

Split-Step Simulations to Assess the Effects of Atmospheric Boundary Layer Turbulence on the Dose Variability of N-Waves and Shaped Booms

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Abstract

The effects of atmospheric boundary layer turbulence on the loudness variability of a sonic boom N-wave and shaped boom are examined with split-step simulations. The shaped boom is representative of a design iteration of the NASA X-59 aircraft. Inhomogeneous atmospheric boundary layer turbulence is generated in the computational domain by a Fourier synthesis method. The N-wave and shaped boom are propagated through turbulent fields representing eight different convection levels measured at the NASA Kennedy Space Center and the NASA Armstrong Flight Research Center. Probability density functions of the formation of caustic regions along the propagation direction are computed from the N-wave results, and a parameter to collapse the caustic PDFs that accounts for both fluctuation intensities and length scales is proposed. Statistical results concerning loudness metric variability are presented, and the standard deviations of several metrics are shown to collapse across different convection levels of turbulence for small nondimensional propagation distances. The loudness metric distributions are observed to be well approximated by a normal distribution for a given range of propagation distances, and become increasingly skewed as distance increases. A model function for the dose variability is proposed, and the function parameters are found to be related to the convection level of the turbulence. The model for the dose variability distribution is compared to simulation data that were not used to find the regression parameters of the model. At several nondimensional propagation distances, agreement is observed between the model and the simulation data. These results indicate that the model may be suitable for providing quick estimates of noise dose variability in the primary carpet region across a wide range of atmospheric boundary layer conditions.

Keywords: Sonic boom, Atmospheric acoustics, Atmospheric turbulence

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1. Introduction

Sonic boom flight tests indicate that turbulence present in the atmosphere leads to spiking and rounding of an N-wave signal [1, 2, 3]. N-wave spiking is exemplified in ground measurements shown in Fig. 1 from a single flyover at sixteen different microphone locations obtained during the Sonic Booms in Atmospheric Turbulence (SonicBAT) campaign [3]. Pierce [4] proposed that the physical mechanism that leads to spiking and rounding of sonic booms is the combination of focusing and diffraction of the wavefront caused by an inhomogeneous field of velocity and temperature fluctuations. Experiments conducted by Davy and Blackstock [5] qualitatively support this theory, and various other theoretical approaches have been proposed with similar qualitative support for spiking and rounding caused by turbulence [6, 7]. These past theoretical approaches have generally modeled the interaction of turbulence with the sonic boom from the viewpoint of geometrical acoustics, the acoustical analogue to geometrical optics, or with first-order scattering theory. Approximations of these theories, detailed by Pierce and Maglieri [8], limit their effectiveness in correctly predicting shock amplitude fluctuations and anomalously large shock wave rise times. Recent investigations of sonic boom propagation through turbulence have focused on numerical simulations, which have the advantage of avoiding some of the approximations in previous theories.

The objective of the present investigation is to utilize recent developments in computational approaches by simulating N-wave and shaped boom propagation in atmospheric boundary layer (ABL) turbulence to assess the impact of the turbulence intensity and integral length scale on the statistics of associated loudness metrics. To better understand these effects, a length scale parameter to nondimensionalize the propagation distance, previously reported by Carr et al. [9] for homogeneous turbulence, is modified to account for varying ABL conditions. This parameter is shown to collapse the standard deviations of five metrics associated with sonic boom perception for small propagation distances, which is helpful for the development of regression models of the evolution of the metric standard deviations. The simulation data gathered here is then leveraged to develop model probability density functions (PDFs) of these five metrics for both shaped waveforms and N-waves propagating through turbulence. Thus, the two key results of this investigation are 1) the development of the model PDFs for the purposes of quickly predicting the effects of atmospheric turbulence on sonic boom waveforms, and 2) the improved understanding of atmospheric turbulence effects on sonic boom waveforms achieved by examining simulation data and modifying the length scale parameter for ABL turbulence.

There is renewed interest among regulators and industry in the potential of future supersonic commercial aircraft, in lieu of future shortcomings in the U.S. aviation infrastructure [10]. Current regulations prevent the operation of civil aircraft in the supersonic regime when flying over land, due in part to community annoyance issues arising from sonic booms. Sonic boom N-waves generated by traditional aircraft configurations cause widespread noise complaints when impacting communities [11, 12]. In recent decades, aircraft shaping has been shown to alter the sonic boom signal created by an aircraft in supersonic flight [13, 14, 15, 16]. The NASA X-59 aircraft is being developed, as part of the Quesst mission, with boom shaping techniques in an attempt to reduce community annoyance levels. A primary objective of the Quesst mission is to provide flight regulators and standards committees with community test data from X-59 flight tests, such

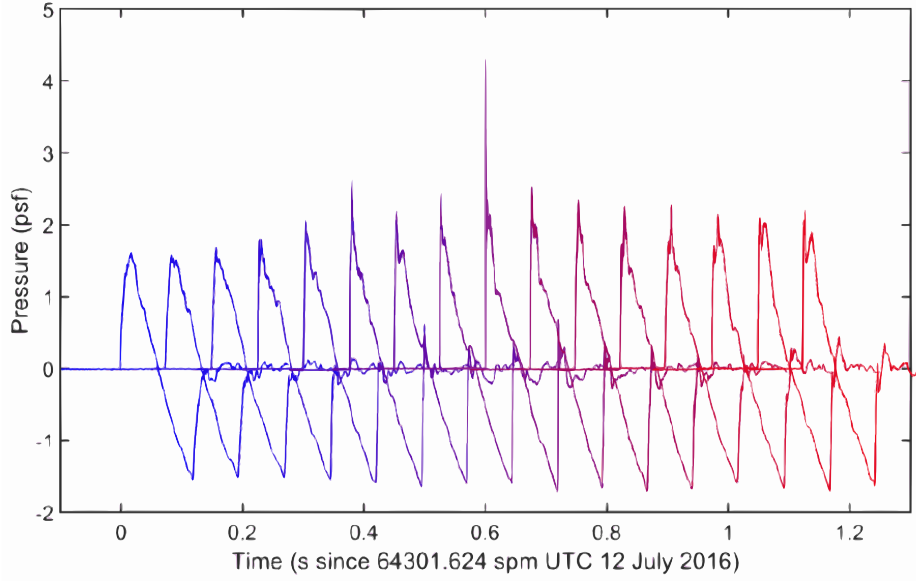


Figure 1: Ground measurements along the primary array for the BAT004 flyover (see Ref. [3]) at the NASA AFRC during SonicBAT. Signatures overlaid on the same plot with channel 1 in blue (left) and channel 16 in red (right). Figure 48 of Bradley et al. [3].

that current regulations may be reassessed using community annoyance data from the X-59 shaped boom.

When conducting flight test planning and survey design, it is useful to have an estimate of noise levels that communities will be subjected to [17, 18]. Geometrical acoustics codes, such as PCBoom [19, 20], offer a fast and effective numerical prediction of sonic boom noise levels by propagating an aircraft pressure signature down to the ground, accounting for nonlinear distortion, atmospheric absorption, and atmospheric stratification. However, these propagation codes have difficulties capturing the focusing, defocusing, and diffraction effects of atmospheric turbulence on the sonic boom. Methods have been devised for simulating turbulence effects within the geometrical acoustics approximation. Examples include the approach taken by Blanc-Benon et al. [21] to compute a probability density function related to the formation of caustics in a turbulent field, or the multiple scattering approach by Lonzaga [22] to predict the average waveform distortion. Although these approaches are useful for predicting the formation of caustics or the average sonic boom overpressure amplitude, they presently have no clear extension to predicting the variance of sonic boom loudness metrics in a turbulent field.

Recent numerical investigations have incorporated turbulence effects into sonic boom predictions; however, not without the shortcomings of either the parabolic approximation [23, 24, 25] or a turbulence model not representative of the atmospheric boundary layer (ABL) [26, 27, 9]. The Khokhlov-Zabolotskaya-Kuznetsov (KZK) equation [28, 29, 30] is commonly used for directional sound beams propagating in a medium in which diffraction effects are nonnegligible. In order to account for turbulence effects on sound propagation, Blanc-Benon et al. [31] integrated the KZK equation in time, and terms representing scalar and vector turbulence were included by Averianov et al. [32, 33]. Stout [23] developed a code to solve the augmented KZK equation

in two and three dimensions in a computational domain with a turbulent field meant to represent the ABL. A significant result of this effort was the development of Finite Impulse Response (FIR) filters, which convolve a waveform with a response function that represents the effect of atmospheric turbulence on the waveform. The code and FIR filters have been used in the SonicBAT [3] investigation to assess the variability and change in sonic boom metrics due to the influence of turbulence. However, the filters were not developed with shaped boom signals representative of future supersonic aircraft designs, and thus may not be applicable for shaped boom predictions.

The nature of the KZK equation and its parabolic approximation prevents accurate computation of the diffraction process as sonic boom waveforms propagate through turbulence. Coulouvrat [34] proposed several scalar wave equations of higher order than the KZK equation. Among these equations is a consistent second-order accurate governing equation that is the starting point of our analysis in Section 2. One-way numerical computations of reduced versions of this governing equation have been shown to accurately account for diffraction effects of ultrasound propagation through a circular aperture [26, 27]. The method has been extended to sonic boom propagation in homogeneous turbulence by Luquet [35]. The investigations by Dagrau et al. [26] and Luquet et al. [27] demonstrated the capability of the one-way approach to simulate sonic boom propagation while including the lowest order diffraction effects; however, not much focus was placed on examining the variability of loudness metrics in these simulation studies.

Loudness metrics provide a quantifiable measure of the potential annoyance that a sonic boom may cause to observers. Loubeau et al. [36] conducted a study of eleven different noise metrics and their ability to accurately predict annoyance levels of shaped sonic boom signals. The study used data from seven different human response studies including five indoor studies and two outdoor studies. Results show that five noise metrics stand out as being more correlated with human annoyance levels. These five metrics are perceived level (PL), B, D, and E weighted sound exposure levels, and ISBAP (Indoor Sonic Boom Annoyance level Predictor). The computational study presented in the SonicBAT report [3] found that noise metrics of sonic boom waveforms tend to decrease in average value as turbulence levels increase, and conversely, their variability increases. Probability distributions of the measured PL at both the NASA Kennedy Space Center (KSC) and the NASA Armstrong Flight Research Center (AFRC) indicate an approximately normal distribution of the metric with respect to different flight passes and measurement locations. This finding has been supported by predictions with a KZK-type equation [23, 37].

Community test data from the NASA X-59 aircraft will be leveraged to develop a dose-response curve, which relates a dose (in this context, “dose” refers to a sonic boom loudness metric level inferred at the observer location) to a participant’s level of annoyance (the response). Bayesian multilevel logistic regression (BMLR) has been proposed as a method to model the dose-response curve from community survey data [38]. However, the model assumes no variability in the noise dose estimate, where dose variability typically refers to the standard deviation of the particular sonic boom loudness metric under consideration. As suggested by the SonicBAT flight test results, ABL turbulence can contribute significantly to variability in the estimated noise dose for a participant. Variability of the noise dose estimate will flatten the dose-response curve resulting from the BMLR model [39]. Doeblér et al. [40] proposed a method to correct for noise dose variability in the BMLR model, using dose variabilities estimated from two previous sonic boom flight tests [41, 42]. When estimating dose variability, consideration was given to all sonic boom events

regardless of the atmospheric conditions at the time. However, previous computational simulations suggest that varying turbulence levels, and varying propagation distances through turbulence, will alter the magnitude of the standard deviation of several sonic boom loudness metrics. Therefore, it is likely that dose variability itself will vary at different times of day and different regions of the sonic boom carpet, and a combination of simulations and ground measurements may be required to obtain improved estimates of dose variability.

In this investigation, numerical simulations of sonic boom propagation are performed through ABL turbulence that is representative of two climate regions, the hot-humid climate at KSC and the hot-dry climate at AFRC. We provide insight into the effect of turbulence on the loudness levels of both N-waves and shaped booms, and propose approximate models of their distributions. Such models provide quick estimates of dose variability for engineers involved in survey design or flight test planning of future supersonic aircraft designs.

The remainder of this paper is structured as follows. Section 2 provides an overview of the computational approach used to simulate sonic boom propagation in the ABL. The process for generating representative turbulent fields in our simulations is highlighted in Sec. 3. A length scale to normalize the propagation direction is formulated in Sec. 4. Section 5 presents the standard deviations and distributions of several loudness metrics for both N-waves and shaped booms in each atmospheric condition, along with the nonlinear regression model of the distributions. Finally, conclusions are discussed in Sec. 6.

2. Computational Approach

The prediction approach taken here is to first obtain a nonlinear waveform near the source, using an F-function calculation for the N-wave and a CFD result for a design iteration of the NASA X-59 for the shaped boom. PCBoom is then utilized to propagate the source waveform down to the ABL altitude, z_i . An illustration of this propagation is provided in Fig. 2. PCBoom is a NASA sonic boom propagation code used to predict acoustic ray paths and account for nonlinear distortion, thermoviscous as well as molecular absorption, and geometrical spreading [20, 43, 44]. We then use the PCBoom prediction as input to our propagation simulations, which simulate propagation from z_i to ground level ($z = 0$) and account for ABL turbulence effects on the sonic boom. Figure 2b provides an illustration of our 2D computational domain along with the incident ray at z_i .

The ABL coordinate system is Cartesian (x, y, z) , where x and y are parallel to the ground, and z is the altitude in the ABL. The height of the ABL is denoted by z_i . The flow and atmospheric properties are defined in (x, y, z) . The sonic boom wavefront is incident at the top of the ABL at an angle θ_{elv} relative to the ground plane. The simulations performed here in the ABL do not use a geometrical acoustics assumption. Straight line propagation from the top of the ABL to the ground is assumed, thus, neglecting the effect of mean atmospheric stratification in the ABL region. Since propagation in the ABL is only a small portion of the total propagation distance of the sonic boom waveform, it is likely that stratification effects in this region are negligible. It is worth noting however, that stratification effects on the boom may be non-negligible for grazing propagation near the lateral cutoff region. The assumption here for long propagation distances to the cutoff region is that the stratification effects are not significantly affecting the sonic boom

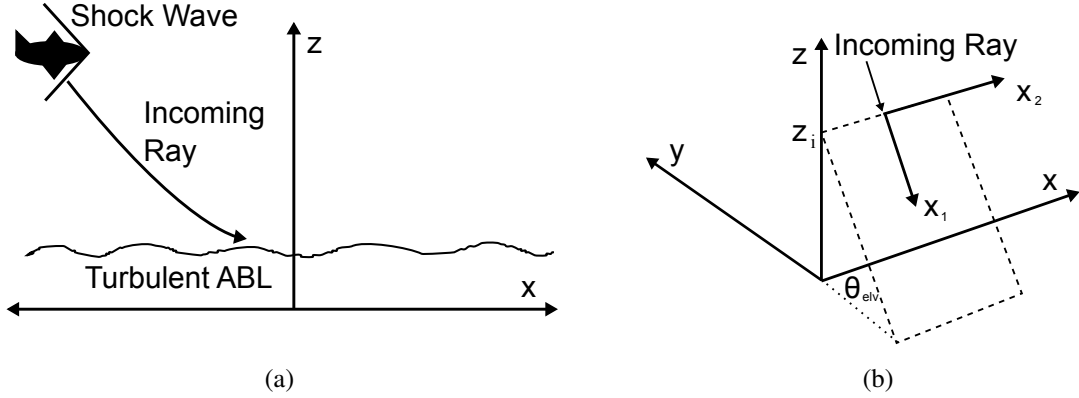


Figure 2: (a) Illustration of ray propagation from source to ABL, and (b) coordinate system of simulations in the ABL.

loudness variability in the carpet region. This assumption is made to justify the use of straight line propagation in this region, and straight line propagation must be assumed in order to generate synthetic turbulence in the domain according to the method in Sec. 3.

We perform 2D simulations, because recent investigation of sonic boom propagation through homogeneous turbulence has suggested that loudness metrics statistics obtained from 2D simulations are in good agreement with statistics obtained from 3D simulations (see [23], [45], and [46]). The assumption is made that the statistics for simulations through inhomogeneous turbulence would have similar agreement. Simulations performed in 2D are less computationally intensive than 3D simulations, so we can perform more simulations to assess more meteorological setpoints and obtain better converged statistics than if 3D simulations were performed. The coordinate system of the computations is (x_1, x_2) , where x_1 is the propagation direction and x_2 is the transverse coordinate. Computations begin at $x_1 = 0$. The waveform is propagated in the x_1 direction until the ground is reached. We do not consider ground reflections in our simulations, and more information on the simulations is provided in Sec. 3.4.

The time it takes the sonic boom to propagate a distance of ℓ is approximately $t_\ell = \ell c_0^{-1}$, where c_0 is the speed of sound at standard sea-level conditions. Suppose that ℓ is the size of a turbulent eddy in the ABL that moves with velocity u_c , such that the characteristic time scale is $t_c = \ell u_c^{-1}$. Then, if $t_\ell/t_c \ll 1$, we can assume that there is very little dynamical evolution of the turbulent eddy for the duration of propagation of the sonic boom. This inequality holds true if $u_c/c_0 \ll 1$, which is true for typical ABL conditions [47]. Thus, by Taylor's frozen field hypothesis [48], the turbulent field is considered to be only a function of the spatial coordinates. This is a common assumption used in previous sonic boom propagation studies [49, 23, 35, 9].

2.1. Governing Equation

The equations of mass, momentum, energy, and state govern the propagation of the sonic boom in the turbulent ABL. Coulouvrat [34] performed a decomposition on the governing equations and

combined them to obtain

$$\begin{aligned} \frac{1}{\check{c}^2} \frac{\check{D}^2 p}{\check{D} t^2} - \check{\rho} \frac{\partial}{\partial x_i} \left(\frac{1}{\check{\rho}} \frac{\partial p}{\partial x_i} \right) = & -2 \frac{\partial \check{u}_j}{\partial x_i} \int_{-\infty}^t \frac{\partial^2 p}{\partial x_i \partial x_j} dt' - 2 \frac{\partial \check{u}_j}{\partial x_i} \left(\check{u}_k \int_{-\infty}^t \int_{-\infty}^{t'} \frac{\partial^3 p}{\partial x_j \partial x_k \partial x_i} dt'' dt' \right. \\ & \left. + \frac{\partial \check{u}_k}{\partial x_j} \int_{-\infty}^t \int_{-\infty}^{t'} \frac{\partial^2 p}{\partial x_k \partial x_i} dt'' dt' + \frac{\partial^2 \check{u}_i}{\partial x_j \partial x_k} \int_{-\infty}^t \int_{-\infty}^{t'} \frac{\partial p}{\partial x_k} dt'' dt' \right) + \frac{\delta}{\check{c}^4} \frac{\partial^3 p}{\partial t^3} + \frac{\beta}{\check{\rho} \check{c}^4} \frac{\partial^2 p^2}{\partial t^2}, \quad (1) \end{aligned}$$

where variables with a breve are associated with the mean flow and turbulence in the ABL, p is the acoustic pressure, $\check{u}_i$ is the flow velocity, $\check{c}$ is the speed of sound, $\check{\rho}$ is the density, δ is the diffusivity of sound [50], β is the nonlinear parameter, x_i are the spatial coordinates, and t is the time. The material derivative is defined as,

$$\frac{\check{D}}{\check{D} t} = \frac{\partial}{\partial t} + \check{u}_i \frac{\partial}{\partial x_i}. \quad (2)$$

For a weakly nonlinear wave, Eq. 1 is accurate to second-order, $O(\epsilon M^2, \epsilon^2)$, where ϵ is the acoustic Mach number defined as $\epsilon = \|u_i\| \check{c}^{-1}$, u_i is the acoustic velocity, and $M = \|\check{u}_i\| \check{c}^{-1}$ is the flow Mach number. In the ABL, the acoustic Mach number is considered to be on the order of M^2 [34].

