Structural Architectures for Self-Erecting Lunar Towers

Jacob G. Daye and Andrew J. Lee
North Carolina State University, Raleigh, NC

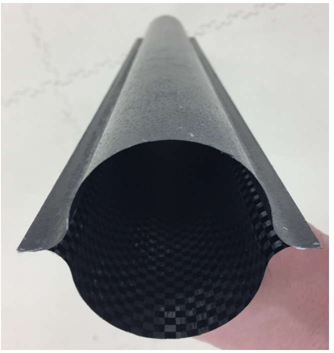
Juan M. “Johnny” Fernandez
NASA Langley Research Center, Hampton, VA

Background and Objectives

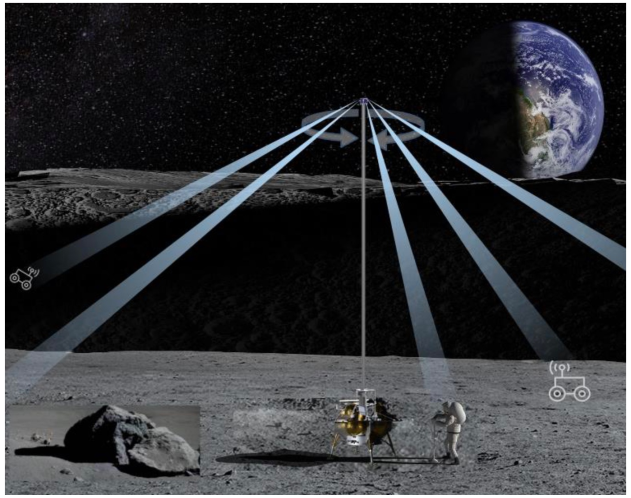
COROTUB – Corrugated Rollable Tubular Boom
CTM – Collapsible Tubular Mast
SELTI – Self-Erectable Lunar Tower for Instruments

- Develop framework for determining allowable-mass support limit for extraterrestrial self-deploying towers
- Parametrically optimize structural performance (deflection and tip mass)
- Compare allowable tip mass between cable-stayed and reference designs with COROTUB and CTM booms

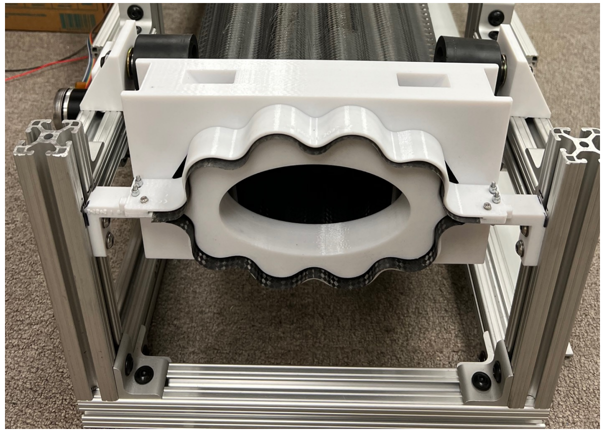
COROTUB deployment mechanism



CTM cross-section

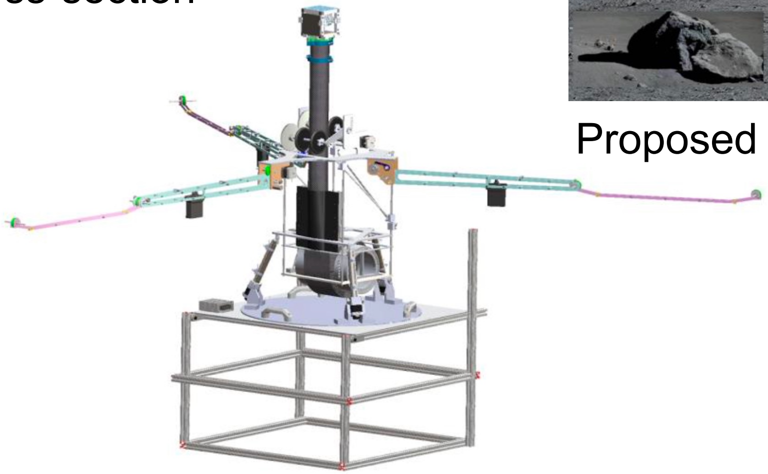


Proposed SELTI applications [1]



