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# Hardware-Efficient Quantum Optimization Layered Algorithms and Experiments

**Daive Venturelli, M. Sohaib Alam, Matthew J Reagor, Bram Evert, Shon Grabbe, Benjamin P Hall, Mark Hodson, Ryan M LaRose, P. Aaron Lott, Eleanor G Rieffel, James Sud, Zihui Wang, Filip A Wudarski**



## **Practical Verification of Quantum Properties in Quantum-Approximate-Optimization Runs**

M. Sohaib Alam, Filip A. Wudarski, Matthew J. Reagor, James Sud, Shon Grabbe, Zihui Wang, Mark Hodson, P. Aaron Lott, Eleanor G. Rieffel, and [Daive Venturelli](#)

*Phys. Rev. Applied* **17**, 024026 – Published 9 February 2022

## **Mixer-Phaser Ansatz for Quantum Optimization with Hard Constraints.**

LaRose, Ryan, Eleanor Rieffel, and [Daive Venturelli](#)  
arXiv:2107.06651 (2021)

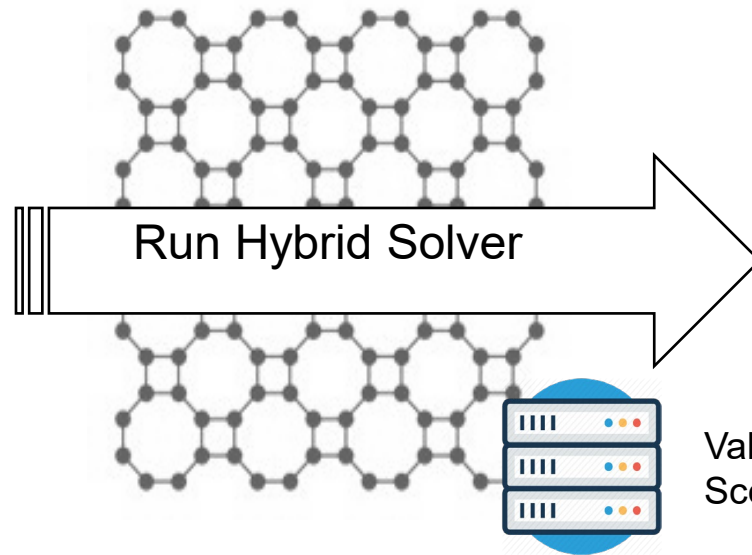


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# About the DARPA ONISQ SAAM Project

**Mandate:** push QAOA-Like algorithms in solving scheduling-like problems at moderate scale and algorithm complexity, i.e.  $\approx 100$ s of variables and  $\approx 1000$ s of gates *(still 2 years to go)*

Compiled SAAM  
Algorithm



Validate (is it quantum?)  
Score Performance (is there advantage?)



# Lessons learned from quantum annealing benchmarking for applications

2013-2017: very intense worldwide effort to understand D-Wave and D-Wave-like quantum annealing capabilities and prospects

## nature physics

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Published: 28 February 2014

### Evidence for quantum annealing with more than one hundred qubits

Sergio Boixo, Troels F. Rønnow, Sergei V. Isakov, Martinis & Matthias Troyer

Nature Physics 10, 218–224 (2014) | Cite this article

11k Accesses | 405 Citations | 204 Altmetric

## PHYSICAL REVIEW X

### Entanglement in a Quantum Annealing Processor

T. Lanting et al. Phys. Rev. X 4, 021041 – Published 29 May 2014

Open Access

### What is the Computational Value of Finite-Range Tunneling?

Vasil S. Denchev, Sergio Boixo, Sergei V. Isakov, Nan Ding, Ryan Babbush, Vadim Smelyanskiy, John Martinis, and Hartmut Neven Phys. Rev. X 6, 031015 – Published 1 August 2016

## PHYSICAL REVIEW X

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## Science

Current Issue First release papers Archive About

HOME > SCIENCE > VOL 345, NO. 6195 > DEFINING AND DETECTING QUANTUM SPEEDUP

### Defining and detecting quantum speedup

TROELS F. RØNNOW, ZHIHUI WANG, JOSHUA JOB, SERGIO BOIXO, SERGEI V. ISAKOV, DAVID WECKER, JOHN M. MARTINIS

SCIENCE • 19 Jun 2014 • Vol 345, Issue 6195 • pp. 420–424 • DOI:10.1126/science.1252219

258 291

#### How to benchmark a quantum computer

Quantum machines offer the possibility of performing certain computations faster than their classical counterparts. However, how to define quantum speedup is a topic of debate. Rønnow et al. describe methods for benchmarking the difference in computational power between classical and quantum processors. They define various types of quantum speedup and consider quantum processors that are designed to solve a specific class of problems.

