Modified Cascading Generalized Inverse Control Allocation

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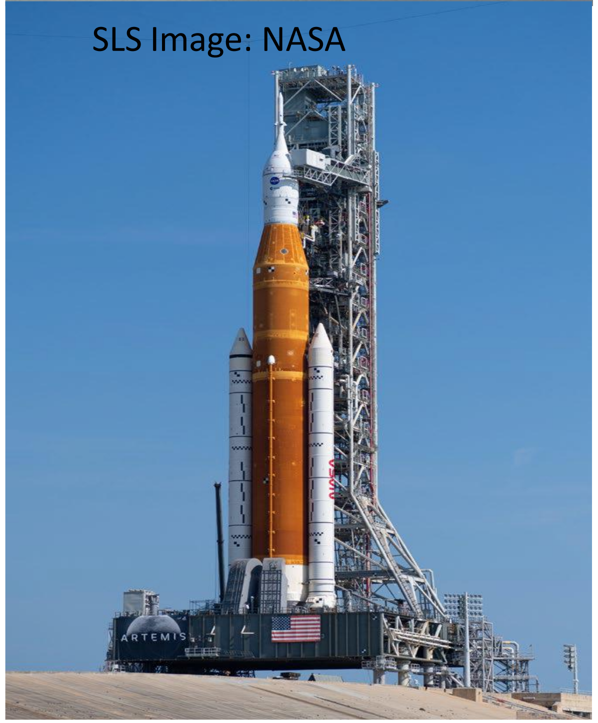


Motivation

Classical design separates into roll/pitch/yaw axes

- Control effectors designed to effect one axis (minimize cross axis coupling)
- Controllers designed to primarily use effector groups per one axis (exception ARI)
- Provided no redundancy & greatly reduced use of available control authority

Modern aircraft/controller utilize Control Allocation across axes

- Modern aircraft over-actuated (more controls than controlled variables)
- Simplifies controller design (virtual controls)
- Enhances usage of available control authority
- Inherent redundancy and safety

Urban Air Mobility (UAM) vehicles require Control Allocation: over-actuated, simplify design, energy limited,…

Problem Formulation

- **Consider the general aircraft dynamics problem:**

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u}, \boldsymbol{t}) \qquad \mathbf{x} \in \mathbb{R}^{n}, \boldsymbol{u} \in \mathbb{R}^{m}, \boldsymbol{t} \in \mathbb{R}$$

$$\boldsymbol{y} = \boldsymbol{h}(\boldsymbol{x}), \qquad \boldsymbol{y} \in \mathbb{R}^{n}$$

- **Simplifying to the linearized system (around trim condition):**

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u}_0, \boldsymbol{t}_0) + \boldsymbol{B}\boldsymbol{u}$$

$$\boldsymbol{y} = \boldsymbol{h}(\boldsymbol{x})$$

where $\boldsymbol{B} = \begin{bmatrix} \frac{\partial f_1}{\partial u_1} & \cdots & \frac{\partial f_1}{\partial u_m} \\ \vdots & \ddots & \vdots \\ \frac{\partial f_n}{\partial u_1} & \cdots & \frac{\partial f_n}{\partial u_m} \end{bmatrix}$

- **Given desired virtual control (moment) $\boldsymbol{m} = \boldsymbol{B}\boldsymbol{u}$, we solve the constrained optimization:**

$$\min_{\boldsymbol{u}} \boldsymbol{J}(\boldsymbol{u}) = \boldsymbol{u}^{T}\boldsymbol{W}\boldsymbol{u}, \;\; \textbf{subject to: } \boldsymbol{m} = \boldsymbol{B}\boldsymbol{u}, \quad \underline{\boldsymbol{u}}_i \leq \boldsymbol{u}_i \leq \overline{\boldsymbol{u}}_i \; \forall\, \boldsymbol{i} \in \{1, 2, \ldots \boldsymbol{m}\}$$

Control Allocation History

- Seminal work by Durham on weighted least squares (single generalized inverse methods), boundary object searching
- Durham: single generalized inverse always strict subset of Attainable Moment Set (i.e., set of possible moments given set of allowable controls)