COROTUB plug shape

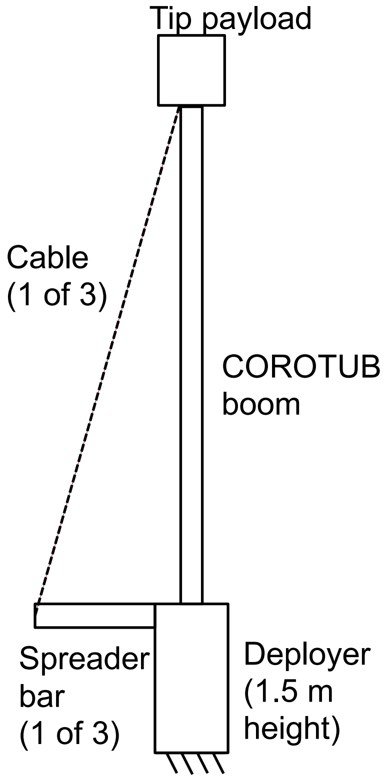
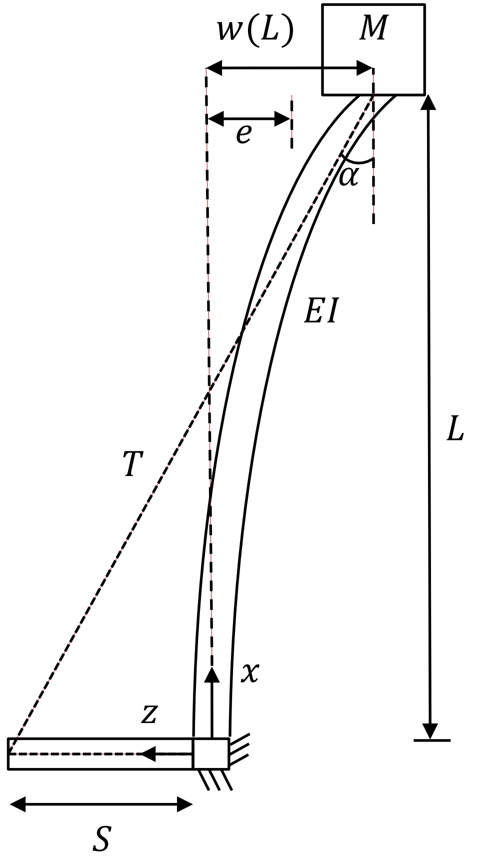
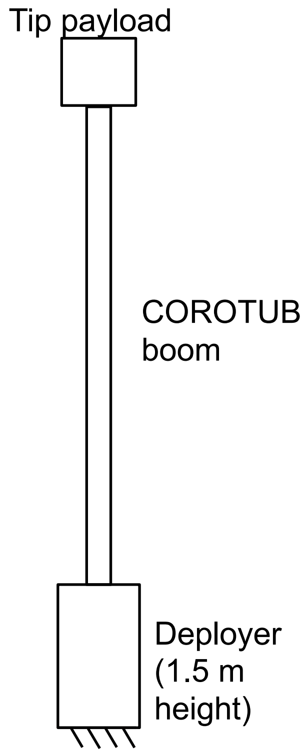
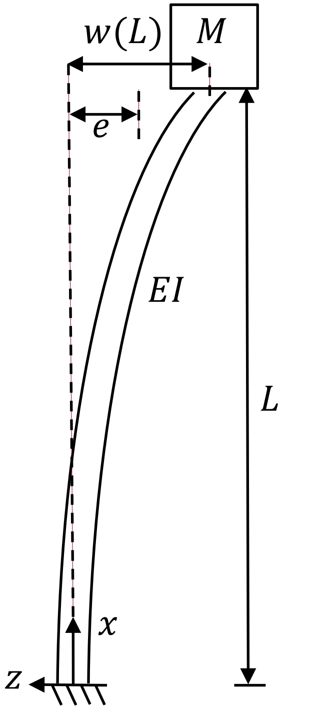
COROTUB cross-section developing from spool



Deployable rigging system [1] – primary tower design for this analysis.

[1] Miller et al., 2023

SELTI Structural Designs

<table>
<tr><th colspan="2">Design A</th><th colspan="2">Design B</th></tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Primary Tower Design</td>
<td>Cable-stayed and cantilever beam-column with initial eccentricity and tip mass</td>
<td>Reference Tower Design</td>
<td>Cantilever beam-column with initial eccentricity and tip mass</td>
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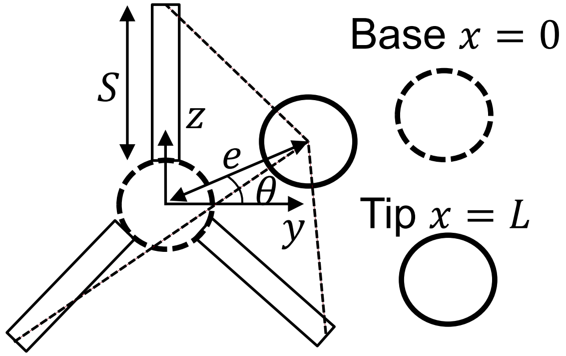
Lateral deflection described by polar coordinates:

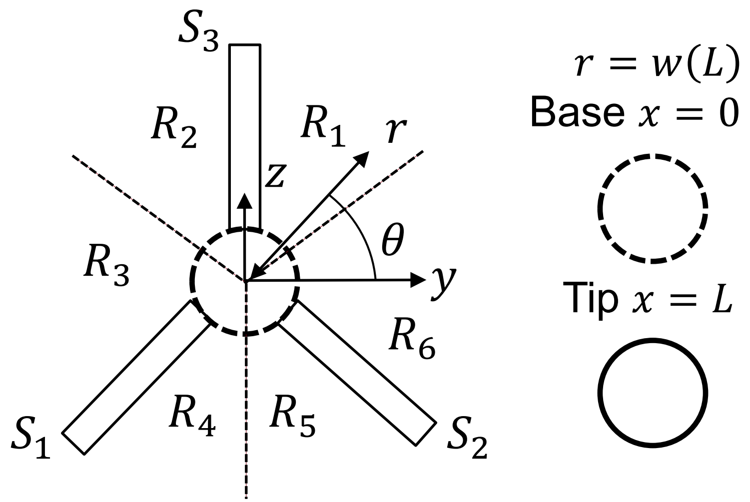
$$r = w(L)$$
$$y = r * \sin\theta$$
$$z = r * \cos\theta$$



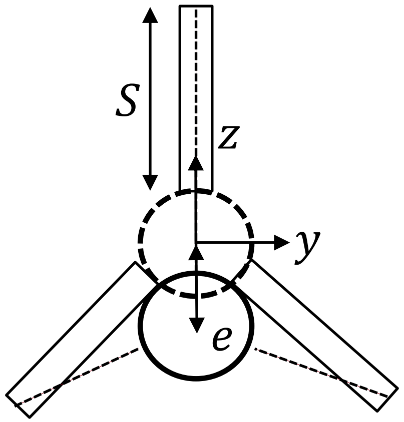
Top-down projection of tip-mass deflection for design A

General Deflection Definition – Radial Symmetry

- For the general definition, two cable tensions T_1 and T_2 contribute towards both corrective and compressive forces
  - Third cable offers no corrective force, so not considered (zero tension).
  - T_1 refers to the 'long' cable and T_2 is the 'short' cable
- Six regions of radial symmetry arise from three-bar design
  - R_1 considered for analysis. Can be extended to general deflection by symmetry.



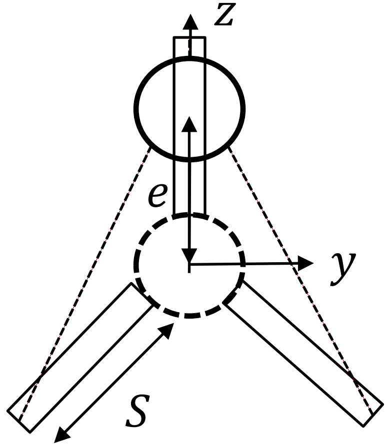
Region boundaries described by two limiting cases:



$\theta = 30°, 150°, 270°$

Case 1 – Deflection is directly **opposite** one spreader bar:

$$T_1 \neq 0, T_2 = 0$$



$\theta = 90°, 210°, 330°$

Case 2 – Deflection is directly **along** one spreader bar:

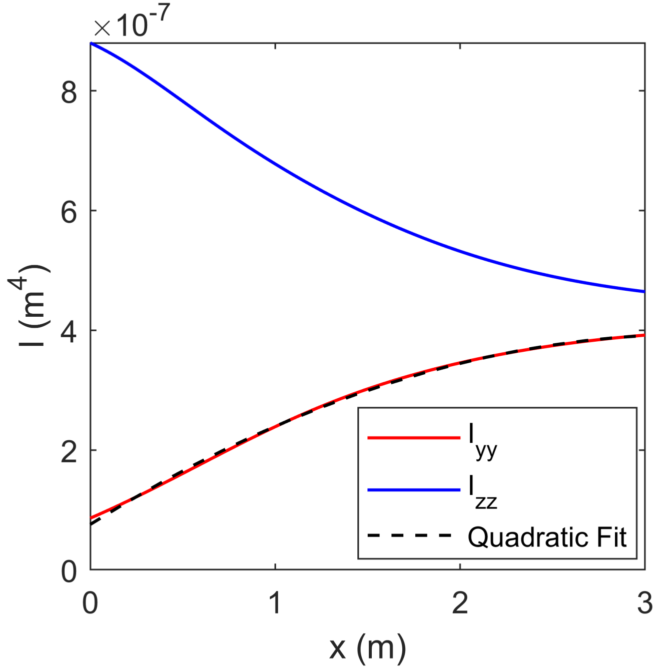
$$T_1 = T_2$$

Regions of radial symmetry and corresponding cable assignments

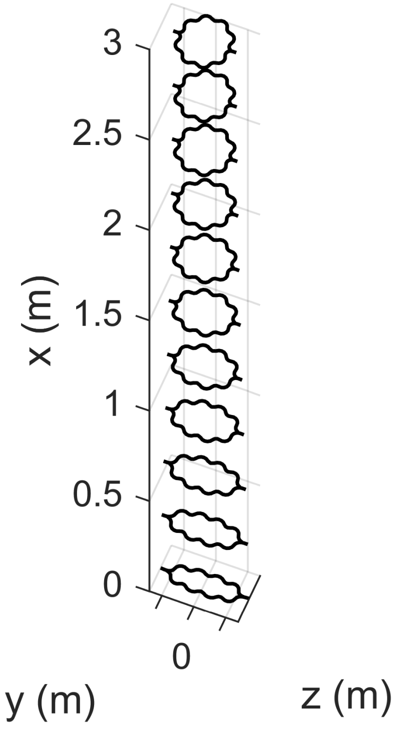
Region	Long cable, T_1	Short Cable, T_2	θ range
R_1	S_1	S_2	$30° < \theta < 90°$
R_2	S_2	S_1	$90° < \theta < 150°$
R_3	S_2	S_3	$150° < \theta < 210°$
R_4	S_3	S_2	$210° < \theta < 270°$
R_5	S_3	S_1	$270° < \theta < 330°$
R_6	S_1	S_1	$330° < \theta < 30°$

COROTUB Profile and Properties

- Plug shape enforces smaller cross section than nominal shape; second moment of area, $I \rightarrow I(x)$
  - Cross-section assumed constant beyond transition length, L_T @ $x = 3\ m$
- Quadratic fit of I_{yy} chosen as conservative analytical estimate for deflection and tip mass model



Second moment of area distribution for transition region of COROTUB



COROTUB cross-section shape from plug (x=0) to nominal shape (x=3)

Property	Symbol	Value(s)	Units
Boom Length	L	5 to 30	m
Boom Deflection Limit	w_{lim}	0.03	None
Boom Max Eccentricity	e_{max}	0.0275	None
Boom Linear Mass Density	ρ	0.15	kg/m
Boom Elastic Modulus	E	80.7	GPa
Boom Deflection Direction	θ	30 to 90	°
Normalized Spreader Bar Length	$\hat{S}$	0 to 0.2	None

$I_{yy} < I_{zz}$; A conservative estimate of M_A uses a quadratic fit of I_{yy}, therefore:

$$I(x) = \begin{cases} -2.937 \times 10^{-8} x^2 + 1.932 \times 10^{-7} x + 7.572 \times 10^{-8} \mathrm{m}^4 & 0 \leq x \leq 3\,\mathrm{m} \\ 3.918 \times 10^{-7}\,\mathrm{m}^4 & x \geq 3\,\mathrm{m} \end{cases}$$

Development of Analytical Model

Compute total compressive cable tension T_x and corrective cable tension T_h

Estimate critical buckling load with Rayleigh-Ritz method

Compute corrective deflection with beam equations

T_x, T_h

P_{cr}

$w_c(L, T_h)$

Total tip deflection:

$$w(L) = \frac{-e}{1 - \frac{1}{P_{cr}}\left(T_x + Mg + \frac{3}{10}\rho L\right)} + w_c(L)$$

$$-e\left(\frac{1}{1 - \frac{P}{P_{cr}}}\right)$$

Initial eccentricity amplified by load factor [2]

$$P = T_x + Mg + \frac{3}{10}\rho L$$

Total Compressive Load

Solve for tip mass:

$$M_A = \frac{P_{cr}}{g}\left(\frac{e}{w(L) - w_c(L)} + 1\right) - \frac{T_x}{g} - \frac{3}{10}\rho L$$