Retaining the flow interaction terms of $O(\epsilon M^2)$ on the right-hand side of Eq. 1 can cause numerical instabilities when the Mach number is not sufficiently small [34]. Therefore, a mixed-order governing equation is the starting point of our numerical analysis,

$$\frac{1}{\check{c}^2} \frac{\check{D}^2 p}{\check{D} t^2} - \check{\rho} \frac{\partial}{\partial x_i} \left(\frac{1}{\check{\rho}} \frac{\partial p}{\partial x_i} \right) = -2 \frac{\partial \check{u}_j}{\partial x_i} \int_{-\infty}^t \frac{\partial^2 p}{\partial x_i \partial x_j} dt' + \frac{\delta}{\check{c}^4} \frac{\partial^3 p}{\partial t^3} + \frac{\beta}{\check{\rho} \check{c}^4} \frac{\partial^2 p^2}{\partial t^2}, \quad (3)$$

where the $O(\epsilon M^2)$ terms have been retained only on the left-hand side. The same terms have been neglected in previous sonic boom simulations [27].

Direct computation of Eq. 3 requires significant computational resources to capture both the forward and backward scattering. In Section 2.2, a partially one-way equation based on the approach of Luquet [35] is formulated that neglects backward scattering, significantly reducing the computational resources required to perform the simulations.

2.2. Partially One-Way Equation

By neglecting back scattering, the solution to Eq. 3 can be marched forward along the propagation direction. Parabolic, wide-angle parabolic, and extra wide-angle parabolic approximations are commonly made to the second derivatives of acoustic pressure in the propagation direction to make the equation more suitable for marching schemes, such as the Crank-Nicolson method [51]. When these approximations are made to the leading order diffraction terms, the accuracy of the computations are limited to scattering angles that satisfy the parabolic assumption. To avoid this restriction, no approximations are made to the lowest order diffraction terms in the foregoing analysis following the approach of Dagrau et al. [26]. Eq. 3 is now recast in the form of a wave equation,

$$\begin{aligned} \frac{\partial^2 p}{\partial t^2} - c_0^2 \frac{\partial^2 p}{\partial x_i \partial x_i} = & (2c_0 c'' + c''^2) \frac{\partial^2 p}{\partial x_i \partial x_i} - 2\check{u}_i \frac{\partial^2 p}{\partial x_i \partial t} - \check{u}_j \frac{\partial}{\partial x_j} \left(\check{u}_i \frac{\partial p}{\partial x_i} \right) \\ & - \frac{\check{c}^2}{\check{\rho}} \frac{\partial \check{\rho}}{\partial x_i} \frac{\partial p}{\partial x_i} - 2\check{c} \frac{\partial \check{u}_j}{\partial x_i} \int_{-\infty}^t \frac{\partial^2 p}{\partial x_i \partial x_j} dt' + \frac{\delta}{\check{c}^2} \frac{\partial^3 p}{\partial t^3} + \frac{\beta}{\check{\rho} \check{c}^2} \frac{\partial^2 p^2}{\partial t^2}, \quad (4) \end{aligned}$$

where c_0 is the speed of sound at ground level and $c'' = \check{c} - c_0$. The terms on the right-hand side of Eq. 4 are associated with mean flow, turbulence, nonlinear distortion, and atmospheric absorption. We consider only two spatial dimensions, x_1 and x_2 . Thus, the indices i and j take on values of 1 and 2 only.

It is advantageous to consider Eq. 4 in terms of the delayed time $\tau = t - c_0^{-1}x_1$. The longitudinal (x_1) second derivative operator on the right-hand side of Eq. 4 does not lend itself to one-way integration methods. Thus, we make the approximation that this operator may be replaced by the other terms in the Helmholtz equation, as such

$$\frac{\partial^2 p}{\partial x_1^2} = \frac{2}{c_0} \frac{\partial^2 p}{\partial x_1 \partial \tau} - \frac{\partial^2 p}{\partial x_2^2} + O(\epsilon M). \quad (5)$$

This approximation is similar to a wide-angle approximation; however, the lowest order terms on the left-hand side of Eq. 4 remain untouched and the approximation is only made to higher order terms. Thus, applying this approximation to Eq. 4 yields

$$\begin{aligned} \frac{\partial^2 p}{\partial x_1 \partial \tau} - \frac{c_0}{2} \frac{\partial^2 p}{\partial x_1^2} - \frac{c_0}{2} \frac{\partial^2 p}{\partial x_2^2} &= \frac{2c_0 c'' + c''^2}{2c_0^3} \frac{\partial^2 p}{\partial \tau^2} - \frac{\check{u}_1}{c_0} \left[\frac{\partial^2 p}{\partial x_1 \partial \tau} - \frac{1}{c_0} \frac{\partial^2 p}{\partial \tau^2} \right] - \frac{\check{u}_2}{c_0} \frac{\partial^2 p}{\partial x_2 \partial \tau} \\ &\quad - \frac{\check{u}_1}{2c_0} \frac{\partial \check{u}_1}{\partial x_1} \left[\frac{\partial p}{\partial x_1} - \frac{1}{c_0} \frac{\partial p}{\partial \tau} \right] - \frac{\check{u}_2}{2c_0} \frac{\partial \check{u}_1}{\partial x_2} \left[\frac{\partial p}{\partial x_1} - \frac{1}{c_0} \frac{\partial p}{\partial \tau} \right] - \frac{\check{u}_1}{2c_0} \frac{\partial \check{u}_2}{\partial x_1} \frac{\partial p}{\partial x_2} - \frac{\check{u}_2}{2c_0} \frac{\partial \check{u}_2}{\partial x_2} \frac{\partial p}{\partial x_2} \\ &\quad - \frac{\check{u}_1^2}{2c_0} \left[\frac{1}{c_0^2} \frac{\partial^2 p}{\partial \tau^2} - \frac{\partial^2 p}{\partial x_2^2} \right] - \frac{\check{u}_1 \check{u}_2}{c_0} \frac{\partial}{\partial x_2} \left[\frac{\partial p}{\partial x_1} - \frac{1}{c_0} \frac{\partial p}{\partial \tau} \right] - \frac{\check{u}_2^2}{2c_0} \frac{\partial^2 p}{\partial x_2^2} - \frac{\check{c}^2}{2c_0 \check{\rho}} \frac{\partial \check{\rho}}{\partial x_1} \left[\frac{\partial p}{\partial x_1} - \frac{1}{c_0} \frac{\partial p}{\partial \tau} \right] \\ &\quad - \frac{\check{c}^2}{2c_0 \check{\rho}} \frac{\partial \check{\rho}}{\partial x_2} \frac{\partial p}{\partial x_2} - \frac{\check{c}^2}{c_0} \frac{\partial \check{u}_1}{\partial x_1} \int_{-\infty}^{\tau} \left[\frac{1}{c_0^2} \frac{\partial^2 p}{\partial \tau'^2} - \frac{\partial^2 p}{\partial x_2^2} \right] d\tau' - \frac{\check{c}^2}{c_0} \left(\frac{\partial \check{u}_2}{\partial x_1} + \frac{\partial \check{u}_1}{\partial x_2} \right) \int_{-\infty}^{\tau} \frac{\partial}{\partial x_2} \left[\frac{\partial p}{\partial x_1} - \frac{1}{c_0} \frac{\partial p}{\partial \tau'} \right] d\tau' \\ &\quad - \frac{\check{c}^2}{c_0} \frac{\partial \check{u}_2}{\partial x_2} \int_{-\infty}^{\tau} \frac{\partial^2 p}{\partial x_2^2} d\tau' + \frac{\delta}{2c_0^3} \frac{\partial^3 p}{\partial \tau^3} + \frac{\beta}{2\rho_0 c_0^3} \frac{\partial^2 p^2}{\partial \tau^2}, \quad (6) \end{aligned}$$

where index notation has been dropped for clarity. Equation 6 is the equation to be solved numerically.

2.3. Split-Step Computations

The resulting partially one-way equation can be categorized into terms accounting for each of the physical effects present: diffraction, heterogeneities, nonlinear distortion, and absorption. The governing partially one-way equation is expressed as

$$\frac{\partial^2 p}{\partial x_1 \partial \tau} = \mathcal{D}(p) + \mathcal{H}(p) + \mathcal{N}(p) + \mathcal{A}(p), \quad (7)$$

where each of the terms on the right-hand side is expressed fully in Appendix A. Diffraction effects are computed with the angular spectrum method [52]. Heterogeneous effects on the sonic boom are computed with a combination of an exact solution for the phase terms and Crank-Nicolson integration of the transverse terms [35]. Nonlinear effects are computed with a Burgers-Hayes algorithm [53] and attenuation from atmospheric absorption is computed assuming no dispersion, according to the absorption factor provided by Bass et al. [54]. The numerical methods are described in more detail in Luquet [35] and Carr [55].

Fractional step methods [56] are employed to compute the solution to differential equations of the form,

$$\frac{df}{dt} = \mathcal{L}(f), \quad (8)$$

where $\mathcal{L}$ is an operator. When computing the solution of Eq. 8, operator splitting may increase speed or accuracy of the integration in time [56]. Strang [57] proposed a splitting technique of second-order accuracy to solve the linear hyperbolic problem,

$$\frac{\partial f}{\partial t} = A \frac{\partial f}{\partial x} + B \frac{\partial f}{\partial y} \quad \text{with} \quad f(0) = f_0. \quad (9)$$

The solution is a composition of Lax-Wendroff [58] operators L ,

$$S_k = L_{k/2}^x \circ L_k^y \circ L_{k/2}^x, \quad (10)$$

where k is the step size.

Strang [57] applied the proposed splitting technique to split the problem into several one-dimensional problems and solve with Lax-Wendroff integration. Here, we will not split Eq. 7 into several one-dimensional problems; rather, the terms in Eq. 7 are split into several groups representing different physical effects. This is the approach adopted by Luquet [35], where the composed solution for the acoustic pressure is

$$p(x_1 + \Delta x_1, x_2, x_3, \tau) = p_{\Delta x_1/2}^N \circ p_{\Delta x_1/2}^{H+A} \circ p_{\Delta x_1}^D \circ p_{\Delta x_1/2}^{H+A} \circ p_{\Delta x_1/2}^N + O(\Delta x_1^3). \quad (11)$$

3. Synthetic Atmospheric Boundary Layer Turbulence

3.1. Model of Turbulence in the Convective Boundary Layer

The convective ABL is composed of the surface layer, mixed layer, and the interfacial layer that separates the ABL and the free troposphere. In Fig. 3, an illustration of the ABL during daytime convective conditions is shown. In the surface layer, gradients of the velocity and temperature are well described by the Monin-Obukhov Similarity Theory (MOST) [59]. The surface layer comprises approximately the lowest 10% of the ABL during the daytime. The mixed layer is dominated by convection and is often referred to as the convective layer during daytime conditions. In the convective layer, the mean wind speed and temperature are approximately constant with altitude. The temperature inversion height, z_i , is generally between 0.5 to 2 km during the daytime. For the purpose of this investigation, we consider z_i to be the interface between the free troposphere and the ABL. The input waveforms to our simulations are the results of propagation through the atmosphere to an altitude of z_i using PCBoom. We restrict our attention to daytime conditions, when the flow is unstable, since sonic boom test flights are typically flown during the daytime. We do not consider the effect of intermittency in our turbulence model and do not attempt to model the wind and temperature gradients in the inversion layer.

In the atmospheric surface layer, $z \leq 0.1z_i$, the intensity and length scale of the temperature fluctuations are influenced by T_* and L_o , which in turn are influenced directly by u_* and Q_0 . The Obukhov length scale is L_o , u_* is the friction velocity, Q_0 is the heat flux from the ground into the

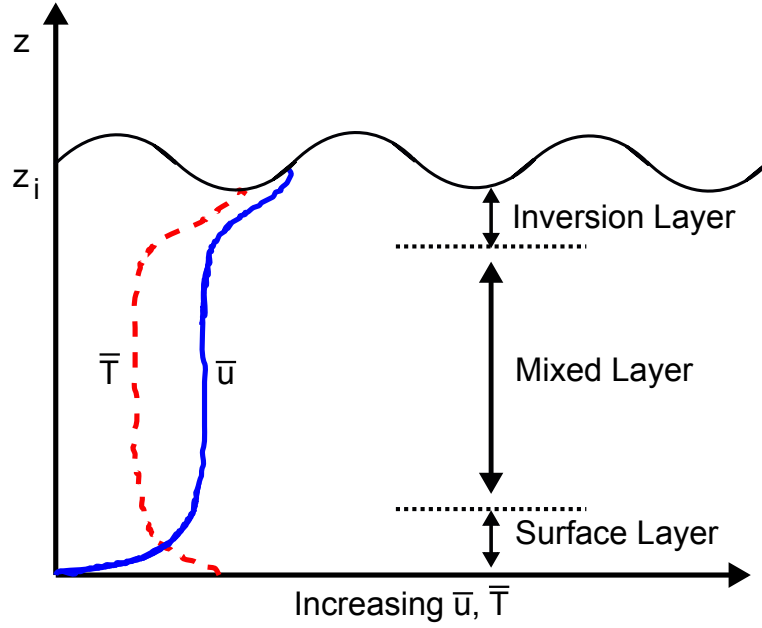


Figure 3: Illustration of the ABL during convective conditions.

atmosphere, and $T_* = -Q_0 u_*^{-1}$ is the temperature scale. The temperature scale, T_* , will be negative when Q_0 is positive, which is true for convective conditions. The expressions for the standard deviation of the temperature fluctuations σ_T , and the von Kármán length scale L_T , determined by Ostashev and Wilson [60] are

$$\frac{\sigma_T^2(z)}{T_*^2} = \frac{4}{[1 - 10\xi]^{2/3}} \quad (12)$$

and

$$\frac{L_T(z)}{z} = 2 \frac{1 - 7\xi}{1 - 10\xi}, \quad (13)$$

where $\xi = zL_o^{-1}$ is the nondimensional altitude.

Experimental evidence [61] indicates that the horizontal spectra of velocity fluctuations do not obey MOST in unstable conditions. For the velocity fluctuations, there are two production mechanisms to consider. The first is turbulence generated by mean shear near the surface, in which the length scale of the fluctuations increases as a function of altitude. The standard deviation and von Kármán length scale of the shear generated turbulence are provided by Ostashev and Wilson [51] as

$$\frac{\sigma_s^2}{u_*^2} = 3.0 \quad (14)$$

and

$$\frac{L_s}{z} = 1.8. \quad (15)$$

The second mechanism is bouyancy, where temperature gradients cause hotter air near the surface to rise in altitude. Bouyancy driven turbulence is dominated by mixed-layer similarity, and the expressions for the standard deviation and length scale are, respectively,

$$\frac{\sigma_b^2}{w_*^2} = 0.35 \quad (16)$$

and

$$\frac{L_b}{z_i} = 0.23, \quad (17)$$

where w_* is the mixed layer velocity scale [51].

The synthetic velocity fluctuations are generated with the Generalized Random Phase Method (GRPM) [62] using a von Kármán spectral model that accounts for both shear-driven and bouyancy-driven fluctuations. The 1D autospectra for the von Kármán model are [62]

$$\hat{R}_{xx}(k_x, z, z') = \frac{2\sigma_u^2 L_u^2}{\sqrt{\pi}\Gamma(1/3)} \left(\frac{\zeta/2}{1 + k_x^2 L_u^2} \right)^{5/6} \left[K_{5/6}(\zeta) - \frac{\zeta}{2} K_{1/6}(\zeta) \right], \quad (18)$$

$$\hat{R}_{zz}(k_x, z, z') = \frac{2\sigma_u^2 L_u^2}{\sqrt{\pi}\Gamma(1/3)} \left(\frac{\zeta/2}{1 + k_x^2 L_u^2} \right)^{5/6} \left[\frac{4}{3} K_{5/6}(\zeta) - \frac{\zeta/2}{1 + k_x^2 L_u^2} K_{11/6}(\zeta) \right], \quad (19)$$

and

$$\hat{R}_T(k_x, z, z') = \frac{2\sigma_T^2 L_T^2}{3\sqrt{\pi}\Gamma(1/3)} \left(\frac{\zeta/2}{1 + k_x^2 L_u^2} \right)^{5/6} \left[\frac{11}{3} K_{5/6}(\zeta) - \frac{\zeta}{1 + k_x^2 L_T^2} K_{11/6}(\zeta) \right], \quad (20)$$

where Γ is the Gamma function, $\zeta = \|z - z'\| L_u^{-1} (1 + k_x^2 L_u^2)^{1/2}$, $\sigma_u^2 = \sigma_s^2 + \sigma_b^2$, and $\sigma_u^2 L_u = \sigma_s^2 L_s + \sigma_b^2 L_b$. The expressions for σ_u and L_u effectively interpolate between the shear and bouyancy induced turbulence. This technique was proposed by Højstrup [63] and is commonly used for sound propagation simulations in ABL turbulence [23, 64, 65, 24].

The length scales and standard deviations in Eqs. 12 through 17 are functions of the altitude, z . However, the computational domain is inclined at an angle, θ_{elv} , from the horizon, which means that the propagation direction, x_1 , is not aligned with z . The length scales and standard deviations may be formulated as a function of x_1 if z is replaced by $(d - x_1) \sin(\theta_{\text{elv}})$, where d is the total propagation distance. The transverse direction x_2 is assumed to be aligned with x , and therefore, the turbulent fluctuations along the x_2 direction are homogeneous.

The expressions for the von Kármán length scales and standard deviations of the temperature and shear-driven fluctuations provided in Eqs. 12 through 15 are valid in the surface layer. In a daytime convective ABL, the mean profiles of temperature and velocity are approximately constant

in the mixed layer region [47, 66]. Past investigations of sound propagation in the ABL [65, 64] have assumed that σ_T , L_T , σ_s , and L_s are constant for $z \geq 0.1z_i$, and equal to their value at $0.1z_i$. We make the same assumption here, since these past investigations have shown good agreement with measurements. For the buoyancy-driven fluctuations, the model is valid for $z \leq 0.9z_i$, but we will assume here that it is valid for $z \leq z_i$, since intermittency is neglected.