- Many, many allocation algorithms:
  - Generalized inverses (pseudo inverse): Moore-Penrose, weighted
  - Linear programming: (non)-direction preserving, Bodson's, Simplex
  - Quadratic programming: various
  - Iterative: daisy chaining, redistributed pseudo-inverse, null-space augmentation, PAN, Cascading Generalized Inverse (CGI) <=> Focus of this research

Cascading Generalized Inverse (Legacy) (1 of 2)

- Cascading Generalized Inverse (CGI) is an iterative weighted least squares

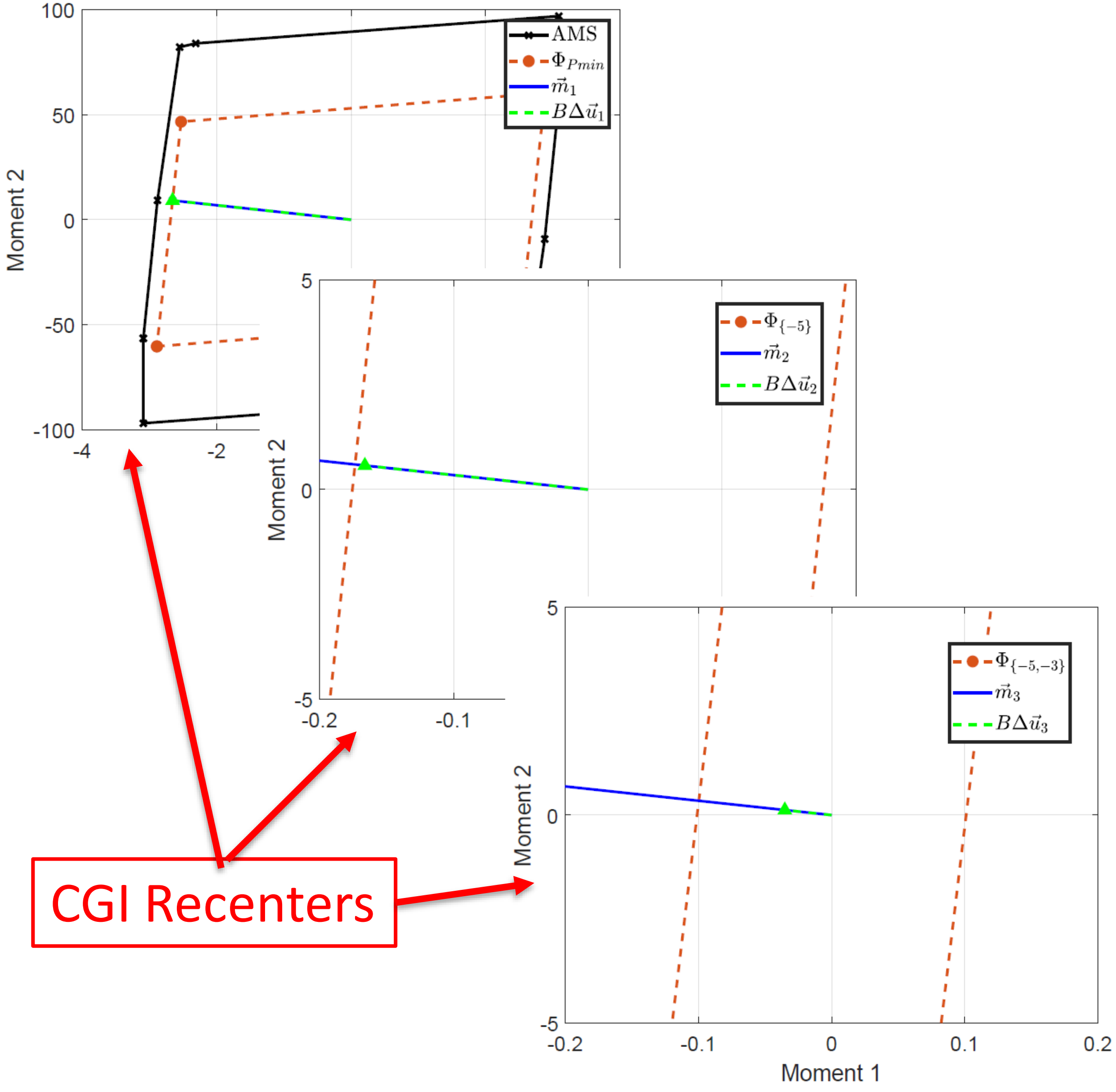
$$P = W^{-1}B^T(BW^{-1}B^T)^{-1}$$
$$P = B^T(BB^T)^{-1}$$

