

# Candidate Performance Metrics for Generalized Control for Autonomous Flight

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**Contingency management is the most challenging aspect of autonomous flight. In order to accommodate the most flexible response to unpredicted events and unexpected circumstances, i.e. contingencies, a new integrated path planning, trajectory following, flight control architecture is required that would maximize the safe operating envelope. For this highly integrated generalized control architecture, a new set of performance metrics that extends beyond traditional stability and performance is required. This paper proposes a candidate set of new performance metrics relevant to urban air mobility mission scenarios.**

## I. Introduction

New aerospace missions and technologies are driving rapid development of autonomous flight capabilities. There is a growing interest among both established and emerging aerospace companies in autonomous cargo delivery across a variety of distances with autonomous flight necessitated by economic feasibility. Another emerging sector, which partially overlaps with autonomous cargo, is urban air mobility (UAM) [1, 2] with its proliferation of distributed electric propulsion (DEP) based vehicle configurations (also known as eVTOL, electric vertical takeoff and landing). The nature of autonomous flight itself suggests a need for closer integration between the traditional flight control and trajectory planning.

DEP and the emerging UAM sector generate new challenges and opportunities for control and trajectory planners. UAM class vehicles typically exhibit three phases of flight — rotor-borne vertical takeoff/landing, fixed-wing like cruise, and a transition phase between the two. The interactions between multiple propulsors, wings, and flight modes result in complex, highly nonlinear flight dynamics with disparate control effectors whose efficacy changes dramatically across the flight envelope [3, 4]. In addition to complex vehicle flight dynamics, operations in an urban environment provide their own challenges. It is highly likely that UAM vehicles will experience dynamic temporary flight restrictions as part of their normal operations, will deal with intruders, as well as some degree of propulsive failures due to the novel nature of this technology. Arguably from control-centric perspective, events that can precipitate contingency management [5, 6] can be reduced to two invariant behaviors - alteration in control strategy and trajectory replanning. These factors require potential alteration in control strategy and dynamic, robust trajectory replanning capabilities in order to ensure safe flight and minimize disruption to the local air traffic management.

The remainder of this document is organized as follows. Section II provides an overview of current metrics used for control design and analysis. Section III describes suggested additional metrics for generalized control that go beyond current state-of-practice. Section IV briefly outlines a selected UAM mission scenario to illustrate the need for new generalized control metrics. This paper is intended to outline limitations with the current metrics and pose the challenge to the aerospace community to develop new metrics to better reflect challenges of emerging vehicle configurations.

## II. Control Metrics

Automatic feedback control systems have been known and used for more than 2000 years dating back to 3rd century B.C. application on water clocks. Longevity and popularity of feedback control can be attributed to its powerful

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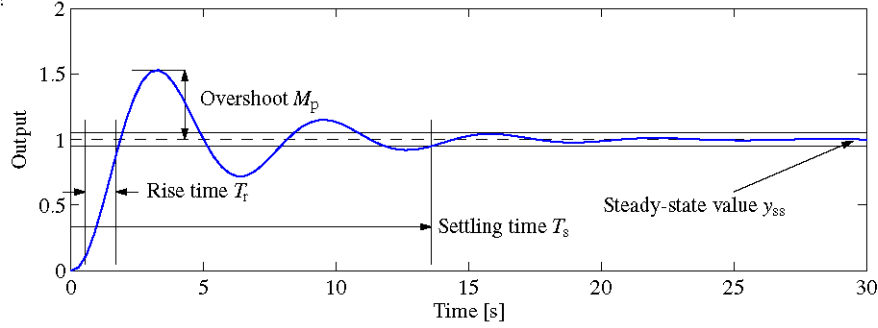
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**Fig. 1 Dynamical system performance metrics**

fundamental properties of command signal following, robustness to uncertainty, disturbance attenuation and system behavior shaping (e.g. equilibrium points, stability, limit cycles, etc.).

In designing control systems, there are several fundamental issues that transcend the boundaries of specific applications. Although they may differ for each application and may have different levels of importance, these issues are generic in their relationship to control design objectives and procedures. Central to these issues is the requirement to provide satisfactory performance in the face of modeling errors, system variations, and uncertainty. Indeed, this requirement was the original motivation for the development of feedback systems. Feedback is only required when system performance cannot be achieved because of uncertainty in system characteristics. As the automatic feedback control evolved along with the flight vehicles, a set of related standards have been developed. The following section will briefly touch on current standards and requirements with the subsequent section briefly discussing additional metrics used in typical state-of-practice by the aerospace industry.