3.2. Generation of Synthetic Turbulence

The GRPM method of Wilson [62] is briefly reviewed in this section to provide context on how we generate inhomogeneous turbulence in the computational domain. The velocity fluctuation field associated with turbulence is denoted by u'_i . Lumley [67] introduced proper orthogonal decomposition (POD) as a technique for representing turbulent velocities by an orthonormal basis, such that

$$u'_i(x_1, x_2, x_3) = \sum_{n=1}^{\infty} a_n \phi_i^{(n)}(x_1, x_2, x_3). \quad (21)$$

Since we only consider velocity fluctuations in the (x_1, x_2) plane, we will neglect the x_3 dimension. POD aims to maximize the projection of u'_i onto ϕ_i . Lumley [67] showed that this is achieved with the following eigenvalue problem,

$$\int \int R_{ij}(x_1, x_2; x'_1, x'_2) \phi_j^{(m)}(x'_1, x'_2) dx'_1 dx'_2 = \lambda_{\text{eig}}^{(n)} \phi_i^{(n)}(x_1, x_2), \quad (22)$$

where ϕ are the eigenvectors and λ_{eig} the eigenvalues. This general technique is used in other areas of the natural sciences, and also goes by Karhunen-Loève decomposition, principal component analysis, and empirical orthogonal function analysis.

In the case of atmospheric boundary layer turbulence, where it is assumed that the turbulent fluctuations along coordinates parallel to the ground plane are homogeneous, the POD technique can be leveraged to generate a kinematic field of turbulence. In this situation, a Fourier transform is applied to Eq. 22 in the x_1 and x_2 coordinates to obtain,

$$\int \hat{R}_{ij}(k_1, x_2, x'_2) \hat{\phi}_j^{(m)}(k_1, x'_2) dx'_2 = \lambda_{\text{eig}}^{(n)}(k_1) \hat{\phi}_i^{(n)}(k_1, x_2). \quad (23)$$

The von-Kármán model for $\hat{R}_{ij}$, provided in Eqs. 18, 19, and 20 for ABL coordinates, is used to compute the eigenvalues and eigenvectors.

Provided with $\lambda_{\text{eig}}^{(n)}(k_1)$ and $\hat{\phi}_i^{(n)}(k_1, x_2)$, a Fourier velocity field satisfying the von-Kármán correlation model can be constructed. We first note that the correlation tensor can be expressed as

$$\hat{R}_{ij}(k_1, x_2, x'_2) = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \langle a_n(k_1) a_m^*(k_1) \rangle \hat{\phi}_i^{(n)}(k_1, x_2) \hat{\phi}_j^{*(m)}(k_1, x'_2). \quad (24)$$

Additionally, the correlation tensor itself has a decomposition of the form

$$\hat{R}_{ij}(k_1, x_2, x'_2) = \sum_{n=1}^{\infty} \lambda_{\text{eig}}^{(n)}(k_1) \hat{\phi}_i^{(n)}(k_1, x_2) \hat{\phi}_j^{*(n)}(k_1, x'_2), \quad (25)$$

which satisfies Eq. 23 subject to the orthogonality condition,

$$\int \hat{\phi}_j^{*(n)}(k_1, x'_2) \hat{\phi}_j^{(m)}(k_1, x'_2) dx'_2 = \delta_{nm}, \quad (26)$$

where δ_{nm} is the Kronecker delta function. Thus, a requirement for coefficients $a_n(k_1)$ is that

$$\langle a_n(k_1) a_m^*(k_1) \rangle = \lambda_{\text{eig}}^{(n)}(k_1) \delta_{nm}. \quad (27)$$

The coefficients satisfying this relation can be generated by setting,

$$a_n(k_1) = \gamma_n \sqrt{\lambda_{\text{eig}}^{(n)}(k_1)}, \quad (28)$$

where γ_n are complex random numbers drawn from a uniform distribution with standard deviation of 1. The Fourier velocity fields are generated by computing the summation,

$$\hat{u}'_i(k_1, x_2) = \sum_{n=1}^N \gamma_n \sqrt{\lambda_{\text{eig}}^{(n)}(k_1)} \hat{\phi}_i^{(n)}(k_1, x_2), \quad (29)$$

and the inverse Fourier transform is performed to obtain u_1 and u_2 .

3.3. Turbulence Data and Simulated Conditions

According to Wilson [68], the convective boundary layer is considered to have weak convection levels when the ratio of inversion height to L_o is on the order of one, $\log(-z_i L_o^{-1}) \sim 0$. Wilson also defined moderate convection as $\log(-z_i L_o^{-1}) \sim 1$ and strong convection as $\log(-z_i L_o^{-1}) \gtrsim 2$. The parameters u_* , Q_0 , and z_i for our simulations are determined from previous experimental measurements. For the purposes of this investigation, we consider weak convection to be $\log(-z_i L_o^{-1}) \leq 0.5$, moderate convection to be $0.5 < \log(-z_i L_o^{-1}) \leq 1.5$, and strong convection to be $\log(-z_i L_o^{-1}) > 1.5$. The exact ranges of the convective parameter, $\log(-z_i L_o^{-1})$, corresponding to weak, moderate, and strong convection levels were defined in order to categorize each ABL setpoint.

The turbulence intensity depends on u_* and Q_0 , and the von Kármán length scales depend on z_i . Measurements of these parameters were made during the SonicBAT flight test campaign, and the remaining parameters (w_* , T_* , and L_o) are computed from their definitions found in Ostashev and Wilson [51]. The parameter space of the ABL that we consider here is based on measurements made at the NASA Kennedy Space Center (KSC) and the NASA Armstrong Flight Research Center (AFRC) during the SonicBAT campaign. The flight passes considered here were chosen based on the convection levels of the ABL, with the goal of considering several cases corresponding to weak, moderate, and strong convection. The ABL parameters for a few of the flight passes at KSC considered here are listed in Table 1, arranged in order of increasing $\log(-z_i L_o^{-1})$. The number after the letters “KSC” denotes the flight pass from the SonicBAT report. Three ABL setpoints from AFRC are also considered, representing strong convection levels. These setpoints do not correspond to flight passes, but rather meteorological measurements performed on different days. They are denoted with numbers 1, 2, and 3.

Table 1: ABL conditions of several flight passes during SonicBAT at KSC and AFRC.

Flight No.	Q_0 (K·m/s)	u_* (m/s)	z_i (m)	w_* (m/s)	T_* (K)	$\log(-z_i L_o^{-1})$
KSC20	0.010	0.345	411.6	0.512	-0.029	0.115
KSC1	0.025	0.375	457.3	0.720	-0.067	0.451
KSC12	0.095	0.468	640.2	1.253	-0.203	0.885
KSC6	0.225	0.390	457.3	1.484	-0.577	1.342
KSC17	0.188	0.315	487.8	1.428	-0.597	1.571
AFRC1	0.306	0.461	1000.0	2.138	-0.664	1.601
AFRC2	0.293	0.378	1347.0	2.324	-0.775	1.968
AFRC3	0.272	0.252	1344.0	2.258	-1.079	2.459

For each setpoint, 20 individual realizations of turbulence are generated, and the sonic boom loudness metrics, overpressure, and rise time are probed at 2048 points for each x_1 plane in each simulation. This results in a total of 40,960 predictions per x_1 plane that are used to calculate statistics. The combination of spatial averaging and ensemble averaging is justified by the homogeneity of the turbulent field in the x_2 direction and by the fact that we start with the same initial waveform for the N-wave and shaped boom simulations, respectively. Also, this type of averaging approach has been previously used by Stout [23]. The standard deviations along x_1/d , or equivalently z/z_i , for each atmospheric setpoint were computed from the synthesized fields to validate that the standard deviations of the generated fields agree with the prescribed values. Figure 4 shows the computed σ_T and σ_u as a function of z/z_i compared to the prescribed functions (Eq. 12 and $\sigma_u^2 = \sigma_s^2 + \sigma_b^2$). The computed standard deviations agree quite well with the model values, and do not deviate from the prescribed values by more than 0.1 K or 0.1 m/s. The overall difference between the prescribed and synthesized values is no more than 3% for any setpoint. This was determined by computing the L2 norm.

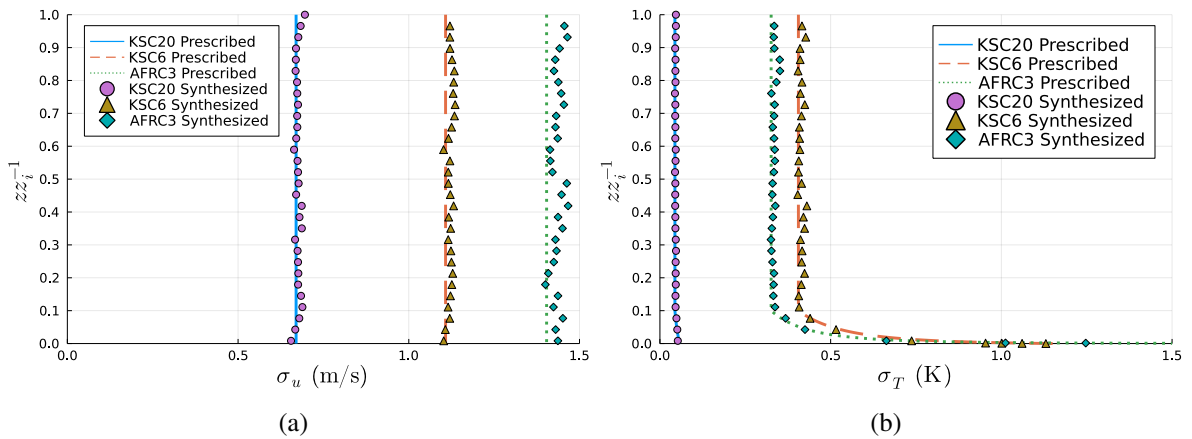


Figure 4: Standard deviations of the velocity and temperature fluctuations for three different ABL setpoints compared to model values: (a) σ_u and (b) σ_T .

Table 2 shows the standard deviations and von Kármán scales of the turbulence generated in the mixed-layer region of the ABL using Eqs. 12 through 17. Each flight number corresponds to turbulence generated for meteorological data shown in Table 1 for the same flight number. The weak convection setpoints (KSC20 and KSC1) correspond to lower temperature and velocity fluctuation intensity. In contrast, the strong convection setpoints (AFRC1-3) have the largest standard deviations of the velocity fluctuation, and thus correspond to stronger turbulence intensities. A similar trend is observed for the length scales, which are largest for the AFRC setpoints and smallest for the weakly convective setpoints.

Table 2: Turbulence intensities and von Kármán length scales in the mixed layer region of the model ABL corresponding to meteorological measurements obtained during the SonicBAT program.

Flight No.	σ_T (K)	L_T (m)	σ_u (m/s)	L_u (m)
KSC20	0.044	68.3	0.67	78.3
KSC1	0.086	71.2	0.78	89.2
KSC12	0.198	94.1	1.10	129.8
KSC6	0.406	65.2	1.11	96.7
KSC17	0.354	69.1	1.01	105.0
AFRC1	0.385	141.5	1.50	215.8
AFRC2	0.341	189.4	1.52	297.4
AFRC3	0.327	188.4	1.41	302.6

3.4. Simulation Conditions and Initial Waveforms

The flight conditions considered here correspond to a sonic boom N-wave produced by an F-18 aircraft and a shaped sonic boom signal produced by a design iteration of the NASA X-59. Both aircraft were assumed to be in steady level flight at an altitude of 53,200 ft, with a cruise speed of $M = 1.34$ for the F-18 and $M = 1.40$ for the X-59. This represents the on-design condition of the X-59. As for the F-18, it was set to the same flight altitude of the X-59 for the purposes of obtaining a predicted N-wave above the ABL in PCBoom that has a similar amplitude to the X-59 waveform. Rays were propagated from both source conditions at an emission angle, ϕ , of 0° , i.e., directly beneath the aircraft, to an altitude of 1 km above the ground in PCBoom. The waveforms at 1 km and $\phi = 0^\circ$ are shown in Fig. 5. They are assumed to be representative of waveforms measured just above the ABL. The N-wave prediction was obtained assuming a standard atmospheric profile with no wind, and the X-59 prediction was obtained using balloon measurements of the atmosphere in Galveston, Texas during QSF18 [42]. The atmospheric data from Galveston is presented in Carr [55]. Before propagation through the ABL, both waveforms are filtered with Lanczos resampling to prevent Gibbs oscillations from appearing in the signal, more details on the filter are given in Appendix A.

For different emission angles and ABL altitudes (ranging from 0.5 to 2 km) there will be slight changes to the waveform amplitudes and durations than what is observed in Fig. 5. However, we will assume that these slight changes in waveform characteristics do not significantly alter the way that turbulence affects both waveforms. Thus, the waveforms in Fig. 5 represent the nominal

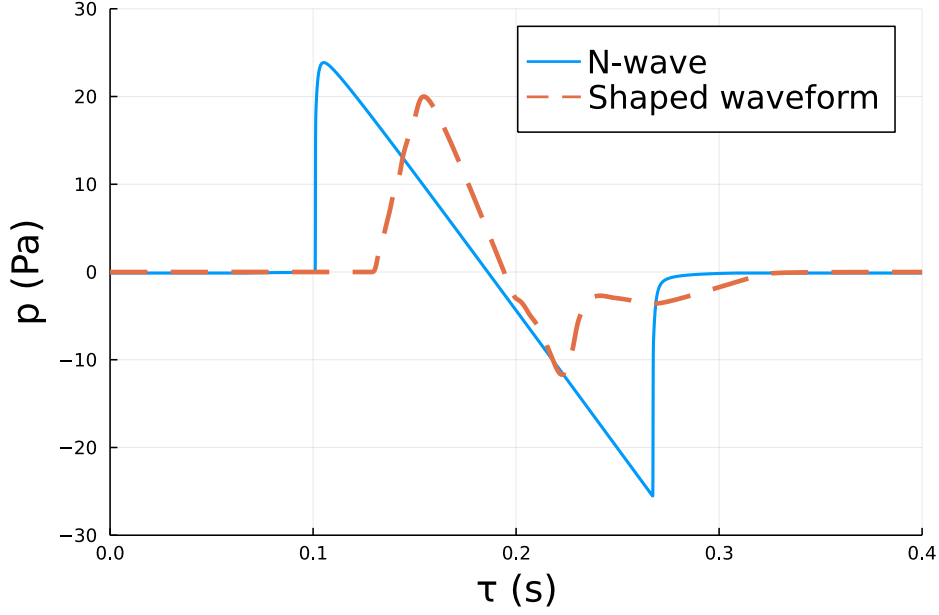


Figure 5: N-wave and X-59 waveform obtained by PCBoom at 1 km altitude.

condition (initial condition) for each elevation angle. Results obtained for the computed loudness metrics are normalized by the nominal condition. The nominal PL condition is 95.5 dB for the N-wave and 68.9 dB for the X-59 prediction.

Simulations for both waveforms were performed for $\theta_{\text{elv}} = 1^\circ, 2^\circ, 5^\circ, 10^\circ, 15^\circ, 25^\circ$, and 35° , for each ABL setpoint. The elevation angles were chosen to correspond to the range of elevation angles between the ray at z_i and ground level that may be observed in the primary carpet region, according to PCBoom simulations. Elevation angles of 25° and 35° are generally representative of booms directly undertrack of the flight path, whereas the elevation angle will tend to decrease as the lateral distance from the flight path increases, until it becomes very small near the lateral cutoff. Thus, by varying the elevation angle, we can simulate propagation through ABL turbulence in different regions of the primary carpet.

The maximum grid point spacing was less than $0.1L_u$ for all simulations performed with $\theta_{\text{elv}} \geq 5^\circ$, and the KSC setpoints at $\theta_{\text{elv}} = 2^\circ$. Simulations performed at $\theta_{\text{elv}} = 1^\circ$ and 2° for the AFRC setpoints were less than $0.2L_u$, and were required in order to obtain probability density functions of the caustic regions for the N-wave. A slightly coarser grid did not affect the computation of the caustic PDFs. This was confirmed by testing grid independence of the caustic PDF result for AFRC3 with $\theta_{\text{elv}} \geq 2^\circ$, and observing no noticeable change in the caustic PDFs between the $0.2L_u$ results and $0.1L_u$ results. Additional simulation details are given in Appendix A.

4. Parameterization of Turbulence Effects on Sonic Boom Waveforms

4.1. Effective Focal Length of Turbulence

A length scale, ℓ_f , was proposed by Carr et al. [9] to nondimensionalize the distance along the propagation direction for simulations with homogeneous velocity fluctuations. This length

scale incorporated the effect of varying turbulence rms velocity on the refraction of the sonic boom waveform. Nondimensionalizing the propagation distance in this manner is advantageous for the development of models describing the effect of turbulence on the sonic boom waveform, as it attempts to collapse data onto a single curve for a wide range of turbulence conditions. A similar scaling parameter, the refraction length, was used by Yuldashev et al. [49] for sonic boom simulations through homogeneous turbulence. Broadly speaking, both length scales attempt to characterize the ability of inhomogeneities in the refraction index to produce caustics. The present investigation considers inhomogeneous fields of velocity and temperature fluctuations. Thus, the index of refraction is no longer determined solely by the root-mean-square velocity and the mean sound speed, but must also account for fluctuations of the sound speed. In addition, the turbulent statistics are functions of z , or x_1 , in the lowest 10% of the ABL. The equivalent focal distance of the field will no longer be a scalar quantity, it will be a function of the altitude of the wavefront.

Past investigations of sound propagation through turbulence have considered the index of refraction of the turbulent field to be [51],

$$n = 1 + \frac{u' + c'}{c_0}, \quad (30)$$

when considering both velocity fluctuations and temperature fluctuations. According to the model of the ABL considered here (Eqs. 12 through 17), the intensity of c' is a function of z . The standard deviation of the velocity and temperature can be used as a measure of the intensity of u' and c' . In the absence of humidity fluctuations, c' is related to T' by $c'c_0^{-1} = 0.5T'T_0^{-1}$. The focal length of a spherical lens of radius a is given by Davy and Blackstock [5] to be $f = 0.5na/(n - 1)$. Thus, assuming the effective focal length of the turbulent field takes a similar form, we get

$$\ell_f(z) = \frac{C_1}{2} \mathcal{L}(L_u, L_T(z)) \frac{1 + \sigma_u c_0^{-1} + 0.5\sigma_T(z)T_0^{-1}}{\sigma_u c_0^{-1} + 0.5\sigma_T(z)T_0^{-1}}. \quad (31)$$

The diameter of an equivalent spherical lens, $\mathcal{L}(L_u, L_T)$, is approximated by the variance weighted average of the von Kármán length scales,

$$\mathcal{L}(z) = \frac{\left(\frac{\sigma_u}{c_0}\right)^2 L_u + \left(\frac{\sigma_T}{2T_0}\right)^2 L_T}{\left(\frac{\sigma_u}{c_0}\right)^2 + \left(\frac{\sigma_T}{2T_0}\right)^2}. \quad (32)$$

With this formulation, ℓ_f is not a constant value for each atmospheric setpoint, but rather a function of the altitude.