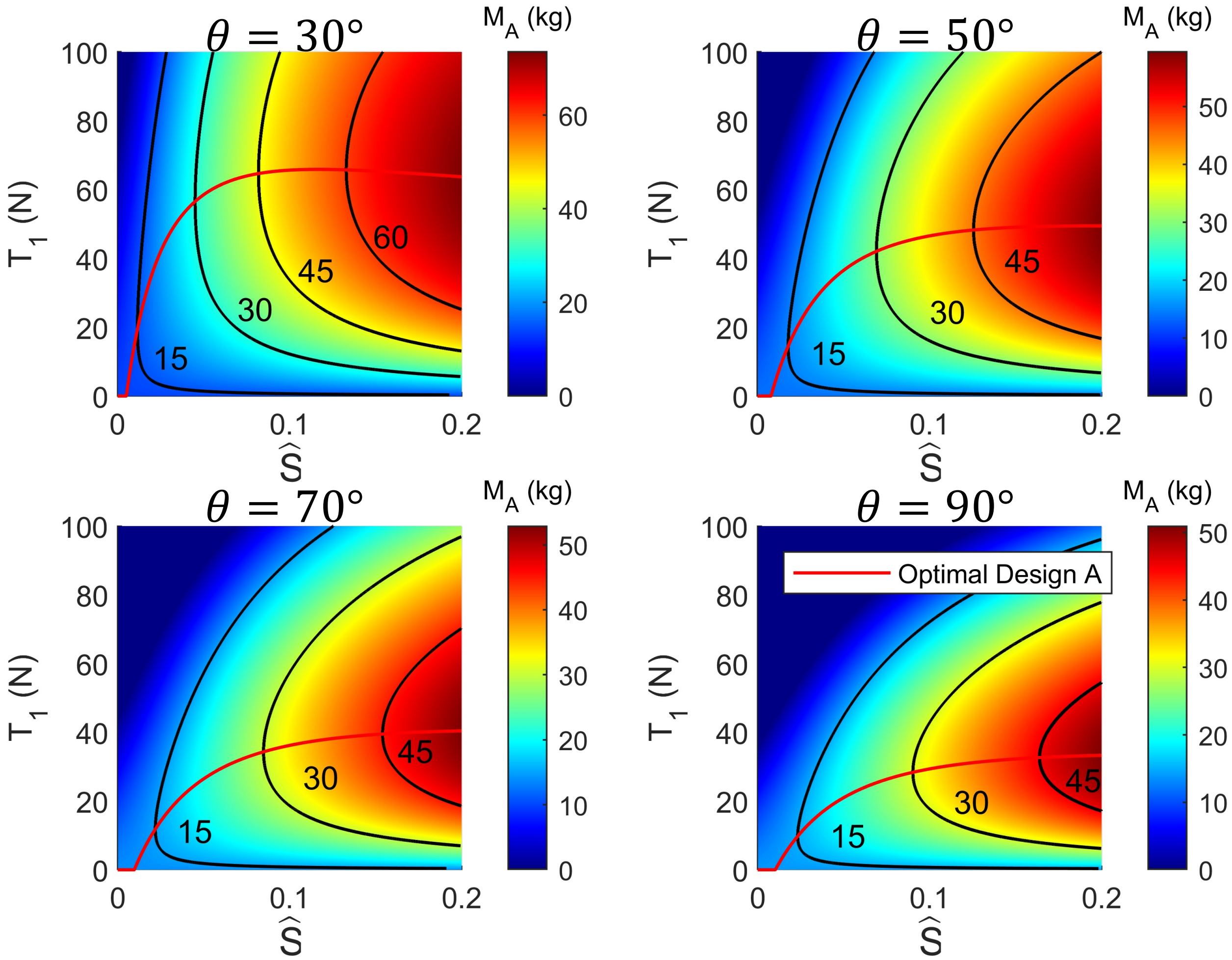
Taking $T_x = T_h = 0$ gives design B:

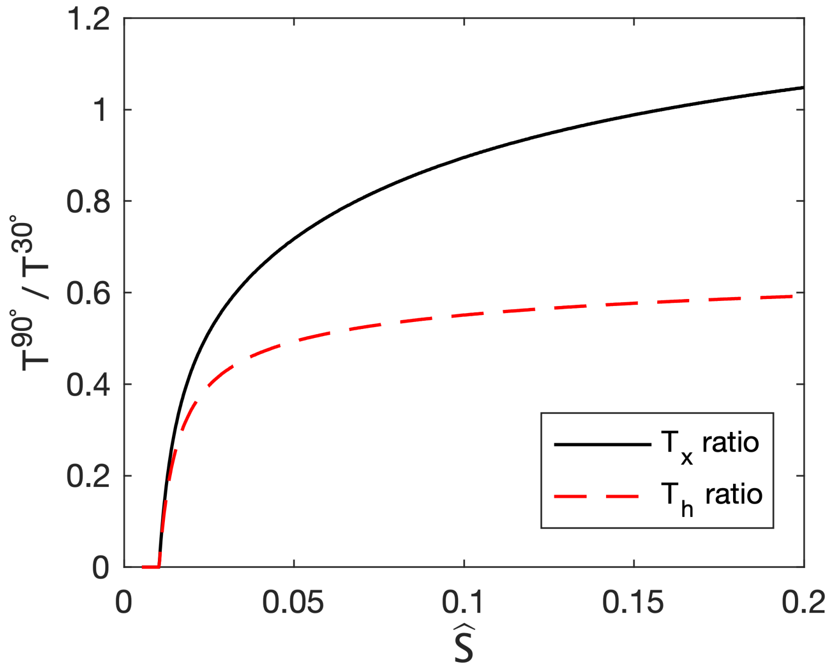
$$M_B = \frac{P_{cr}}{g}\left(\frac{e}{w(L)} + 1\right) - \frac{3}{10}\rho L$$