### Experimental investigation of performance differences between coherent Ising machines and a quantum annealer

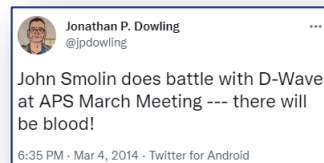
RYAN HAMERLY, TAKAHIRO INAGAKI, DETELI MCMAHON, DAVID VENTURELLI, ALIBEZA MARANDI, TATSUHIRO ONODERA, EDWIN NG, CARSTEN LANGROCK, KENSUKE INABA, YOSHISHISA YAMAMOTO

#### Benchmark Study of Quantum Algorithms for Combinatorial Optimization: Unitary versus Dissipative

Krishanu Sanjar, Artur Scherer, Satoshi Kato, Sam Reifenstein, Navid Ghadermarzy, Willem B. Kravtsov, Yoshitaka Inui, Edwin Ng, Tatsuhiro Onodera, Pooya Ronagh, and Yoshihisa Yamamoto

<sup>1</sup>QIB Information Technologies (IQIB), Vancouver, BC, Canada  
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<sup>3</sup>Physics & Informatics Laboratories, NTT Research Inc, Sunnyvale, CA, USA  
<sup>4</sup>E. L. Ginzton Laboratory, Stanford University, Stanford, CA, USA  
<sup>5</sup>School of Applied and Engineering Physics, Cornell University, Ithaca, NY, USA  
<sup>6</sup>Institute for Quantum Computing, University of Waterloo, Waterloo, ON, Canada  
<sup>7</sup>Department of Physics & Astronomy, University of Waterloo, Waterloo, ON, Canada

Is the machine quantum?  
Is the theory correlation? Is there tunneling? Is there entanglement?

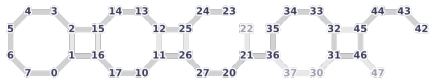


How would I know if I have an advantage?  
Asymptotically, at application scale?  
Versus “quantum inspired”?  
At best parameters?\*





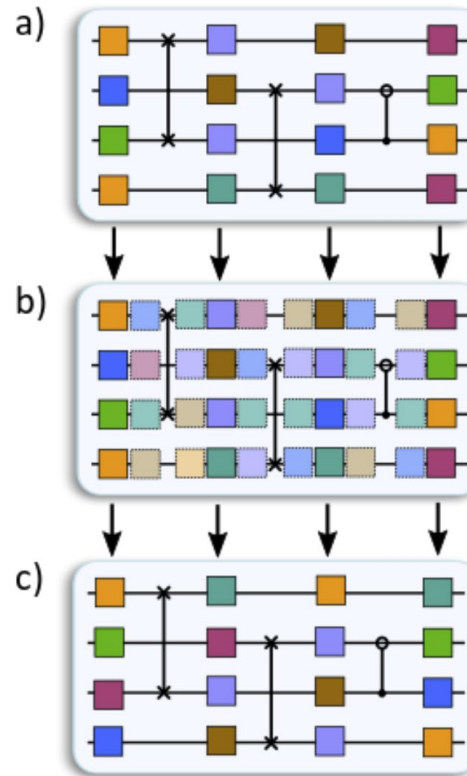
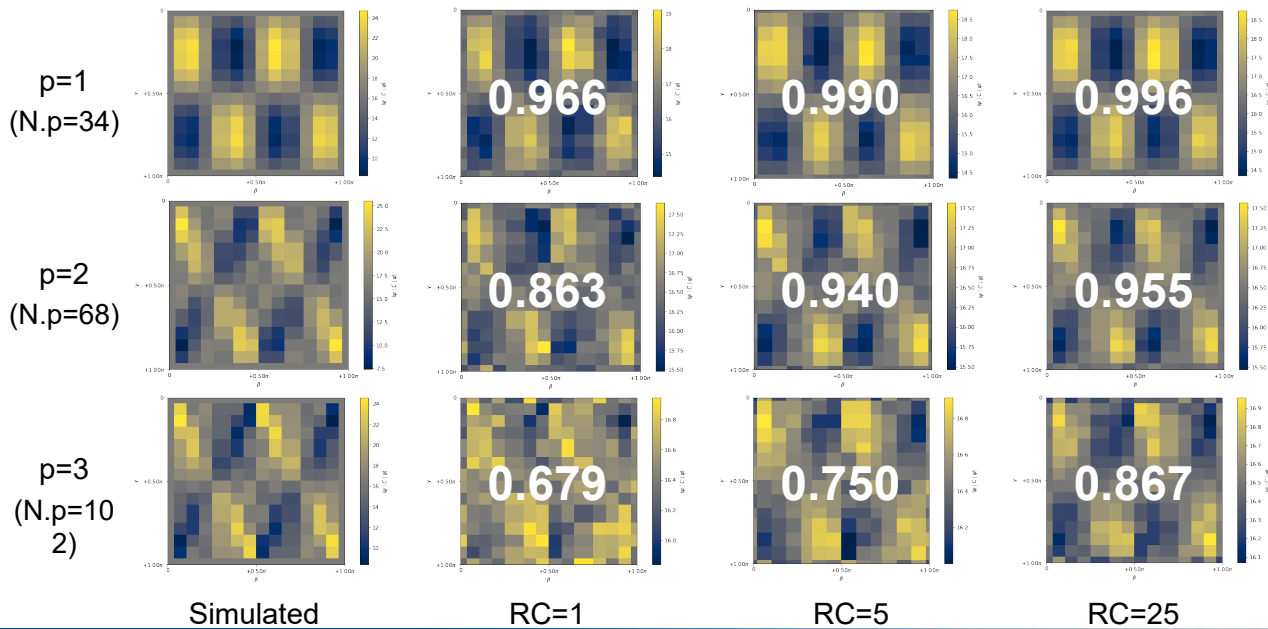
# Test of Quantumness: Correlation with Theory for Native-Aspen Graph QAOA MaxCut



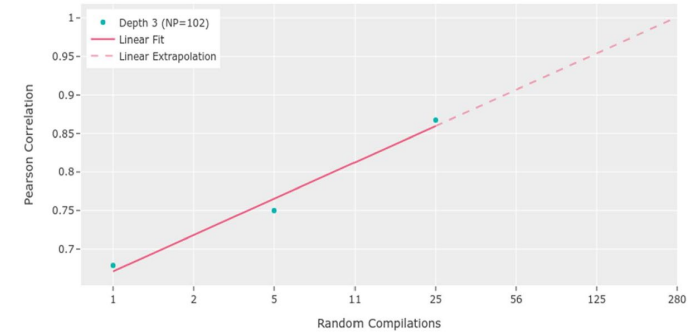
Randomized Compilation provided by True-Q  
Phys. Rev. A 94, 052325 (2016)  
Phys. Rev. X 11, 041039 (2021)



MaxCut QAOA on Native Chip Layout N=34, p=3



These tests are indirect evidence of substantial impact of coherent errors.