The mixed layer, $0.1z_i \leq z \leq 0.9z_i$, comprises the majority of the simulation domain for all conditions considered in this study. The model for the standard deviations and length scales of turbulence considered here assumes that they remain constant in the mixed layer, and that the horizontal directions are homogeneous. Thus, it may be appropriate to scale the propagation direction by the value of ℓ_f in the mixed layer region, where it is constant. The assumption being that the inhomogeneity in the last 10% of propagation does not significantly affect the results. In the mixed layer, the length scales depend on the convective parameter such that

$$L_T L_o^{-1} = 0.2z_i L_o^{-1} \frac{1 - 0.7z_i L_o^{-1}}{\Omega}, \quad (33)$$

where $\Omega = 1 - z_i L_o^{-1}$,

$$L_s L_o^{-1} = 0.18 z_i L_o^{-1}, \quad (34)$$

and

$$L_b L_o^{-1} = 0.23 z_i L_o^{-1}, \quad (35)$$

according to Eqs. 13, 15, and 17 with $z = 0.1 z_i$. This leads to an expression for L_u given by

$$L_u L_o^{-1} = \frac{0.54 u_*^2 + 0.08 w_*^2}{3.0 u_*^2 + 0.35 w_*^2} z_i L_o^{-1}. \quad (36)$$

The standard deviations σ_u and σ_T in the mixed layer region can also be expressed in terms of the convective parameter, the friction velocity, the mixed layer velocity scale, and the temperature scale. By setting $\xi = z_i L_o^{-1}$ in Eqs. 12, 14, and 16 and letting $\sigma_u^2 = \sigma_s^2 + \sigma_b^2$ we obtain

$$\frac{\sigma_T}{2T_0} = \frac{T_*}{T_0} \frac{1}{\Omega^{1/3}}, \quad (37)$$

and

$$\frac{\sigma_u}{c_0} = \sqrt{3.0 \frac{u_*^2}{c_0^2} + 0.35 \frac{w_*^2}{c_0^2}}. \quad (38)$$

The diameter term, $\mathcal{L}$, can now be expressed as

$$\mathcal{L}_m L_o^{-1} = \frac{\left[0.54 \left(\frac{u_*}{c_0} \right)^2 + 0.08 \left(\frac{w_*}{c_0} \right)^2 \right] \Omega^{5/3} + 0.2 \left(\frac{T_*}{T_0} \right)^2 \Omega_1}{\left[3 \left(\frac{u_*}{c_0} \right)^2 + 0.35 \left(\frac{w_*}{c_0} \right)^2 \right] \Omega^{5/3} + \left(\frac{T_*}{T_0} \right)^2 \Omega} z_i L_o^{-1}, \quad (39)$$

in the mixed layer region, where $\Omega_1 = 1 - 0.7 z_i L_o^{-1}$. The parameter ℓ_f now depends solely on the friction velocity u_* , mixed layer velocity scale w_* , temperature scale T_* , and the convection parameter $z_i L_o^{-1}$. After some algebra, we see that ℓ_f in the mixed layer region is

$$\ell_f = C_1 \frac{z_i}{2} \frac{\left[0.54 \left(\frac{u_*}{c_0} \right)^2 + 0.08 \left(\frac{w_*}{c_0} \right)^2 \right] \Omega^{5/3} + 0.2 \left(\frac{T_*}{T_0} \right)^2 \Omega_1}{\left[3 \left(\frac{u_*}{c_0} \right)^2 + 0.35 \left(\frac{w_*}{c_0} \right)^2 \right] \Omega^{5/3} + \left(\frac{T_*}{T_0} \right)^2 \Omega} \times \frac{\Omega^{1/3} + \sqrt{3 \left(\frac{u_*}{c_0} \right)^2 + 0.35 \left(\frac{w_*}{c_0} \right)^2} \Omega^{1/3} + \frac{T_*}{T_0}}{\sqrt{3 \left(\frac{u_*}{c_0} \right)^2 + 0.35 \left(\frac{w_*}{c_0} \right)^2} \Omega^{1/3} + \frac{T_*}{T_0}}. \quad (40)$$

4.2. Empirical Correction

The probability density functions of the first caustic locations were computed for each ABL setpoint for the N-wave simulations. The shaped boom simulations did not experience as many high amplitude events as the N-wave simulations, thus the N-wave simulations are a better indicator of caustic formations. A caustic region was determined to be a region where the sonic boom overpressure amplitude experiences a 50% increase in magnitude compared to the nominal condition. The length scale, ℓ_f , for each case was computed using Eq. 40, which is valid for the mixed layer region, and assumed to be constant in the entire field. In order to accurately compute the PDF of the caustic locations, we must propagate the waveform to a distance where caustics have a very low probability of occurring. If the domain does not extend far enough, then the computed probability at each propagation distance will be higher than the true probability of a caustic appearing, since the entire domain of possibilities is not considered. Since ℓ_f in the mixed layer is rather large, this meant we had to run additional simulations at an elevation angle of one degree for the KSC setpoints.

Figure 6 shows the computed PDFs of the first caustic location along the normalized propagation direction for each atmospheric setpoint. It is clear that the PDFs for the KSC setpoints are in agreement across varying convection levels, from weak to moderate. However, the AFRC cases are in disagreement with the KSC setpoints. One significant difference between the AFRC and KSC setpoints is the ABL height, z_i , which determines the von Kármán scales of turbulence in the mixed layer. The increase in the turbulence length scales between the KSC and AFRC cases resulted in differences in the PDFs, since AFRC1 and KSC17 have convection levels that are relatively close in magnitude (see Table 1) and the velocity fluctuation intensities of KSC6 and AFRC3 differ by only 0.3 m/s. Thus, the ℓ_f parameter in the mixed layer region is able to collapse the PDF of the caustic locations across different rms fluctuation magnitudes (σ_u) only for similar turbulence integral scales (L_u). This was observed previously in Carr et al. [9] and confirmed here for the case of an inhomogeneous ABL.

As it stands, with ℓ_f computed using Eq. 40, predictions of the caustic location PDF for the N-wave simulations do not collapse across different values of the ABL altitude, z_i . The PDF of the caustic locations for the AFRC setpoints exhibits a much higher probability of finding a caustic region at $x_1 = 0.5\ell_f$ than the KSC setpoints, and a smaller probability of observing a caustic for $x_1\ell_f^{-1} > 1.5$. The higher probability densities occurring around $x_1\ell_f^{-1} = 0.5$ for the AFRC setpoints are likely due to the increase in the turbulence integral scale in the mixed layer region, caused by the increase in the ABL height, z_i . The ℓ_f parameter is not able to collapse the data for significant increases in the turbulence integral scale.

It is likely that the expression for $\mathcal{L}/2$ is not an appropriate representation of the radius of curvature of the effective lens that represents the turbulence. In lieu of a more suitable expression for $\mathcal{L}$, an empirical modification is applied to the current form of ℓ_f in order to collapse the caustic PDFs. The constant C_1 is assumed to be a function of the ABL height, $C_1 = F(z_i)$. Then, several simulations are performed at different z_i values to determine the values of C_1 required at each z_i to collapse the caustic PDFs. A line of best fit of these data will then provide the functional form of C_1 . The setpoints KSC6, KSC12, AFRC1, and AFRC3 were used to obtain $C_1(z_i)$. Once the optimal C_1 values to collapse the PDFs were determined, a line of best fit was computed by least

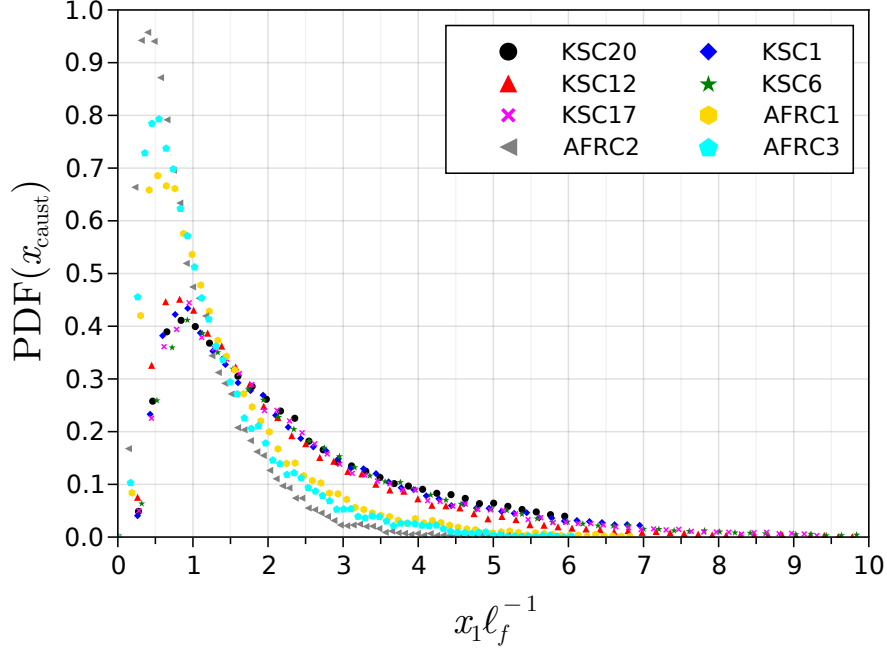


Figure 6: Probability density functions of the location of the first caustic along the propagation direction for each setpoint.

squares regression. The line of best fit is shown in Fig. 7. The appropriate expression for C_1 is,

$$C_1(z_i) = a_1 - b_1 \log(z_i), \quad (41)$$

where $a_1 = 0.74 \pm 0.03$ and $b_1 = 0.208 \pm 0.009$. The exact numerical values used in our predictions are shown in Fig. 7. Although the z_i values used to obtain this expression range from 457.3 m to 1344.0 m, $C_1(z_i)$ is assumed to follow Eq. 41 for typical values of z_i measured during SonicBAT (approximately 200 m to 2000 m) [3]. It is not recommended that Eq. 41 be used outside of this range, without additional simulations or experimental testing. Eq. 41 replaces C_1 in Eq. 40 to collapse the PDFs for all of the cases. These PDFs are shown in Fig. 8. The agreement is significantly improved compared to the previous results.

For the remainder of this investigation, ℓ_f is computed in the mixed layer with $C_1 = C_1(z_i)$, according to Eq. 41. The values of ℓ_f in the mixed layer region are shown in Table 3. For each setpoint considered, ℓ_f is between 2 to 4 km. This implies that, using PCBoom or other ray-tracing sonic boom prediction software, an estimate for the ground locations where a caustic is most likely to occur can be determined by examining the ray path propagation distance from z_i to the ground. If the propagation distance through the ABL is between $0.5\ell_f$ and $1.5\ell_f$, then the simulations performed here suggest that the probability of a caustic forming in this region is higher than anywhere else in the field. Additionally, for regions of the sonic boom carpet where the propagation distance through turbulence is greater than $5\ell_f$, there is a low probability of a caustic forming. It should be noted that once a caustic forms, it can persist for several hundreds of meters according to previous simulations (see [23, 37, 9]). So regions where caustics have a significant effect upon perception of the boom may extend further beyond what is indicated by Fig. 8.

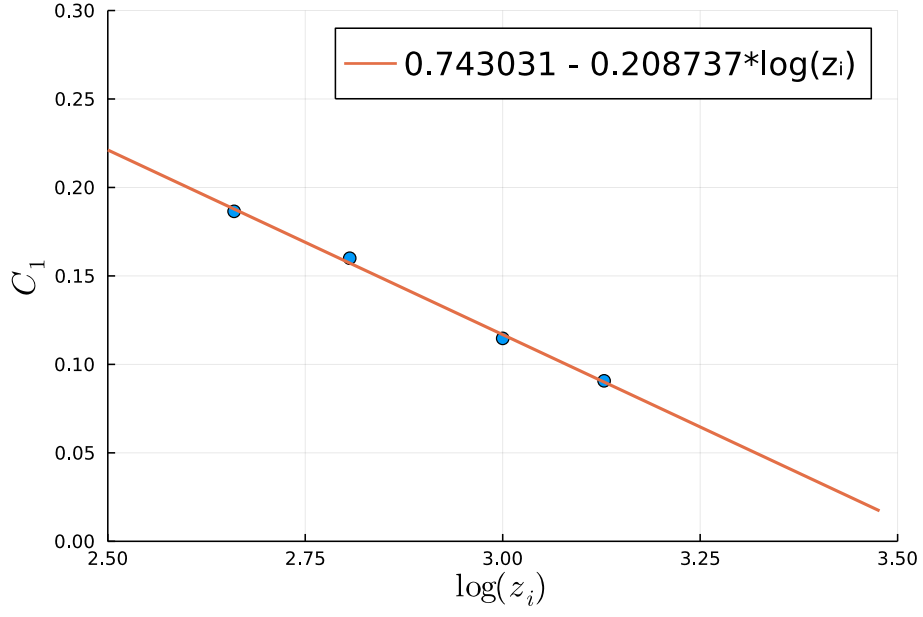


Figure 7: The optimal C_1 coefficients for four different z_i values, and a line of best fit computed by least squares regression.

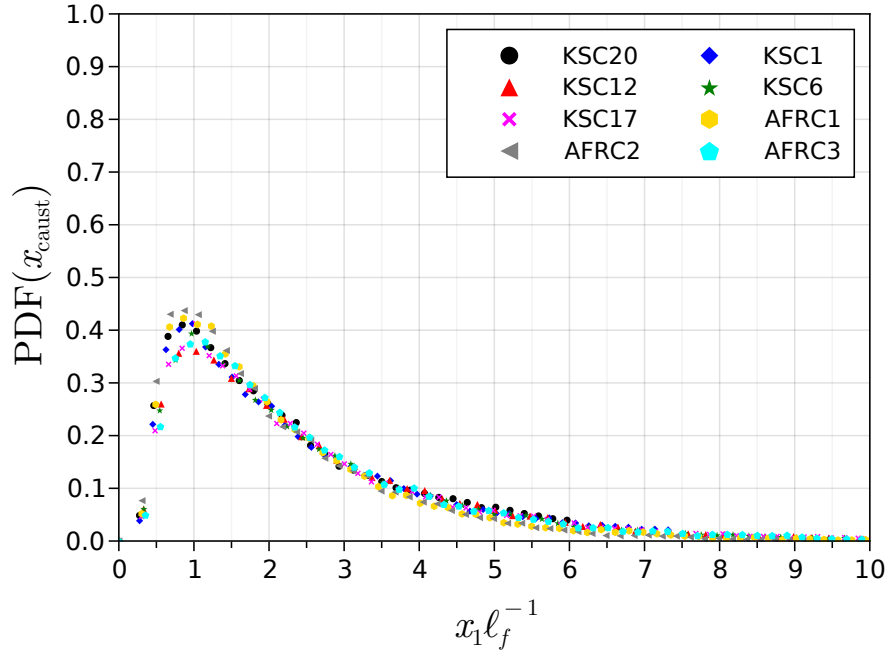


Figure 8: Probability density functions of the location of the first caustic along the propagation direction for each setpoint. Computed with $C_1 = C_1(z_i)$ in ℓ_f .

Table 3: Values of ℓ_f in the mixed layer region for each ABL setpoint considered.

Flight No.	ℓ_f (m)
KSC20	3890
KSC1	3540
KSC12	2948
KSC6	2369
KSC17	2763
AFRC1	2565
AFRC2	2724
AFRC3	3014

4.3. Sonic Boom Waveforms at Several Scaled Distances

The ability of ℓ_f to parameterize the effect of turbulence on sonic boom is visually apparent in N-waves and shaped booms from the simulations performed here. Figure 9 displays sonic boom N-waves and shaped booms extracted along the centerline ($x_2 = 0$ m) at several nondimensional propagation distances for ABL setpoints of KSC20, KSC6, and AFRC3 corresponding to weak, moderate, and strong convection conditions. At the same $x_1 \ell_f^{-1}$ locations, the degree to which turbulence has affected the N-wave appears to be similar. That is, at $x_1 \ell_f^{-1} = 0.25$ for all setpoints, there is little alteration in the N-wave due to turbulence. The effects of turbulence become more significant at $x_1 \ell_f^{-1} = 0.5$ and 1.0 . Significant post-boom fluctuations are present for $x_1 \ell_f^{-1} \geq 2$ at all three ABL setpoints. The shaped booms exhibit smaller amplitude changes, for the same turbulence realization. However, post-boom fluctuations only appear to be significant for $x_1 \ell_f^{-1} \geq 2$, similar to the N-wave.

5. Dose Variability

Statistics of the noise metrics considered here were observed to be in approximate agreement along $x_1 \ell_f^{-1}$ for various elevation angles of propagation. An elevation angle of 2° for the KSC setpoints and 5° for the AFRC setpoints provides an optimum balance of obtaining statistical information about the loudness metrics for long propagation distances while maintaining a sufficient spatial resolution of the grid. Thus, the standard deviations and cumulative probabilities of ΔPL , ΔISBAP , ΔBSEL , ΔDSEL , and ΔESEL presented in this section correspond to an elevation angle of 2° for the KSC setpoints and 5° for the AFRC setpoints, where $\Delta\chi(x_1, x_2) = \chi(x_1, x_2) - \chi(x_1 = 0)$ is the computed loudness metric (χ stands for any one of the loudness metrics considered here: PL, ISBAP, BSEL, DSEL, ESEL) at (x_1, x_2) relative to the nominal χ (the χ of the initial waveform). The average loudness metric level relative to the nominal is denoted $\overline{\Delta\chi}$ and the standard deviation is denoted $\sigma_{\Delta\chi}$. There is no reflection factor applied to the results obtained here; thus, the metrics presented are free-field metrics.