[2] Timoshenko, 1961

Optimization of Allowable Tip Mass – L = 15m

- Optimal allowable tip mass is found by maximizing $T_h(T_1)$ and minimizing $T_x(T_1)$
- A critical limit is identified where optimal $T_1 < 0$ for small values of $\hat{S} = S/L$
  - In this region, we set $T_1 = 0$ and by extension, $M_A = M_B$

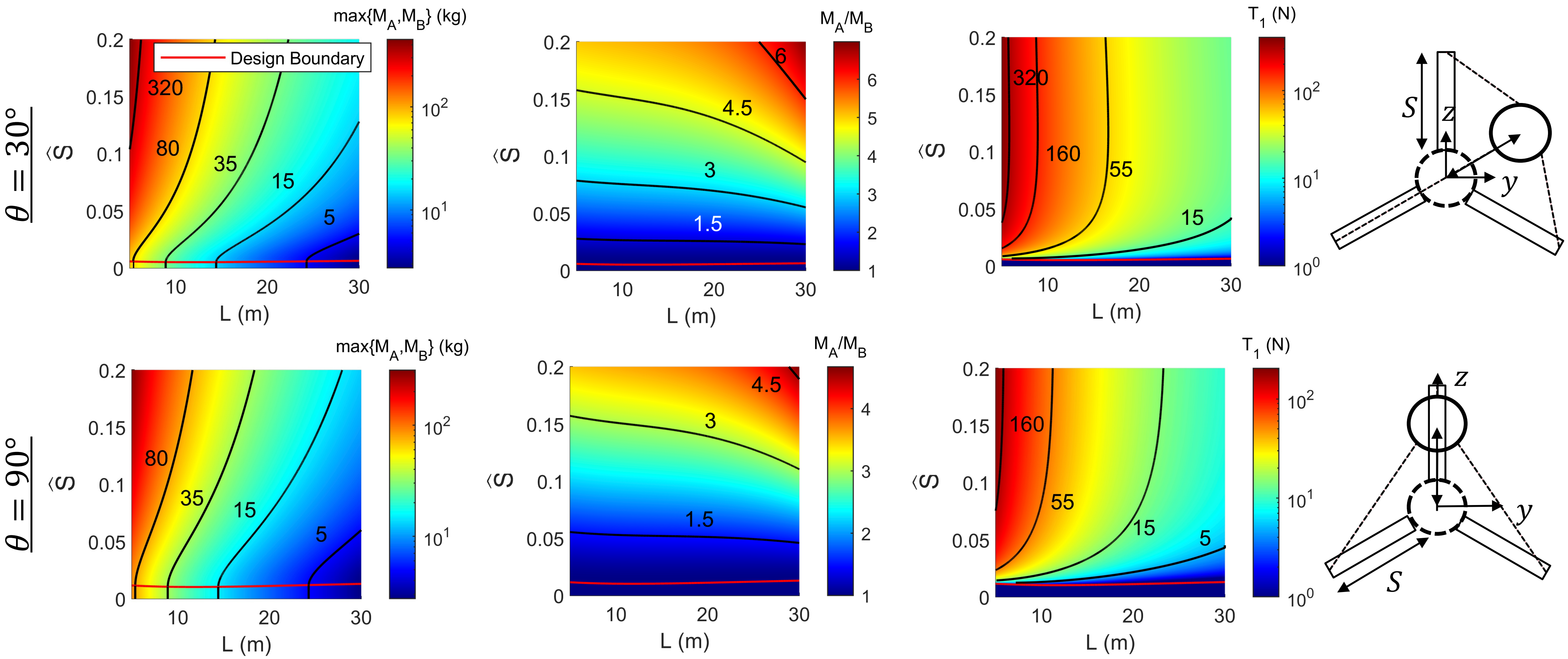




M_A is found to decrease with increasing θ (R_1).
For all regions, M_A decreases as θ approaches case 2.

Full Comparison – COROTUB

- Design A outperforms design B for most of the design space: where optimal $T_1 \neq 0$
- Below design boundary, design B would be favorable due to excess mass of cable system.
- Design boundary is found to increase with θ; approximately double from case 1 to 2.