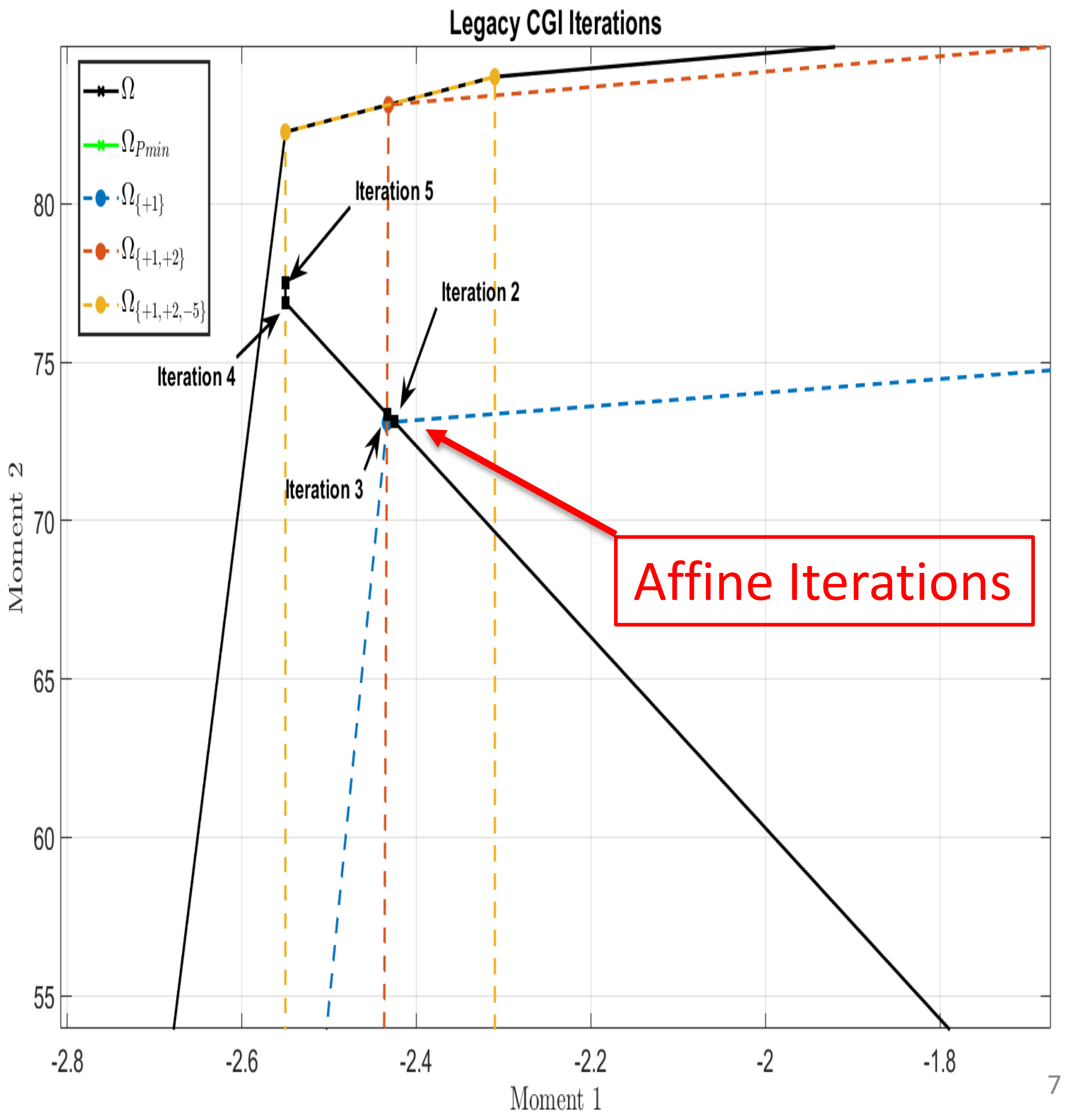
- Series of single matrix (weighted) generalized inverses
  - Utilize current generalized inverse until effector saturates
  - If saturated, remove effector and its moment contribution, then generate next generalized inverse
  - Continue until desired moment achieved, or no unsaturated effectors remain