The average values of each of the metrics considered are not presented here. There are several reasons why we omit presenting the average values. The first is the omission of geometrical

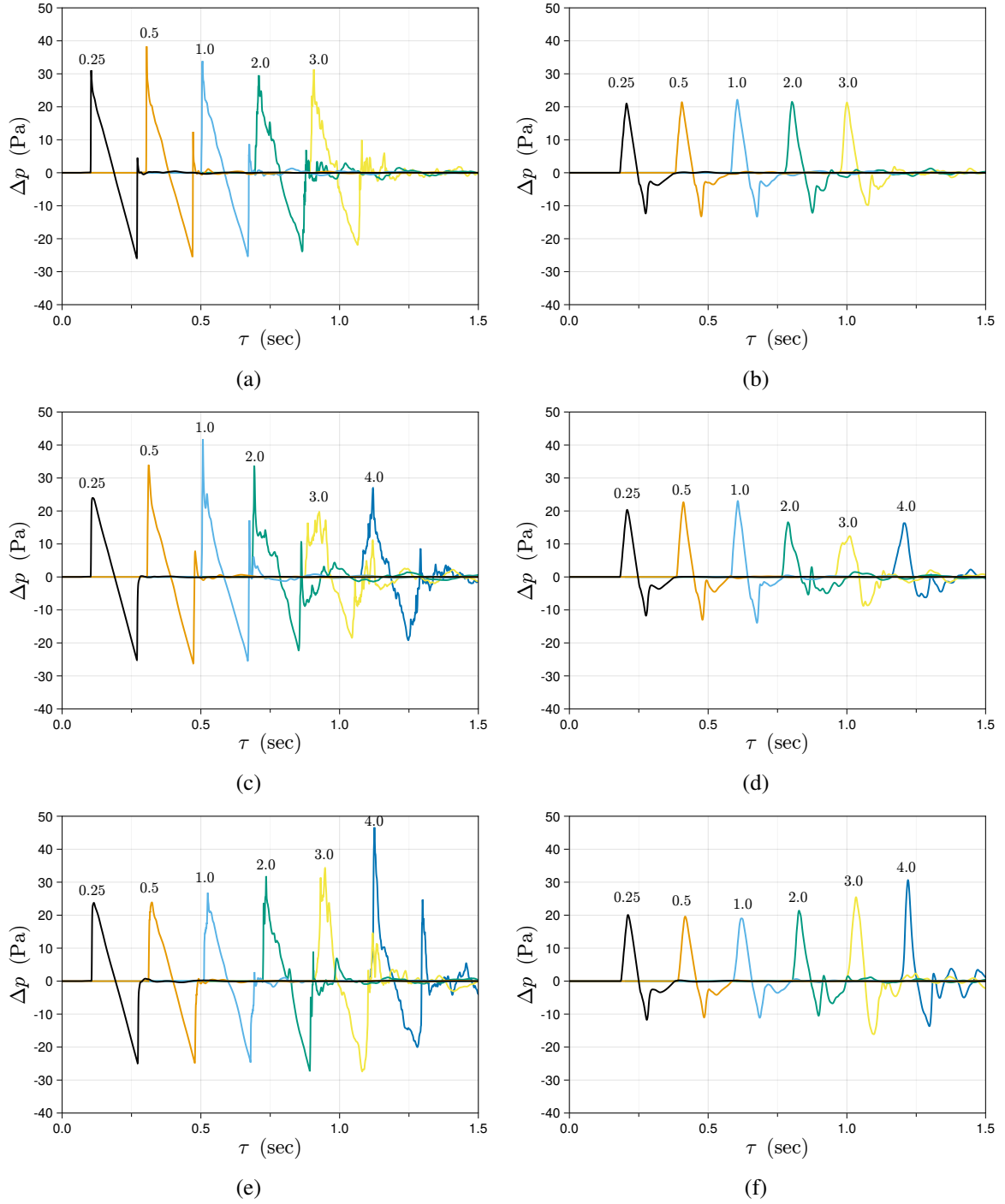


Figure 9: Waveforms extracted along the propagation direction at a lateral location of $x_2 = 0$ m for several $x_1 \ell_f^{-1}$ locations (noted above the leading shock for each waveform) and a single realization of turbulence at the KSC20 setpoint: (a) N-waves and (b) shaped booms, KSC6 setpoint: (c) N-waves and (d) shaped booms, and AFRC3 setpoint: (e) N-waves and (f) shaped booms.

spreading and ABL stratification in the present simulations. Spreading of the sonic boom will decrease its amplitude along the propagation direction, leading to a consistent decrease in loudness levels (if other propagation effects are neglected). Although spreading is most significant near the source, and propagation in the ABL represents only a small portion of the total propagation distance of the sonic boom waveform, the neglect of spreading will still have a slight effect on the predicted average sonic boom loudness levels. Stratification effects will also have an average effect upon the waveform amplitude and period (distance between leading and trailing shocks). This effect will be more pronounced at smaller elevation angles. Additionally, the convective ABL model used here assumes that only the lowest 10% is stratified. Thus, refraction due to stratification would only be present for a small portion of the total propagation. The second reason concerns the Lanczos resampling procedure that is applied to the initial waveforms, in order to make them more suitable for propagation with the spectral method used to account for turbulence fluctuations and diffraction effects. The Lanczos resampling procedure essentially acts as a low pass filter, smoothing out the shocks present in the sonic boom waveforms to prevent the presence of Gibbs oscillations during propagation. The smoothing procedure will impact the average loudness levels, and was found only to have a slight impact on the maximum attained loudness variability. Simulations performed with no filter had maximum ΔPL standard deviation levels that were approximately 0.3 to 0.5 dB lower for the N-wave, and 0 to 0.25 dB lower for the shaped boom, than simulations with the filter, but there were no differences in the initial slope of the results along with the shape of the curves and the collapse with respect to ℓ_f . Thus, many of the important features of the results presented here were insensitive to the Lanczos resampling procedure, and the maximum loudness variability levels presented here are likely closer to the true levels since the simulations without filtering contained Gibbs oscillations in the signal, which are nonphysical. A third and final argument for the omission of the average loudness levels concerns the inability of the present simulations to recreate other environmental conditions of a testing site, such as the presence of buildings or an abundance of trees and foliage that may impact the reflection factor of the ground, besides turbulence conditions. These effects will likely have an average impact on the noise dose levels at a specific testing location. Thus, it is argued that information about the average loudness levels be extracted from measurements made at a specific testing site. Previous simulations of sonic boom propagation through turbulence do indicate, however, that turbulence has the effect of decreasing the average loudness level of the boom (see [23, 37]).

5.1. Simulation Results

Figures 10 through 14 display the computed standard deviations of PL, ISBAP, BSEL, DSEL, and ESEL for the N-wave and shaped boom in each ABL condition. The standard deviations of each loudness metric were observed to change as a function of the nondimensional propagation distance, $x_1 \ell_f^{-1}$. This implies that three factors related to turbulence effects contribute to the observed dose variability in any given region of the sonic boom carpet. Listed in no particular order, these are

1. the propagation distance through turbulence,
2. the rms velocity and rms temperature, and
3. the von Kármán length scales (or correspondingly, integral length scales) of the velocity and temperature fluctuations.

All three factors are accounted for in the ℓ_f parameter, and a collapse of the standard deviations along $x_1 \ell_f^{-1}$ is observed when $x_1 \ell_f^{-1} \leq 0.5$.

The maximum attained dose variability for each loudness metric varies depending on the metric and waveform considered. However, the dose variability for the waveforms considered here is generally ≤ 4 dB for each ABL setpoint and metric. Yuldashev et al. [37] similarly obtained dose variabilities of ≤ 4 dB for similar N-wave overpressures, while Stout [23] obtained dose variability for the PL slightly larger than 4 dB for an N-wave with 50 Pa overpressure. The DSEL metric appears to have the lowest dose variability, and is thus the least sensitive to turbulence of the metrics considered here. This result is in agreement with previous simulations (see Carr et al. [9] and Leconte et al. [46]) as well as noise metric sensitivity studies for sonic boom conducted by Loubeau et al. [36] and Loubeau [69] that indicate BSEL and DSEL are the metrics least sensitive to atmospheric turbulence. For $\sigma_{\Delta PL}$, the N-wave results do not vary significantly as the convection level is increased. However, the maximum $\sigma_{\Delta PL}$ tends to increase with convection level for the shaped boom, and the spread between ABL setpoints is larger than what is observed for the N-wave. The shaped boom results tend to have a larger spread of the maximum standard deviation attained for each loudness metric considered in Figs. 10 through 14. It should also be noted that the maximum standard deviation of PL observed here for the N-wave and shaped boom are similar in magnitude to estimates of the PL standard deviation obtained from the WSPR and QSF18 flight tests [41, 42, 40], which provides some level of confidence in the simulation results.

There is a common trend among the dose variability curves for each metric examined; the dose variability initially increases linearly and then gradually decreases. The linear increase has been observed previously in simulations by Stout [23], Carr et al. [9], and Leconte et al. [46] for homogeneous turbulence. Physically, we interpret this as a consequence of the sensitivity of the loudness metrics to the focusing and defocusing effect of turbulence on the sonic boom overpressure. For the N-wave, the leading and trailing shocks are known to be susceptible to significant amplitude increases (spiking and caustic formation) when propagating through atmospheric turbulence [4]. As we may observe from Fig. 8, caustic formation is concentrated around $x_1 \ell_f^{-1} = 1$. Thus, it is no surprise that the dose variability peaks at the same nondimensional propagation distance, since the shock amplitude will also incur its most significant variability at this location due to the increased likelihood of caustic formation. For $x_1 \ell_f^{-1} > 1$, the continued effect of turbulence attenuates energy from the high frequency components of the N-wave, as has been observed in previous simulations [9], leading to rounding and smoothing of the waveform. The effects of thermoviscous absorption and molecular relaxation are also considered here, which contributes to the smoothing of shocks present in the waveforms. Thus, for $x_1 \ell_f^{-1} > 1$ the likelihood of caustic formation begins to decrease and the waveforms become more rounded, leading to a decrease in dose variability. A similar process occurs for the shaped boom, in spite of the fact that the initial waveform does not contain the distinct leading and trailing shocks of a typical N-wave. This may be why the rate of increase of the dose variability is lower in general for the shaped boom as opposed to the N-wave. The dose variability for the shaped boom also tends to deviate from the initial linear increase more rapidly than the N-wave for cases of weak to moderate convection. The rate of decrease in dose variability beyond the maximum location is also generally smaller in magnitude than the N-wave fall off. These differences in the dose variability curves for the shaped boom, as opposed to the N-wave, are pronounced for the BSEL, DSEL, and ESEL results.

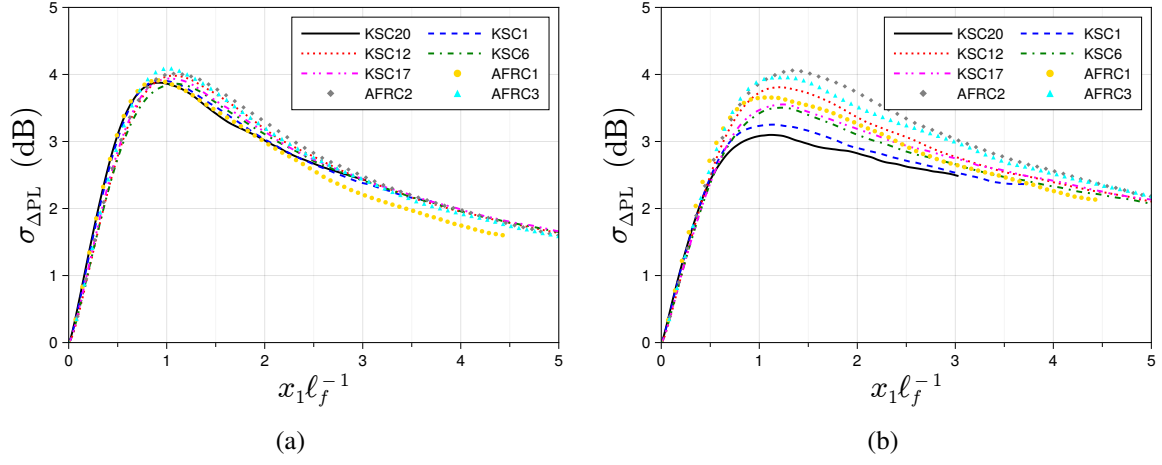


Figure 10: Standard deviation of PL for the (a) N-wave and (b) shaped boom.

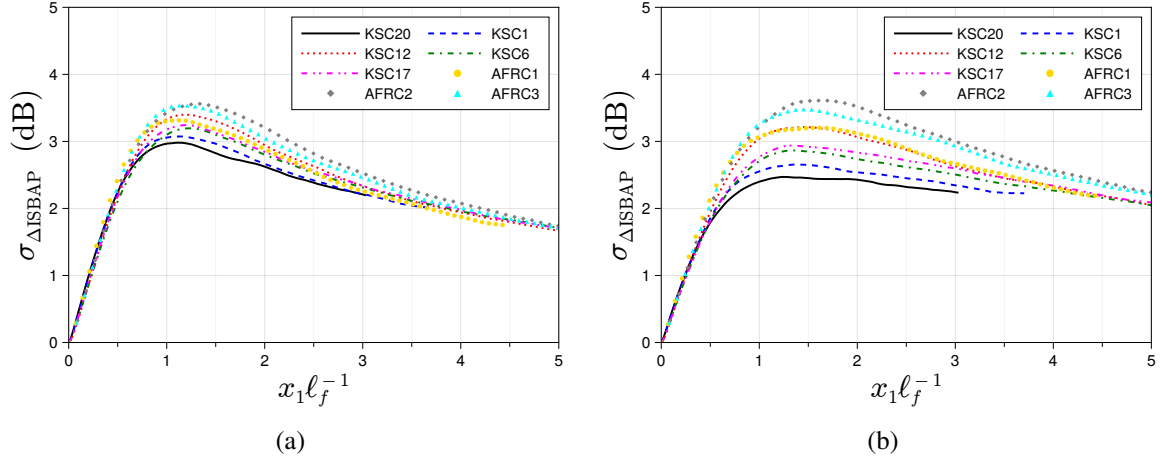


Figure 11: Standard deviation of ISBAP for the (a) N-wave and (b) shaped boom.

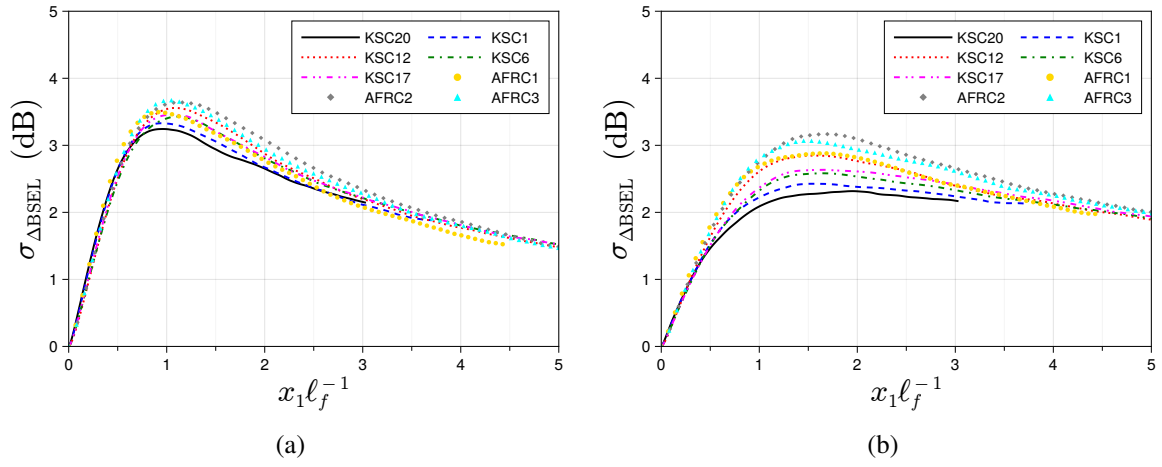


Figure 12: Standard deviation of BSEL for the (a) N-wave and (b) shaped boom.

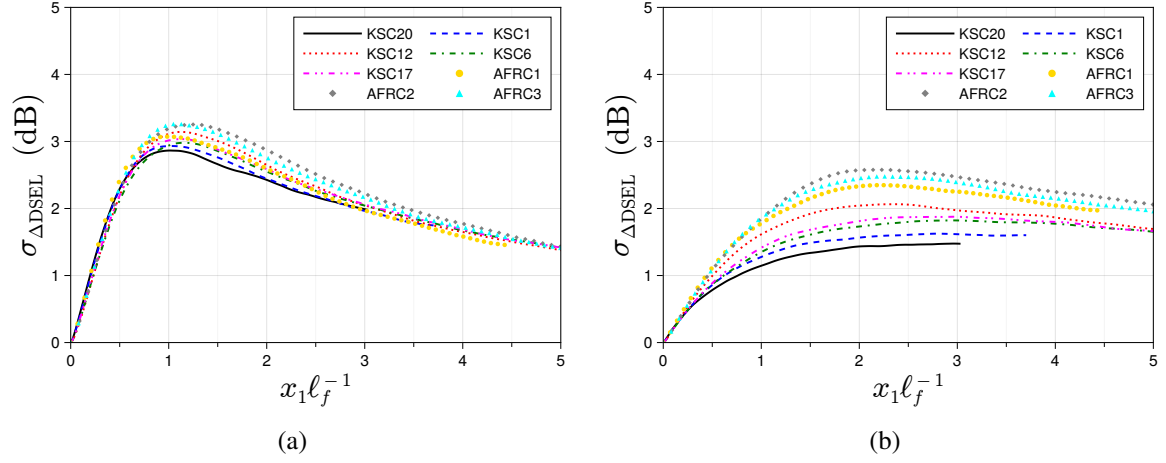


Figure 13: Standard deviation of DSEL for the (a) N-wave and (b) shaped boom.

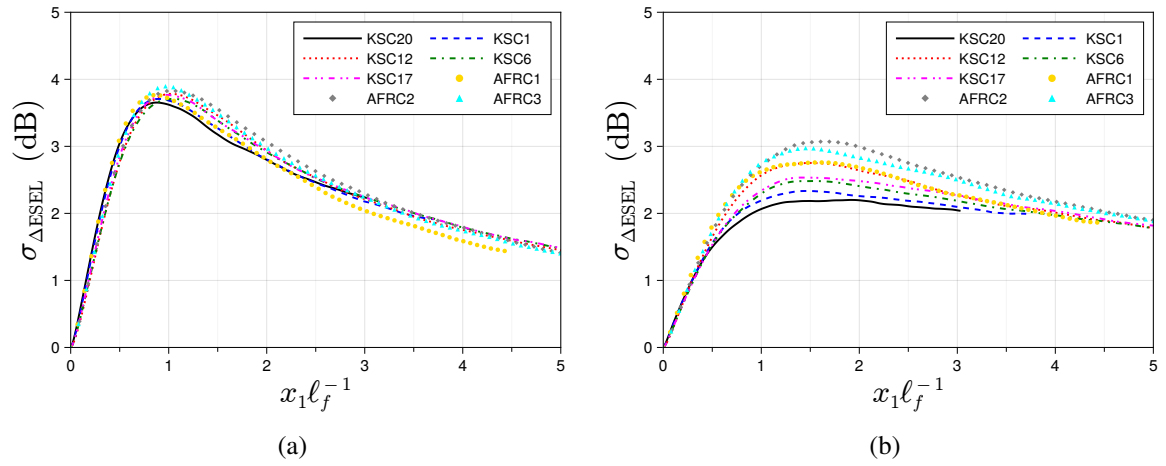


Figure 14: Standard deviation of ESEL for the (a) N-wave and (b) shaped boom.