#### A. Current Standards Requirements

All design and analysis start with a model, a representation of the dynamical system. There might be multiple models to represent the dynamical system with different levels of fidelity depending on the phenomena of interest. Typical dynamical system models are nonlinear and difficult to analyze; however, many insights can be derived from system approximations. In aerospace, a nonlinear system is approximated by a linear time-invariant (LTI) model that allows for exploitation of a rich set of tools associated with linear system analysis. The standard requirements and metrics are based on these linear systems and tools. The assumption is that an LTI model provides a good approximation of the system behavior in a sufficiently large neighborhood around an operating point so that linear system based metrics are valid to design and analyze automatic feedback controllers.

There is a well developed set of standard control metrics that is utilized to assess stability and robust performance of these designs [7–10]. The dynamical system performance metrics are based on time domain quantities such as rise and settling time as well as maximum overshoot of the command and steady state error to a step response (see Fig. 1).

A critical property of feedback systems is their robustness; that is, their ability to maintain performance in the face of uncertainties. In particular, it is important that a closed-loop system remains stable despite differences between the model used for design and the actual physical system. These differences result from variations in modeled parameters as well as system elements which are either approximated, aggregated, or ignored in the design model.

Stability and robustness metrics required by the regulatory authorities are frequency domain based, tracing sinusoidal signal behavior through the system. These metrics, represented by gain and phase margins, describe amplitude and phase between input and output. The gain and phase margin, properties of LTI systems, are checked one loop at a time. Additionally, bandwidth limitations are imposed as an insurance against uncertainty which grows with frequency due to unmodeled high frequency dynamics. Once linear system performance, stability and robustness metric analysis have been satisfied, it is an accepted practice to check these in a nonlinear Monte Carlo simulation with predetermined uncertainty for a select number of dynamic parameters and atmospheric disturbances. For missiles and launch vehicles, that constitute current autonomous vehicle class, Monte Carlo simulation is also used to determine the dispersion around a nominal predefined trajectory.

## B. Typical State-of-Practice

For multiloop systems, individual loops may be analyzed using single-input single-output (SISO) scalar techniques, but these methods fail to capture the essentially multivariable nature of many systems. For example, scalar methods may ignore variations which simultaneously affect multiple loops. For this reason much of aerospace industry has adapted internal analysis tools, beyond the requirements documents, that address the multivariable nature of the aerospace systems.

Since the 1980's there has been a major development of design and analysis tools dealing with multi-input multi-output (MIMO) dynamical systems with uncertainty that better reflect the more complex dynamics of advanced aircraft configurations. The most prevalent methods are frequency based and are an extension of classical SISO scalar approaches such as Bode, Nyquist and Nichols. These frequency domain techniques are largely based on singular values and generalize the Bode and Nyquist techniques for multivariable systems. These are being widely used in the aerospace industry and can be found in any number of references [11–14].

The main difference between a scalar (SISO) system and a MIMO system is the presence of directions in the latter. Directions are relevant for vectors and matrices, but not for scalars. The singular value decomposition (SVD) provides a useful way of quantifying multivariable directionality. In order to address the directionality in the system, the MIMO analysis techniques exploit the concepts of singular values, singular vectors, and the spectral norm of a matrix. The significance of this approach is that it naturally provides multivariable interpretation to many key classical control concepts.

Recall that any complex  $m \times n$  matrix  $\mathbf{A}$  may be factored into a singular value decomposition (SVD)

$$\mathbf{A} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^*$$

where  $\mathbf{U}$  is the  $m \times m$  unitary matrix of output singular vectors  $u_i$  and  $\mathbf{V}$  is the  $n \times n$  unitary matrix of input singular vectors  $v_i$ ;  $\mathbf{\Sigma}$  is an  $m \times n$  matrix with  $p = \min(m, n)$  non-negative singular values,  $\sigma_i(\mathbf{A})$ , arranged in descending order along its main diagonal; the other entries are zero.