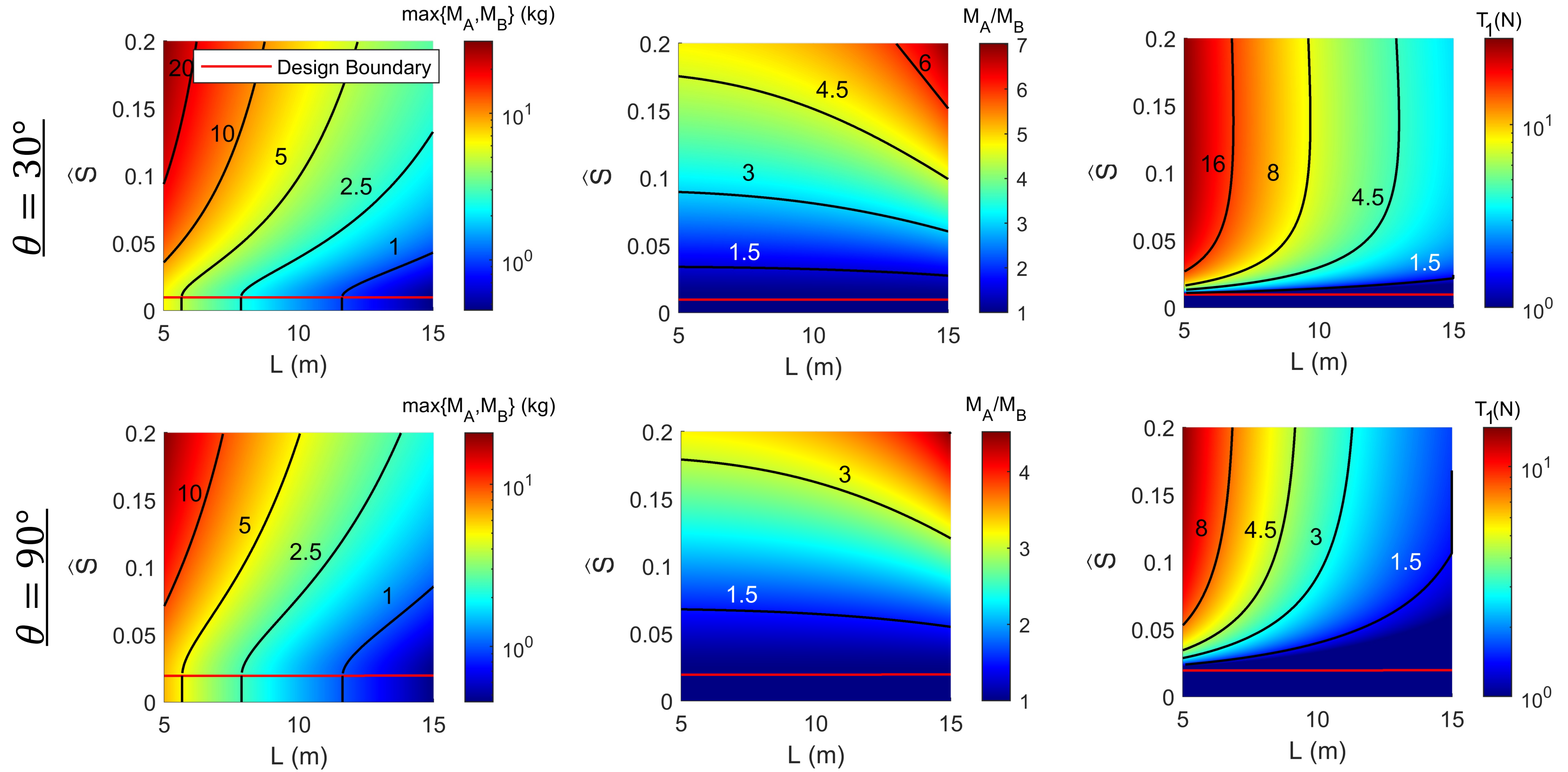
Case 1 is thus best-case loading

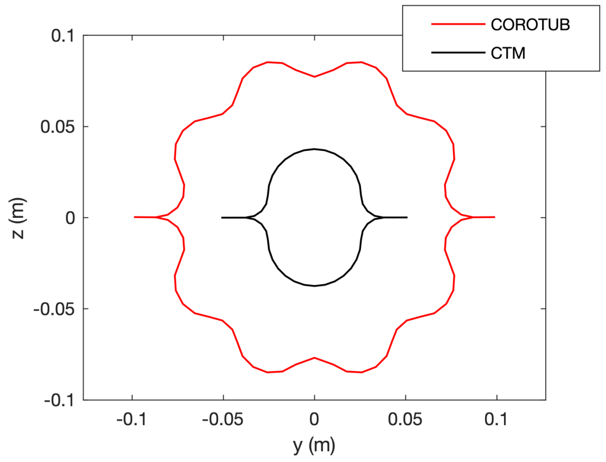
Case 2 is thus worst-case loading

Full Comparison – CTM

Process for analysis is the same as for COROTUB, but certain properties are changed. Notably, $I \rightarrow$ constant