In recent work on incorporating noise dose variability into dose-response modeling of shaped booms, the dose variability distributions have been assumed to be Gaussian [40]. However, skewness relative to a normal distribution was observed in PL measurements made at KSC during the SonicBAT campaign [3], suggesting that PL may not always follow a normal distribution. The normality of the PL and ISBAP distributions for both waveforms are investigated here by qualitative examination of the cumulative probability distributions along $x_1 \ell_f^{-1}$. Figures 15 and 16 display the distributions of $\Delta\text{PL} - \overline{\Delta\text{PL}}$ and $\Delta\text{ISBAP} - \overline{\Delta\text{ISBAP}}$ for both waveforms at $x_1 \ell_f^{-1} = 1, 2$, and 3. In general, it is observed that the loudness metric distributions are either peaked or have some slight skewness to the right or left of a normal distribution for $x_1 \ell_f^{-1} \lesssim 1$, are normally distributed for $1 \lesssim x_1 \ell_f^{-1} \lesssim 3$, and are skewed to the right of a normal distribution for $x_1 \ell_f^{-1} \gtrsim 3$, and this is true for both waveforms. Although qualitatively, the shaped boom distributions tend to adhere more closely to a true normal distribution. Quantitative examination of the skewness of PL for both waveforms supports the trends observed in the cumulative probability distributions. The skewness is close to 0 between $1 \leq x_1 \ell_f^{-1} \leq 2$, and then is greater than 0.25 for $x_1 \ell_f^{-1} > 3$ for the N-wave simulations. The kurtosis of the N-wave results is within a range of 2.5 to 4 for $x_1 \ell_f^{-1} > 0.25$, which is close to the expected value of 3 for a normal distribution. For the shaped boom, the skewness is between -0.2 and 0.35 for $x_1 \ell_f^{-1} > 0.25$ and the kurtosis is between 2.6 and 3.3 for the same range, which indicates that the shaped boom results more closely adhere to a normal distribution.

Nondimensional propagation distances less than 1 are likely to occur for regions directly undertrack of the flight path, where the elevation angle between the wavefront normal and ground is larger, or when the convection level is significantly weak such that the integral scales and rms velocity are sufficiently small and the turbulence has little impact on altering the boom. Thus, it is likely that a larger portion of the sonic boom carpet will have $x_1 \ell_f^{-1} < 1$ for weaker convection levels, in contrast to moderate and strong convection levels. As the turbulence rms velocity and length scales are further increased, regions of the primary carpet, particularly near the lateral cut-off, will have $x_1 \ell_f^{-1} \gtrsim 3$. Our simulations indicate that regions with $x_1 \ell_f^{-1} \gtrsim 3$ will likely observe dose variability distributions that are skewed to the right of a normal distribution. This implies that stronger convection levels, like those observed at AFRC, will likely lead to skewness in the loudness metric distributions at significant lateral distances from the flight path. It is important to note that Leconte et al. [45] found that the largest disagreement between 2D and 3D simulations in the cumulative probability plots occurs for cases of very large boom amplification. Thus, our results for the highest cumulative probabilities may not be the most representative of what would occur in true 3D propagation. However, these events represent only a small subset of outcomes, and for modeling the dose variability distributions we are most concerned with representing the majority of outcomes that occur with 10% to 90% cumulative probability.

5.2. Model of Dose Variability

Although the distributions deviate from a true normal distribution for $x_1 \ell_f^{-1} \lesssim 1$ and $x_1 \ell_f^{-1} \gtrsim 3$, the range of cumulative probabilities between 10% and 90% follow a normal distribution for the $x_1 \ell_f^{-1}$ distances examined. This was also observed in previous simulations of sonic boom through turbulence [23, 9]. Additionally, measurements during SonicBAT made at AFRC indicate a strong

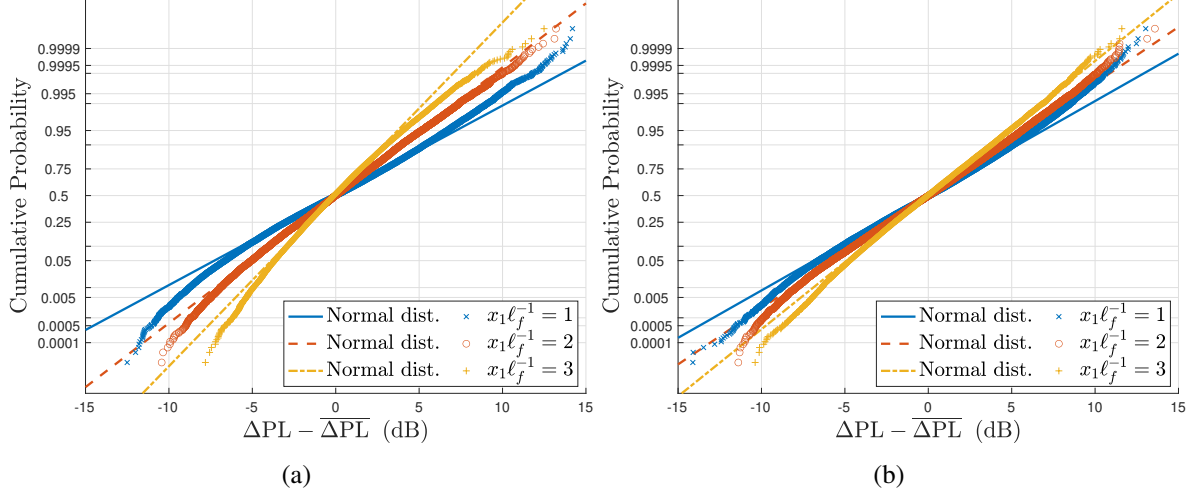


Figure 15: Cumulative probability plots of PL for (a) N-wave and (b) shaped boom for the case of strong convection (AFRC3).

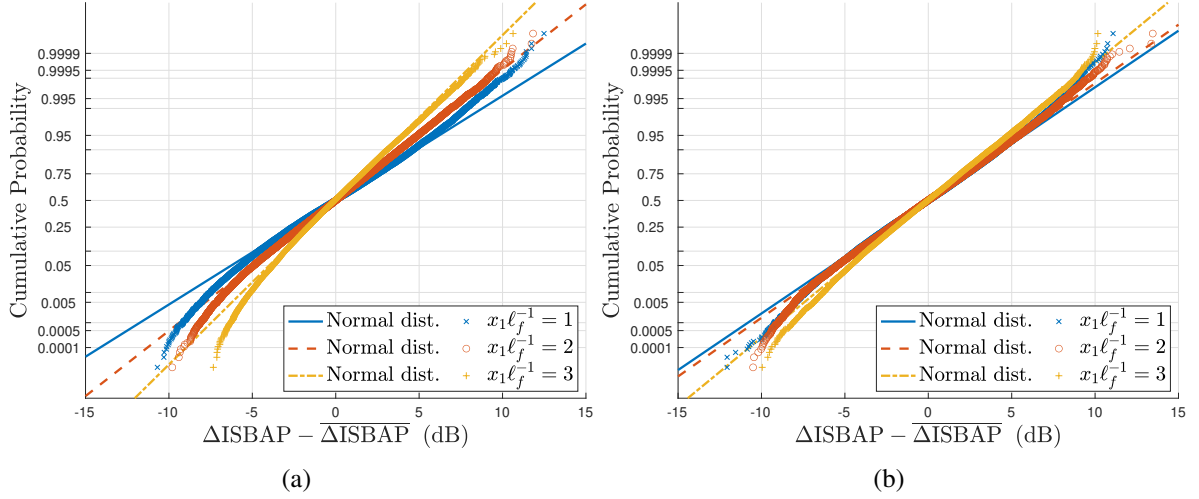


Figure 16: Cumulative probability plots of ISBAP for (a) N-wave and (b) shaped boom for the case of strong convection (AFRC3).

normality of the PL distributions. Thus, in the development of a dose variability distribution model presented in this section, we have chosen to assume a normal distribution of the dose variability corresponding to each metric considered.

The standard deviations of the loudness metrics presented in Figs. 10 through 14 were shown to increase linearly for small $x_1 \ell_f^{-1}$ until reaching a maximum value, and then undergo a gradual decrease as $x_1 \ell_f^{-1}$ increased beyond the location of maximum loudness variation. A function suitable for describing such curves is

$$\sigma_{\Delta\chi} = \frac{Ax_1 \ell_f^{-1}}{B + (x_1 \ell_f^{-1})^\alpha}, \quad (42)$$

where the parameters A , B , and α are determined by a regression fit to the simulation data. Each of the parameters in Eq. 42 may be related to the convection level parameter, $\log(-z_i L_o^{-1})$, for each waveform and metric considered. Their relationship to the convection level is described by

$$A = A_1 B, \quad (43)$$

$$B = B_1 + B_2 \log(-z_i L_o^{-1}), \quad (44)$$

and

$$\alpha = \alpha_1 + \alpha_2 \log(-z_i L_o^{-1}). \quad (45)$$

Thus, knowledge of the constants A_1 , B_1 , B_2 , α_1 , and α_2 determined by regression will yield an approximate model for the dose variability. Tables 4 and 5 provide these values for each metric of the N-wave and shaped boom simulations, respectively.

Table 4: Regression parameters for the N-wave in the dose variability model.

Metric	A_1	B_1	B_2	α_1	α_2
PL	7.129	1.025	0.123	1.973	0.041
ISBAP	5.458	1.130	0.204	1.836	0.064
BSEL	6.344	0.960	0.169	1.903	0.067
DSEL	5.438	1.002	0.205	1.872	0.071
ESEL	7.130	0.912	0.135	1.959	0.059

In Figs. 17 and 18, the regression model is compared to $\sigma_{\Delta\text{PL}}$ and $\sigma_{\Delta\text{ISBAP}}$ for both waveforms at three different ABL setpoints. The model captures the trend of the dose variability, with good quantitative agreement for the purposes of dose variability estimation for flight test planning. The largest discrepancy occurs for the KSC6 setpoint of the shaped boom simulations, where there is an approximately 0.25 dB disagreement in $\sigma_{\Delta\text{PL}}$ in the range $1.5 \leq x_1 \ell_f^{-1} \leq 3$ and an approximately 0.5 dB disagreement in the $\sigma_{\Delta\text{ISBAP}}$ at $x_1 \ell_f^{-1} = 2$. Although the regression model will have slight disagreements to the simulation data at certain setpoints and propagation distances, the ability to

Table 5: Regression parameters for the shaped boom in the dose variability model.

Metric	A_1	B_1	B_2	α_1	α_2
PL	6.320	0.939	0.249	1.677	0.094
ISBAP	4.747	1.007	0.443	1.549	0.141
BSEL	3.815	1.390	0.382	1.563	0.148
DSEL	2.180	1.122	1.357	1.280	0.272
ESEL	3.924	1.173	0.399	1.554	0.147

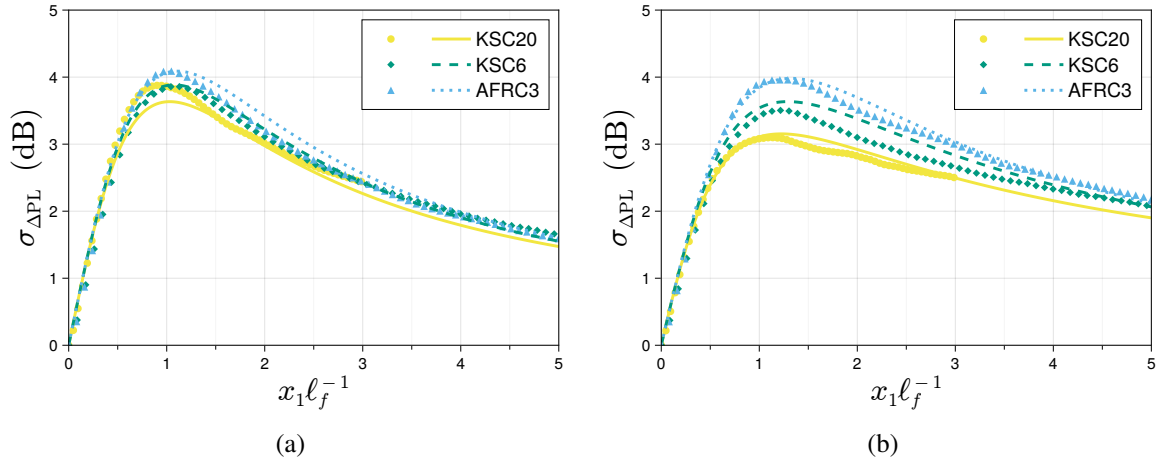


Figure 17: Standard deviation of PL for 3 different convection levels obtained from simulations (discrete points) compared to the regression model (solid lines): (a) N-wave results and (b) shaped boom results.

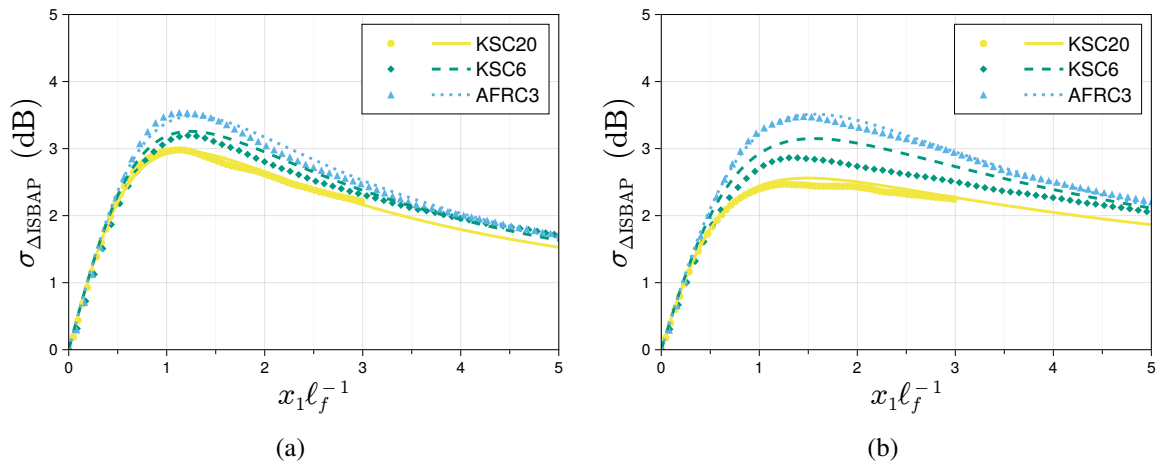


Figure 18: Standard deviation of ISBAP for 3 different convection levels obtained from simulations (discrete points) compared to the regression model (solid lines): (a) N-wave results and (b) shaped boom results.

quickly estimate the dose variability with Eq. 42 rather than perform new simulations for each ABL condition considered during flight test planning far outweighs the shortcomings.

The dose distribution is modeled assuming a normal distribution,

$$f(\Delta\chi - \overline{\Delta\chi}) = \frac{1}{\sigma_{\Delta\chi} \sqrt{2\pi}} e^{-\frac{1}{2} \left(\frac{\Delta\chi - \overline{\Delta\chi}}{\sigma_{\Delta\chi}} \right)^2}. \quad (46)$$

Comparisons are now made between Eq. 46, using the regression model for $\sigma_{\Delta\text{PL}}$, and PL distributions computed from simulations for the shaped boom at two elevation angles, 10° and 25° . Although only comparisons with PL distributions for the shaped boom are shown here, the model comparisons to the N-wave simulations and other sonic boom metrics in Tables 4 and 5 are similar to the shaped boom PL results. The simulation data obtained at these elevation angles were not used to compute the regression parameters for the model in Eq. 42. We observe, in Fig. 19 that the model provides a sufficient approximation of the PL distribution at ground level for simulations performed at several convection levels and elevation angles. These setpoints represent three different meteorological conditions and two approximate regions of the sonic boom carpet, the undertrack and off-track regions. In Figs. 19a, 19b, and 19c, some left skewness is observed in the simulation results. This is most pronounced at $x_1 \ell_f^{-1} = 0.25$, where the model has the most significant disagreement with simulations results, due to the normal distribution assumption. However, the simulation results at $x_1 \ell_f^{-1} = 0.46$ and 0.61 are well approximated by the model. The simulation results at $x_1 \ell_f^{-1} = 1.06, 1.12$, and 2.59 appear to be slightly peaked relative to the model, indicating the model's tendency to overpredict the variance of the distribution when $x_1 \ell_f^{-1} \geq 1$.

6. Conclusion

Split-step simulations of the propagation of two sonic boom waveforms, an N-wave and a shaped boom from a design iteration of the NASA X-59 aircraft, were performed through fields of ABL turbulence representative of meteorological conditions measured at the NASA KSC and AFRC during SonicBAT. Inhomogeneous turbulence in the computational domain was generated with the generalized random phase method, using a model for the velocity and temperature fluctuation statistics provided by Ostashev and Wilson [51], based on previous ABL measurements in combination with MOST. A parameter previously proposed by Carr et al. [9] to collapse N-wave caustic PDFs onto a single curve in a homogeneous field of velocity fluctuations, ℓ_f , was modified to incorporate temperature fluctuations in the ABL, as well as changing von Kármán length scales in the mixed layer. An empirical correction was applied to ℓ_f in order to obtain an improved collapse of the caustic PDFs.