It is not feasible to continue to increase the number of randomized compilations as N increases (p=3 N=34 for 25 RCs takes 43 minutes, 35 of which is programming time)



# Test of Quantumness: Signatures of non-classicality, single particle effects

“Tunneling” aka “Ability to Superpose” aka “Single-particle Quantum Coherence”

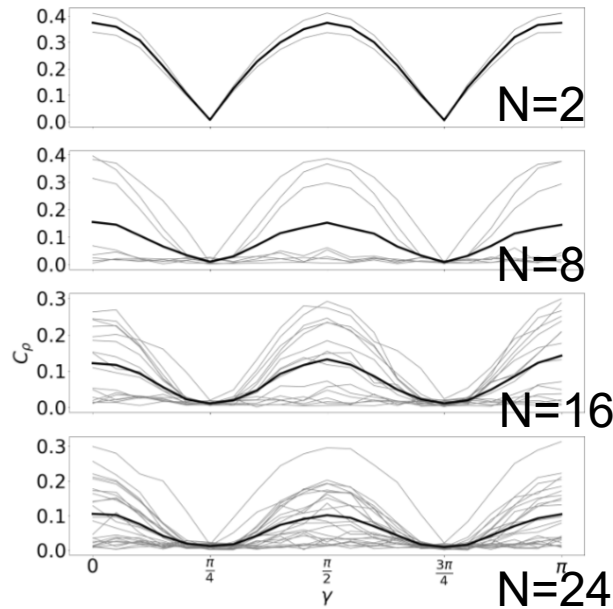
Maximum value of the reduced density matrix for a single qubit after a QAOA run as a metric of “*survived single particle coherence*”.

$$\rho = \begin{pmatrix} \rho_{11} & \rho_{12} \\ \rho_{21} & \rho_{22} \end{pmatrix}$$

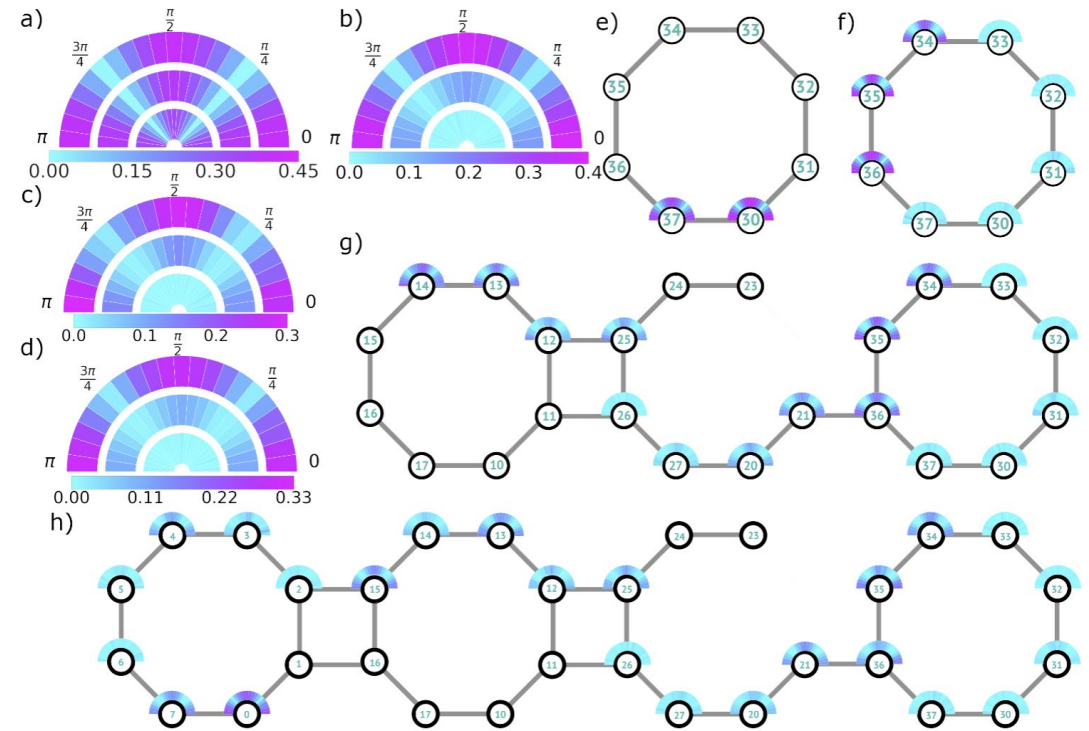
$$C_\rho = |\rho_{12}|$$

Obtainably by standard tomographic reconstructions (MLE)

$$\rho = \frac{1}{2}(\mathbb{1} + \langle X \rangle X + \langle Y \rangle Y + \langle Z \rangle Z)$$



Coherences as a function of  $\gamma$  for a fixed value of  $\beta = \pi/8$  for qubits (thin, gray lines) in a linear chain QAOAMaxCut circuit of (top to bottom) 2, 8, 16 and 24 qubits. The thick (black) line represents the mean coherence across all the qubits



2D view – could inform compiler on inhomogeneities



# Test of Quantumness: Signatures of non-classicality, multi-particles effects

**Quantum Entanglement** – can we construct a **witness** that uses the same measurement overhead (3x) as the tunneling test?

