zero for given alpha is directly obtained from model distribution

Probability distributions for derived quantities

Model distributions can used to obtain conditional distributions on derived quantities of interest

- **Example:** trim angle of attack found using *Bayes' Theorem*

$$p(\alpha = \alpha_{\text{trim}}) = p(\alpha \,|\, C_{\text{m,cg}} = 0) \propto p(C_{\text{m,cg}} = 0 \,|\, \alpha)\ \boxed{p(\alpha)}$$

- Probability of pitching moment being zero for given alpha is directly obtained from model distribution

- Using an "uninformative" prior for the probability of angle of attack yields the desired result

Probability distributions for derived quantities

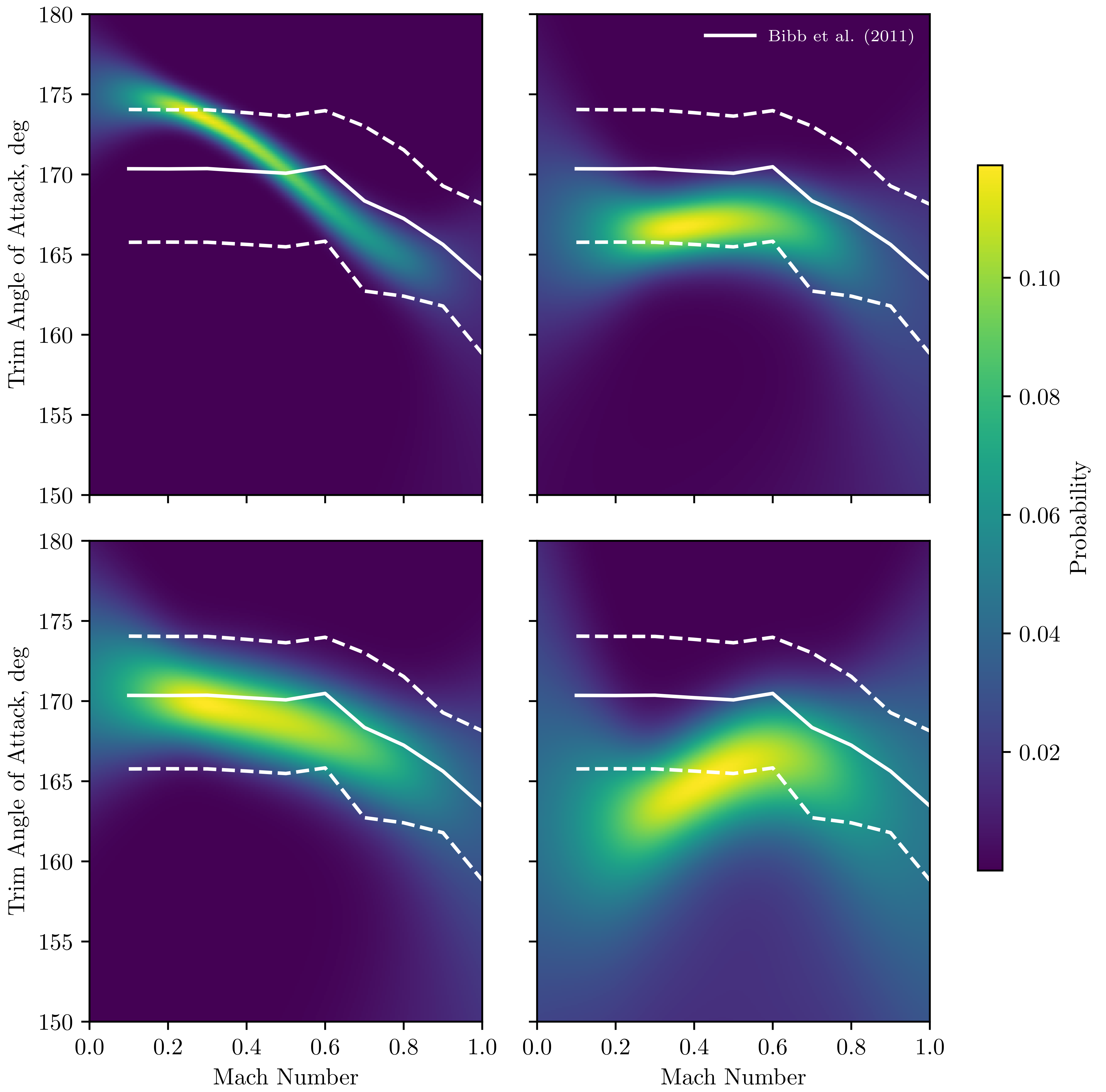
Model distributions can used to obtain conditional distributions on derived quantities of interest

- **Example:** trim angle of attack found using *Bayes' Theorem*

$$p(\alpha = \alpha_{\text{trim}}) = p(\alpha \,|\, C_{\text{m,cg}} = 0) \propto p(C_{\text{m,cg}} = 0 \,|\, \alpha)\; p(\alpha)$$

- Probability of pitching moment being zero for given alpha is directly obtained from model distribution

- Using an "uninformative" prior for the probability of angle of attack yields the desired result