A collapse of the standard deviations of several loudness metrics was observed across the entire range of convection levels examined here for $x_1 \ell_f^{-1} \leq 0.5$. The maximum attained dose variability for PL was observed to fall between 3 and 4 dB for both the N-wave and shaped boom. The remaining loudness metrics considered were not as sensitive to turbulence effects as the PL, and DSEL appears to be the least sensitive. The distributions of PL and ISBAP presented in Figs. 15 and 16 indicate that these two metrics most closely follow a normal distribution in the range $1 \leq x_1 \ell_f^{-1} \leq 3$. For $x_1 \ell_f^{-1} < 1$, the distributions were observed to be either peaked relative to a

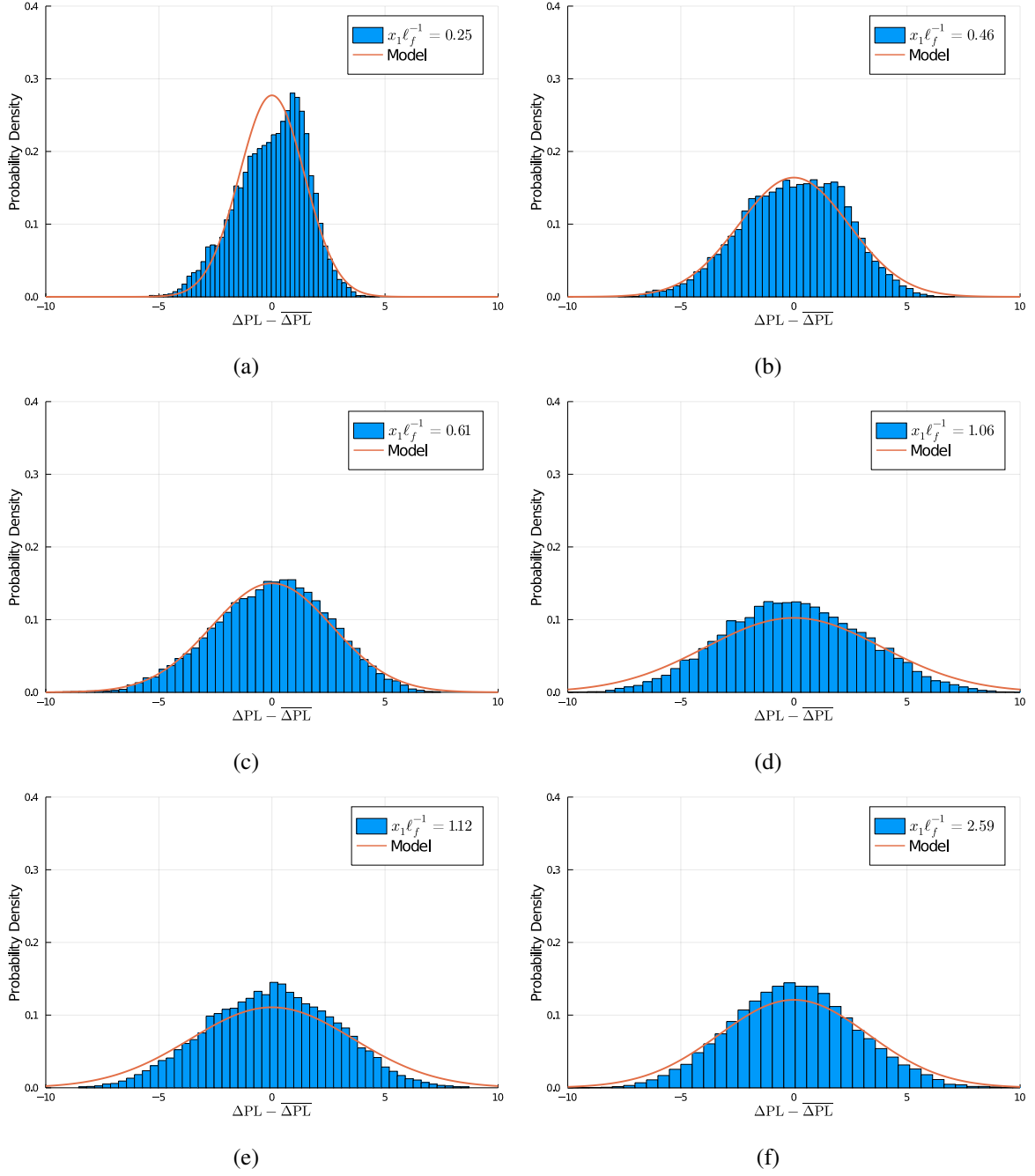


Figure 19: Regression model comparison to simulation data performed at ground level for (a) KSC20 $\theta_{\text{elv}} = 25^\circ$, (b) KSC6 $\theta_{\text{elv}} = 25^\circ$, (c) KSC20 $\theta_{\text{elv}} = 10^\circ$, (d) AFRC3 $\theta_{\text{elv}} = 25^\circ$, (e) KSC6 $\theta_{\text{elv}} = 10^\circ$, (f) AFRC3 $\theta_{\text{elv}} = 10^\circ$ for the shaped boom.

normal distribution, or skewed to the left. For $x_1 \ell_f^{-1} > 3$, there was a consistent skewness to the right, prevalent in the simulation results.

Although there is skewness present in the simulation results for the loudness metric distributions, cumulative probabilities between 10% and 90% generally agree quite well with a normal distribution for the entire propagation range considered here. Thus, it is likely that a normal distribution assumption is sufficient for the purpose of noise dose estimation. The regression model proposed for the loudness metric distributions leverages this assumption. The function in Eq. 42 was observed to agree with the simulation results when nonlinear regression is performed to determine the parameters A , B , and α from the $\theta_{\text{elv}} = 5^\circ$ results. A regression model is obtained when the parameters A , B , and α are related directly to the convection level for each metric and waveform considered. This approximate model of the dose variability in combination with the normal distribution assumption was found to agree well with simulation results for $\theta_{\text{elv}} = 10^\circ$ and 25° . Thus, the regression model is able to approximate the distributions of several sonic boom loudness metrics considered here, based on the results of our simulations across a range of meteorological conditions. The regression model will only perform as well as the simulations are able to capture the appropriate physics of sonic boom propagation in the ABL. Some limitations of the current model are the use of Lanczos resampling on the initial waveforms and the neglect of: spreading of the sonic boom waveform, 3D effects, ground reflections, and the effect of urban environments on ABL turbulence. Due to these limitations, it is likely that a combination of simulation data and ground measurement data from past or future flight tests will be required to improve the estimates of the regression parameters in Tables 4 and 5. However, the current regression parameters determined from our simulations, along with Eqs. 42 and 46, are likely useful in providing a quick estimate of noise dose variability at various proposed flight test survey locations and meteorological conditions.

Acknowledgement

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Appendix A. Terms of The Partially One-Way Equation

The diffraction term,

$$\mathcal{D}(p) = \frac{c_0}{2} \frac{\partial^2 p}{\partial x_i \partial x_i}, \quad (\text{A.1})$$

accounts for variations in the pressure in the transverse directions. The nonlinear and absorption terms are, respectively,

$$\mathcal{N}(p) = \frac{\beta}{2\rho_0 c_0^3} \frac{\partial^2 p^2}{\partial \tau^2} \quad (\text{A.2})$$

and

$$\mathcal{A}(p) = \frac{\delta}{2c_0^3} \frac{\partial^3 p}{\partial \tau^3}. \quad (\text{A.3})$$

The nonlinear term accounts for variations in the wave speed that are amplitude-dependent, and the absorption terms model the attenuation effects of thermoviscous absorption and molecular relaxation. Molecular relaxation is accounted for using the absorption coefficient expression provided by Bass et al. [54] and the algorithm is detailed in Carr [55]. The terms involving flow effects are contained in the $\mathcal{H}$ operator, which can be split into two terms containing “phase effects” ($\mathcal{H}_1$) and “coupling effects” ($\mathcal{H}_2$),

$$\mathcal{H}(p) = \mathcal{H}_1(p) + \mathcal{H}_2(p). \quad (\text{A.4})$$

Terms involving only derivatives with respect to x_1 and τ are contained in $\mathcal{H}_1$, and the remaining terms are contained in $\mathcal{H}_2$. The $\mathcal{H}_1$ terms are,

$$\begin{aligned} \mathcal{H}_1(p) = & \frac{2c_0 c'' + c''^2}{2c_0^3} \frac{\partial^2 p}{\partial \tau^2} - \frac{\check{u}_1}{c_0} \left[\frac{\partial^2 p}{\partial x_1 \partial \tau} - \frac{1}{c_0} \frac{\partial^2 p}{\partial \tau^2} \right] - \frac{\check{u}_1}{2c_0} \frac{\partial \check{u}_1}{\partial x_1} \left[\frac{\partial p}{\partial x_1} - \frac{1}{c_0} \frac{\partial p}{\partial \tau} \right] \\ & - \frac{\check{u}_2}{2c_0} \frac{\partial \check{u}_1}{\partial x_2} \left[\frac{\partial p}{\partial x_1} - \frac{1}{c_0} \frac{\partial p}{\partial \tau} \right] - \frac{\check{u}_1^2}{2c_0} \frac{1}{c_0^2} \frac{\partial^2 p}{\partial \tau^2} - \frac{\check{c}^2}{2c_0 \check{\rho}} \frac{\partial \check{\rho}}{\partial x_1} \left[\frac{\partial p}{\partial x_1} - \frac{1}{c_0} \frac{\partial p}{\partial \tau} \right] - \frac{\check{c}^2}{c_0^3} \frac{\partial \check{u}_1}{\partial x_1} \frac{\partial p}{\partial \tau}. \end{aligned} \quad (\text{A.5})$$

The $\mathcal{H}_2$ terms are,

$$\begin{aligned} \mathcal{H}_2(p) = & -\frac{\check{u}_2}{c_0} \frac{\partial^2 p}{\partial x_2 \partial \tau} - \frac{\check{u}_1}{2c_0} \frac{\partial \check{u}_2}{\partial x_1} \frac{\partial p}{\partial x_2} - \frac{\check{u}_2}{2c_0} \frac{\partial \check{u}_2}{\partial x_2} \frac{\partial p}{\partial x_2} + \frac{\check{u}_1^2}{2c_0^2} \frac{\partial^2 p}{\partial x_2^2} - \frac{\check{u}_1 \check{u}_2}{c_0} \frac{\partial}{\partial x_2} \left[\frac{\partial p}{\partial x_1} - \frac{1}{c_0} \frac{\partial p}{\partial \tau} \right] \\ & - \frac{\check{u}_2^2}{2c_0} \frac{\partial^2 p}{\partial x_2^2} - \frac{\check{c}^2}{2c_0 \check{\rho}} \frac{\partial \check{\rho}}{\partial x_2} \frac{\partial p}{\partial x_2} + \frac{\check{c}^2}{c_0} \frac{\partial \check{u}_1}{\partial x_1} \int_{-\infty}^{\tau} \frac{\partial^2 p}{\partial x_2^2} d\tau' - \frac{\check{c}^2}{c_0} \left(\frac{\partial \check{u}_2}{\partial x_1} + \frac{\partial \check{u}_1}{\partial x_2} \right) \int_{-\infty}^{\tau} \frac{\partial}{\partial x_2} \left[\frac{\partial p}{\partial x_1} - \frac{1}{c_0} \frac{\partial p}{\partial \tau'} \right] d\tau' \\ & - \frac{\check{c}^2}{c_0} \frac{\partial \check{u}_2}{\partial x_2} \int_{-\infty}^{\tau} \frac{\partial^2 p}{\partial x_2^2} d\tau'. \end{aligned} \quad (\text{A.6})$$

The matrix equation formed by the $\mathcal{H}_2$ terms is solved with a Thomas algorithm and a periodic boundary condition in x_2 , and is detailed further in Carr [55]. The periodic boundary condition may be utilized since the turbulence is homogeneous in the x_2 direction.

In our simulations, for elevation angles at 2° and 5° , Δx_1 and Δx_2 fall between values of 5.6 to 9 meters, and $\Delta x_1 = \Delta x_2$ for every simulation. For larger elevation angles, Δx_1 and Δx_2 fall between values of 0.35 to 3.78 meters. The sampling frequency of the N-wave is 6.6 kHz and the sampling frequency of the shaped boom is 4.5 kHz.

The window used for the Lanczos resampling procedure is

$$L(y) = \begin{cases} 1 & \text{if } y = 0, \\ \frac{a \sin(\pi y) \sin(\pi y/a)}{\pi^2 y^2} & \text{otherwise,} \end{cases} \quad (\text{A.7})$$

where $y = a(i/m - 0.5)$, m is the number of points used for the length of the window, i iterates through these points, and a is a window width parameter. The parameters m and a may be tuned to remove Gibbs oscillations while retaining as much frequency content of the waveforms as possible. For our simulations, m and a were chosen to be 20 and 2, respectively.

Appendix B. Probability Plots of Additional Loudness Metrics and Regression Model Comparison

Figure B.1 displays the cumulative probability plots of BSEL, DSEL, and ESEL at three different nondimensional propagation distances for the N-wave and shaped boom simulations. The trends observed for BSEL, DSEL, and ESEL, are consistent with what is observed for PL and ISBAP. However, the distributions for the shaped boom simulations tend to follow a normal distribution at all three $x_1 \ell_f^{-1}$ distances examined.

Figure B.2 displays comparisons of the regression model for the dose variability (Eq. 42) to simulation data for BSEL, DSEL, and ESEL. Good agreement between the model and the simulation data is observed across the weak, moderate, and strong convection levels. The largest disagreement occurs for the moderate convection case of the shaped boom DSEL results.

References

- [1] H. H. Hubbard, D. J. Maglieri, V. Huckel, D. A. Hilton, Ground Measurements of Sonic-Boom Pressures for The Altitude Range of 10,000 to 75,000 Feet, NASA-TR-R-198, 1964, <https://ntrs.nasa.gov/citations/19640014910>.
- [2] D. J. Maglieri, T. L. Parrot, D. A. Hilton, W. L. Copeland, Lateral-Spread Sonic-Boom Ground-Pressure Measurements from Airplanes at Altitudes to 75,000 Feet and Mach Numbers to 2.0, NASA-TN-D-2021, 1963, <https://ntrs.nasa.gov/citations/19640000542>.
- [3] K. A. Bradley, C. M. Hobbs, C. B. Wilmer, V. W. Sparrow, T. A. Stout, J. M. Morgenstern, K. H. Underwood, D. J. Maglieri, R. A. Cowart, M. T. Collmar, H. Shen, P. Blanc-Benon, Sonic Booms in Atmospheric Turbulence (SonicBAT): The Influence of Turbulence on Shaped Sonic Booms, NASA/CR-2020-220509, 2020, <https://ntrs.nasa.gov/citations/20200002482>.
- [4] A. D. Pierce, Spikes on sonic-boom pressure waveforms, J. Acoust. Soc. Am. 44 (1968) 1052–1061, <https://doi.org/10.1121/1.1911195>.
- [5] B. A. Davy, D. T. Blackstock, Measurements of the refraction and diffraction of a short N wave by a gas-filled soap bubble, J. Acoust. Soc. Am. 49 (1971) 732–737, <https://doi.org/10.1121/1.1912410>.
- [6] S. C. Crow, Distortion of sonic bangs by atmospheric turbulence, J. Fluid Mech. 37 (1969) 529–563, <https://doi.org/10.1017/s0022112069000711>.
- [7] A. R. George, Propagation of sonic booms and other weak nonlinear waves through turbulence, Phys. Fluids 14 (1971) 548, <https://doi.org/10.1063/1.1693471>.
- [8] A. D. Pierce, D. J. Maglieri, Effects of atmospheric irregularities on sonic-boom propagation, J. Acoust. Soc. Am. 51 (1972) 702–721, <https://doi.org/10.1121/1.1912904>.
- [9] A. N. Carr, J. B. Lonzaga, S. A. E. Miller, Numerical prediction of loudness metrics for N-waves and shaped sonic booms in kinematic turbulence, J. Acoust. Soc. Am. 151 (2022) 3580–3593, <https://doi.org/10.1121/10.0011514>.
- [10] Beyond Traffic 2045 Trends and Choices, United States Department of Transportation, 2017, <https://www.transportation.gov/policy-initiatives/beyond-traffic-2045-final-report>.
- [11] P. N. Borsky, Community Reactions to Sonic Booms in the Oklahoma City Area, Report No. 101, National Opinion Research Center New York, 1965, <https://apps.dtic.mil/sti/citations/AD0613620>.
- [12] D. Suisman, The Oklahoma City sonic boom experiment and the politics of supersonic aviation, Radical History Review 2015 (2015) 169–195, <https://doi.org/10.1215/01636545-2800022>.
- [13] C. Darden, A study of the limitations of linear theory methods as applied to sonic boom calculations, in: 28th Aerospace Sciences Meeting, American Institute of Aeronautics and Astronautics, 1990, <https://doi.org/10.2514/6.1990-368>.
- [14] M. Siclari, C. Darden, CFD prediction of the near-field sonic boom environment for two low boom HSCT configurations, in: 22nd Fluid Dynamics, Plasma Dynamics and Lasers Conference, American Institute of Aeronautics and Astronautics, 1991, <https://doi.org/10.2514/6.1991-1631>.

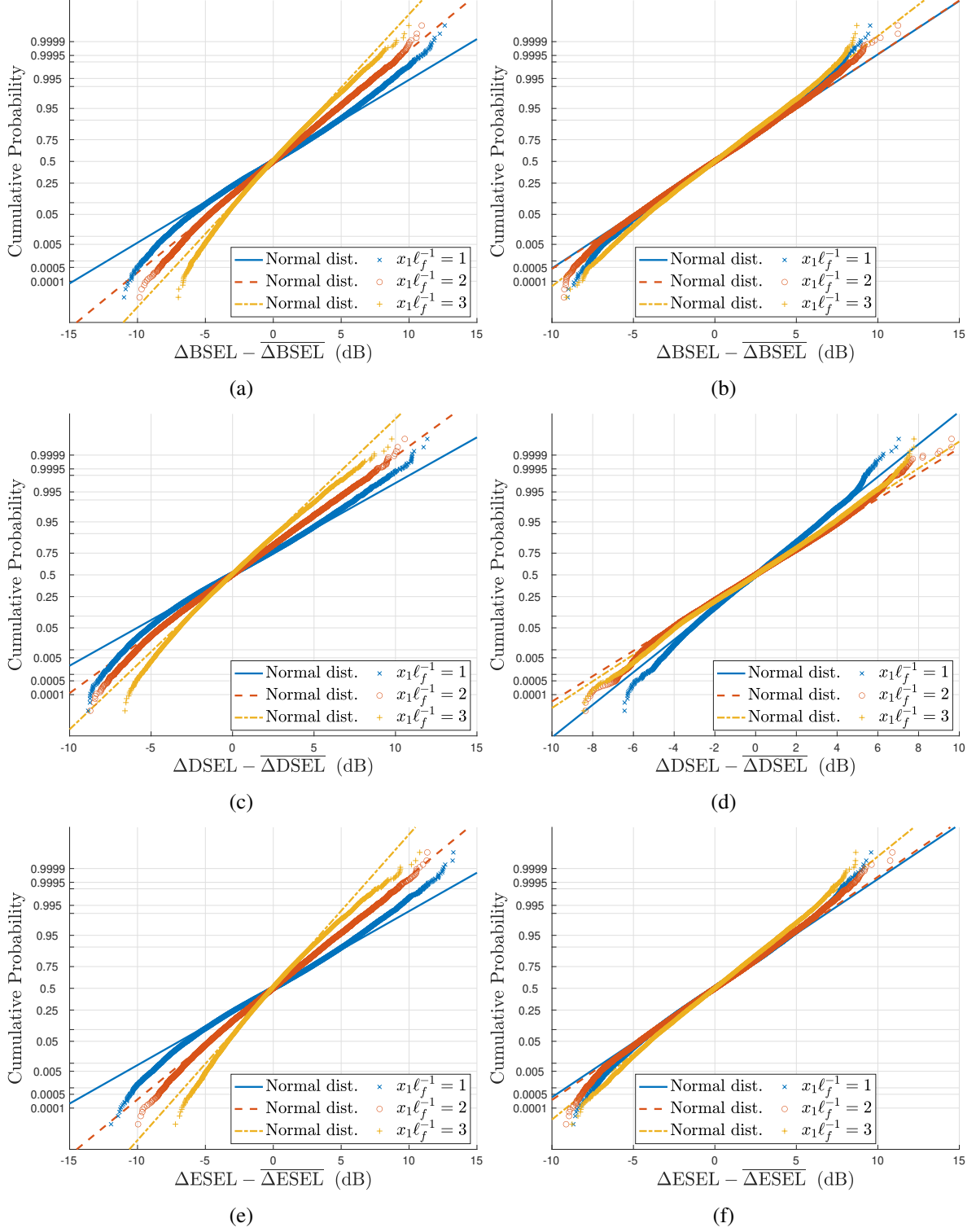


Figure B.1: Cumulative probability plots of BSEL for the (a) N-wave and (b) shaped boom, DSEL for the (c) N-wave and (d) shaped boom, and ESEL for the (e) N-wave and (f) shaped boom in the case of strong convection (AFRC3).