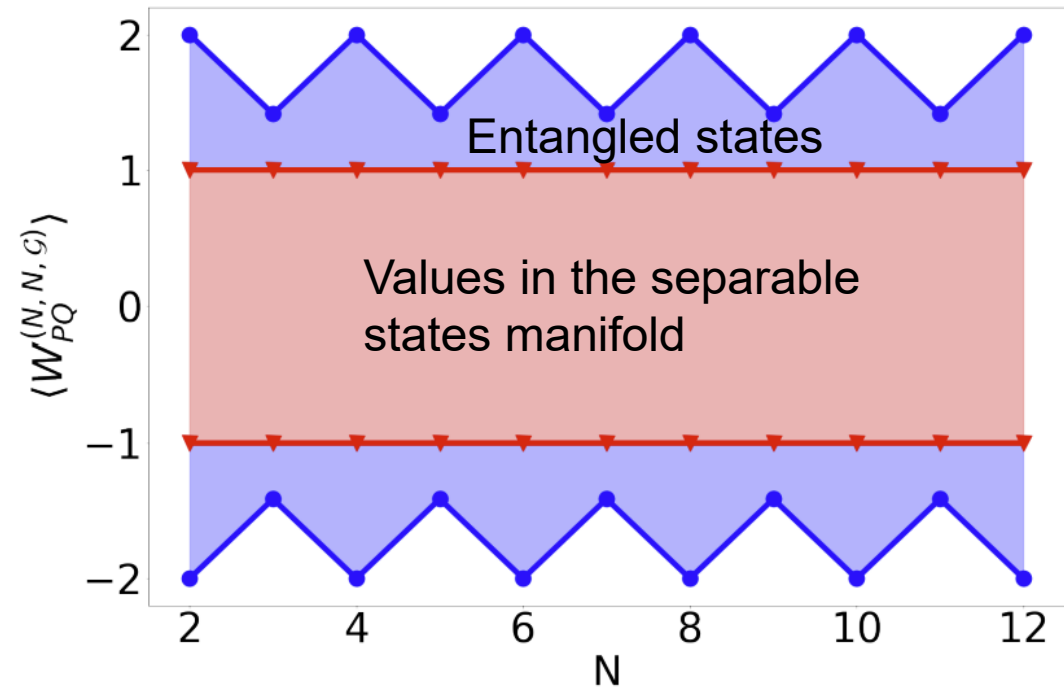
These families of observables require measurements only in 3 bases

$$W_{PQ}^{(k,N,\mathcal{G})} = \sum_{\langle i_1, \dots, i_k \rangle} \left( \bigotimes_{j=i_1}^{i_k} P_j + \bigotimes_{j=i_1}^{i_k} Q_j \right)$$

where  $P, Q \in \{X, Y, Z\}$  and  $P \neq Q$ .

$$W_{XYZ}^{(k,N,\mathcal{G})} = \sum_{\langle i_1, \dots, i_k \rangle} \left( \bigotimes_{j=i_1}^{i_k} X_j + \bigotimes_{j=i_1}^{i_k} Y_j + \bigotimes_{j=i_1}^{i_k} Z_j \right)$$

Can act as *entanglement witnesses*  $|\langle W_M^{(k,N,\mathcal{G})} \rangle_{sep}| \leq |E_k|$