The following notations for singular values are often adopted:

$$\overline{\sigma}(\mathbf{A}) := \sigma_{\max}(\mathbf{A}) = \sigma_1 = \text{the largest singular value of } \mathbf{A}$$

and

$$\underline{\sigma}(\mathbf{A}) := \sigma_{\min}(\mathbf{A}) = \sigma_p = \text{the smallest singular value of } \mathbf{A}.$$

Geometrically, the singular values of a matrix  $\mathbf{A}$  are precisely the lengths of the semiaxes of the hyperellipsoid  $E$  defined by

$$E = \{y : y = Ax, x \in C^n, \|x\| = 1\}.$$

Thus  $v_1$  is the direction in which  $\|y\|$  is largest for all  $\|x\| = 1$ ; while  $v_n$  is the direction in which  $\|y\|$  is smallest for all  $\|x\| = 1$ . From the input/output point of view,  $v_1$  ( $v_n$ ) is the highest (lowest) gain input (or control) direction, while  $u_1$  ( $u_m$ ) is the highest (lowest) gain output (or observing) direction [13].

Hence, it is often convenient to introduce the following alternative definitions for the largest singular value  $\overline{\sigma}$ :

$$\overline{\sigma}(\mathbf{A}) := \max_{\|x\|=1} \|Ax\|,$$

and for the smallest singular value  $\underline{\sigma}$ :

$$\underline{\sigma}(\mathbf{A}) := \min_{\|x\|=1} \|Ax\|,$$

The minimum singular value of the matrix  $\mathbf{A}$  gives a measure of how close  $\mathbf{A}$  is to being singular, i.e. a measure of robustness. To determine system robustness the return difference matrix plays a key role. The degree of robustness of the multivariable system is determined by how close to singular the return difference matrix is by computing its singular values versus frequency. This measure of closeness to singularity is used in forming a multivariable gain margin, similar to the classical gain margin. Thus, classical gain and phase margins are used to measure the robustness

of SISO systems to perturbations in the feedback loop, while singular values are used in measuring the robustness of multivariable systems.

Once the linear system analysis has been satisfactorily completed across the envelope of interest, directed Monte Carlo analysis is used to verify the results of the linear analysis within the bounds of uncertainty region of interest.

In summary, the current state of practice for flight control design focus on modifying vehicle dynamics to satisfy linear SISO system metrics provided in the standards as well as other metrics that would test for cross-loop coupling, uncertainty in vehicle dynamics and nonlinear behavior. There are no required standard metrics in the latter case and specifics vary among companies. The process in general consists of:

- 1) Creating LTI models at different points spanning the flight envelope
- 2) Designing a controller at each point to satisfy performance (time domain - rise time, overshoot, settling time, etc.)
- 3) Checking stability and robustness at each design point
- 4) Gain scheduling between each design point
- 5) Run nonlinear Monte Carlo simulation to validate stability and robustness.

### **III. Control Metrics for Autonomous Flight**

The contention of this paper is that with the emergence of new configurations with complex multi-modal dynamics and expansion of autonomous missions, an expanded set of metrics covering a tighter coupling between dynamics, flight control, trajectory following and replanning is necessary to maximize performance and safety.

The previous section described current standards for flight control, based on piloted vehicles, and additional testing for MIMO systems that is typically done for internal clearance inside aerospace companies. This section discusses additional metrics that might be used by some in the aerospace sector as well as discuss the need for new metrics that emphasize the tighter integration of flight control with trajectory planning.

#### **A. Additional Control Metrics**

With the emergence of new DEP VTOL aircraft, the complexity of aero-propulsive interactions for these configurations emphasizes the need to properly deal with dynamic uncertainty both in design and analysis of the flight controller. These dynamic systems are inherently MIMO and should be analyzed as such. Performing controller analysis just to fulfill time domain performance specifications and gain/phase stability margins will miss destabilizing interactions. Moreover, there are tight margins between physical capability and what is likely required for safe and efficient operations. In these cases, it is potentially more damaging to be too conservative in assessment of stability and performance under uncertainty than too optimistic. For example, if a trajectory to avoid obstacles and land cannot be found due to conservatism on the vehicle control performance assessment, there is a significant chance of a catastrophic outcome. The implication is that not only the uncertainty in a system, with its multiple sources, must be modeled appropriately but the analysis of stability and performance must be done as accurately as practically feasible.