CGI Limitations:

1) **Unable to achieve some desired outcomes**
2) **Intermittently fails to provide optimal allocation**
3) **Can fail to preserve moment direction near boundary**

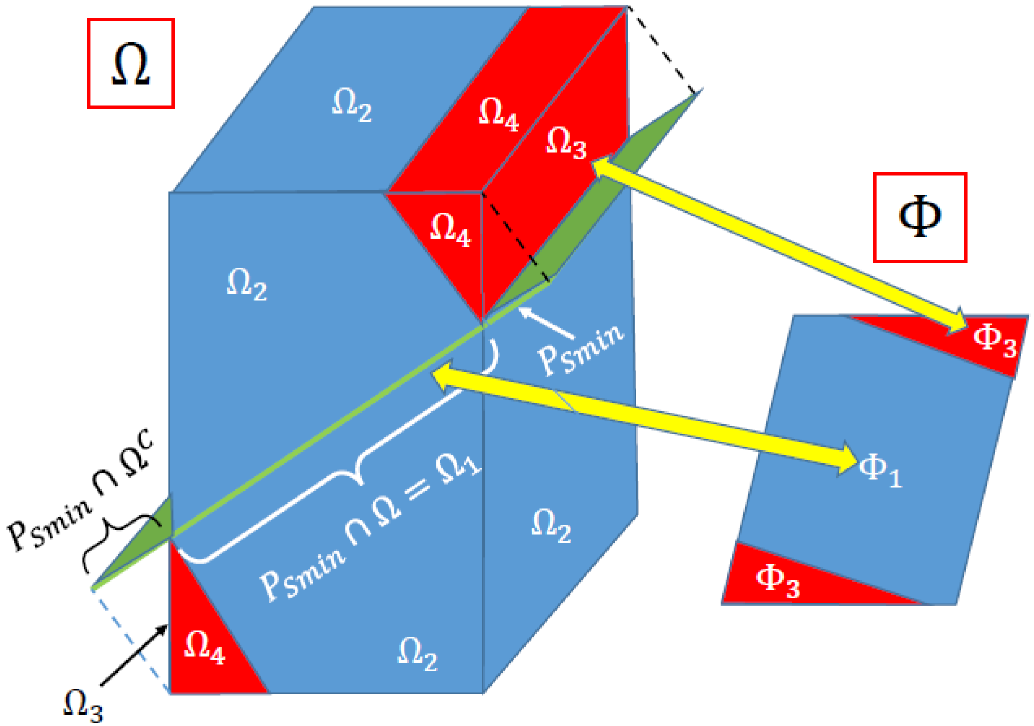
Cascading Generalized Inverse (Legacy) (2 of 2)





Modified Cascading Generalized Inverse (New) (1 of 2)

- Modified Cascading Generalized Inverse (MCGI) is a version of CGI (has same basic core algorithm)
- Incorporates effector unsaturation: add back an effector removed in a previous iteration to allow it to come off an effector limit



Scalar Difference Quadratic
(Compares Null space components)