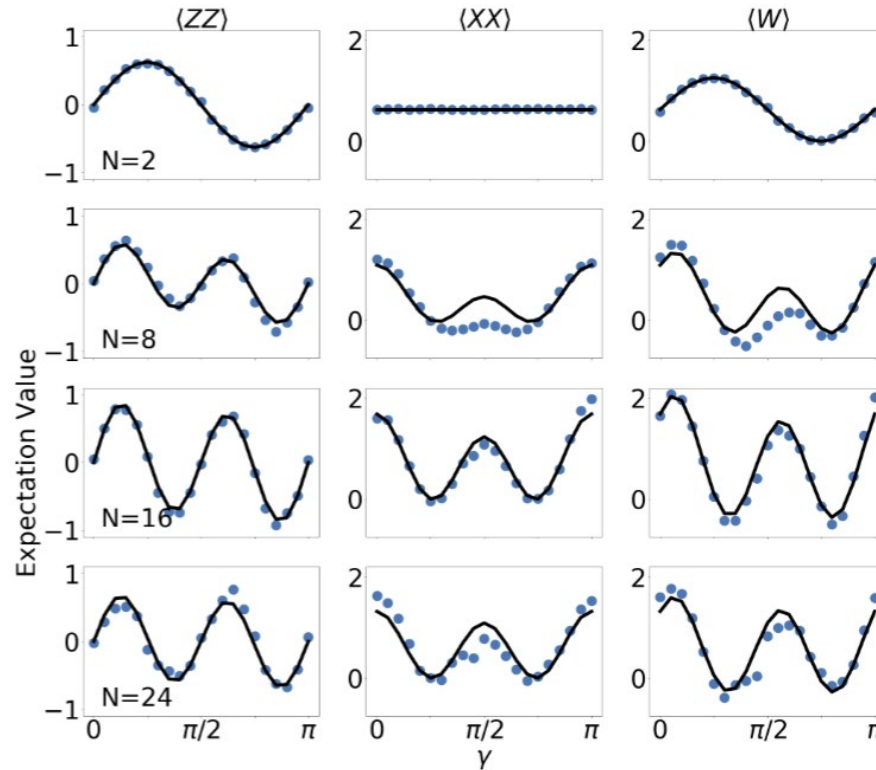




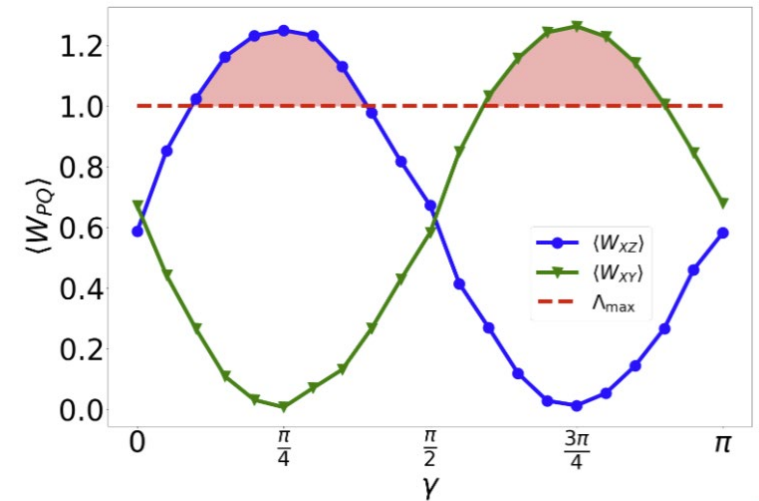
# Test of Quantumness: Signatures of non-classicality, multi-particles effects – N=2

## Quantum Entanglement

After randomized compiling, experimental results show that the designed witnesses expectation value fit the theory assuming a global depolarizing noise channel. For N=2, fitted noise is sufficiently low, for larger N fitted noise is too high to beat the separable threshold for a line-graph witness.



**For 2 qubit circuits the violation is well detected**, for  $N \geq 3$  no detection (active noise mitigation, entanglement beyond witness, too fragile type of entanglement)





# Quantum Alternating Operator Ansatz

(See Hadfield et al 2019)

## Bitflip mixers

- Maximum Cut
- Max-SAT, Min-SAT, NAE-SAT
- Set Splitting
- MaxE3LIN2

...

## XY mixers

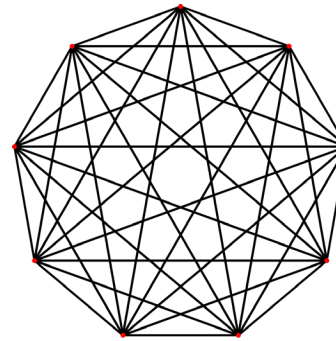
- Max-ColorableSubgraph
- Graph Partitioning
- Maximum Bisection
- Max Vertex k-Cover

...

Enforcing the same number of bits=1 is the same as doing two spin-flips

$$XY = \sigma^+ \sigma^- + \sigma^- \sigma^+$$

$$|001\rangle \xrightarrow{XY(2,3)} a|001\rangle + b|010\rangle \xrightarrow{XY(2,3)} a'|001\rangle + b'|010\rangle + c'|100\rangle$$



## From the Quantum Approximate Optimization Algorithm to a Quantum Alternating Operator Ansatz

Stuart Hadfield\*, Zhihui Wang<sup>+,\*\*</sup>, Bryan O’Gorman<sup>+,†,‡</sup>, Eleanor G. Rieffel<sup>+</sup>, Davide Venturelli<sup>+,\*\*</sup>, Rupak Biswas<sup>+</sup>

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<sup>\*\*</sup> USRA Research Institute for Advanced Computer Science (RIACS), Mountain View, CA

<sup>†</sup> Stinger Ghaffarian Technologies, Inc., Greenbelt, MD

<sup>‡</sup> Berkeley Quantum Information and Computation Center and Departments of Chemistry and Computer Science, University of California, Berkeley, CA

September 12, 2017

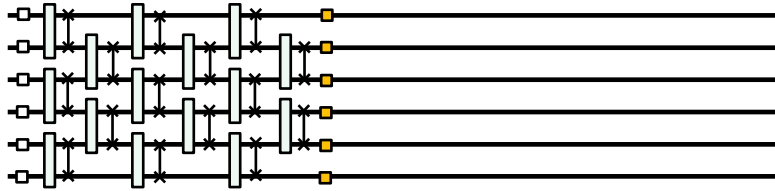
The next few years will be exciting as prototype universal quantum processors emerge, enabling implementation of a wider variety of algorithms. Of particular interest are quantum heuristics, which require experimentation on quantum hardware for their evaluation, and which have the potential to significantly expand the breadth of applications for which quantum computers have an established advantage. A leading candidate is Farhi et al.’s Quantum Approximate Optimization Algorithm, which alternates between applying a cost-function-based Hamiltonian and a mixing Hamiltonian. Here, we extend this framework to allow alternation between more general families of operators. The essence of this extension, the Quantum Alternating Operator Ansatz, is the consideration of general parameterized families of unitaries rather than only those corresponding to the time-evolution under a fixed local Hamiltonian for a time specified by the parameter. This ansatz supports the representation of a larger, and potentially more useful, set of states than the original formulation, with potential long-term impact on a broad array of application areas.