Often, stability is not the only property of a closed-loop system that must be robust to perturbations. Typically, there are exogenous disturbances acting on the system (wind gusts, sensor noise) that result in tracking and regulation errors. Under perturbation, the effect that these disturbances have on error signals can greatly increase. In most cases, long before the onset of instability, the closed-loop performance will degrade to the point of unacceptability. Hence, there is a need for another metric called “robust performance” test, which indicates the worst-case level of performance degradation associated with a given level of perturbations.

Recall from previous section that typically in practice singular values are used to test stability and performance in the presence of uncertainty. It is important to note that this uncertainty takes an unstructured form. However, if a system is subject to multiple sources of uncertainties, in order to use singular value analysis, it is necessary to reflect all sources of uncertainties from their known point of occurrence to a single reference location in the loop. Such reflected uncertainties invariably have a great deal of structure, which must then be “covered up” with a large, arbitrarily more conservative perturbation in order to maintain a simple bounded representation at the reference location. Clearly such a representation introduces significant conservatism to the analysis thus it is highly desirable to treat uncertainties as they are and where they are in the system. This leads to structured singular value (SSV), which is denoted by  $\mu$ , that provides the unified framework to treat the robust stability and robust performance problems for systems with multiple sources of uncertainties. Detailed discussion of SSV can be found in [13]; for completeness the definition is provided below.

Consider uncertainty set  $\Delta \subset C$  defined as

$$\Delta = \{\text{diag}[\delta_1 I_{r_1}, \dots, \delta_s I_{r_s}, \Delta_1, \dots, \Delta_F], : \delta_i \in \mathbb{C}, \Delta_j \in C^{m_j \times m_j}\}$$

Similar to the unstructured uncertainty case, the reciprocal of the largest singular value of a matrix is a measure of the smallest perturbation matrix. For structured uncertainty  $\Delta \in \Delta$  this measure is Structured Singular Value denoted by  $\mu_\Delta(M)$ .

**Definition:** For  $M \in C^{n \times n}$ ,  $\mu_\Delta(M)$  is defined as

$$\mu_\Delta(M) := \frac{1}{\min\{\overline{\sigma} : \Delta \subset \Delta, \det(I - M\Delta) = 0\}}$$

unless no  $\Delta \in \Delta$  makes  $I - M\Delta$  singular in which case  $\mu_\Delta(M) := 0$ .

SSV is a powerful framework that removes conservatism associated with unstructured uncertainty and singular value analysis. Just like, the reciprocal of the largest singular value of a matrix is a measure of the smallest “destabilizing” perturbation matrix, SSV provides a measure of “destabilizing” perturbation for a system with structured uncertainty. SSV is proposed as an additional metric used for analysis of system robust stability and performance.

In addition to importance of properly representing and analyzing system stability and performance, the emerging eVTOL configurations are typically over actuated and would benefit from considering system directional controllability. To that end, the recent emergence of control allocation has demonstrated the ability to improve performance while simultaneously simplifying the design cycle for transitioning vehicles and those with multiple variants (e.g., F/A-18 Magic Carpet, Space Launch System, F-35, etc.). Broadly speaking, control allocation is a process to utilize a virtual control (e.g., desired moments and/or accelerations) within the control loops, while separately determining what effectors to use in providing the desired virtual control. The general nonlinear control problem linearized around a trim condition  $(\mathbf{u}_0, \mathbf{t}_0)$  with the desired virtual control  $\nu$  is:

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{u}, \mathbf{t}) \implies \mathbf{x} = \mathbf{f}(\mathbf{x}, \mathbf{u}_0, \mathbf{t}_0) + \nu, \quad \mathbf{x} \in \mathcal{X}^n, \mathbf{u}, \nu \in \mathcal{U}^m, \mathbf{t} \in \mathcal{T} \quad (1)$$

$$\mathbf{y} = \mathbf{h}(\mathbf{x}), \quad \mathbf{y} \in \mathcal{Y}^n \quad (2)$$