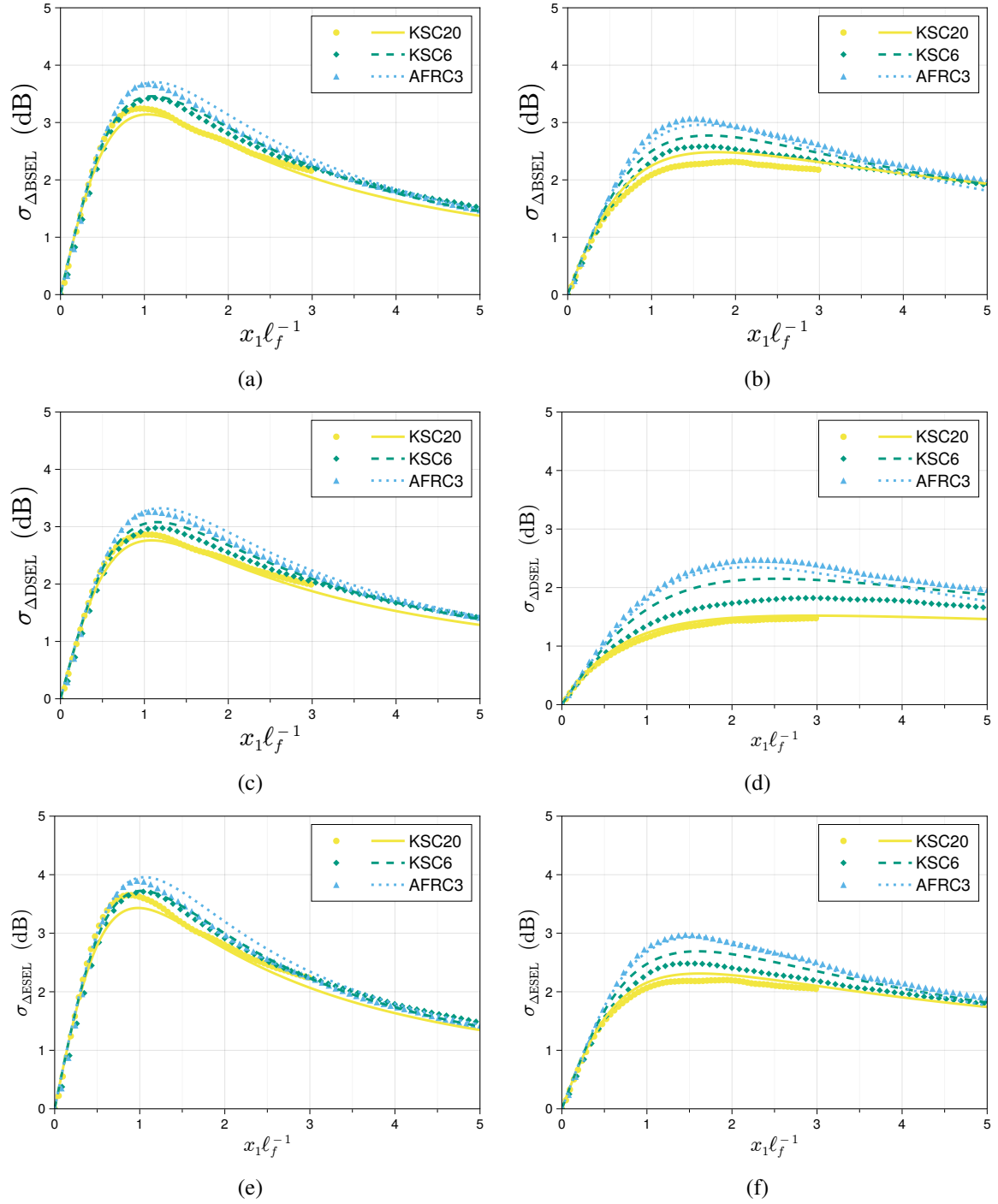


Figure B.2: Standard deviation of the (a) N-wave BSEL, (b) shaped boom BSEL, (c) N-wave DSEL, (d) shaped boom DSEL, (e) N-wave ESEL, and (f) shaped boom ESEL at 3 different convection levels obtained from simulations (discrete points) compared to the regression model (solid lines).

- [15] M. J. Siclari, C. M. Darden, Euler code prediction of near-field to midfield sonic boom pressure signatures, *J. Aircr.* 30 (1993) 911–917, <https://doi.org/10.2514/3.46434>.
- [16] J. Pawlowski, D. Graham, C. Boccadoro, P. Coen, D. Maglieri, Origins and overview of the shaped sonic boom demonstration program, in: 43rd AIAA Aerospace Sciences Meeting and Exhibit, American Institute of Aeronautics and Astronautics, 2005, <https://doi.org/10.2514/6.2005-5>.
- [17] W. Doebler, A. Loubeau, J. B. Lonzaga, Simulated x-59 on- and off-design sonic boom levels in realistic atmospheres for noise dose range estimation, *J. Acoust. Soc. Am.* 149 (2021) A102–A102, <https://doi.org/10.1121/10.0004645>.
- [18] W. Doebler, A. Loubeau, Updated noise dose range of NASA’s X-59 aircraft estimated from propagation simulations, *J. Acoust. Soc. Am.* 152 (2022) A86–A86, <https://doi.org/10.1121/10.0015630>.
- [19] K. J. Plotkin, M. Downing, J. Page, USAF single event sonic boom prediction model: PCBOOM, *J. Acoust. Soc. Am.* 95 (1994) 2839–2839, <https://doi.org/10.1121/1.409605>.
- [20] J. B. Lonzaga, Recent enhancements to NASA’s PCBoom sonic boom propagation code, in: AIAA Aviation 2019 Forum, American Institute of Aeronautics and Astronautics, 2019, <https://doi.org/10.2514/6.2019-3386>.
- [21] P. Blanc-Benon, D. Juvè, G. Comte-Bellot, Occurrence of caustics for high-frequency acoustic waves propagating through turbulent fields, *Theor. Comput. Fluid Dyn.* 2 (1991) 271–278, <https://doi.org/10.1007/bf00271467>.
- [22] J. B. Lonzaga, Distortion and losses of sonic booms through turbulent fields, *J. Acoust. Soc. Am.* 149 (2021) A74–A74, <https://doi.org/10.1121/10.0004558>.
- [23] T. Stout, Simulation of N-wave and shaped supersonic signature turbulent variations, Ph.D. dissertation, Pennsylvania State University, 2018.
- [24] T. A. Stout, V. W. Sparrow, P. Blanc-Benon, Evaluation of numerical predictions of sonic boom level variability due to atmospheric turbulence, *J. Acoust. Soc. Am.* 149 (2021) 3250–3260, <https://doi.org/10.1121/10.0004985>.
- [25] T. A. Stout, V. W. Sparrow, Atmospheric turbulence effects on shaped and unshaped sonic boom signatures, *J. Acoust. Soc. Am.* 151 (2022) 3280–3290, <https://doi.org/10.1121/10.0011393>.
- [26] F. Dagrau, M. Rénier, R. Marchiano, F. Coulouvrat, Acoustic shock wave propagation in a heterogeneous medium: A numerical simulation beyond the parabolic approximation, *J. Acoust. Soc. Am.* 130 (2011) 20–32, <https://doi.org/10.1121/1.3583549>.
- [27] D. Luquet, R. Marchiano, F. Coulouvrat, Long range numerical simulation of acoustical shock waves in a 3d moving heterogeneous and absorbing medium, *J. Comput. Phys.* 379 (2019) 237–261, <https://doi.org/10.1016/j.jcp.2018.11.041>.
- [28] R. V. Khokhlov, K. A. Naugol’nykh, S. I. Soluyan, Waves of moderate amplitudes in absorbing media, *Acustica* 14 (1964) 248–253.
- [29] E. A. Zabolotskaya, R. V. Khokhlov, Quasi-plane waves in the nonlinear acoustics of confined beams, *Sov. Phys. Acoust.* 15 (1969) 35–40.
- [30] V. P. Kuznetsov, Equations of nonlinear acoustics, *Sov. Phys. Acoust.* 16 (1971) 467–470.
- [31] P. Blanc-Benon, B. Lipkens, L. Dallois, M. F. Hamilton, D. T. Blackstock, Propagation of finite amplitude sound through turbulence: Modeling with geometrical acoustics and the parabolic approximation, *J. Acoust. Soc. Am.* 111 (2002) 487–498, <https://doi.org/10.1121/1.1404378>.
- [32] M. V. Averijanov, V. A. Khokhlova, O. A. Sapozhnikov, P. Blanc-Benon, R. O. Cleveland, Parabolic equation for nonlinear acoustic wave propagation in inhomogeneous moving media, *Acoust. Phys.* 52 (2006) 623–632, <https://doi.org/10.1134/s1063771006060017>.
- [33] M. Averijanov, P. Blanc-Benon, R. O. Cleveland, V. Khokhlova, Nonlinear and diffraction effects in propagation of N-waves in randomly inhomogeneous moving media, *J. Acoust. Soc. Am.* 129 (2011) 1760–1772, <https://doi.org/10.1121/1.3557034>.
- [34] F. Coulouvrat, New equations for nonlinear acoustics in a low Mach number and weakly heterogeneous atmosphere, *Wave Motion* 49 (2012) 50–63, <https://doi.org/10.1016/j.wavemoti.2011.07.002>.
- [35] D. Luquet, 3D simulation of acoustical shock waves propagation through a turbulent atmosphere. Application to sonic boom, Theses, Université Pierre et Marie Curie - Paris VI, 2016,

<https://tel.archives-ouvertes.fr/tel-01360574>.

- [36] A. Loubeau, S. R. Wilson, J. Rathsam, Updated evaluation of sonic boom noise metrics, *J. Acoust. Soc. Am.* 144 (2018) 1706–1706, <https://doi.org/10.1121/1.5067578>.
- [37] P. V. Yuldashev, M. M. Karzova, V. A. Khokhlova, P. Blanc-Benon, Numerical simulation of a nonlinear parabolic equation for analyzing the perceived loudness statistics of sonic boom wave after propagation through atmospheric turbulent layer, *Acoust. Phys.* 67 (2021) 26–37, <https://doi.org/10.1134/s1063771021010061>.
- [38] J. Lee, J. Rathsam, A. Wilson, Bayesian statistical models for community annoyance survey data, *J. Acoust. Soc. Am.* 147 (2020) 2222–2234, <https://doi.org/10.1121/10.0001021>.
- [39] R. D. Horonjeff, An examination of dose uncertainty and dose distribution effects on community noise attitudinal survey outcomes, *J. Acoust. Soc. Am.* 150 (2021) 1691–1701, <https://doi.org/10.1121/10.0005949>.
- [40] W. Doebler, A. B. Vaughn, K. Ballard, J. Rathsam, The effect of modeling dose uncertainty on low-boom community noise dose-response curves, *J. Acoust. Soc. Am.* 150 (2021) A259, <https://doi.org/10.1121/10.0008216>.
- [41] J. A. Page, K. K. Hogdon, P. Krecker, R. Cowart, C. Hobbs, C. Wilmer, C. Koenig, T. Holmes, T. Gaugler, D. L. Shumway, J. L. Rosenberger, D. Philips, Waveforms and Sonic Boom Perception and Response (WSPR): Low-Boom Community Response Program Pilot Test Design, Execution, and Analysis, NASA/CR–2014-218180, 2014, <https://ntrs.nasa.gov/citations/20140002785>.
- [42] J. A. Page, K. H. Hogdon, R. P. Hunte, D. E. Davis, T. A. Gaugler, R. Downs, R. A. Cowart, D. J. Maglieri, C. Hobbs, G. Baker, M. Collmar, K. A. Bradley, B. Sonak, D. Crom, C. Cutler, Quiet Supersonic Flights 2018 (QSF18) Test: Galveston, Texas Risk Reduction for Future Community Testing with a Low-Boom Flight Demonstration Vehicle, NASA/CR–2020-220589, 2020, <https://ntrs.nasa.gov/citations/20200003223>.
- [43] J. A. Page, J. B. Lonzaga, M. J. Shumway, S. R. Kaye, R. S. Downs, A. Loubeau, W. J. Doebler, PCBoom Version 7 Technical Reference, NASA/TM-20220007177, 2022, <https://ntrs.nasa.gov/citations/20220007177>.
- [44] J. A. Page, J. B. Lonzaga, M. J. Shumway, S. R. Son, R. S. Downs, A. Loubeau, W. J. Doebler, PCBoom Version 7.3 User’s Guide, NASA/TM-20220016207, 2023, <https://ntrs.nasa.gov/citations/20220016207>.
- [45] R. Leconte, J.-C. Chassaing, F. Coulouvrat, R. Marchiano, Simulation of the propagation of classical and low booms through turbulent wind fluctuations, *J. Acoust. Soc. Am.* 149 (2021) A73, <https://doi.org/10.1121/10.0004556>.
- [46] R. Leconte, J.-C. Chassaing, F. Coulouvrat, R. Marchiano, Propagation of classical and low booms through kinematic turbulence with uncertain parameters, *J. Acoust. Soc. Am.* 151 (2022) 4207–4227, <https://doi.org/10.1121/10.0011771>.
- [47] R. Stull, An introduction to boundary layer meteorology, Kluwer Academic Publishers, Dordrecht Boston, 1988.
- [48] G. I. Taylor, The spectrum of turbulence, *Proc. R. Soc. London. Ser. A - Math. and Phys. Sci.* 164 (1938) 476–490, <https://doi.org/10.1098/rspa.1938.0032>.
- [49] P. V. Yuldashev, S. Ollivier, M. M. Karzova, V. A. Khokhlova, P. Blanc-Benon, Statistics of peak overpressure and shock steepness for linear and nonlinear N-wave propagation in a kinematic turbulence, *J. Acoust. Soc. Am.* 142 (2017) 3402–3415, <https://doi.org/10.1121/1.5015991>.
- [50] M. J. Lighthill, Waves in fluids, Cambridge University Press, Cambridge, England; New York, 1978.
- [51] V. E. Ostashev, D. K. Wilson, Acoustics in moving inhomogeneous media, CRC Press, Taylor & Francis Group, Boca Raton, FL, 2016.
- [52] J. Goodman, Introduction to Fourier optics, Roberts & Co, Englewood, Colorado, 2005.
- [53] F. Coulouvrat, A quasi-analytical shock solution for general nonlinear progressive waves, *Wave Motion* 46 (2009) 97–107, <https://doi.org/10.1016/j.wavemoti.2008.09.002>.
- [54] H. E. Bass, L. C. Sutherland, A. J. Zuckerwar, D. T. Blackstock, D. M. Hester, Atmospheric absorption of sound: Further developments, *J. Acoust. Soc. Am.* 97 (1995) 680–683, <https://doi.org/10.1121/1.412989>.
- [55] A. N. Carr, Sonic Boom Propagation in the Turbulent Atmospheric Boundary Layer, Ph.D. dissertation, University of Florida, 2022.
- [56] R. I. McLachlan, G. R. W. Quispel, Splitting methods, *Acta Numer.* 11 (2002) 341–434,

- <https://doi.org/10.1017/s0962492902000053>.
- [57] G. Strang, On the construction and comparison of difference schemes, *SIAM J. Numer. Anal.* 5 (1968) 506–517, <https://doi.org/10.1137/0705041>.
 - [58] P. Lax, B. Wendroff, Systems of conservation laws, *Comm. Pure Appl. Math.* 13 (1960) 217–237, <https://doi.org/10.1002/cpa.3160130205>.
 - [59] A. S. Monin, A. M. Obukhov, Osnovnye zakonomernosti turbulentnogo peremeshivaniya v prizemnom sloe atmosfery (basic laws of turbulent mixing in the atmosphere near the ground), *Trudy geofiz. inst. AN SSSR* 24(151) 24 (1954) 163–187.
 - [60] V. E. Ostashev, D. K. Wilson, Relative contributions from temperature and wind velocity fluctuations to the statistical moments of a sound field in a turbulent atmosphere, *Acust. Acta Acust.* 86 (2000) 260–268.
 - [61] J. Kaimal, J. Finnigan, *Atmospheric Boundary Layer Flows: Their Structure and Measurement*, Oxford University Press, 1994.
 - [62] D. K. Wilson, Turbulence Models and the Synthesis of Random Fields for Acoustic Wave Propagation Calculations., ARL-TR-1677, Army Research Laboratory, 1998, <https://apps.dtic.mil/sti/tr/pdf/ADA352483.pdf>.
 - [63] J. Højstrup, Velocity spectra in the unstable planetary boundary layer, *J. Atmos. Sci.* 39 (1982) 2239–2248, [https://doi.org/10.1175/1520-0469\(1982\)039<2239:vsitup>2.0.co;2](https://doi.org/10.1175/1520-0469(1982)039<2239:vsitup>2.0.co;2).
 - [64] M. J. Kamrath, V. E. Ostashev, D. K. Wilson, M. J. White, C. R. Hart, A. Finn, Vertical and slanted sound propagation in the near-ground atmosphere: Amplitude and phase fluctuations, *J. Acoust. Soc. Am.* 149 (2021) 2055–2071, <https://asa.scitation.org/doi/10.1121/10.0003820>.
 - [65] V. E. Ostashev, M. J. Kamrath, D. K. Wilson, M. J. White, C. R. Hart, A. Finn, Vertical and slanted sound propagation in the near-ground atmosphere: Coherence and distributions, *J. Acoust. Soc. Am.* 150 (2021) 3109–3126, <https://asa.scitation.org/doi/10.1121/10.0006737>.
 - [66] J. Wyngaard, *Turbulence in the Atmosphere*, Cambridge University Press, 2010.
 - [67] J. L. Lumley, The structure of inhomogeneous turbulent flows, in: A. M. Yaglom, V. I. Tatarsky (Eds.), *Atmospheric Turbulence and Radio Wave Propagation*, Nauka, Moscow, 1967.
 - [68] D. K. Wilson, A turbulence spectral model for sound propagation in the atmosphere that incorporates shear and buoyancy forcings, *J. Acoust. Soc. Am.* 108 (2000) 2021–2038, <https://doi.org/10.1121/1.1311779>.
 - [69] A. Loubeau, Review of noise metric sensitivities for analysis of quiet supersonic overflight, *J. Acoust. Soc. Am.* 151 (2022) A87, <https://doi.org/10.1121/10.0010746>.