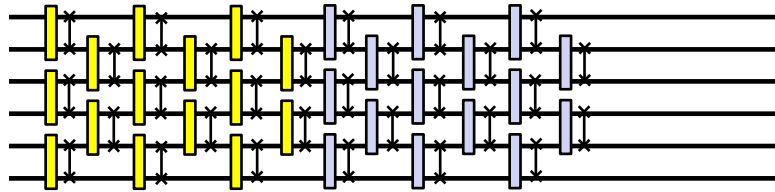


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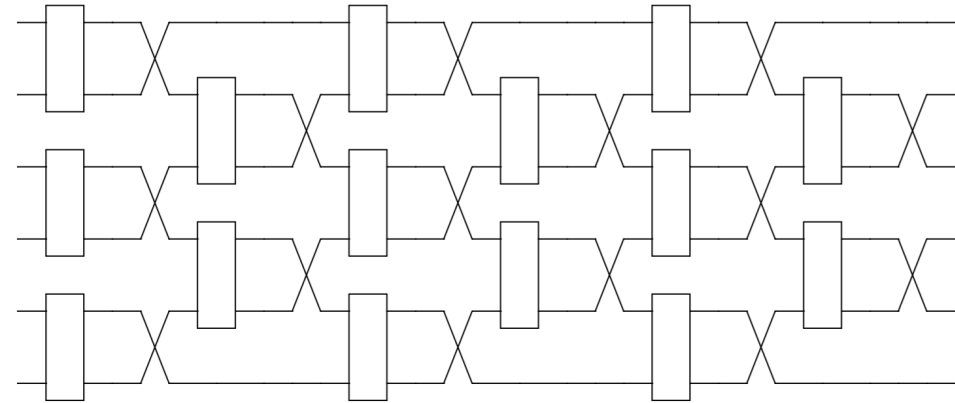
# Fully Connected Problems and SWAP Networks



QAOA  $p=1$   
(Fahri et al. 2020)  
- Unconstrained -

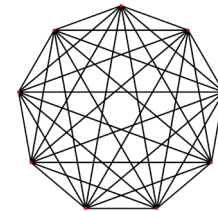


QAOA-XY  $p=1$   
(Hadfield et al. 2019)

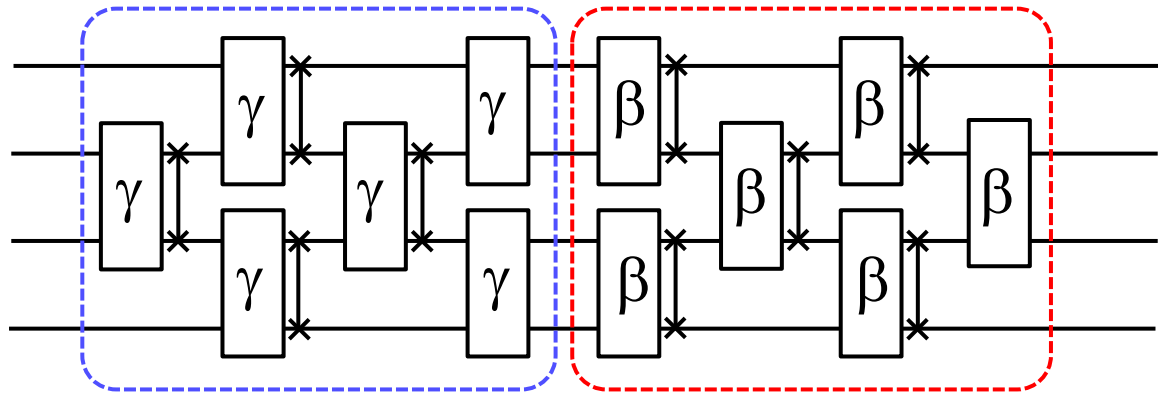


Swap network depth  $N$  with  $N(N-1)/2$  gates

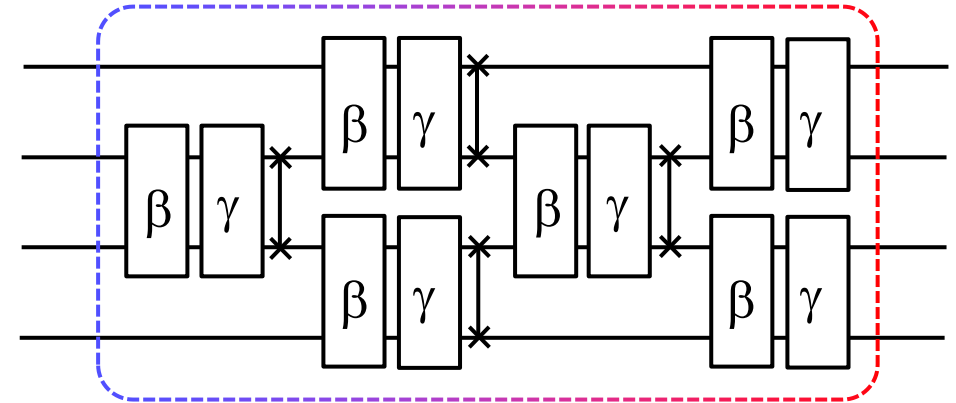
See Kivlichan Phys.Rev.Lett 120, 110501 (2018)  
and O’Gorman et al. ArXiv:1905.05118 (2019)



# QAOA-like Hardware Efficient Ansatz with XY Mixers



## Mixer-phaser



If instead of doing

- a PS round  $\text{Exp}[i\gamma J_{nm} ZZ]$  and a
  - MIX round  $\text{Exp}[i\beta(XX+YY)]$
- we do a phaser-mixer round
- $\text{Exp}[i\beta(XX+YY) + i\gamma J_{nm} ZZ]$