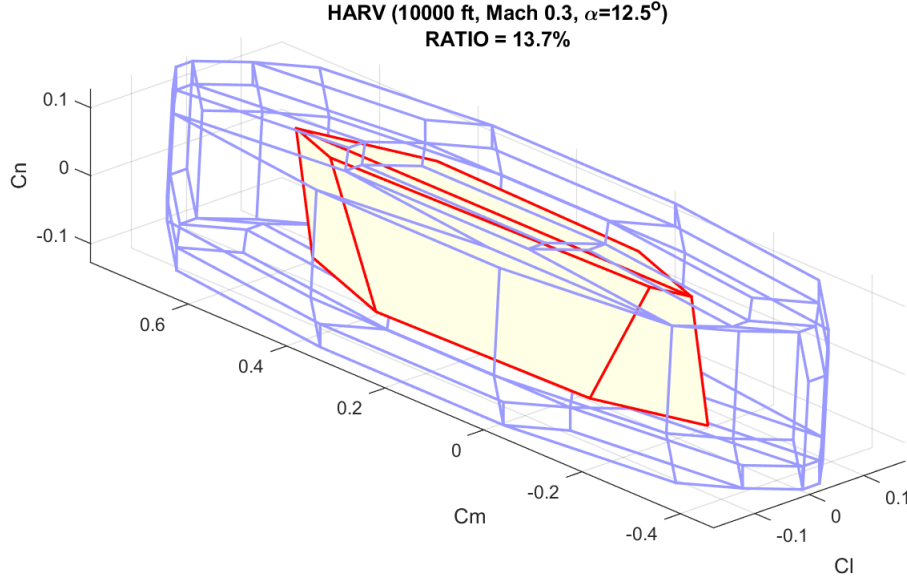
Separate from the control design solution implemented to produce the desired virtual control  $\nu$ , the linearized allocation problem is typically defined as finding the optimal control  $\mathbf{u}$  such that  $\nu = \mathbf{B}\mathbf{u}$ , subject to effector positions and/or rate limits where:

$$B = \begin{bmatrix} \frac{\partial \mathbf{f}_1}{\partial \mathbf{u}_1} & \dots & \frac{\partial \mathbf{f}_1}{\partial \mathbf{u}_m} \\ \vdots & \ddots & \vdots \\ \frac{\partial \mathbf{f}_n}{\partial \mathbf{u}_1} & \dots & \frac{\partial \mathbf{f}_n}{\partial \mathbf{u}_m} \end{bmatrix}. \quad (3)$$

The  $B$  matrix in Eq. (3) is referred to as the control effectiveness or the input sensitivity matrix. The allocation optimal cost function is often a weighted quadratic cost on the control.

Regardless of control architecture, it is important to ask what are the appropriate metrics to describe the portion of total control authority available for use in this architecture? Similarly, how can one be certain that this particular control architecture has access to all available (theoretical) control authority for a particular vehicle configuration at a given flight condition? The current state of practice for control allocation using a linearized control effectiveness matrix is to generate an attainable moment set (AMS),  $\Phi$ . This set is generated as the convex hull of moments generated by the application of all combinations of saturated effectors to the input sensitivity matrix. For the single  $B$  matrix allocation problem (e.g., Moore-Penrose pseudo-inverse), it is well known that only a strict subset of the AMS is available for a given effector weighting matrix. The current performance metric is a volumetric ratio of the weighted allocators attainable set to the (linearized) theoretical AMS. An example of this metric is shown in Fig. 2 [15]. In this figure, the theoretical AMS is compared against the AMS subset generated by the Moore-Penrose pseudo-inverse (i.e., the optimization weighting is the identity matrix) [15] with the result that identity matrix weighing only allows for reaching  $\approx 14\%$  of the available control authority.

However, this metric has serious limitations. First, the metric relies on a linearized approximation of the true non-linear aero-propulsive model. Next, the metric is only valid in the vicinity of a particular linearized trim condition.



**Fig. 2 Attainable moment subset**

Thus any attempt to use this metric to quantify available control authority usage throughout the flight envelope would necessitate examination of many trim conditions that span the flight envelope. There is a wide array of allocation algorithms, some of which guarantee access to the entire AMS but fail to preserve weighted optimal allocation (e.g., Durham’s Direct method)[16, 17], and others that preserve optimal allocation throughout the AMS (e.g., modified cascading generalized inverse (MCGI), affine generalized inverse (AGI)) [18–20].