- Specific trends found to be identical to COROTUB analysis
- In general, reduced flexural stiffness results in lower allowable tip mass across entire design space
  - CTM tower found to have a practical upper limit on boom tower length due to reduced stiffness.





CTM cross-section at the plug compared to COROTUB

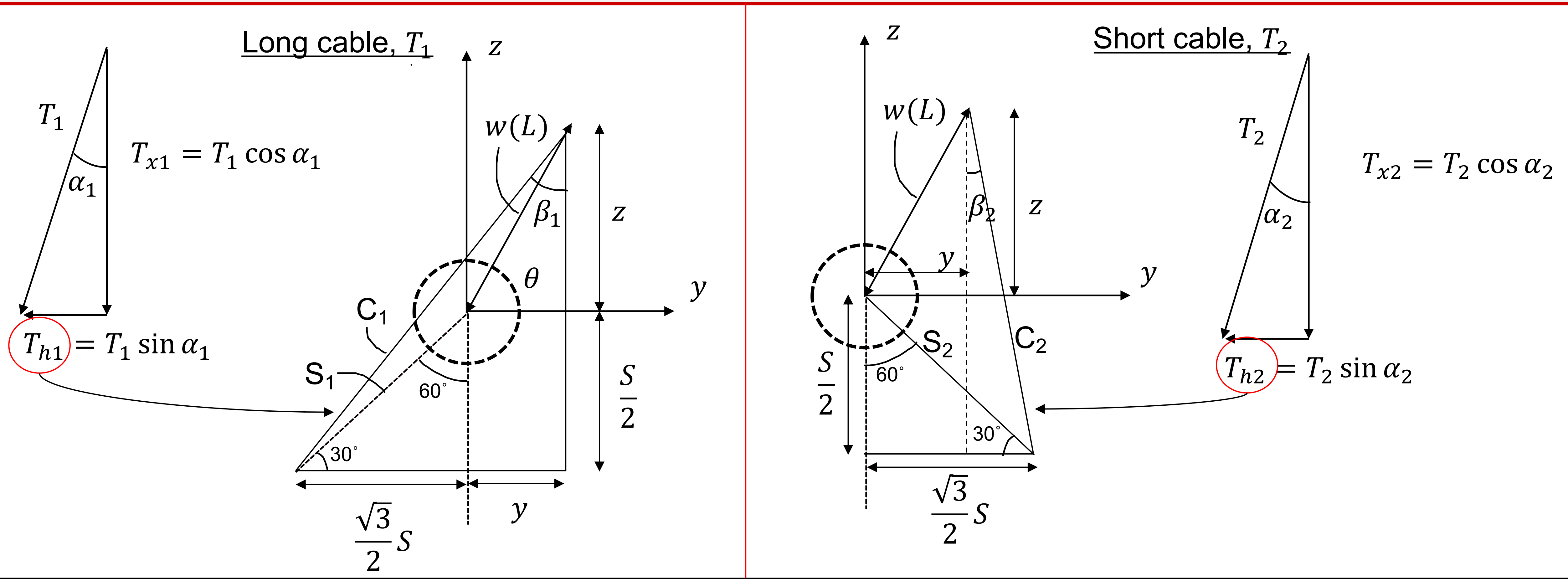
Conclusions

- Framework developed for quantifying the structural benefit of a cable-stayed support system for lunar towers.
  - Primary comparison metric is allowable tip mass
- Cable-stayed design outperforms unsupported tower for most tower heights and spreader bar lengths.
  - Optimizing cable tension reveals design boundary is low normalized spreader bar length.
- Deflection model is described by six regions of radial symmetry and bounding cases
  - Deflection directly opposite one spreader bar yields highest allowable tip mass
- Deflection limits imposed are considered conservative; results likely underestimate practical allowable tip mass.

Thank you!

Appendix

Decomposition of Tension forces



- Decomposing tension into vertical, *x*, and horizontal, *h*, components gives expressions for corrective T_h and compressive T_x loads for both cables T_1 and T_2.
- Expressions for T_x, T_h, and T_2, are found in terms of T_1, $w(L)$, and θ, simplifying inputs to the model.