Mixer-Phaser Ansätze for Quantum Optimization with Hard Constraints

Ryan LaRose,<sup>1,2,3</sup> Eleanor Rieffel,<sup>1</sup> and Davide Venturelli<sup>1,2,\*</sup>

<sup>1</sup>Quantum AI Laboratory (QuAIL), NASA Ames Research Center

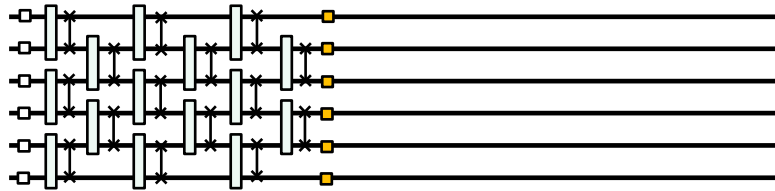
<sup>2</sup>USRA Research Institute for Advanced Computer Science (RIACS)

<sup>3</sup>Department of Computational Mathematics, Science, and Engineering, Michigan State University

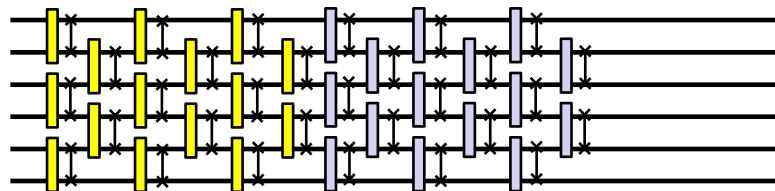


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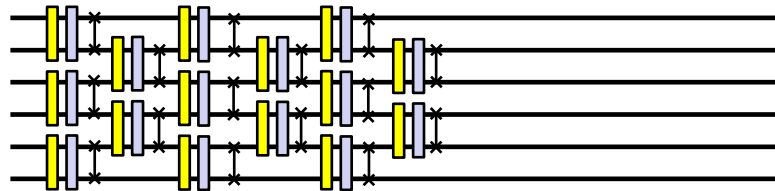
# QAOA-like Hardware Efficient Ansatz with XY Mixers



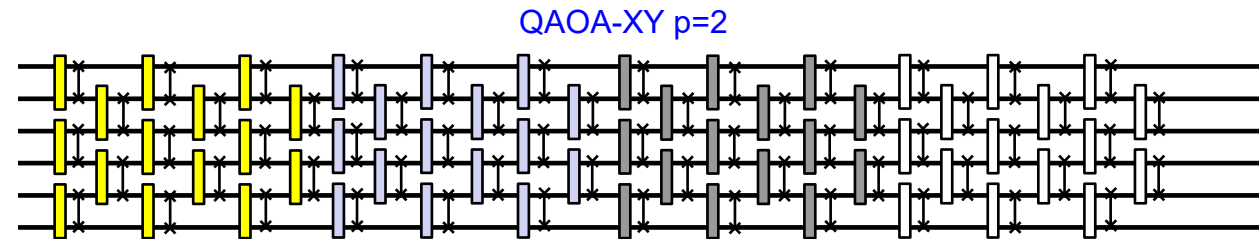
QAOA  $p=1$   
(Fahri et al. 2020)  
- Unconstrained -



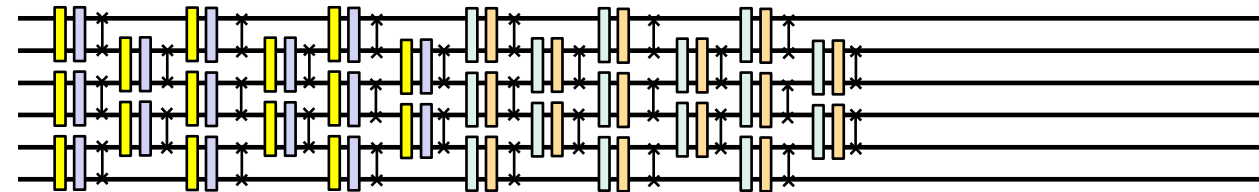
QAOA-XY  $p=1$   
(Hadfield et al. 2019)



QAMPA  $p=1$   
(LaRose et al. 2021)



QAOA-XY  $p=2$



QAMPA  $p=2$

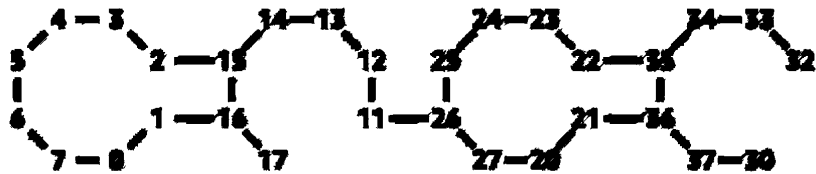




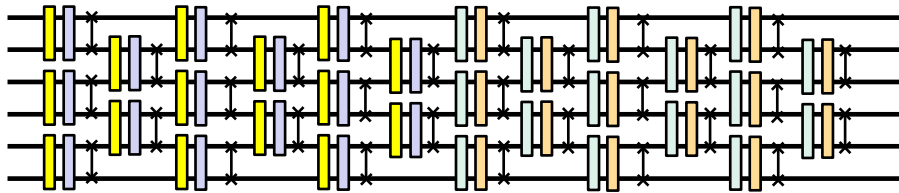
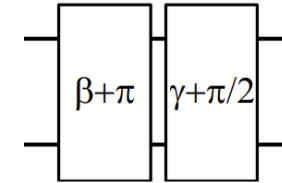
# Synthesis of QAMPA gate on superconducting QPUs

SWAP is for free because it is renormalized in the XY and ZZ parameters!