The aforementioned volumetric ratio of the AMS is only part of the story. Due to the criticality of optimal allocation for eVTOL aircraft (e.g., minimum energy usage for a desired virtual control  $\nu$ ), an equally important metric is warranted for magnitude of degradation from the true optimal allocation throughout a given AMS at a specific flight condition (i.e., fixed  $B$  matrix). Unfortunately, analytic solutions to the linearized, constrained, weighted allocation problem are generally only known for portions of the AMS (e.g., AMS boundary cases and interior cases for which no effectors are saturated), however brute-force methods exist to determine the minimal allocation throughout the AMS. So a more complete picture of control architecture performance at a flight condition is to simultaneously compare the volumetric percentage of the AMS achieved, while comparing the current architecture’s allocation to the true optimal allocation throughout the AMS (or compare to the allocation performance of another candidate control architecture’s allocation).

## B. Trajectory/control metrics

As has been mentioned previously, the contention is that autonomous flight requires additional metrics, especially those dealing with trajectories. Currently, typical control design targets conventional aircraft configurations operating in essentially linear regions of vehicle dynamics around an operating point with very generous margins for tracking trajectories consisting of line segments with very benign curvature transitions or high performance controllers, still operating in local linear regions, supporting human generated and tracked nonlinear trajectories. However, as new missions and configurations develop, both the high performance trajectories and the controllers following them will be required.

Typical trajectory tracking metrics consist of Euclidean distance from a nominal trajectory for a nominal and uncertain system and root mean square (RMS) errors in the presence of environmental disturbances, combined with system uncertainty. An example of such trajectory metrics is borrowed from [21] and is illustrated in Fig. 3 and Fig. 4. Two interesting phenomena of note from this example. First, both Fig. 3b and Fig. 4b provide an example case for a robust performance metric, by demonstrating how exogenous disturbances acting on the system, in this case turbulence, can result in growing tracking error. The second phenomena illustrates a need for a more integrated control/trajectory metric. Note the difference in tracking error between Fig. 3a and Fig. 4a which is driven strictly by the difference

in trajectory, different commanded accelerations over the same time interval. Using standard metrics, the controller performance is perfectly adequate; however, a more aggressive trajectory exposes limitations in performance, potentially in both controller and vehicle, as well as hints at the nonlinear nature of the true vehicle dynamics.

Performance metrics for generalized control, a term describing an integrated flight control and trajectory planning, must cover a wide range of situations, capturing the system response to all inputs and disturbances for all system configurations, both nominal and off-nominal. This requirement on metrics is difficult without principled assumptions and approximations. In the case of linear systems, these requirements can become simple: the separation principle allows control and state observer design to be carried out independently, and linear systems only have one equilibrium point. Without these properties, nonlinear control metrics cannot, in general, be computed for a system or controller in all cases. Instead, the metric can only capture the behavior of the system around a specific equilibrium point, in response to a specific set of inputs and disturbances. In mature and well understood systems, many of these complications can be overlooked without consequence, since experience and careful system design have accumulated towards robust system designs that minimize undesirable nonlinear behavior [22].

The new class of vehicle under consideration is not as well understood, only recently developed and with rapidly changing designs. Furthermore, the dynamic nature of these systems makes it less likely that analysis around fixed equilibria will sufficiently capture system performance. For example, in a tilt-wing vehicle, stability at an arbitrary point during a wing-tilt transition is of less interest than ensuring that the entire transition maneuver is performed safely and robustly [4].

This dependence on the trajectory leads to two new design considerations: the first is that the robustness of the system is dependent on how close the system state is to the designed trajectory, in the same way that the system might be expected to perform better closer to a controller design point. In this way, performance in terms of trajectory tracking error, is linked to robustness. The second consideration is that the trajectories from planners be compatible with the system dynamics. If stability is computed around a nominal trajectory, that trajectory must have properties that make it conducive to being tracked by the vehicle dynamics. These considerations suggest that:

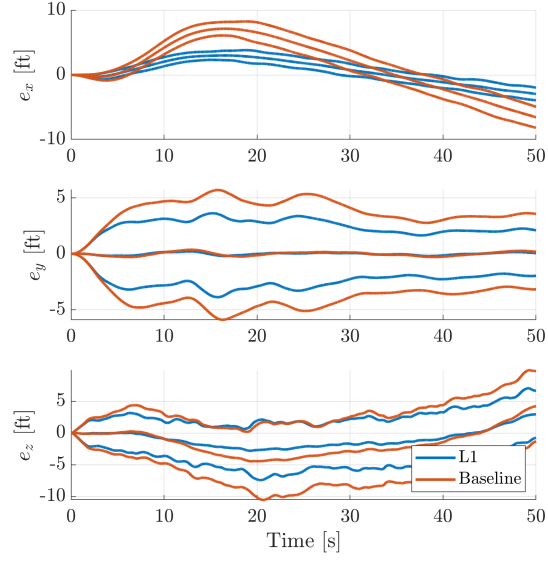
- 1) The interplay between the levels of control must increase, guidance, control, sensor processing must all be considered together, rather than individually
- 2) The interplay between the GNC systems and the dynamics must be a priori predictable
- 3) Some metrics on system performance should be tied to specific trajectories or classes of trajectories.

In recent years, there has been a great deal of research coming from academia that focuses on combining vehicle control with trajectory planning to generate safe and agile flight. By definition, such flight is represented by nonlinear system dynamics that trajectory planning takes advantage of. Some of the tools that show promise are control contraction metrics ([23, 24]) and control barrier functions ([25]). Such developments are fortuitous for the new eVTOL vehicle configurations that must operate in complex, cluttered urban environments and do so safely and efficiently. These vehicles must take full advantage of their physical capabilities and, hence, be treated as nonlinear systems for at least parts of their trajectories. The implication is that practical metrics for this class of nonlinear systems are necessary both for designing trajectories under system uncertainty and exogenous disturbances as well as for accurate analysis of their robust stability and robust performance. Development of such metrics is the challenge this paper would like to pose to the aerospace controls community.

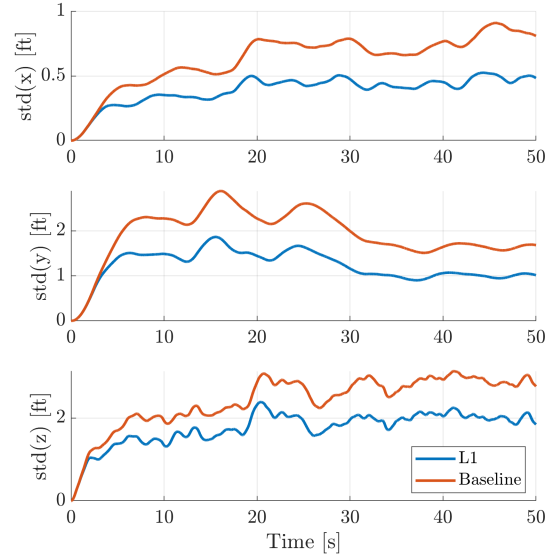
## IV. Summary Discussion

This section summarizes the discussion regarding a need for additional control metrics with an illustrative UAM scenario and associated contingencies. Here the term control is used in the most general sense to include both flight control and trajectory planning, what has been termed generalized control in the previous section.

The illustrative scenario is as follows. Consider a DEP VTOL vehicle flying between two urban vertiports or from an urban vertiport to a suburban destination. The vehicle will likely encounter turbulence and a dynamic wind field, common to urban environment. Because the vehicle has new DEP technology, it has higher than average probability of encountering a propulsive fault that decreases propulsive thrust in at least one propulsor, during any of the vehicle flight modes. It is also possible that the vehicle would encounter an uncooperative intruder across its trajectory, Canadian geese do not carry a transponder or fly a predictable trajectory. Additionally, the vehicle may encounter a pop-up weather cell that it will need to avoid. Fig. 5 illustrates the complexity of the operating environment from which a subset of likely contingencies have been chosen. These potential contingencies, happening individually or in some combination, would necessitate a trajectory replanning that is tightly coupled with vehicle dynamic capabilities provided by automatic flight control and cognizant of the external obstacles and disturbances. The resulting system stability and

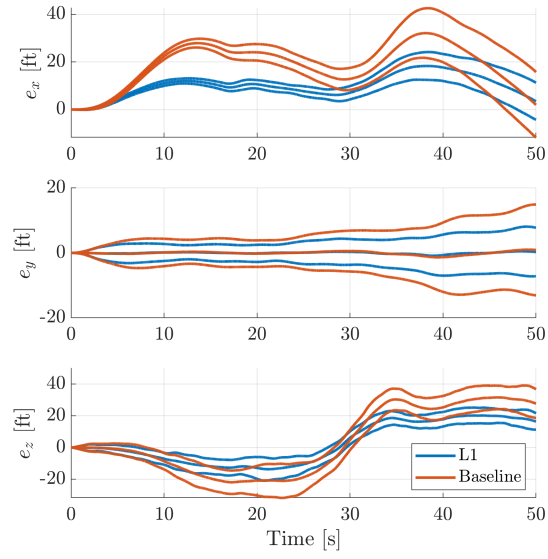


(a) Distribution of tracking errors over time (mean, standard deviation for each controller).