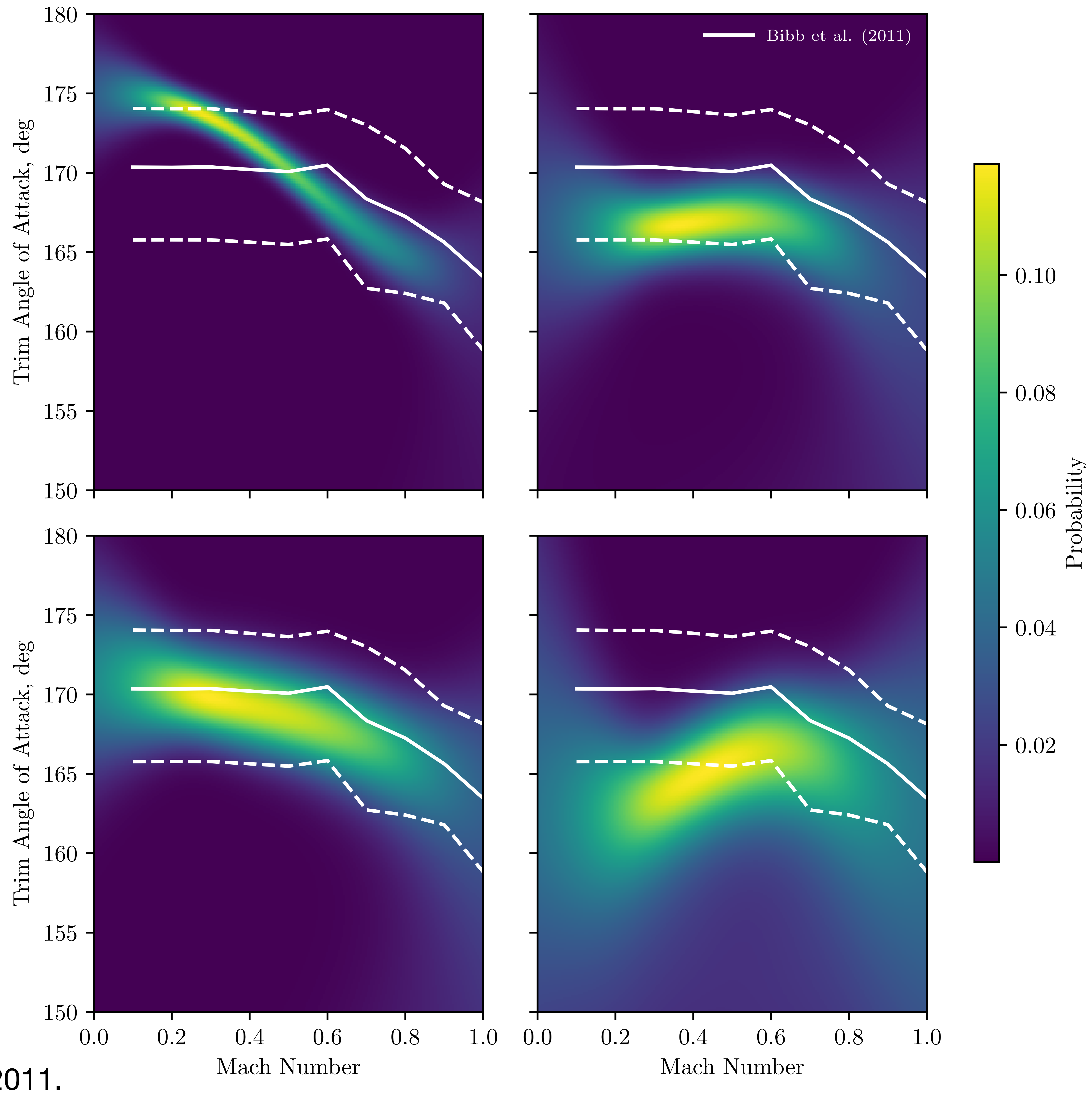
Probability distributions for derived quantities

Model distributions can used to obtain conditional distributions on derived quantities of interest

- **Example:** trim angle of attack found using *Bayes' Theorem*

$$p(\alpha = \alpha_{\text{trim}}) = p(\alpha \,|\, C_{\text{m,cg}} = 0) \propto p(C_{\text{m,cg}} = 0 \,|\, \alpha)\; p(\alpha)$$

- Probability of pitching moment being zero for given alpha is directly obtained from model distribution
- Using an "uninformative" prior for the probability of angle of attack yields the desired result
- Orion v0.60 DB nominal and 100% CI bounds [1] provided for comparison



[1] Bibb, Walker, Brauckmann, Robinson. *29th AIAA Applied Aero. Conf.,* No. 2011-3507, 2011.

Concluding Remarks

Concluding Remarks

- Presented multihierarchy Gaussian Process models for probabilistic aerodatabases

Concluding Remarks

- Presented multihierarchy Gaussian Process models for probabilistic aerodatabases
- Increased flexibility for aerodatabase designers to incorporate expert judgement with a consistent treatment of uncertainty

Concluding Remarks

- Presented multihierarchy Gaussian Process models for probabilistic aerodatabases
- Increased flexibility for aerodatabase designers to incorporate expert judgement with a consistent treatment of uncertainty
- Future improvements:
    - More complex fidelity structures, incorporating more corrections
    - Improve model accuracy by incorporating nonlinear basis functions into correction weights
    - Incorporate nonlinear autoregressive GPs

Concluding Remarks

- Presented multihierarchy Gaussian Process models for probabilistic aerodatabases
- Increased flexibility for aerodatabase designers to incorporate expert judgement with a consistent treatment of uncertainty
- Future improvements:
    - More complex fidelity structures, incorporating more corrections
    - Improve model accuracy by incorporating nonlinear basis functions into correction weights
    - Incorporate nonlinear autoregressive GPs
- When no flight data is available, could attempt to use previously flown vehicles as surrogate for correction weights

Multihierarchy Gaussian Process Models for Probabilistic Aerodynamic Databases using Uncertain Nominal and Off-Nominal Configuration Data

2023 AIAA SciTech Forum
January 23-27, 2023 - National Harbor, Maryland

James B. Scoggins, Thomas J. Wignall, Tenavi Nakamura-Zimmerer, Karen Bibb
NASA Aerofusion-MLUQ Early Career Initiative, NASA Langley Research Center