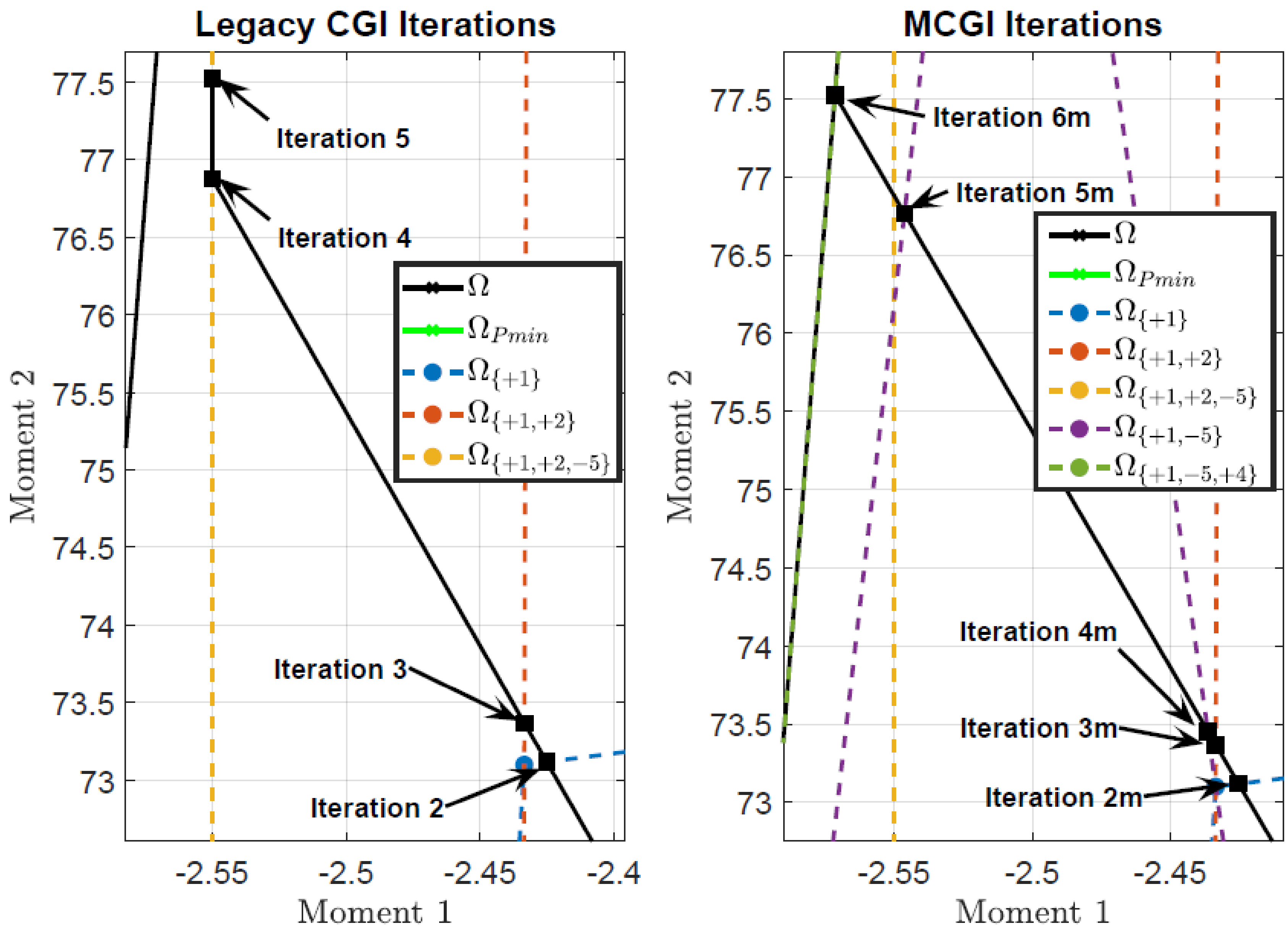
$$\mathcal{F}(a) = \left\|\vec{c}_{\underline{0}} + \vec{c}_{\underline{1}}\right\|_2^2 - \left\|\vec{c}_0 + \vec{c}_1\right\|_2^2 = c_a a^2 + c_b a + c_c \in \Re, \text{ where } a_z = -c_b/2c_a$$

$$c_a = \hat{u}_{des}^T B^T \left(P_{\underline{T_i}}^T P_{\underline{T_i}} - P_{T_i}^T P_{T_i}\right) B \hat{u}_{des}$$

$$c_b = 2\hat{u}_{des}^T B^T \left(P_{\underline{T_i}}^T \left(I - P_{\underline{T_i}} B\right) - P_{T_i}^T \left(I - P_{T_i} B\right)\right) \vec{u}_{i-1}$$

$$c_c = \vec{u}_{i-1}^T \left(\left(P_{T_i} B\right)^T \left(I - P_{T_i} B\right) - \left(P_{\underline{T_i}} B\right)^T \left(I - P_{\underline{T_i}} B\right) + \left(P_{T_i} - P_{\underline{T_i}}\right) B\right) \vec{u}_{i-1}$$