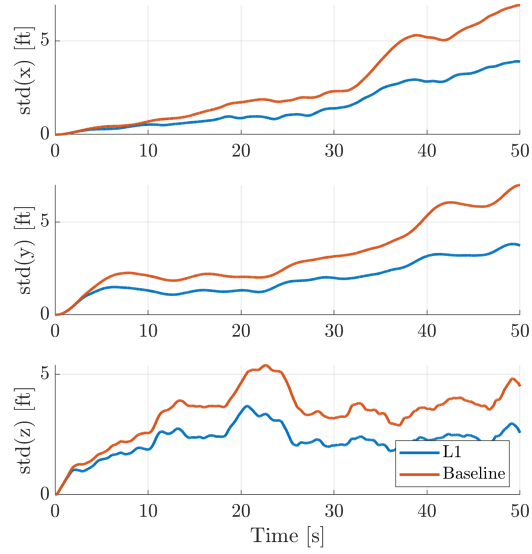


(b) Deviation of tracking errors over time (standard deviation of error due to turbulence).

**Fig. 3** Turbulence + wind (x-direction), 50 runs. Straight acceleration: 0 to 50 fps.

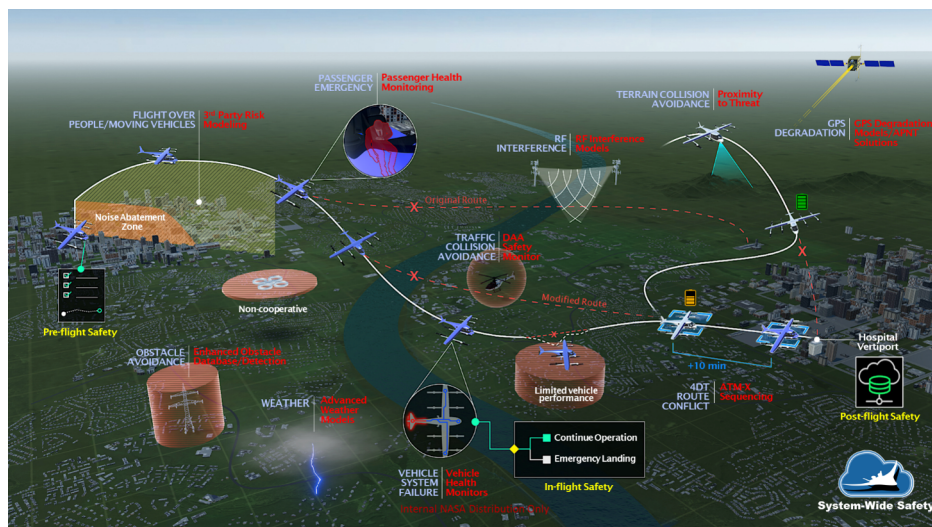


(a) Distribution of tracking errors over time (mean, standard deviation for each controller).



(b) Deviation of tracking errors over time (standard deviation of error due to turbulence).

**Fig. 4** Turbulence + wind (x-direction), 50 runs. Straight acceleration: 0 to 200fps.



**Fig. 5** An example UAM scenario with sample contingencies.

performance would be nonlinear and must be analyzed as such. Any significant conservatism can place the vehicle closer to unsafe operating conditions than if the system analysis were more reflective of reality.

Considerations of multi-modal vehicle dynamics, highly nonlinear during transition, complex operating environment, and the combination of likely contingencies drive the need to establish additional generalized control metrics that can be evaluated in a practical manner and produce non-conservative results. The suggestion of incorporating SSV and directional controllability analysis is an attempt to better understand vehicle dynamic capabilities with a given controller and provide an insight into vehicle dynamics. The question of additional appropriate metrics that would quantify generalized control robust stability and robust performance is wide open. This paper is written to challenge the aerospace control community in generating such metrics.

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