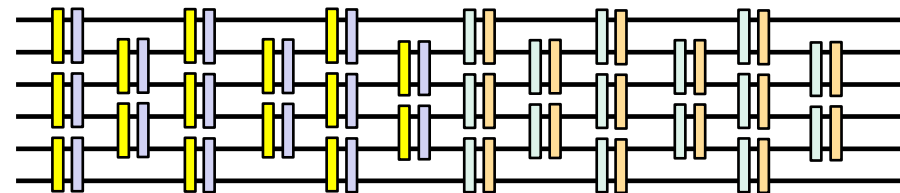
$$e^{i\pi/4}ZZ(\gamma+\pi)XY(\beta+\pi/2)=ZZ(\gamma)XY(\beta)SWAP$$



$$\begin{pmatrix} e^{i\frac{\phi}{2}} & 0 & 0 & 0 \\ 0 & i e^{-i\frac{\phi}{2}} \sin[\frac{\theta}{2}] & e^{-i\frac{\phi}{2}} \cos[\frac{\theta}{2}] & 0 \\ 0 & e^{-i\frac{\phi}{2}} \cos[\frac{\theta}{2}] & i e^{-i\frac{\phi}{2}} \sin[\frac{\theta}{2}] & 0 \\ 0 & 0 & 0 & e^{i\frac{\phi}{2}} \end{pmatrix}$$



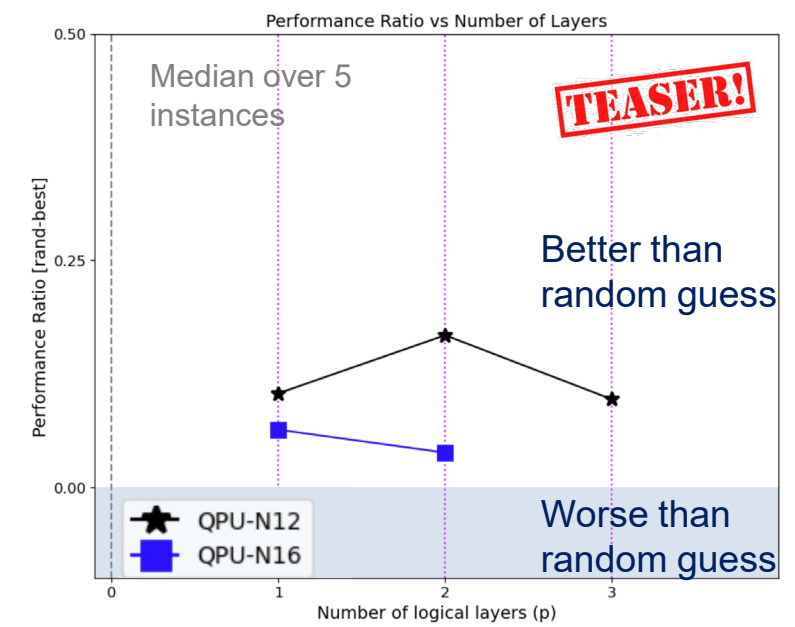
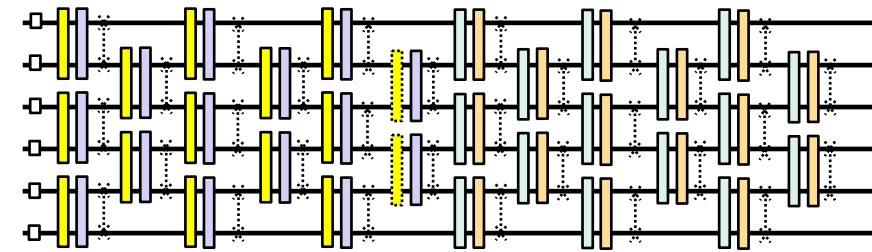
QAMPA p=2



QAMPA p=2 (compilation simplified)

# Current Work inspired by Experimental Results on Rigetti Aspen-11 QPU

- **Probability to keep the hamming weight subspace is minimal 1%** for  $N=8$   $p=3$  and gets to 0.1% for  $N=12$ , assuming based on indirect evidence that most of the errors are coherent, we could consider the XY gates as mixers for an unconstrained problem (coherent leakage beyond the feasible subspace).
- Experimental tests are done with QAMPA-H where **initialization consists of hadamards gate**, and where some extra parameters are added (TBA) targeting the resolution of unconstrained SK model
- **$N=12$ ,  $16$   $p=2-3$**  could be pushed on previous QPU to observe results significantly over the best random-guess ( $\approx 500$  gates).  $N=20$  is overwhelmed by noise using a very basic parameter setting strategy
- Improvements will come with **new parameter setting, and less noisy chip** (Aspen-M-1 that has 80 qubits)





# Conclusions – with resources to watch

- Designed scalable protocols and benchmarks for quantumness of runs of quantum optimization algorithms, tested on Rigetti Aspen. Detected visual correlation, tunneling for  $\approx 30$  qubits and entanglement for  $\approx 2$  qubits.
- Designed, numerically and experimentally benchmarked an hardware efficient ansatz to improve over QAOA in experiments. Obtained beyond random guessing performance improving with  $p$  for  $N \approx 16$

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