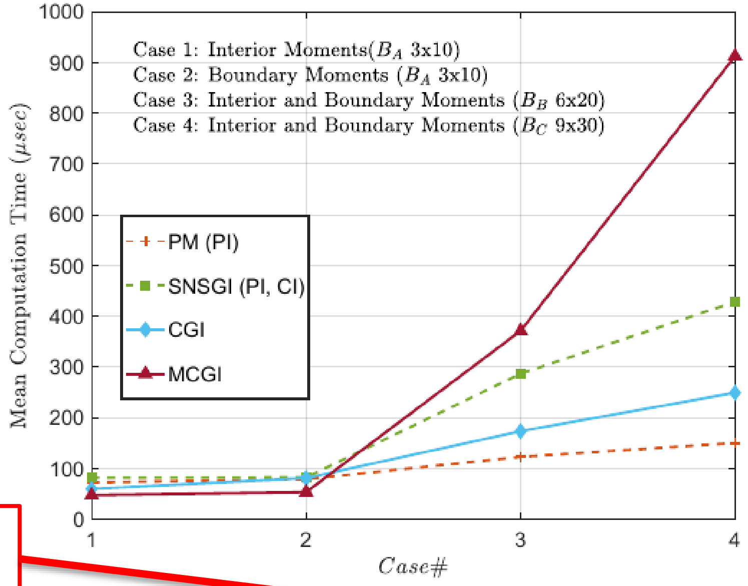
Modified Cascading Generalized Inverse (New) (2 of 2)



MCGI Unsaturation:

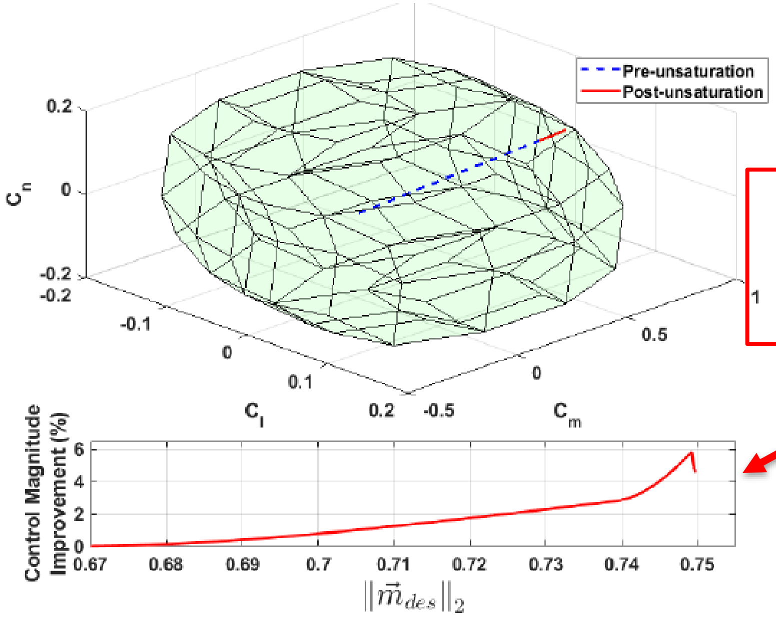
- Preserves moment direction
- Obtains maximal moment
- Obtains optimal allocation

Results and Impact



(a) Algorithm Mean Computation Time

MCGI drawback



(b) Single Unsaturation Case MCGI to CGI Comparison

MCGI allocation improvement

Table 1 Boundary Validation Statistics, $B_A \in \Re^{3\times10}$, 1000 Cases

	$V_m(\%)$	$\mu_m(\%)$	$\sigma_m(\%)$	$V_u(\%)$	$\mu_u(\%)$	$\sigma_u(\%)$
PM (PI)	35.5	+0.447	0.749	39.8	-3.77	5.16
SNSGI (PI, IC)	100	$+1.83\times10^{-11}$	3.75×10^{-11}	4.0	-5.92×10^{-07}	1.32×10^{-06}
CGI	34.3	+0.319	0.633	34.3	-8.07	7.21
MCGI	100	$+1.31\times10^{-12}$	8.32×10^{-12}	100	-1.31×10^{-10}	5.62×10^{-10}
DA	100	REF SOURCE *	-	100	REF SOURCE *	-

1) V_m, V_u: % validation cases. Failure (red) and success (black)
2) Mean (μ_m) and Std Dev (σ_m) of moment error metric E_m
3) Mean (μ_u) and Std Dev (σ_u) of control error metric E_u
* Unique DA solution used as reference control

MCGI obtains entire AMS

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Questions?

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